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# THE EQUIVALENT SELECTIVITY OF TRANSFORMER-COUPLED LOOP-ANTENNA INPUT CIRCUITS

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# THE EQUIVALENT SELECTIVITY OF TRANSFORMER-COUPLED LOOP-ANTENNA INPUT CIRCUITS

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May 13, 1949

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### ABSTRACT

The over-all or equivalent selectivity of the simple transformer-coupled loop-antenna circuit is considered. From the analysis, design criteria are obtained. Graphs of equivalent circuit  $Q$  are so presented that intermediate design values can be determined by interpolation. Conclusions are that the net over-all circuit  $Q$  may be somewhat higher than that of the loop antenna for particular cases of elements with high  $Q$  values or for low values of transformer coupling. The magnitude of the over-all net  $Q$  is, in large measure, dependent on the  $Q$  values of the three elements in the circuit (loop, transformer primary, and transformer secondary). The relative changes in the input circuit selectivity with variation in the  $Q$  value of each circuit element, the influence of transformer coupling, and the effect of primary to loop-inductance ratio are discussed.

### PROBLEM STATUS

This report completes one phase of the problem; work on other phases is continuing. Criteria determining optimum loop inductance will be considered in detail in a subsequent report.

### AUTHORIZATION

NRL Problem No. R10-43R (BuShips Problem S1083.1).

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## UNITS AND SYMBOLS

(Throughout this report, inductances are expressed in henries, capacitances in farads, and resistances and reactances in ohms.)

- $C$  = Capacitance of the variable (tuning) capacitor.
- $L_o$  = Inductance of the loop antenna alone.
- $L_p$  = Transformer primary inductance alone.
- $L_2$  = Transformer secondary inductance alone.
- $L$  = Transformer secondary inductance with the primary circuit connected.
- $k$  = Transformer coefficient of coupling.
- $r_o$  = Resistance of the loop antenna alone.
- $r_p$  = Resistance of the transformer primary winding alone.
- $r_2$  = Resistance of the transformer secondary winding alone.
- $R$  = Resistance of the secondary circuit with the primary circuit connected.
- $Q_o$  = "Figure of merit" of the loop antenna alone.
- $Q_p$  = "Figure of merit" of the transformer primary winding alone.
- $Q_2$  = "Figure of merit" of the transformer secondary winding alone.
- $Q_x$  = "Figure of merit" of the transformer secondary circuit with the primary circuit connected.
- $\omega$  = Angular velocity:  $2\pi$  times frequency.
- $f_r$  = Frequency at which the circuit is resonant.
- $f_1$  = Lower frequency at which the power dissipated in the circuit is half the value at resonance.
- $f_2$  = Higher frequency at which the power dissipated in the circuit is half the value at resonance.

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## THE EQUIVALENT SELECTIVITY OF TRANSFORMER-COUPLED LOOP-ANTENNA INPUT CIRCUITS

### INTRODUCTION

Effective underwater reception generally involves the collection of VLF radio-frequency energy in a loop antenna which is properly transformer-coupled to a receiver. The criteria for optimum voltage amplification and signal-to-noise ratio for such circuits are given in an earlier report.<sup>1</sup> That report, however, does not directly concern itself with the selectivity of such circuitry.

A properly designed transformer-coupled loop-antenna input circuit with its secondary branch resonated by a suitable variable condenser is practically identical to a single-tuned circuit of variable frequency with one mode of oscillation. The effective or net Q of such a circuit is essentially a direct criterion of its selectivity against undesired adjacent-channel signals, its bandwidth, and its rise and decay time characteristics under modulated signal conditions. Knowledge of the input circuit Q value is therefore a necessary factor in the design of a complete receiving system, since it affects performance as regards cross-modulation, signal-to-noise ratio and signal-to-noise ratio depression, one-signal selectivity, the tracking of the cascaded selective circuits, and the maximum rate of keying or modulation to which the receiver will respond satisfactorily.

The material in this report has been prepared as an aid to the easy and rapid determination of the net or equivalent Q of the transformer-coupled-type loop-antenna input circuit as it appears at the transformer secondary terminals. The bandwidth of the input circuit at the grid of the first tube at 3 decibels below maximum response will be <sup>2</sup>

$$\left( f_2 - f_1 \right)_{3 \text{ db}} = \frac{f_r}{Q_x} .$$

<sup>1</sup> Fratianni, S. V., "Theory and Design of Resonant Transformer-Coupled Loop-Antenna Input Systems for VLF Reception," (Restricted), NRL Report R-3281, April 28, 1948.

<sup>2</sup> For a list of units and symbols, see page vi.

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At 10 decibels below maximum response

DECLASSIFIED  $(f_2 - f_1)_{10 \text{ db}} = \frac{3 f_r}{Q_x}$

The time for decay of a transient oscillation by 40 decibels in the circuit will be

$$t_{40} = \frac{1.46 Q_x + 1}{f_r}$$

#### DETERMINATION OF TRANSFORMER-COUPLED LOOP-CIRCUIT EQUIVALENT OR NET Q

If a loop antenna is connected to the primary winding of a suitable coupling transformer and it is assumed that the individual Q's of the circuit inductive elements are 10 or greater, it can be determined from Figure 1 (where circuit (B) is the equivalent of circuit (A)<sup>3,4</sup>) that the

