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Silver Spring, Maryland

APL/JHU TG-20
January 14, 1947

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By

An Informal Committee

of

THE APPLIED PHYSICS LABORATORY

of

THE JOHNS HOPKINS UNIVERSITY

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NUCLEAR POWERED FLIGHT

AS OF APRIL 1, 1947 THE CLASSIFICATION OF THIS DOCUMENT HAS BEEN CHANGED TO [redacted]

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FIRST PROGRESS REPORT

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14 January 1947

TO: L. R. Hafstad

FROM: A. E. Ruark

SUBJECT: Transmittal of Progress Report entitled "Nuclear-Powered Flight", by an Informal Committee of the Applied Physics Laboratory of the Johns Hopkins University.

In accordance with your verbal instructions of about 9 June 1946, the Committee has considered the general problem of air vehicles driven by nuclear power. Three copies of the subject report are respectfully submitted herewith. A first draft was submitted October 25, 1946. Since that time many errors have been corrected and much new material has been added. The initial distribution is indicated in the report.

Your comments and those of other interested persons will be appreciated by the Committee. Review by suitable members of APL is hereby requested.

It is believed that any further work on this subject at APL should be carried on by a small staff with fresh instructions, and that the existing large committee should be discharged in the near future.

FOR THE COMMITTEE

Arthur E. Ruark, Chairman;
Technical Supervisor
for Research Laboratory.

AER:rh

Encl. 3 -- Copies 1, 2, and 3 of subject report.

[redacted]

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This document contains [redacted] national defense of the U [redacted] ionage Act, 50 U. S. C. [redacted] ion of its contents [redacted] rized persons is prohibited by law.

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CHAPTER V. SUPERSONIC NUCLEAR-POWERED RAM-JETS AND TURBO-JETS

Introduction and Summary

In order to estimate the feasibility of employing nuclear-energy to power ram-jet missiles, it becomes necessary to make comparisons between previously conceived designs employing conventional fuels and possible designs for utilizing such energy.

Conventional Design. Part A of this chapter is therefore devoted to a description of a long-range ram-jet-propelled guided missile, using gasoline as a fuel. This is essentially a summary of a previous report issued by this laboratory. It is concluded that such a design can have a range of 3,070 miles when flown at a Mach number of 2.2 and an altitude of 70,000 feet. The statement is also made that a range of 5,000 miles is in the realm of possibility if certain increases in ram-jet performance can eventually be obtained.

The answer to the question of nuclear ram-jet feasibility should emerge from an integrated design procedure which takes into account the following factors, simultaneously.

(1) Neutron-design of the reactor and its controls.

(2) The mechanical and chemical properties of the hot reactor.

(3) Heat transfer to the gas stream.

(4) Aerodynamic design of a structure to carry the payload and reactor.

In practice, we cannot yet carry through a straightforward design including all these factors. It is necessary to lay aside the unsolved questions of mechanical and chemical integrity and to make designs covering a range of operating temperatures which does not completely rule out the use of desirable reactor materials, and which, in the light of existing experience, may provide sufficient thrust to meet our speed requirement. Aside from these considerations, it is desirable to have on record basic calculations concerning heat transfer to a gas stream, by radiation and by convection, in single-tube and multi-tube reactors. It is also desirable to study the aerodynamic characteristics, particularly the internal drag of such reactors, the reactor weights and volumes involved, and the effect of these factors on the size and weight of the bird.

Designs Using Nuclear Energy. Part B is concerned with the problems of heat transfer in single and multi-tube convective reactors and in radiative-heated reactors.

Part C presents the propulsion equations leading to the relations between gas exit temperature and net thrust coefficients for any type of reactor-heated ram-jet or turbo-jet.

In Part D the aerodynamic relations necessary for flight under different conditions are derived and relations are given for the weights that can be supported in flight at specified velocity and altitude, given certain ram-jet designs and thrust coefficients.

In Part E the results of the previous sections are combined to give the preliminary design of a convective-heated ram-jet that is capable of flight at 50,000 ft. altitude at a Mach number of two. The results give the total weight that can be supported in flight as a function of the wall temperature in the reactor. For a given payload, curves are presented showing the percentage of the total weight that is available for structure weight (i.e. the total weight less that of the payload and reactor) for different wall temperatures of the reactor. On the basis of these curves it is concluded that it is possible to build a convective-heated ram jet if the wall temperatures are above 3600°R and the reactor material can be sufficiently compacted.

Part F gives the arguments why a radiative-heated ram jet seems incapable of flight except under conditions of very large sizes with consequent large uranium requirements.

In Part G the analysis of a supersonic turbo jet with exhaust heating capable of flight at a Mach number of 1.4 is presented. It seems likely that such a design is feasible if reactor wall temperatures of at least 4000°R are used but the uranium requirements are twice as large as those in the ram jet since two reactors are required. The structure of the vehicle is also much more complicated than that of the ram jet because of the addition of the turbine-compressor unit.

PART A. RAM-JETS WITH CONVENTIONAL FUEL

By R. J. Vicars

Reference 1 indicates that, of the types of vehicles studied, a ram-jet employing a Kantrowitz-Donaldson type diffuser is the most efficient long range missile. Accordingly, a missile was designed to operate at the optimum Mach number of 2.20 and optimum air-to-fuel ratio of 18 to 1 with gasoline, and calculations indicate that a range of 3,070 miles may be expected. This missile is capable of transporting a spherical warhead of 12,000 pounds requiring 113 cubic feet of space. It begins self-powered flight at an altitude of 68,000 feet and may be expected to climb approximately 3,000 feet during the course of its flight. A thrust coefficient of 0.928 is required and seems attainable at a burning efficiency of 80%. This is discussed in detail in Reference 2.

The following table summarizes the weights and performance of a ram-jet eight feet in diameter and 80 feet long, carrying a 12,000 lb. warhead and operating at an air-to-fuel ratio of 18:1.

TABLE I. SUMMARY OF WEIGHTS AND PERFORMANCE

Weight of structure	48,230 lbs.
Weight of warhead	12,000
Weight of fuel	76,000
Weight of controls	500
Total gross weight	136,730 lbs.
Total empty weight	60,730 lbs.
Range	3,070 miles
Flight altitude (initial)	68,000 ft.
Flight Mach number	2.2

The results of the calculations indicate the range that may be expected from this long range ram-jet propelled missile without undue development of new techniques. The powered range only has been considered - the increment of range due to launching and gliding being negligible by comparison. Preliminary analysis of the glide path of the missile indicated that the deceleration was so great and the resulting velocity so low, that, to make a suitable weapon, the missile should be powered into the target.

Little consideration has been given as yet to vehicles for launching these missiles, but it is estimated that a typical launching vehicle would weigh about one and a half to two times the weight of the missile.

The following basic assumptions were necessary for the analysis of the conventional fueled ram-jet missile and seem well within reach.

(1). Present quality of burning can be maintained at the operating altitude.

(2). Materials will withstand operation at equilibrium temperatures for three hours.

(3). The Kantrowitz-Donaldson type diffuser is suitable for the desired pressure recovery.

(4). Gasoline type fuel may be used satisfactorily under flight conditions.

It can be expected that a substantially greater range can be obtained for this type of missile by the use of improved fuels of the Borohydride type, by the use of the Oswatitsch type diffuser to secure the desired pressure recovery, and other improved techniques. It is believed that eventually with the development of these techniques, it will be possible to achieve a range of 5,000 miles, the desired tactical range of operation for such a missile.

