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A SPHERICAL BALLOON WITH INTERNAL CORNER REFLECTOR FOR RADAR

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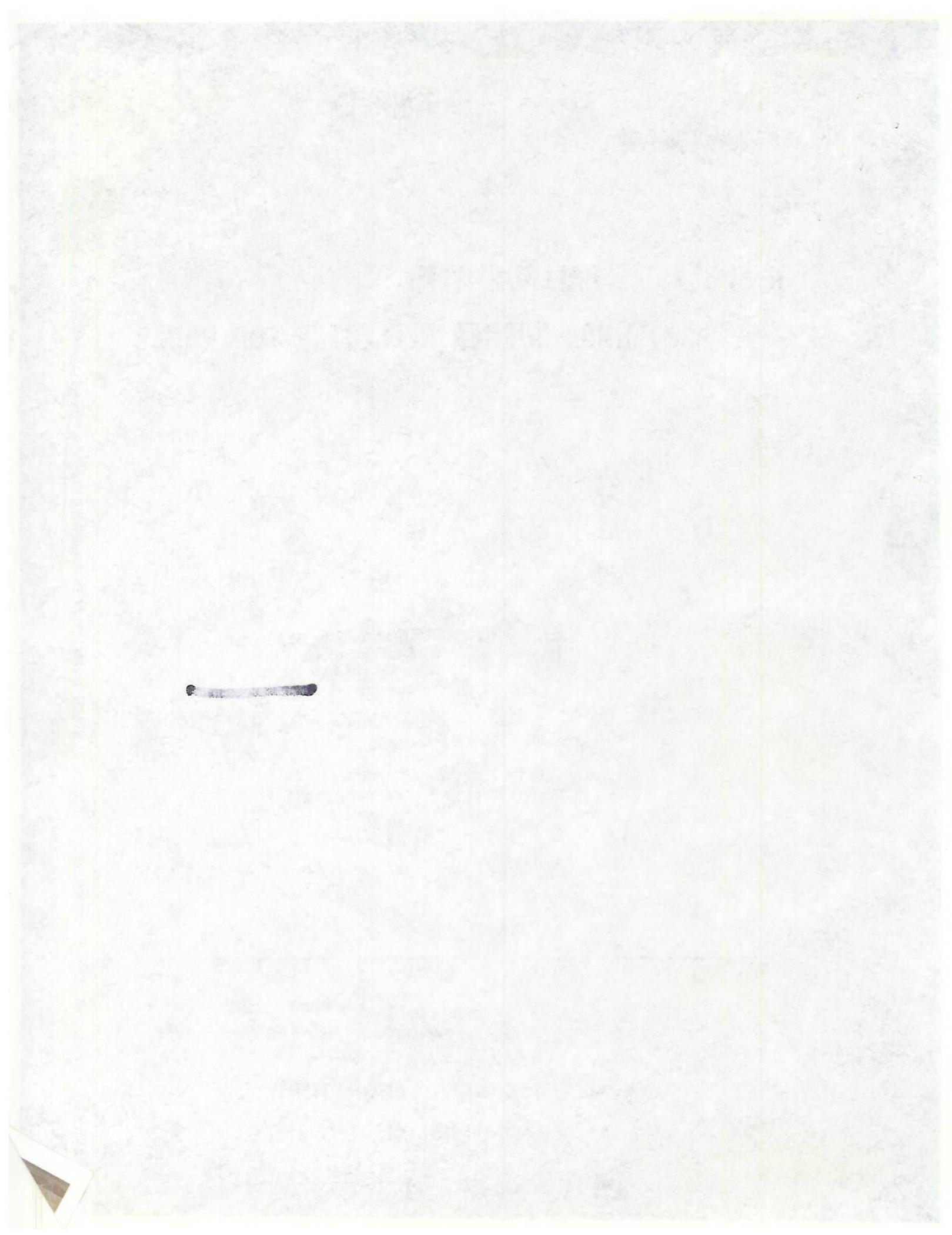
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A SPHERICAL BALLOON WITH INTERNAL CORNER REFLECTOR FOR RADAR

Sam K. Brown, Jr.

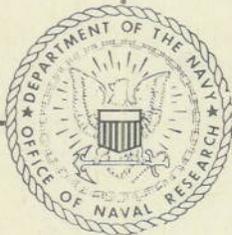
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June 30, 1948

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ABSTRACT

An eight-foot-diameter spherical balloon with an internal octahedral corner reflector for radar reflections has been developed.

Operational tests indicated that the reflector is efficient at 3000 Mc with probably similar characteristics between 800 and 6000 Mc. Response at 9100 Mc was considered satisfactory though no quantitative tests were made.

The balloon, when floating on water, was detected to a range of 20,000 yards when observed with an SG-3 radar at 3700 Mc and also with a Mark 8 Model 3 radar at 9100 Mc.

PROBLEM STATUS

This is an interim report on this problem.

AUTHORIZATION

NRL Problem No. R06-26R (BuShips Problem S1263X-C).