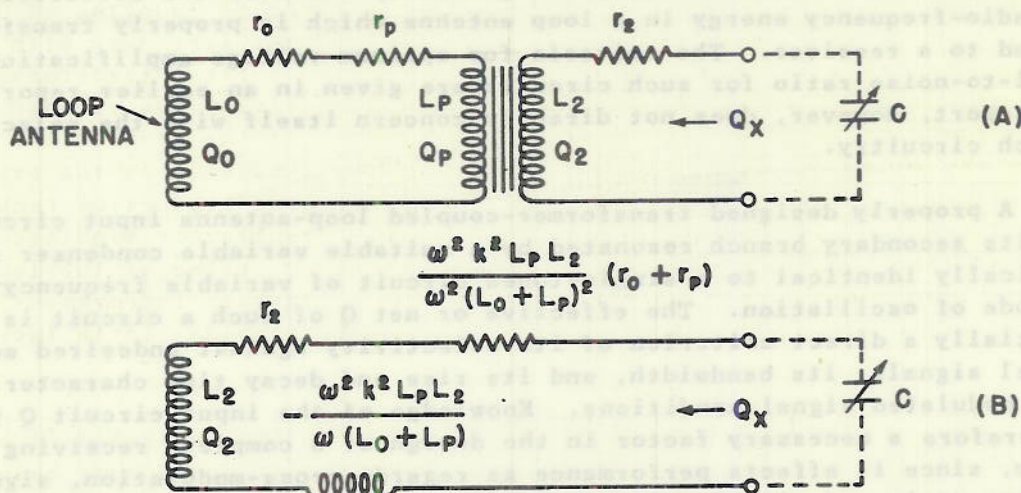


Figure 1- Transformer-coupled loop-antenna circuit

net or effective Q ratio ( $Q_x/Q_0$ ) can be expressed as indicated by equation (6)<sup>5</sup>:

$$\frac{Q_x}{Q_0} = \frac{Q_2/Q_0 \left[ 1 - k^2 \left( \frac{1}{1+L_0/L_p} \right) \right]}{1+k^2 \left[ \frac{Q_2}{Q_0} \left( \frac{1}{1+L_0/L_p} \right) \left( \frac{1}{1+L_p/L_0} \right) + \frac{Q_2}{Q_p} \left( \frac{1}{1+L_0/L_p} \right)^2 \right]} \quad (6)$$

<sup>3</sup> Terman, F. E., "Radio Engineer's Handbook," McGraw-Hill, 1943, p. 150.

<sup>4</sup> Fratianni, op. cit., p. 17.

<sup>5</sup> For derivation of the equations, see Appendix.



This equation is simplified in the special case when the ratio  $L_o/L_p$  equals unity.  $L_p$  often can be assumed to equal  $L_o$  as an aid in the quick and approximate evaluation of the net  $Q_x$ . For instance, this inductance ratio can vary from approximately 0.5 to 1.1 and still not greatly affect the circuit signal-to-noise factor.<sup>6</sup> For the special case of  $L_o/L_p = 1$ , equation (6) becomes

$$\frac{Q_x}{Q_o} = \frac{Q_2/Q_o \left(1 - k^2/2\right)}{1 + \frac{1}{2}k^2 \left(\frac{Q_2}{Q_o} + \frac{Q_2}{Q_p}\right)} \quad (7)$$

The usefulness of equation (7) is perhaps best illustrated by an example. Assume a loop antenna with a  $Q_o$  of approximately 30 at 15 kc. If  $Q$  values of 100 and 50 are obtained for the transformer secondary and primary windings, respectively, with a transformer coefficient of coupling of 0.9 with the loop disconnected, the net  $Q_x$  looking into the secondary with the loop connected will be 28.6. This indicates that no further transformer  $Q$  improvement is profitable, because the circuit selectivity has reached the limit of the loop antenna  $Q$ . Further  $Q$  improvement of the transformer can increase the over-all circuit selectivity slightly, but only at the expense of a considerably larger transformer and more stringent production requirements. Therefore, if reasonable  $Q$  values for the transformer windings are chosen, so that the net over-all  $Q_x$  is close to the  $Q_o$  value of the loop antenna, the design will usually be satisfactory. Further improvement in the transformer  $Q$ 's in the above example would also provide little increase in over-all receiver sensitivity.

For illustration of the effect on  $Q_x$  caused by the variation of element  $Q$  in the coupling network, the curves of Figures 2 and 3 have been plotted. These graphs show particular cases derived from equation (7).

Figure 2 indicates that improvement of  $Q_p$  or  $Q_o$  for a fixed  $Q_2$  value of 100 will provide significantly higher values of  $Q_x$  up to a  $Q_p$  of about 100. For values of  $Q_p$  greater than 100, the rate of improvement in  $Q_x$  is small, thereby yielding small return for the larger physical size of the primary winding demanded by the greater values of  $Q_p$ . This graph indicates that up to a certain point, the ability of  $Q_x$  to increase with an increase in value of  $Q_p$  (for a constant value of  $Q_2$ ) is limited by the value of  $Q_o$ . Hence, to effect significant improvement in this particular system beyond the point at which the  $Q_p$  value is equal to 100, the value of  $Q_o$  must be increased. It is interesting to note that if  $Q_o$  and  $Q_p$  are interchanged (i.e., if  $Q_p$  is fixed at 20, 30, and 100 and if  $Q_o$  is the variable of the ordinate scale), the curves will be unaltered. The curves then show the variations of net  $Q$  with changes in the value of  $Q_o$ . For increased values of  $Q_o$  up to about 100, the improvement in the net  $Q$  value will be rapid and profitable. For further increase in the net  $Q$  value,  $Q_p$  should be increased.

<sup>6</sup> Fratianni, loc. cit.



