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Navy Department - Office of Research and Inventions

NAVAL RESEARCH LABORATORY
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SHIP-SHORE RADIO DIVISION - M. and D. F. SECTION

20 January 1946

# SENSITIVITY AND ACCURACY COMPARISONS OF DAU VS. DAJ GONIOMETERS FOR DAJ-a INSTALLATIONS

By S. F. George and E. H. Flath

FR-2707

\- Report R-2707 -



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Ship-Shore Radio Division

Commodore H. A. Schade, USN
Director, Naval Research Laboratory

Preliminary Pages ...a-d
Numbered Pages ......17
Plates ..............19
Distribution List ...e

NRL Problem S1046R-C



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[redacted] Samuel H. George

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## ABSTRACT


As a part of the modernization and re-conversion of the DAJ high frequency Naval Shore Direction Finding Stations, it was necessary to determine the most suitable goniometers to use in the new DAJ-a installations. Two general types of measurements were made on a DAU goniometer, a set of four used DAJ goniometers from the Dupont Station and a set of four new DAJ goniometers from the factory. These tests were: (1) The effect of these goniometers on over-all sensitivity, and (2) the bearing error introduced by the goniometers themselves. A preliminary investigation was conducted to confirm the procedure used for measuring the sensitivities through balanced goniometer inputs by the use of the standard available unbalanced signal generators. From sensitivity considerations it is immaterial whether the single wide-band DAU goniometer or the four individual DAJ goniometers are employed, provided the 330 ohm resistors are removed from across the DAU stators. The appendix discusses theoretically and constructionally the building of the goniometer accuracy test rig designed for this problem. From accuracy considerations it seems advisable to recommend the use of the DAU wide-band goniometer in Bands 1, 2, and 3 and the DAO goniometer in Band 4. It was found that in Band 4 the octantal spacing error of the #4 DAJ Adcock Array largely cancelled out the octantal error of the DAO goniometers.


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## INTRODUCTION

1. Preliminary to the modernization of the DAJ shore direction finder installation, tests were conducted at the Laboratory to determine whether the individual-band DAJ goniometers or the single wide-band DAU goniometer would provide the greatest over-all sensitivity for the DAJ-a equipment. The results of these measurements indicated that an increase in sensitivity from one to five decibels could be realized by using the existing individual DAJ goniometers in place of the new DAU goniometers. Although such an increase in sensitivity is not large, it was considered at the time that it was sufficient to warrant the use of the four separate DAJ goniometers. This was especially true in the DAO Array which covered the highest frequency band, since it had been the experience of the Laboratory that the DAU goniometer produced excessive errors and uncertain or reversed sense above 22 megacycles. As reported in Reference 2, representatives of Op-20-G3, BuShips, and NRL concurred in the recommendation that individual DAJ goniometers be used in all four bands.

2. Subsequent to the Pilot modernization of a DAJ to a DAJ-a, at Amagansett, Long Island, by NRL engineers, a report of field tests conducted by representatives of Op-20-G3 indicated that by a minor modification in the DAU goniometers, i.e., by removing the 330 ohm resistor from across each stator coil, the over-all sensitivity of the equipment could be increased sufficiently to recommend the use of the wide-band DAU goniometers instead of the individual-band DAJ goniometers. It should be mentioned here that the present unmodified DAU goniometers have a 330 ohm resistor across each stator coil. These resistors were needed in the DAU installations to damp a loop and transmission line resonance condition, but they are not required for good operation of the DAJ-a installations. Additional field tests by Op-20-G3 were made at the Imperial Beach, California, Supplemental Radio Station to determine the over-all deviation of the installation using both sets of goniometers throughout the working frequency range of the equipment. These calibration tests revealed no significant differences in error produced by the two sets of goniometers.

3. In the light of these findings, the Laboratory was requested to extend the investigation of the goniometers with respect to their effect on over-all sensitivity and bearing accuracy. The work is covered by Problem S1046R-C which was requested in Reference 1, and resulted from the conferences reported in Reference 3 and 4. This report concerns the work on goniometer sensitivity and accuracy tests, the results, and specific recommendations.

## GONIOMETERS TESTED AND COMPARISONS MADE

4. Before making any tests, the question arose as to whether the aging and use of the old DAJ goniometers could in any way affect their sensitivity and accuracy. In order to make these tests conclusive, two distinct sets of individual DAJ goniometers were tested: (1) a new and unused set from the factory and (2) the old and used set from the Dupont Station:

[redacted]

| New Factory Goniometers | Old Dupont Goniometers |
|---|---|
| DAL Serial #27 | DAL Serial #24 |
| DAM Serial #18 | DAM Serial #23 |
| DAN Serial #18 | DAN Serial #2 |
| DAO Serial #27 | DAO Serial #29 |

The Model DAU Serial #33 goniometer was used to represent the single wide-band units. Sensitivity and accuracy tests were made throughout the working frequency range of the goniometers. The tests on the DAU goniometer were made both with and without the 330 ohm resistors across the goniometer stators.

GONIOMETER SENSITIVITY TESTS

Investigation of Measurement Techniques

5. The correct measurement technique for testing the sensitivity of direction finder equipment through goniometers having balanced input by the use of unbalanced signal generators had not been too strong heretofore. Prior to testing the goniometers a rather complete investigation was carried out by the Laboratory and reported in R-2692 "Balanced Input Sensitivity Measurements Using Unbalanced Signal Generator" dated 10 November 1945. The conclusions of this work were briefly that the use of balanced-to-unbalanced transformers was not necessary in the frequency range of 1.5 through 30 megacycles. Correct sensitivity figures can be obtained by any of six different unbalanced input circuits, provided the dummy antenna is suitable. In particular, feeding the goniometer stators through equal resistors from the signal generator hot and ground terminals to the balanced stator terminals gives the correct sensitivity value provided: (1) the signal generator output impedance is low and (2) the signal generator output voltage reading is the open-circuited voltage.

Methods Used in Sensitivity Tests

6. For the sensitivity measurements, the signal was introduced into only one stator, with the rotor so oriented as to be in its position of maximum coupling to the stator being fed. The rotor is thus in a position of minimum (theoretically zero) coupling to the unused stator which is left open. The signal was introduced through two 70 ohm resistors, one from the "hot" lead of the signal generator to a balanced stator terminal and the other from the ground lead to the other balanced stator terminal. The Model 65-B Standard Signal Generator (Serial #710) was used exclusively for all of the measurements. The output impedance of this generator varies from 5 ohms at 1.5 and 2 megacycles to almost 25 ohms at 30 megacycles. An input resistance of 140 ohms might be desired since the goniometers are constructed to be fed from transmission lines of 140 ohms surge impedance, but it was felt unwarranted to keep changing resistance values as the signal generator output impedance varied with frequency. As a result the input resistance varied from 145 to 165 ohms throughout the frequency range. The rotor output was conducted through the standard recommended 50 ohm cable into the DAU receiver (Serial #33). The receiver output was the d-c voltage from the second de-

[redacted]

tector, thus enabling cw sensitivity to be measured, and requiring no modulation or BFO. The receiver gain was adjusted for the accepted 20 db signal plus noise-to-noise ratio, using standard 10 volts signal plus noise d-c output.

Results of Sensitivity Measurements

7. The sensitivity figures were converted into decibels above one microvolt to afford an easier comparison between goniometers. In Plate 1 are graphs of sensitivity using the DAU goniometer both with and without the 330 ohm resistors across the stators. The DAU goniometer is better from sensitivity considerations without the 330 ohms, the average improvement being about 1 db in Band 1, 1.5 db in Band 2, and 2.5 db in Bands 3 and 4. A comparison of the sensitivities using the new DAJ goniometers from the factory with those used at the Dupont Station is shown in Plate 2. The difference is less than 1 db which is within the limit of observational and experimental accuracy. Plate 3 shows a comparison of the sensitivities using the DAU goniometer without the 330 ohm resistor and the Dupont DAJ goniometers. In Band 1 the DAL is from 0.5 and 2.5 db better than the DAU goniometer. In Bands 2 and 3 there is no significant difference between the DAM, DAN and the DAU goniometers. In Band 4 the DAU goniometer is better by over 2 db up to 22 megacycles but above that frequency the DAO goniometer is better by as much as 2 db.

GONIOMETER ACCURACY TESTS

Method of Testing

8. A rather extensive discussion of the problem involved in testing goniometers in the frequency range covered by the DAJ-a equipments is given in the Appendix and the development of the goniometer test rig used in these accuracy tests is considered in some detail. The method consists essentially of a voltage dividing network used to produce voltages of controlled variation across the two stators of the goniometer, thus giving selected spaced bearings about the azimuth of 360 degrees. In the final test rig adopted, three equal resistances are used for this voltage division as illustrated schematically in Fig 2 of Plate 16 and the actual bearings obtainable are given in Table 1. The two equal 70 ohm ($Z_s/2$) resistors are used to simulate the 70 ohm twin-coax transmission lines employed in the installations. The goniometer accuracies so obtained are the same as would result by using a dummy antenna and transmission line (as is demonstrated in the Appendix) so long as the impedances which both stators see are identical.

9. Although the test rig used for these measurements is not the ultimate in design, it is considered more than adequate for the purpose intended. Checks on the Model FH-4 equipment as mentioned in the appendix indicate that the error in bearing angle of the test rig does not exceed one degree and the maximum phase shift between stator voltages is less than 10 degrees at 30 megacycles. Four complete sets of error tests were made. Both the unused factory DAJ goniometers and the used Dupont DAJ goniometers were checked at representative frequencies throughout the range of the DAJ-a equipments. The DAU goniometer was tested both with and without the 330

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ohm resistors across the stators and at the same representative frequencies. The initial zero degree setting of each goniometer was made at the lower end of the four bands, thus duplicating the procedure used in the actual installations. The bearings were read on a 360 degree dial attached to the rotor. This dial was rotated until a minimum d-c voltage output was noted on a meter across the second detector of the DAU receiver. The readings were repeatable to within ± 1 degree.

Results of Tests

10. The results of the goniometer accuracy tests are presented graphically in Plates 4 through 13. Each plate, representing a single frequency, contains three separate curves of goniometer error in degrees versus the correct bearing: (1) The DAU goniometer, (2) DAJ goniometers from the Factory, and (3) DAJ goniometers from Dupont. No attempt was made to correct the plates by using the FH-4 calibration, since the FH-4 was considered to have at least ±1 degree error in bearing presentation. In the first three bands, from 1.5 through 14 megacycles, none of the goniometers exhibit errors in excess of 2.5 degrees. In Band 4 the errors begin to increase with frequency. The DAJ goniometers do not show errors in excess of 2.5 degrees through the entire band, but the zero is displaced at 30 megacycles by as much as 2 degrees. The DAU goniometer zero is likewise displaced by about a degree at 22 megacycles and by 2 or 3 degrees at 30 megacycles. The maximum bearing error in the DAU goniometer rises from 3.5 degrees at 22 megacycles to over 8 degrees at 30 megacycles. In general, it is noted that all of the curves show the conventional octantal error trend, but the error is negative in the first octant. This may be expected in a multi-turn distributed winding goniometer, but it is the reverse of a one turn or closely wound transformer systém (reference 7).

Explanation of Error

11. No attempt will be made in this report to give a complete explanation of the numerous causes for goniometer errors, but it seems in order here to mention briefly why the errors probably increase with frequency in the DAU goniometer. First, the stators, while being magnetically in space quadrature are not electrostatically shielded. This might permit capacitive coupling causing leakage between the two stators, which would explain the shifting of the zero. Secondly, there might be capacitive coupling between the rotor and stators, since these are not electrostatically shielded either. This could cause the conventional octantal error noted in the curves, destroying the pure sine wave coupling low by the introduction of odd harmonics.

Adcock Spacing Error Calculation

12. The fact that the bearing error produced in both the DAU and DAJ goniometers is negative in the first octant combined with the fact that the spacing error of an Adcock array is positive in the first octant brings up the very interesting question as to what extent these two errors cancel and thus neutralize one another. Consequently, the spacing error was computed for the #4 DAJ-a Array at 14, 22, and 30 megacycles. The well-known error equation -

$$\varepsilon = \arctan \frac{\sin\left(\frac{\pi \ell}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi \ell}{\lambda}\cos\theta\right)} - \theta \quad \ldots\ldots\ldots\ldots (1)$$

[redacted]

was used where $\theta$ is the correct angle of arrival of the signal in degrees, d is the distance between the opposite monopoles in meters, $\lambda$ is the wave length in meters, and then $\delta$ is the error in degrees. The results of the computation are shown in Plate 15. It is noted that the error curves are octantal with maximum error at 22-1/2 degrees. The spacing error is not particularly significant at 14 or 22 megacycles, but at 30 megacycles the error attains the peak value of over 3.5 degrees. A comparison of Plates 11, 12, 13 and 15 shows that to a large extent the goniometer errors do cancel the spacing errors. For the DAJ goniometers the resultant error is very small even at 30 megacycles, but with the DAU goniometer there is still a maximum resultant error of about 4.5 degrees. This cancellation of errors may be very helpful in explaining the low deviation curves obtained in actual field calibrations in Band 4 using the #4 DAJ Array.