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A SPHERICAL BALLOON WITH INTERNAL CORNER REFLECTOR FOR RADAR

INTRODUCTION

One of the more common devices for reflecting electromagnetic energy is the corner reflector consisting of three mutually perpendicular electrically conducting planes. When the dimensions of the corner are greater than about 3λ the performance follows closely the optical analogy of three mirrors arranged in like manner.

The chief advantage of the corner reflector is its ability to produce reflections over a relatively broad angle. For this reason, it finds wide use as aids to navigation, means of identification in air-sea rescue, and in deceptive devices for radio countermeasures.

Several applications make it desirable that a radar reflector, such as a corner type, be capable of floating on the surface of water and so designed as to properly orientate itself as frequently as possible with the viewing radar.

An inflated spherical balloon fulfills the characteristics of flotation and orientation. For reflecting properties, it was hoped that a corner reflector might be placed inside a balloon which would be flexible in construction yet would assume proper shape upon balloon inflation.

DESCRIPTION OF BALLOON

A large portion of the development must be accredited to the Goodyear Tire and Rubber Company, Akron, Ohio, who, from basic designs supplied by the Laboratory, produced the balloon with internal corner reflector.

The general specifications for the balloon were as follows:

- (a) It should contain a spherical octahedral corner reflector whose surfaces were electrically conducting and integral to the outer balloon surface.
- (b) With the balloon inflated, the internal surfaces of each corner should have sides perpendicular within 0.4λ , measured at the extremity of each face.
- (c) The balloon when inflated with helium should be air buoyant.
- (d) The size of the balloon should be at least four feet in diameter but no greater than necessary for free lift.

To fulfill the air buoyancy requirement, it was necessary to use a balloon eight feet in diameter. This permitted a four-foot edge distance (apex to hypotenuse) for each of the eight internal corners.

The balloon itself was made from one layer of rubberized nylon while the conducting internal surfaces were made of two layers of the same material with 0.001-in. aluminum foil between.

In the deflated condition, the balloon occupied a space of approximately 3 cubic feet and weighed 16 pounds.

LABORATORY TESTS

These tests were confined to a checking of the physical dimensions of the inflated balloon to determine approximately the perpendicularity of the corner surfaces.

An inspection window was conveniently located in one octant adjacent to the balloon throat such that when a flash light was inserted in the throat, one could observe the interior planes constituting one corner.

It was seen that the surfaces were quite flat with no apparent waves. This indicated that if measurements were made at the exterior edge of each plane, a representative figure of surface perpendicularity and flatness could be obtained.

The balloon was inflated with air until taut then carefully positioned so that plane AECFA (Figure 1) was in a vertical position. With point A as the zero point, deviations of plane ABCDA from the horizontal were determined at 24 equally spaced points. The average deviation from the mean was 0.36 cm. The other planes (EBFDE and AECFA) exhibited the same degree of flatness. These measurements, of necessity, were exterior to the balloon and represent only plane edge deviations; however, it was felt they were reasonably indicative of the flatness of the whole plane.

Investigators of reflector response have indicated that surface flatness for corner reflectors should be within $\lambda/25$ for the most effective reflectivity. The surface flatness deviation figure of 0.36 cm for the corner surfaces of the balloon is practically $\lambda/25$ for the 10-cm band of frequencies.

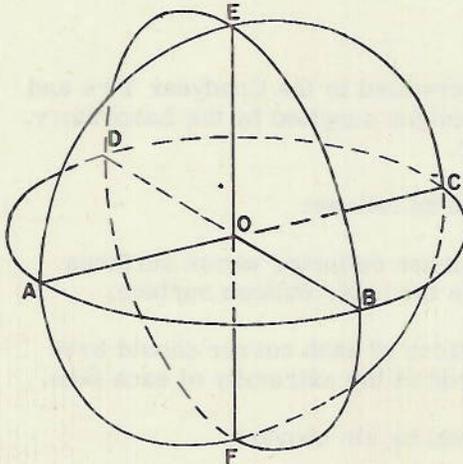


Fig. 1 - Sketch showing internal octahedral corner reflector of spherical balloon

In checking perpendicularity of internal surfaces, a bold assumption as to the position of the center of the octahedron (point O) must be made. However, the external measurements of the arc lengths of the corners indicated maximum deviations from an arc length mean for a given corner of 2 cm. In other words, the outer edge of each corner surface was not greater than 2 cm from being perpendicular with the other surfaces. Theory indicates that when surfaces deviate at their edge by 0.4λ from perpendicularity, the reflected power is 3 db down from incident power. Since a 2-cm deviation represents 0.4λ where λ is 5 cm, the upper frequency limit of the spherical octahedral corner for effective reflectivity would be 6000 M

Inflation pressures of at least 1 psi provided sufficient rigidity to the internal surfaces. The above pressure can be ascertained by striking the balloon exterior. A healthy bounce occurs