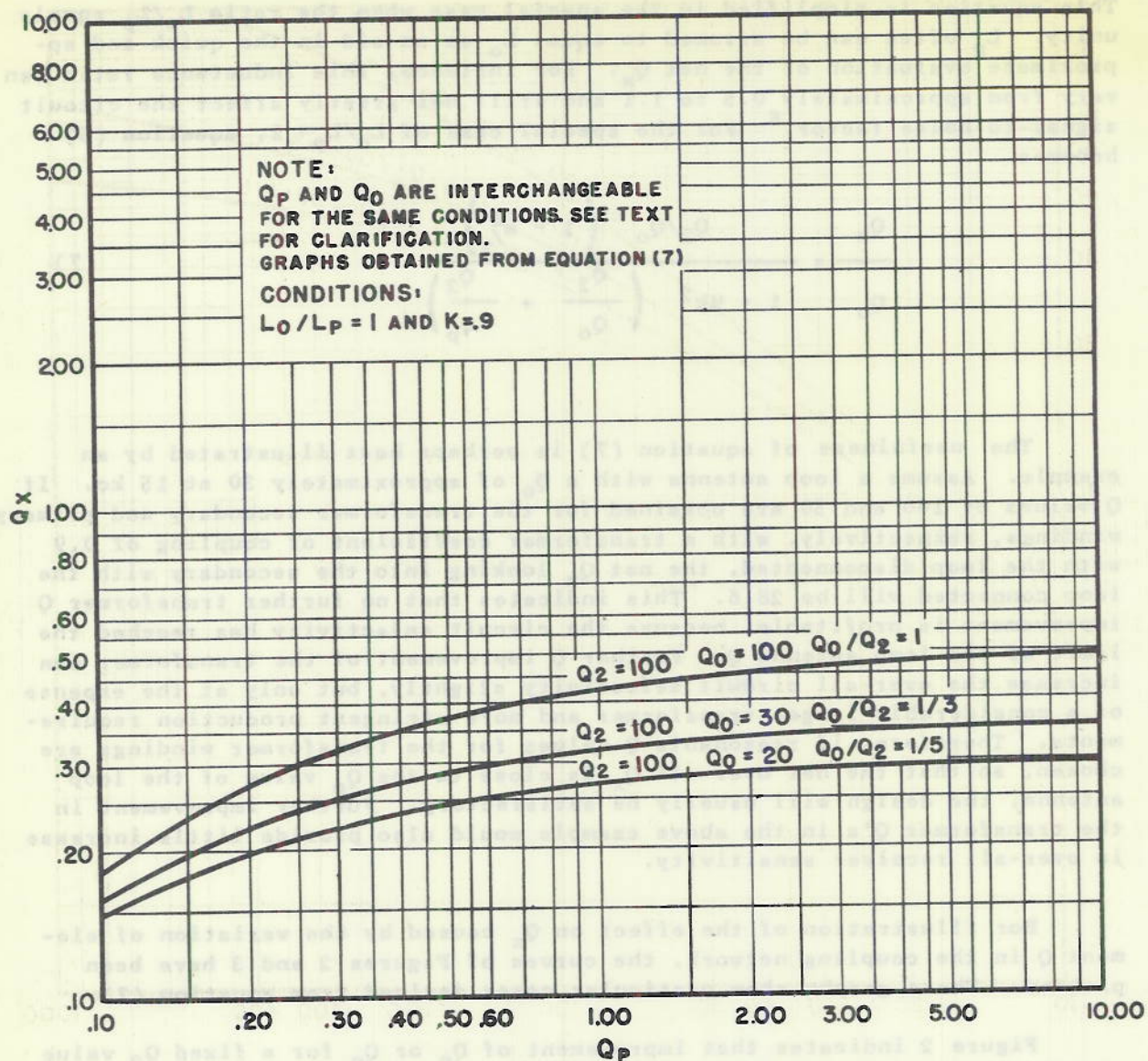


Figure 2- Variation of net transformer secondary  $Q_x$  with changes of transformer primary  $Q_p$

Figure 3 shows the improvement obtained in  $Q_x$  for increasing values of  $Q_2$  with a constant  $Q_p$  value of 100. It is evident from this graph that for low values of  $Q_2$  a large change in  $Q_0$  (from 20 to 100) has a very small effect on  $Q_x$ , while a small improvement in  $Q_2$  yields a considerable increase in  $Q_x$ . This effect is less, however, when  $Q_2$  reaches the vicinity of 100. In this region, the improvement in  $Q_x$  obtained by increasing either the  $Q_2$  or  $Q_0$  values is more nearly the same. Increase of  $Q_2$  to values over 100 is not profitable unless  $Q_0$  approaches a value of 100 or higher.

#### Q DESIGN CRITERIA FOR LOOP-ANTENNA TRANSFORMER-COUPLED CIRCUITS

To aid in quick determination of the value of the net  $Q_x$  for various designs, equation (6) has been plotted as shown in Figures 4, 5, 6 and 7.



