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7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Texas at Austin
101 East 27th Street
Suite 5.300
Austin, TX 78712 -1532

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INVESTIGATOR(S):

Name: Francois Baccelli
Email: Francois.Baccelli@austin.utexas.edu
Phone Number: 5124717028
Principal: Y

Organization: **University of Texas at Austin**
Address: 101 East 27th Street, Austin, TX 787121532
Country: USA

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Submitted By: Francois Baccelli
Email: Francois.Baccelli@austin.utexas.edu
Phone: (512) 471-7028

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Participant Type: PD/PI
Participant: Francois Baccelli
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Participant Type: PD/PI
Participant: Sriram Vishwanath
Person Months Worked: 3.00
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Participant: Jae Oh Woo
Person Months Worked: 12.00 **Funding Support:**
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Final Report

François Baccelli and Sriram Vishwanath

THE LAWS OF OPINION DYNAMICS IN SOCIAL SYSTEMS

The University of Texas at Austin

1 Background

Understanding the theoretical underpinnings of opinion dynamics is an important component of developing a complete picture of the manners in which social systems evolve and behave. Although an extremely interdisciplinary and complex problem, the modeling of opinion dynamics can be broken down to some essential components. Indeed, a variety of tools, ranging from combinatorics to game theory have already been applied to this domain in order to distill and model the main components and interactions within social systems, as detailed below.

A majority of existing work in this domain is initialized with a setup where a (static) *social graph* determines the set of all allowable forms of interactions between agents in the system. Subsequently, the dynamics proceeds based on interactions among neighbors in this graph.

In addition to these, there is a growing body of work on settings where no such graph dictates interactions among agents, and/or where the graph is not static. Examples of interactions to be modeled include online interactions in forums such as Reddit, Quora where groups are self-selective in involvement, bringing like-minded people together to interact with one another. In the offline world, conferences, meetings serve a similar purpose of bringing people together, and the relationship structure among agents evolves due to these self-selecting interactions among agents. Along a similar vein, Twitter represents a dynamic graph where agents "follow" or "unfollow" one another based on proximity of opinions, much more in line with forums and gatherings than static graph models.

Bounded confidence, the other family of well known models for opinion dynamics, models interactions in systems where agents interact with one another based on the proximity of their opinions. In the bounded confidence opinion dynamics, agents interact over time in a pairwise manner. Each interaction triggers an opinion update which is again a (possibly linear) function of the opinions of the agents involved; however the interaction is only effective if the opinion difference is within a threshold "distance". Thus, this model allows for opinion-dependent social exchanges and the incorporation of each agent's internal views.

Our main achievement is a stochastic model featuring pairwise interactions among agents, generalizing the mechanism of interactions currently studied in the context of bounded confidence opinion dynamics models and bringing it closer, in spirit, to existing graph-based interaction models. In addition to opinion-dependent social exchanges, our model incorporates the inherent stochasticity in interactions, imperfect exchange of opinions as well as self-beliefs, which capture the endogenous evolution of opinions innate to each agent. In addition, this model can be combined to graph-based and bounded confidence dynamics.

We characterized the conditions under which these dynamics are stable, in a mathematical sense, and analyzed the implications of this result from a sociological perspective. Overall, our work builds a stronger connection between the two bodies of work on graph-based and bounded confidence based dynamics, in addition to providing a stochastic generalization of both.

2 Main Research Achievements

2.1 Initial Objectives

By incorporating opinion-dependent interactions or exchanges, bounded confidence dynamics take a good first step towards modeling opinion evolution in social systems, but this class of models and their existing variants do not yet capture some of the inherent characteristics of opinion dynamics in social systems. We analyzed the components we found to be missing in existing models, and subsequently described ways in which our model incorporates them.

First, existing models assume a deterministic and thresholded behavior of agents in considering opinions of other agents. On contrary, in real life, social interactions possess a fair degree of inherent randomness, and lack sharp thresholds in terms of interactions and overall behavior.

Second, in most bodies of existing work, it is often assumed that each agent has full knowledge of the opinions of the agents it interacts with. However, in practice, opinions may not be known exactly, and there may be an associated error in estimation. This estimation error can substantially impact the process of incorporation of other agents' opinions in both space and time.

Third, error/noise in estimating the opinion of an agent can also directly impact the actual opinion update process.

Fourth, each agent may possess its own innate self-beliefs that influence its opinion, in addition to external interactions with other agents within the social system.

Fifth, not all agents that share similar opinions may interact with one other, as they may not gain the opportunity to do so. In addition, the strengths of friendships, and therefore, the extent of interaction between all agents may not be the same.