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PART B. HEAT TRANSFER IN REACTOR-HEATED RAM- AND TURBO-JETS

By A. G. Carlton and C. F. Meyer

1. Outline and preview of Results.

We consider first the problem of convective heat transmission in a ram-jet consisting of a diffuser followed by a hot cylindrical tube. The ram-jet is assumed to be similar in general design to fuel-burning ram-jets, and is considered as operating at an altitude of 50,000 to 60,000 feet, an atmospheric temperature of 400° Rankine and a flight Mach number of 2. The heat is supplied by maintaining the reactor surface at a constant high temperature.

The finding is that length-over-diameter ratios of 100 or more are required, so that the use of a single cylindrical tube would be infeasible on the basis of convective transfer alone. Two ways to avoid the difficulty are then considered:

(a) Use of a multi-tube reactor, to increase the ratio of heated surface to reactor volume.

(b) Use of an opacifier in the gas stream of a single tube reactor. At 3000° K, one square foot of hot wall radiates 426 kilowatts, and if any considerable fraction of this can be caught in the gas stream by adding a suitable absorbing smoke, the proposal is by no means fanciful.

It also turns out for a multi-tube heater that the length diameter ratio, of an individual tube must be 100 or more, but this is perfectly feasible as the diameters will be of the order of 0.1 feet or less, so that reactor lengths of the order of 5 to 10 feet are permissible. Later it will be shown that the number of tubes required for an 8 ft. diameter ram-jet will be of the order of 5000 to 15,000.

Next heat transfer in the reactor material is considered. The gradients required are large enough to rule out the use of a single-tube annular reactor at once. However, the radiative case is presented in detail, because there may be other applications for the analysis, and because it is desirable that the possibilities be assessed, quite independent of the difficulty about heat transfer in the solid material of the reactor. The calculations made are as follows:

(a) Radiation transfer from hot walls to smoke.

(b) Transfer from smoke to gas. This is rapid and efficient. The air-smoke ratio is large enough so that total smoke weight is not serious. However, the particle size required is smaller than that encountered in ordinary smokes.

It is concluded from this study that adequate heat transfer from hot walls to a gas stream can be obtained either by the use of a multi-tube convective heater or by a single tube reactor using a reasonable amount of smoke as an "opacifier" in the gas stream. However, consideration of the heat transfer through the reactor material indicates that it will seriously limit the use of a single tube reactor.

2. Units and Symbols.

Units are feet, pounds, hours and °R throughout, except for r^*, particle radius in microns, and λ, mean free path in microns. Frequently used symbols are as follows:

G - mass velocity per unit cross-section.

μ - viscosity

q - volume heat flow (Btu)/(ft^3) (hr).

σ - Stefan-Boltzmann constant = 1.73×10^{-9} B.T.U. per hour per sq. ft. of wall surface.

T - Temperature.

F - fraction of the radiant energy absorbed along the mean beam length.

h - heat transfer coefficient.

D - tube diameter.

r - particle radius, feet; r* - particle radius, microns.

N - number of particles per cubic foot.

v - speed of gas flow.

M - Mach number.

ρ - density; ρ_p - mass of particles in cubic foot of gas.

c - specific heat at constant pressure; γ - ratio of gas specific heats.

ϵ_p - particle absorption coefficient.

k - heat conductivity.

R_{as} - mass ratio of air to smoke particles.

Subscripts:

w, wall; p, particle; i, intake; d, diffuser; e, exit; t total; o, stagnation.

3. Required Power Input from Reactor to Jet.

The required power output of the reactor is given by

$$P_o = Gc\ (T_e - T_d)$$

where G is the rate of mass flow, and c the specific heat of the gas at constant pressure. Let D_r be the outside diameter of the reactor and Γ the fraction of its gross cross-section occupied by the gas stream. Referring G to intake conditions,

$$G = \rho_i\, v_i\, \frac{A_i}{A_d}\, \tfrac{1}{4}\pi D_r^2\, \Gamma \qquad (1)$$

Therefore

$$P_o = \rho_i\, v_i\, \frac{A_i}{A_d}\, \tfrac{1}{4}\pi D^2\, \Gamma c\ (T_e - T_d). \qquad (2)$$

Now the volume of the reactor is given by

$$V = \tfrac{1}{4}\pi D_r^2\ (1 - \Gamma)\ L.$$

Therefore

$$\frac{P}{V} = \frac{\rho_i\, v_i\, (A_i/A_d)\, \Gamma c\ (T_e - T_d)}{(1 - \Gamma)\ L} \qquad (3)$$

Let us assume the following representative values as an example.

$\rho_i = .008\ lb/ft^3$

$v_i = 2000$ ft/sec

$A_i/A_d = 1/2$

$c = 0.25$ BTU/lb. ^oR

$T_e - T_d = 2800^oR$

$L = 7.14$ ft.

$\Gamma = 0.5$

Then $P_o/V = 784$ BTU/sec ft^3 = 825 kw/ft^3 = 0.295 kw/cm^3. (3a)

This is the power per unit volume for a reactor that will propel a ram-jet of these characteristics. The formula applies to reactors with any number of tubes.

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4. Convective Heating in a Cylindrical Tube.

Let h be the heat transfer coefficient from the hot tube wall to the gas, in btu/(hr)(ft)^2 (°R), i.e., the heat flow per unit wall surface, for 1° temperature differential. The following empirical heat transfer equation for turbulent motion appears to be the best available, according to McAdams, Reference 6.

$$h = 0.023\ c\ G^{0.8}\ \mu^{0.2}\ D^{-0.2}\ (c\ \mu/k)^{-0.6}. \qquad (4)$$

Here D is the tube diameter, k the thermal conductivity and μ the viscosity of the gas. Equation (4) is dimensionless. $c\ \mu/k$ is Prandtl's number, which is relatively indenendent of temperature and pressure; as an average value for air, we shall employ 0.74. $\mu^{0.2}$ does not change greatly throughout the tube, and will be considered a constant, 0.55. Thus, from (4),

$$h = 0.0151\ c\ G^{0.8}/D^{0.2}, \qquad (4a)$$

which is now confined to the system of units used in this paper.

By Bernoulli's equation, the heat absorbed in an element of length dL is $\frac{1}{4}\pi D^2 Gc\ dT_o$ where T_o is the stagnation temperature; the heat transferred from the surface of the elementary cylinder is $h\pi D(T_w - T_o)dL$. The heat transferred from the surface must equal the heat absorbed by the gas. Thus, using 4a,

$$dT_o = 0.0604\ \frac{T_w - T_o}{D^{1.2} G^{0.2}}\ dL. \qquad (5)$$

For simplicity it will be assumed that the cross-section at the exit of the diffuser is the same as the cross-section of the gas-stream or streams in the reactor. (See Figure 6-6). The total mass flow is constant along the entire ram-jet, so

$$G = \rho v = \rho_i \frac{A_i}{A_d} C_{si} M_i, \qquad (6)$$

where v is speed, ρ_i is atmospheric density, A_i/A_d is the ratio of the intake cross-section to the diffuser exit cross-section, C_{si} is the atmospheric speed of sound, and M_1 is flight Mach number. At 60,000 ft. the speed of sound is 3,500,000 ft/hr. Substituting in (5) and then integrating,

$$dT_o = 0.003 \frac{(T_W - T_o)dL}{D^{1.2} (\rho_i M_i A_i/A_d)^{0.2}}$$

$$Ln \frac{T_W - T_{do}}{T_W - T_{eo}} = \frac{0.003 L/D}{(D \rho_i M_i A_i/A_d)^{0.2}} \quad (5a)$$

For completeness we write the equation in terms of all the relevant parameters;

$$Ln \frac{T_W - T_{do}}{T_W - T_{eo}} = \frac{0.092 \mu^{0.2} (c\mu/k)^{-0.6}}{D \rho_i (A_i/A_d) C_{si} M_i^{0.2}} \cdot L/D \quad (5b)$$

The variables in the denominator appear only to the 0.2 power, so that we can replace them by average values. For an altitude of 60,000/ft, ρ_i = 0.008, M_i = 2, D = 6, A_i/A_d = 0.5. Then (5a) becomes

$$Ln \frac{T_W - T_{do}}{T_W - T_{eo}} = 0.0056 L/D, \quad (5c)$$

$$T_{eo} = T_W - (T_W - T_{do}) e^{-0.0056 L/D.}$$

Now $T_o = T (1 \frac{\gamma - 1}{2} M^2)$, according to Cook (Reference 7), where T is the static temperature and γ the ratio of the specific heats. Before heating, γ = 1.4, and the atmospheric temperature is about 400°R, thus T_{do} = 720°.