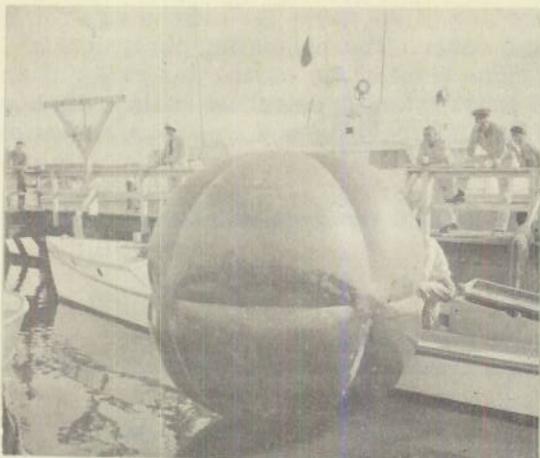


Figure 2

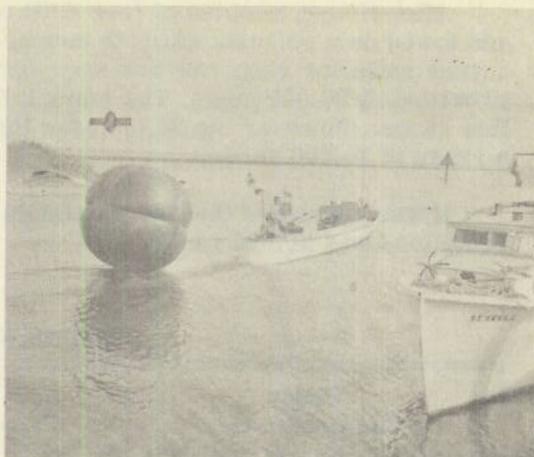


Figure 3

at pressures above 1 psi. Figure 2 shows the balloon inflated with air and floating on the surface of water. It was found that the effect of normal temperature variation on balloon pressure did not markedly affect the perpendicularity of internal surfaces.

As a final test the balloon was inflated with helium to determine its lift in free air. The lift was slightly greater than one pound.

FIELD TESTS

The balloon reflector was subjected to several tests at the Chesapeake Bay Annex of the Laboratory to determine the following performance characteristics:

- (a) Seaworthiness
- (b) Ease of handling
- (c) Range at which balloon could be seen with 10-cm and 3-cm radar
- (d) Measurement of radar cross section.

Measurements of radar range were made using an SG-3 and Mark 8 Model 3 radar whose characteristics are given in Table 1. The SG-1 radar was used for radar cross section measurements.

The balloon was inflated with air and with an appropriate sling towed behind a 66-foot boat out into Chesapeake Bay in the manner shown in Figure 3. At a range of 5000 yards the balloon was cast adrift. The prevailing wind was "off shore" with approximate velocity of 5 knots.

The attitude of the radar echoes indicated that the balloon was in random motion with frequent orientation permitting triple reflection from the spherical corners. Occasional sharp flashes were observed which were probably due to orientation of one of the three large flat planes of the balloon.

After fifteen minutes of free drift on the surface of the water the balloon was retrieved and towed on a constant azimuth course from the radar. The fluctuating pip typical of corner reflector response was seen above the noise level of the radars to a range of approximately 20,000 yards. The Mark 8 Model 3 radar failed to detect the balloon pip beyond this range. However the SG-3 radar indicated the flat sheet flashes, previously mentioned, to a range of 24,000 yards.

It was concluded that the maximum useful range of the balloon under the particular test conditions was 20,000 yards.

TABLE I
PERTINENT DATA OF RADARS

Radar Characteristics	SG-1	SG-3	Mark 8 Mod 3
Frequency	3050 Mc	3700 Mc	9100 Mc
Peak pulse power	50 kw	500 kw	25 kw
Polarization	horizontal	horizontal	horizontal
Antenna height above surface of water (mean low tide)	137 ft	137 ft	137 ft

Radar cross section measurements of the balloon reflector were then made at S-band using an SG-1 radar. Proper equipment was not available for similar tests on X-band.

These measurements were made by comparing the pip height of the target balloon with the pip height of a standard target whose radar cross section was known.

For comparison purposes, the output from a pulsed r-f signal generator was introduced into the wave guide system of the radar producing an artificial pip on the "A" scope of the radar. The pip height could be varied by means of a calibrated attenuator and its position on the base line controlled by a delay control in the system which synchronized the signal generator and radar.

The balloon was towed to a position in Chesapeake Bay at a range of 3000 yards from the SG-1 radar. At this position the balloon was orientated such that one octant would focus towards the radar antenna as the balloon was jockeyed about a central position. Measurements were then made of the radar echo. The balloon was next rotated to a broadside position to observe the flat sheet response. Resultant pip heights of this response were not recorded.

This same procedure was followed at discrete ranges to 17,000 yards on both outbound and inbound courses. The decibel difference between the balloon radar echoes and the standard target echoes was then plotted as a function of range (Figure 4) and the effective radar cross section computed from the graph according to the method of NRL Report R-2232.* As shown in Figure 4, the computed value for far zone cross section is 2.5×10^{12} square meters. For comparison purposes, a typical echo response curve for a destroyer has been shown.

