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CW ATTENUATION AND PHASE MEASUREMENTS OF COAXIAL TRANSMISSION LINES

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ABSTRACT

The problem of measuring the attenuation of low-loss transmission lines between 10 and 300 megacycles has been investigated, and there has been adopted a method of measurement which reduces the measurement of attenuation to the linear measurement of the distance between two points on a slotted line. For low values of attenuation, the slotted line need not be more than a small fraction of a wavelength long. Experimental data obtained from a test cable indicates that the attenuation varies at low frequencies according to the equation $\alpha = (4.152 \pm 0.024)10^{-2}\sqrt{f}$ where α is expressed in db/100 feet and f in Mc. The velocity of phase propagation through the test cable was measured at approximately 50, 80, and 200 Mc; no significant variation was observed.

PROBLEM STATUS

This is an interim report; work is continuing on the general problem.

AUTHORIZATION

NRL Problem H09-01R-1

CONTENTS

CW ATTENUATION AND PHASE MEASUREMENTS OF COAXIAL TRANSMISSION LINES

INTRODUCTION

The problem of measuring the attenuation and phase velocity of coaxial transmission lines was investigated in the frequency interval of 10 Mc to 300 Mc. A survey of the literature showed several methods of approach to the solution of this type of problem. Due to the low frequencies involved, all methods that require the use of a slotted line a half wavelength or more in length were immediately eliminated as being impractical. Methods which proposed a direct measure of the Q of a resonant section of transmission line were considered but rejected. One of these methods was the S-Function method described by Chandler Stewart, Jr.[1] and indicated by him as being inaccurate above 100 Mc. The method finally adopted might be called the "Width of Standing-Wave Minimum" method.[2]

The equipment required to make the measurements consisted of an r-f signal generator, a short slotted line and probe, a detector, and a calibrated attenuator. To measure the attenuation of a test cable, the far end was short circuited and the open end was connected in series with the slotted line and the r-f signal generator. The slotted-line probe was connected through a calibrated variable attenuator to a detector. The length of line and r-f frequency were so chosen as to cause a null in the voltage standing-wave pattern to fall within the slot of the slotted line. By measuring the distance between the points near the null where the voltage was 3 db higher than at its minimum value, the attenuation of the test cable was determined by the equation

$$\delta' = \alpha x$$

where δ' is equal to 1/2 the distance in radians between the 3 db points, x is the length of cable under test, and α is the attenuation of the cable per unit length.

The velocity of phase propagation was measured by locating the position of the null on the slotted line at several accurately known frequencies. Since the length of the

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[1] Stewart, C., Jr. "The S-Function Method of Measuring Attenuation of Coaxial Radio-Frequency Cable," Electrical Engineering, Transactions, 64:616-619, Sept. 1945

[2] King, D. D. "Impedance Measurements on Transmission Lines," Proc. I.R.E., 35:509-514, May 1947

transmission line and the location of the null could be measured accurately, the wavelength in the transmission line could be calculated accurately. The velocity of phase propagation is defined [3] as the product of frequency and wavelength. Because these measurements were made at several frequencies, any variation in the velocity of phase propagation within the frequency range of this investigation could be detected.

ATTENUATION

Theory of Measurement

To show how the relationship $\delta' = \alpha x$ was derived, we proceed as follows. The voltage along a transmission line is:

$$V = Z_o (Ae^{-\gamma x} - Be^{\gamma x})\, e^{j\omega t}.$$

This is the general expression for the voltage at any point, x, and any time, t, in a transmission line of characteristic impedance, Z_o, terminated in an impedance,[4] Z_R. The first term in the above equation represents the voltage of the wave traveling from the generator to the load. The second term represents the reflected wave. The factor $e^{j\omega t}$ represents the time variation of the voltage and for our purposes can be omitted. Since we are interested in the case when Z_R is a perfect short circuit, we can equate A to B or set $B/A = 1$.

