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GAIN OF ELECTROMAGNETIC HORNS

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ABSTRACT

Recent experimental evidence indicates that the measured gain of pyramidal electromagnetic horns may be considerably in error if the measurements are carried out at short distances, and if the aperture-to-aperture separation between horns is used in the gain formula

$$G = (4\pi R/\lambda)\sqrt{P_R/P_T}\,.$$

Further experimental verification of this effect has been obtained, and a theory has been developed which is in good quantitative agreement with present experimental data and demonstrates the physical reasons why the previous "far-field" criterion of $2D^2/\lambda$ is invalid.

Curves are presented from which the error in gain measured at any distance may be obtained and applied as a correction.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problem R09-53
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GAIN OF ELECTROMAGNETIC HORNS

INTRODUCTION

The absolute gain of electromagnetic horns is usually measured by making use of two identical horns, one being employed as a transmitter and one as a receiver (Figure 1). If the aperture-to-aperture separation is "R" and the receiving horn is in the "far field" of the transmitting horn, then the power per unit area arriving at the receiving horn is $P_A = (P_T/4\pi R^2)\, G_T$, where P_T is the total power transmitted and G_T is the gain of the transmitting horn. The absorption cross section which the receiving horn presents to the incident wave is $A_R = (\lambda^2/4\pi)\, G_R$, where G_R is the gain of the receiving horn when used as a transmitter and where λ is the wavelength. Hence the power received is $P_R = P_A A_R = (P_T/4\pi R^2)\, G_T\, (\lambda^2/4\pi)\, G_R$. Solving for $\sqrt{G_T G_R}$, $\sqrt{G_T G_R} = (4\pi R/\lambda)\, \sqrt{P_R/P_T}$. If the receiving and transmitting horns are identical, then $G_T = G_R = G$; hence

$$G = \frac{4\pi R}{\lambda} \sqrt{P_R/P_T} \quad . \tag{1}$$

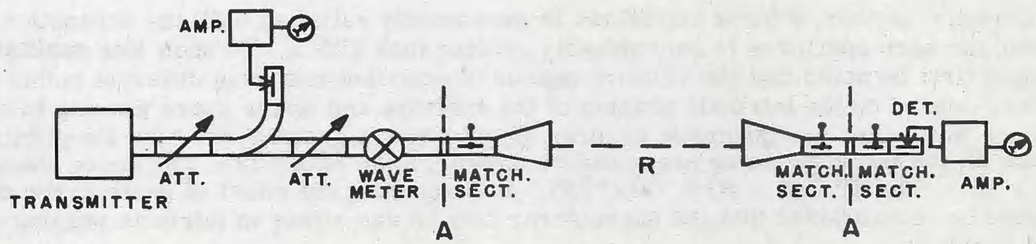


Figure 1 - Experimental setup for gain measurements

The quantities R and λ can easily be measured. The ratio of P_R to P_T can be determined by connecting a matched detector directly to the transmitter and then to the receiving horn. Actually, quantities proportional to P_R and P_T are measured, but for a given detector the proportionality constant is the same in both instances and hence divides out in the ratio.

It should be noted that in the true Fraunhofer field P_R falls off as $1/R^2$; hence, R cancels out in the expression for G, and as would be expected, the measured gain is independent of R. In making gain measurements the first question which naturally arises is: What is the minimum aperture separation (R_{min}) for which the true Fraunhofer gain will be obtained; that is to say, at what point does P_R begin to fall off as $1/R^2$? Only at aperture separations of R_{min} or greater will the gain formula (Equation 1) give the correct far-field gain when the experimental values of R, λ, P_R, and P_T are substituted.

The commonly accepted criterion for this minimum separation is $R_{min} = 2D^2/\lambda$, where D is the larger horn aperture dimension. This criterion is arrived at either by intuitive reasoning based on simple vector diagrams (the maximum phase error across the aperture at this distance being $\lambda/16$), or through a calculation of the transmitting gain for some simple aperture, e.g., a circular in-phase aperture. It will be shown that the reasoning which leads to this criterion is in error, and that the required minimum aperture separation may actually be many times $2D^2/\lambda$.

The failure of the $2D^2/\lambda$ criterion was first noted experimentally by W. C. Jakes,[1] whose results indicated that an error of the order of one db may occur at $2D^2/\lambda$, and that the true Fraunhofer gain may not be realized even at distances several times $2D^2/\lambda$.

The purpose of the present work was (a) to investigate the physical reasons responsible for this gain variation at short distances and to establish some criterion for calculating the minimum aperture separation for which the true Fraunhofer gain will be obtained, and (b) to obtain accurate values for the gains of a set of horns which covers the microwave frequency band and can be used as primary gain standards.

THEORY

Two assumptions are implicit in the gain formula: (a) the power arriving at the receiving aperture varies as $1/R^2$, and (b) the wave striking the receiving horn is sensibly plane, so that the effective cross section of the receiving horn is $(\lambda^2/4\pi)\, G_\infty$, where G_∞ is the true Fraunhofer gain.

Actually, neither of these conditions is necessarily satisfied until the separation between the horn apertures is considerably greater than $2D^2/\lambda$. To show this qualitatively, it should first be noted that the relative phases of contributions from different points in the aperture depend on the intrinsic phasing of the aperture and on the space phasing in exactly the same way—both are quadratic errors. Considering a sectoral horn for simplicity (Figure 2), the intrinsic phase error can be shown[2] to be $-k(x^2/2\ell)$. The space phase error is $-k(r-R) = -k(\sqrt{R^2+x^2} - R) \cong -k(x^2/2R)$. In discussing the effect of phase error on gain, it should be remembered that the phase error may be due either to intrinsic phasing or to space phasing.