CONCLUSIONS

13. Removing the 330 ohm resistors from across the stators of the DAU goniometer improves the over-all sensitivity of the DAJ-a equipment by from 1 to 2.5 db throughout the frequency range. There was no significant difference in sensitivity between the new factory DAJ and the used Dupont DAJ goniometers. The over-all improvement in sensitivity from using the DAJ goniometers in place of the DAU goniometer without the 330 ohm resistors across the stators was not appreciable. In Band 1 it was from 3 to 0 db, in Bands 2 and 3 it was less than 1 db, and in Band 4 it was from 3 to 0 db up to 22 Mc and from 0 to -2 db from 22 to 30 Mc.

14. A goniometer accuracy test rig covering the high frequency range was developed based upon a resistance voltage dividing network. The details of the problems encountered are considered at large in the appendix. The test rig was entirely suitable for the problem at hand and was as accurate as could be checked on the British FH-4 equipment.

15. The goniometer test rig developed has very definitely proved that the resistance voltage dividing method is practical in the high frequency range. It is considered that further refinements in design and construction might enable the extension of the applicability through at least part of the VHF range and might also increase the accuracy of the bearings.

16. It was demonstrated theoretically that the goniometer errors obtained by using the test rig are those to be actually existent in field installations provided the stators see like impedances from the transmission lines from the cathode followers. The test rig as used would apply to any goniometer system in which the bearings were not a function of this stator impedance. In other cases the impedance labelled in Figure 2 of Plate 16 would necessarily have to be a dummy antenna simulating the exact stator input impedance.

17. There was no observed difference in the accuracy of the DAU goniometer with and without the 330 ohm resistors across the stators. In Bands 1, 2, and 3 none of the goniometers tested exhibited bearing errors in excess of 2.5 degrees. In Band 4 the DAU goniometer grew progressively worse until at 30 megacycles it produced errors as large as 8 degrees. The DAO gonio-

[redacted]

meters did not give errors in excess of 3 degrees throughout Band 4, but the new factory DAO performed better than the used Dupont DAO. For the most part all of the errors were octantal in nature with some axial displacement at the highest frequencies.

18. For all of the frequencies tested, the unused factory DAJ goniometers showed less error than the used Dupont DAJ goniometers which definitely indicated that mechanical wear added to bearing errors.

19. The octantal spacing error for the DAO #4 Adcock Array was computed for 14, 22, and 30 megacycles. This error is positive in the first octant and increases with frequency to a maximum of about 3.5 degrees at 30 megacycles. The goniometer error, which is also octantal, was negative in the first octant, so that the two errors tend to neutralize one another. It happens that for the DAO goniometer the two errors practically cancel.

RECOMMENDATIONS

20. From sensitivity considerations above, no specific recommendations may be offered concerning the desirability of one goniometer over another. It is recommended, however, that whenever the DAU goniometer be used in conjunction with DAJ-a installations the 330 ohm resistors be removed from across the two stators.

21. In consideration of the results of the goniometer accuracy tests and also in view of mechanical advantages not mentioned in this report, it is recommended that the DAU goniometer might be employed in Bands 1, 2, and 3 of the DAJ-a equipments. It is further recommended that from accuracy considerations the DAO goniometers be used in Band 4. It would be desirable to provide new or rebuilt DAO goniometers for Band 4 since mechanical wear seemed to increase the bearing errors.

REFERENCES

1. BuShips conf. letter Serial No. 1435(925D) R&D 25D-96 dated 14 February 1945 to NRL: Request for Assignment of Problem S1046R-C.

2. NRL conf. ltr. C340-40/45(301B) dated 31 March 1945 to BuShips: Minutes of Conference at NRL on 7 February 1945 on DAJ Modernization.

3. Conf. report by Lt. Kane (Serial 976) Ser. GM-976-1944 dated 24 April 1945 on Matters Relative to DAJ-a: Phase (a) Comparison of DAU vs. DAJ Goniometer Sensitivities.

4. NRL conf. memo. C340-139/45 (301B) dated 14 July 1945: Minutes of Conference with A. E. Parker, CRE, on performance of Model DAJ-a at Supplemental.

5. NRL interim report on Problem S1046R-C (342:RA-WM) C-342-28/45 dated 27 March 1945 on Installation Changes Covering Modernization of Existing Model DAJ Equipment.

[redacted]

6. British Intelligence Report #1215 dated 2 October 1944: A Long and Meduim Wave Artificial Adcock Aerial Unit.

7. Keen, Ronald, "Wireless Direction Finding", Iliffe, London 1938.

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# Theoretical and Constructional Consideration of Apparatus for Determination of Goniometer Errors in the High Frequency Range

## Introduction

1. Prior to this investigation the Laboratory had not developed a suitable means of determining goniometer errors in the high frequency range. A rather uncertain method which had been used in the past consisted in "backing up" the unknown goniometer against the FH-3 goniometer and noting the difference in bearing for zero differential voltage. The uncertainties and disadvantages of this method of analysis are too numerous to discuss here, but suffice it to say that the technique has been considered with disfavor, especially since the accuracy of the FH-3 goniometer is not definitely known. Furthermore, the FH-3 goniometer does not completely cover the high frequency range, it being designed for the DAR band from 1 to 20 megacycles.

2. In accordance with the request of BuShips in reference 1 for an accuracy check of the DAU and DAJ goniometers over their working frequency range of from 1.5 to 30 megacycles, the Laboratory investigated methods for the determination of goniometer errors in the high frequency range. In reference 6, the British had reported a method of voltage variation of known amounts by constructing a resistance voltage-dividing network. The apparatus was reported to have been used successfully over the low and medium frequency ranges, but the set-up prohibited its use at high frequencies. The Laboratory decided to follow a two-course program: (1) begin a long-term project to develop mutual-inductance attenuators and (2) attempt to modify the British system to apply to the high frequency range. The first of these projects is still under investigation and will be the subject of a subsequent report. The latter project has been completed at least tentatively and is the subject of this Appendix.

## Specific Requirements

3. In order to understand fully the problems encountered and the specific requirements in the development of goniometer accuracy test apparatus which will be sufficiently reliable, a brief theoretical analysis of the goniometer principle will be useful. The conventional inductive goniometer consists of two fixed "stator" coils placed in space quadrature to one another to eliminate electromagnetic coupling and a single rotating "search" coil which moves about in the field produced by the stator in such a mechanical manner as to simulate the pick-up from a small rotating loop in a similar field in space. Assuming a voltage to exist in either stator, then, the rotor developes a sinusoidal voltage variation as it moves continuously through 360 degrees. In Figure 1 of Plate 16 is shown a simple schematic diagram of the arrangement of the goniometer coils.

4. To make the analysis general, the coupling law will be taken in its most general form:

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$$M_1 = K\sqrt{L_1 L_r}\sum A_n \sin n\theta \qquad (1)$$

$$M_2 = K\sqrt{L_2 L_r}\sum B_n \cos n\theta \qquad (2)$$

5. Here, $M_1$ and $M_2$ are mutual inductances as indicated in Figure 1 of Plate 16 and $K$ is the maximum coefficient of coupling between the stator and rotor. Also, $A_n$ and $B_n$ are constants depending upon the configuration and electrical characteristics of the goniometer also assume:

$$Z_1 = Z_{11} + j\omega L_1 \qquad Z_2 = Z_{22} + j\omega L_2$$

Mesh currents have been established, as indicated in Figure 1 of Plate 16, to facilitate the analysis: $E_1$ and $E_2$ are assumed to be in phase.

$$E_1 = I_1 Z_1 - j\omega M_1 I_r \qquad (3)$$

$$E_2 = I_2 Z_2 - j\omega M_2 I_r \qquad (4)$$

$$0 = I_r (Z_r + j\omega L_r) - j\omega M_1 I_1 - j\omega M_2 I_2 \qquad (5)$$

Solving equations (3) and (4) for $I_1$ and $I_2$ in terms of $I_r$:

$$I_1 = \frac{E_1 + j\omega M_1 I_r}{Z_1} \qquad (6)$$

$$I_2 = \frac{E_2 + j\omega M_2 I_r}{Z_2} \qquad (7)$$

Substituting equations (6) and (7) in equation (5):

$$I_r (Z_r + j\omega L_r) = j\omega M_1 \left(\frac{E_1 + j\omega M_1 I_r}{Z_1}\right) + j\omega M_2 \left(\frac{E_2 + j\omega M_2 I_r}{Z_2}\right) \qquad (8)$$

$$I_r Z_r + j\omega L_r I_r = \frac{j\omega M_1 E_1}{Z_1} - \frac{\omega^2 M_1^2 I_r}{Z_1} + \frac{j\omega M_2 E_2}{Z_2} - \frac{\omega^2 M_2^2 I_r}{Z_2} \qquad (9)$$

Solving for $I_r$:

$$I_r = \frac{j\omega\left(\frac{M_1 E_1}{Z_1} + \frac{M_2 E_2}{Z_2}\right)}{Z_r + j\omega L_r + \omega^2\left(\frac{M_1^2}{Z_1} + \frac{M_2^2}{Z_2}\right)} \qquad (10)$$

[redacted]

For a null, let $I_r$ be zero, thus $E_r = 0$; then

$$M_1E_1Z_2 + M_2E_2Z_1 = 0 \qquad (11)$$

Equation (11) indicates that the bearing angle $\theta$ definitely depends upon the stator impedances if they are not equal, which is to be expected. Now let $Z_{11} = Z_{22}$ and then:

$$M_1E_1 + M_2E_2 = 0 \qquad (12)$$

$$\frac{M_1}{M_2} = -\frac{E_2}{E_1} \qquad (13)$$

Therefore:

$$\frac{K\sqrt{L_1L_r}\sum_{n=1}^{\infty} A_n \sin N\theta}{K\sqrt{L_2L_r}\sum_{n=1}^{\infty} B_n \cos N\theta} = -\frac{E_2}{E_1} \qquad (14)$$

6. Equation (14) shows that the bearing angle, $\theta$, is independent of the stator input impedance provided it is the same for both stators.

7. Assuming the maximum coefficient of coupling to be identical between each stator and the rotor and that the self inductances of the stators are equal, the equation reduces to:

$$\frac{\sum_{n=1}^{\infty} A_n \sin N\theta}{\sum_{n=1}^{\infty} B_n \cos N\theta} = -\frac{E_2}{E_1} \qquad (15)$$

8. For the case in which the pure sine law relations hold, i.e., no goniometer error, n is unity and $A_n = B_n$ :

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{E_2}{E_1} \qquad (16)$$

This equation shows that the bearing is a function of the voltages appearing across the stators when: (1) these voltages have the same phase, and (2) there is no error in the goniometer. In other words, by producing in the stators two voltages $E_1$ and $E_2$ of the same phase, equation (16) can be used to determine the bearing which would be obtained in an ideal goniometer. This provides a standard upon which the correct bearings can be calculated from known values of $E_1$ and $E_2$. From this theoretical discussion it is seen that the bearing does not change with the input impedance provided the input impedance is the same for both stators. Furthermore it makes no difference what type of component is used for the voltage division, provided the impedance of both stators remains practically constant throughout the entire frequency range. It was decided that a resistance division would be most suitable over the 20 to 1 frequency range studied since it could be made more independent of frequency. Further, it was desirable to make the resistance very low compared with 140 ohms so that the input impedance would remain as constant as possible for all combinations of voltage. The schematic diagram in Plate 16, Figure 2 indicates the general principle of the test apparatus desired. The equal voltage terminals are marked by the numerals and the goniometer stators are marked AB and CD respectively. By placing these stator terminals at various taps on the resistance dividing circuit, the different bearings are produced. The correct bearings for any setting may be computed from equation (15). A list of the bearings obtainable and the settings required may be found in Table 1.

Development of Test Apparatus

9. From the theoretical discussion presented in the last section it seemed advisable to use resistances of small value to effect the voltage division. A brief consideration revealed that three equal resistors in series would be the simplest circuit giving sufficient bearings for a good error curve. Consequently, the first test rig consisted of three carbon rod resistors physically placed along three sides of a square and electrically connected in series across the output of a signal generator. Each of these resistors was made from a piece of carbon rod one-fourth inch in diameter and two and one-half inches long, and each had a d-c resistance of 0.07 ohms. The resistance network was provided with four taps, one at each end of the set and the other two between the resistors. At each tap was placed a 70 ohm resistor leading to a bananna plug which extended through a shield box to provide external taps. The different outputs were taken from these external plugs into the stator coils of the goniometer. This test set-up was found to have considerable error and blur up to 50 percent.

10. One of the first and obvious revisions was to shift the location of the four 70 ohm resistors. These were placed in series with the balanced stator leads and the subsequent voltage division taken at the remote ends of the resistors. This resulted in terminating each stator in 140 ohms at all times. The blur was greatly reduced, but the performance was far from satisfactory. The chief disadvantage was in making the three carbon rods exactly the same r-f resistance, especially since 0.07 ohm was such a small value.