$$\therefore V = Z_o A (e^{-\gamma x} - e^{\gamma x}) = -2Z_o A \sinh(\alpha x + j\beta x)$$

where $\gamma \equiv \alpha + j\beta$ is the propagation factor. Expanding, we have

$$V = -2Z_o A (\sinh \alpha x \cos \beta x + j \cosh \alpha x \text{ sine } \beta x).$$

The absolute value of V is

$$|V| = 2\,|Z_o|\,|A| \sqrt{\sinh^2 \alpha x \cos^2 \beta x + \cosh^2 \alpha x \sin^2 \beta x}.$$

If αx is small, then we can replace $\sinh^2 \alpha x$ by $(\alpha x)^2$ and $\cosh^2 \alpha x$ by 1.

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[3] Terman, F. E. "Radio Engineers' Handbook," First Edition, p. 176, McGraw-Hill Book Co., Inc., New York, 1943

[4] Slater, J. C. "Microwave Transmission," First Edition, p. 24, McGraw-Hill Book Co., Inc., New York, 1942

We have then

$$|V| = 2\,|Z_0|\,|A|\sqrt{(\alpha x)^2\cos^2\beta x + \sin^2\beta x.}$$

Since we are interested in the voltage around the minimum voltage point, we can replace βx by $n\pi + \delta$, where δ is a small angular displacement. As a result we can replace $\cos^2 \beta x$ by 1 and $\sin^2\beta x$ by δ^2.

$$\therefore |V| = 2\,|Z_0|\,|A|\sqrt{(\alpha x)^2 + \delta^2}.$$

Let us consider the value of $|V|$ when it has increased by a factor of $\sqrt{2}$ from its minimum value. At minimum, $|V| = 2\,|Z_0|\,|A|\,\alpha x$ since $\delta = 0$.

$$2\sqrt{2}\,|Z_0|\,|A|\,\alpha x = 2\,|Z_0|\,|A|\sqrt{(\alpha x)^2 + (\delta')^2}$$

$$2(\alpha x)^2 = (\alpha x)^2 + (\delta')^2$$

$$\delta' = \pm\,\alpha x$$

δ' represents the angular displacement in radians along the line from the position of the minimum to the point at which the voltage in the standing wave is 3 db higher. The resulting figure is the total attenuation of the line under test expressed in nepers. To convert nepers to decibels, we multiply the number of nepers by 8.686.

It has been shown that the voltage around a null can be represented by the equation $|V| = 2\,|Z_0|\,|A|\sqrt{(\alpha x)^2 + \delta^2}$. The ratio of the voltage at a distance δ away from the null to the minimum voltage in the null is as follows:

$$\frac{|V|}{|V_0|} = \frac{2\,|Z_0|\,|A|\sqrt{(\alpha x)^2 + \delta^2}}{2\,|Z_0|\,|A|\sqrt{(\alpha x)^2}} = \sqrt{1 + \frac{\delta^2}{(\alpha x)^2}}.$$

Expressing this voltage ratio in db, we have

$$\text{db} = 10 \log\left[1 + \frac{\delta^2}{(\alpha x)^2}\right].$$

The second derivative of this equation shows that the slope of its curve is a maximum at the 3-db level. Furthermore, the equation when plotted is very nearly linear between 1 and 5 db. This information is useful in determining the reliability of the experimental data.

Experimental Techniques

The particular line on which measurements were made was manufactured by Andrew Corporation of Chicago, Illinois. It was a solid 7/8-inch, copper transmission line having a characteristic impedance of 51.5 ohms. The center conductor of the line was supported by steatite beads spaced six inches apart. Preliminary calculations indicated that the attenuation to be expected in this line would vary with frequency from approximately 0.25 to 1.0 db per 100 feet of length.

