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Time-Dependent Networks: Prediction, Disruption, and Control Final Report: FY2019-FY2023

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1 Technical Objective of the Research

We solve the problem of predicting and controlling nonlinear dynamical networks under random perturbations and targeted attacks, with applications to general communication networks. We expect our new theory and tools to impact the control of networked sensors and complex coupled systems with broad science benefits to national defense by identifying general mechanisms and pathways through which nonlinear dynamical networks collapse, and how distributed control and adaptive topology can be used to retain network functionality. Dynamic networks, such as mobile sensors, transportation and computer networks, collaborating autonomous agents, and social networks, are ubiquitous, rely on network communication, and underlie vital processes relevant for defense, national security, and infrastructure, including: Maritime Battlespace Environments, Autonomous Systems, Irregular Warfare, and Information Dominance. Such networks are constantly subjected to perturbations that cause network connections and behaviors to change in time, and in ways that make long-term prediction and control very difficult. Relevant examples are ad-hoc mobile networks that operate in noisy and hostile environments, computer networks that can be infiltrated by internal viruses, and power grids that are coupled to constantly fluctuating input from renewable sources. Such electronic networks for defense are vulnerable to non-lethal attacks that may cause large changes in network functionality, or loss of function altogether. However, the overwhelming majority of what is known about complex networks with noisy and nonlinear dynamics pertains to static network limits, where dynamical perturbations, adaptivity, and other crucial time-dependent effects are ignored.

The main objective of this proposal is to develop the necessary mathematical and numerical tools to handle stochastic prediction of dynamic communicating networks, and develop a solid probabilistic framework for adaptive network control in the presence of targeted and random perturbations. Theory will be tested in high-fidelity mobile-network communication simulations/experiments such as mixed-reality experiments with an emphasis on targeted disruption, environmental factors, communication fade out, and coordinated mobile agents.

2 Technical Approach

The main approach is to develop a general mathematical and computational framework based on statistical physics that will predict and control the effects of perturbations on nonlinear dynamic communication networks. To make progress, we will leverage our previous work on static networks,where we have developed a novel theory of large fluctuations due to general perturbations. In parallel with theoretical development will be mixed-reality experiments for mobile communicating agents, such as those used in modeling communication networks of multi-agent dynamics. The effects of network fade-out, multi-cast communication, environmental noise, distributed time delays, and targeted disruptions will be emulated

The predicted scaling results will also be validated using a class of general network models such as collaborating autonomous agents, delay coupled lasers, virus spread in computer networks, and social networks such as recruitment and epidemic outbreaks in large populations. The effects of delay on networked dynamical systems will also be developed numerically and theoretically.

We present our results as a function of each fiscal year funded.

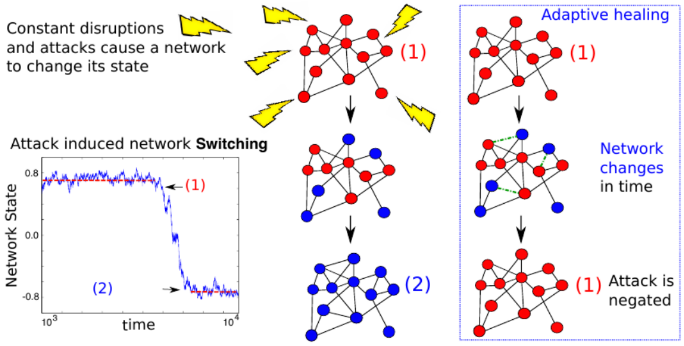

Figure 1: Schematic of Network Research. External and internal sources of noise perturb the dynamics and operations of networks. The main goals are to describe how uncertainties impact the long term limit of the networks by describing the paths in high dimensional space, and then using that knowledge to eliminate the effects of noise, and design new monitoring and controls to maintain the pristine operational state.

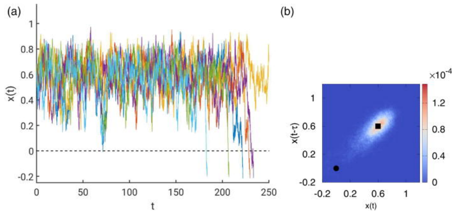

Figure 2: (a) Sample switching time series in two-peak delay distribution and (b) a probability density of switching obtained by numerical simulations. Black squares (circles) denote the attractor (saddle point). See [14] for details.