11. For a further modification the same general arrangement of elements was used, but the carbon rods were each replaced by four matched ten ohm resistors in parallel, giving three 2.5 ohm resistances in series across the signal generator. A measurement of the voltages across the taps showed that the voltage outputs varied when the stator leads were reversed. It was believed that this was due to the fact that one side of the signal generator and consequently one side of the test rig was grounded. To avoid this an unbalanced-balanced transformer was placed between the test rig and the signal generator, and another voltage measurement was made. This measurement showed a decrease in the difference of readings by reversing the leads, but not a complete elimination. By slotting the holes through which the plugs extended, thus reducing circulating currents, the difference was completely removed. However, the test apparatus still showed appreciable error. The impedance of the resistance dividing network was measured at 30 megacycles on a General Radio 916-A bridge and was found to be $80 + j200$ ohms. This

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indicated that the network was far from being a pure resistance bit; that there was actually considerable inductance present.

12. In order to reduce the effects of inductance and capacitance, so that the stator impedances could be maintained practically constant, a special design was made, and the final complete system including the balance-to-unbalanced transformer used to isolate the resistances from the system ground is shown in Plate 17. The actual voltage dividing network is shown assembled in Plate 18. It is made up of three sets of four carbon resistors in parallel, each resistor having a value of ten ohms, and each set 2.5 ohms. The four resistors which made up a set were mounted between two brass caps, 7/8 inch in diameter, 1/4 inch in length, along the circumference of a 1/2 inch circle (see Plate 19). These brass caps were tapped to enable a connecting rod to be threaded into it. The connecting links were made of brass and the complete set-up joined into one unit by use of the connecting links. This unit was slightly under six inches in length. The connecting links contained the leads to the taps and also the provisions for mounting the set. The leads of the resistors were mounted through the brass cap thus giving a negligible lead length to the resistor. Impedance measurements were also made on this final test rig using a General Radio 916A Impedance Bridge. The following results were obtained:

| At 1 Megacycle: from taps | Complex Impedance | Impedance |
|---|---|---|
| 1-2 | 2.5 + j0 | 2.5 ∠0° |
| 2-3 | 2.5 + j0 | 2.5 ∠0° |
| 3-4 | 2.5 + j0 | 2.5 ∠0° |
| 1-3 | 5.0 + j0 | 5.0 ∠0° |
| 1-4 | 7.5 + j0 | 7.5 ∠0° |
| At 15 Megacycle: from taps | | |
| 1-2 | 2.5 + j0.8 | 2.6 ∠17.7° |
| 2-3 | 2.5 + j0.8 | 2.6 ∠17.7° |
| 3-4 | 2.5 + j0.8 | 2.6 ∠17.7° |
| 1-3 | 5.0 + j1.7 | 5.3 ∠18.8° |
| 1-4 | 7.5 + j3.2 | 8.1 ∠23.1° |
| At 30 Megacycle: from taps | | |
| 1-2 | 2.6 + j1.5 | 3.0 ∠30° |
| 2-3 | 2.6 + j1.5 | 3.0 ∠30° |
| 3-4 | 2.6 + j1.5 | 3.0 ∠30° |
| 1-3 | 5.2 + j3.8 | 6.4 ∠36.2° |
| 1-4 | 7.8 + j6.5 | 10.1 ∠39.8° |

[redacted]

## Calibration of Test Rig

13. In order to obtain some idea as to the accuracy of the test rig constructed, the bearings were compared on the British FH-4 equipment which is in actuality nothing more than a complex oscillograph with two identical balanced receivers. A definite advantage of the FH-4 is that when properly aligned the bearing indicated is the resultant only of voltage magnitude applied to the receiver inputs and is not a function of the phase difference between the two input signals. The phase difference, however, is discernible from a broadening of the pattern into an ellipse, which method of determining phase shift is frequently employed in conjunction with conventional oscillographs. The particular FH-4 used for this comparison exhibited a certain amount of i-f leakage between the output circuits feeding the oscilloscope plates which could not be completely neutralized. This condition caused an indication uncertainty of about a degree, so the comparison cannot be considered more accurate than ± 1 degree. The results of the calibration are shown in Plate 14. The difference between the calculated test rig bearing and the FH-4 observed bearing does not exceed two degrees throughout the working frequency range of the DAJ-a equipment. Because this difference was so small, error corrections were not applied to the subsequent data taken. It was further observed that the maximum phase shift between the inputs from the test rig at any frequency was 10 degrees, found at 30 megacycles at a calculated bearing indication of 161.6 degrees.

## Effect of Phase Shift Upon Null-Type Bearing

14. In order to determine the effect of a phase shift between stator input voltages on a null-type direction finder, it will be necessary to extend the theory which led to equation (16). To do this, just multiply both members of equation (10) by $Z_r$ so that $E_r = I_r Z_r$ and thus:

(16) (3)

$$I_r Z_r = E_r = \frac{j\omega\left[\frac{M_1 E_1}{Z_1} + \frac{M_2 E_2}{Z_2}\right]}{Z_r + j\omega L_r + \omega^2\left[\frac{M_1^2}{Z_1} + \frac{M_2^2}{Z_2}\right]} Z_r \qquad (17)$$

Assume

$$Z_1 = Z_2 \quad \text{and} \quad M_1 = C_1 \sin\theta$$
$$M_2 = C_1 \cos\theta$$

Then

$$E_r = \frac{j\omega C_1 (E_1 \sin\theta + E_2 \cos\theta)}{Z_1 Z_r + jZ_1\omega L_r + \omega^2 C_1^2} Z_r \qquad (18)$$

Introducing a phase shift $\phi$ into the voltage of one of the stators, the voltages become:

$$E_1 \;,\; E_2\,\varepsilon^{j\phi} \;,\; \text{and } E_r$$

[redacted]

Then

$$E_r = \frac{j\omega C_1 Z_r}{Z_1 Z_r + jZ_1 \omega L_r + \omega^2 C_1^2} \left[E_1 \sin\theta + E_2 \; {}^{j\phi} \cos\theta\right] \qquad (19)$$

Let:

$$\frac{j\omega C_1 Z_r}{Z_1 Z_r + jZ_1 \omega L_r + \omega^2 C_1^2} = C_2$$

This is done since the only variable of interest is $\theta$. It must be remembered, however, that $C_2$ here is a variable with .

Therefore:

$$E_r = C_2 \left[E_1 \sin\theta + E_2 \, \varepsilon^{j\phi} \cos\theta\right] \qquad (20)$$

A simple inspection of equation (20) shows that $E_r$ is now a complex voltage. Setting $E_r$ equal to zero would therefore be meaningless from the practical standpoint since the ordinary goniometer cannot provide for reducing both the real and imaginary components of a voltage to zero simultaneously. In other words, an inductive goniometer acts not as a phase discriminator but as a magnitude discrimination of the resultant rotor voltage. It thus becomes obvious that it is the absolute value of $E_r$ with which the goniometer rotor output is concerned. Expanding the factor $\varepsilon^{j\phi}$ in equation (20) in terms of a complex trigonometric expression:

$$E_r = C_2 \left[E_1 \sin\theta + E_2 \cos\phi \cos\theta + jE_2 \sin\phi \cos\theta\right] \qquad (21)$$

The absolute value of $E_r^2$ is given by:

$$E_r^2 = C_2 \left\{(E_1 \sin\theta + E_2 \cos\phi \cos\theta)^2 + E_2^2 \sin^2\phi \cos^2\theta\right\} \qquad (22)$$

Although it may seem obvious at this point that $|E_r^2|$ can never be zero for an actual goniometer, a short theoretical digression will prove the argument.

Suppose $|E_r^2|$ be zero; then:

$$E_1^2 \sin^2\theta + 2E_1E_2 \cos\phi \cos\theta \sin\theta + E_2^2 \cos^2\theta = 0 \qquad (23)$$

Divide all terms by $\cos^2\theta$ :

$$E_1^2 \tan^2\theta + 2E_1E_2 \cos\phi \tan\theta + E_2^2 = 0 \qquad (24)$$

Solve for $\tan\theta$ :

$$\tan\theta = \frac{-2E_1E_2 \cos\phi \pm \sqrt{4E_1^2E_2^2 \cos^2\phi - 4E_1^2E_2^2}}{2E_1^2} \qquad (25)$$

[redacted]

$$\tan \theta = \frac{-2E_1E_2 \cos \phi \pm j2E_1E_2 \sin \phi}{2E^2} \qquad (26)$$

$$\tan \theta = \frac{-E_2 \cos \phi \pm jE_2 \sin \phi}{E_1} \qquad (27)$$

But equation (27) is not physically realizable since θ must be a real angle. Therefore, $|E_r^2|$ cannot be zero. This condition is generally referred to in the literature as "blur". There is no null, and the bearing indication consists of a minimum signal. This bearing value may be determined theoretically by taking the partial derivative of $|E_r^2|$ with respect to θ and setting the result equal to zero for a minimum. Operating upon equation (22)

$$\frac{2\,|E_r|}{C_2}\frac{\partial |E_r|}{\partial \theta} = 2(E_1 \sin\theta + E_2 \cos\phi \cos\theta)(+E_1 \cos\theta - E_2 \cos\phi \sin\theta) - 2E_2^2 \sin^2\phi \sin\theta \cos\theta = 0 \qquad (28)$$

Setting $\frac{\partial |E_r|}{\partial \theta}$ equal to zero:

$$E_1^2 \sin\theta\cos\theta + E_1E_2 \cos\phi \cos^2\theta - E_1E_2 \cos\phi \sin^2\theta - E_2^2 \cos^2\phi \sin\theta \cos\theta - E_2^2 \sin^2\phi \sin\theta \cos\theta = 0 \qquad (29)$$

Collecting and re-arranging terms:

$$(E_1^2 - E_2^2) \sin\theta \cos\theta = E_1 E_2 \cos\phi (\sin^2\theta - \cos^2\theta) \qquad (30)$$

$$(E_1^2 - E_2^2) \sin 2\theta = 2E_1E_2 \cos\phi \cos 2\theta \qquad (31)$$

Finally:

$$\tan 2\theta = \frac{2E_1E_2 \cos\phi}{E_1^2 - E_2^2} \qquad (32)$$

Equation (32) shows that the observed bearing angle θ is a function of the phase difference $\phi$ as well as the voltage magnitudes $E_1$ and $E_2$. By letting $\phi$ be zero and expressing $\tan 2\theta$ in terms of θ, equation (32) is seen to be identical to equation (16). The error   in degrees, due to the introduction of the phase shift $\phi$ between the two stator voltages is given by:

$$= 1/2 \text{ arc tan } - \frac{2E_1E_2 \cos\phi}{E_1^2 - E_2^2} - \text{arc tan } - \frac{E_2}{E_1}$$

[redacted]

This error function is based upon the assumption that the magnitudes of the voltages do not change but there is only a phase shift between them. The following calculation is for a phase shift $\phi = 10^\circ$ when $E_1 = 3E_2$:

$$= 1/2 \text{ arc tan } - \frac{2\,(3E_2^{\,2})\ \text{Cos } 10^\circ}{9E_2^{\,2} - E_2^{\,2}} - \text{arc tan } - \frac{E_2}{3E_2}$$

$$= 1/2 \text{ arc tan } - \frac{6\ (0.985)}{8} - \text{arc tan } - 1/3$$

$$= 1/2 \text{ arc tan } -0.739 - \text{arc tan } -0.333$$

$$= 1/2\ (36.4) \pm 18.4$$

$$= 0.2 \text{ degree}$$

From this computation it is seen that the maximum phase shift, encountered in the test rig only, produces an error of but two-tenths of one degree. Therefore, the phase differénces inherent in the test rig constructed are not sufficient to cause significant bearing errors.