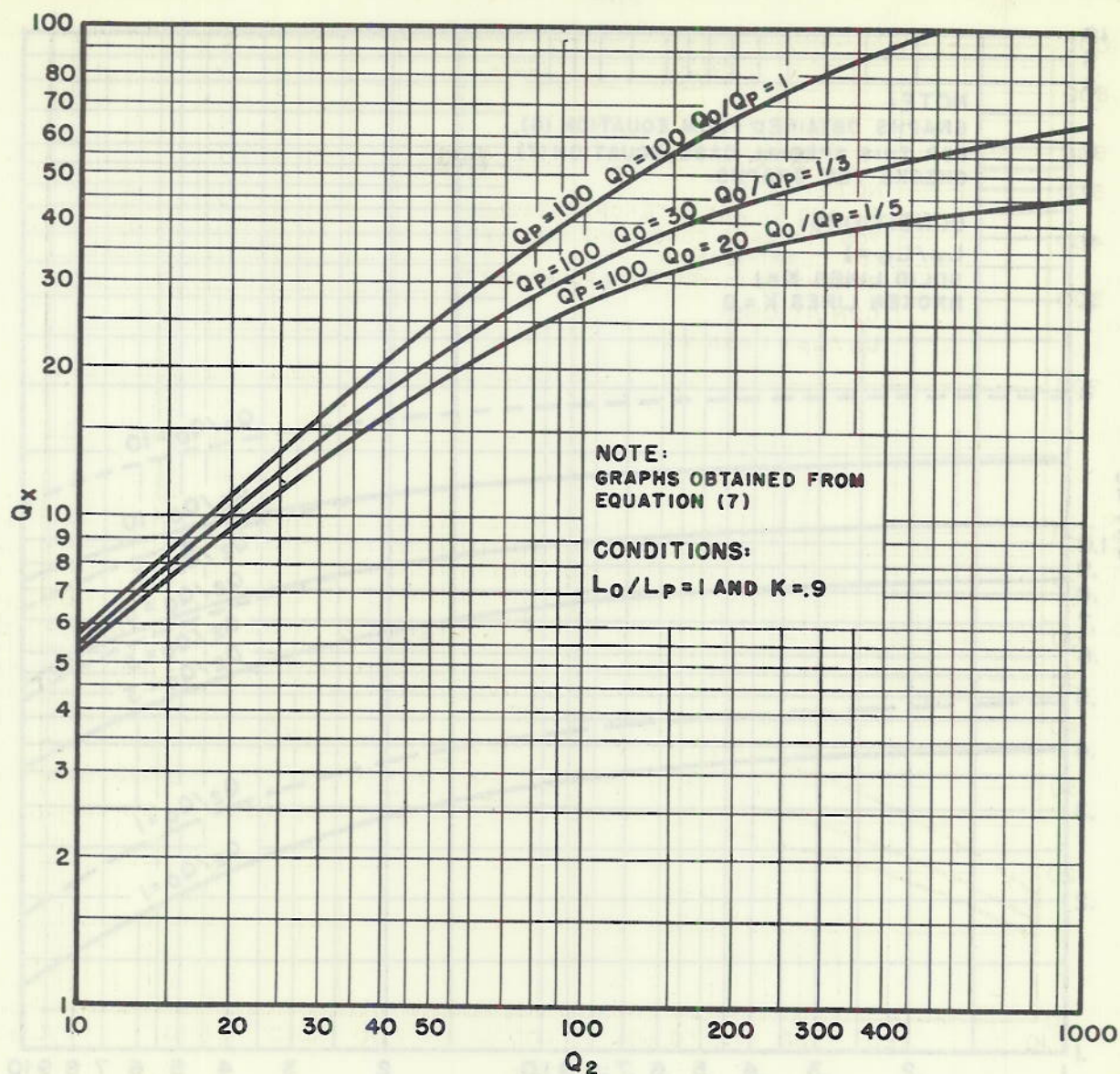


Figure 3- Variation of net transformer secondary  $Q_x$  with changes of transformer secondary  $Q_2$

These graphs provide information for the cases that have parameters ( $Q$  and  $K$  values) as given in the plots. Enough progressive values are shown so that the  $Q$  information for any intermediate values can be determined with reasonable accuracy by interpolation. The graphs show the variation of the net circuit  $Q$  ratio,  $Q_x/Q_0$ , with respect to the transformer  $Q$  ratio,  $Q_2/Q_p$ , for  $L_p/L_0 = 1, 0.8, 0.6$ , and  $0.4$  (Figures 4, 5, 6 and 7 respectively).

In Figure 4 (and similarly in the other graphs), it can be seen that, as the transformer coefficient of coupling is reduced, the net circuit selectivity,  $Q_x$ , will increase. Also, if the transformer  $Q$  values are increased while a constant value of loop-antenna  $Q_0$  is maintained, the value of  $Q_x$  increases. It is also evident that it is possible to have a somewhat



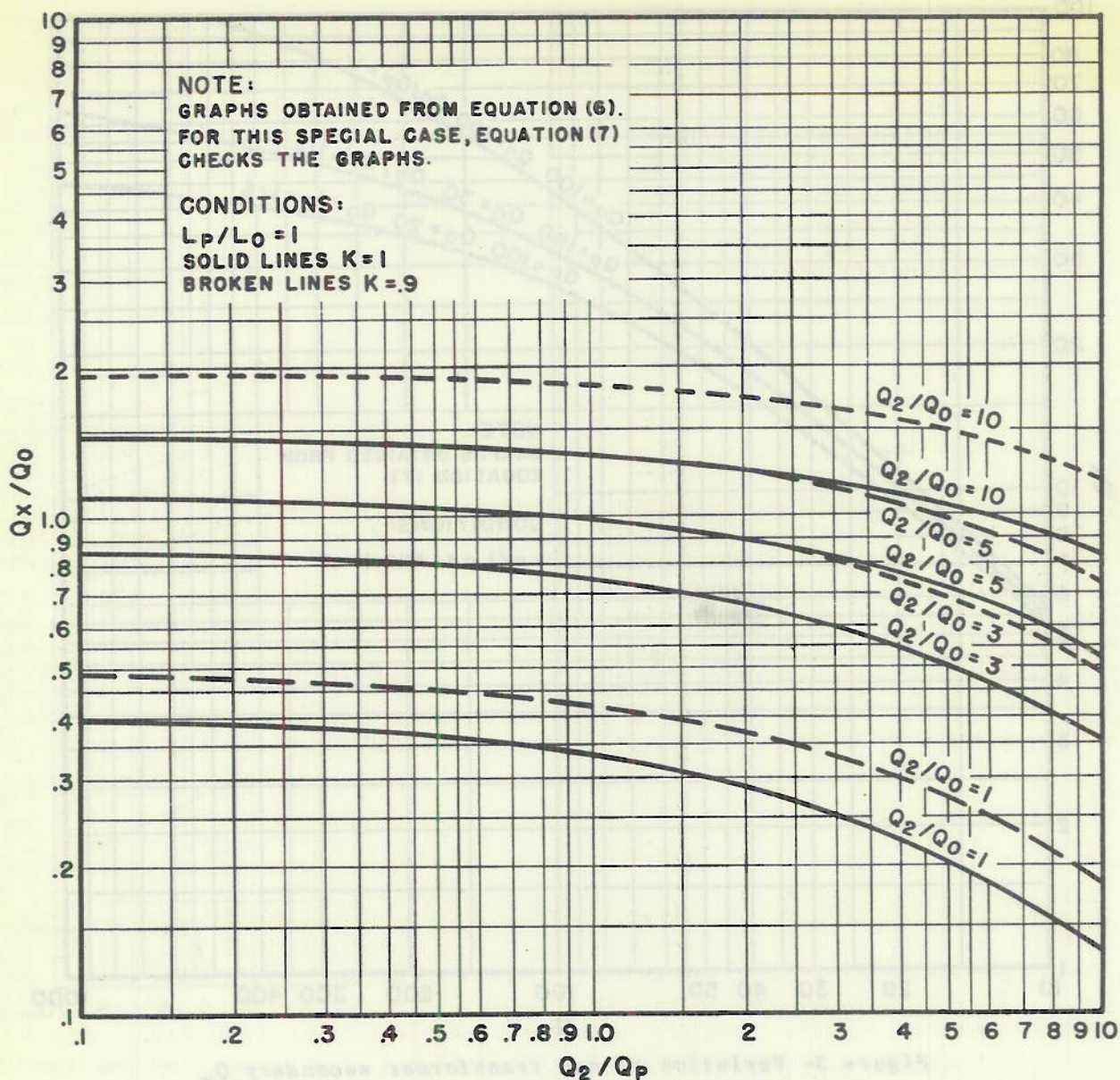


Figure 4- Variation of the ratio  $Q_x/Q_o$  with changes in the ratio  $Q_2/Q_p$  for a primary inductance ratio of 1

