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**APPLYING MULTI-CRITERIA DECISION
ANALYSIS METHODS TO THE NAVY PROGRAM
OBJECTIVE MEMORANDUM PROCESS**

by

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TO THE NAVY PROGRAM OBJECTIVE MEMORANDUM PROCESS**

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ABSTRACT

The U.S. Navy's annual Program Objective Memorandum (POM) describes changes in the content and priorities of Navy programs, given fiscal constraints. The POM process involves many stakeholders, echelons, and criteria, and it consists of multiple stages developed in a hierarchical manner. One way of viewing the annual POM process is as a three-level hierarchy corresponding to levels of decision-making: Thrusts, Programs, and Projects. Each Thrust represents a general Navy-wide objective supported by multiple Programs. Each Program consists of a set of fundable supporting Projects. Thrusts, Programs, and Projects together make up alternatives in the decision process. There are currently no formal methods that the Navy uses in the POM process to assist in decision-making. This thesis develops a formal methodology based upon Multi-Criteria Decision Analysis. This methodology consists of two stages: 1) use pairwise comparisons to evaluate alternatives that include each Thrust, each Thrust-related Program, and each Program-related Project, and 2) output the ratings of each alternative in a set, then take the ratings outputted from the first stage and use optimization methods to allocate the budget to Projects. We outline two different rating methods to use in the optimization stage: cardinal model and ordinal model. The purpose of our research is to help standardize the decision-making process associated with the annual POM.

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GLOSSARY

ALNAV: Message directed to all Navy and Marine Corps units

Child alternative: alternative that supports a parent alternative

Criteria: touchstones or yardsticks that we use to compare alternatives

Criteria weight: amount of importance the decision-makers put on a criteria when evaluating alternatives in $P(i)$ or $Q(j)$

$d_{h,j}$: pairwise comparison between criterion h and j

Hierarchy: structure of the problem to be solved, contains multiple tiers

Intensity: amount of preference of one alternative over another

Multi-Objective Decision Analysis (MODA): a method that enables us to break the big problem of choosing the best alternative in the presence of multiple criteria into several smaller problems that can be handled more effectively

Pairwise comparisons: relative preference of decision-maker between two alternative made as a ratio

Parent Alternative: alternative that alternatives in the next lowest tier support. i is a parent alternative to all Programs in $P(i)$ and j is a parent alternative to all Projects in $Q(j)$

Program: An entity associated with one or more Thrust

Project: An entity associated with one or more Program. Element in the lowest tier of the hierarchy.

R^j : Ratio-scale matrix, is a $m \times m$ square matrix, where m is the number of alternatives we are comparing.

$r_{i,k}^j$: pairwise comparison between alternatives i and k with respect to criteria j

Rating: value of one alternative in how well it supports a parent alternative. (synonym for *relative value*)

Ratio-scale matrix: matrix of pairwise comparisons for alternatives in regards to criteria, or pairwise comparisons of criteria weights

Relative value: value of one alternative in how well it supports a parent alternative. (synonym for *rating*)

Stakeholder: a decision-maker that has a stake in a decision-making problem

Thrust: A overarching objective of the Navy. Element in the highest tier of the hierarchy.

Tier: one level of a hierarchy

D : Ratio-scaled matrix of pairwise comparisons of criteria

$P(i)$: the set of Programs that support Thrust i as

$Q(j)$: the set of Projects that support Program j

v_i^j : The relative value of each alternatives with respect to criterion j .

EXECUTIVE SUMMARY

The U.S. Navy's Program Objective Memorandum (POM) is a collection of briefs, presentations, and other documents that describe how shifts in planning priorities are reflected in changes in the content and priorities of Navy's Programs given fiscal constraints. Each annual POM is essential to the Navy, as it lays out the vision for the Navy's present and future needs. Even though the POM process is important to the future of the Navy, there is currently no formal methodology used during the annual creation of the POM. This thesis seeks to create a formal well-structured process aimed at aiding decision-making and removing potential biases.

The difficulty in creating a formal methodology for the POM is that there are multiple echelons that decision-makers must consider. For example, there may be an objective the Navy needs to accomplish, such as "deter and/or prepare for conflict with country X." However, the Navy cannot invest in a general fund labeled "deter and/or prepare for conflict with X." It must instead invest in specific aircraft, ships, submarines, training, manpower, cyber capabilities, etc., that will support not only deterring/preparing for a conflict with country X, but also other objectives, such as "nuclear deterrence," "security of the mainland," "supporting friends and allies" and so on. The Navy consists of numerous assets supporting a variety of objectives, constituting a hierarchy of multiple tiers.

In this thesis, we explain the hierarchy in terms of three tiers, but this could be expanded to include more tiers depending on how Navy leadership chooses to organize the POM problem. The top tier of the hierarchy consists of the Navy's overall objectives, which we call *Thrusts*. Examples of Thrusts are: "nuclear deterrence," "security of the mainland," and "supporting friends and allies." The middle tier of the hierarchy consists of more specific areas called *Programs*, that support the Thrusts. Examples of Programs are: "anti-submarine warfare," "anti-air warfare," and "cyber operations." We sort the Programs into sets according to which Thrust they support. That is, each Thrust i has a set $P(i)$ that contains Programs that support it. Note that a Program may be included in multiple sets if it supports multiple Thrusts. The lowest tier is made up of *Projects* that are

specific fundable investments with some associated cost. Examples of Projects are: “F/A-18 Super Hornet,” “Arleigh Burke class guided missile destroyer,” and “Ohio class ballistic missile submarine.” The Projects are sorted into sets according to which Program they support. That is, each Program j has a set $Q(j)$ that contains Projects that support it. Again, a Projects may be in multiple sets if it supports multiple Programs. The general term we use for any Thrust, Program, or Project is *alternative*.

With the hierarchy created, we perform the first stage, which we call the Multi-Objective Decision Analysis (MODA) stage or Multi-Criteria Decision Analysis (MCDA) stage. Each set of alternatives (Thrusts, $P(i)$, and $Q(j)$) is associated with a set of relevant criteria for the evaluation of the alternatives in that set. The MODA stage compares alternatives in a set based on how well they perform with respect to the various associated criteria. Examples of criteria relevant for alternatives in the set $P(\text{Anti-Submarine})$ are: “detection range,” “detection reliability,” “maximum time on station,” and “offensive power against submarines.” The MODA stage can apply to one individual stakeholder or multiple stakeholders, with the only modification being that if multiple stakeholders are involved each stakeholder must be given a weight that will determine how much importance the MODA model places on their opinion. This weight may be dependent on the set of alternatives and specific criterion. Each stakeholder makes pairwise comparisons between alternatives, for each criterion, in each set of alternatives (Thrust, $P()$ and $Q()$). All stakeholders compare each alternative to every other alternative in a set based on individual criteria and provide a ratio $r_{i,k}^j$ representing their preference of alternative i over alternative k with respect to criterion j ; either more preferred ($r_{i,k}^j > 1$) or less preferred ($r_{i,k}^j < 1$). The $r_{i,k}^j$ ratios are combined into ratio-scale matrices R^j , one matrix for each criterion j . Then all stakeholders compare the criteria based on how important they believe each criterion to be. The stakeholders provide a ratio $d_{j,h}$ of their preference for criterion j either more than ($d_{j,h} > 1$) or less than ($d_{j,h} < 1$) criterion h . The $d_{j,h}$ ratios are combined to form a single ratio-scale matrix D for each set of alternatives. Like regression analysis in statistics, the alternatives’ ratings, as well as the criteria weights, in the various sets are obtained as normalized ratings obtained from least squares: ratings obtained by minimizing

the Euclidean distance to the entries in the elicited D and R^j matrices. Note that if multiple stakeholders are considered, each stakeholder is given a weight to denote how much their input is weighted, and that is taken into account in the least squares minimization. The least squares provide, for each criterion j and alternative i in a set a relative rating v_i^j , and a weight w_j for each criterion j . To derive an overall rating for each alternative i , we take the weighted sum, $\sum_j w_j * v_i^j$.

With each alternative's rating in each respective set as output from the MODA stage, the next step is to perform the budgeting optimization stage. This stage takes the ratings of alternatives obtained from the MODA stage and identifies the Projects to fund that maximize Navy-wide overall rating while staying within the given budget. There are two implementations of the budgeting optimization stage: cardinal and ordinal. Both implementations take the relative ratings of alternatives in sets and transform them into relative values within that tier. The cardinal implementation does this using the exact ratings obtained from the MODA stage. The ordinal implementation is a relaxation of the cardinal implementation as it treats the ratings obtained from the MODA stage as ordinal ranks and derives overall values from the ranks. We explain both of the implementations in more detail in Chapter III. With the values of alternatives in each tier, the budgeting optimization stage identifies the most valuable Projects to the entire system based on the preferences expressed in the MODA stage. After both stages are explained in detail, an example is worked through in Chapter IV.

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I. INTRODUCTION

A. PREFACE

The U.S. Navy's annual Program Objective Memorandum (POM) is a collection of briefs, presentations, and other supporting documents that describe how shifts in planning priorities are reflected in changes in the content and priorities of Navy programs, given fiscal constraints. The POM process involves many stakeholders, echelons, and criteria, and it consists of multiple stages developed in a hierarchical manner. Although the process functions as an initial stage in the creation of the Navy's budget, there is currently no known formal methodology in the POM process to assist in collective decision-making. We intend to develop in our research a formal methodology, based on Multi-Criteria Decision Analysis (MCDA).

The main goal of the modeling effort in this thesis is to use MODA and other decision-supporting methods to turn the POM process into a consistent, unbiased process. We intend to create a final product that can be implemented immediately in the creation of the POM. However, even if our proposed model is not implemented in its entirety by decision-makers, it can still provide insights, and could be used as an aid to the decision-making process.

B. BACKGROUND AND LITERATURE REVIEW

Next, we describe some terminology and general background used in this thesis.