3 Yearly Progress

3.1 FY19

3.1.1 Delay communication in networks:

An important challenge in working with multi-agent systems is that in the presence of noise and communication delay, interactions between the individual agents can lead to unexpected complex system-wide behaviors.[10, 11, 8, 19, 20] We considered a generic model of two weakly coupled dynamical systems with communication delay, and showed how noise on one system can drive large fluctuations in the other through the coupling interface. Moreover, the large fluctuation happens while the first system exhibits only small random oscillations. [14, 20]In the case of systems with multiple locally stable equilibria, we show how the expected time to transition between two equilibrium states scales as a function of the coupling and delay between the two systems. In addition, our results show that the probability of switching in the noise-free system scales inversely as the square of reduced noise intensity amplitude, rendering the virtual probability of switching to be an extremely rare event. Our results are confirmed using Monte Carlo simulations.

The results done in collaboration with Profs. Thomas Carr and Klimka Szwaykowska, were published and presented at 2018 11th International Symposium on Mechatronics and its Applications (ISMA).[20]

3.1.2 Noise induced switching with general delay distributions

We considered the problem of calculating the noise-induced switching rates in systems with delay-distributed kernels and Gaussian noise. A general variational formulation for the switching rate is derived for any distribution kernel, and the obtained equations of motion and boundary conditions represent the most probable, or optimal, path, which maximizes the probability of escape. Explicit analytical results for the switching rates for small mean time delays are obtained for the uniform and bi-modal (or two-peak) distributions. They suggest that increasing the width of the distribution leads to an increase in the switching times even for longer values of mean time delays for both examples of the distribution kernel, and the increase is higher in the case of the two-peak distribution. Analytical predictions are compared to the direct numerical simulations and show excellent agreement between theory and numerical experiment.The work was done in collaboration with Prof. Yuliya Kyrychko and published in CHAOS. [14]

3.1.3 On general noise induced large fluctuations in networks

We studied rare phase slips due to noise in synchronized Kuramoto oscillator networks. In the small-noise limit, we demonstrate that slips occur via large fluctuations to saddle phase-locked states. For tree topologies, slips appear between subgraphs that become disconnected at a saddle-node bifurcation, where phase-locked states lose stability generically. This pattern is demonstrated for sparse networks with several examples. Scaling laws are derived and compared for different tree topologies. On the other hand, for dense networks slips occur between oscillators on the edges of the frequency distribution. If the distribution is discrete, the probability-exponent for large fluctuations to occur scales linearly with the system size. However, if the distribution is continuous, the probability is a constant in the large network limit, as individual oscillators fluctuate to saddles while all others remain fixed. In the latter case, the network's coherence is approximately preserved.

The research was was published in CHAOS and done in collaboration with Dr.Jason Hindes.[11, 8]

3.1.4 On stochastic model reduction

We derived a general theory of stochastic model reduction which is based on a normal form coordinate transform method of A.J. Roberts. This nonlinear, stochastic projection allows for the deterministic and stochastic dynamics to interact correctly on the lower-dimensional manifold so that the dynamics predicted by the reduced, stochastic system agrees well with the dynamics predicted by the original, high-dimensional stochastic system. The method may be applied to any system with well-separated time scales. In this article, we consider a physical problem that involves a singularly perturbed Duffing oscillator as well as a biological problem that involves the prediction of infectious disease outbreaks. The research was done in collaboration with Profs. Eric Forgoston and Lora Billings of Montclair State University, and published as an invited article in a book entitled Stochastic PDEs and Modeling of Multi-scale Complex System 2019.[2]