Values of L/D are listed below for various values of T_W and T_{eo}.

	Wall Temperature, T_W		
T_{eo}	4500	5000	5500
3000	166	140	120
3500	242	197	159
4000	370	265	208

Length/diameter ratios such as called for in the above table are out of the question in a ram-jet consisting of a single large tube, because of the great weight of reactor implied. Thus we must consider multi-tube reactors.

The only change that occurs in the above analysis when we consider multi-tube reactors is the smaller diameter of the tube. Later considerations using optimum reactor design indicate that the tube diameters should be of the order of 0.1 feet or less. If we carry out the above calculations for a tube of this diameter we obtain the following values of L/D.

	Wall Temperature, T_w		
T_{eo}	4500	5000	5500
3000	74	62	53
3500	107	87	71
4000	164	108	92

These values of L/D are reasonable and lead to values of L from 5.3 ft. to 16 ft. Thus it appears possible to build a reactor of reasonable size and weight which would have adequate heat transfer to furnish exit gas temperatures in the ranges noted above.

5. Heat Transfer in the Reactor Material.

Before considering the heat transfer from the hot walls to the gas stream in a radiative-heated ram-jet we shall study the problem of heat transfer through the wall material of the reactor.

The power output of a ram-jet operating at a velocity v_i is given (Section 3) by the relation

$$P_o = \rho_i v_i A_i c \, (T_e - T_d)$$

The maximum possible input to gas from the radiating wall of area S is given by

$$P_i = \sigma S T_w{}^4. \qquad (6)$$

The efficiency of conversion of radiant energy into gas energy is given by P_o/P_i.

For a jet 6 feet in diameter, with $A_i/A_d = 0.5$, operating at 60,000 ft. altitude and at a velocity of 2000 ft/sec, we take the following values:

$$\rho_i = .008 \ \frac{lb}{ft^3} ; \quad A_i = 14 \ ft^2;$$

$$c = 0.25; \ T_e - T_d = 2800°R;$$

$$\sigma = 1.73 \times 10^{-9}; \ S = 1700 \ ft^2, \text{ for } L/D = 15; \ T_w = 5000°R.$$

Then,

$$P_o = 5.62 \times 10^8 \ Btu/hr = 1.48 \times 10^5 \ KW.$$

$$P_i = 18.4 \times 10^8 \ Btu/hr = 4.85 \times 10^5 \ KW.$$

Eff. = 30.5%

As a matter of fact, the power not transferred to the gas returns to the walls, so that the output of the reactor is only P_o, instead of P_i. Thus the heat transfer necessitates a temperature gradient through the pile wall, given by

$$dT/dx = P_o/Sk \qquad (7)$$

Data on the thermal conductivity k for materials which will stand very high temperatures are scanty above 3000°R. Values of k for graphite (Reference 19) and tungsten (Reference 12) are given below:

GRAPHITE		TUNGSTEN	
T, °R	k, $Btu/hr^{-1} ft^{-2} in(°R)^{-1}$	T, °R	k
1750	517	1800	585
2650	290	2700	689
4090	101	3600	779
4990	85	4500	843
5350	87	5040	883

The estimated values of k at 5000°R are

$$k_c = 86, \; k_w = 860.$$

To supply the power requirement, the temperature gradients needed are

3840°R/in. for carbon;

384°R/in. for tungsten.

These figures show how difficult it is to get the heat needed through any considerable thickness of reactor material. The figures tell us unambiguously that the use of a single tube reactor of carbon is impossible. The figures for tungsten are given only for comparison, because it has been studied so much by the lamp industry. We see no way to make a high- temperature tungsten pile, -- since it oxidizes so readily and is a very poor moderator.

There appears to be no data available on the heat conductivity of BeO at high temperatures but it is probably much less than that of carbon. Nevertheless we shall proceed to analyze the problem of heat transfer to a gas stream containing an "opacifier". This will enable us to drive more nails into the coffin of a radiative-heated ram jet missile, although someday it may have applications to some other type of vehicle such as a "satellite".

6. <u>Radiant Heat Transfer from a Hot Tube to a Gas Stream Containing Smoke.</u>

The calculations in Section 5 indicate that it will be necessary for the gas stream to absorb about 30% of the radiation from the hot tube walls. The most promising means of absorbing radiation is the use of a dense smoke in the gas stream. The heat transfer takes place in two stages, radiation from the wall to the smoke particles, followed by conduction from the particle to the gas. Section 7 shows that the heat transfer from the smoke particles to the gas is a rapid and efficient process under reasonable operating conditions. Because of this fortunate circumstance we consider the transfer as direct from the hot wall to the gas, the absorbing characteristics of the smoke determining the portion of the radiation which flows into the gas. The net outward heat flow per unit wall surface is

$$\sigma (T_w^4 - T^4) F \qquad (21)$$

where F is the fraction of the radiation absorbed by the gas in one average trip from one hot wall to another. This formula will be derived from the assumptions that the wall is black and that the smoke and the gas together act as a gray body of temperature T. Considering the case of two plane hot walls, and calling them 1 and 2, we note that, per hour and per sq. ft.,

Wall 1 emits σT_w^4

Wall 1 receives from wall 2 $\sigma T_w^4 (1 - F)$

Wall 1 receives $\sigma T^4 F$ from the gray gas and smoke because of Kirchhoff's law. The sum of these terms yields (21). (In the case of a central reactor with a perfectly reflecting cylinder outside, the argument is similar, the radiation returning from the reflecting wall replacing the radiation from the opposite wall. This device has been considered, but since its properties are roughly similar to those of the annular reactor, it will not be discussed further).

The heat flow q per unit gas volume is the heat flow per unit wall surface times the ratio of the radiating surface to the gas volume,

$$q = \sigma (T_w^4 - T^4) F (4/D). \tag{22}$$

The quantity of heat flowing into a unit volume of gas in its passage through one foot is q/v, where the speed v of gas flow is

$$v = MC \sqrt{Tg},$$

with M the Mach number and $C\sqrt{Tg}$ the speed of sound. Neglecting the difference between T and T_o (the stagnation temperature), which does not exceed 10 per cent, we have

$$dT/dL = q/\rho vc$$
$$= \frac{\sigma (T_w^4 - T^4) F (4/D)}{\rho MC \sqrt{T} \quad c} \tag{23}$$

Here M and ρ can be expressed in terms of their values at the diffuser point as follows:

$$\rho/\rho_d = v_d/v = M_d\sqrt{T_d}/M\sqrt{T},$$
$$M/(1+\gamma M^2) = K\sqrt{T}, \tag{24}$$

where K is a constant. We obtain

$$M = (1 - \sqrt{1-4\gamma K^2T}) \;/2\gamma K\sqrt{T} = K\sqrt{T} \; f \; (\gamma K^2T), \tag{24a}$$

where f (γK^2T) is a slowly varying function of its variable, as shown in the following tabulation:

T:	700	2000	3500°R
f:	1.1	1.2	1.6

The constant K in (24) is conveniently evaluated at the diffuser point, where $K = M_d/\sqrt{T_d}\,(1+\gamma_d M_d^2)$. The lesser of the two roots in M must be selected to yield the correct value at the diffuser point. (Note that (25) has no real root for $T > 1/4\gamma K^2$; this merely states that the gas cannot be heated above this temperature without affecting the intake conditions).

