REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
23-09-2021	Final Report	5-Jul-2018 - 30-Jun-2021

4. TITLE AND SUBTITLE
Final Report: Extending Accelerated Optimization into the PDE Framework

5a. CONTRACT NUMBER
W911NF-18-1-0281

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
Georgia Tech Research Corporation
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Atlanta, GA 30332 -0420

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
73030-CS.9

12. DISTRIBUTION AVAILIBILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Anthony Yezzi
UU	UU	UU	UU		19b. TELEPHONE NUMBER 404-385-1017

RPPR Final Report
as of 26-Jan-2022

Agency Code: 21XD

Proposal Number: 73030CS **Agreement Number: W911NF-18-1-0281**
INVESTIGATOR(S):

Name: Anthony Yezzi
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Country: USA
DUNS Number: 097394084 EIN: 580603146
Report Date: 30-Sep-2021 Date Received: 23-Sep-2021
Final Report for Period Beginning 05-Jul-2018 and Ending 30-Jun-2021
Title: Extending Accelerated Optimization into the PDE Framework
Begin Performance Period: 05-Jul-2018 **End Performance Period:** 30-Jun-2021
Report Term: 0-Other
Submitted By: Anthony Yezzi Email: anthony.yezzi@ece.gatech.edu
Phone: (404) 385-1017

STEM Degrees: 1 **STEM Participants:** 2

Major Goals: The primary goal is to extend momentum based methods (Heavy Ball, and Nesterov) from the finite dimensional framework into the infinite dimensional framework of gradient based optimization over the manifolds of functions, curves, surfaces, diffeomorphisms, and other distributed unknowns. This includes developing 1) a theory of PDE methods for accelerated gradient optimization 2)formulation of the specific PDE equations for the application of this theory in several optimization contexts 3) numerical methods for solving accelerated PDE systems 4) theoretical analysis of convergence rates and performance gains 5) development of efficient implementations amenable to parallel
computing.

Accomplishments: Mathematical formulations as well as numerical implementations for PDE based accelerated gradient optimization were first completed and evaluated for six different applications.

1) Image denoising with L2, TV, and Beltrami style regularization.
2) Debluring and inpainying in the Beltrami framework
3) Active contour based image segmentation
4) Active surface based multi-view stereo 3D reconstruction
5) Diffeomorphic non-rigid image registration (directed toward medical imagery)
6) Efficient solution of the obstacle problem

In addition, for cases and 1, 2, and 7 above, convergence rate analysis was carried out which yielded easily computed expressions for optimal damping coefficients in the corresponding accelerated PDE systems.

The mathematical framework was then successfully extended into the new application areas of region based image segmentation (based on fuzzy region competition) as well as the completely novel area of shaped based inversion of radar data. Two different methodologies for radar shape inversion were explored for proof of concept, one in the frequency domain (with 2D simulations) and another in the time domain (with 3D simulations) .

Application of the active contour and active surface frameworks were then developed more maturely in the context of medical image segmentation and stereo reconstruction. Further, multi-threading has now been integrated into the initial image denoising
and 3D mulltiview stereo reconstruction applications

Next, a more efficient, completely linear version of the initially formulated diffeomorphic image registration methodology was developed which was not only more robust, but also immediately amenable to parallel multi-thread or GPU based implementations.

Finally, the PDE acceleration benefits were applied to medical image segmentation methodologies which combined active contours and voxel-based classifiers based on deep convolutional neural networks.

Training Opportunities: Postdoctoral researcher, Dr. Samuel Bignardi, and doctoral student Alper Yildirim were funded
on this project to develop accelerated PDE based methodologies for shape inversion using radar.
Alper Yildirim successfully defended his PhD after the second year of the project. During the final year, a new graduate student, Huizong Yang, was hired who began (and continues) working on accelerated PDE active surface methods for 3D reconstruction from both RGB and RGBD images.

Results Dissemination: In addition to published results (see "Products" section), the PI Yezzi presented aspects of this work
a the following venues

1) Interdisciplinary Distinguished Seminar Series at North Carolina State University on September 21, 2018 the presentation "Accelerated Optimization in the PDE Framework"

2) Georgia Tech Institute for Robotics and Intelligent Machines Distinguished Seminar Series, Atlanta, October 24, 2018.

3) Northeastern University Invited Colloquium, Boston, October 26, 2018.

4) United Technologies Research Corporation invited colloquium, Farmington, CT, November 12, 2018.

5) IPAM Workshop on High Dimensional Hamilton-Jacobi PDE's at UCLA (originally
scheduled to be delivered on premise but changed to remote presentation due to COVID-19) on
April 23, 2020.