Our model aims to capture the above mentioned five missing elements into a stochastic framework generalizing bounded-confidence opinion dynamics, as detailed next.

2.2 Main Achievements

By allowing *opinion-dependent probabilistic* exchange of opinions among agents, our model incorporates the inherent stochasticity as desired by the first and second characteristics above. In addition, the impact of erroneous opinion estimates on updates and the impact of self-belief in shaping an agent's opinion, as discussed in the third and fourth characteristics, are captured by the introduction of terms that we call *endogenous* processes. Finally, an *interaction* process and pairwise *strengths of influence* cover the notion of social proximities and their impact on opinion dependent social exchanges. Our basic paper on the matter [5] was published electronically by Transactions Automatic Control.

2.2.1 Analytical Tools for Stochastic Opinion Dynamics

The key new achievement we obtained with Jae Oh Woo is an analytic understanding of our model [8]. This was made possible by introducing a continuous time version of this class of dynamics, which allows one to connect this framework to that of partial differential equations. These partial differential equations have a diffusive part coming from the self-belief component, and a non local part coming from the updates resulting from interactions. We first obtained a new characterization of the stationary regime of stochastic opinion dynamics on graphs based on Mellin Transforms. The Mellin transforms are used to solve the partial differential equations satisfied by the opinion differences. Lately, we succeeded in extending this approach to the stochastic bounded-confidence opinion dynamics setting. New and intriguing new phenomena appear in this case such as the fusion of opinions. We now have a full classification of behaviors and a full characterization of the steady state distribution for power law interaction functions as those initially discussed in [5]. This opens new perspectives to complement the primarily qualitative theory developed in the first phase of the project with a quantitative counterpart. Our paper on this topic was published in the Journal of Applied probability.

2.2.2 Stochastic Opinion Dynamics as Particle Systems

A Graduate Student Natasa Dragovic worked on an extension of bounded-confidence opinion dynamics to situations where the number of agents is extremely large. The idea is to use techniques of particle systems to analyze such situations. Her work [6] consisted in analyzing bounded-confidence opinion dynamics for agents with initial opinions forming a Poisson point process in the infinite Euclidean plane. The problem is extremely difficult in general. We proved first results on the case where interactions are asymmetrical with a node following its closest neighbor only. A paper is under preparation.

2.3 Student and Postdoctoral Fellow Advising

(a) Graduate Advisees

- Dr. Baccelli advised a graduate student, Natasha Dragovic, of the department of Mathematics working on the extension of the bounded confidence model dynamics to the infinite Euclidean space. N. Dragovic is now

(c) Postdoctoral Fellows

- Dr. Baccelli and Dr. Vishwanath co-advised a postdoctoral fellow Jae Oh Woo working on the analytical theory for opinion dynamics. This postdoctoral fellow left in August 17. He now works with Industry in California.
- Dr. Baccelli and Dr. Vishwanath hired a new postdoctoral fellow Deepanshu Vasal who joined in September 17. A new research direction was selected for his research. It consisted of analyzing situations where interactions are based on the distances between actions rather than opinions.

2.4 Visibility

2.4.1 Special Lectures, Keynote addresses and Colloquia

Dr. F. Baccelli gave the following keynote lectures:

- Keynote Lecture, *ISWCS'15*, Brussels, August 2015.

- Invited lecture at the *Stanford University Probability Seminar*, April 2015.
- Keynote Lecture, *Stochastic Networks'16*, San Diego, June 2016.
- Colloquium at *UT Dallas*, September 2016.
- Keynote lecture at **WITMSE'17**, Paris, France, September 17.
- Keynote lecture at **2nd Symposium on Spatial Networks**, Oxford, UK, September 2017.
- Keynote lecture at **IEEE WiOpt'17**, Paris, France, May 17.
- Keynote lecture at **2nd Symposium on Spatial Networks**, Oxford, UK, September 2017.

Dr. S. Vishwanath gave the following invited lectures:

- Talk at Texas A&M, October 2016.
- Talk at UCSD, February 2017.

Dr. J.O. Woo gave the following lectures and posters:

- Poster at ITA 2017, UCSD, February 2017.
- Presentation at ISIT, Aachen, Germany, June 2017.
- Talk at the probability seminar at Korea Advanced Institute of Science and Technology, Daejeon, South Korea, June 2017.
- Talk at the Discrete Math seminar at Korea Advanced Institute of Science and Technology, Daejeon, South Korea, June 2017.

2.4.2 Honors, Awards

F. Baccelli

- Member of the French Academy of Sciences, Paris.
- Honorary doctorate, Department of Mathematics, Heriot Watt University, Edinburgh, UK, awarded in November 2016.