We shall require the number of particles of opacifier per unit volume, N. This is given by

$$N/N_d = \rho/\rho_d.$$

where N_d is determined by the amount of smoke we choose to have at the tail end of the central body.

For the absorption coefficient F per average path from wall to wall, we consider the mean beam length to be D, which turns out to be the correct value for infinite tube length with constant wall temperature and constant gas temperature. The absorption coefficient of a particle in a unidirectional radiation stream is $\pi r^2 \epsilon_p$, where r is the particle radius (equivalent radius if particle is not spherical), and ϵ_p the absorption coefficient of the material in the form of particles. Thus

$$F = 1-e^{-N\pi r^2 \epsilon_p D} = 1-e^{-K_1 T_d/T} \qquad (25)$$

where $K_1 = N_d \pi r^2 \epsilon_p D (1+\gamma_d M_d^2)/f$, by the equations preceding.

Making the indicated substitutions in (23), we obtain:

$$\frac{dT}{dL} = \frac{4\sigma}{C}\frac{T_w^4 - T^4}{K_2}(1-e^{-K_1 T_d/T}). \qquad (26)$$

It is now our task to study the integral,

$$I \equiv \int_{T_d}^{T_e} (T_w^4 - T^4)^{-1}(1-e^{-K_1 T_d/T})^{-1}dT = 4\sigma L/CK_2, \qquad (27)$$

where $K_2 = D\sqrt{T_d}\,\rho_d\, c.$

The factor $(T_w^4 - T^4)^{-1}$ varies slowly unless T closely approaches T_w. Hence we take its mean value f_1 out of the integral.

$$I = T_w^{-4} f_1 \int_{T_d}^{T_e} (1-e^{-K_1 T_d/T})^{-1} dT. \qquad (28)$$

The limits of f_1 are given by

$$(1 - T_d^4/T_w^4)^{-1} \leq f_1 \leq (1 - T_e^4/T_w^4)^{-1}. \qquad (29)$$

The integral (28) is evaluated by introducing the variable $x = T/T_d$, expanding and integrating term by term. Reference 16 gives a series for $x\,(e^x - 1)$, valid when the absolute value of x is less than 2π. Let $X = T_e/T_d$; then the result is

$$\frac{T_d\,(X^2 - 1)}{2\,K_1}\left[1 + \frac{K_1}{X+1} + \frac{K_1^2 \log X}{6\,X^2 - 1} + \sum_2^{\infty} \frac{(-1)^i\, B_i K_i^{2i}\,(X^{2-2i} - 1)}{(i - 1)(2i)!\,(X^2 - 1)}\right]$$

Here the B's are the Bernoulli numbers:

$$B_1 = 1/6,\ B_2 = 1/30,\ B_3 = 1/42,\ \text{etc.}$$

We call the bracketed expression $\Phi(K_1, X)$. Thus, for $K_1 < 2\pi$,

$$I = \frac{T_d\,(X^2 - 1)\,f_1}{T_w^4\,K_1}\,\Phi(K_1, X).$$

In general the terms after the first three can be neglected; e.g., if $X = 5$, the coefficient of K_1^4 is $\frac{(1/30)(.04 - 1)}{(1)\,(24)\,(24)} = -\frac{1}{18{,}000}$, and K_1^4 cannot exceed $(2\pi)^4 = 1550$.

From (30) and (27),

$$X^2 - 1 = \frac{8\,\sigma\,T_w^4 K_1 L}{C\,f_1 K_2 T_d\,\Phi(K_1,X)} \tag{30}$$

$$= \frac{8\,\sigma\,T_w^4}{C\,T_d^{3/2}}\cdot\frac{K_1 L/D}{f_1 M_d\,\rho_d c\,\Phi(K_1,X)} \tag{31}$$

Now, roughly, $\sigma/C = 1.0 \times 10^{-14}$; $c = 0.25$, $M_d = 0.2$,

$\rho_d = 3\rho_i = 0.024$; $T_d = 700°R$; f and f_1 is about 1.2.

Let us try the value $L/D = 15$, and obtain values of K_1 for various wall temperatures. We have,

$$T_w = 2200\sqrt[4]{\frac{X^2-1}{K_1}\,\Phi(K_1,X)} \tag{32}$$

We put $X = 5$ for $T_e = 3500°$. Then

$$T_w = 4800\,\sqrt[4]{\Phi(K_1,5)/K_1}\,, \tag{33}$$

$$\Phi(K_1,5)/K_1 = \frac{1}{K_1} + \frac{1}{6}\left(1 + \frac{K_1}{15}\right). \tag{34}$$

From these values we obtain the following dependence of wall temperature on K_1:

K_1:	.5	.6	.75	1.0
T_w:	6100	5700	5400	5000

Thus for a wall temperature of 5000°R, K_1 is about 1.0. We are now in a position to estimate the air-smoke ration R_{as}, which must not be exceeded if radiant transfer is to be sufficient for our purpose. From the definition of K_1, at $T_w = 5000°R$ we have

$$N_d \pi r^2 \epsilon_p D(1 + \sigma_d M_d^2)/f = 1.0, \tag{35}$$

Since the air-smoke ratio is given by

$$R_{as} = \frac{\rho_{gd}}{N_d \frac{4}{3} \pi r^3 \rho_p} \tag{36}$$

where ρ_p is the effective density of the smoke particles, we have

$$N_d \pi r^2 = \frac{3 \rho_{gd}}{4 \rho_p} \frac{S}{A} \frac{1}{r}. \tag{37}$$

Also, $(1 + \gamma_d M_d^2) \approx 1.05$, and $f \approx 1.2$. Then from (35)

$$\frac{\rho_d}{\rho_p} \frac{S}{A} \frac{\epsilon_p}{r} D = 1.6 \tag{38}$$

Assuming that ρ_p = 80 lb./cu. ft., we get

$$R_{as} = 1.8 \times 10^{-4} \frac{\epsilon_p}{r} D = 54 \frac{\epsilon_p}{r^*} D \tag{39}$$

The dependence of the air-smoke ratio on various factors is more clearly seen by combining (31), (35), and (36) to obtain:

$$R_{as} = \frac{6\sigma}{C_{cg}} \frac{T_w^4 \sqrt{T_d}}{T_e^2 - T_d^2} \frac{\epsilon_p}{r \rho_p} \frac{1 + \gamma_d M_d^2}{M_d} \frac{L}{f f_1 \epsilon} \tag{40}$$

In this equation the first group consists of constants, the second of temperature factors, the third of particle characteristics; the fourth depends on M_d, i.e., on M_1 and A_1/A_d, and the fifth group is the length of the heater tube times factors which are roughly constant.