1. Decision Theory and Alternatives

The process of choosing among different options with knowledge of the differences between the options is what makes up decision theory. In the words of D. Warner North (1968), "Decision theory is the formalization of common sense." Decision theory is used to minimize the consequences of an unfavorable outcome (North 1968). The options to choose from in a decision problem are called *alternatives*. An assumption we make in decision-making is that the decision-maker can compare two alternatives, that are not necessarily intrinsically better or worse than one another, and choose which one is

preferable. Along with this, if the decision-maker compares more than two alternatives, the order of preference for the alternatives should be transitive (North 1968). For example, a decision-maker might need to choose between three different aircraft to invest in. If aircraft “X” is preferred over aircraft “Y,” and aircraft “Y” is preferred over aircraft “Z,” then it should follow that the decision-maker prefers aircraft “X” over aircraft “Z.” This transitive property enables a decision-maker to rank alternatives in order from most preferable to least preferable. However, people do not always see alternatives as transitive so we discuss how to get around the issue of people not rating alternatives transitively.

2. Multi-Objective Decision Analysis (MODA)

Now that alternatives are defined the method the decision-maker uses to compare alternatives needs to be defined. In this thesis the POM is made into a multi-objective decision analysis (MODA) problem. Note that some literature refers to multiple-*criteria* decision analysis (MCDA) rather than multi-*objective* decision analysis, but both terms have the same meaning. MODA problems compare alternatives based on multiple criteria, and combine the comparisons of the criteria to obtain cardinal ratings or rank ordering (Cook and Kress 1996). The MODA process adds structure to the decision problem by rating alternatives based on specific criterion and then combining the ratings into an overall rating (Guitouni et. al 1998). There are many different models to solve MODA problems. However, Guitouni claims the goal of all models is “homo-economicus,,” that a rational decision-maker “always prefers the solution that maximises his welfare” (1998). Therefore, even when a solution is found for the decision problem, the solution cannot be defined as being “optimal.” It can only be said that the decision is to the “satisfaction of the decision-maker” (Guitouni et. al 1998). By applying MODA to problems that do not have cut and dry optimal solutions, decision-makers can find solutions that best satisfies their wishes.

3. Multiple Stakeholders

Before multiple stakeholders can be introduced into our problem, the term *stakeholder* must first be defined. The definition from the Merriam-Webster dictionary that we use is, “one that has a stake in an enterprise.” In *A Stakeholders Approach to MCDA*, Banville describes three different types of decision processes where stakeholders are

involved (Banville et. al 1998). Type A is autocratic, with one authority making decisions and all of the stakeholders giving input to the authority. Type C is consultive where some stakeholders are consulted, but the end decision is made by the single authority in charge. Type G is a group project where a group of stakeholders make the decision (Banville et. al 1998). As additional stakeholders are introduced to the decision-making process the amount of disagreement increases as stakeholders have differing ideas on how to rank alternatives. With more stakeholders of different standings in an organization, the need to determine how much weight the opinion of each stakeholder holds may become an issue. For example, in a military context those with a lower official rank might have their opinion allocated a lower weight than those of a higher rank.

4. Analytic Hierarchy Process (AHP)

One way to structure a MODA problem is to take the problem and split the alternatives into different criteria to make it more manageable. By having multiple criteria in the problem, the decision-maker does not have to compare alternatives overall, but can compare each all alternatives based on specific criterion (Saaty 1990). To control the scales of measurement of judging alternatives by particular criteria, the numbers are normalized so they all have ratings from zero to one. Having normalized cardinal ratings, rather than ordinal rankings, is important to the analytical process as it enables decision-makers to see the magnitude of difference between two alternatives. There are two kinds of measurements used in the AHP: relative and absolute. Relative measurement compares alternatives directly against each other while absolute evaluates how important or valuable an alternative is objectively, possibly using a Likert scale from little importance or value to extreme importance or value (Saaty 1990).

5. Program Objective Memorandum (POM)

The Program Objective Memorandum (POM) is a fiscally-constrained prioritization exercise in which services change the contents of Programs to reflect planning priorities as part of the Planning, Programming, Budgeting, and Execution (PPBE) process. The PPBE is the DOD's resource allocation process (Blickstein et. al 2016). The POM specifically is part of the Programming phase of the PPBE. Each service

submits a POM, along with a Budget Estimate Submission (BES) that evaluates the fiscal viability of Programs, to the office of the Secretary of Defense. The POM is created with a five-year outlook adjusting the existing force structure to achieve as many strategic goals as possible, in both the near term and the long term (Blickstein et. al 2016).

Each service branch submits their own POM to the Secretary of Defense. The Chief of Naval Operations (CNO) is responsible for the Navy's POM. There are currently no known formal methods used to assess decision-making, so guidance from the CNO is used as a framework for the POM. There may not always be clear explicit instructions from the CNO, so the creators must refer to publications and public speeches from the CNO as guidance for what the CNO wants (Blickstein et. al 2016). Guidance from the Secretary of the Navy and CNO is often vague and may contradict previous guidance. The N80 office has the job of interpreting what the CNO actually wants, and adjusting the POM to fit the CNO's vision. The CNO may also send the POM back for revision if it does not reflect his or her vision (Blickstein et. al 2016).

The Navy POM is submitted on July 30 each year. The Office of Management and Budget and the Office of the Secretary of Defense then review the services POMs over the course of October to December (Blickstein et. al 2016). In December the overall DOD budget request is sent to the Office of Management and Budget for final approval. On the first Monday in February the President submits the budget to Congress and the FYDP (Future Year Defense Program) is updated. Then in April the DOD sends their new DPG (Defense Planning Guidance) to the services so the services can plan for the next year. The approximate timelines are when packages are submitted, however the creation of a POM is a continual process with Strategy and Guidance being stressed in the beginning, Warfighting Integration/Program Development in the middle of the cycle, and Fiscal Integration at the end of the cycle (Blickstein et. al 2016). All three of these topics are examined throughout the entire process, but they are stressed at different points of the year.

6. POM Stakeholders

There are many stakeholders that are interested in the Navy's POM. To understand what the roles of the stakeholders are, some terminology players in the POM process must

be defined. The two types of sponsors for the POM are resource sponsors and requirements sponsors. According to Blickstein et. al (2016) some typical responsibilities of resource sponsors include:

- “representing Fleet requirements associated with the sponsor’s capability portfolio,”
- “define, advocate for, and defend specified subsets of the Navy’s capability requirements,”
- “defining and translating the capability requirements portfolio into program and resource requirements,”
- “vertically integrating the sponsor’s subsets of the Navy’s program portfolio into complete capabilities,”
- “developing, submitting, and defending [Strategic System Programs (SSPs)] for the sponsor’s program portfolio to the N8 and the CNO,”
- “integrating capabilities and resources for the program portfolio,” and
- “integrating the aspirations of applicable [Program Executive Offices (PEOs), Program Manager (PM), and System Commands (SYSCOMs)] into the sponsor’s internal SPP and eventually combined SPP, including statements of requirements maintenance and modernization and other support elements.”

Notable resource sponsors are Expeditionary Warfare (N95), Surface Warfare (N96), Undersea Warfare (N97), and Air Warfare (N98).

According to Blickstein et. al (2016) some typical responsibilities of requirements sponsors include:

- “highlighting issues of collective importance and that usually require collective of centralized actions to mitigate or resolve”
- “advocating and/or assessing a set of requirements”

Some of the most important areas requirements sponsors must highlight are manpower, maintenance, intelligence, and operations. These are all areas that resource sponsors utilize and hold in great importance, but since they are areas of collective importance they are managed as one entity. Since the resources that requirements sponsors manage are generally spread out among multiple resource sponsors, requirements sponsors generally do not own the resources they are in charge of (Blickstein et. al 2016).

The numbered directorates all have some input into the Navy's POM. The N8 office, however, has the most direct impact on the POM. It is considered the first among equals, and owns nothing so it is considered impartial. N80 is the execution arm within N8. N80 makes the final decision of what goes into the POM. N81 is in charge of analytic capability and capacity and studies in the Navy (Blickstein et. al 2016). Since N81 takes an analytic approach it should be less political. N82 is in charge of the budget. It looks at the executability of programs, the ability to execute given the desired funding, and looks at funded programs that are not executing well and moves funding where it has potential to be better spent. The N82 is important to the POM because without a budget the POM is merely wishful thinking (Blickstein et. al 2016).

Some other directorates that are important to the POM include N1 (Personnel, Manpower, and Training), N2/N6 (Information Warfare), N4 (Fleet Readiness and Logistics), and N9 (Warfare Systems). The N1 office is a resource sponsor and a requirements sponsor as it deals with manpower, personnel, training, and previously education. The N2 office and N6 office are resource sponsors as they deal with Information Warfare. The N4 office is both a resource sponsor and a requirements sponsor as it is in charge of fleet readiness, logistics, shore readiness, navy expeditionary medical services, and environmental Programs. The N9 office is in charge of warfare systems, which is the largest proportion of the Navy's total obligations, and advocate for them each year (Blickstein et. al 2016). All of the directorates have input, but the N8 office uses all the information it is given and creates the POM.

Along with the directorates, insight from the fleet also informs decisions in the POM. Fleet Commanders and Type Commanders give insight on what is critical in operations (Blickstein et al. 2016). Generally, if Fleet or Type Commanders communicate

that a requirement such as personnel, training, or maintenance is a critical need they are not denied their request. System Commanders also give insight on the development of the POM as they can provide insight on realistic cost, design, and engineering requirements as they have more technical and historical knowledge of the problems faced (Blickstein et. al 2016).

The Marine Corps creates its own POM that is sent to the Secretary of the Navy separately from the Navy's. However, the Marine Corps has an interest in the Navy's POM in areas that impact joint operations (Blickstein et. al 2016). The Navy supplies many aircraft and amphibious ships that the Marines use therefore they give input on what they would like included in the POM. Since they are so intertwined the Commandant of the Marine Corps and the CNO coordinate so all of their requirements are met (Blickstein et. al 2016).