* W. S. Ament, M. Katzin, and F. C. Macdonald, "Radar cross section of ship targets, II", NRL Report R-2232, 18 February 1944.

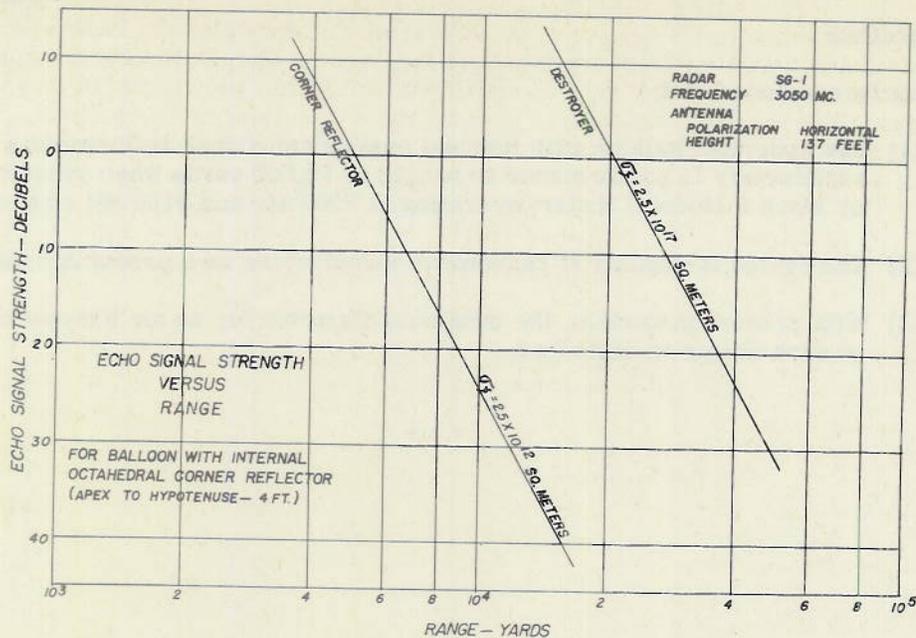


Figure 4

DISCUSSION

In assessing the properties of this balloon with internal reflector, several factors encountered during tests should be discussed.

The reflector surfaces proved to be sufficiently perpendicular and flat to give good triple reflection response at 10 cm. Though the 3-cm response looked good it was impossible to evaluate this properly due to a lack of appropriate test equipment. In general, it can be said that the device is efficient as a reflector from 800 Mc to 6000 Mc with usable response to at least 9100 Mc.

Inflation of the balloon was accomplished with a one-third horse power air blower in three minutes. Two men were required for proper handling. An idea of inflation problems can be obtained from the two photographs of Figures 2 and 5.



Figure 5

Under average wind conditions and when free, the balloon assumed random positions in regard to corner orientation.

The Goodyear Company has made a four-foot diameter balloon which is now being subjected to field tests. Due to its weight, however, this balloon is not air buoyant.

The theoretical relationships involved in the design of a spherical corner reflector have been worked out in the appendix.

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CONCLUSIONS

It can be concluded that:

- (1) The spherical balloon with internal octahedral corner reflector has proven satisfactory in performance to ranges of 20,000 yards when viewed with an SG-3 or Mark 8 Model 3 Radar, operating at 3700 Mc and 9100 Mc respectively,
- (2) The device is capable of reasonable manufacture on a production basis,
- (3) With present materials, the minimum diameter for an air buoyant balloon of such design is eight feet.

* * *

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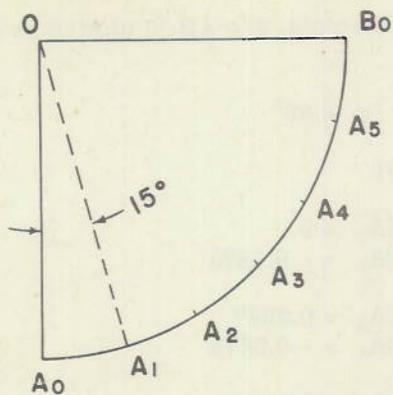


Figure 5 - Projection of an Arc

The effective area of the circular corner then becomes the area common to the projection of the entrance pupil and exit pupil on the $\xi\eta$ plane. Since each pupil or aperture is bounded by an arc, the projection on this plane is determined by the projection of several points on the arc which can arbitrarily be positioned 15 degrees apart, as shown in Figure 5.