7. Absorption of Radiation by Particles.

In Section 6 the absorption of radiation was considered from the simplified point of view that upon impact with a particle radiation is scattered forwards, in other words, scattering was really neglected. This leads to the expression

$$F = 1 - 1^{-N\pi r^2 \epsilon_p D} \tag{41}$$

for the fractional absorption in the length D. Ruark suggested that the average path length might be considerably increased due to scattering, since the F-value desired is about 30 percent. If the scattering were perfectly random the absorption could be calculated by the methods of ordinary diffusion theory. Consider the radiation as photons moving through a medium having N scattering and partially absorbing particles per unit volume. Let each particle have a geometrical cross-section of b and absorptivity ϵ_p. The mean free path of a photon will be given by

$$\ell = \frac{1}{Nb(1 - \epsilon_p)} \tag{42}$$

since the photon velocity is large compared to particle velocities. The mean diffusion length L_d is given by (ref. 14)

$$L_d = (\nu \tau)^{1/2} \tag{43}$$

where ν is the diffusion constant and τ is the mean lifetime before capture of a photon. Now if the photons are randomly scattered the diffusion constant is given by

$$\nu = \ell V/3, \tag{44}$$

where V is the velocity of light, and the mean life is

$$\tau = \frac{1}{N\epsilon_p bV} \tag{45}$$

Therefore we have

$$L_d = \frac{1}{\sqrt{3\epsilon_p(1-\epsilon_p)}\, bN} \tag{46}$$

Now the steady-state diffusion equation is

$$\nabla^2 n_\nu = n_\nu/\nu\tau \tag{47}$$

where n_ν is the number of photons per unit volume. For a plane case the solution is,

$$n_\nu = n_{\nu o} e^{-x/L_d} \tag{48}$$

Therefore, $$F = 1 - e^{-\sqrt{3\epsilon_p(1-\epsilon_p)}\, bNx} \tag{49}$$

For our case where D = x and $b = \pi r^2$

$$F = 1 - e^{-\sqrt{3\epsilon_p(1-\epsilon_p)}\,\pi r^2 ND} \tag{50}$$

This equation breaks down unless ϵ_p is very small. For the case in which ϵ_p is about 1 we can write

$$L_d = \ell = \frac{1}{Nb}$$

or $$F = 1 - e^{-N\pi r^2 D} \tag{50a}$$

since if $\epsilon_p = 1$ the photon will be absorbed at the first collision. Thus the form used in Section 6 is valid providing that ϵ_p is approximately equal to 1.

On the other hand, the expression (50) is only valid for random scattering of the photons. This is obviously not the case for photons. The work of Rayleigh (ref. 15) showed that for particles of the sizes we must employ the scattering is very much in the forward direction. He distinguishes the following cases:

Case 1. $r \ll$ wavelength; forward and backward scatterings are equal.

Case 2. $r \approx$ wavelength; scattering primarily forward.

Case 3. $r \gg$ wavelength; scattering primarily backward.

For our case the values of r and of the wavelength are approximately the same. Rayleigh discusses the case in which r is equal to the wavelength divided by π, which is fairly representative of our conditions. In this case the proportion of radiation scattered forward is very large, being on a relative basis, 1 at 0°, 0.6 at 30° and .01 at 90° as measured back from the forward direction. Thus the correct form of the diffusion path should take this factor into account. This has not been done in this treatment since most of the other factors are so unreliable at present. The large ratio of forward to backward scattering helps to reduce the influence of scattering on the results of Section 7.

$$\rho_1 V_1 = 17 \text{ lb/ft}^2\text{/sec} \tag{2}$$

and since we have chosen a configuration with

$$A_1/A_2 = 1/3 \tag{3}$$

we have

$$\rho_2 V_2 = \rho_1 V_1 \; A_1/A_2 = 5.667 \text{ lb/ft}^2\text{/sec} \tag{4}$$

Now the subsonic diffuser reduces the stream velocity from the initial Mach number of

$$M_o = .904 \tag{5}$$

to $$M_2 = .217 \tag{6}$$

in accordance with the relation given by equation (9) of Chapter 5C. The efficiency of the diffusion process is taken to be 0.8. The equation connecting the Mach numbers across the compressor stage is (ref. 3, equation 15)

$$\frac{M_3}{M_2} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma+1}{2\gamma}} \tag{7}$$

A compression ratio of three is considered typical for turbojets and since $\gamma = 1.4$

$$M_3 = (1/3)^{6/7} M_2 = .390M_2 = .0847 \qquad (8)$$

The equation connecting stagnation temperatures across the compressor is (equation 17, loc. cit.):

$$\frac{T_3^{(s)}}{T_2^{(s)}} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\gamma-1}{2} M_2^2}\right) \qquad (9)$$

$$= 3^{2/7} \frac{5 + M_3^2}{5 + M_2^2} = 1.358 \qquad (10)$$

where the stagnation temperature, which remains constant except where work is done on or heat is added to the gas stream, is the following function of static temperature and Mach number:

$$T^{(s)} = T\,(1 + \frac{\gamma-1}{2} M^2). \qquad (11)$$

Now since the flight temperature T_1 is 393°R and Mach number $M_0 \cong 0.9$, we have:

$$T_2^{(s)} = T_1^{(s)} = 393\,(1 + \frac{.81}{5}) \cong 460. \qquad (12)$$

Hence, by equation (10):

$$T_3^{(s)} = 625°R. \qquad (13)$$

This is then the temperature at the input to the reactor. Since the gas temperature at the exit of the reactor must not exceed 2000°R, a constraint imposed by practical gas turbine design, we see that the temperature difference, ΔT, through which the reactor raises the gas is:

$$\Delta T = 2000 - 625 = 1375°R. \quad (14)$$

Now in order that the gas stream in the nuclear turbo jet receive the same energy as was available in the form of heat value of the gasoline in the gasoline burning prototype, the mass flow must satisfy:

$$\dot{Q} = \dot{m} C_p \Delta T, \quad (15)$$

that is $$\dot{m} = \frac{\dot{Q}}{C_p \Delta T} = \frac{2.37 \times 10^4}{.26 \times 1375} = 66.3 \text{ lb/sec} \quad (16)$$

where the values of $\dot{Q}$ and ΔT are from equations 1 and 14 respectively and where an average value of C_p was taken to be .26 BTU/lb.deg.F.

Since $\dot{m} = \rho VA$, the intake area follows at once with the aid of equation 2, namely:

$$A_1 = \frac{66.3}{17} = 3.90 \text{ sq. ft.} \quad (17)$$

And since $A_1/A_2 = 1/3$, the gas stream cross-sectional area at the diffuser exit and in the reactor must be:

$$A_2 = A_3 = 11.7 \text{ sq. ft.} \quad (18)$$

Now heat transfer equation 5, Chapter 5B when integrated is:

$$\log \frac{T_w - T_3^{(s)}}{T_w - T_4^{(s)}} = \frac{.0604}{(\rho_3 V_3)^{0.2}} \frac{L}{d^{1.2}} \qquad (19)$$

where L, d are the length and diameter in feet of an individual tube of the convective heater and the mass current density, ρV, is in lb/ft^2/hr. The solution for $L/d^{1.2}$ yields:

$$\frac{L}{d^{1.2}} = \frac{(20400)^{.2}}{.0604} \log \frac{4000 - 625}{4000 - 2000} \qquad (20)$$

where a wall temperature of 4000°R in the reactor is assumed. Hence

$$\frac{L}{d^{1.2}} = 63.05 \qquad (21)$$

This equation expresses the constraint imposed upon the reactor design by the heat transfer equation. Another condition is that expressed by equation 18 which gives the free gas stream cross-sectional area, namely:

$$A_3 = 11.7 \text{ sq. ft.} \qquad (22)$$

A third condition may be supplied by considering optimum nuclear reactor design. As will be seen presently, these three conditions determine the reactor configuration.