3.2 FY20

3.2.1 On time dependent networked delay communicating agents

Torus bifurcations of large-scale swarms having range dependent communication delay Dynamical emergent patterns of swarms are now fairly well established in nature and include flocking and rotational states. Recently, there has been great interest in engineering and physics to create artificial self-propelled agents that communicate over a network and operate with simple rules, with the goal of creating emergent self-organizing swarm patterns. In this subtopic, we show that when communicating networks have range dependent delays, rotational states, which are typically periodic, undergo a bifurcation and create swarm dynamics on a torus.[16, 17, 6] The observed bifurcation yields additional frequencies into the dynamics, which may lead to quasi-periodic behavior of the swarm. The resulting observed attractors appear as breathing ring states, with new modes governed by the introduction of one or more new frequencies. A paper has been written and has appeared in collaboration with NRL code 6792, 5514, and George Mason university Mathematics Department, Georgia Tech Research Institute, and University of Pennsylvania Mechanical Engineering Department.[16]

Unstable modes and bi-stability in delay-coupled swarms It is known that introducing time delays into the communication network of mobile-agent swarms produces coherent rotational patterns, from observed in both theory and experiments. Often such spatiotemporal rotations can be bistable with other swarming patterns, such as milling and flocking. Yet, most known bifurcation results related to delay-coupled swarms rely on inaccurate mean-field techniques. As a consequence, the utility of applying macroscopic theory as a guide for predicting and controlling swarms of mobile robots has been limited. To overcome this limitation, we perform an exact stability analysis of two primary swarming patterns in a general model with time-delayed interactions. By correctly identifying the relevant spatio-temporal modes, we are able to accurately predict unstable oscillations beyond the mean-field dynamics and bi-stability in large swarms—laying the groundwork for comparisons to robotics experiments. The research has been done in collaboration between NRL codes 6792, 5514, and George Mason university Mathematics Department.[6]

Delay communicating swarm experiments We have successfully increased the number of agents flown in the LASR Prototyping Highbay to 28 Crazyflie micro-UAV. Our mixed-reality experiments typically consist of 8 - 10 robots and 10 - 20 simulated agents, which is a significant improvement of the ratio of simulated agents to robots. Likewise, steps have been taken to use the built-in Peer to Peer (P2P) communication between robots. Currently this is focused on broadcast communication, and will be limited by bandwidth. To address the limited bandwidth issue, we are considering how we can add into our models different sensor considerations and time dependent network configurations which will lessen the burden on the communication network.[1]

3.2.2 Power networks driven by non-Gaussian fluctuations

Many networks must maintain synchrony despite the fact that they operate in noisy environments. Important examples are stochastic inertial oscillators, which are known to exhibit fluctuations with broad tails in many applications, including electric power networks with renewable energy sources, as well as communication networks in real environments. Such non-Gaussian fluctuations can result in rare network desynchronization, which could cause large amplitude fluctuations and hinder performance. Here we have built a general theory for inertial oscillator network desynchronization by non-Gaussian noise. We compute the rate of desynchronization and show that higher moments of noise enter at specific powers of coupling throughout the network. The main result is that of either speeding up or slowing down the desynchronization rate exponentially depending on how noise statistics match the statistics of a network's slowest mode.[8]

As a result our formulation, we use our theory to introduce a technique that drastically reduces the effective description of network desynchronization. Most interestingly, when instability is associated with a single edge, the reduction is to one stochastic oscillator, which is an amazing reduction.

The main application of the theory based on inertial oscillators is that of predicting the effect of renewable energy sources on large power grid networks. The research involved Jason Hindes (NRL), Philippe Jacquod (University of Applied Sciences of Western Switzerland), and Ira B. Schwartz(NRL), and has appeared in Physical Review E.[8]

3.2.3 Predicting the impact of asymptomatic transmission, non-pharmaceutical intervention and testing on the spread of COVID19

We were assigned the task of developing models to predict and minimize the risk of the outbreak of COVID-19. To this end, we introduced a novel mathematical model to analyze the effect of removing non-pharmaceutical interventions on the spread of COVID19 as a function of disease testing rate. We find that relaxing interventions has a strong impact on the size of the epidemic peak as a function of intervention removal time. We showed that it is essential for predictive models to explicitly capture transmission from asymptomatic carriers and important to obtain precise information on asymptomatic transmission by testing. The asymptomatic reservoir, reported to account for as much as 85% of transmission, will contribute to resurgence of the epidemic if public health interventions are removed too soon. Use of more basic models that fail to capture asymptomatic transmission can result in large errors in predicted clinical caseload or in fitted epidemiological parameters and, therefore, may be unreliable in estimating the risk of a second wave based on the timing of terminated interventions. The work was done in collaboration with IBM Almaden, and is in preprint form at the moment.[18]