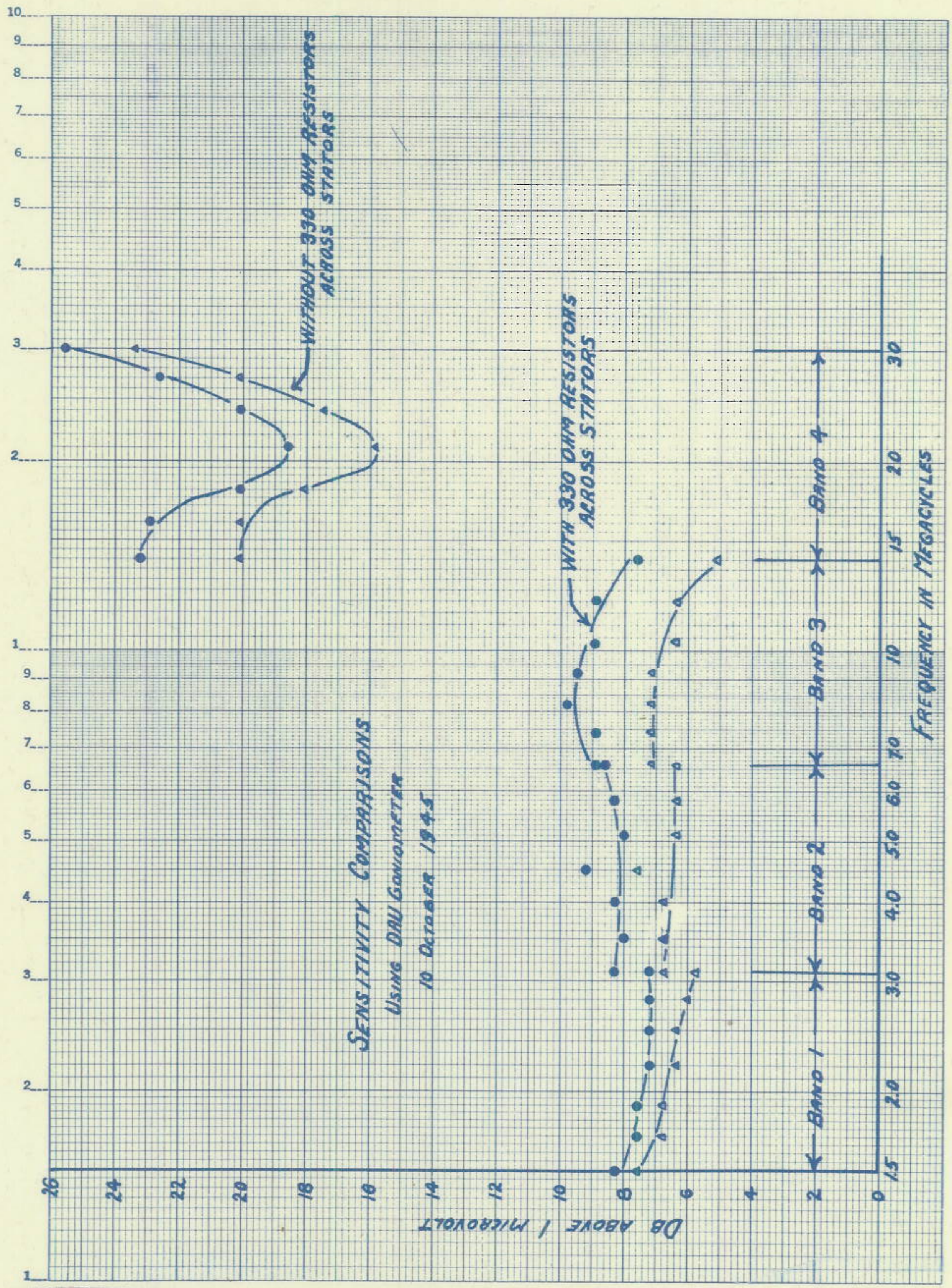
[redacted]

## TABLE I

Terminal Settings and Bearings Obtained
From Test Rig Constructed

| Stator Terminal Settings on Test Rig (See Plate 16) | | | | Bearing Obtained In Degrees |
|---|---|---|---|---|
| A | B | C | D | |
| 1 | 4 | 4 | 4 | 0 |
| 1 | 4 | 3 | 4 | 18.4 |
| 2 | 4 | 3 | 4 | 26.6 |
| 1 | 4 | 2 | 4 | 33.7 |
| 2 | 4 | 2 | 4 | 45.0 |
| 2 | 4 | 1 | 4 | 56.3 |
| 3 | 4 | 2 | 4 | 63.4 |
| 3 | 4 | 1 | 4 | 71.6 |
| 4 | 4 | 1 | 4 | 90.0 |
| 4 | 3 | 1 | 4 | 108.4 |
| 4 | 3 | 2 | 4 | 116.6 |
| 4 | 2 | 1 | 4 | 123.7 |
| 4 | 2 | 2 | 4 | 135.0 |
| 4 | 1 | 2 | 4 | 146.3 |
| 4 | 2 | 3 | 4 | 153.4 |
| 4 | 1 | 3 | 4 | 161.6 |
| 4 | 1 | 4 | 4 | 180.0 |

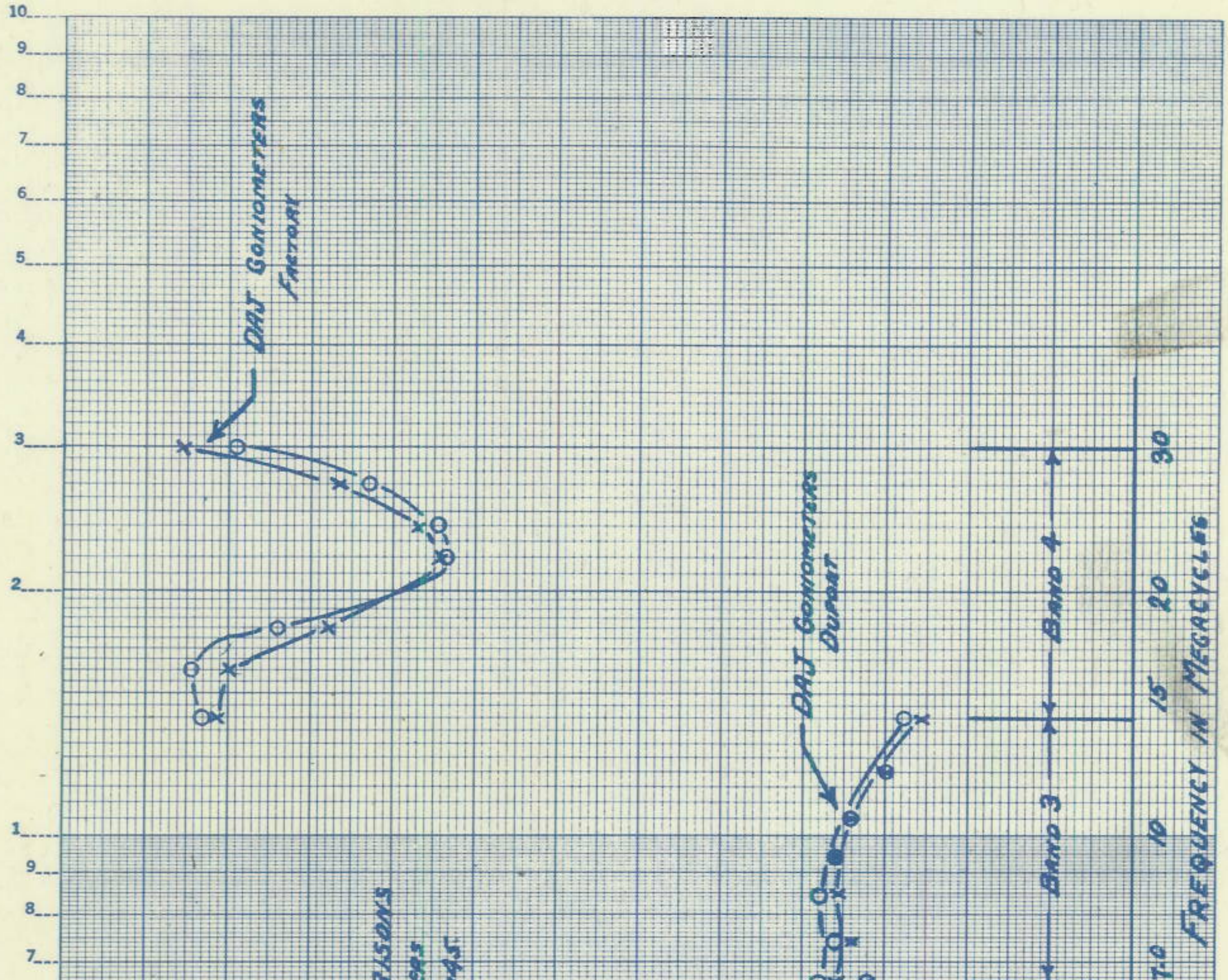
[redacted]




[redacted]

R-2707

PLATE 1

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R - 2707

PLATE 2

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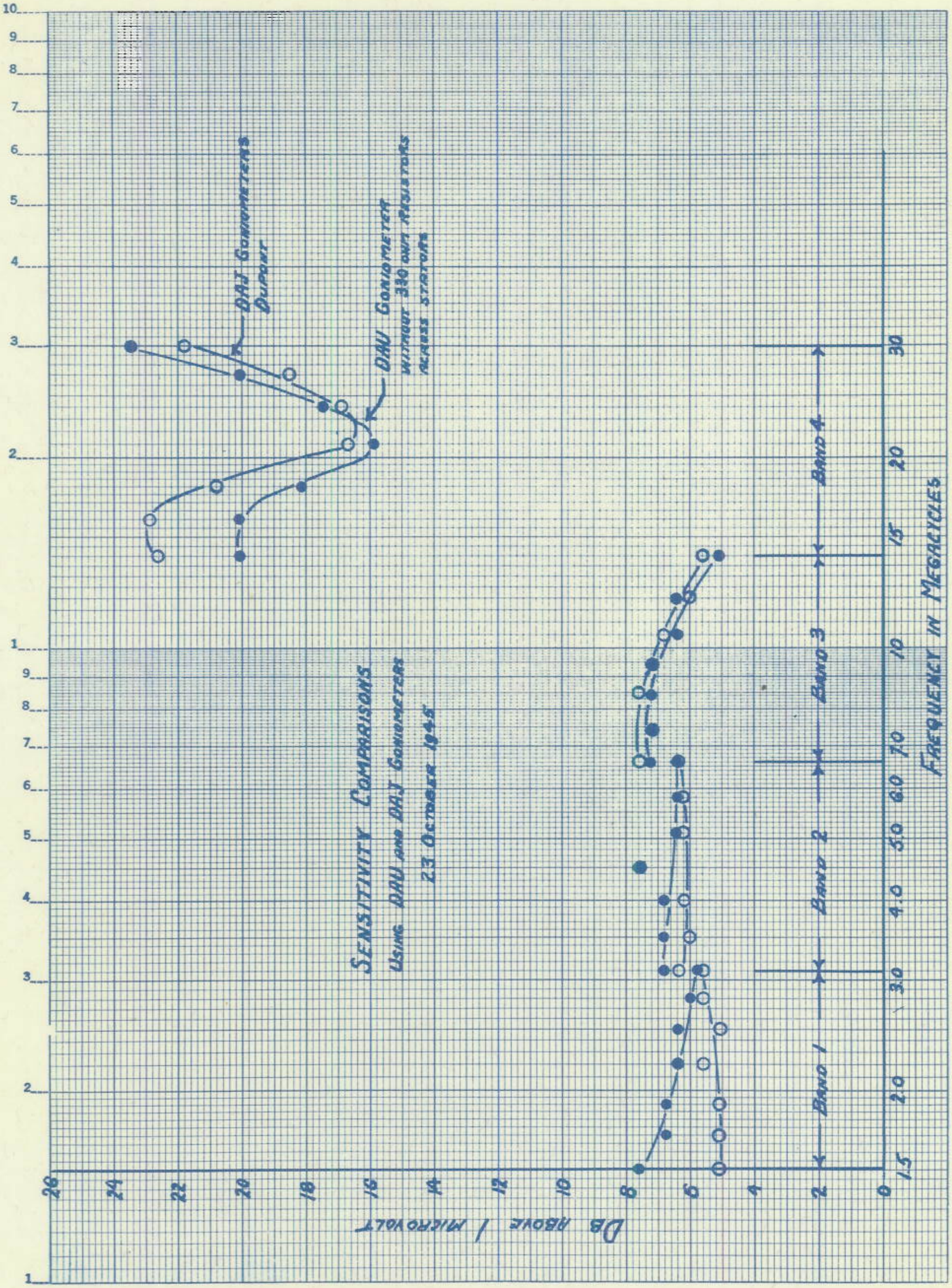