greater net circuit  $Q$  than that of the loop-antenna circuit alone. To obtain this condition, however, the transformer  $Q$ 's generally have to be rather high, which usually necessitates large dimensions in the transformer. For example, if the loop  $Q_o$  is 50 and  $L_p/L_o = 1$ , with a transformer  $K$  value of unity, the primary and secondary  $Q$ 's of the transformer would have to be more than 5 times the loop  $Q$ , or over 250, to produce a value of  $Q_x$  which is over 50. For transformer  $K$  values slightly less than unity, the primary and secondary  $Q$ 's of this transformer could be somewhat less than 250, but from the standpoint of size and weight of the transformer, these  $Q$ 's would still be too great for most practical cases.



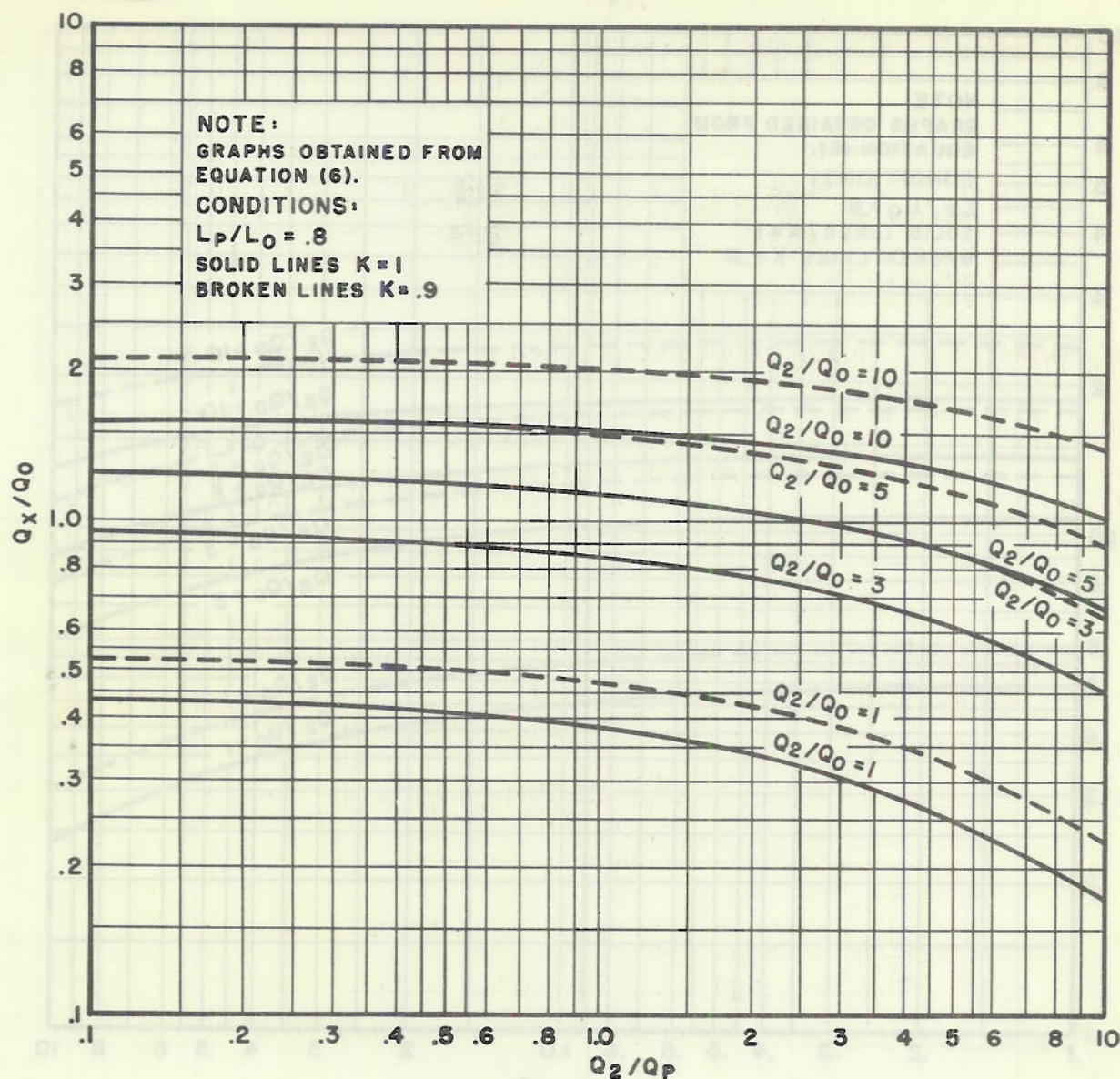


Figure 5- Variation of the ratio  $Q_x/Q_o$  with changes in the ratio  $Q_2/Q_p$  for a primary inductance ratio of .8

Although the graphs of Figures 4 through 7 provide enough information for Q design criteria of loop-antenna transformer-coupled circuits, they do not indicate at a glance the change in net Q with the change in Q value of each separate element of the circuit. This information is provided more clearly in Figure 8 for the special case of  $L_p/L_o = 1$  and  $K=1$ . The graph is derived from equation (7) by setting  $Q_p$  equal to  $4Q_o$ ,  $3Q_o$ ,  $2Q_o$ ,  $Q_o$ ,  $1/2Q_o$ ,  $1/3Q_o$  and  $1/4Q_o$  and is so arranged that all the Q's involved are presented with respect to a common denominator  $Q_o$ . The graph shows the variation of the ratio  $Q_x/Q_o$  with changes in the ratio  $Q_2/Q_o$  for the various constant values of  $Q_p/Q_o$ . The curves indicate that increasing the Q value of any transformer