As a follow on, we have also developed a new optimization control scheme to minimize the risk defined by the epidemic peak. We have put out a proposal to ONR and other agencies to tackle the problem of risk minimization on Navy ships and shore installations. The work was being done in collaboration with NSWC Carderock and NRL.

3.3 FY21

3.3.1 On time dependent networked delay communicating swarms

Delay communicating swarm experiments We have successfully increased the number of agents flown in the LASR Prototyping Highbay to 28 Crazyflie micro-UAV. Our mixed-reality experiments typically consist of 8 - 10 robots and 10 - 20 simulated agents, which is a significant improvement of the ratio of simulated agents to robots. Likewise, steps have been taken to use the built-in Peer to Peer (P2P) communication between robots. Currently this is focused on broadcast communication, and will be limited by bandwidth. To address the limited bandwidth issue, we are considering how we can add into our models different sensor considerations and time dependent network configurations which will lessen the burden on the communication network.

3.3.2 Social time dependent networks in modeling human contacts in epidemic models

Periodic closure control of epidemics Without vaccines and treatments, societies must rely on non-pharmaceutical intervention strategies to control the spread of emerging diseases such as COVID-19. Though complete lockdown is epidemiologically effective, because it eliminates infectious contacts, it comes with significant costs. Several recent studies have suggested that a plausible compromise strategy for minimizing epidemic risk is periodic closure, in which populations oscillate between wide-spread social restrictions and relaxation. However, no underlying theory has been proposed to predict and explain optimal closure periods as a function of epidemiological and social parameters. In this work we developed such an analytical theory for SEIR-like model diseases, showing how characteristic closure periods emerge that minimize the total outbreak, and increase predictably with the reproductive number and incubation periods of a disease– as long as both are within predictable limits. Using our approach we demonstrated a sweet-spot effect in which optimal periodic closure is maximally effective for diseases with similar incubation and recovery periods. Our results compare well to numerical simulations, including in COVID-19 models where infectivity and recovery show significant variation. The paper is done in collaboration with Ira B. Schwartz and Jason Hindes(NRL) and Simone Bianco(IBM Almaden), and has appeared in PLOS ONE: https://doi.org/10.1371/journal.pone.0244706 [4]

Predicting noise driven epidemic coutbreaks The COVID-19 pandemic has demonstrated how disruptive emergent disease outbreaks can be and how useful epidemic models are for quantifying risks of local outbreaks. Here we developed an analytical approach to calculate the dynamics and likelihood of outbreaks within the canonical Susceptible-Exposed-Infected-Recovered and more general models, including COVID-19 models, with fixed population sizes. We computed the distribution of outbreak sizes including extreme events, and show that each outbreak entails a unique, depletion or boost in the pool of susceptibles and an increase or decrease in the effective recovery rate compared to the mean-field dynamics -- due to finite-size noise. Unlike extreme events occurring in long-lived metastable stochastic systems, the underlying outbreak distribution depends on a full continuum of optimal paths, each connecting two unique non-trivial fixed-points, and thus represents a novel class of extreme dynamics. The research was done in collaboration with Ira B. Schwartz and Jason Hindes(NRL) and Michael Assaf(Hebrew University of Jerusalem).[3]