The transmission line was short circuited at the far end by a flat, silver-plated, brass plate which was silver soldered to the inner conductor and bolted to one of the flange connectors at the end of the outer conductor. The open end of the line was connected to a slotted line which was fed by an r-f signal generator (Figure 1). Between the probe on the slotted line and the detector was placed a calibrated variable attenuator. It was necessary to use a receiver as a detector since the signal strength near the minimum of the standing-wave voltage pattern was very low. The output from the receiver was observed on a Ballentine voltmeter. The slot in the slotted line was approximately 12 inches long. The characteristic impedance of the slotted line was chosen to equal that of the transmission line thereby minimizing reflections at their junction.

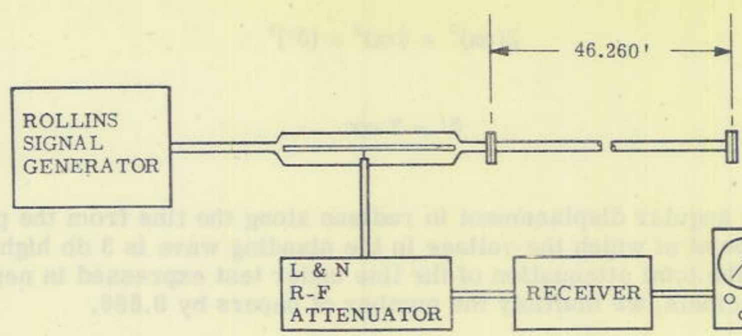

Figure 1 - Block diagram of equipment used to measure attenuation of a transmission line

A length of transmission line (about 50 feet) was chosen so that at approximately 10 Mc the first null in the voltage standing-wave pattern resulting from a short circuit at its far end fell within the slot length of the slotted line. Using the probe, the value of the minimum voltage in the null was observed at the output of the detector. This value was recorded and then the amount of attenuation in the probe lead was increased in one-db steps from 1 to 5 db. At each step, the probe was moved from right to left about the minimum until the output meter read the same as it did previously in the null. The position of the probe at these points was recorded.

The data obtained represents db levels as a function of the probe position in the slotted line and when plotted should fall on a straight line as indicated by the theory. By plotting these data on graph paper (Figure 2) and drawing the straight lines as indicated, any appreciable deviation in frequency, r-f power level from the signal generator, receiver sensitivity, or error in the variable attenuator would have shown up by causing the points to depart from the straight lines. For our calculations of αx, the distance $2\delta'$ was measured between the two straight lines at the 3-db level.

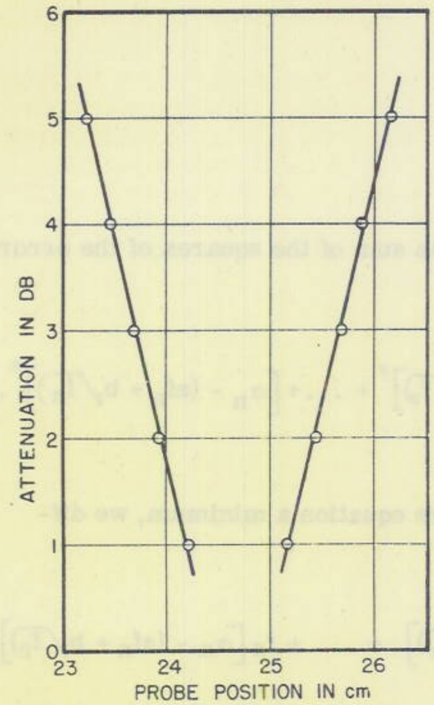

Figure 2 - Plot of a voltage standing-wave pattern near a null as a function of probe position in the slotted line. Frequency = 127.02 Mc, $2\delta' = 196$ cm

Since the calculation of the attenuation from the data depended upon the wavelength at which the measurements were made, the frequencies read off the signal generator dial were corrected. This was done by noting the position of the null on the slotted line and calculating the true frequency from a knowledge of the electrical length of the transmission line, which was obtained from measurements on the phase characteristics of the line.