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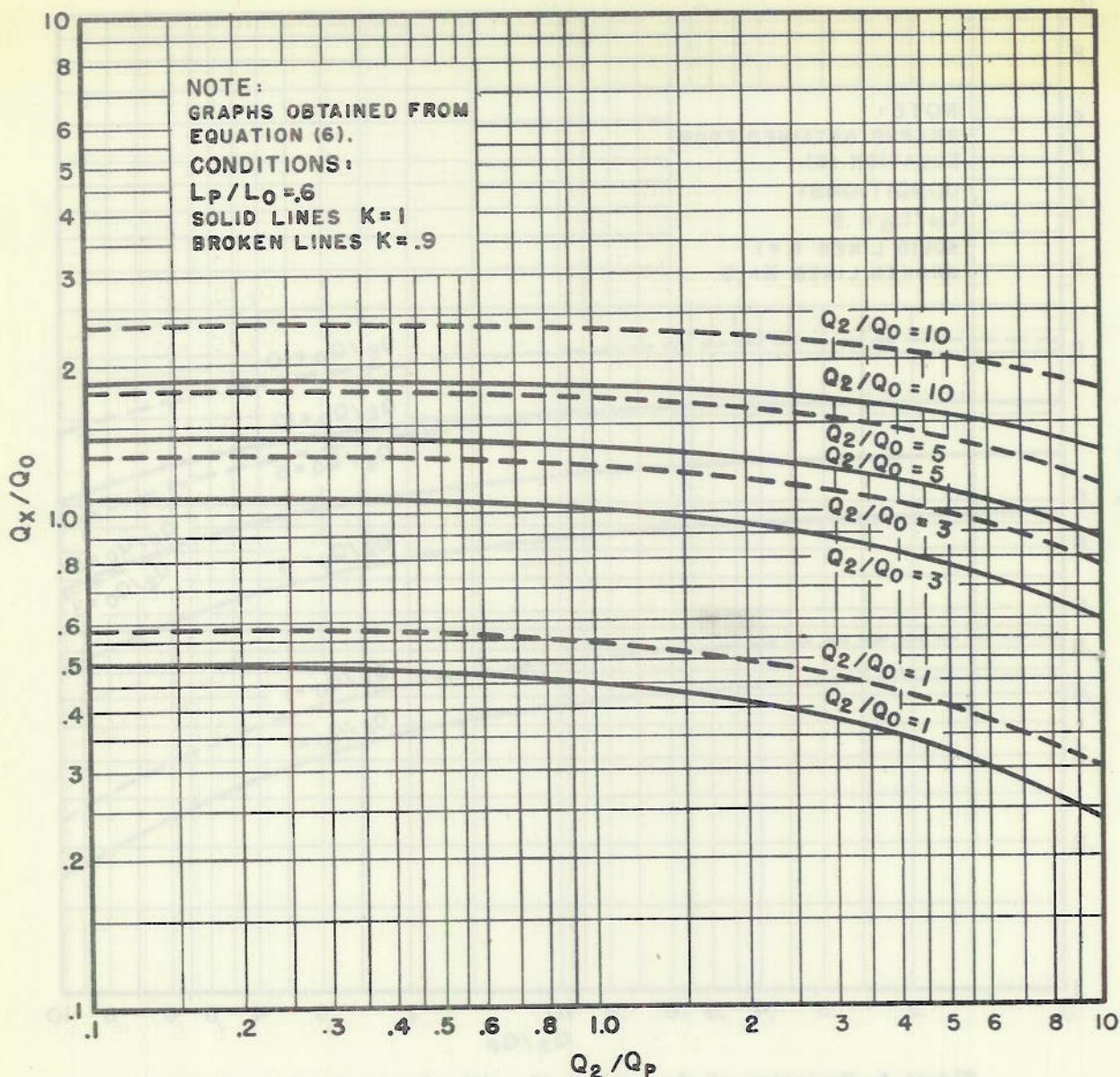


Figure 6- Variation of the ratio  $Q_x/Q_0$  with changes in the ratio  $Q_2/Q_p$  for a primary inductance ratio of .6

element will result in a higher value of net  $Q$ . It can be seen from this graph that for the high values of  $Q_2/Q_0$ , the family of curves begins to flatten out, particularly for the cases of  $Q_p/Q_0$  less than unity. This indicates that in this portion of the graph, further increase of the  $Q_2$  value will result in only small increase in the value of net  $Q$ , whereas an increase in the transformer primary  $Q_p$  value will provide a large increase in  $Q_x$  value. Also, for small values of  $Q_2/Q_0$ , the slopes of the curves become greater and the spacing between the curves becomes smaller. This indicates that, in this portion of the graph, increasing the  $Q_2$  value is profitable and that increasing the  $Q_p$  value has much less effect on the net  $Q$ . Therefore, at a point somewhere between these extremes (approximately where the