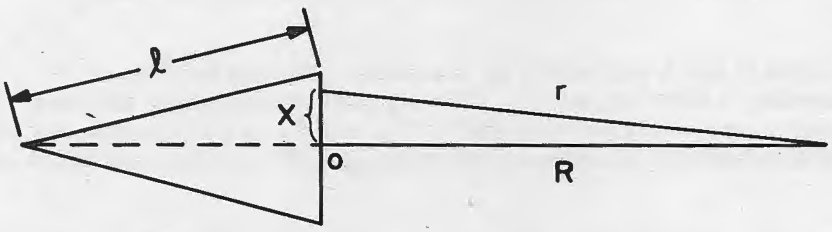


Figure 2 - Physical dimensions for computing the phase errors

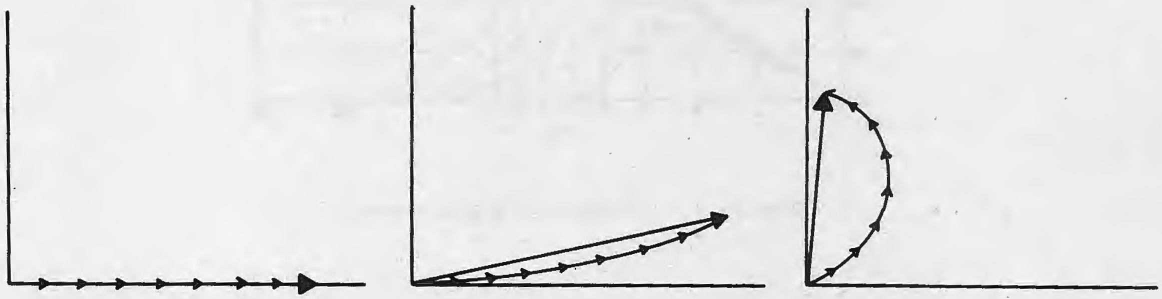
[1] Jakes, W. C., Proc. I.R.E., 39 (No. 2): 160-162, February 1951

[2] Schelkunoff, S. A., "Electromagnetic Waves," New York: Van Nostrand, 361:88, 1943

Let us first make a qualitative comparison between the gains of transmitting apertures having maximum phase errors of 0, $\lambda/8$, and $\lambda/4$, respectively. In the case of zero phase error, the contributions from the various points of the aperture add in phase as shown in Figure 3a. In going from zero phase error to a phase error of $\lambda/8$, the vectors are all rotated through small angles less than about 40^0. The resultant would not be expected to differ materially from the in-phase case (Figure 3b).

Going from a phase error of $\lambda/8$ to an error of $\lambda/4$, each vector is again rotated through an angle of less than about $40°$, but in addition to this rotation, the upper vectors are all rotated by the lower ones. This means that many of the vectors are rotated through large angles, and the resultant for $\lambda/4$ may differ considerably from the resultant for $\lambda/8$ (Figure 3c).

Hence in going from a zero phase error to a $\lambda/8$ phase error the gain changes by only a small amount, whereas in going from $\lambda/8$ to $\lambda/4$ (again a change of $\lambda/8$) the gain changes by a considerable amount.



(a) zero phase error (b) phase error of $\lambda/8$ (c) phase error of $\lambda/4$

Figure 3 - Qualitative illustration of the effect of phase error on gain

This variation may be seen more quantitatively from Schelkunoff's gain curves [2] (Figure 4). For a fixed aperture size "a" the phase error depends only on the slant height "ℓ." As an example, for $a/\lambda = 6$ the slant height may be changed from $\ell = \infty$ (phase error = 0) to $\ell = 30\lambda$ (phase error = 0.15λ), which represents a change in phase error of 0.15λ, with only about 2 percent loss in gain. However, in going from $\ell = 30\lambda$ (phase error = 0.15λ) to $\ell = 15\lambda$ (phase error = 0.30λ), also a change in phase error of 0.15λ, the gain decreases by about 12 percent. Thus, the larger the initial phase error, the more sensitive the gain becomes to further variations in phase error.

In the case of an electromagnetic horn, as an observer moves in from the Fraunhofer region to the Fresnel region, the space phasing effectively adds a quadratic phase error to the intrinsic quadratic error of the aperture, that is to say, it makes the wave front appear more curved. On the basis of the preceding argument, the measured gain would be expected to decrease, the decrease being a function of the intrinsic phasing as well as the space phasing. Hence the point at which the true Fraunhofer gain is realized cannot be specified by a simple expression involving the aperture dimensions alone, e.g., $2D^2/\lambda$.

To investigate the problem quantitatively, let us first calculate the amplitude of the field at any arbitrary point of the receiving aperture. The apertures of the transmitting and receiving horns are separated by a distance R, origins are chosen at the centers of the apertures, and points in the apertures are denoted by (x,y) and (ξ, η), respectively. The x and ξ axes are parallel to each other and to the E-vector, and the y and η axes are parallel to each other and to the H-vector. Pertinent dimensions are shown in Figure 5.