Analysis of Results

Data was obtained at approximately 10-Mc intervals between 10 Mc and 260 Mc when measuring the attenuation of the test cable (Figure 3). The points should ideally fall on a curve, the equation[5] of which has the form $\alpha = af + b\sqrt{f}$. The dielectric losses are associated with the linear term af and the resistive losses with the term $b\sqrt{f}$. To determine the coefficients a and b so that the equation would best fit the experimental points, the method of least squares was applied to the data.

The experimentally determined values for α do not all fall on a smooth curve. The differences or the "errors" between the experimentally determined values of α and the curve represented by the equation $\alpha = af + b\sqrt{f}$ can be expressed as follows:

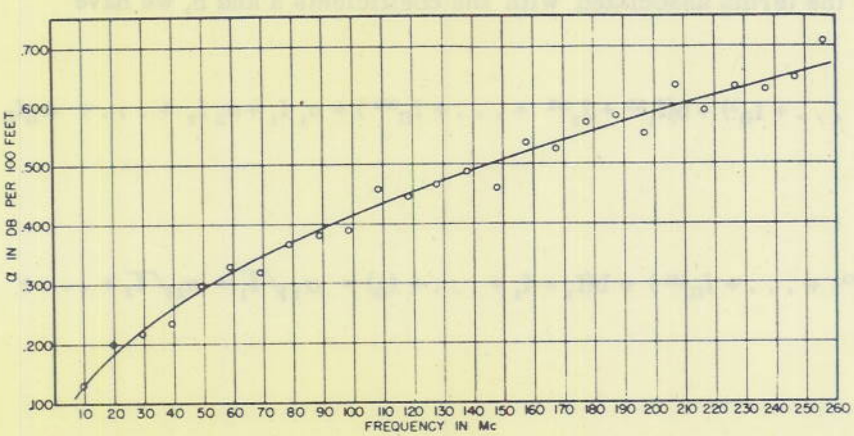

Figure 3 - Plot of the experimentally determined values for α as a function of frequency. The curve represents the equation $\alpha = (4.152 + 0.024)10^{-3}\ \sqrt{f}$ where f is expressed in Mc

[5] Jones, T. I. "The Measurement of the Characteristics of Concentric Cable at Frequencies between 1 and 100 Megacycles Per Second, I.E.E., Vol. 89, Part III, pp. 213-220, December 1942

$$\alpha_1 - (af_1 + b\sqrt{f_1}) = e_1$$

$$\alpha_2 - (af_2 + b\sqrt{f_2}) = e_2$$

$$\vdots$$

$$\alpha_n - (af_n + b\sqrt{f_n}) = e_n \, .$$

It is desired to determine values for a and b such that the sum of the squares of the errors is a minimum. Expressed mathematically

$$\sum_{i=1}^{n} (e_i)^2 = \left[\alpha_1 - (af_1 + b\sqrt{f_1})\right]^2 + \left[\alpha_2 - (af_2 + b\sqrt{f_2})\right]^2 + \ldots + \left[\alpha_n - (af_n + b\sqrt{f_n})\right]^2 .$$

Since it is desired to made the left-hand side of the above equation a minimum, we differentiate as follows:

$$\frac{\partial}{\partial a}\left(\sum e^2\right) = 0 = f_1\left[\alpha_1 - (af_1 + b\sqrt{f_1})\right] + f_2\left[\alpha_2 - (af_2 + b\sqrt{f_2})\right] + \ldots + f_n\left[\alpha_n - (af_n + b\sqrt{f_n})\right]$$

$$\frac{\partial}{\partial b}\left(\sum e^2\right) = 0 = \sqrt{f_1}\left[\alpha_1 - (af_1 + b\sqrt{f_1})\right] + \sqrt{f_2}\left[\alpha_2 - (af_2 + b\sqrt{f_2})\right] + \ldots + f_n\left[\alpha_n - (af_n + b\sqrt{f_n})\right] .$$