All of the previous stakeholders have input into the POM, but in the end, it belongs to the CNO who holds the final decision-making power on what ends up in the POM. The CNO can choose how involved to be in the decision-making process and may give direct guidance on what to include in the POM or could leave it up to the N8 to piece together what is important (Blickstein et. al 2016). *Navy Planning, Programming, Budgeting and Execution. A Reference Guide for Senior Leaders, Managers, and Action Officers* cites the CNO's "ALNAV announcements, sailing and navigation instructions, similar policy and intention documents, and transmittals" as documents which the POM should be based on. The CNO can reject a POM and have it reworked until it is something he or she approves of. Once the CNO approves the POM it is submitted to the Secretary of the Navy (Blickstein et. al 2016).

7. POM Development (Navy versus Army versus Air Force)

Each service creates a POM to submit to the Secretary of Defense. However, each branch creates its POM independently using different structures than the others. A study by Johnathan R. Paden from the Air Command and Staff College describes each of the services' approaches. The Air Force uses what it calls "The Air Force Corporate Structure," which uses stakeholders from different mission areas to get expertise from each area in the

decision-making process. The Army uses a sequential process with multiple levels that allows multiple pairs of eyes to look at the POM in case a previous level overlooked it (Paden 2018). Paden describes the Navy model as a “hub-and-spoke model” since the N80 is in charge of creating the POM, and has no stake in any program so it relies on sponsors to provide information (Paden 2018). This “hub-and-spoke model” should allow for a more analytical system without biases affecting the decision. However, without a model to base this decision on, the Navy’s creation process is not as efficient as possible.

II. RANKING ALTERNATIVES IN EACH TIER

A. OVERVIEW

Similarly to the Analytic Hierarchy Process (AHP) briefly discussed in Chapter I and described in (Saaty 1990), the POM process is also imbedded in a hierarchy, albeit different than the one used in the AHP. While the hierarchy in the AHP comprises a mix of goals, criteria, and alternatives, the tiers of the hierarchy in the POM process correspond to levels of decision-making (e.g., senior executives, intermediate management, junior decision-makers) and the decision alternatives that correspond to each of those levels. In this thesis, we focus on alternatives that make up three tiers of the POM hierarchy, which we call: *Thrusts*, *Programs*, and *Projects*. At the top tier, we define a Thrust as a Navy-wide overarching objective such as “deter/prevail in a conflict against China,” “control the North Atlantic” or “defend the homeland from attack.” U.S. Navy Thrusts for a given POM cycle are determined by guidance from senior naval officials, such as the Secretary of the Navy and the CNO. A Thrust is associated with a set of one or more Programs at the second tier. We denote the set of Programs that support Thrust i as $P(i)$. Note that a Program may be associated with more than one Thrust, which means that the sets $P(i)$ may intersect. For example, a Program labeled “submarine community” may support the Thrusts: “detering/prevaling in a conflict against the China” and “defending the homeland,” but in different ways and to different extents and impacts. At the lowest tier, a Project is an alternative associated with one or more Programs.

We denote the set of Projects that support Program j as $Q(j)$. As with the sets $P(i)$, the sets $Q(j)$ may intersect too. An example of a Project in the submarine Program is a specific platform such as the Virginia Class submarine. At any tier, we call an alternative a parent alternative to alternatives in the next lowest tier if the alternative in the lower tier supports the alternative in the higher tier. Thus, Thrust i is a parent alternative to all Programs in the set $P(i)$, and Program j is a parent alternative to all Projects in the set $Q(j)$. Note that an alternative may have multiple parent alternatives as the alternatives may support multiple alternatives in the tier above them. See Figure 2.1. for a diagram of a possible hierarchy.

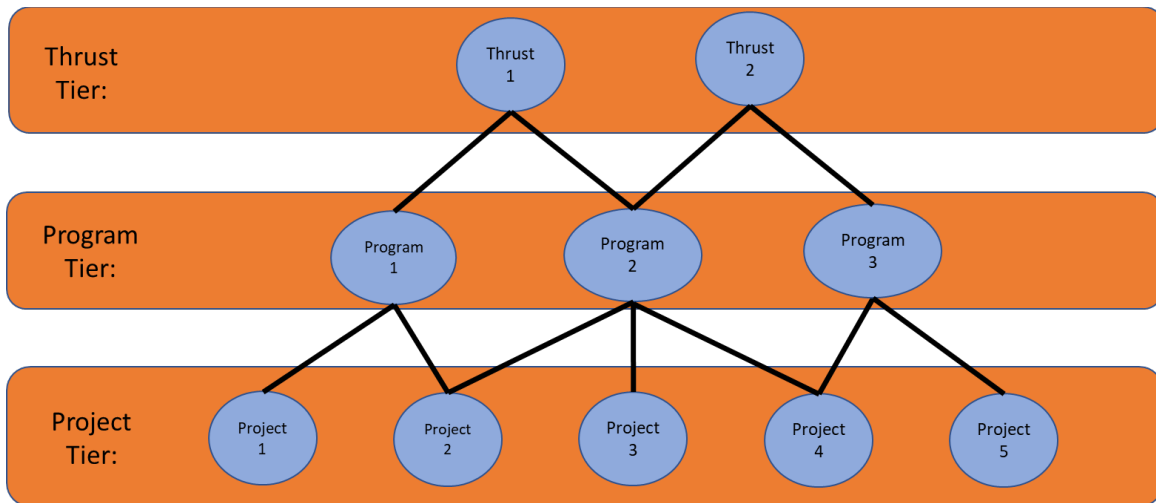


Figure 2.1. Notional hierarchy showing three tiers of alternatives that are interconnected. The three tiers are: Thrusts, Programs, and Projects.

We can add or remove tiers from the hierarchy depending on the scope of the POM and the resolution of the funded entities. At each tier, the alternatives—Thrusts, Programs, or Projects—are evaluated with respect to criteria that are specific to the sets they are a part of. For example, the set of criteria associated with the Projects in $Q(\text{Project 1})$ may be different than the set of criteria applicable to the Projects in $Q(\text{Project 2})$. Criteria are touchstones or yardsticks that we use to compare alternatives. Many times, the alternatives we compare are not definitively better with respect to all of the criteria. Multi-Objective Decision Analysis (MODA) is a method that enables us to break the big problem of choosing the best alternative in the presence of multiple criteria into several smaller problems that can be handled more effectively. The alternatives are evaluated with respect to each criterion and each alternative obtains a criteria-dependent rating, also referred to as a relative rating. The criteria are weighted according to their in-context importance relative to the set of evaluated alternatives. For example, in the submarine example we can have two Projects that belong to the submarine Program, Ohio class submarine and Virginia class submarine. One criterion for submarines could be “effectiveness of nuclear deterrence,” which the Ohio class should be rated much higher than the Virginia class. However, for the criterion “Effectiveness of anti-surface warfare” the Virginia class will most likely be rated higher than the Ohio class. This process is performed by each

stakeholder independently, and the consensus of the ratings of alternatives with respect to a certain criterion, as well as the consensus weights of the criteria, are obtained by a group decision process.

In the next sections, we describe how we obtain the ratings of the alternatives.

B. EVALUATION OF THE ALTERNATIVES BY A SINGLE STAKEHOLDER

We use pairwise comparisons for evaluating the relative ratings of alternatives with respect to a criterion, as well as determining criteria weights. The pairwise comparisons of the alternatives represent the stakeholder's relative preferences and are recorded in ratio-scale matrices, one for each criterion. To create a ratio-scale matrix of the alternatives for a certain criterion we take each pair of alternatives and ask the stakeholder to compare them independently with respect to the criterion as if no other criteria exist. Note since criteria are specific to alternatives the stakeholder compares only the alternatives in whatever set the criterion is specific to, i.e., the set of alternatives in $P(i)$ or $Q(j)$. The stakeholder compares the alternatives in ratio form describing the intensity of preference for one alternative over the other. We denote the intensity of preference of alternative i over alternative k with respect to criterion j as $r_{i,k}^j$. Once the stakeholder decides upon $r_{i,k}^j$ ratios for all pairs we combine them into a matrix R^j where $r_{i,k}^j$ is the element in the i th row, and k th column, demonstrated in Table 1 for the case of five alternatives.

Table 2.1. The matrix R^j

$r_{1,1}^j$	$r_{1,2}^j$	$r_{1,3}^j$	$r_{1,4}^j$	$r_{1,5}^j$
$r_{2,1}^j$	$r_{2,2}^j$	$r_{2,3}^j$	$r_{2,4}^j$	$r_{2,5}^j$
$r_{3,1}^j$	$r_{3,2}^j$	$r_{3,3}^j$	$r_{3,4}^j$	$r_{3,5}^j$
$r_{4,1}^j$	$r_{4,2}^j$	$r_{4,3}^j$	$r_{4,4}^j$	$r_{4,5}^j$
$r_{5,1}^j$	$r_{5,2}^j$	$r_{5,3}^j$	$r_{5,4}^j$	$r_{5,5}^j$

In general, the ratio-scale matrix, R^j , is a $m \times m$ square matrix, where m is the number of alternatives we are comparing. From the construction of R^j it is clear that $r_{i,k}^j = \frac{1}{r_{k,i}^j}$ for all alternatives i and k as the ratios are inverted when the stakeholder switches the order of the pair. Similarly, the diagonal of R^j is always 1's as those entries are comparing an alternative to itself. We say R^j is transitive if $r_{i,t}^j = r_{i,k}^j * r_{k,t}^j$. This means that the stakeholder is consistent in how they compare the alternatives. The objective is to elicit from R^j normalized relative ratings v_i^j that represent the rating of each alternative with respect to criterion j . If consistency is present throughout, then R^j uniquely determines the v_i^j relative ratings. However, in the real world, we cannot assume the stakeholder is always consistent when asked to make these types of comparisons. Therefore, we use least squares to find the relative ratings v_i^j that are closest to the given ratios for each criterion j . That is, we minimize the difference (Euclidean distance) between the ratio of relative ratings, $\frac{v_i^j}{v_k^j}$, and the pairwise comparisons $r_{i,k}^j$ the stakeholder made.