A general expression is now determined for the position in the $\xi\eta$ plane of the projection of any point (x, y, z) . Let each point be represented by a vector which in turn is resolved into its $x, y,$ and z component vectors. Since the projection of a vector is the resultant of the projection of its component vectors, each component can be treated separately. Thus

$$\xi = -x \sin\theta + y \cos\theta \quad (1)$$

$$\eta = -x \cos\theta \sin\phi - y \sin\theta \sin\phi + z \cos\phi \quad (2)$$

Relations (1) and (2) may be extended further to simplify substitutions of θ and ϕ . For example, the x and y components of arc points A_0, \dots, B_0 are expressed as

$$\begin{array}{ll} A_0 = (1, 0, 0) & A_4 = (0.5000, 0.8660, 0) \\ A_1 = (0.9659, 0.2588, 0) & A_5 = (0.2588, 0.9659, 0) \\ A_2 = (0.8660, 0.5000, 0) & B_0 = (0, 1, 0) \\ A_3 = (0.7071, 0.7071, 0) & \end{array}$$

Then (1) and (2) are rewritten as follows:

$$\begin{array}{l} \xi A_0 = -\sin\theta \\ \eta A_0 = -\cos\theta \sin\phi \end{array} \quad (3)$$

$$\begin{array}{l} \xi A_1 = -0.9659 \sin\theta + 0.2588 \cos\theta \\ \eta A_1 = -0.9659 \cos\theta \sin\phi - 0.2588 \sin\theta \sin\phi \end{array} \quad (4)$$

$$\begin{array}{l} \xi A_2 = -0.8660 \sin\theta + 0.5000 \cos\theta \\ \eta A_2 = -0.8660 \cos\theta \sin\phi - 0.5000 \sin\theta \sin\phi \end{array} \quad (5)$$

$$\begin{array}{l} \xi A_3 = -0.7071 \sin\theta + 0.7071 \cos\theta \\ \eta A_3 = -0.7071 \cos\theta \sin\phi - 0.7071 \sin\theta \sin\phi \end{array} \quad (6)$$

$$\begin{array}{l} \xi A_4 = -0.5000 \sin\theta + 0.8660 \cos\theta \\ \eta A_4 = -0.5000 \cos\theta \sin\phi - 0.8660 \sin\theta \sin\phi \end{array} \quad (7)$$

$$\begin{array}{l} \xi A_5 = -0.2588 \sin\theta + 0.9659 \cos\theta \\ \eta A_5 = -0.2588 \cos\theta \sin\phi - 0.9659 \sin\theta \sin\phi \end{array} \quad (8)$$

$$\begin{array}{l} \xi B_0 = \cos\theta \\ \eta B_0 = -\sin\theta \sin\phi \end{array} \quad (9)$$

In like manner, the expressions (1) and (2) may be extended to include arcs BC and CA. A given value of θ and ϕ is then applied to the three sets of equations and

the projected points plotted as shown in Figure 6. For example, arc AB is plotted as seen below

Given $\theta = 45^\circ$

$\phi = 36^\circ$

Substituting in (3) through (9)

$\xi A_0 = -0.7071$
 $\eta A_0 = -0.4155$

$\xi A_3 = 0$
 $\eta A_3 = -0.5876$

$\xi A_1 = -0.5000$
 $\eta A_1 = -0.5088$

$\xi A_4 = 0.2587$
 $\eta A_4 = -0.5675$

$\xi A_2 = -0.2587$
 $\eta A_2 = -0.5675$

$\xi A_5 = 0.5000$
 $\eta A_5 = -0.5088$ (10)

$\xi B_0 = 0.7071$
 $\eta B_0 = -0.4155$

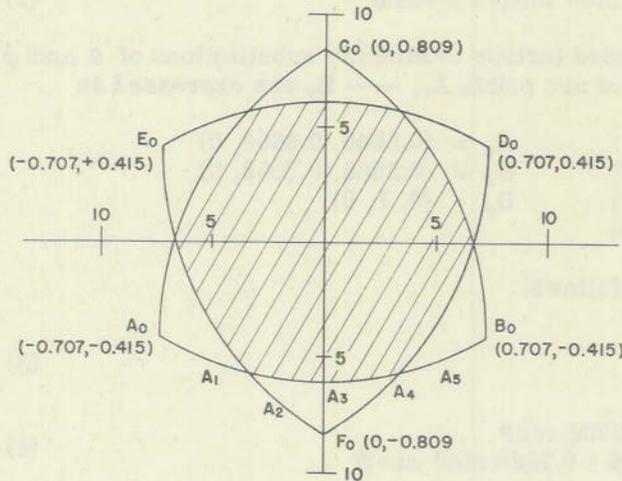


Figure 6 - Effective Reflection Area Plotted on XY Coordinates

Expressions for arc ED, EF, and FD need not be considered since their projections are again the mirror image of the projections of arc AB, BC, and CA.

Smooth curves are drawn through the projected points and the area common to the entrance and exit pupil determined by means of a planimeter.

This procedure was followed for values of $\theta = 9^\circ, 18^\circ, 27^\circ, 36^\circ$ and 45° (symmetry exists about 45°) and $\phi = 9^\circ, 18^\circ, 27^\circ, \dots, 63^\circ$. Curves were plotted for each of the 35 line-of-sight positions and the effective

areas determined. Each projection was plotted on axes 20 inches long, as shown in Figure 6. The edge distance of the corner AO was then considered as unity or 10 inches for the purpose of computations.