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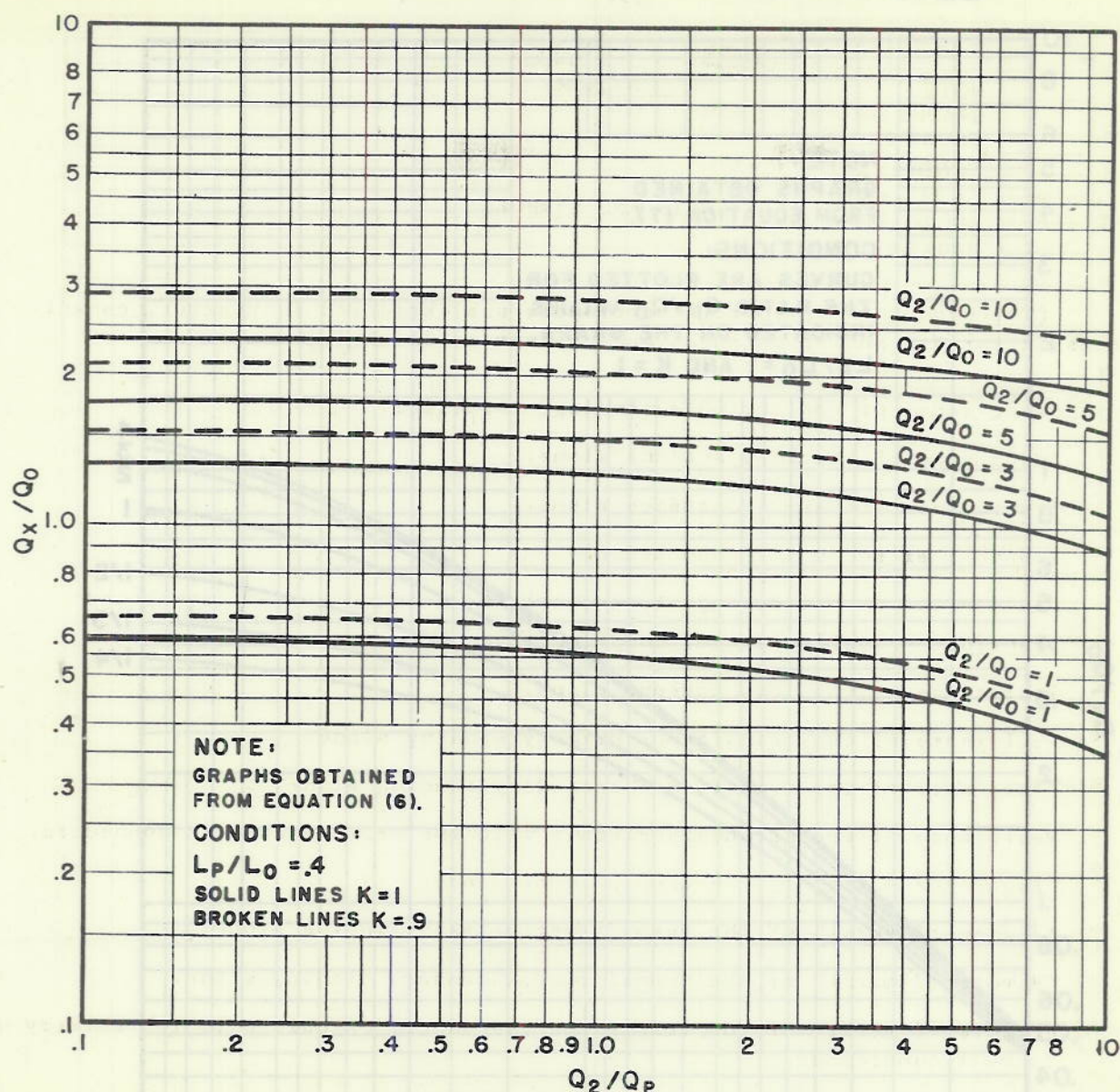


Figure 7- Variation of the ratio  $Q_x/Q_0$  with changes in the ratio  $Q_2/Q_p$  for a primary inductance ratio of .4

ratio  $Q_2/Q_0 = 2$  and  $Q_p/Q_0 = 1$ ), the effect on net  $Q$  will be equal when either  $Q_2$  or  $Q_p$  values are slightly increased or decreased. Considering that normally every effort is made in loop-transformer design to have the ratio  $Q_p/Q_0$  equal to at least unity, the curves indicate that improvement in  $Q_2$  value will be effective in increasing  $Q_x$  for almost any initial value of  $Q_2$ .

#### CONCLUSIONS

The selectivity of a resonant transformer-coupled loop-antenna input system, as represented by net  $Q$  ( $= Q_x$ ), is dependent on the relative  $Q$



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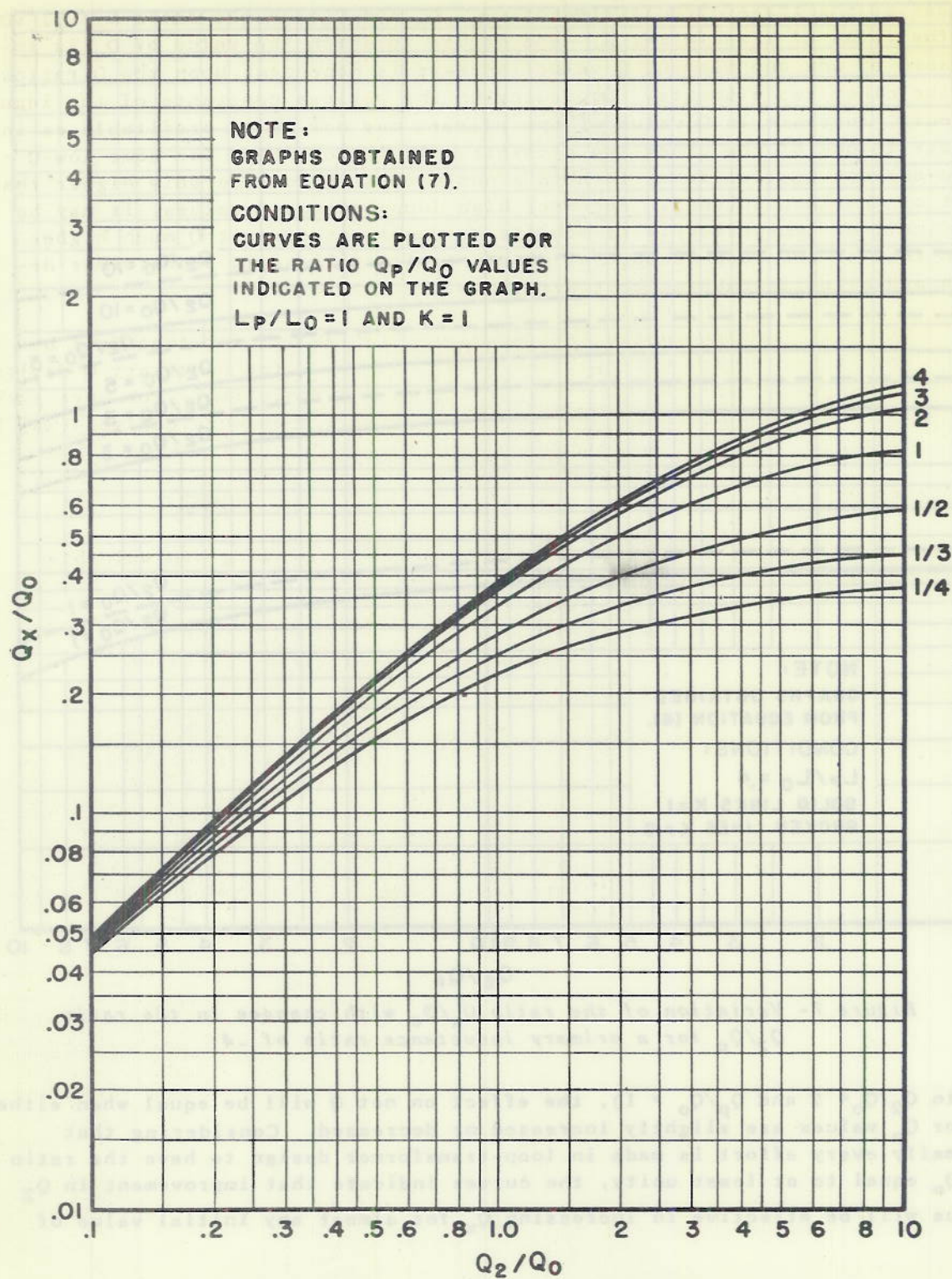