3.3.3 On interacting networked swarms and swarm instabilities

Colliding networked swarms Swarming patterns that emerge from the interaction of many mobile agents are a subject of great interest in fields ranging from biology to physics and robotics. In some application areas, multiple swarms effectively interact and collide, producing complex spatiotemporal patterns. Recent studies have begun to address swarm-on-swarm dynamics, and in particular the scattering of two large, colliding swarms with nonlinear interactions.To build on early numerical insights, we developed a self-propelled, rigid-body approximation that can be used to predict the parameters under which colliding swarms are expected to form a milling state. Our analytical method relies on the assumption that, upon collision, two swarms oscillate near a limit cycle, where each swarm rotates around the other while maintaining an approximately constant and uniform density. Using this approach we are able to predict the critical swarm-on-swarm interaction coupling, below which two colliding swarms merely scatter, as a function of physical swarm parameters. We show that the critical coupling gives a lower bound for all impact parameters, including head-on collision, and corresponds to a saddle-node bifurcation of a stable limit cycle in the uniform, constant density approximation. Our results are tested and found to agree with both small and large multi-agent simulations. The team consists of

Ira B. Schwartz and Jason Hindes(NR, Victoria Edwards and M. Ani Hsieh(UPENN) .

DOI: 10.1103/PhysRevE.103.062602 [5]

Swarm shedding-A new discovery Understanding swarm pattern-formation is of great interest because it occurs naturally in many physical and biological systems, and has artificial applications in robotics. In both natural and engineered swarms, agent communication is typically local and sparse. This is because, over a limited sensing or communication range, the number of interactions an agent has is much smaller than the total possible number. A central question for self-organizing swarms interacting through sparse networks is whether or not collective motion states can emerge where all agents have stable and coherent dynamics. In this work we introduce the phenomenon of swarm shedding in which weakly-connected agents are ejected from stable milling patterns in self-propelled swarming networks with finite-range interactions. We show that swarm shedding can be localized around a few agents, or delocalized, and entail a simultaneous ejection of all agents in a network. Despite the complexity of milling motion in complex networks, we successfully build mean-field theory that accurately predicts both milling state dynamics and shedding transitions. The latter are described in terms of saddle-node bifurcations that depend on the range of communication, the inter-agent interaction strength, and the network topology. The team consists of Ira B Schwartz, George Stantchev and Jason Hindes(NRL, Victoria Edwards(UPENN), and Klimka Szwaykowska Kasraie(GTRI), and has appeared in Nature Scientific Reports.

https://doi.org/10.1038/s41598-021-92748-1 [7]

3.4 FY22

3.4.1 On time dependent networked delay communicating swarms

KL reduced order modeling of delay - coupled communicating agents We have used model-based generated time series of swarms to examine on what manifold the dynamics lives. We find that that as a function of repulsion forces between agents, the KL dimension increases sharply from 4 modes to over 100 modes at a critical value. We have also examined the dynamics of the of the reduced order models, and find that for most of the higher dimensional dynamics, it is chaotic. For low dimensional dynamics, it appears to be either regular periodic motion, or motion on a torus.[16] The results appeared in CHAOS as an editor's choice article.[13]

3.4.2 Social time dependent networks in modeling human contacts in epidemic models

COVID-19 modeling of outbreak distributions The COVID-19 pandemic has demonstrated how disruptive emergent disease outbreaks can be and how useful epidemic models are for quantifying risks of local outbreaks. Here we developed an analytical approach to calculate the dynamics and likelihood of outbreaks within the canonical Susceptible-Exposed-Infected-Recovered and more general models, including COVID-19 models, with fixed population sizes. We computed the distribution of outbreak sizes including extreme events, and show that each outbreak entails a unique, depletion or boost in the pool of susceptibles and an increase or decrease in the effective recovery rate compared to the mean-field dynamics -- due to finite-size noise. Unlike extreme events occurring in long-lived metastable stochastic systems, the underlying outbreak distribution depends on a full continuum of optimal paths, each connecting two unique non-trivial fixed-points, and thus represents a novel class of extreme dynamics. The research was done in collaboration with Ira B. Schwartz and Jason Hindes(NRL) and Michael Assaf(Hebrew University of Jerusalem), and has now sappeared in PRL as an editor's choice article. DOI:https://doi.org/10.1103/PhysRevLett.128.078301[3]