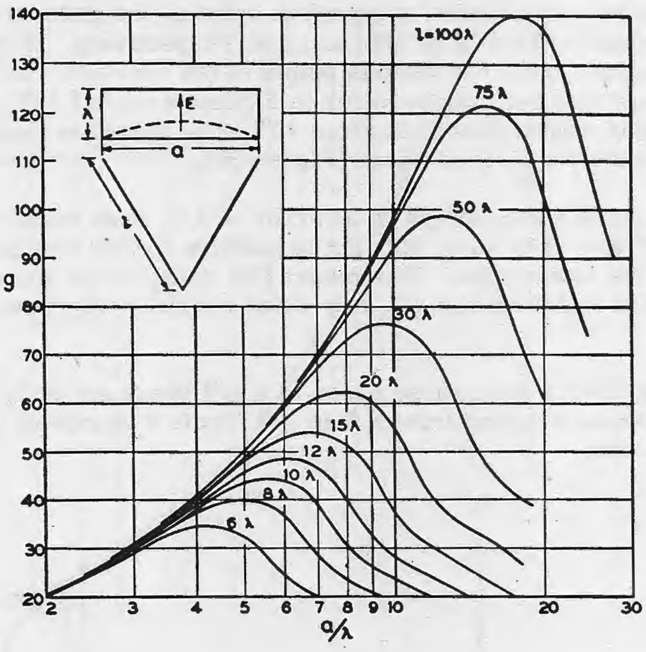


Figure 4 - Schelkunoff's gain curves

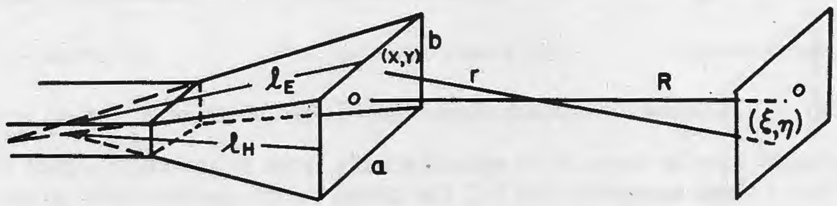


Figure 5 - Physical dimensions for calculating the gain

Assuming the field at the aperture of the transmitting horn is the same as though the horn were continued,[2] the aperture distribution is given by

$$e(x,y) = E_0 \cos\frac{\pi y}{a}\, e^{-jk\left(x^2/2\ell_E + y^2/2\ell_H\right)}$$

where ℓ_E and ℓ_H are the E- and H-plane slant heights, respectively.

From Figure 5

$$r = \sqrt{R^2 + (x-\xi)^2 + (y-\eta)^2} \cong R + \frac{(x-\xi)^2 + (y-\eta)^2}{2R}.$$

Hence, in the Fresnel approximation

$$\mathcal{E}(\xi,\eta) = \frac{E_0}{\lambda R}\, e^{-jkR} \iint \cos\frac{\pi y}{a}\, e^{-jk\left(x^2/2\ell_E + y^2/2\ell_H\right)} e^{-jk\frac{(x-\xi)^2 + (y-\eta)^2}{2R}}\, dx\, dy.$$

Denoting $\left|\mathcal{E}(\xi,\eta)\right|$ by $E(\xi,\eta)$,

$$E(\xi,\eta) = \frac{E_0}{2\lambda R}\left|\int_{-a/2}^{+a/2}\left[e^{j\frac{\pi y}{a}} + e^{-j\frac{\pi y}{a}}\right] e^{-j\frac{\pi}{2}\left[\frac{2}{\lambda}\left(\frac{1}{\ell_H}+\frac{1}{R}\right)y^2 - \frac{4\eta}{\lambda R}\,y\right]}dy\right.$$

$$\left.\times\int_{-b/2}^{+b/2} e^{-j\frac{\pi}{2}\left[\frac{2}{\lambda}\left(\frac{1}{\ell_E}+\frac{1}{R}\right)x^2 = \frac{4\xi}{\lambda R}\,x\right]}dx\right|$$

$$= \frac{E_0}{2\lambda R}\left|\int_{-a/2}^{+a/2} e^{-j\frac{\pi}{2}\left(Ay^2 - By\right)}dy + \int_{-a/2}^{+a/2} e^{-j\frac{\pi}{2}\left(Ay^2 - Cy\right)}dy\right|$$

$$\times\left|\int_{-b/2}^{+b/2} e^{-j\frac{\pi}{2}\left(Dx^2 - Fx\right)}dx\right|$$

where

$$A = \frac{2}{\lambda}\left(\frac{1}{\ell_H}+\frac{1}{R}\right),\quad B = 2\left(\frac{2\eta}{\lambda R}+\frac{1}{a}\right),\quad C = 2\left(\frac{2\eta}{\lambda R}-\frac{1}{a}\right),$$

$$D = \frac{2}{\lambda}\left(\frac{1}{\ell_E}+\frac{1}{R}\right),\ \text{and}\ F = \frac{4\xi}{\lambda R}\ .$$

Completing the square in each component and factoring,

$$E(\xi,\eta) = \frac{E_0}{2\lambda R}\left| e^{j\frac{\pi}{2}\left(B^2/4A\right)}\int_{-a/2}^{+a/2} e^{-j\frac{\pi}{2}\left(\sqrt{A}y - B/2\sqrt{A}\right)^2}dy\right.$$

$$\left.+\ e^{j\frac{\pi}{2}\left(C^2/4A\right)}\int_{-a/2}^{+a/2} e^{-j\frac{\pi}{2}\left(\sqrt{A}y - C/2\sqrt{A}\right)^2}dy\right|$$

$$\times\left| e^{j\frac{\pi}{2}\left(F^2/4D\right)}\int_{-b/2}^{+b/2} e^{-j\frac{\pi}{2}\left(\sqrt{D}x - F/2\sqrt{D}\right)^2}dx\right|$$

Let

$$\frac{\pi}{2}\left(B^2/4A\right) = \epsilon + Q$$

and

$$\frac{\pi}{2}\left(C^2/4A\right) = \epsilon - Q$$

where

$$\epsilon = \frac{\pi\lambda}{4}\,\frac{R\ell_H}{R + \ell_H}\left[\frac{4\eta^2}{\lambda^2 R^2} + \frac{1}{a^2}\right],\quad Q = \frac{\pi\eta}{a}\,\frac{\ell_H}{R+\ell_H}\,.$$