Table 3 of Chapter 2 gives, for an average number of 2.1 neutrons emitted per fission, the following approximate critical sizes for optimum solid reactor design:

$$r_c = 54.\ \text{cm} = 1.78\ \text{ft.}$$

(23)

$$L_c = 99.\ \text{cm} = 3.25\ \text{ft.}$$

For a reactor perforated by tubes we need to divide the above numbers by $1 - \Gamma$, hence:

$$r_c = \frac{1.78}{1 - \Gamma}\ \text{ft.} \qquad (24)$$

$$L_c = \frac{3.25}{1 - \Gamma}\ \text{ft.} \qquad (25)$$

where Γ is the ratio of the gas stream free cross-sectional area to the total cross-section in the reactor. But from the definition of Γ we have:

$$\Gamma \pi r^2 = A_3$$

so that the constraint expressed by equation 22 becomes:

$$r = \sqrt{\frac{11.7}{\pi \Gamma}} = \frac{1.93}{\sqrt{\Gamma}} \qquad (26)$$

The value of Γ such that both equation 24 and equation 26 yield the same value of r is given by:

$$\frac{1.78^2}{(1 - \Gamma)^2} = \frac{1.93^2}{\Gamma} \qquad (27)$$

This requires that

$$\Gamma = 0.410 \quad , \quad r = 3.02\ \text{ft.}$$

Hence, by equation 25,

$$L = \frac{3.25}{1-\Gamma} = 5.51 \text{ ft.}$$

The volume of reactor is therefore:

$$\text{Vol.} = (1-\Gamma)\frac{A_3}{\Gamma} L = 1.439 \times 11.7 \times 5.51 = 92.77 \text{ cu.ft.}$$

The weight data are:

	Carbon Reactor	BeO Reactor
Reactor weight, lbs.	12,800	17,500
Uranium weight, lbs.	50	44

Since L = 5.51 ft, equation 21 gives:

$$d = \left(\frac{5.59}{63}\right)^{.8333} = .131 \text{ ft.} = 1.57 \text{ in.}$$

Finally, the number of tubes necessary to accommodate the mass flow is given by

$$n = \frac{A_3}{\pi d^2/4} = 867.9 \text{ or } 868.$$

The preceding results define the reactor completely. The method of calculation, it will be recalled, was to design the reactor so as to supply the same amount of energy to the air stream as would be available from the gasoline consumption in the commercial gasoline-burning prototype turbo-jet. This is a very general treatment and one should therefore examine the validity of the major assumptions involved. One such assumption is that the thrust of the pure air stream in the nuclear vehicle and that of the air-gasoline exhaust products in the gasoline prototype are not too dissimilar. Now both models have approximately the same exhaust temperatures, due to the 1500°F. limitation at the turbine. Also, because of this limitation, turbo-jets operate at extremely lean mixtures, air-fuel ratios being of the order of 60 or more, assuring a molecular weight of the exhaust gases little different from that of air alone. Hence it is not likely that the thrusts of the two designs will be appreciably different. One other factor is the drag due to the reactor tubes. It is not believed, however, that this will be sufficiently greater than in the conventional turbo-jet burner to cause much error in the calculations. For the present reactor design with L/d ratio of 42, equation 13 of Chapter 5C yields a drag per unit cross-sectional area of approximately one dynamic head, -- a figure which is in good agreement with experimental values obtained from laboratory tests on conventional ram-jet burners.

The power added to the gas stream turns out to be approximately 33,600 H.P., while the power output of the engine (thrust times velocity) is 6400 H.P. This gives an overall efficiency of 19%.

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CHAPTER IV. PRELIMINARY REPORT ON NUCLEAR ENERGY FOR ROCKET PROPULSION

by

F. T. McClure and R. B. Kershner

1. Introduction

In the comparison of fuels for use in rocket propulsion, probably the most significant parameter is the so-called effective gas velocity or, equivalently, the specific impulse. This quantity is defined by the ratio of the thrust to the mass rate of discharge of propulsive gas and is mainly a function of the thermodynamic properties of the gas. A convenient formula for the specific impulse, I, of a gas, is

$$(1) \qquad I\ \left[\text{lb(force)-sec/lb(mass)}\right] = 9.302\ I_r \sqrt{nT_c}$$

where n = inverse of the molecular weight of the gas (combustion products), T_c = chamber gas temperature in degrees Kelvin, and I_r, the reduced specific impulse, is a function only of the ratio of specific heats of the gas, the ratio of chamber pressure to atmospheric pressure, and the area expansion ratio of the rocket nozzle. The function I_r is graphed over a range of all three variables in ABL-SR-10 (OSRD No. 5548), "The Reduced Specific Impulse of Ideal Gases", Nancy Marmer and F. T. McClure. Numerically it varies between 1.6 and 2.6 for the usual range of the variables.

Other things being equal a rocket loaded with propellant of high specific impulse has a greater range than a corresponding rocket with a low specific impulse propellant. A major portion of the effort of rocket development work has been aimed at obtaining fuels with a high specific impulse. Fuels in common use now have an impulse of 180 to 250 lb.sec/lb.

Equation (1) shows that a high specific impulse requires high gas temperature and low molecular weight. Since available construction materials seem to place an upper limit on the temperature not much above temperatures obtained with current fuels (3000 to 3500°K, i.e. 5400 - 6300°R) it appears that significant improvements can be attained only by the use of fuels with a lower molecular weight.*

The importance of low molecular weight is well recognized and accordingly considerable attention has been paid to the use of hydrogen as a rocket fuel. The problem is, then, to find a means for heating hydrogen to a temperature in the neighborhood of 3000°K in as economical a manner as possible. The most obvious means for accomplishing this is to burn a portion of the hydrogen, for example with oxygen, to supply the necessary heat. With the use of the hydrogen-oxygen combination an optimum specific impulse may be expected with approximately a 5-to-1 mole ratio, (See "Fuel Systems for Jet Propulsion" by A. W. Lemmon, Jr., Report of the Gilliland Committee, and "Calculated Performance of Hydrogen and Oxygen as Jet Motor Propellant", Aerojet Engineering Corporation, Technical Memorandum No. RTM-23.) With 5-to-1 mole ratio and an operating pressure of 50 atmospheres, an impulse of 395 lb.sec/lb at sea level is predicted. With this ratio of hydrogen to oxygen a temperature of 2760°K is obtained but the mean molecular weight is about 8.4 due to the formation of a considerable amount of water vapor in

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*The upper allowable limit on the gas temperature might be raised still further by improved cooling of the walls by film methods or the like. If, however, the gas is to be heated by heat transfer from a wall (by black radiation or conduction) its temperature cannot be raised above that of that wall, so that cooling does not solve the problem.

the reaction products. With a greater amount of oxygen the temperature is higher but the impulse is lower due to the overbalancing effect of the increase in molecular weight. Conversely, with less oxygen the molecular weight is lower but the decrease in temperature is sufficient to reduce the specific impulse. The combination of hydrogen and oxygen in a mole ration of 5-to-1 gives a higher specific impulse than is predicted for any other fuel so far investigated.

Clearly, a means for heating hydrogen to a high temperature without increasing the molecular weight would give a very significant increase in the specific impulse. In fact, hydrogen alone at a temperature of 2500° Kelvin and a pressure ratio of 50 would give a sea level impulse of about 730 lb.sec/lb. It must be borne in mind, however, that the mechanism for heating the hydrogen constitutes a dead weight in the rocket which somewhat reduces the effectiveness of the gain in specific impulse. In particular, if the weight of the energy source required to produce a certain thrust was greater than the thrust produced, the resulting rocket would not rise in spite of the high specific impulse. The problem is to produce an energy source with very high power per unit weight. Recent developments in nuclear energy reactors suggest consideration of these devices as a promising means for heating hydrogen for rocket propulsion purposes.