The resulting family of curves is plotted in Figure 7. Using these determined effective areas some useful relationships were established.

Let

A_{ca} = surface area of circular corner (AOB + BOC + COA)

R = edge distance of corner

R = 10 inches = 25.4 cm.

$A_{ca} = \frac{\pi R^2 \times 3}{4} = \frac{\pi (25.4)^2 \times 3}{4} = 1520 \text{ sq. cm.}$ (11)

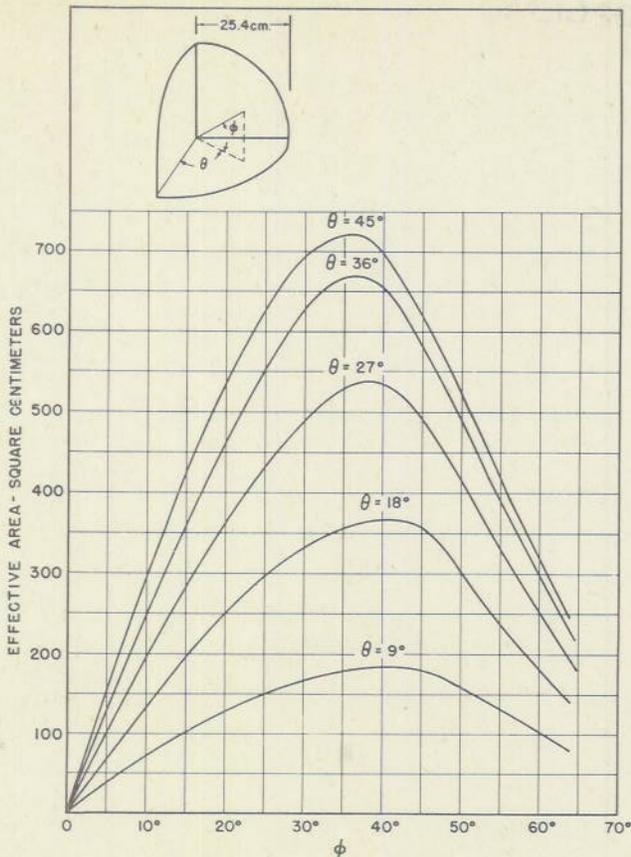


Figure 7 - Effective Areas for Various Angles of Incidence

The maximum effective area occurs when $\theta = 45^\circ$ and $\phi = 36^\circ$ (actually $\phi = 35^\circ 15.9'$). From Figure 6, the effective area for this incident angle is 720 sq. cm.

Let A_{ce} = effective area of circular corner

$$\text{Then } \frac{A_{ce}}{A_{ca}} = \frac{720}{1520} = 0.474 \quad (12)$$

To establish a comparison, this same ratio is determined for a triangular corner.

Let A_{te} = effective area of triangular corner

a = edge distance of corner (apex to outer edge)

A_{ta} = surface area of corner

It can be shown that

$$A_{te} = \frac{a^2}{\sqrt{3}} \quad (13)$$

$$A_{ta} = \frac{3 \times a^2}{2} \quad (14)$$

and

$$\frac{A_{te}}{A_{ta}} = \frac{a^2}{\sqrt{3}} \times \frac{2}{3a^2} = 0.385 \quad (15)$$

The ratio of effective area to surface area of a circular corner to the similar ratio for the triangular corner can then be found by dividing (12) by (15)

or

$$\frac{A_{ce}}{A_{ca}} \div \frac{A_{te}}{A_{ta}} = \frac{0.474}{0.385} = 1.23 \quad (16)$$

Perhaps more useful would be the relationship of maximum effective area to corner edge distance.

From (12)

$$A_{ce} = 0.474 A_{ca} \quad (17)$$

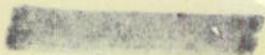
Substituting (11) into (17)

$$A_{ce} = 0.474 \frac{(\pi R^2 \times 3)}{4} \quad (18)$$

$$A_{ce} = 0.356 \pi R^2$$

In the design of a circular corner the information contained in equation (18) and the curves of Figure 7 become useful.

* * *



The distance between the
 points when $\theta = 45^\circ$ is
 $2\sqrt{2} \sin 45^\circ = 2$
 Hence, the effective area for the
 current is $2 \sin 45^\circ$.