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI
Participant: Anthony Yezzi
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Samuel Bignardi
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)

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Participant: Alper Yildirim
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Huizong Yang
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
National Academy Member: N

ARTICLES:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Journal of Mathematical Imaging and Vision
Publication Identifier Type: DOI Publication Identifier: https://doi.org/10.1007/s10851-019-00910-2
Volume: 62 Issue: First Page #: 10
Date Submitted: 6/30/20 12:00AM Date Published: 1/1/20 5:00AM
Publication Location:
Article Title: Accelerated variational PDE's for effiient solution of regularized inversion problems
Authors: Minas Benyamin, Jeff Calder, Ganesh Sundaramoorthi, Anthony Yezzi
Keywords: PDE Acceleration, denoising, inpainting, image restoration
Abstract: We further develop a new framework, called PDE Acceleration, by applying it to calculus of variations problems defined for general functions on R^n, obtaining efficient numerical algorithms to solve the resulting class of optimization problems based on simple discretizations of their corresponding accelerated PDE's. While the resulting family of PDE's and numerical schemes are quite general, we give special attention to their application for regularized inversion problems, with particular illustrative examples on some popular image processing applications. The method is a generalization of momentum, or accelerated, gradient descent to the PDE setting. For elliptic problems, the descent equations are a nonlinear damped wave equation, instead of a diffusion equation, and the acceleration is realized as an improvement in the CFL condition from dt~dx^2 (for diffusion) to dt~dx (for wave equations). We work out several explicit as well as a semi-implicit numerical schemes.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 26-Jan-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Research in Mathematical Sciences
Publication Identifier Type: DOI Publication Identifier: https://doi.org/10.1007/s40687-019-0197-x
Volume: 6 Issue: 35 First Page #: 1
Date Submitted: 6/30/20 12:00AM Date Published: 10/31/19 12:56PM
Publication Location:
Article Title: PDE Acceleration: A convergence rate analysis and applications to obstacle problems
Authors: Jeff Calder, Anthony Yezzi
Keywords: PDE Acceleration, obstacle problem
Abstract: This paper provides rigorous convergence rate and complexity analysis for a general class of PDE's to solve calculus of variations problems for functions on \R^n using the recently introduced framework ofPDE acceleration. PDE acceleration grew out of a variational interpretation of momentum methods, such as Nesterov's accelerated gradient method and Polyak's heavy ball method, that views acceleration methods as equations of motion for a generalized Lagrangian action. Its application to several regularized inversion problems yields equations of motion in the form of a damped nonlinear wave equation rather than nonlinear diffusion arising from gradient descent. These \emph{accelerated PDE's} can be efficiently solved with simple explicit finite difference schemes where acceleration is realized by an improvement in the CFL condition from dt~dx^2 for diffusion equations to dt~dx for wave equations. In this paper, we prove a linear convergence rate for PDE acceleration for strongly convex
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Journal of the Optical Society of America A
Publication Identifier Type: DOI Publication Identifier: 10.1364/JOSAA.377896
Volume: 37 Issue: 4 First Page #: 568
Date Submitted: 6/30/20 12:00AM Date Published: 3/1/20 5:00AM
Publication Location:
Article Title: Closed-form solution for thin lens image irradiance under arbitrary solid angle
Authors: Robert D. Friedlander, Anthony J. Yezzi
Keywords: irradiance equation, thin lens model, shape from defocus, multiview stereo
Abstract: Optical imaging systems are found everywhere in modern society. They are integral to computer vision, where the goal is often to infer geometric and radiometric information about a 3D environment given limited sensing resources. It is helpful to develop relationships between these real-world properties and the actual measurements that are taken, such as 2D images. We derive a new relationship between object radiance and image irradiance based on power conservation and a thin lens imaging model. The relationship has a closed-form solution for in-focus points and can be solved via numerical integration for points that are not focused. It can be thought of as a generalization of Horn’s commonly accepted irradiance equation. Through both ray tracing simulations and comparison to the intensity values of actual images, our equation shows better accuracy than Horn’s equation. An improvement is most notable for large lenses and near-focused images where the pinhole assumption breaks down.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 26-Jan-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Computers in Biology and Medicine
Publication Identifier Type: DOI Publication Identifier: 10.1016/j.compbiomed.2020.103701
Volume: 120 Issue: First Page #: 103701
Date Submitted: 6/30/20 12:00AM Date Published: 5/1/20 4:00AM
Publication Location:
Article Title: Development of a new fully three-dimensional methodology for tumours delineation in functional images