In this report a simple quantitative discussion of the advantages of a rocket operated by hydrogen heated by a nuclear energy reactor will be given. For comparison purposes a hydrogen-oxygen rocket will be used as the prototype of "conventional" rockets. While a hydrogen-oxygen propulsion system has not yet been successfully used, it seems amply clear that the problems of development cannot be more difficult than those to be expected in the nuclear energy case. In particular the difficult problem of handling liquid hydrogen is common to both.

One feature of rocket design which reduces slightly the advantage of a low molecular weight fuel is the fact that such a fuel is likely to have low density and thus require a disproportionately large tank and structure weight for its storage in the rocket. Neglect of this point is likely to give a misleading impression of the relative advantages of different fuels. For example, in the case of a bi-fuel rocket with a large discrepancy between the densities of the two fuel components, the optimum ratio of the two fuel components is not the ratio which gives the maximum specific impulse. As mentioned before, the optimum specific impulse with the hydrogen-oxygen rocket is expected with approximately 5-to-1 mole ratio of hydrogen to oxygen. This implies a substantially larger volume of hydrogen than oxygen and, correspondingly, a disproportionately large weight of the hydrogen tanks. While shifting to lower hydrogen-oxygen ratios decreases the specific impulse and thus increases the fuel weight required, it might also overcompensate by decreasing the total fuel volume and hence decreasing the required tank and structure weight. The optimum ratio is that which gives the minimum sum of fuel weight and tank and structure weight.

Determination of this optimum requires a knowledge of the required velocity and an exact relation between tank and structure weight and fuel volume. The last relation is not well established but recent estimates of the Douglas Aircraft Corporation and the Glenn L. Martin Company ("Proposal for structural study of high altitude test vehicle", Glen L. Martin Company, Engineering Report 2373, May, 1946, and "Consideration of a high altitude space vehicle (Hall project)" Report ES-20515, El Segundo Engineering Department, Douglas Aircraft Company, March 28, 1946.) for the design of a satellite rocket have indicated that a tank and structure weight as low as one pound per cubic foot might be obtainable by the use of recent aircraft engineering design. Assuming this value, a rough analysis indicates that the two influences of changing the weight ratio of hydrogen to oxygen almost exactly compensates when fuel ratios are varied from 5-to-1 down to 3-to-1. Fuel ratios in this range lead to almost the same payload-range relation, at least for ranges up to satellite. For an escape rocket which with a single stage hydrogen-oxygen rocket is on the borderline of feasibility, the small effect of varying the hydrogen-oxygen balance may become very significant.

2. Requirements for Long Range Rockets

In this section we compare the design requirements of a hydrogen-oxygen rocket and a hydrogen-nuclear energy rocket to obtain various ranges. The ranges considered are 1000, 5000, and 10,000 miles. This last range is very nearly equivalent to a satellite rocket. In addition, an "escape" rocket is included. In calculating the velocity necessary for attaining these ranges air drag was neglected and effectively instantaneous burning was assumed. As a result the rockets described would not actually attain the ranges given but comparison should still be essentially valid. However, it should be noted that the assumption of sea level impulse throughout burning will make an error which will at least partially compensate those mentioned above.

Table 1 gives, then, the required initial velocities for a drag free shell with the prescribed range. These are obtained from the formula

(2) $$V = 36,670 \sqrt{\frac{\sin \Theta}{1 + \sin \Theta}}$$

where V is the velocity in ft./sec, 36,670 is the escape velocity in ft/sec, and Θ is 1/2 the range in radians.

Table 1

Range (miles)	1000	5000	10,000	escape
Velocity (H/sec)	12,300	22,300	25,600	36,700

The fuel required for a rocket to attain a given velocity is calculated from the well known rocket formula

(3) $$V = 32.16 \; I \; \log_e \left(\frac{\text{weight with fuel}}{\text{weight without fuel}} \right)$$

Values of the specific impulse, I, for a hydrogen-oxygen rocket with a mole ratio of 5-to-1 and for rockets propelled by hydrogen heated (by a nuclear reactor) to 2500°K, 2060°K and 1630°K respectively, are given in Table 2. The operating pressure was taken as 50 atmospheres in all cases.

Table 2

Code Number	A	B	C	D
Fuel	$5H_2$-to-$1O_2$	H_2 + N.E.	H_2 + N.E.	H_2 + N.E.
Gas Temperature (°K)	2760	2500	2060	1630
Specific Impulse (lb.sec/lb)	395	730	665	590

Table 3 gives the percent fuel, the percent tanks and supporting structure, and the remaining percent, β, for rockets of the four types A, B, C, D to attain the velocities given in Table 1. The percent fuel is calculated from (3) and the percent tanks and structure are obtained from the assumption of one pound of tank and structure weight per cubic foot of fuel.

The remainder, β, is the percent of weight available for rocket motor and nozzle, pumps, control, payload and (except in case A) nuclear reactor.

Table 3

Weight distribution of various long-range rockets

Velocity	Code Number	% H_2	% O_2	% Tanks & Structure	β
12300	A	15	47	4	34
	B	41	---	9	50
	C	44	---	10	46
	D	48	---	11	41
22300	A	20	63	5	12
	B	61	---	14	25
	C	65	---	15	20
	D	69	---	16	15
25600	A	21	66	6	7
	B	66	---	15	19
	C	70	---	16	14
	D	74	---	17	9
36700	A	23	71	6	0
	B	79	---	18	3
	C	82	---	19	(impossible)
	D	86	---	19	(impossible)

It will be noticed that the value of β for the nuclear energy rockets is almost always greater than for the "conventional" rocket. The difference between the value of β for cases B, C, D and the value of β for case A represents the weight percentage available for the nuclear reactor, if the nuclear rocket is just to compete with the "conventional" prototype.

3. Energy Considerations

In this section we give a preliminary survey of the energy requirements for a nuclear heated hydrogen rocket. We consider case B in which the hydrogen is heated to 2500°K.

To vaporize one gram of hydrogen at its boiling point and heat the resulting gas to 2500°K, at constant pressure, requires approximately 9400 gm-cals. (see, for example, NDRC Report A-116, "Thermodynamic Properties of Propellant

Gases", J. O. Hirschfelder, F. T. McClure, C. F. Curtiss, D. W. Osborne).

Thus the energy required, E, is given by

(4) $E \doteq 3.93 \times 10^{11}$ ergs/gm.

From Table 3 it is seen that the total weight of a rocket using hydrogen-nuclear energy must be at least $1/.16 \doteq 6.25$ times the weight of the reactor if the rocket is to out-perform a conventional rocket even at 1000 miles range. Thus, if W is the weight of the nuclear reactor the rocket weight is greater than 6.25W. Allowing an initial thrust of 2 g (over 1 g is required to rise at all) the thrust, F, must exceed 12.5 W. Since thrust equals the product of the specific impulse, I, and the mass rate of discharge, $\dot{m}$, we have

(5) $730 \cdot \dot{m} \doteq 12.5\ W$

where $\dot{m}$ is in grams/sec if W is in grams.

From (4) the mass rate, $\dot{m}$, requires an energy rate of

$$\dot{m}\,E \doteq 3.93\,\dot{m} \times 10^{11} \text{ ergs/sec.}$$

Hence, from (5),

(6) $\frac{\dot{m}\,E}{W} \doteq 6.74 \times 10^{9}$ ergs/sec-gm.

Thus a power output of .674 K.W. per gram of reactor is required. This is equivalent to 305 KW, or 410 horsepower, per pound of reactor.