Changing the variables, let

$$\alpha = \sqrt{A}\,y - B/2\sqrt{A}\,,\qquad dy = d\alpha/\sqrt{A}$$

$$\beta = \sqrt{D}\,y - C/2\sqrt{A}\,,\qquad dy = d\beta/\sqrt{A}$$

$$\psi = \sqrt{D}\,x - F/2\sqrt{D}\,,\qquad dx = d\psi/\sqrt{D}$$

$$E(\xi,\eta) = \frac{E_0}{2\lambda R\sqrt{AD}}\left|\left[\int_{\psi_1}^{\psi_2} e^{-j\frac{\pi}{2}\psi^2}\,d\psi\right]\left[e^{jQ}\int_{\alpha_1}^{\alpha_2} e^{-j\frac{\pi}{2}\alpha^2}\,d\alpha + e^{-jQ}\int_{\beta_1}^{\beta_2} e^{-j\frac{\pi}{2}\beta^2}\,d\beta\right]\right| \tag{2}$$

where

$$\alpha_1 = -\frac{1}{\sqrt{2}}\left[\frac{a}{\sqrt{\lambda}}\sqrt{\frac{R+\ell_H}{R\ell_H}} + \left(\frac{\sqrt{\lambda}}{a} + \frac{2\eta}{R\sqrt{\lambda}}\right)\sqrt{\frac{R\ell_H}{R+\ell_H}}\right],$$

$$\alpha_2 = +\frac{1}{\sqrt{2}}\left[\frac{a}{\sqrt{\lambda}}\sqrt{\frac{R+\ell_H}{R\ell_H}} - \left(\frac{\sqrt{\lambda}}{a} + \frac{2\eta}{R\sqrt{\lambda}}\right)\sqrt{\frac{R\ell_H}{R+\ell_H}}\right],$$

$$\beta_1 = -\frac{1}{\sqrt{2}}\left[\frac{a}{\sqrt{\lambda}}\sqrt{\frac{R+\ell_H}{R\ell_H}} + \left(\frac{\sqrt{\lambda}}{a} - \frac{2\eta}{R\sqrt{\lambda}}\right)\sqrt{\frac{R\ell_H}{R+\ell_H}}\right],$$

$$\beta_2 = +\frac{1}{\sqrt{2}}\left[\frac{a}{\sqrt{\lambda}}\sqrt{\frac{R+\ell_H}{R\ell_H}} - \left(\frac{\sqrt{\lambda}}{a} - \frac{2\eta}{R\sqrt{\lambda}}\right)\sqrt{\frac{R\ell_H}{R+\ell_H}}\right],$$

$$\psi_1 = -\frac{1}{\sqrt{2\lambda R}}\left[b\sqrt{\frac{R+\ell_E}{\ell_E}} + 2\xi\sqrt{\frac{\ell_E}{R+\ell_E}}\right],$$

$$\psi_2 = +\frac{1}{\sqrt{2\lambda R}}\left[b\sqrt{\frac{R+\ell_E}{\ell_E}} - 2\xi\sqrt{\frac{\ell_E}{R+\ell_E}}\right].$$

These can be converted to Fresnel integrals

$$\int_{x_1}^{x_2} e^{-j\frac{\pi}{2}x^2}\,dx = \int_0^{x_2} e^{-j\frac{\pi}{2}x^2}\,dx - \int_0^{x_1} e^{-j\frac{\pi}{2}x^2}\,dx$$

$$= \left[C(x_2) - C(x_1)\right] - j\left[S(x_2) - S(x_1)\right].$$

Equation (2) then reads

$$E(\xi,\eta) = \frac{E_0\sqrt{\ell_E \ell_H}}{4\sqrt{(R+\ell_E)(R+\ell_H)}}\; Y(\eta)\,X(\xi) \qquad (3a)$$

where

$$Y(\eta) = \left(\left\{\cos Q\left[C(\alpha_2) + C(\beta_2) - C(\alpha_1) - C(\beta_1)\right] + \sin Q\left[S(\alpha_2) + S(\beta_1) - S(\alpha_1) - S(\beta_2)\right]\right\}^2 + \left\{\cos Q\left[S(\alpha_2) + S(\beta_2) - S(\alpha_1) - S(\beta_1)\right] - \sin Q\left[C(\alpha_2) + C(\beta_1) - C(\alpha_1) - C(\beta_2)\right]\right\}^2\right)^{\frac{1}{2}}$$

$$X(\xi) = \left\{\left[C(\psi_2) - C(\psi_1)\right]^2 + \left[S(\psi_2) - S(\psi_1)\right]^2\right\}^{\frac{1}{2}}. \qquad (3b)$$

To determine the average power per unit solid angle radiated by the transmitting horn in the direction of the receiving horn, the Poynting vector must be integrated over the receiving aperture. The expression for $|E(\xi,\eta)|^2$ obtained from Equations (3) is unfortunately too complicated to integrate directly, but calculation of the aperture field for a number of different horns shows that $Y(\eta)$ and $X(\xi)$ can be approximated by

$$Y(\eta) = Y\left(\frac{a}{2}\right) + \left[Y(0) - Y\left(\frac{a}{2}\right)\right]\cos^{\frac{1}{2}}\left(\frac{\pi\eta}{a}\right) \quad \text{and}$$

$$X(\xi) = X\left(\frac{b}{2}\right) + \left[X(0) - X\left(\frac{b}{2}\right)\right]\cos^{\frac{1}{2}}\left(\frac{\pi\xi}{b}\right)$$

where $Y(0)$ and $Y\left(\frac{a}{2}\right)$, and $X(0)$ and $X\left(\frac{b}{2}\right)$ are the values of $Y(\eta)$ and $X(\xi)$ at the center and edges of the aperture, respectively. The cosinusoidal term is quite small, so that deviations in this term from exact $\cos^{1/2} x$ behavior are unimportant. Alternatively, Equation (2) can be expanded in a Taylor series in ξ and η, thus obtaining an expression which can be averaged. The author has done this, but the present expressions are more symmetric and the numerical difference between the two methods is small.