Combining the terms associated with the coefficients a and b, we have

$$a(f_1^2 + f_2^2 + \ldots + f_n^2) + b(f_1^{3/2} + f_2^{3/2} + \ldots + f_n^{3/2}) = \alpha_1 f_1 + \alpha_2 f_2 + \ldots + \alpha_n f_n$$

and

$$a(f_1^{3/2} + f_2^{3/2} + \ldots + f_n^{3/2}) + b(f_1 + f_2 + \ldots + f_n) = \alpha_1\sqrt{f_1} + \alpha_2\sqrt{f_2} + \ldots + \alpha_n\sqrt{f_n}$$

or

$$a \sum_{i=1}^{n} f_i^2 + b \sum_{i=1}^{n} f_i^{3/2} = \sum_{i=1}^{n} \alpha_i f_i$$

and

$$a \sum_{i=1}^{n} f_i^{3/2} + b \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} \alpha_i \sqrt{f_i} .$$

Solving these two equations simultaneously for a and b, we get

$$a = \frac{\sum \alpha f \sum f - \sum \alpha\sqrt{f} \sum f^{3/2}}{\sum f^2 \sum f - \left(\sum f^{3/2}\right)^2}$$

and

$$b = \frac{\sum f^2 \sum \alpha\sqrt{f} - \sum f^{3/2} \sum \alpha f}{\sum f^2 \sum f - \left(\sum f^{3/2}\right)^2} .$$

Evaluating these summations from the experimental data where f was expressed in Mc and α in db per 100 feet of cable length, and, solving the above equations, the following values for a and b were determined:

$$a = 0.44 \times 10^{-4}$$

$$b = 4.094 \times 10^{-2}.$$

To determine the reliability of these constants it was decided to evaluate the probable errors in a and b. Following the procedure outlined by W. N. Bond,[6] equations for the probable errors in a and b were derived and evaluated. They are

$$r_a = \pm r \sqrt{\frac{\sum f}{\sum f^2 \sum f - \left(\sum f^{3/2}\right)^2}} = \pm 0.889 \times 10^{-4}$$

and

$$r_b = \pm r \sqrt{\frac{\sum f^2}{\sum f^2 \sum f - \left(\sum f^{3/2}\right)^2}} = \pm 0.1172 \times 10^{-2} .$$

[6] Bond, W. N. "Probability and Random Error," p. 105, Edward Arnold and Co., London, 1935

where $r = \pm 0.6745 \sqrt{\sum e^2/(n-2)}$ and n is the number of experimental points (in this case 26). The final equation for α then is

$$\alpha = (0.4 \pm 0.9)\, 10^{-4}\, f + (4.09 \pm 0.12)\, 10^{-2} \sqrt{f}.$$

Since the value for a as determined by the method of least squares is so small and its uncertainty is so great, doubt arose as to the advisability of including it in the equation representing the curve which best fits the data. Consequently, it was decided to find the constant in a new equation which did not include the linear term in f. The new equation considered was

$$\alpha = b' \sqrt{f}.$$

Applying the method of least squares to this equation, a value for b′ was found in terms of α and f, such that

$$b' = \frac{\sum_{i=1}^{n} \alpha_i f_i^{\,1/2}}{\sum_{i=1}^{n} f_i}.$$

Solving, it was found that

$$b' = 4.1522 \times 10^{-2}.$$

To determine the probable error in b′, the following equation was derived:

$$r_{b'} = \pm \frac{r}{\sqrt{\sum f}}$$

where

$$r = \pm 0.6745 \sqrt{\frac{\sum (e')^2}{n-1}}.$$

Solving, we have

$$r_{b'} = \pm 2.3983 \times 10^{-4}.$$

Therefore, the new equation for α is

$$\alpha = (4.152 \pm 0.024)\, 10^{-2} \sqrt{f}.$$

The probable error in b′ is ± 0.58 percent.