[redacted] R-2707 PLATE 3

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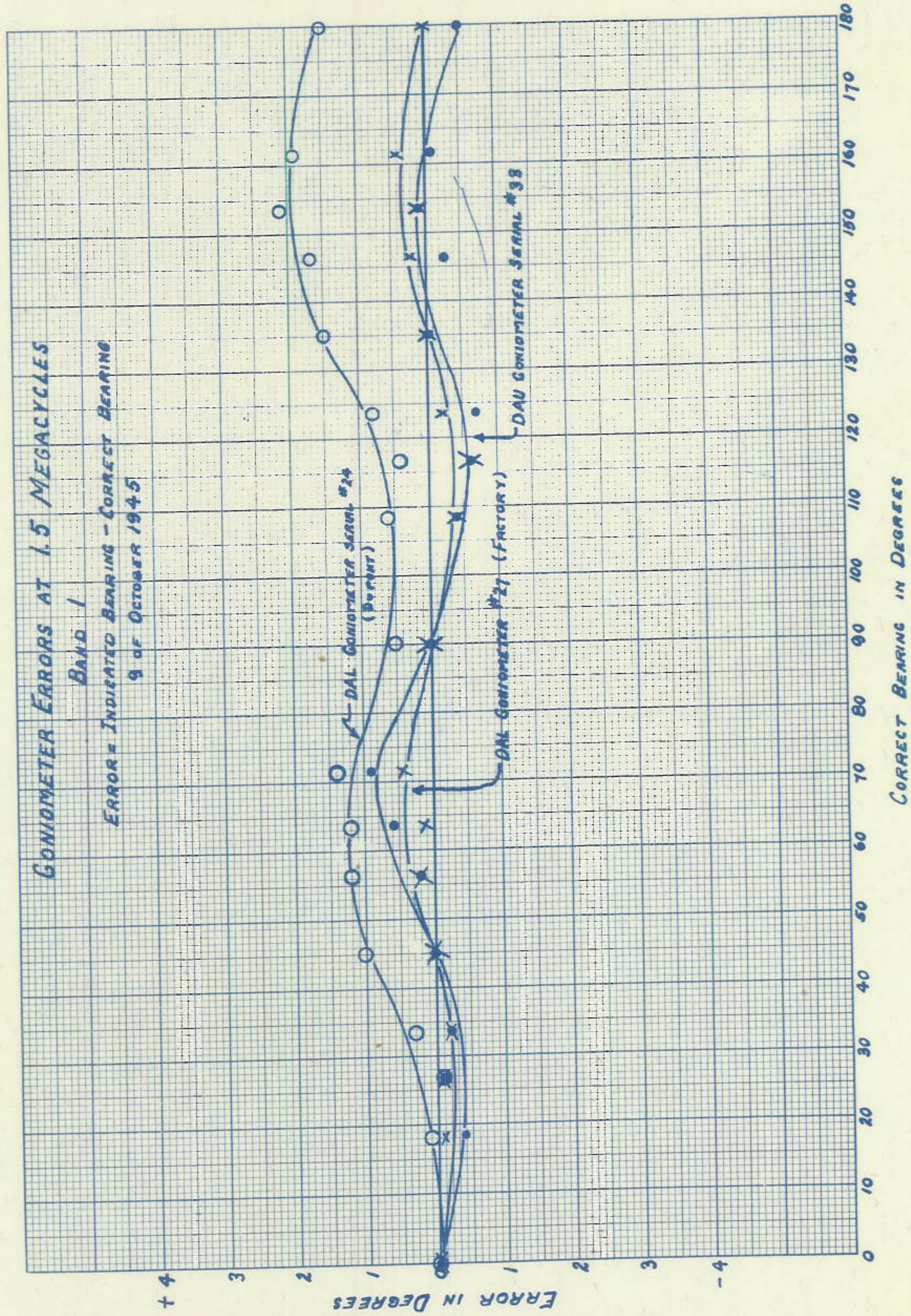


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R-2707

PLATE 4

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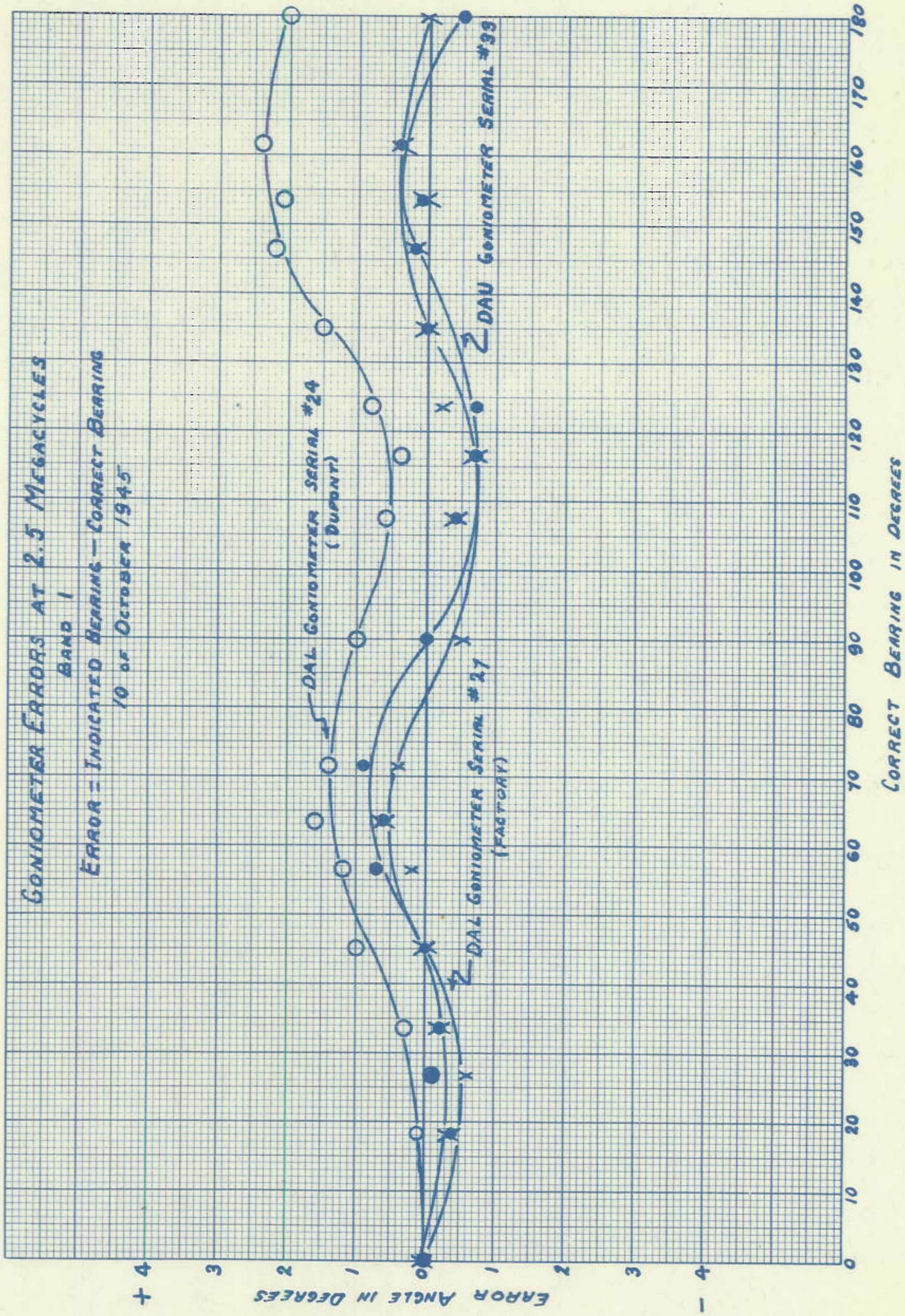


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R-2707

PLATE 5

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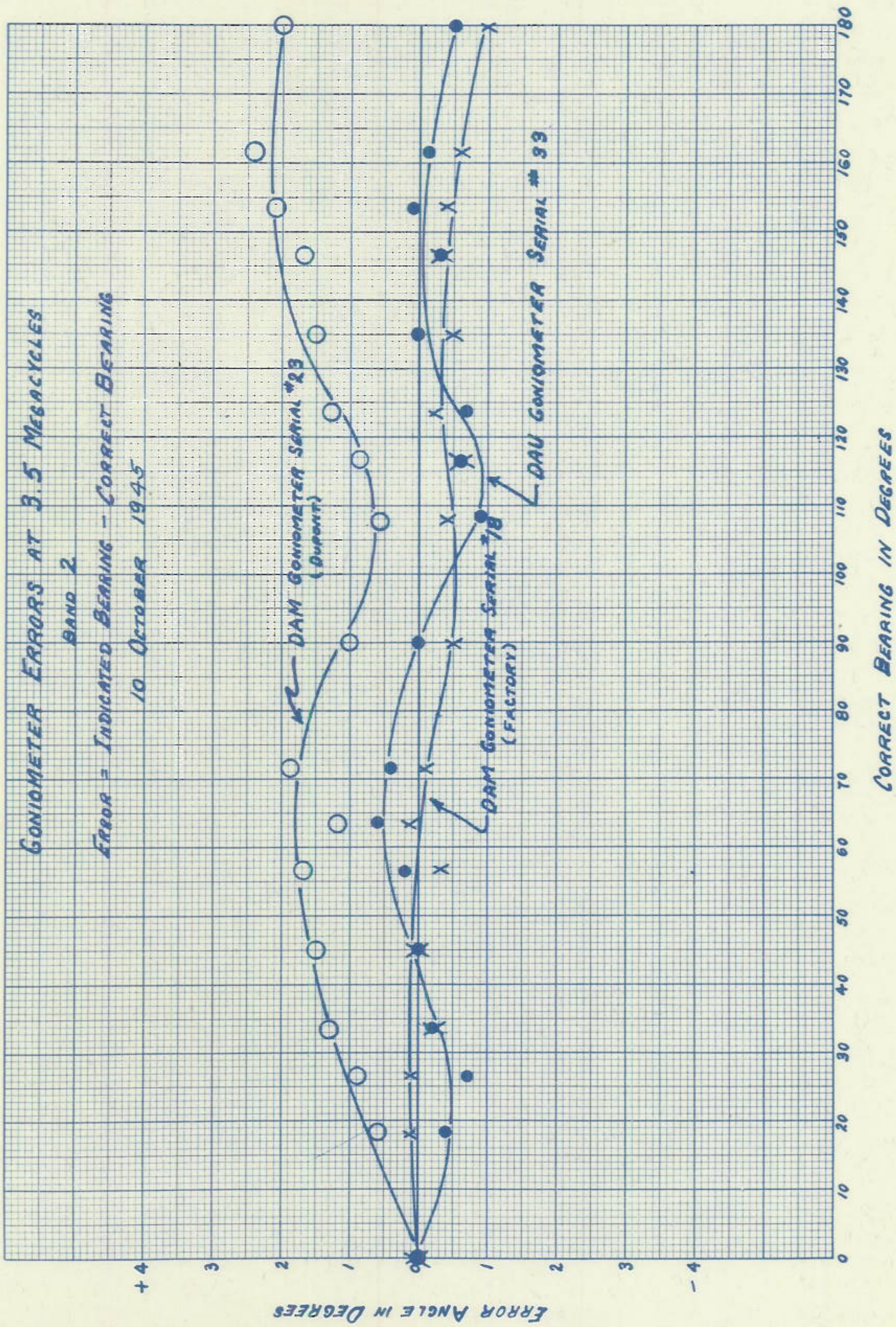


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R - 2707

PLATE 6

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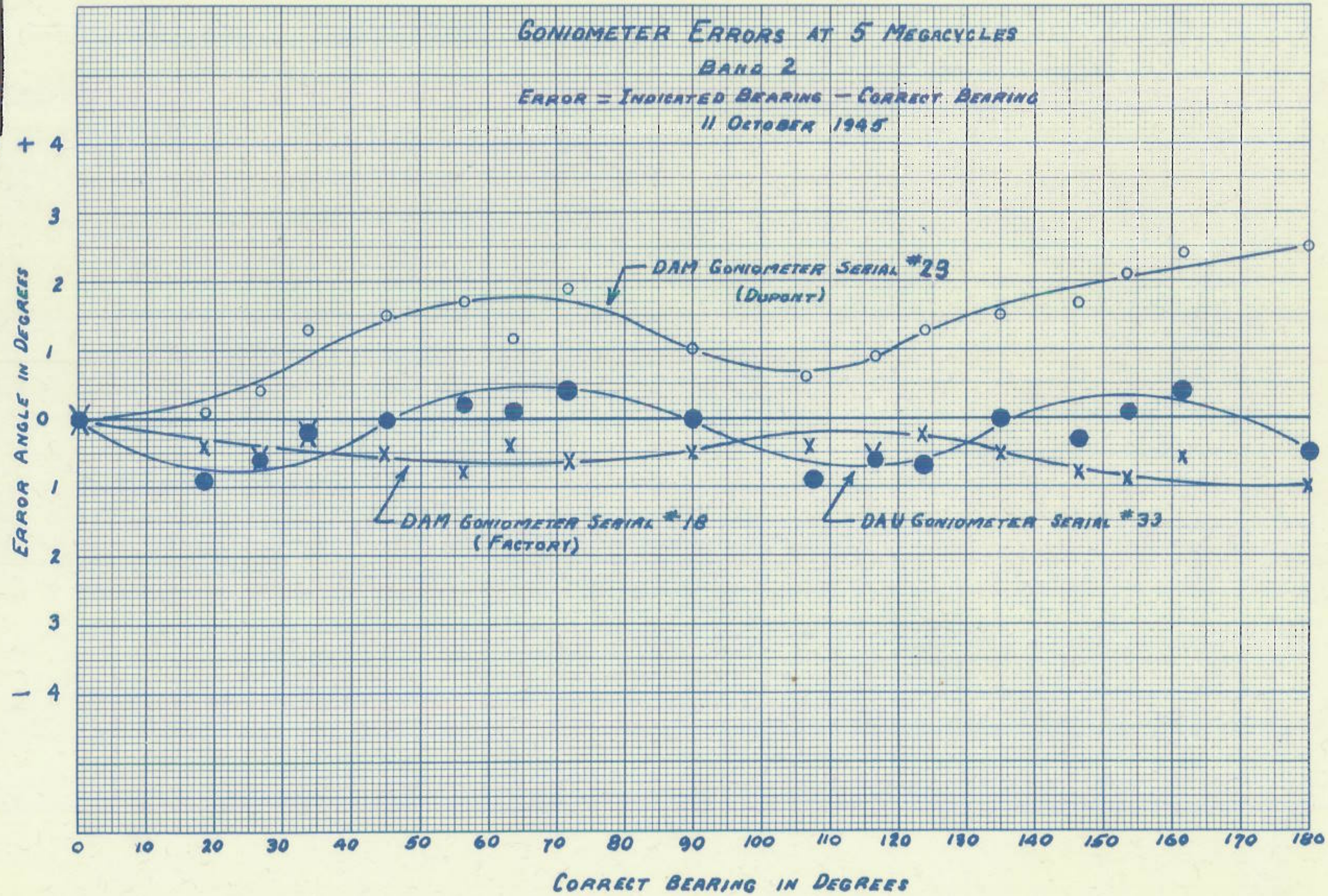