Using these expressions for $Y(\eta)$ and $X(\xi)$, the average value of E^2 over the receiving aperture is

$$\overline{E^2_{ap}} = \frac{E_0^2\,\ell_E\,\ell_H}{(R+\ell_E)(R+\ell_H)}\;\theta^2\,\omega^2$$

where

$$\theta^2 = \frac{1}{4}\left\{Y^2\left(\frac{a}{2}\right) + 1.526\left[Y(0) - Y\left(\frac{a}{2}\right)\right]Y\left(\frac{a}{2}\right) + 0.636\left[Y(0) - Y\left(\frac{a}{2}\right)\right]^2\right\} \text{and}$$

$$\omega^2 = \frac{1}{4}\left\{X^2\left(\frac{b}{2}\right) + 1.526\left[X(0) - X\left(\frac{b}{2}\right)\right]X\left(\frac{b}{2}\right) + 0.636\left[X(0) - X\left(\frac{b}{2}\right)\right]^2\right\}.$$

Taking the ratio of the receiving cross section to transmitting gain to be the same as that in the plane-wave case, i.e., $A(R)/G(R) = \lambda^2/4\pi$, the power received is

$$P_R = \frac{1}{2}\sqrt{\epsilon/\mu}\ \overline{E^2_{ap}}\,(\lambda^2/4\pi)\,G(R).$$

Substituting this in the gain formula, Equation (1), we have

$$G(R) = \frac{2\pi R^2\sqrt{\epsilon/\mu}\ \overline{E^2_{ap}}}{P_T} = \frac{2\pi R^2\sqrt{\epsilon/\mu}\ \overline{E^2_{ap}}}{(1/4)\sqrt{\epsilon/\mu}\ abE_0^2} = \frac{8\pi R^2\ \overline{E^2_{ap}}}{abE_0^2}\,. \tag{4}$$

The ratio of this gain measured at a distance R to the gain which would be measured at infinity is given (from Equations 3 and 4) by

$$\frac{G(R)}{G(\infty)} = \frac{8\pi R^2\ \overline{E^2_{ap}}\Big/ab\ E_0^2}{\left(8\pi\ell_H\ell_E/ab\right)\left[C^2(r) + S^2(r)\right]\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}} \text{ and}$$

$$\frac{G(R)}{G(\infty)} = \frac{\theta^2}{\left(1 + \ell_H/R\right)\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}}\ \frac{\omega^2}{\left(1 + \ell_E/R\right)\left[C^2(r) + S^2(r)\right]}$$

where

$$r = \frac{b}{\sqrt{2\lambda\ell_E}},\quad u = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\ell_H}}{a} + \frac{a}{\sqrt{\lambda\ell_H}}\right), \text{ and } v = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\ell_H}}{a} - \frac{a}{\sqrt{\lambda\ell_H}}\right).$$

Thus $G(R)/G(\infty)$ is a function of five independent variables, a, b, ℓ_H, ℓ_E, and R, and each of the two factors is a function of three independent variables (R is common to both factors). This dependence makes it difficult to express the results graphically, so that a separate calculation would have to be made for each set of horns tested. Since the calculation is tedious, it is fortunate that this expression can be rewritten in terms of four new variables,

$$M = \frac{8\lambda R}{a^2},\quad H = \frac{8\lambda\ell_H}{a^2},\quad P = \frac{8\lambda R}{b^2}, \text{ and } E = \frac{8\lambda\ell_E}{b^2}.$$

The first two depend on H-plane dimensions only, and the second two on E-plane dimensions only. The quantities M and P represent the reciprocals of the maximum phase errors across the aperture due to the space phasing in the H- and E-planes, respectively. The quantities H and E represent the reciprocals of the maximum phase errors across the aperture due to intrinsic phasing in the H- and E-planes, respectively.

In terms of these new variables, the final result becomes

$$\frac{G(R)}{G(\infty)} = \frac{\omega^2}{\left(1 + \frac{E}{P}\right)\left[C^2(r) + S^2(r)\right]} \times \frac{\theta^2}{\left(1 + \frac{H}{M}\right)\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}}$$

where

$$r = 2\big/\sqrt{E},\quad u = \frac{\sqrt{H}}{4} + \frac{2}{\sqrt{H}},\quad v = \frac{\sqrt{H}}{4} - \frac{2}{\sqrt{H}},$$

$$\theta^2 = \frac{1}{4}\left\{Y^2\left(\frac{a}{2}\right) + 1.526\left[Y(0) - Y\left(\frac{a}{2}\right)\right]Y\left(\frac{a}{2}\right) + 0.636\left[Y(0) - Y\left(\frac{a}{2}\right)\right]^2\right\},$$

$$\omega^2 = \frac{1}{4}\left\{X^2\left(\frac{b}{2}\right) + 1.526\left[X(0) - X\left(\frac{b}{2}\right)\right]X\left(\frac{b}{2}\right) + 0.636\left[X(0) - X\left(\frac{b}{2}\right)\right]^2\right\},$$

$$Y\left(\frac{a}{2}\right) = \left(\left\{\cos Q\left[C(h) + C(\ell) - C(g) - C(k)\right] + \sin Q\left[S(h) + S(k) - S(\ell) - S(g)\right]\right\}^2 + \left\{\cos Q\left[S(h) + S(\ell) - S(g) - S(k)\right] - \sin Q\left[C(h) + C(k) - C(\ell) - C(g)\right]\right\}^2\right)^{\frac{1}{2}};$$

$$Y(0) = \left\{\left[C(f) - C(e)\right]^2 + \left[S(f) - S(e)\right]^2\right\}^{\frac{1}{2}},$$

$$X\left(\frac{b}{2}\right) = \left\{\left[C(t) - C(p)\right]^2 + \left[S(t) - S(p)\right]^2\right\}^{\frac{1}{2}},$$

$$X(0) = \left[C^2(m) + S^2(m)\right]^{\frac{1}{2}},$$

$$h = +2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}} - \frac{2}{M}\sqrt{\frac{HM}{H+M}},$$

$$\ell = +2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}} + \frac{2}{M}\sqrt{\frac{HM}{H+M}},$$

$$g = -2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}} - \frac{2}{M}\sqrt{\frac{HM}{H+M}},$$

$$k = -2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}} + \frac{2}{M}\sqrt{\frac{HM}{H+M}},$$

$$f = +2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}},$$

$$e = -2\sqrt{\frac{H+M}{HM}} - \frac{1}{4}\sqrt{\frac{HM}{H+M}},$$

$$p = -2\sqrt{\frac{E+P}{EP}} - 2\sqrt{\frac{E/P}{E+P}},$$

$$t = +2\sqrt{\frac{E+P}{EP}} - 2\sqrt{\frac{E/P}{E+P}},$$

$$m = +2\sqrt{\frac{E+P}{EP}},$$

$$Q = \frac{\pi}{2}\,\frac{H}{H+M},$$

$$M = \frac{8\lambda R}{a^2},\quad H = \frac{8\lambda \ell_H}{a^2},\quad P = \frac{8\lambda R}{b^2},\ \text{and}\ E = \frac{8\lambda \ell_E}{b^2}.$$

(If all dimensions are given in wavelengths, set λ = 1 wherever it appears.)