Figure 8- Variation of the ratio  $Q_x/Q_0$  with changes in the ratio  $Q_2/Q_0$

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values of the three circuit elements  $Q_o$ ,  $Q_p$  and  $Q_2$ ; on the ratio  $L_p/L_o$ ; and on the value of transformer K. The degree to which the value of  $Q_x$  is increased by any one element Q value increase is dependent upon the Q ratios of the other two elements. Depending on the various constants of the input circuit, increase in Q value of one element may not be as profitable as increase in one of the other two elements. It is possible, for some low-Q loop-antenna applications, to have a net circuit Q considerably higher than that of the loop antenna. For very high loop-antenna Q values, it may be difficult or impracticable to achieve or use a net circuit Q much higher than that of the loop because of practical limitations in transformer design and circuit-tracking considerations.

For transformer-coupled loop systems with values of Q that fall between the values shown in the curves of this report, the necessary Q design criteria can be determined from the graphs by interpolation. For more exact values, the equations themselves (from which the graphs were derived) may be used instead.

#### ACKNOWLEDGMENT

Acknowledgment is made to Mr. Emerick Toth who has given valuable suggestions and comments throughout this work.

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values of the three circuit elements  $R$ ,  $C$ , and  $L$  on the ratio  $I_0/I_1$  and on the value of transformer  $K$ . The degree to which the value of  $I_0$  is increased by any one element  $Q$  value increase is dependent upon the  $Q$  value of the other two elements. Depending on the various constants of the input circuit, increase in  $Q$  value of one element may not be as profitable as in other elements. It is possible, for some low- $Q$  circuits, to have a net circuit  $Q$  considerably higher than that of the loop antenna. For very high loop-antenna  $Q$  values, it may be difficult or impracticable to achieve or use a net circuit  $Q$  much higher than that of the loop antenna of practical limitations in transformer ratio and circuit-breaking considerations.

For transformer-coupled loop system with values of  $Q$  that fall between the values shown in the curves of this report, the necessary  $Q$  design criteria can be determined from the graphs by interpolation. For more exact values, the equations themselves (from which the graphs were derived) may be used instead.

ACKNOWLEDGMENT

Acknowledgment is made to Mr. Eustice Fort and his given valuable suggestions and comments throughout this work.

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## APPENDIX

Determination of the Ratio of  
Net Secondary Circuit  $Q_x$  to Loop-Antenna  $Q_o$

The "Figure of Merit" ( $Q$ ) of a resonant circuit is usually expressed as the ratio of the net reactive component  $X_L$  or  $X_C$  to the net resistive component  $R$ . Therefore, from Figure 1, the following equation can be written:

$$Q_x = \frac{\omega L_2 - \frac{\omega^2 k^2 L_p L_2}{\omega(L_o + L_p)}}{r_2 + \frac{\omega^2 k^2 L_p L_2}{\omega^2 (L_o + L_p)^2} (r_o + r_p)} \quad (1)$$

Dividing numerator and denominator of equation (1) by  $1/r^2$  and noting from Figure 1 that  $Q_2 = (\omega L_2)/(r_2)$ , the following equation is obtained:

$$Q_x = \frac{Q_2 - Q_2 k^2 \frac{L_p}{L_o + L_p}}{1 + \frac{Q_2 L_p k^2}{\omega (L_o + L_p)^2} (r_o + r_p)} \quad (2)$$

From Figure 1(A), the following are obtained

$$r_o = \frac{\omega L_o}{Q_o} \quad (3)$$

$$r_p = \frac{\omega L_p}{Q_p} \quad (4)$$

Upon rearranging terms and substituting equations (3) and (4)', the expression becomes:

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$$Q_x = \frac{Q_2 \left[ 1 - k^2 \left( \frac{1}{1 + (L_o)/(L_p)} \right) \right]}{1 + Q_2 \frac{L_p k^2}{(L_o + L_p)^2} \left( \frac{L_o}{Q_o} + \frac{L_p}{Q_p} \right)} \quad (5)$$

Dividing both sides of equation (5) by  $Q_o$  and rearranging terms for further clarification, the expression becomes:

$$\frac{Q_x}{Q_o} = \frac{\frac{Q_2}{Q_o} \left[ 1 - k^2 \left( \frac{1}{1 + (L_o)/(L_p)} \right) \right]}{1 + k^2 \left[ \frac{Q_2}{Q_o} \left( \frac{1}{1 + (L_o)/(L_p)} \right) \left( \frac{1}{1 + (L_p)/(L_o)} \right) + \frac{Q_2}{Q_p} \left( \frac{1}{1 + (L_o)/(L_p)} \right)^2 \right]} \quad (6)$$

For the special case when the ratio  $L_o/L_p = 1$ , equation (6) becomes:

$$\frac{Q_x}{Q_o} = \frac{\frac{Q_2}{Q_o} \left( 1 - \frac{k^2}{2} \right)}{1 + \frac{1}{4} k^2 \left( \frac{Q_2}{Q_o} + \frac{Q_2}{Q_p} \right)} \quad (7)$$

\* \* \*