Let A_e = effective area of the
 circular current
 $A_e = 2 \sin 45^\circ = \sqrt{2}$
 $A_e = \sqrt{2}$

The magnetic field at the center of the
 circular current is $B = \frac{\mu_0 I}{2R}$

Let A_e = effective area of the
 circular current
 $A_e = \pi R^2 \sin^2 \theta$
 $A_e = \pi R^2 \sin^2 45^\circ = \frac{\pi R^2}{2}$
 $R = \sqrt{\frac{2A_e}{\pi}}$

Substituting the value of R in the
 expression for B , we get
 $B = \frac{\mu_0 I}{2 \sqrt{\frac{2A_e}{\pi}}} = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt{A_e}}$

Substituting the value of $A_e = \sqrt{2}$, we get
 $B = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt{\sqrt{2}}} = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

The value of magnetic field at the center
 of the circular current is $B = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

Substituting the value of $I = 10$ A, we get
 $B = \frac{\mu_0 \times 10 \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}} = \frac{\mu_0 \times 10 \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

Hence, the magnetic field at the center of the
 circular current is $B = \frac{\mu_0 \times 10 \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

Substituting the value of $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, we get
 $B = \frac{4\pi \times 10^{-7} \times 10 \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}} = \frac{4\pi \times 10^{-6} \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

Hence, the magnetic field at the center of the
 circular current is $B = \frac{4\pi \times 10^{-6} \times \sqrt{\pi}}{2\sqrt{2} \sqrt[4]{2}}$

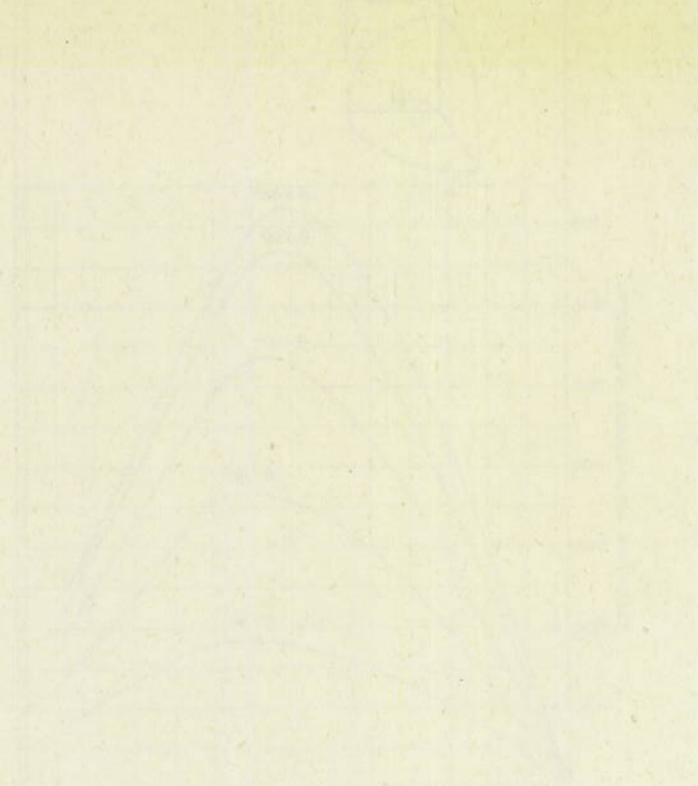


Figure 1: Diagram of a circular current loop of radius R in a horizontal plane. A point is marked at the center of the loop. A vertical line is drawn from the center to a point on the loop, forming an angle θ with the normal to the plane of the loop.

The magnetic field at the center of the circular current is $B = \frac{\mu_0 I}{2R}$

The effective area of the circular current is $A_e = \pi R^2 \sin^2 \theta$

Substituting the value of $R = \sqrt{\frac{2A_e}{\pi}}$ in the expression for B , we get

$B = \frac{\mu_0 I}{2 \sqrt{\frac{2A_e}{\pi}}} = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt{A_e}}$

Hence, the magnetic field at the center of the circular current is $B = \frac{\mu_0 I \sqrt{\pi}}{2\sqrt{2} \sqrt{A_e}}$

APPENDIX

EFFECTIVE AREA OF A CIRCULAR RADAR CORNER REFLECTOR

With the advent of radar came the need for a compact target reflector with large radar cross section. This can be realized in a flat plate, whose dimensions are large compared to the wavelength, as long as it is "viewed" with the line of sight exactly perpendicular to the plate. A device called a corner reflector, on the other hand, will provide large equivalent flat-plate area and at the same time give reflections from almost any direction of illumination. This latter feature is the principal advantage of such a reflector. The small glass reflectors in the highway markers are examples of this corner principle.

Corner reflectors consist of three intersecting, mutually perpendicular planes, as illustrated by Figures 1 and 2. An incident ray directed into the reflector will, over a broad angle, be triply reflected and emerge in the direction from which it came. As the incident ray moves out of the region where triple reflection occurs, double reflection (from two planes whose line of intersection is nearly normal to the line of sight) and single reflection (from a plane nearly normal to the line of sight) begin to contribute to the radar cross section. Due to the parenthetical limitations above only the triple reflection zone is of interest to the reflector designer.

When an incident ray enters the aperture or entrance pupil of the corner (surface ABC, Figure 2) it will, after triple reflection, appear to emerge from a point whose coordinates are the negative of the point of entrance. If the negative of every point on ABC is plotted, the surface DEF results which is called the reflected or exit pupil. Every entering ray and every leaving ray will pass through the entrance pupil. For a ray to be reflected in the same direction from which it came, however, it must appear to emerge from the exit pupil as well. Thus the effective or flat plate area of a corner reflector is the area common to both the entrance and exit pupil when projected on a plane perpendicular to the line of sight.