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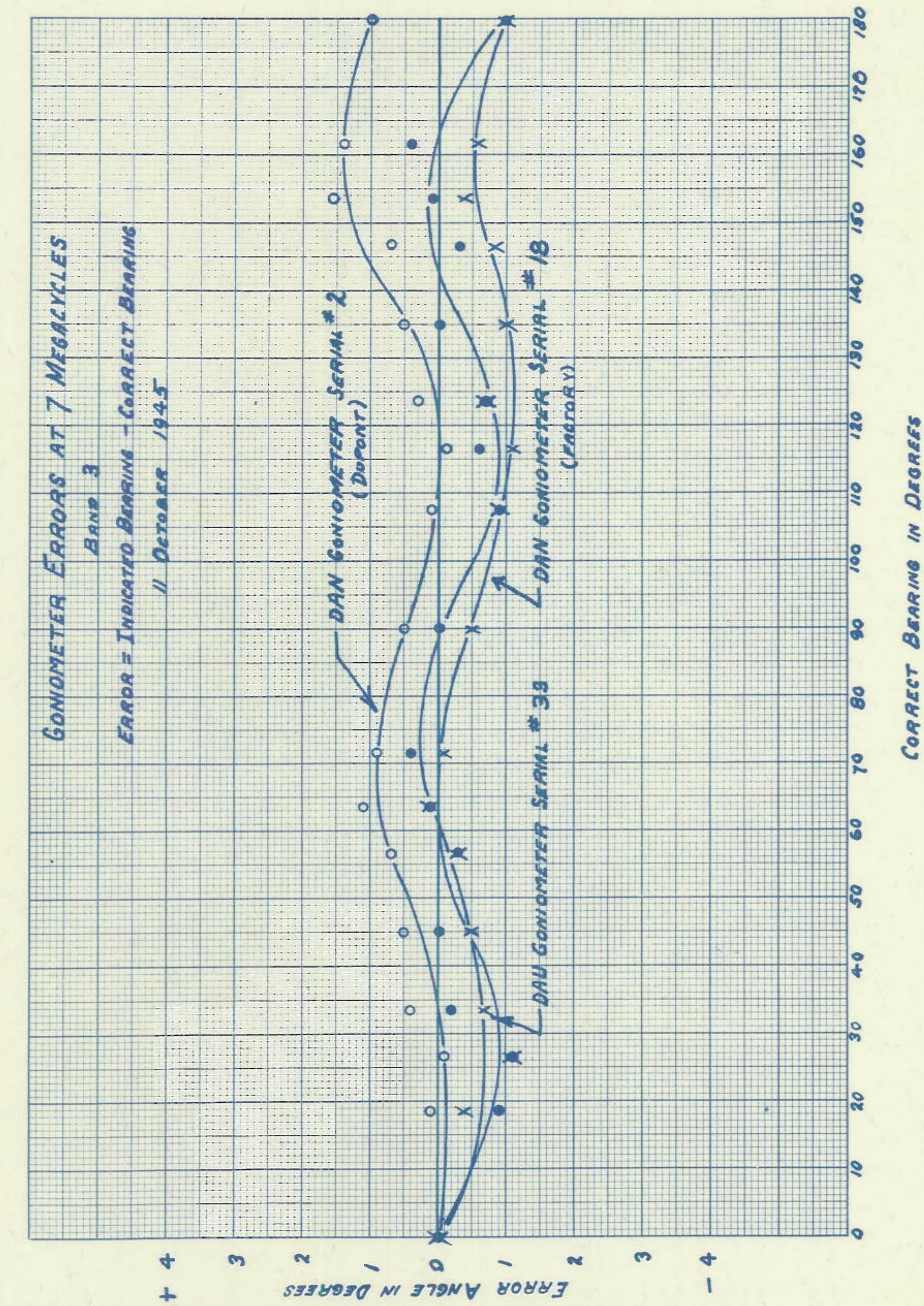
R-2707

PLATE 7

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R - 2707

PLATE 8

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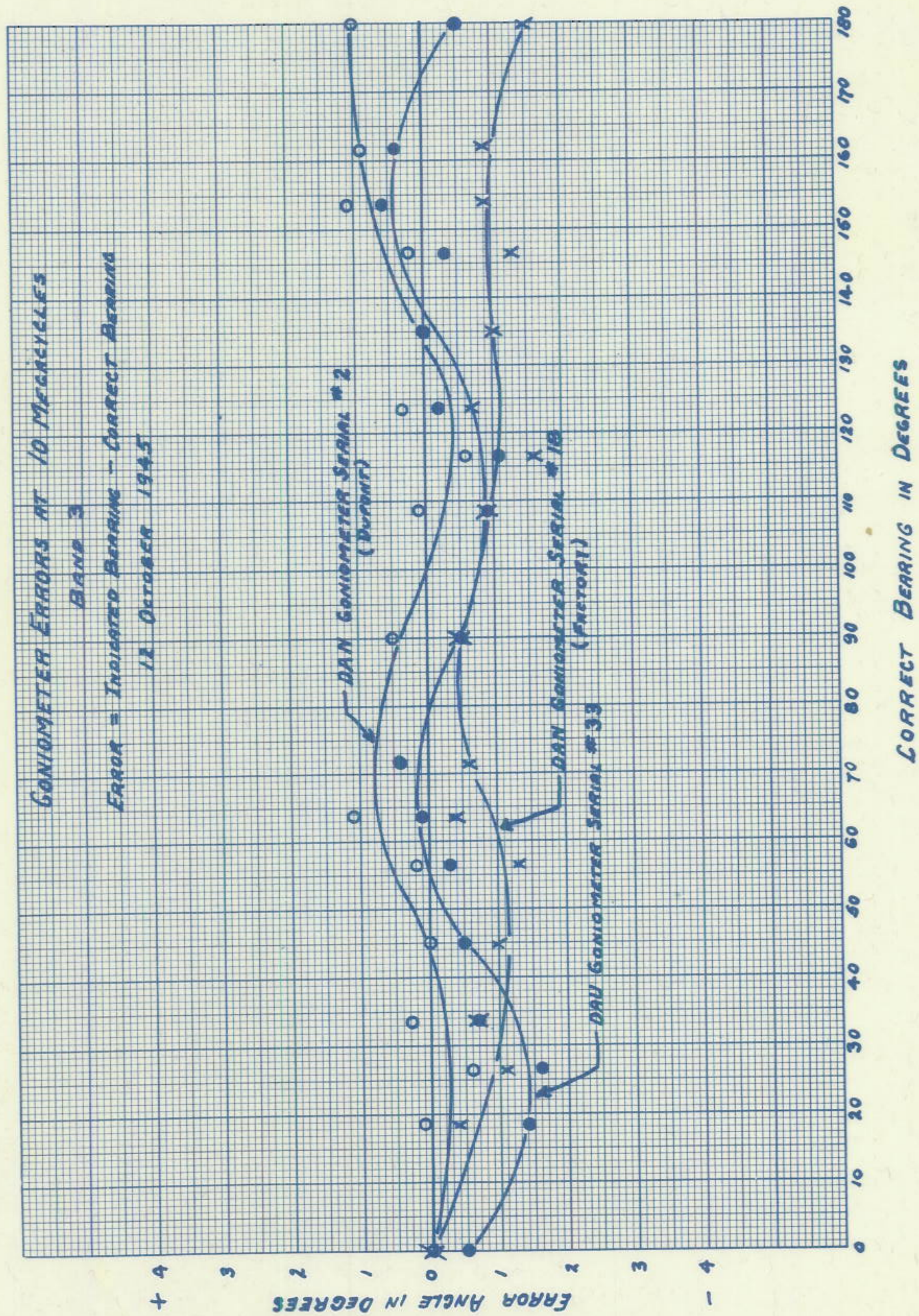