Since the expression for $G(R)/G(\infty)$ has the form of a product of two factors, one depending on M and H alone, and one depending on P and E alone, each factor can be plotted separately

in db as a one-parameter family of curves. The total correction to the gain will then be given by the sum of the corrections read from the two graphs (Figures 6 and 7).

Since essentially the same assumptions are made in calculating $G(R)$ and $G(\infty)$, it might reasonably be expected that the percentage error is nearly the same for each, and hence the percentage error in the ratio is small. That this is true has been verified experimentally in a number of cases.

USE OF CURVES

In measuring the gain of a particular set of horns, it is important to know the minimum aperture separation between horns which will give the correct far-field gain figure. In order to calculate H (and E) from the horn dimensions, the proper curves on the two graphs should be selected for the particular horn in question. To find the minimum aperture separation for which the true Fraunhofer gain will be measured, each curve is followed out to the zero correction line. In this manner, two values are obtained for R, one for each plane, and the larger is the required minimum aperture separation.

Often this distance will be found prohibitively large, either because adequate space is not available in which to perform the measurements, or because serious reflections are encountered when the distance to neighboring objects becomes comparable with the aperture-to-aperture separation. At some wavelengths, errors due to reflections may be appreciable even after considerable precautions have been taken to minimize them.

Here the alternative is to make the measurements at shorter distances and then to correct the measured values so that the true gain may be obtained. This correction factor is taken directly from the curves (by adding the corrections in db read from the two separate curves) for each distance at which measurements are carried out.

To afford the convenience and accuracy of linear interpolation, $\log M/8 = \log \lambda R/a^2$ and $\log P/8 = \log \lambda R/b^2$ are plotted on the H- and E-plane graphs, respectively, instead of M and P themselves.

For example, suppose it is desired to measure the gain of a horn having the dimensions $a = 4.69\lambda$, $b = 3.78\lambda$, $\ell_H = 6.52\lambda$, $\ell_E = 5.94\lambda$, and a measurement is to be carried out at $R = 2a^2/\lambda$. What correction to the measured value is necessary to give the true gain?

Calculating E, H, $\log \frac{\lambda R}{a^2}$, and $\log \frac{\lambda R}{b^2}$, the following results are obtained:

$$E = \frac{8\lambda \ell_E}{b^2} = \frac{(8)\,(5.94)}{(3.78)^2} = 3.326, \quad H = \frac{8\lambda \ell_H}{a^2} = \frac{(8)\,(6.52)}{(4.69)^2} = 2.371,$$

$$\log \frac{\lambda R}{a^2} = \log 2 = 0.301, \text{ and } \log \frac{\lambda R}{b^2} = \log \frac{2a^2}{b^2} = \log \frac{(2)\,(4.69)^2}{(3.78)} = 0.488.$$

Looking at the H-plane curves for $H = 2.371$ and $\log \frac{\lambda R}{a^2} = 0.301$ shows a correction of 0.47 db. Looking at the E-plane curves for $E = 3.326$ and $\log \frac{\lambda R}{b^2} = 0.488$ shows a correction of 0.49 db. Hence 0.96 db must be added to the "gain" measured at this distance to obtain the correct gain.