We formulate the model as follows:

(1) Evaluation of Relative Ratings with Respect to Each Criterion by One Stakeholder

Indices

i, k – Index of an alternative, $i, k = 1, \dots, I$

j – Index of a criteria, $j = 1, \dots, J$

Variables

v_i^j – Relative rating of alternative i with respect to criterion j

Parameters

$r_{i,k}^j$ – Pairwise comparison ratio between alternative i and alternative k with respect to criterion j .

Constraints

$$\sum_i^I v_i^j = 1$$

$$v_i^j \geq 0 \forall i \in I$$

Objective Function

$$\text{Min } \sum_{i,k} \left(\frac{v_i^j}{v_k^j} - r_{i,k}^j \right)^2$$

Once we obtain relative ratings for the alternatives with respect to each criterion, we ask the stakeholder to do a similar pairwise comparison process with the criteria weights. We form a single matrix D with elements $d_{j,h}$, using pairwise comparisons of all pairs of criterion j and criterion h . As with R^j , we expect that there may be some inconsistencies in the pairwise comparisons, so we solve, once again, a least squares minimization problem to derive weights w_j for each criterion j .

We formulate the model as follows:

(2) Evaluation of Weights of Criterion By One Stakeholder

Indices

j, h – Set of criteria, $j, h = 1, \dots, J$

Variables

w_j – Weight of criterion j

Parameters

$d_{j,h}$ – Pairwise comparison ratio between criterion j and criterion h .

Constraints

$$\sum_j w_j = 1$$

$$w_j \geq 0 \forall j \in J$$

Objective Function

$$\text{Min } \sum_{h,j} \left(\frac{w_h}{w_j} - d_{h,j} \right)^2$$

Once we have derived relative ratings for alternatives with respect to each criterion, and weights for each criterion we combine the two to find the overall rating of the alternative:

$$\text{Overall Rating(Alternative } i) = \sum_j w_j * v_i^j \quad (1)$$

C. EVALUATION OF ALTERNATIVES BY MULTIPLE STAKEHOLDERS

In the decision-making process of the POM, there are different stakeholders that wish their opinion to be accounted for. Ideally, the stakeholders have a discussion and reach a consensus on pairwise comparisons for each alternative. However, when many different stakeholders are involved, it is not realistic to assume a consensus can be reached by discussion alone while maintaining that each stakeholder leaves the discussion feeling that their opinion was accurately taken into account. The model described in Section B of this chapter can be extended for obtaining consensus among stakeholders regarding alternative relative ratings and criterion weights. The additional inputs required when multiple stakeholders are involved are the weight given to the opinion of each stakeholder. These weights, which represent how much we value the stakeholder's input, are typically determined by a third party based on organizational rank and expertise in the area of

interest. In the development of the POM, this means a more junior officer who works with the development of a Project and has a deep understanding of actual capabilities does not have their opinion discarded by a more senior officer who has a different agenda to fulfill. These stakeholders' weights, in evaluating the merits of alternatives and determining criteria weights, may be dependent on the subject matter as some stakeholders may be experts in only some areas and not in others.

We formulate the model as follows:

(1) Evaluation of Relative Ratings of Alternatives with Respect to a Given Criterion by Multiple Stakeholders

Indices

i, k – Index of an alternative, $i, k = 1, \dots, I$

j – Index of a criterion, $j = 1, \dots, J$

s – Index of a stakeholder, $s = 1, \dots, S$

Variables

v_i^j – The consensus relative rating of alternative i with respect to criterion j

Parameters

$r_{i,k}^j(s)$ – Pairwise comparisons made by stakeholder s between alternative i and alternative k with respect to criterion j .

l_s^j – Weight given to the opinion of stakeholder s with respect to criterion j .

Constraints

$$\sum_i v_i^j = 1$$

$$v_i^j \geq 0 \forall i \in I$$

Objective Function

$$\text{Min } \sum_s \sum_{i,k} \left(\frac{v_i^j}{v_k^j} - r_{i,k}^j(s) \right)^2 * l_s^j$$

For deriving weights of criterion for multiple stakeholders we formulate the model as follows:

(2) Evaluation of Weights of Criterion By Multiple Stakeholders

Indices

j, h – Set of criteria, $j, h = 1, \dots, J$

s – Set of stakeholders, $s = 1, \dots, S$

Variables

w_j – The consensus weight of criterion j

Parameters

$d_{h,j}(s)$ – Pairwise comparison ratio between criterion h and criterion j for stakeholder s .

l_s – Weight given to the opinion of stakeholder s in the ranking process of the criteria.

Constraints

$$\sum_j w_j = 1$$

$$w_j \geq 0 \quad \forall j \in J$$

Objective Function

$$\text{Min } \sum_s \sum_{r,j} \left(\frac{w_h}{w_j} - d_{h,j}(s) \right)^2 * l_s$$

We are now in the same situation as we were with a single stakeholder and can derive an overall rating for each alternative by multiplying the weight of each criterion by the relative rating of the alternative with respect to that criterion and summing over all criteria.

$$\text{Overall Rating(Alternative } i) = \sum_j w_j * v_i^j \quad (2)$$

D. MODA MODEL EXAMPLE

We walk through a small example of the MODA model now with only one stakeholder. Figure 2.2 illustrates the hierarchy with two Programs and three Projects that the MODA model is applied.

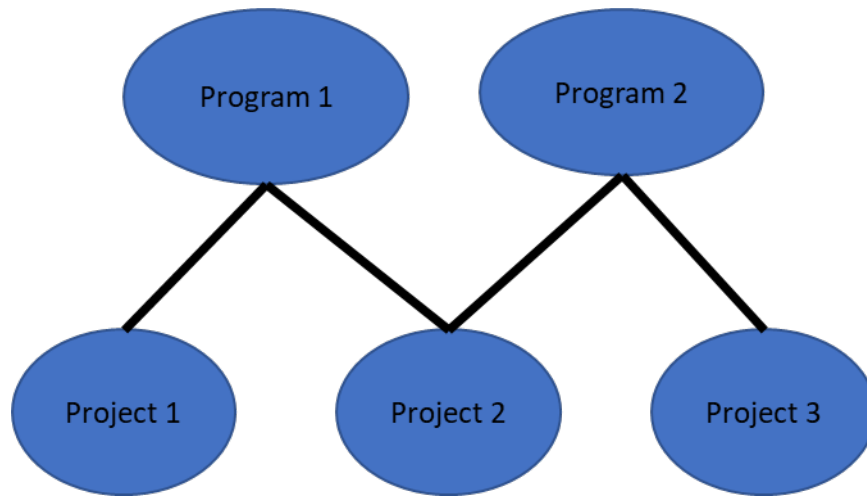


Figure 2.2. Example hierarchy for the MODA model. The model contains two Programs and three Projects.

For the comparisons between the alternatives in $Q(\text{Program 1})$, we use three criteria. For the comparisons between the alternatives in $Q(\text{Program 2})$, we use two criteria. The ratio-scale matrices are given in Tables 2.2 through Table 2.6.

Table 2.2. The matrix $R^{Criterion\ 1} Q(\text{Program 1})$

	Project 1	Project 2
Project 1	1	2
Project 2	1/2	1

Table 2.3. The matrix $R^{Criterion\ 2} Q(\text{Program 1})$

	Project 1	Project 2
Project 1	1	4/3
	3/4	1

Table 2.4. The matrix $R^{Criterion\ 3} Q(\text{Program 1})$

Stakeholder B	Project 1	Project 2
Project 1	1	7/3
Project 2	3/7	1

Table 2.5. The matrix $R^{Criterion\ 1} Q(\text{Program 2})$

Stakeholder C	Project 1	Project 2
Project 2	1	5/9
Project 3	9/5	1

Table 2.6. The matrix $R^{Criterion\ 2} Q(\text{Program 2})$

Stakeholder A	Project 1	Project 2
Project 2	1	3/5
Project 3	5/3	1

We calculate the least squares of the ratio-scale matrices to obtain the ratings of the alternatives shown in Table 2.7 and Table 2.8.

Table 2.7. The relative ratings with respect to each criterion for Program 1

Programs/Thrusts	Criterion 1	Criterion 2	Criterion 3
Project 1	0.66	0.57	0.7
Project 2	0.33	0.43	0.3
Project 3	0	0	0

Table 2.8. The relative ratings with respect to each criterion for Program 2

Programs/Thrusts	Criterion 1	Criterion 2
Project 1	0	0
Project 2	0.36	0.38
Project 3	0.64	0.62

We have the stakeholder create ratio-scale matrices comparing the criterion for each set of Projects. These matrices are shown in Table 2.9 and Table 2.10.

Table 2.9. The matrix $R^{Weights}$ Program 1

	Criterion 1	Criterion 2	Criterion 3
Criterion 1	1	4/5	3/2
Criterion 2	5/4	1	10/9
Criterion 3	2/3	9/10	1

Table 2.10. The matrix $R^{Weights}$ Program 2

	Criterion 1	Criterion 2
Criterion 1	1	7/6
Criterion 2	6/7	1

We calculate the least squares of the ratio-scale matrices to obtain weights for the criteria. For Program 1 the criteria are weighted: criterion 1 = 0.35, criterion 2 = 0.37, criterion 3 = 0.28. For Program 2 the criteria are weighted: criterion 1 = 0.54, criterion 2 = 0.46.