Authors: Albert Comelli, Samuel Bignardi, Alessandro Stefano, Giorgio Russo, Maria Gabriella Sabini, Massimo I
Keywords: active contours, active surfaces, medical image segmentation
Abstract: Delineation of tumours in Positron Emission Tomography (PET) plays a crucial role in accurate diagnosis and radiotherapy treatment planning. In this context, it is of outmost importance to devise efficient and operator-independent segmentation algorithms capable of reconstructing the tumour three-dimensional (3D) shape. In previous work, we proposed a system for 3D tumour delineation on PET data (expressed in terms of Standardized Uptake Value - SUV), based on a two-step approach. Step 1 identified the slice enclosing the maximum SUV and generated a rough contour surrounding it. Such contour was then used to initialize step 2, where the 3D shape of the tumour was obtained by separately segmenting 2D PET slices, leveraging the slice-by-slice marching approach. The segmentation in step 2 is now performed by evolving an active surface directly in the 3D space which performs the shape reconstruction on the whole stack of slices simultaneously, leveraging cross-slice information.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Inverse Problems
Publication Identifier Type: Other Publication Identifier:
Volume: 37 Issue: 2 First Page #: 025004
Date Submitted: 9/23/21 12:00AM Date Published: 1/5/21 5:00AM
Publication Location:
Article Title: A Feasibility Study of Radar-Based Shape and Reflectivity Reconstruction Using Variational Methods
Authors: Samuel Bignardi, Anthony Yezzi, Alper Yildirim, Chris Barnes, Romeil Sandhu
Keywords: Radar processing, shape inversion, accelerated optimization
Abstract: Remote sensing radar techniques provide highly detailed imaging. Nevertheless, these techniques don’t embed any explicit shape, and shape reconstruction from radar typically relies on applying computer vision techniques, originally designed for optical images, to radar imaging products. Shape reconstruction directly from raw data would be desirable in many applications, e.g. in computer vision and robotics. Nevertheless, shape inversion has seldom be attempted in the radar context, as high frequency signals lead to energy functionals dominated by tightly packed narrow local minima. We investigate the feasibility of shape reconstruction by inversion of pulse-compressed radar signals. Motivated by geometric methods that have matured within the fields of image processing and computer vision, we pose the problem in a variational context obtaining a partial differential equation for the evolution of an initial shape toward the shape–reflectivity combination that best reproduces the data.
Distribution Statement: 3-Distribution authorized to U.S. Government Agencies and their contractors
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 26-Jan-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: SIAM Journal on Imaging Sciences
Publication Identifier Type: DOI Publication Identifier: 10.1137/19M1304210
Volume: 13 Issue: 4 First Page #: 2029
Date Submitted: 9/23/21 12:00AM Date Published: 11/1/20 4:00AM
Publication Location:
Article Title: Accelerated Optimization in the PDE Framework Formulations for the Active Contour Case
Authors: Anthony Yezzi, Ganesh Sundaramoorthi, Minas Benyamin
Keywords: partial differential equations, acceleration, Nesterov, mass transport optimization, gradient descent, variational, manifolds
Abstract: Following the seminal work of Nesterov, accelerated optimization methods have been used to powerfully boost the performance of first-order, gradient based parameter estimation in scenarios where second-order optimization strategies are either inapplicable or impractical. Not only does accelerated gradient descent converge considerably faster than traditional gradient descent, but it also performs a more robust local search of the parameter space by initially overshooting and then oscillating back as it settles into a final configuration, thereby selecting only local minimizers with a basis of attraction large enough to contain the initial overshoot. This behavior has made accelerated and stochastic gradient search methods particularly popular within the machine learning community. We show how their formulation may be further extended to infinite dimensional manifolds (starting here with the geometric space of curves and surfaces).
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: Advances in Neural Information Processing Systems (NeurIPS)
Date Received: 04-Oct-2019 Conference Date: 02-Dec-2018 Date Published:
Conference Location: Montreal, Canada
Paper Title: Variational PDEs for Acceleration on Manifolds and Application to Diffeomorphism
Authors: Ganesh Sundaramoorthi, Anthony Yezzi
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: IEEE Conf. Computer Vision and Pattern Recognition (CVPR)
Date Received: 04-Oct-2019 Conference Date: 16-Jun-2019 Date Published:
Conference Location: Long Beach, CA
Paper Title: PDE Acceleration for Active Contours
Authors: Anthony Yezzi, Ganesh Sundaramoorthi, Minas Benyamin
Acknowledged Federal Support: **Y**

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Partners

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I certify that the information in the report is complete and accurate:
Signature: Anthony Yezzi
Signature Date: 9/23/21 6:43PM

Nothing to report in the uploaded pdf

(see accomplishments)