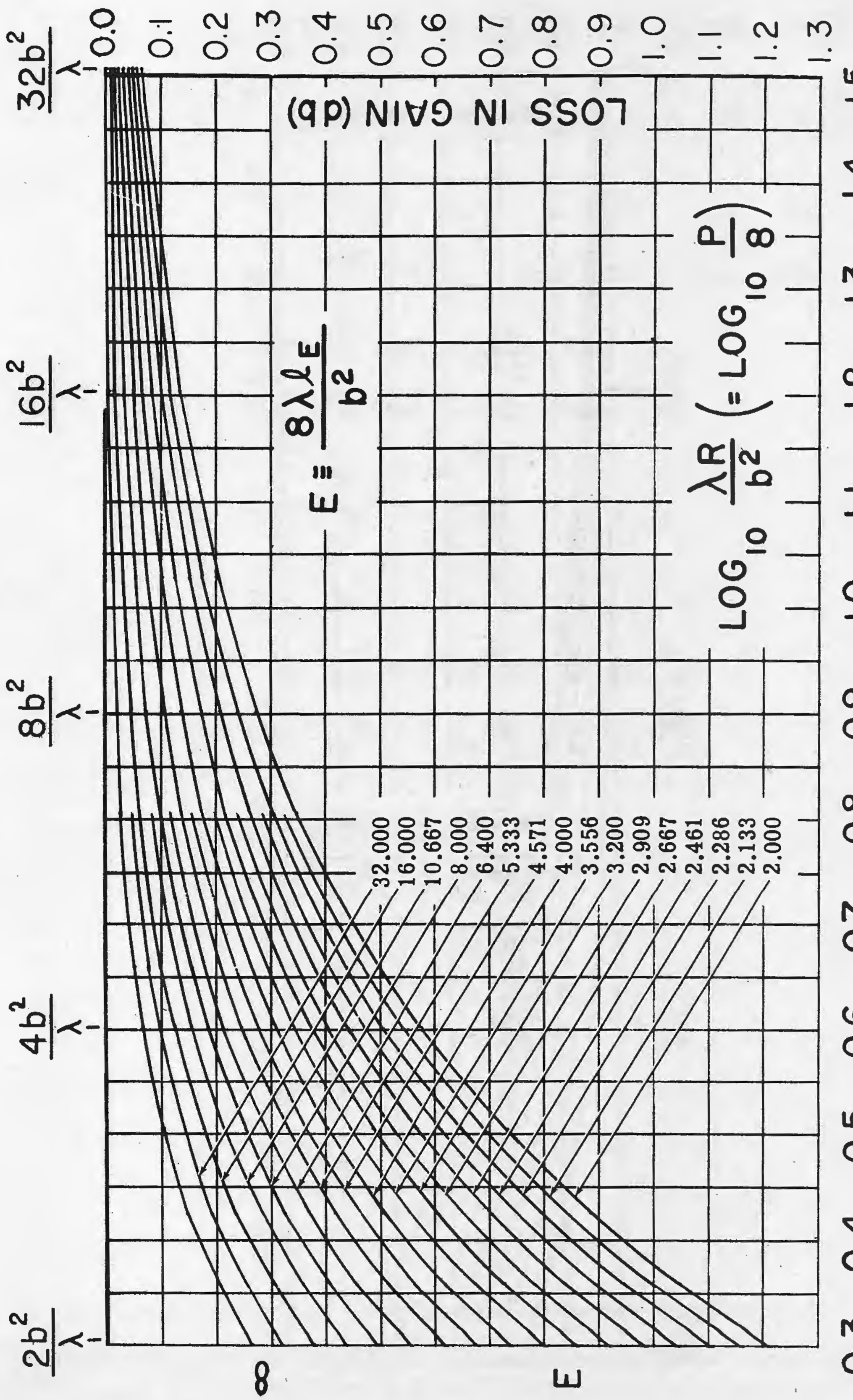


Figure 6 - E-plane correction curves

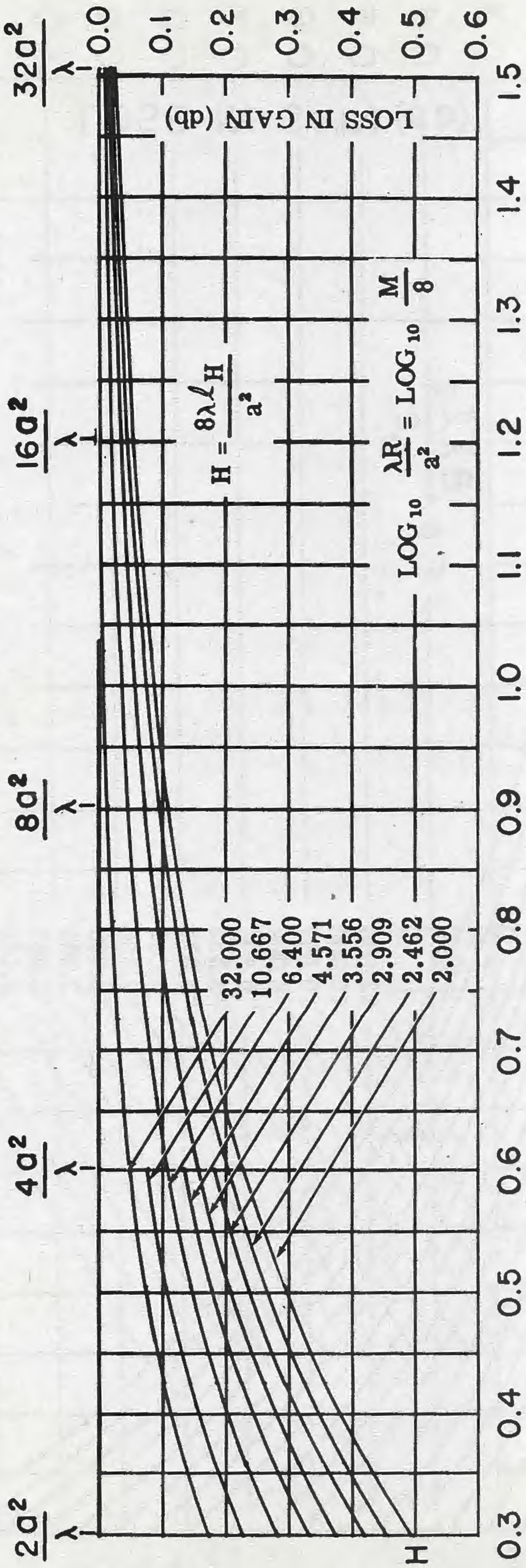


Figure 7 - H-plane correction curves

EXPERIMENTAL RESULTS

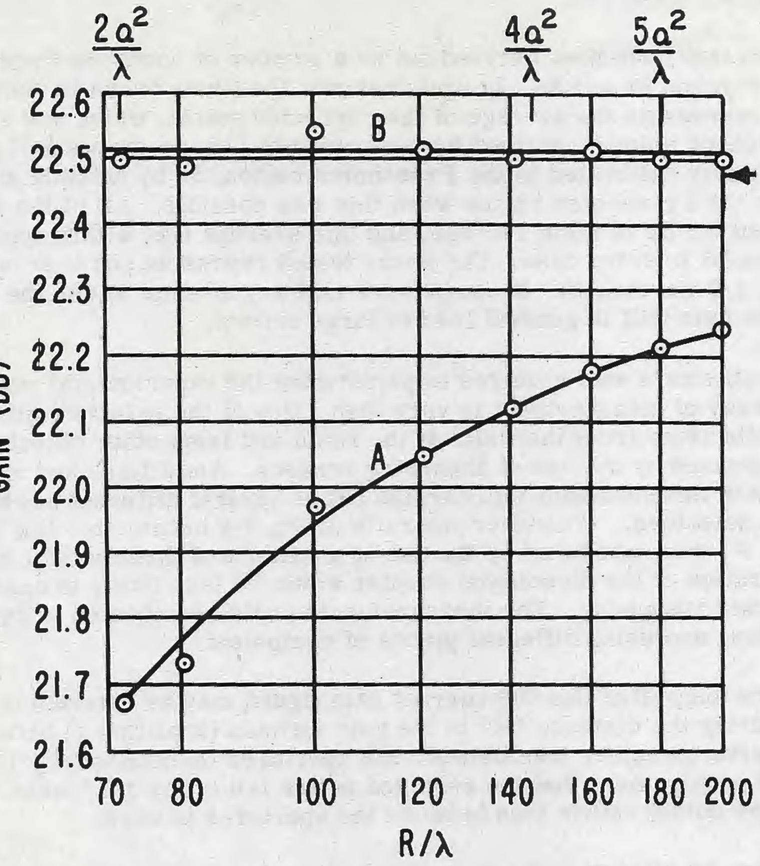
Measurements have been carried out on a number of horns, and some of the results are given in Figures 8a and 8b. In each instance the lower curve is uncorrected, and the upper curve represents the average of the corrected points, which are also shown. The arrows are "check points" obtained by comparing the horn with a small gain standard which was actually calibrated in the Fraunhofer region, or by actually measuring the gain of the horn in the Fraunhofer region when this was possible. All of the corrected points lie within about 0.1 db of their average, and this average lies within approximately 0.1 db of the check point in every case. The horns tested represent phase errors ranging from about 1/16 to 1/2 wavelength. It can be seen that any attempt to use the uncorrected points to compute the gain will in general lead to large errors.

Considerable care was required in performing the experimental work, since the desired accuracy of measurement is very high. One of the principal difficulties encountered was due to reflections from the walls of the room and from other objects, but these were carefully minimized by the use of absorbing screens. Amplifiers and meters were calibrated, and each measurement was carried out at several different power levels. Bolometers were used as detectors. Whenever the ratio of P_R/P_T became too low (at large horn separations), P_T was measured by the use of a calibrated directional coupler. It was felt that the calibration of the directional coupler would be less likely to change than that of a variable or fixed attenuator. The measurements could be repeated to better than ±0.1 db on different days and using different pieces of equipment.

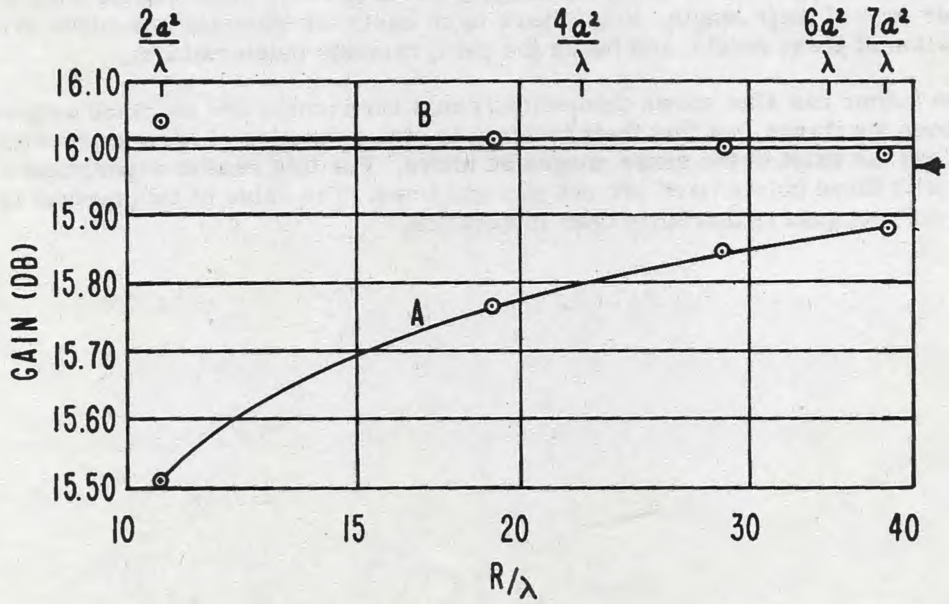
It has been suggested that the correct gain figure may be obtained from the experimental data by measuring the distance "R" in the gain formula (Equation 1) between points located behind the apertures rather than between the apertures themselves.[1] These points are located by the requirement that the received power fall off as $1/r^2$ when the separation "r" between the points rather than between the apertures is used.