3.4.3 On interacting networked swarms and swarm instabilities

Swarm collisions Swarming behavior, where coherent motion emerges from the interactions of many mobile agents, is ubiquitous in physics and biology. Moreover, there are many efforts to replicate swarming dynamics in mobile robotic systems which take inspiration from natural swarms. In particular, understanding how swarms come apart, change their behavior, and interact with other swarms is a research direction of special interest to the robotics and defense communities. Here we develop a theoretical approach that can be used to predict the parameters under which colliding swarms form a stable milling state. Our analytical methods rely on the assumption that, upon collision, two swarms oscillate near a limit-cycle, where each swarm rotates around the other while maintaining an approximately constant density. Using our methods, we are able to predict the critical swarm-swarm interaction coupling (below which two colliding swarms merely scatter) for nearly aligned collisions as a function of physical swarm parameters. We show that the critical coupling corresponds to a saddle-node bifurcation of a limit-cycle in the constant-density approximation. Finally, we show preliminary results from experiments in which two swarms of micro UAVs collide and form a milling state, which is in general agreement with our theory. In addition, we have quantified how the mixing of two swarms emerges using various measures of dimension, such as capacity. We have applied this to both theory and experiments of mixing swarms, and there exists an inertial range which follows a scaling law. The team consisted of Jason Hindes and Victoria Edwards. The results are in a NATO proceedings: DOI: 10.14339/STO-MP-SCI-341-08-PDF[15]

3.5 FY23

[12, 9, 13]

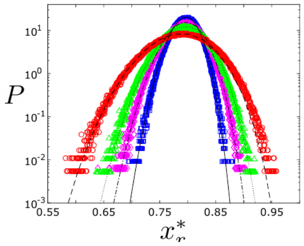

Figure 3: Outbreak statistics for white noise. The simulated white-noise PDFs (from narrowest to widest). White-noise predictions for each combination are shown with curves over- laying the simulation results. See [9] for details.

3.5.1 Noise induced network fluctuations

Fluctuations in Oscillator Networks Oscillatory networks subjected to noise are broadly used to model physical and technological systems. Due to their nonlinear coupling, such networks typically have multiple stable and unstable states that a network might visit due to noise. In this manuscript, we focus on the assessment of fluctuations resulting from heterogeneous and correlated noise inputs on Kuramoto model networks. We evaluate the typical, small fluctuations near synchronized states and connect the network variance to the overlap between stable modes of synchronization and the input noise covariance. Going beyond small to large fluctuations, we introduce the indicator mode approximation, that projects the dynamics onto a single amplitude dimension. Such an approximation allows for estimating rates of fluctuations to saddle instabilities, resulting in phase slips between connected oscillators. Statistics for both regimes are quantified in terms of effective noise amplitudes that are compared and contrasted for several noise models. Bridging the gap between small and large fluctuations, we show that a larger network variance does not necessarily lead to higher rates of large fluctuations.[12]

Parameter noise induced fluctuations We study the effect of noisy infection (contact) and recovery rates on the distribution of outbreak sizes in the stochastic SIR model. The rates are modeled as Ornstein-Uhlenbeck processes with finite correlation time and variance, which we illustrate using data from the RSV 2019-2020 season in the US. In the limit of large populations, we find analytical solutions for the outbreak-size distribution in the long-correlated (adiabatic) and short-correlated (white) noise regimes, and demonstrate that the distribution can be highly skewed with significant probabilities for large fluctuations away from mean-field theory. Furthermore, we assess the relative contribution of demographic and reaction-rate noise on the outbreak-size variance, and show that demographic noise becomes irrelevant in the presence of slowly varying reaction-rate noise but persists for large system sizes if the noise is fast. Finally, we show that the crossover to the white-noise regime typically occurs for correlation times that are on the same order as the characteristic recovery time in the model.[9]

3.5.2 Correlating chaos with KL dimension in colliding swarms

We consider the problem of characterizing the dynamics of interacting swarms after they collide and form a stationary center of mass. Modeling efforts have shown that the collision of near head-on interacting swarms can produce a variety of post-collision dynamics including coherent milling, coherent flocking, and scattering behaviors. In particular, recent analysis of the transient dynamics of two colliding swarms has revealed the existence of a critical transition whereby the collision results in a combined milling state about a stationary center of mass. In the present work, we show that the collision dynamics of two swarms that form a milling state transitions from periodic to chaotic motion as a function of the repulsive force strength and its length scale. We used two existing methods as well as one new technique: Karhunen–Loeve decomposition to show