We finish by combining the relative ratings of the Projects with respect to the criteria and the weights of the criteria to solve for the overall ratings of the Projects in each set. We show this in equations 3–8:

$$\text{Overall Rating}(\text{Project 1} \in Q(\text{Program 1})) = 0.66 * 0.35 + 0.57 * 0.37 + 0.7 * 0.28 = 0.638 \quad (3)$$

$$\text{Overall Rating}(\text{Project 2} \in Q(\text{Program 1})) = 0.33 * 0.35 + 0.43 * 0.37 + 0.3 * 0.28 = 0.359 \quad (4)$$

$$\text{Overall Rating}(\text{Project 3} \in Q(\text{Program 1})) = 0 \quad (5)$$

$$\text{Overall Rating}(\text{Project 1} \in Q(\text{Program 2})) = 0 \quad (6)$$

$$\text{Overall Rating}(\text{Project 2} \in Q(\text{Program 2})) = 0.36 * 0.54 + 0.38 * 0.46 = 0.369 \quad (7)$$

$$\text{Overall Rating}(\text{Project 3} \in Q(\text{Program 2})) = 0.64 * 0.54 + 0.62 * 0.46 = 0.631 \quad (8)$$

III. THE OPTIMIZATION MODELS

A. OVERVIEW OF THE OPTIMIZATION MODELS

Once we derive overall ratings for the alternatives from the MODA model we can begin the process of deciding which Projects to invest in. In addition to the overall ratings relative to each parent alternative we derived in the MODA stage, the other inputs we need for the optimization model stage are the costs associated with funding each Project and the overall budget. We assume that to fund some Project k , we are either all in and investing the required c_k dollars or all out and investing zero dollars. No partial investments are possible. In a follow-on study, we could examine the effect of a new technology having its' effectiveness degraded over time using different objective functions and asking for more comparisons from the stakeholders, but for now, we assume the stakeholders are accounting for the degradation of technology in the pairwise comparisons made in the MODA model. We also assume that we are given a fixed budget B that we cannot change. We do not necessarily need to utilize every dollar, but we are not allowed to go over the budget. We examine two different methods for optimizing the amount of money allocated among Projects based on the MODA model output. The two methods differ according to the input obtained from the MODA model. In one method we use the cardinal ratings directly obtained from the MODA model for all alternatives, as described in Chapter II, and in the other method, which takes the MODA output in a more relaxed manner, we use ordinal rankings for all alternatives, induced from the cardinal ratings. We use the cardinal model if we are more concerned with evaluating value using exact ratings derived from the MODA model, and we use the ordinal model if we want to skip or only use the MODA model as a rough estimate and use more generic rankings.

B. CARDINAL MODEL

The cardinal model uses the ratings obtained from the MODA model, described in Chapter II, for the Thrusts, Programs, and Projects alternatives. This model takes a similar approach to the knapsack problem (Prabhakant 1979). The premise is to fill as much value into the “knapsack” without overfilling it. In our case, the knapsack is the budget, denoted

B , and the items to fill the budget are the cardinal ratings of the Projects' as obtained from the MODA model. The constraining factor for how many Projects we can fit in the budget is the cost of each Project k , denoted c_k .

In the MODA model we have been evaluating each alternative against other alternatives in the parent alternative set $P(i)$ or $Q(j)$, that is the set of Programs that support Thrust i ($P(i)$), or the set of Projects that supports Program j ($Q(j)$) respectively. As stated in Chapter II the sum of the ratings of each alternative in the set that supports a parent alternative, $P(i)$ and $Q(j)$, is always one. We define the following notation used in each tier of the hierarchy. The rating of Thrust i to the system is denoted z_i , the rating of Program j as a member of $P(i)$ is denoted $u_{i,j}$, and the rating of Project k as a member of $Q(j)$ is denoted $v_{j,k}$. It is important to note that if a Program j belongs to, say, $P(i)$ and $P(i')$ (e.g., see Program 2 in Figure 2.1., which belongs to $P(\text{Thrust 1})$ and $P(\text{Thrust 2})$) then the $u_{i,j}$ and $u_{i',j}$ ratings may be quite different. The same observation applied to Projects that belong to two or more Programs (e.g. Project 4 in Figure 2.2. belongs to $Q(\text{Program 4})$ and $Q(\text{Program 5})$). If Program j does not belong in $P(i)$ then $u_{i,j} = 0$, by definition. The same is true for Projects: if Project k does not belong to $P(i)$ then $v_{j,k} = 0$. Now we combine the ratings and calculate the overall value of an alternative Project, considering its own ratings in regards to the various Programs to which it belongs, and consider the overall value of those parent Programs and Thrusts. Numerically, we multiply the value of the parent alternative by the rating of the child alternative with regard to the parent alternative. Since the Thrusts are at the top of the hierarchy we do not need to scale their rating obtained from MODA and use the rating z_i as the overall value of Thrust i . To calculate the overall value to the system of each program, u_j^* we use the equation

$$\text{Value (Program } j) = u_j^* = \sum_i^I z_i * u_{ij} \quad (9)$$

Likewise, to calculate the value to the system of each project, v_k^* we use the following equation,

$$\text{Value (Project } k) = v_k^* = \sum_j^J u_j^* * v_{j,k} \quad (10)$$

We examine a small example of using these formulas looking at Figure 3.1 and Figure 3.2. The equation $z_1 + z_2 = 1$ shows the normalized ratings given directly from the MODA model. The u_j^* equations show how we used the z_i values along with the $u_{i,j}$ ratings to calculate the overall values of the Programs. The v_j^* equations show how we use the u_j^* values along with the $v_{j,k}$ ratings to calculate the overall value of the Projects.

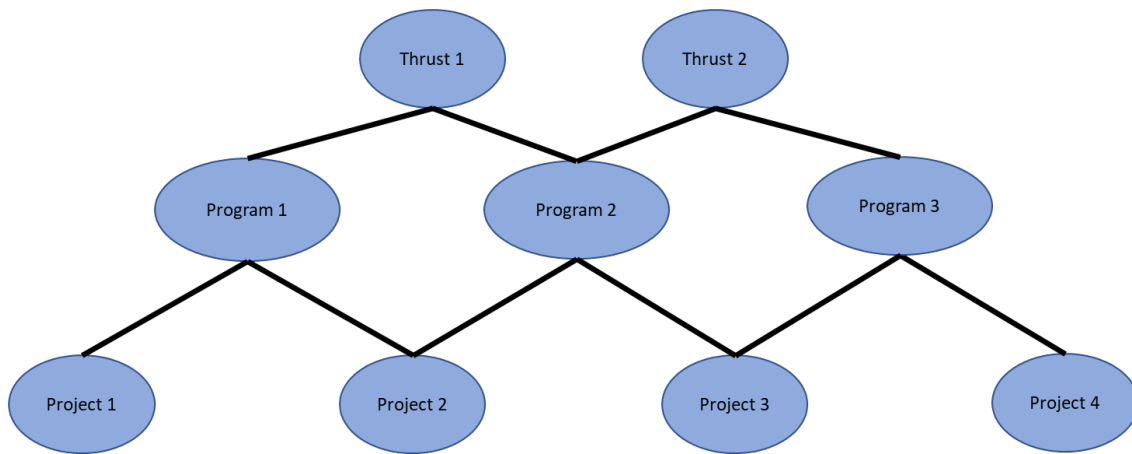


Figure 3.1. Diagram of a hierarchy of three tiers. Each Thrust is the parent alternative to two Programs. Each Program is a parent alternative to two Projects. The alternatives in lower tiers have multiple parent alternatives.

$z_1 + z_2 = 1$ $u_1^* = u_{11} * z_1$ $u_2^* = u_{12} * z_1 + u_{22} * z_2$ $u_3^* = u_{23} * z_2$	$v_1^* = v_{11} * u_1^*$ $v_2^* = v_{12} * u_1^* + v_{22} * u_2^*$ $v_3^* = v_{23} * u_2^* + v_{33} * u_3^*$ $v_4^* = v_{34} * u_3^*$
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Figure 3.2. Values for alternatives are calculated based on the ratings in regard to each parent alternative and the value of the parent alternative in the cardinal model

Now we show an example with given ratings. Suppose Project 1 belongs to $Q(1)$ and $Q(2)$, that is, Project 1 is part of two Programs: 1 and 2. Program 1 belongs to Thrust 1 and 2, but Program 2 only belongs to Thrust 2. Figure 3.3 shows this subset of a larger hierarchy.

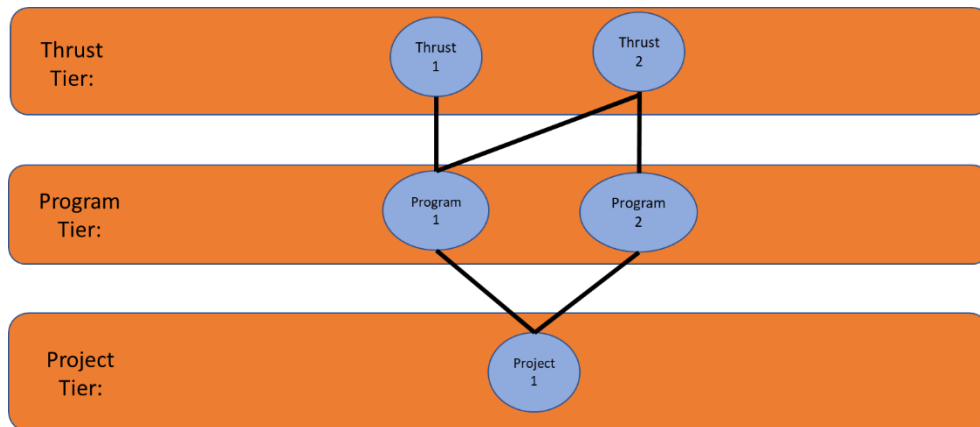


Figure 3.3. Subset of a larger hierarchy containing a Project that has two parent Programs. One of the parent Programs only has one parent Thrust, and the other has two parent Thrusts.