[redacted] R-2707 PLATE 9

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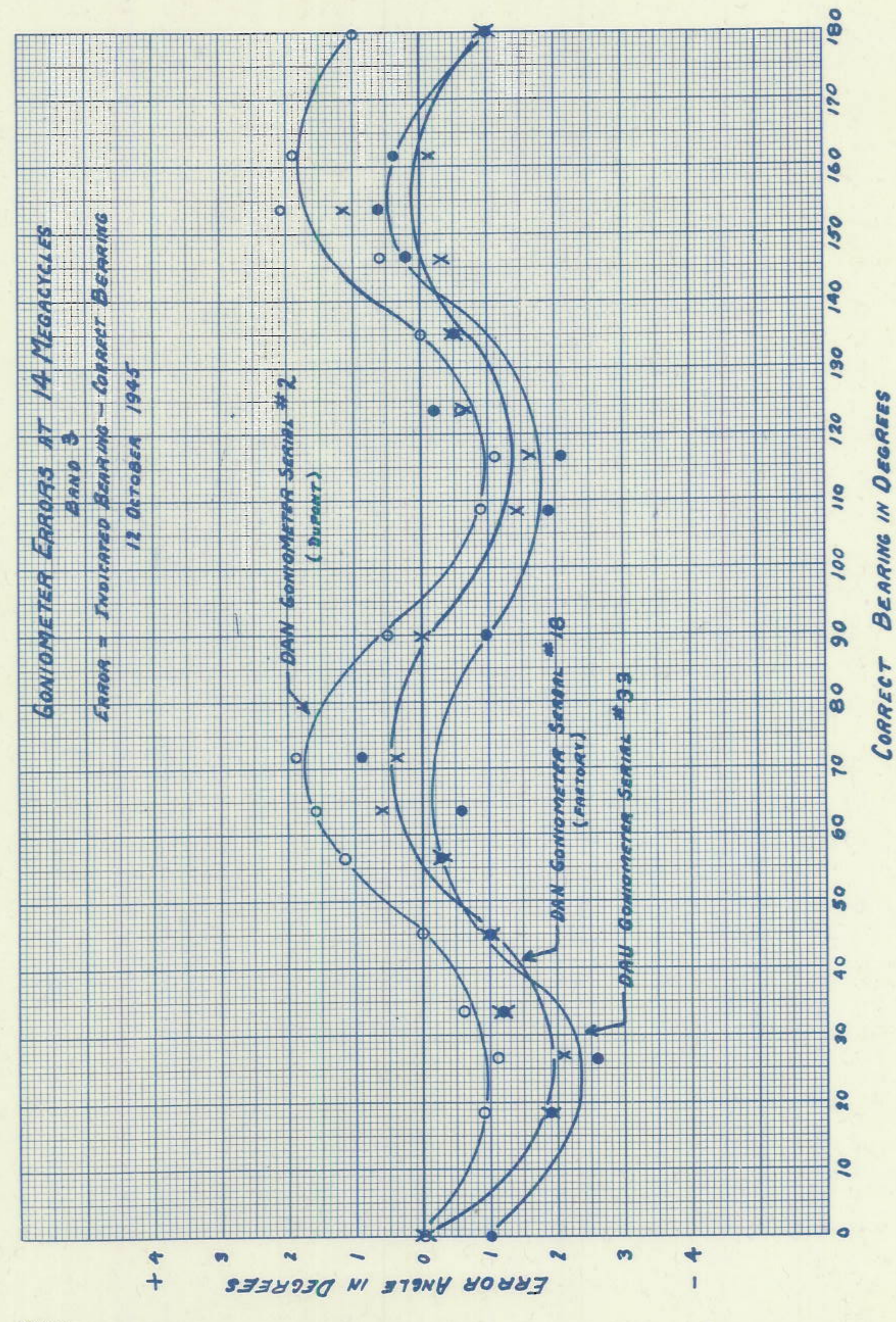
GONIOMETER ERRORS AT 14 MEGACYCLES
BAND 3
ERROR = INDICATED BEARING - CORRECT BEARING
12 OCTOBER 1945
DAN GONIOMETER SERIAL #2
(DUPONT)
DAN GONIOMETER SERIAL #18
(FACTORY)
DAU GONIOMETER SERIAL #33
CORRECT BEARING IN DEGREES
ERROR ANGLE IN DEGREES
+4
3
2
1
0
1
2
3
-4
0
10
20
30
40
50
60
70
80
90
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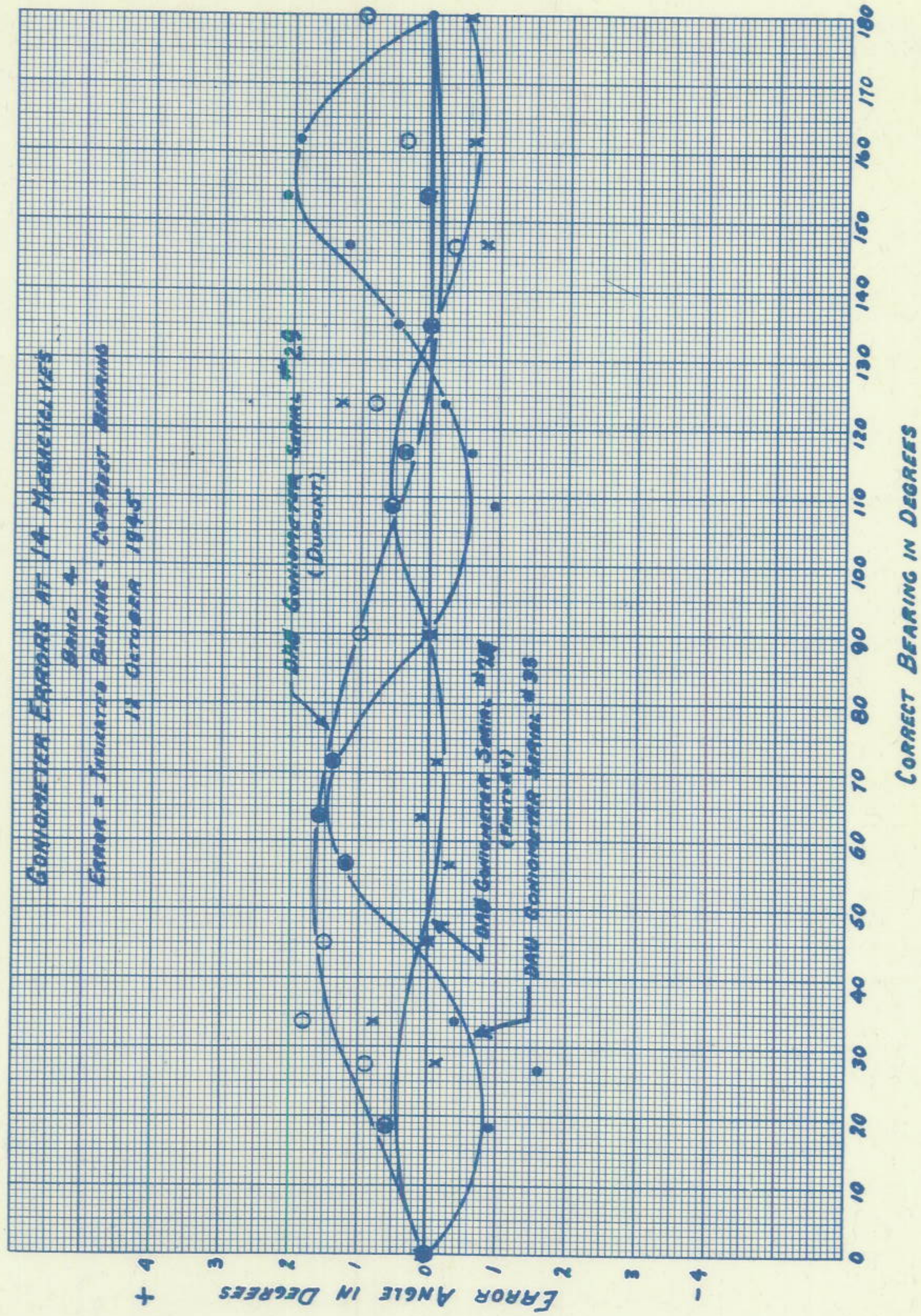
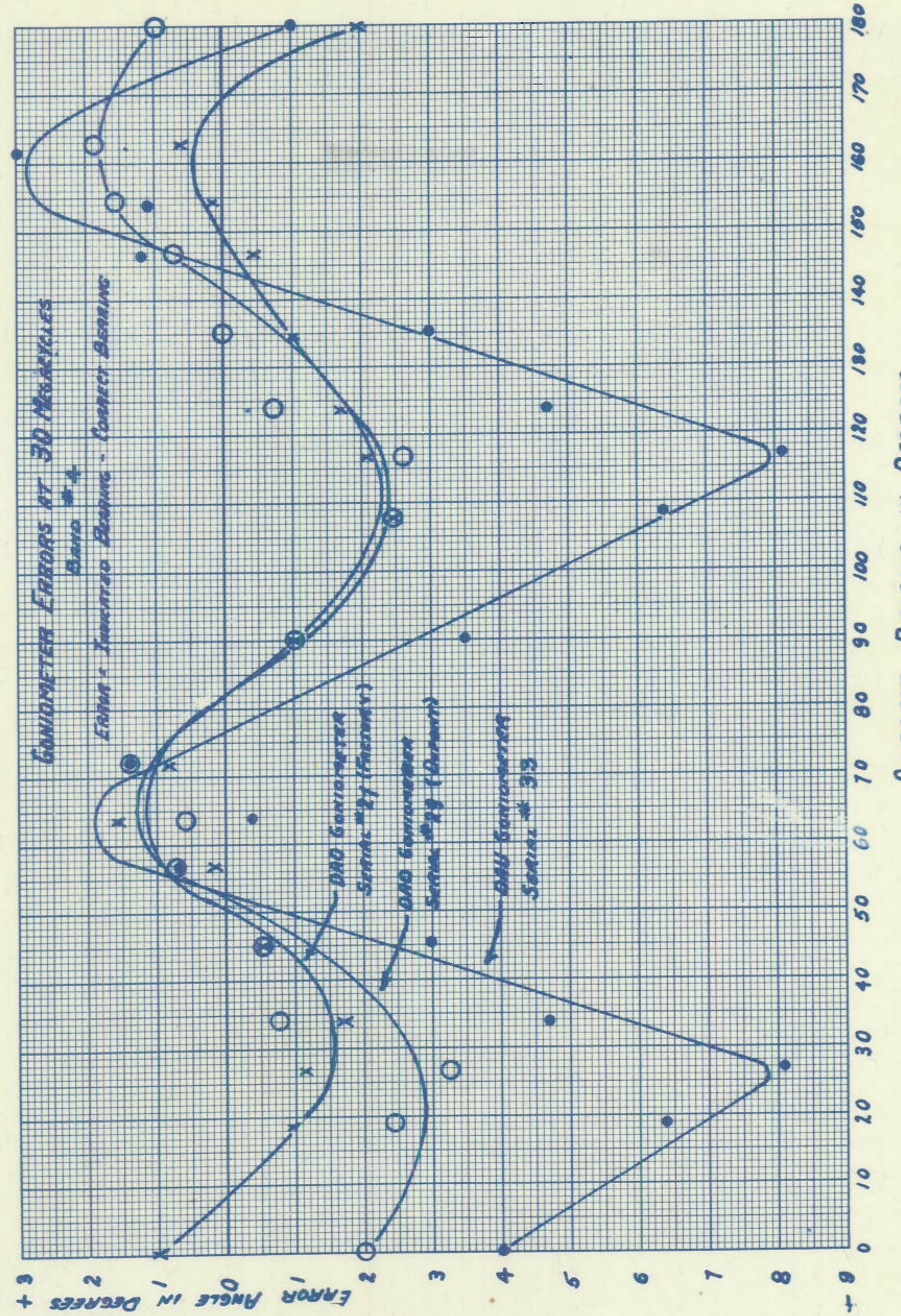
140
150
160
170
180




[redacted] R-2707 PLATE 11

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R-2707

PLATE 13

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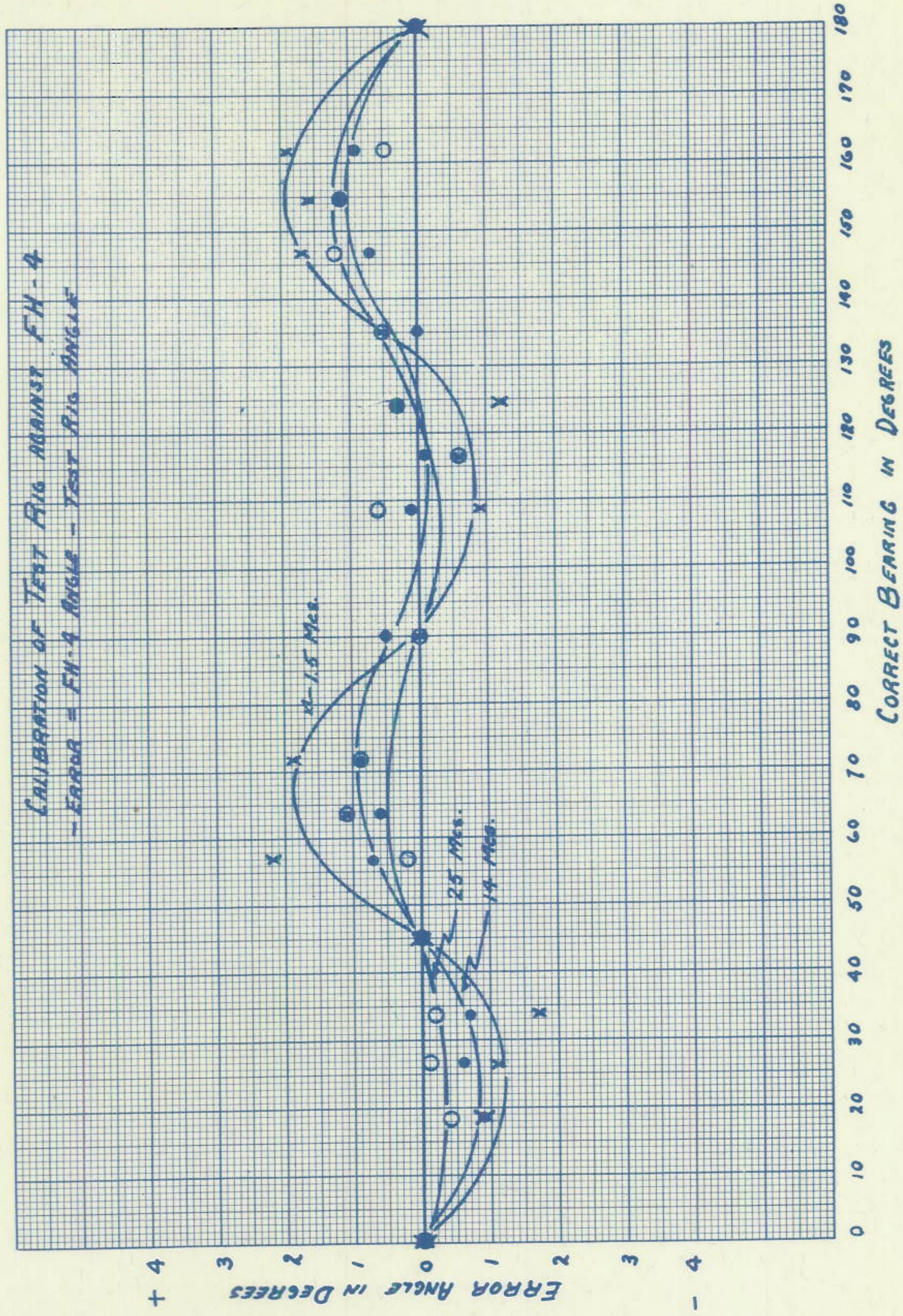