If, however, an attempt is made to locate these points experimentally, the log P_R/P_T vs. log r curve never becomes exactly a stright line with slope -2 for any location of the points. Instead a set of curves is obtained, several of which approximate such a straight line over part of their length. Since there is no basis for choosing one curve over another, the location of these points, and hence the gain, remains indeterminate.

The author has also shown theoretically that such points are not fixed with respect to the horn apertures, but that their location is also a function of horn separation, and hence does not exist in the sense suggested above. For this reason experimental curves plotted with these points fixed are not straight lines. The value of this method in determining the correct gain is therefore open to question.



(a) Horn No. 1



(b) Horn No. 2

Figure 8 - Experimental gain vs. aperture separation for two typical horns

CONCLUSIONS

(a) Further experimental verification of the observed variation in measured gain with aperture separation for electromagnetic horns has been obtained. A theory has been developed which is in good quantitative agreement with the experimental data, and demonstrates the physical reasons why the previous "far-field" criterion of $2D^2/\lambda$ is invalid. The $2D^2/\lambda$ criterion has been replaced by a generic set of curves from which the error in gain measured at any distance may be determined directly and applied as a correction. The minimum aperture separation for which zero correction is required marks the beginning of the true Fraunhofer region.

(b) Making use of the theoretical results where necessary, a series of electromagnetic-horn master-gain standards has been accurately calibrated over a frequency range of 960 to 39,000 Mc. These master standards are now being used to calibrate a set of secondary gain standards by comparison measurements, and the secondary standards will, in turn, be used for measuring the gain of unknown antennas.

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