Assume the MODA model outputs the ratings of Thrusts as $z_1 = 0.4$ and $z_2 = 0.6$, and the ratings of the lower tiers as follows: $u_{1,1} = 0.7$, $u_{2,1} = 0.4$, $u_{2,2} = 0.5$, $v_{1,1} = 0.8$, $v_{2,1} = 0.6$. We calculate the overall value of the Programs and Projects using equations 11–13.

$$u_1^* = z_1 * u_{1,1} + z_2 * u_{2,1} = 0.4 * 0.7 + 0.6 * 0.4 = 0.52 \quad (11)$$

$$u_2^* = z_2 * u_{2,2} = 0.6 * 0.5 = 0.3 \quad (12)$$

$$v_1^* = u_1^* * v_{1,1} + u_2^* * v_{2,1} = 0.52 * 0.8 + 0.3 * 0.6 = 0.596 \quad (13)$$

Going back to the submarine example, if the stakeholders determine that “detering conflict with Russia” is the most important Thrust and they also determine the “submarine community” is the most important Program to “deter conflict with Russia,” then the Projects in the submarine community are valued very highly. However, if the stakeholders decide “detering conflict with Russia” is of low importance the value of the Projects in the “submarine community” decreases even if they have the same high rating in regards to the Thrust.

Now that we have established a method to evaluate the overall value of Projects we can plug the values into an optimization model to maximize the value of the entire set of alternatives. Formally,

Indices

i – Set of Thrusts, $i = 1, \dots, I$

j – Set of Programs, $j = 1, \dots, J$

k – Set of Projects, $k = 1, \dots, K$

Variables

X_k = The decision of whether or not to fund Project k . $X_k = 1$ if Project k is funded and 0 otherwise.

Parameters

z_i – Value of Thrust i to the system.

$u_{i,j}$ – Rating of Program j with regards to Thrust i .

u_j^* – Overall value of Program j .

$v_{j,k}$ – Rating of Project k with regards to Program j .

v_k^* – Overall value of Project k .

c_k – Cost of Project k .

B – Budget

Constraints

$$\sum_k X_k * c_k \leq B$$

$$X_k \in \{0,1\} \forall k$$

Objective Function

$$\text{Max } \sum_k v_k^* * X_k$$

C. ORDINAL METHOD

The ordinal model is based on the output ratings from the MODA model in Chapter II. It does not use the ratings directly as given but only the rank order of the ratings. For each set of alternatives (Thrusts, Programs that belong to the same Thrust ($P(j)$), and Projects that belong to the same Program ($Q(k)$)) we use the obtained ratings from the MODA model to rank the alternatives: highest rating rank 1 and lowest rating the last rank position. For example, if we are given ratings for three Programs in $P(i)$: $u_{i,1} = 0.3$, $u_{i,2} = 0.1$, and $u_{i,3} = 0.6$, we put the Programs in order from highest rating to lowest rating, so in $P(i)$ the rank order is: Program 3, Program 1, Program 2; and then we disregard the

cardinal ratings after the Programs are ranked. The model will derive new ratings based on the rankings. Alternatively, if the MODA model in Chapter II is not utilized, it may be easier for multiple stakeholders to come to a consensus on the rank of alternatives rather than assigning cardinal ratings. The only inputs required to the ordinal model other than the ordinal rankings of the alternatives are constants α, δ , and τ that serve as required buffers between the ranks of the alternatives at the Project, Program, and Thrust tiers respectively. The parameters α, δ , and τ indicate the intensity of preferences among rank positions. Smaller values of α, δ , and τ represents a weak preference in difference between alternatives while larger values of α, δ , and τ indicate a strong preference. This model treats new ratings of alternatives in sets $P(i)$ or $Q(j)$ as a decision variable where the sum of these variables in a certain $P(i)$ or $Q(j)$ equals 1. These ratings are denoted the same as in the cardinal model, $z_i, u_{i,j}$, and $v_{j,k}$ for Thrusts, Programs, and Projects respectively. For a trivial example, we examine a hierarchy if we set $\alpha = 0.2$ and we only have two tiers. In the hierarchy we have two Projects that both support a single Program, Program 1, with Project 1 ranked first and with Project 2 ranked second, then the ordinal model includes a constraint that assigns a gap of at least 0.2 between the ratings $v_{1,1}$ and $v_{1,2}$. Note that if both Project 1 and Project 2 belong to sets of two different Programs they may have different relative positions in the ranking in the two corresponding $Q()$ sets. We additionally constrain the ratings of $v_{j,k}$ to be between zero and one for all Projects k . We impose rating conservation constraints that force the sum of the ratings of the Projects in $Q(j)$ to be equal to the sum of the ratings of Program j in regards to each Thrust that Program j supports and the sum of the ratings of all Programs in $P(i)$ to be equal to the rating of Thrust i . That is $\sum_{k \in Q(j)} v_{j,k} - \sum_{i=1}^I u_{i,j} = 0$, and $\sum_{j \in P(i)} u_{i,j} - w_i = 0$. Once ratings are assigned at all tiers the model calculates the overall value of each Project, V_k by summing all of the ratings assigned in regards to Programs, $v_{i,j}$. Then the model chooses Projects to fund that maintain the rankings of all alternatives at all tiers and maximize the sum of the value of the Projects.

We can write the optimization problem formally,

(1) Ordinal Budgeting Model

Indices

i – Set of Thrusts, $i = 1, \dots, I$

j – Set of Programs, $j = 1, \dots, J$

k – Set of Projects, $k = 1, \dots, K$

Sets

$Q(j)$ = The set of Projects in Program j

$P(i)$ = The set of Programs in Thrust i

Variables

$v_{j,k}$ = Rating of Project k relative to Program j . $v_{j,k} = 0$ if $j \notin R_j$.

$V_k = \sum_{j=1}^J v_{j,k}$ = Value of Project k , $k = 1, \dots, K$.

$u_{i,j}$ = Rating of Program j relative to Thrust i . $u_{i,j} = 0$ if $j \notin P_i$.

$z_i = \sum_{j \in P_i} u_{i,j}$ Value of Thrust i .

$X_k = 1$ if Project k is selected for funding and 0 otherwise.

Parameters

α – Rank separation threshold for Projects.

δ – Rank separation threshold for Programs

τ – Rank separation threshold for Thrusts.

B – Budget

Constraints

$0 \leq v_{j,k} \leq 1$, scaling.

If Project k is ranked higher than Project k' with respect to Program j then

$$v_{j,k} - v_{j,k'} \geq \alpha.$$

If Program j is ranked higher than Program j' with respect to Thrust i then

$$u_{i,j} - u_{i,j'} \geq \delta.$$

If Thrust i is ranked higher than Thrust i' then $Z_i - Z_{i'} \geq \tau$.

$\sum_{k \in Q(j)} v_{j,k} - \sum_{i=1}^I u_{i,j} = 0$. This is a rating-conservation constraint for Programs.

$\sum_{j \in P(i)} u_{i,j} - w_i = 0$. This is a rating-conservation constraint for Programs.

$$\sum_{k=1}^K C_k X_k \leq B$$

Objective Function

$$\text{Max } \sum_{k=1}^k V_k X_k$$

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IV. APPLICATION EXAMPLE

A. OVERVIEW

To demonstrate the applicability of the methodology described in Chapters II and III, we now present an example that goes through both the MODA stage and the budgeting optimization stage. In the MODA model, we derive ratings for alternatives in a given tier corresponding to a parent alternative in a higher tier. Then we take the ratings and apply both the cardinal and ordinal budgeting optimization models to choose which projects to fund. Fictional future platforms are used in this example. The ratings and costs used are notional and only intended to be used for demonstrational purposes. Our three-tier example comprises of two Thrusts at the top tier, three Programs at the middle tier, and five Projects at the bottom tier, as shown in Figure 4.1. In our example, we use overarching objectives that the Navy needs to accomplish as Thrusts. The Programs we use are more specific objectives that support one or more objectives specified as a Thrust. The Projects are individual ships and submarines, or squadrons of aircraft that directly support one or more objectives specified as Programs. The Project tier is where a dollar amount is required as these alternatives are directly fundable. As mentioned earlier, the number of tiers may be larger than three, with more alternatives at each tier. To demonstrate the applicability of our modeling approach, and to keep the example within a reasonable level of transparency, we use the hierarchy as shown in Figure 4.1. The costs are notional and are given in hundreds of millions of dollars.

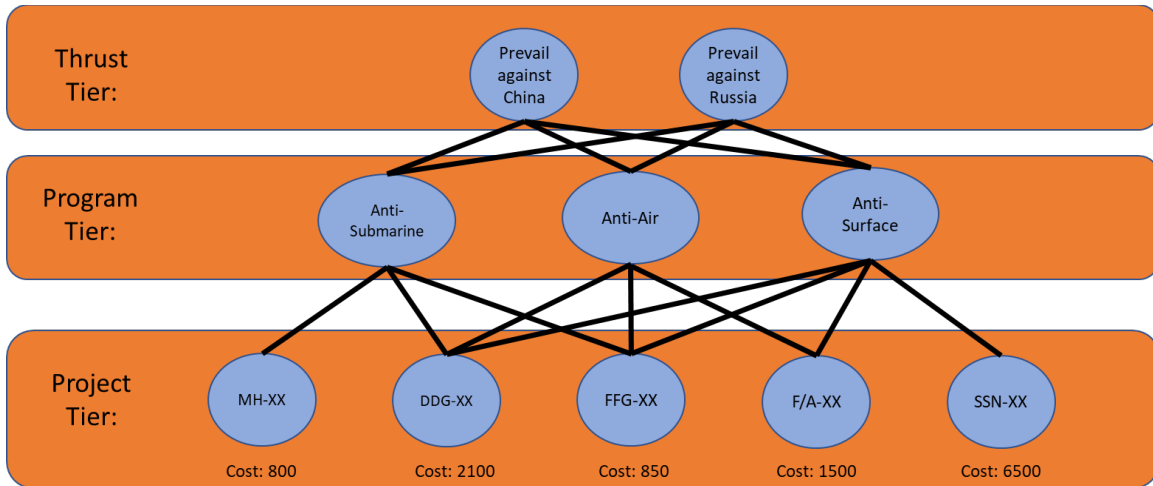


Figure 4.1. Notional hierarchy of future spending in the Navy's POM, including costs of each Project.