2.4.3 Organization of an ARO Conference on Opinion Dynamics

The two PIs organized the Opinion Dynamic Conference, on June 13-14, 2016, at UT Austin. The aim of the conference was to assist the Army in planning future research on Opinion Dynamics by reviewing the main methodological approaches which are currently proposed to analyze, predict and control opinion dynamics `http://www.opiniondynamics.org/index.html` The workshop attracted 31 attendees (including the speakers) from a variety of institutions.

3 Conclusions

The research of the two PIs at the University of Texas at Austin was focused on the **mathematics of social networks**. The PIs and their students and postdocs developed a new approach to opinion dynamics based on models where interactions between agents depend on the geometry of opinions. The practical motivation for such models stems from the observation that opinion sharing is more likely between agents whose opinions are close in some sense. The resulting geometry based dynamics lead to non-linear, stochastic interaction models which are quite fascinating. A large set of mathematical questions ranging from stochastic stability to quantitative analysis to inference have already been identified and continue to be investigated. More precisely, the line of thought described above is currently continued in the direction of analytic characterization within this context. In particular, the work on the Mellin transform approach completed lately has several extensions in higher dimensions. A new line of thought on very large populations of agents will also be investigated. Finally, the PIs are actively investigating data-centric mechanisms for validating the models and assumptions made in our analysis.

The work with Avhishek Chatterjee (former PhD student of ECE, UT Austin) aimed at developing a stochastic version of the bounded confidence model. The stability region of the two models is now completely solved [1], [2], [3].

The work of the Postdoctoral Fellow Jae Oh Woo [8] complemented this stability analysis by an analytical characterization of the stationary distribution of opinion differences. This analysis covers both the graph case and the bounded-confidence case. A large set of mathematical questions beyond stochastic stability and ranging from quantitative analysis to inference have already been identified and will be investigated in the forthcoming years.

The work with N. Dragovic [6] (former PhD student of the Department of Mathematics, UT Austin) developed a new class of particle system models for opinion dynamics.

Finally, the investigation of data-centric mechanisms for validating the models and assumptions made in our analysis developed through the work of the postdoc, Dr. Vasal, who devoted his energies in developing ideas that would enable us to connect modeling and data in greater depth. This work materialized in publications like [7, 9].

This research was presented in selective ECE conferences (like IEEE CDC, IEEE CISS, or IEEE Infocom as well as the ARO workshop on Opinion Dynamics). It was published in top journals of the field (both in IEEE transactions and in math journals like JAP). In addition, a structuring conference was organized on the matter in June 2016.

References

[1] F. Baccelli, A. Chatterjee, S. Vishwanath, "Heavy Tailed Influencers and Stochastic Bounded Confidence Stability", **Proceedings of the Allerton Conference**, UIUC, Oct. 2014.

[2] F. Baccelli, A. Chatterjee, S. Vishwanath, "Stochastic Bounded Confidence Opinion Dynamics", **Proceedings of the Conference on Decision and Control**, Los Angeles, Dec. 2014.

[3] F. Baccelli, A. Chatterjee, S. Vishwanath, "Pairwise Stochastic Bounded Confidence Opinion Dynamics: Heavy Tails and Stability", **Proceedings of IEEE Infocom'15**, Hong Kong, April 2015.

[4] A. Chatterjee, "Understanding Dynamics and Resource Allocation in Social Networks", **Thesis, The University of Texas at Austin**, August 6, 2015.

[5] F. Baccelli, A. Chatterjee, S. Vishwanath, "Pairwise Stochastic Bounded Confidence Opinion Dynamics: Heavy Tails and Stability", **IEEE Transactions on Automatic Control**, April 2017.

[6] N. Dragovic, "On the Poisson Follower Model", **Thesis, The University of Texas at Austin**, August 2020.

[7] D. Vasal, A. Sinha, and A. Anastasopoulos, "A Systematic Process for Evaluating Structured Perfect Bayesian Equilibria in Dynamic Games With Asymmetric Information", **IEEE Trans. Autom. Control.**, 64-1, pp 81–96, 2019.

[8] J.O. Woo, F. Baccelli, S. Vishwanath, "On the Steady State of Continuous Time Stochastic Opinion Dynamics with Power Law Confidence, **Journal of Applied Probability**, 58(3), 2021.

[9] R. K. Mishra, D. Vasal, and S. Vishwanath, "Decentralized Multi-agent Reinforcement Learning with Shared Actions", **55th Annual Conference on Information Sciences and Systems**, CISS, Baltimore, MD, USA, March 24-26, 2021,