To determine which of the two equations fits the data better, we apply the Gauss criterion[7] and evaluate Ω for the two equations. The quantity Ω is defined by the equation

$$\Omega = \frac{\sum e^2}{n - m}$$

where n is the total number of measurements and m is the number of constants in the equation in question. The equation which produces the smaller numerical value for Ω is considered the better equation. The equation $\alpha = af + b\sqrt{f}$ gave a value for Ω equal to 4.408×10^{-4} and the equation $\alpha = b'\sqrt{f}$, 4.368×10^{-4}.

The Gauss criterion indicates, then, that the equation $\alpha = b'\sqrt{f}$ is slightly the better equation. It is further indicated that the measurable losses through this cable at these frequencies are primarily resistive, i.e., the dielectric losses are not reliably detectable from the available data. It might be possible, using the same method of measurement, to obtain more information concerning the dielectric losses if more measurements were made at each frequency from which a more reliable average value for α could be obtained.

The theoretical value for the resistive losses in the coaxial cable was calculated[8] in order that it might be compared with the experimentally determined value for b′. The theoretical value for the coefficient of the term $\sqrt{f}$ was found to be 3.95×10^{-2}. Depending upon the value chosen for the conductivity of copper, the above value will vary approximately ± 1 percent. The difference between the theoretical and experimental value is approximately 5 percent. Consequently, approximately 4 percent of the above difference may be due to the presence of impurities in the copper and/or to losses at the joints in the cable and in the short circuit.

After the data had been obtained (Figure 3) and the results partially analyzed, additional data, shown in Table 1, were taken to: (a) to determine the changes in attenuation with small changes in frequency, and (b) to determine the reproducibility of the previously taken data.

From the results it may be seen that:

(a) The changes in attenuation with small changes in frequency are much smaller than the deviations from the curve of the original data.

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[7] Worthing, A. G. and Geffner, J. "Treatment of Experimental Data," p.261, John Wiley & Sons, New York, 1943

[8] "Reference Data for Radio Engineers," Federal Telephone and Radio Corp., Second Edition, p. 216, 1946

(b) The deviation of the new points from the curve is of the same order as the original data, but the new points deviate from the original points by amounts of this same order.

TABLE 1
Attenuation vs. Frequency

Frequency (Mc)	Attenuation (db/100 feet)
19.560	0.179
19.622	0.176
19.665	0.176
19.743	0.175
19.814	0.175
39.125	0.251
39.225	0.249
39.318	0.254
39.481	0.248
39.614	0.246

Hence, it is concluded that the major cause of the deviations of the experimental points from the curve (Figure 3) is an unknown long-term variation of the conditions of measurement; e.g., a variation of the test-cable attenuation with temperature or humidity. Since it was impossible to house the entire length of the test cable within the temperature-regulated laboratory, the last ten feet of the cable were exposed to weather variations outside the building.

TRANSMISSION LINE PHASE CHARACTERISTIC

Experimental Techniques

When comparing the velocity of phase propagation of the transmission line at different frequencies, it is necessary to know the frequency of the signal generator very accurately. The method used to determine the frequency required the use of two sources of standard frequencies (Figure 4). One of the standards was a crystal-controlled, 10-Mc oscillator and harmonic amplifier and the other a local broadcast station. The output from the crystal oscillator was mixed through an r-f mixer with a signal from the signal generator through the probe and its lead. The difference frequency, which could be varied by varying the r-f signal generator, was mixed with the broadcast station signal until a zero signal beat was produced in a broadcast receiver. When this occurred, the frequency of the signal generator was equal to the harmonic frequency of the crystal oscillator minus the frequency of the local broadcast station. Care was taken to make certain that the signal-generator frequency was below the harmonic frequency.