B. MODA MODEL

For the sake of brevity, we explicitly demonstrate the MODA model only for the Projects belonging to the Anti-Submarine Program. The evaluations of alternatives in other sets (e.g., $P(\dots)$ and $Q(\dots)$) are done in a similar way. The Projects belonging to $Q(\text{Anti_Submarine})$, are: the new class of helicopter (MH-XX), the new class of guided-missile destroyers (DDG-XX), and the new class of guided-missile frigates (FFG-XX). The stakeholders for this evaluation are: a subject-matter expert from The Office of Naval Research (Stakeholder A), a member of the Chief of Naval Operations staff (Stakeholder B), and another senior leader with anti-submarine experience in the fleet (Stakeholder C). Again, the choice of these stakeholders is entirely notional, for demonstration purposes. The set of stakeholders may be larger, and perhaps different in realistic POM evaluations. We give weights to the decisions of the stakeholders as follows: subject-matter expert from The Office of Naval Research a weight of 0.5, the member of the Chief of Naval Operations staff a weight of 0.3, and the senior leader from the fleet a weight of 0.2. The stakeholders rank the alternatives based on four criteria: “detection range,” “detection reliability,” “maximum time on station,” and “offensive power against submarines.” Each stakeholder provides, for each criterion, a ratio-scale matrix representing pairwise comparisons between alternatives. An entry $r_{i,k}^j$ in a matrix indicates the extent alternative i is better (or

worse) than alternative k with respect to criterion j . Table 4.1, Table 4.2, and Table 4.3 present the ratio-scale matrices created by each stakeholder for Q (Anti-Submarine), the Projects that support the anti-submarine Program, with respect to the criterion detection range.

Table 4.1. The matrix $R^{\text{detection range}}$ (Stakeholder A)

Stakeholder A	MH-XX	DDG-XX	FFG-XX
MH-XX	1	4/9	4/5
DDG-XX	9/4	1	10/7
FFG-XX	5/4	7/10	1

Table 4.2. The matrix $R^{\text{detection range}}$ (Stakeholder B)

Stakeholder B	MH-XX	DDG-XX	FFG-XX
MH-XX	1	5/8	7/10
DDG-XX	8/5	1	4/3
FFG-XX	10/7	3/4	1

Table 4.3. The matrix $R^{\text{detection range}}$ (Stakeholder C)

Stakeholder C	MH-XX	DDG-XX	FFG-XX
MH-XX	1	3/4	11/10
DDG-XX	4/3	1	1
FFG-XX	10/11	1	1

To obtain a consensus rating of the alternatives, based on the stakeholders' inputs in Table 4.1, Table 4.2, and Table 4.3, we minimize the least squares to derive relative ratings of each alternative with respect to the criterion of detection range. $r_{i,k}^{\text{detection range}}(s)$ is the entry of the ratio-scale matrix made by stakeholder s that compares alternative i and k based on detection range. l_s^j is the weight decision-maker s has in the decision-making process with respect to criterion j . $v_i^{\text{detection range}}$ for $i =$

MH-XX, DDG-XX, FFG-XX, is the decision variable that denotes the consensus relative ratings of alternative i with respect to criterion j .

$$\begin{aligned} \text{Min } \sum_s \sum_{i,k} \left(\frac{v_i^{\text{detection range}}}{v_k^{\text{detection range}}} - r_{i,k}^{\text{detection range}}(s) \right)^2 * l_s^j \\ \text{s.t } \sum_{i \in Q(\text{Anti-Submarine})} v_i^{\text{detection range}} = 1 \\ v_i^{\text{detection range}} \geq 0 \forall i \end{aligned}$$

Using the ratios from the ratio-scale matrices, we get consensus relative ratings for the Projects in the Anti-Submarine Program, with respect to the criterion of detection range. The relative ratings are, $v_{\text{MH-XX}}^{\text{detection range}} = 0.24$, $v_{\text{DDG-XX}}^{\text{detection range}} = 0.44$, $v_{\text{FFG-XX}}^{\text{detection range}} = 0.32$. Similar ratio-scale matrices and calculations are performed for each of the other criteria in the anti-submarine Program with the only modifications being the pairwise comparisons given by the stakeholders. Table 4.4. shows the obtained consensus relative ratings of the three alternatives with respect to each criterion.

Table 4.4. The ratings with respect to criterion for the Anti-Submarine Program

	Detection Range	Detection Reliability	Max Time on Station	Offensive Power vs. Subs
MH-XX	0.24	0.61	0.05	0.39
DDG-XX	0.44	0.23	0.45	0.28
FFG-XX	0.32	0.16	0.50	0.33

Now that we have ratings with respect to each criterion we need the stakeholders to decide on the weights of each criterion. We do this in a similar way as we did with the same stakeholders making comparisons, but this time comparing the criteria themselves rather than the alternatives.

Table 4.5. The pairwise-comparison Matrix *D* of the criteria for the Projects in *P*(Anti-Submarine) (Stakeholder A)

Stakeholder A	Detection Range	Detection Reliability	Max Time on Station	Offensive Power vs. Subs
Detection Range	1	2/3	8/9	3/7
Detection Reliability	3/2	1	4/3	4/5
Max Time on Station	9/8	3/4	1	3/5
Offensive Power vs. Subs	7/3	5/4	5/3	1

Table 4.6. The pairwise-comparison Matrix *D* of the criteria for the Projects in *P*(Anti-Submarine) (Stakeholder B)

Stakeholder B	Detection Range	Detection Reliability	Max Time on Station	Offensive Power vs. Subs
Detection Range	1	4/9	5/6	3/7
Detection Reliability	9/4	1	1	3/5
Max Time on Station	6/5	1	1	2/3
Offensive Power vs. Subs	7/3	5/3	3/2	1

Table 4.7. The pairwise-comparison Matrix *D* of the criteria for the Projects in *P*(Anti-Submarine) (Stakeholder C)

Stakeholder C	Detection Range	Detection Reliability	Max Time on Station	Offensive Power vs. Subs
Detection Range	1	5/4	5/6	1/5
Detection Reliability	4/5	1	4/3	3/5
Max Time on Station	6/5	3/4	1	1/2
Offensive Power vs. Subs	5	5/3	2	1

We minimize the least squares regarding the pairwise comparisons of the criteria weights to derive the weights for each criterion

$$\begin{aligned} \text{Min } \sum_s \sum_{(r,j)} \left(\frac{w_h}{w_j} - d_{h,j}(s) \right)^2 * l_s \\ \text{s.t } \quad \sum_r w_j = 1 \\ w_j \geq 0 \forall r \end{aligned}$$

Using the ratios from the stakeholders pairwise comparisons we get the weight of detection range = 0.15, the weight of detection reliability = 0.25, the weight of max time on station = 0.21, and the weight of offensive power against submarines = 0.39.

Now that we have the relative ratings of the alternatives in $Q(\text{Anti-Submarine})$ with respect to each criterion and the weights of each criterion, we can calculate the overall rating of each alternative in $Q(\text{Anti-Submarine})$. We calculate the overall rating of each alternative in that Program by taking the weighted (by criteria weights) sum of the alternatives' ratings as follows:

$$\begin{aligned} \text{Overall Rating}(\text{MHXX} \in \text{Anti-Submarine}) \\ = \sum_j^n w_j * v_{MH-XX}^j = 0.15 * 0.24 + 0.25 * 0.61 + 0.21 * 0.05 + 0.39 * 0.39 = 0.3511 \end{aligned} \quad (14)$$

$$\begin{aligned} \text{Overall Rating}(\text{DDGXX} \in \text{Anti-Submarine}) \\ = \sum_j^n w_j * v_{DDG-XX}^j = 0.15 * 0.44 + 0.25 * 0.23 + 0.21 * 0.45 + 0.39 * 0.28 = 0.3272 \end{aligned} \quad (15)$$

$$\begin{aligned} \text{Overall Rating}(\text{FFGXX} \in \text{Anti-Submarine}) \\ = \sum_j^n w_j * v_{FFG-XX}^j = 0.15 * 0.32 + 0.25 * 0.16 + 0.21 * 0.50 + 0.39 * 0.33 = 0.3217 \end{aligned} \quad (16)$$

We repeat this process for the set of Thrusts, for every set of Program alternatives $P(i)$ where $i = \{\text{Prevail against China, Prevail against Russia}\}$, and for Project alternatives $Q(j)$ where $j = \{\text{Anti-Submarine, Anti-Air, Anti-Surface}\}$. The repetition of the MODA model for all of these sets gives use the overall rating of each alternative. It is important to note that the criteria and stakeholders may vary among sets of alternatives,

certain criteria may only apply to certain sets of alternatives, and different stakeholders may have more relevance to and/or knowledge of certain subject matters than others and therefore will be assigned different weights for different sets of alternatives and criteria. As mentioned earlier, the MODA model is applied to Thrusts and any set $P(i)$ and $Q(j)$ in a similar manner as we have applied it to $Q(\text{Anti-Submarine})$. Once the ratings have been obtained for all alternatives in all sets, we are ready to move on to the budgeting optimization models using the given ratings shown in Table 4.8, Table 4.9, and Table 4.10.

C. CARDINAL BUDGETING MODEL

For the cardinal budgeting model, we use the ratings directly from the MODA model. Tables 4.8–4.10 show the ratings for all alternatives. Note that a rating of zero indicates that the alternative is not in the set of alternatives that support that parent alternative.