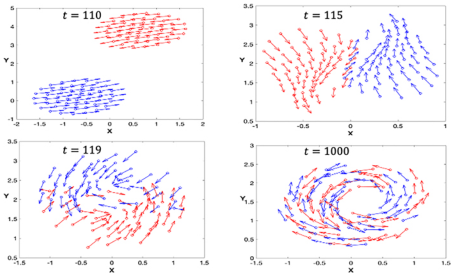

Figure 4: Four time-snapshots for different values of time , showing each colliding swarm with different colors: red and blue circles. Velocities are drawn with arrows.

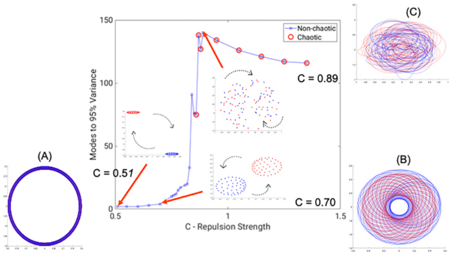

Figure 5: The 0-1 chaos test result (red circles) plotted over KL mode decomposition (blue circles). The x axis represents values of repulsive strength, C, and y axis is the number of KL modes needed to capture 95 percent of the total variance. The figures outside of the graph axis, (A)-(C), are the multi-agent attractor time series projected onto the x–y plane corresponding to the time-series. The figures inside the axes are the corresponding fixed time snapshots showing the individual agents. This is to show that in real time, combined milling of two swarms are fragmented while it is not clear from the multi-agent attractor time-series projected onto the xy plane.

the effective modal dimension chaos lives in, the 0-1 test to identify chaos, and then constrained correlation embedding to show how each swarm is embedded in the other when both swarms combine to form a single milling state after collision. We expect our analysis to impact new swarm experiments which examine the interaction of multiple swarms.[13]

4 Summary of Results

4.1 Theory Advances

- Extended fixed delay noise-induced switching theory to networks with distributed delay.
- Developed a new technique for model reduction in stochastic dynamics
- Discovered and analyzed a new networked multi-agent dynamic phenomena called swarm shedding
- Developed extreme-outbreak-dynamics theory that can be used to predict the full probability distribution of final outbreak sizes and rare events
- Corrected mean-field theory for networked swarms that exhibit bi-stable autonomous patterns.
- Generalized switching and extinction theory for networks with periodic oscillation in links.

4.2 Applications

- Applied our theory to electric power-grid models containing renewable energy sources with broad-tailed fluctuations due to turbulence.
- Analyzed how breathing modes in natural and robotic swarms arise in communication networks with range-dependent delay.
- Verified swarm bifurcation theory in a mixed-reality set of experiments in the LASR high-bay.
- Created optimal control theory for containing the spread of emerging diseases such as COVID-19
- Discovered-analyzed a new networked multi-agent dynamic phenomena called swarm shedding
- Developed extreme-outbreak-dynamics theory that can be used to predict the full probability distribution of final outbreak sizes (e.g., for COVID-19)

4.3 Ongoing Research

- Apply time-dependent network changes to analyze how coherent multi-agent-vehicle swarms become unstable.
- Investigate range-dependent delay effects, due to fade-outs in communication dynamics, which modify multi-agent stability.
- Use the developed large-fluctuation theory for time-dependent networks to study adaptive dynamics that extremize the probability of a given network transition.
- Apply time-dependent network adaptivity to epidemics (such COVID-19) and social recruitment processes, such as terrorist cells.
- Examine noise-induced networked switching in rate tipping adiabatic dynamics.

4.4 Defense Applications and Potential Impact

- The theory predicts disruption to DoD electric power grids, important for communications on land, air or at sea.
- The results on epidemic outbreak prediction are important for monitoring the spread of disease on ships, and generate a pathway to control in large populations, such as the outbreak that occurred on the USS Forester.
- With swarms of agents such as drones becoming more prominent in conflicts, our results on predicting stable emergent formations will be important for future conflicts and the science of autonomy of multi-agent systems.

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