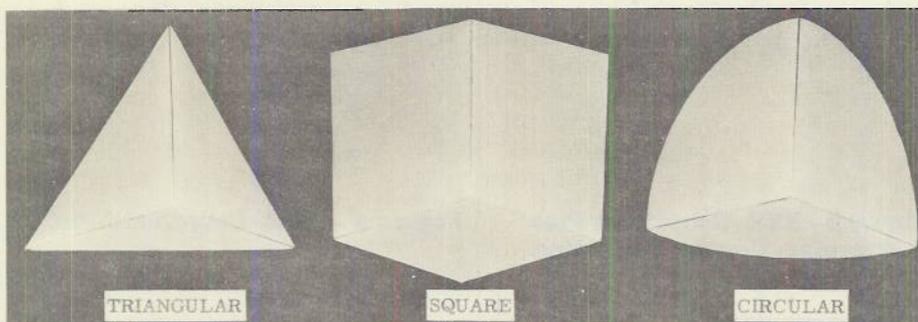


Figure 1 - Three Types of Radar Corner Reflectors

With ABC lying in the plane of the paper and the incident ray perpendicular to it, projections of ABC and DEF will be as shown with the effective area of the corner as abcdef. As the incident angle of the wave front changes so will the effective area.

Determination of this area can be carried out either mathematically or graphically. When dealing with a triangular or square corner reflector, the former method is the simpler. When treating circular corners or any unconventional configuration, the graphical solution becomes the more practical if not the only method.

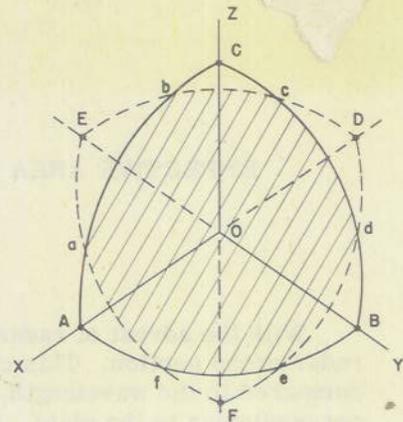


Figure 2 - Flat Projection of Entrance and Exit Pupils

This paper describes the graphical solution of effective area as applied to a circular corner. It follows closely the method used by Crout and Holt of MIT¹ in early investigations of triangular and square corners. Figure 1 shows the three types of reflectors mentioned above.

Two coordinate systems are used in the solution. The first, the XYZ coordinate system, defines the circular corner as shown in Figure 2, with the line of sight or incident ray position determined by θ and ϕ as shown in Figure 3. The second system includes the $\xi\eta$ plane of Figure 4 which lies perpendicular to the line of sight and passes through point O. As shown in Figure 4, the $\xi\eta$ axes, defined by ϕ and θ are mutually perpendicular with the ξ axis always lying in the XY plane.

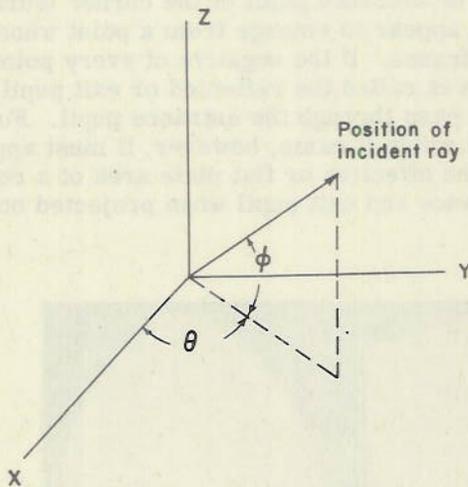


Figure 3 - XYZ-Coordinate Plot of Circular-Corner Reflection

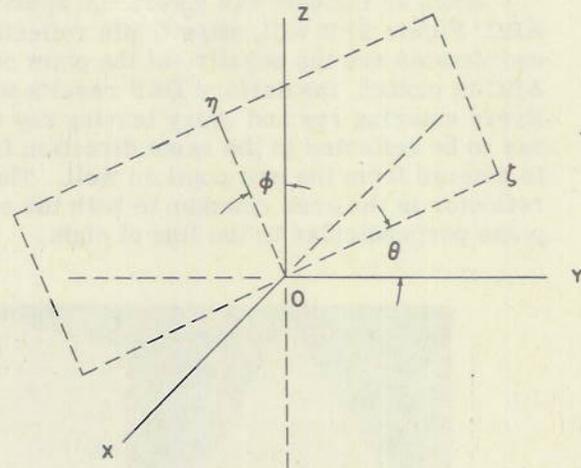


Figure 4 - XYZ-Coordinate Plot of $\xi\eta$ Plane

¹ Radiation Laboratory Report 43-31, The Application of Corner Reflectors to Radar (Theoretical), May 14, 1943.