Table 4.8. The ratings of each Thrust

Strategic Importance/Thrust	Prevail against China	Prevail against Russia
Strategic Importance	0.6	0.4

Table 4.9. The ratings of Programs in regard to each Thrust

Programs/Thrusts	Prevail against China	Prevail against Russia
Anti-Submarine	0.2	0.7
Anti-Air	0.3	0.2
Anti-Surface	0.5	0.1

Table 4.10. The ratings of Projects in regard to each Program

Projects/Programs	Anti-Submarine	Anti-Air	Anti-Surface
MH-XX	0.35	0	0
DD-XX	0.33	0.29	0.25
FFG-XX	0.32	0.09	0.15
F/A-XX	0	0.62	0.24
SSN-XX	0	0	0.36

For the cardinal budgeting model, we need to calculate the overall value of each Project at the lowest tier. Note that this overall value is affected by its rating(s) in the sets of the Program(s) in which it belongs, the relative value of those Programs, as well as the relative value(s) of the Thrusts to which those Programs belong. The ratings of the of the Thrusts are determined directly from the MODA model by the top-tier stakeholders. The overall value of a Program depends on how much relative value it provides to each Thrust to which it belongs to, and the overall values of the corresponding Thrusts. We denote z_i as the rating of Thrust i , $u_{i,j}$ as the rating of Program j in regards to Thrust i , and u_j^* the overall value of Program j . We demonstrate how to calculate the overall value of Programs with examples for both $u_{\text{Anti-Air}}^*$ and $u_{\text{Anti-Submarine}}^*$. Note that the Programs have different ratings in the sets corresponding to the two parent Thrusts. The overall values of Programs are a weighted sum of those two ratings, weighted by the ratings of the two corresponding Thrusts. Equations 17 and 19 are the general formula for $u_{\text{Anti-Air}}^*$ and $u_{\text{Anti-Submarine}}^*$, and equations 18 and 20 apply the ratings from the table to the general formula.

$$u_{\text{Anti-Air}}^* = z_{\text{Prevail against China}} * u_{\text{Prevail against China, Anti-Air}} + z_{\text{Prevail against Russia}} * u_{\text{Prevail against Russia, Anti-Air}} \quad (17)$$

$$u_{\text{Anti-Air}}^* = 0.6 * 0.3 + 0.4 * 0.2 = 0.26 \quad (18)$$

$$u_{\text{Anti-Surface}}^* = z_{\text{Prevail against China}} * u_{\text{Prevail against China, Anti-Surface}} + z_{\text{Prevail against Russia}} * u_{\text{Prevail against Russia, Anti-Surface}} \quad (19)$$

$$u_{\text{Anti-Surface}}^* = 0.6 * 0.5 + 0.4 * 0.1 = 0.34 \quad (20)$$

Now that we demonstrated how to calculate the overall value of Programs we perform a similar calculation to calculate v_k^* , the overall value of Project k , using $v_{j,k}$, the rating of Project k in regards to Program j , and the previously calculated u_j^* . We demonstrate this calculation of the overall value for Project F/A-XX which belongs to two programs: Anti-Air and Anti-Surface. Equation 21 is the general formula for $v_{\text{F/A-XX}}^*$, and equation 22 applies the ratings from the tables derived in previous steps.

$$v_{F/A-XX}^* = u_{\text{Anti-Air}}^* * v_{\text{Anti-Air},F/A-XX} + u_{\text{Anti-Surface}}^* * v_{\text{Anti-Surface},F/A-XX} \quad (21)$$

$$v_{F/A-XX}^* = 0.26 * 0.62 + 0.34 * 0.24 = 0.2428 \quad (22)$$

Now that we have shown how to calculate the overall value of Programs and Projects we will provide the remaining overall values of the other Programs and Projects in Table 4.11 and Table 4.12 respectively.

Table 4.11. Overall value of Programs

Program	Value
Anti-Submarine	0.40
Anti-Air	0.26
Anti-Surface	0.34

Table 4.12. Overall value of Projects

Project	Value
MH-XX	0.1400
DDG-XX	0.2924
FFG-XX	0.2024
F/A-XX	0.2428
SSN-XX	0.1224

We now input the overall value of each Project into the optimization model to decide which Projects to fund. For this example, we set the budget to four hundred million dollars. While not directly shown in this thesis, the optimization model code we used, based on the formulation in Chapter III.B, maximizes overall value while staying within the budget, and we decide to fund MH-XX, DDG-XX, and FFG-XX.

D. ORDINAL BUDGETING MODEL

For the ordinal budgeting model, we start by taking the ratings from the MODA model and turning them into ordinal rankings. Cardinal and ordinal ratings are shown for the same alternatives in Table 4.13, Table 4.14, and Table 4.15.

Table 4.13. Cardinal ratings and ordinal ranks of each Thrust

Strategic Importance/Thrust	Type	Prevail against China	Prevail against Russia
Strategic Importance	Cardinal	0.6	0.4
	Ordinal	1	2

Table 4.14. Cardinal ratings and ordinal ranks of each Program

Programs/Thrusts	Type	Prevail against China	Prevail against Russia
Anti-Submarine	Cardinal	0.2	0.7
	Ordinal	3	1
Anti-Air	Cardinal	0.3	0.2
	Ordinal	2	2
Anti-Surface	Cardinal	0.5	0.1
	Ordinal	1	3

Table 4.15. Cardinal rating and ordinal rank of each Project

Projects/Programs	Type	Anti-Submarine	Anti-Air	Anti-Surface
MH-XX	Cardinal	0.35	0	0
	Ordinal	1	n/a	n/a
DD-XX	Cardinal	0.33	0.29	0.25
	Ordinal	2	2	2
FFG-XX	Cardinal	0.32	0.09	0.15
	Ordinal	3	3	4
F/A-XX	Cardinal	0	0.62	0.24
	Ordinal	n/a	1	3
SSN-XX	Cardinal	0	0	0.36
	Ordinal	n/a	n/a	1

The only inputs required for the ordinal model other than the ordinal rankings of the alternatives are constants α , δ , and τ that serve as required buffers between the ratings of the alternatives at the Project, Program, and Thrust tiers respectively. For this example, we set $\alpha = 0.0001$, $\delta = 0.001$, and $\tau = 0.01$. Note that depending on the ordinal rankings, choosing too large α , δ , and τ constants will render the optimization problem infeasible. The ordinal optimization model takes the ordinal rankings and creates decision variables

associated with the ratings of each alternative. The decision variable ratings are constrained to be at least α , δ , or τ greater than the alternatives ranked lower in the set, and for the value of alternatives to be conserved between tiers.

To demonstrate how we constrain the ordinal budgeting problem we examine the constraints to the ratings of Projects in the set $Q(\text{Anti-Surface})$. The Projects ranked from highest to lowest importance in $Q(\text{Anti-Surface})$ are: SSN-XX, DDG-XX, F/A-XX, FFG-XX. This induces constraints on the ordinal budgeting model,

$$v_{\text{Anti-Surface,SSN-XX}} - v_{\text{Anti-Surface,DDG-XX}} \geq \alpha \quad (23)$$

$$v_{\text{Anti-Surface,DDG-XX}} - v_{\text{Anti-Surface,F/A-XX}} \geq \alpha \quad (24)$$

$$v_{\text{Anti-Surface,F/A-XX}} - v_{\text{Anti-Surface,FFG-XX}} \geq \alpha \quad (25)$$

The constraints shown for $Q(\text{Anti-Surface})$ in equation 23, equation 24, and equation 25 maintain that the relative value for each Project in $Q(\text{Anti-Surface})$ are at least α different from each other. We use α for the set of Projects in $Q(\text{Anti-Surface})$ as α is the threshold for relative value between Projects. For the set of Thrusts we use the constant τ as the threshold, and for each set of Programs we use the constant δ as the threshold. In addition to the previous constraints, we also impose that the ratings of $v_{j,k}$ to be between zero and one for all Projects k . We also include constraints to conserve value. We constrain the sum of the ratings of the Projects in $Q(j)$ to be equal to the sum of the ratings of Program j in regards to each Thrust that Program j supports and the sum of the ratings of all Programs in $P(i)$ to be equal to the rating of Thrust i . That is $\sum_{k \in Q(j)} v_{j,k} - \sum_{i=1}^I u_{i,j} = 0$, and $\sum_{j \in P(i)} u_{i,j} - w_i = 0$. Once ratings are assigned at all tiers the model calculates the overall value of each Project, V_k by summing all of the ratings assigned in regards to Programs, $v_{i,j}$. Along with the constraints, costs are entered into the ordinal budgeting model formulated in Chapter III and using an optimization algorithm solver we find that the Projects to fund that maximize the overall value are: MH-XX, DDG-XX, and FFG-XX. Again, while the code is not directly shown in this thesis, we use the formulation found in Chapter III.C

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V. CONCLUSION

A. SUMMARY

We have developed a method for aiding decision-makers in evaluating and budgeting Projects in the U.S. Navy's Program Objective Memorandum (POM). We started with the MODA model to take multiple stakeholder inputs on the rating of alternatives relative to a parent alternative based on specific criteria and derived overall ratings for each alternative in regards to their parent alternatives. With the overall ratings we applied either the cardinal or ordinal models to aid decision-makers in determining which Projects to fund. The cardinal method used the relative values obtained from the MODA stage to directly calculate the overall value of each alternative, and an optimization model maximizes the total overall value by deciding which Projects to fund. The ordinal method applied a similar optimization concept but relaxed the problem by switching the cardinal overall ratings into rankings that became decision variables in the optimization problem.

B. FUTURE WORK

There is the opportunity for additional work to improve decision-making in the Navy's POM. A problem for the Navy is synchronizing both existing and future requirements in the decision-making process. Specifically accounting for funding that is given without any immediate impact but potential impact in the future, or funding that has high immediate impact that degrades with time. Another aspect to consider is a sensitivity analysis of both the MODA stage and the budgeting optimization stage. Some specific tuning parameters that can be analyzed are modifying inputs such as each stakeholder's weight in the decision-making process, and the α , δ , and τ values in the ordinal budgeting model

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