Reconnecting the probe to a high-frequency receiver and observing its output, the position of the null was located. The frequency of the signal generator was checked again against the broadcast station and the null relocated. This procedure was repeated a minimum of six times at each of three different frequencies (approximately 50 Mc, 80 Mc, and

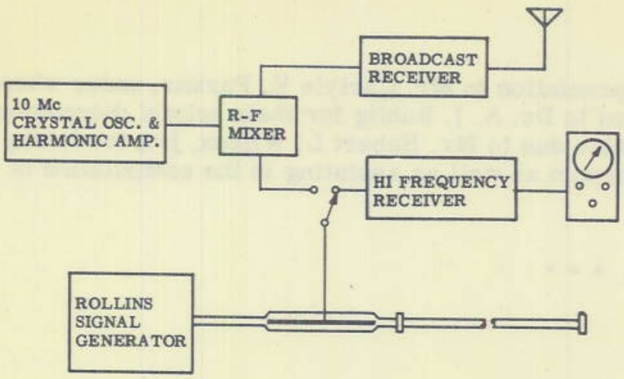

Figure 4 - Block diagram of equipment used to measure phase characteristics of a transmission line

200 Mc) and at each frequency the average position of the null was computed. From these data, the velocity of phase propagation of the transmission line could be compared at these frequencies.

Results

The position of the null on the slotted line was located at 24.83 cm when the signal generator was operating at a frequency of 49.270 Mc. This null was the fifth from the short circuit at the end of the transmission line. Consequently, there were 2-1/2 wavelengths between the located null and the short circuit. The total length of the slotted line between the zero point on its linear scale (Figure 5) and the beginning of the transmission line was 41.48 cm. The difference between this figure and the position of the null, 16.65 cm, is that portion of the 2-1/2 wavelengths, voltage-distribution pattern which fell in the slotted line. Therefore, the difference between 2-1/2 and $16.65/\lambda_o$ is the number of wavelengths which is contained in the transmission line, where λ_o is the wavelength in vacuo. Using a value of the velocity of electromagnetic propagation equal to the velocity of light in a vacuum,[9] $c = 2.99796 \times 10^{10}$ cm/sec, and at 49.270 Mc, $\lambda_o =$ 608.48 cm. The transmission line contains, then, 2.47264 λ. Since the length of the transmission line is 1410.0 cm, $\lambda = 570.24$ cm. The velocity of phase propagation, then, is

$$v = \lambda f = 2.80957 \times 10^{10} \text{ cm/sec.}$$

These calculations were repeated using data obtained at 78.950 Mc and 197.48 Mc. The values obtained for v at these frequencies were 2.80975×10^{10} cm/sec and 2.8096 $\times 10^{10}$ cm/sec. These values for v vary around their average value by ± 0.004 percent. In terms of c, the average value for the velocity of phase propagation is 0.937 c.

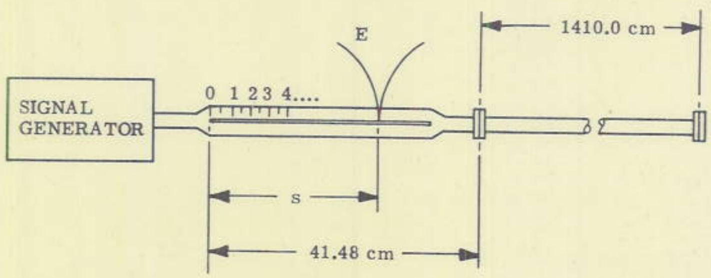

Figure 5 - Block diagram illustrating the location of a null in the slotted line

[9] "Handbook of Chemistry and Physics," 23rd Edition, Chemical Rubber Publishing Co., Cleveland, Ohio, 1939

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Mr. Carlyle V. Parker, under whose supervision this work was carried out, and to Dr. A. J. Ruhlig for their helpful discussion and constructive criticism. Thanks are also due to Mr. Robert L. Wilcox, Jr., for taking the greater part of the data used in this report as well as assisting in the computation of the results.

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