[redacted] R-2707 PLATE 14

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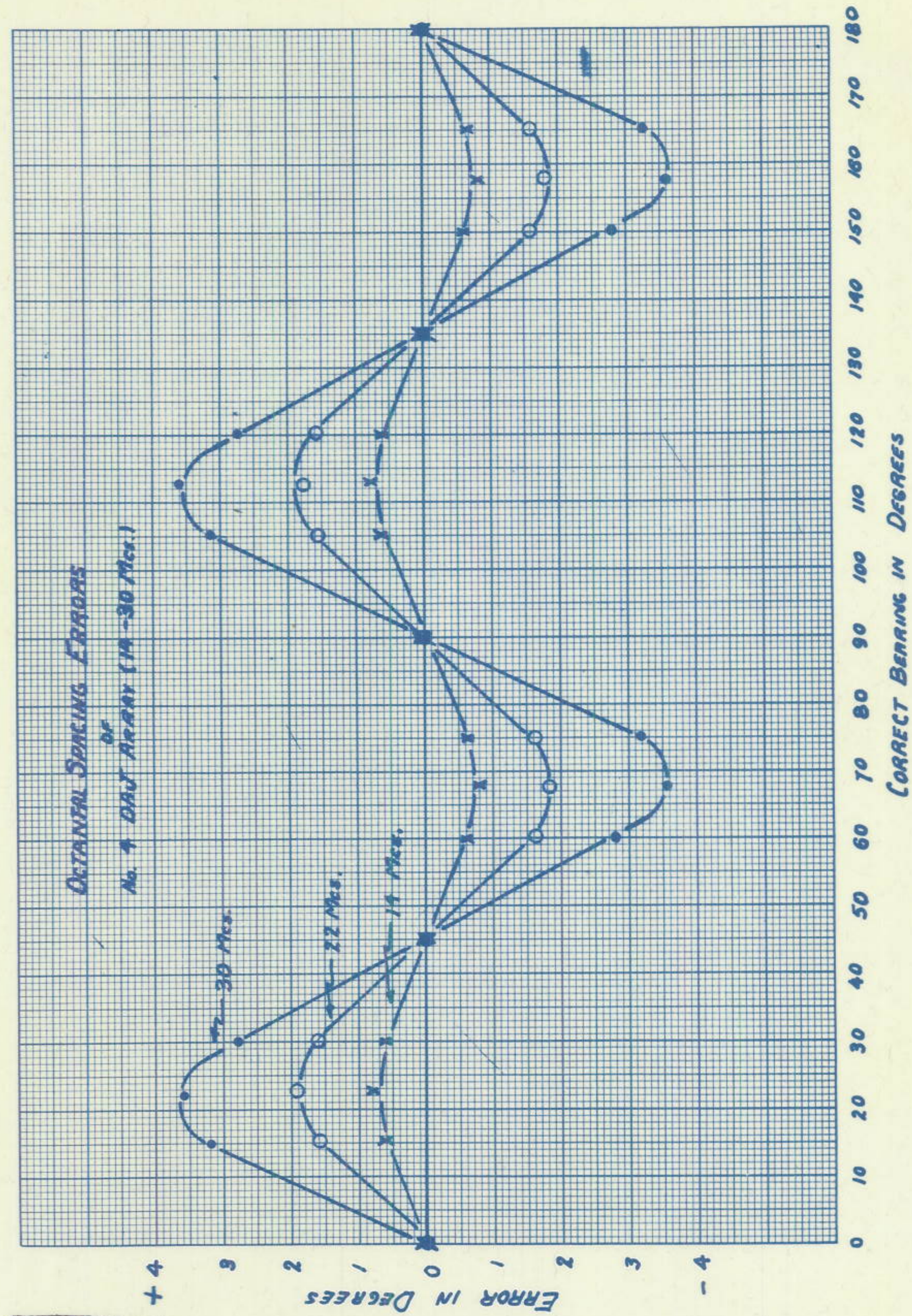


[redacted] R-2707 PLATE 15

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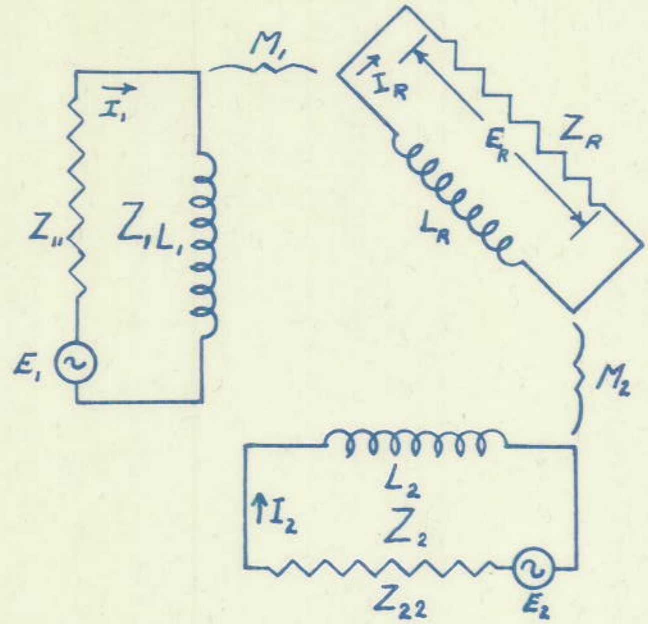



FIG 1 SCHEMATIC DIAGRAM OF
GONIOMETER CIRCUIT

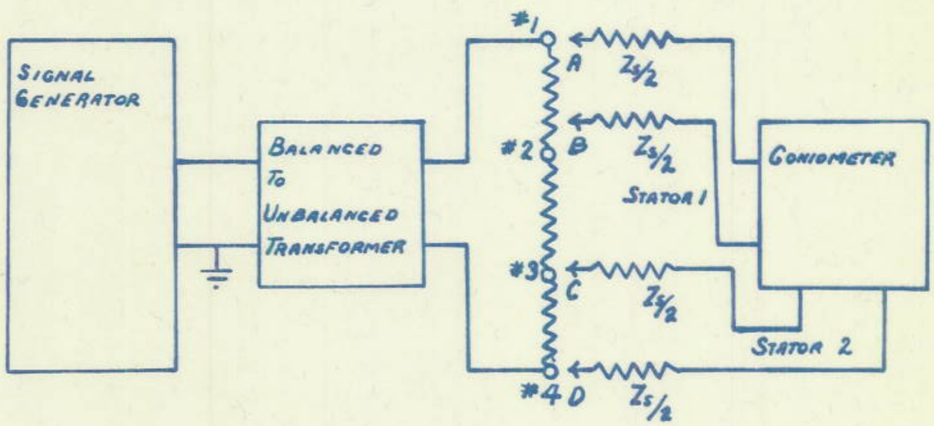
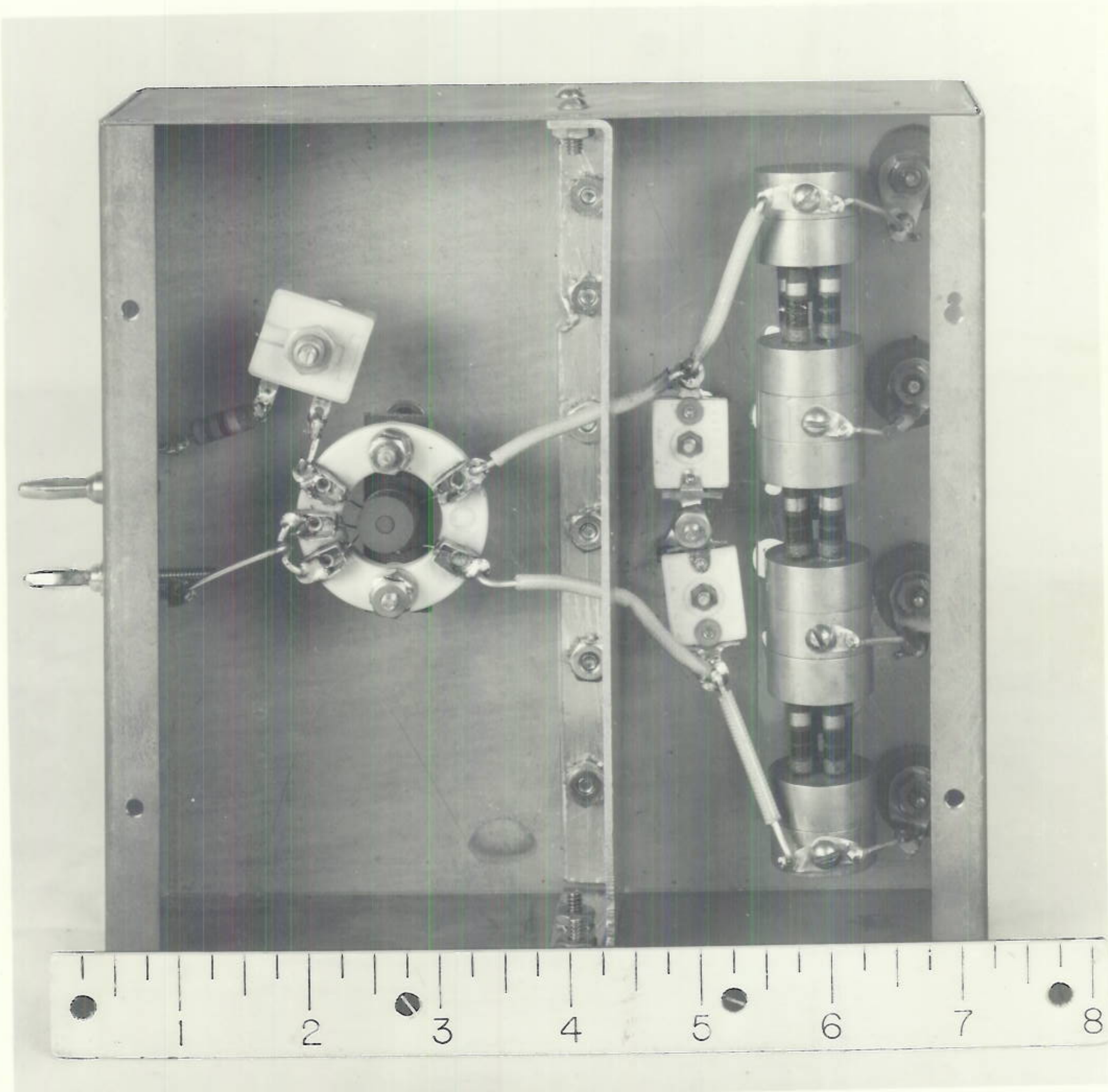
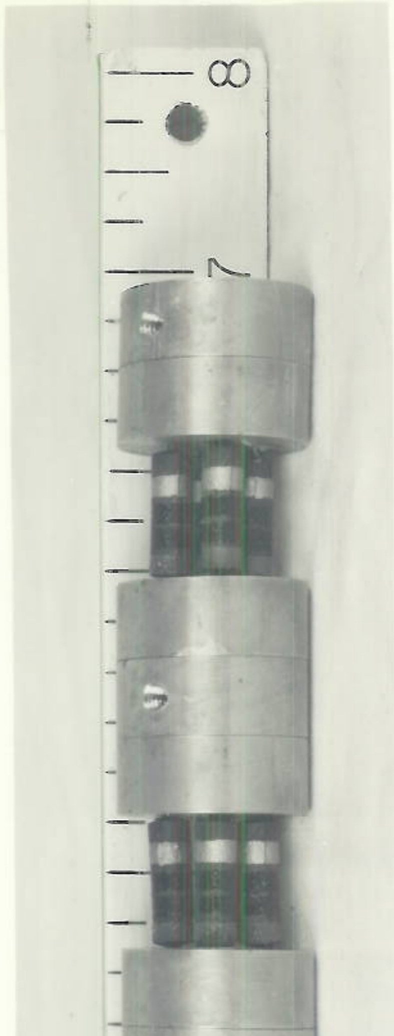


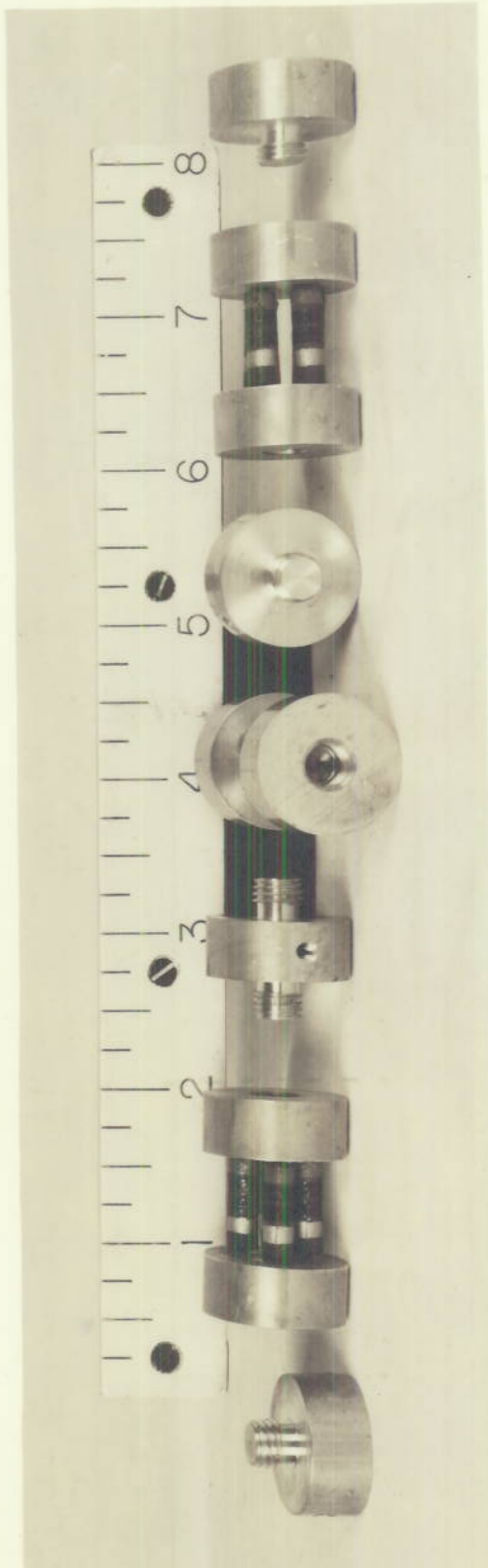

FIG 2 BLOCK DIAGRAM OF
TEST SET-UP




BOTTOM VIEW OF SONIOMETER TEST RIG
WITH BALANCED-TO-UNBALANCED TRANSFORMER.




ASSEMBLED VIEW OF VOLTAGE DIVIDER.



DETAIL VIEW OF VOLTAGE DIVIDER.