It is obvious that the power output of nuclear reactions can greatly exceed the above requirement. The problem is to develop a means for transferring the energy produced into the hydrogen gas in the form of heat.

4. The Heat Exchange Problem.

Let the nuclear energy reactor be of the nuclear fission-chain variety (as opposed to radioactive material). Assume it has a uniform cross-section of arbitrary shape and is characterized by the following parameter:

Cross section area of reactor matter: A_r

Length: L

Total effective heat transfer surface: S

Temperature of surface: T

Density: ρ_r

Then the weight of the reactor is

(7) $$W = \rho_r A_r L$$

Suppose that the heat transfer mechanism operates through the surface of the reactor and is proportional to S (conduction or radiation). Let j be a suitable average rate of energy transfer per unit surface, so that j S is the rate at which energy is made available to the gas. Then, replacing $\dot{n} E$ in (6) by j S and using (7)

$$j S = 6.74 \times 10^9 \rho_r A_r L$$

or

(8) $$j \left(\frac{S/L}{A_r}\right) = 6.74 \times 10^9 \rho_r$$

Now $\left(\frac{S/L}{A_r}\right)$ is a geometrical factor giving the ratio of the perimeter to the area of the reactor cross section. For example, if the reactor consists of a bundle of rods of <u>radius</u> X,

$$\frac{S/L}{A_r} = \frac{2}{X}$$

If the reactor consists of concentric annular cylinders of <u>thickness</u> X then, again,

$$\frac{(S/L)}{A_r} = \frac{2}{X}$$

Thus, for the moment, we rewrite (8) as

(9) $j/X = 3.37 \times 10^9 \rho_r$ ..

It is difficult to decide what density of reactor would be required but we might consider reactors with moderators of BeO or C (graphite), both of which have high melting points. Their densities are about 3 and 2.2 respectively. Actually it seems quite unlikely that BeO would stand up in an atmosphere of hot, high pressure hydrogen. Thermodynamically, graphite also can react with hydrogen but kinetically this heterogeneous reaction may not occur in significant amount during the required operation time. Let us choose, then, the density of graphite for our example (the weight of fissionable and other material is neglected). Then

(10) $j/X = 7.4 \times 10^9$

For large values of X it is easy to show that conduction cannot provide as much heat transfer as is available through radiation if the wall temperature is of the order of 3000°K. Thus we consider first the possibilities of radiative heat transfer. Assuming 100% emissivity of the surface at 3000°K and 100% absorbtion in the gas (neglecting the weight of smoke material necessary to produce high absorbtivity) we are able to obtain a maximum value of X. For, under the above assumptions,

$$j = \sigma(3000)^4$$

where σ is the Stefan-Boltzmann constant ($\sigma = 5.673 \times 10^{-5}$ ergs cm^{-2} $(^\circ K)^{-4}$ sec^{-1}). Then, from (10)

$$X = .62 \text{ cm.}$$

It is seen that radiation will only supply the required energy if X is at most 0.62 cm. The small value of X reflects the requirement of a large ratio of surface to volume of reactor. Considerations of the reactor design imply, however, that the dimensions of the gas spaces within a reactor must be reduced along with the dimensions of the reactor spaces, otherwise the overall density of the reactor would be reduced and the reactor would fall below critical. Thus a high surface to volume for the reactor also implies a high surface to volume for the gas spaces within the reactor. Hence the gas passages are very thin and the absorption of radiation by the gas cannot be expected to approach the 100% assumed above. Allowance for the conceivable absorption attainable, even by the inclusion of smoke in the hydrogen, makes it appear that the possibility of the operation of a nuclear energy rocket depending on radiation for the heat transfer is remote.

On the other hand, with a sufficiently high ratio of surface to volume in the gas passages, heat transfer by conduction exceeds heat transfer by radiation even at 3000°K. In the next sections, therefore, the problem of heat transfer by conduction, in reactors with gas passages with a high surface to volume ratio, is considered in more detail.

5. Heat Exchange by Conduction.

We consider a reactor in the form of a solid cylinder with a number of cylindrical gas passages (pipes) drilled through it lengthwise and arranged in a hexagonal lattice. Then the reactor may be considered as built up from hexagonal cylinders each containing one gas passage (see Fig. 1).

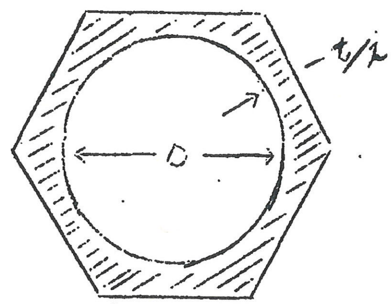


Fig. 1.

Let $A_g = \pi D^2/4$ be the cross-section area of a gas passage and A_r the area of the hexagonal annulus of reactor associated with a single gas passage. Thus A_r is the shaded area in Fig. 1. As usual, let L be the length of the reactor.

We consider the heat transferred by conduction in a single gas passage. We assume a wall temperature of T_w °K and determine the conditions under which the gas will be heated to T°K (from the boiling point) by passage through the pipe. Actually we require that the gas be heated from absolute zero to T°K; the slight additional heating from 0°K to the boiling point largely compensates our neglect of the heat of vaporization of hydrogen.

Under these conditions the equation of energy may be written

$$\text{(11)} \qquad \frac{d}{dx}\left[\int_0^T c_p \, dT + \tfrac{1}{2}v^2\right] = \frac{h(T_w - T)}{A_g \rho v} \, \frac{S}{L}$$

where $S = \pi DL$ is the heat transfer surface, h is the heat transfer coefficient, T_w is the wall temperature, and ρ is the gas density and v its velocity so that $\rho A v = \dot{m}$ is constant. For the heat transfer coefficient, h, we

8. Heat Transfer from Smoke Particles to Gas-Stream.

The heat transfer from particles to gas is given by:

$$q_{pg} = h_{pg}\, 4\pi r^2 N\, (T_p - T_g) \qquad (52)$$

The heat transfer coefficient h_{pg} has not been studied for particles of the size in which we are interested. Some work on the heat transfer to clouds of small spheres of upwards of 300 microns in diameter has been reported by Johnstone, Pigford and Chapin (Reference 8). If there is a relative velocity V between particle and gas, the solution of the heat transfer equation given by the above authors is

$$h_{pg} = \frac{k\, G\,(R,\, P)}{r} \qquad (53)$$

where k is the heat conductivity of the gas, R the Reynolds number ($2rV\rho/\mu$), and P is the Prandtl number.

The function G reduces to unity for small values of the Reynolds number. The Reynolds number is less than 1 for our case because of the small size of the smoke particles and also because of the small relative velocity to be expected. Thus, we are probably safe in assuming that

$$h_{pg} = k/r \qquad (54)$$

providing that r is not comparable with the mean free path of the molecules of the gas.

The mean free path of the gas molecules is given by

$$\frac{W}{\sqrt{2}\,\rho\, 4\pi r_m^2} \qquad (55)$$

where W is the molecular weight, r_m the radius of a molecule and n is the number of molecules per unit volume. Since σ for air the molecular radius is approximately 3×10^{-8} cm, the value of λ in free air at 60000 ft., and 0°C is 0.74 microns. From this point on λ and r will be measured in microns; and the micron value of r will be called r*. k is then in BTU per sq.ft. per hr., per °R, per micron, and h is in corresponding units.

In the diffuser where the density is approximately three times that at intake, the value of λ is about 0.25 microns. The mean free path at any point in the tube is then given by

$$\lambda = \frac{\lambda_d \ \rho_d}{\rho} \quad (56)$$

Preliminary study indicated that the particle size should be of the order of 0.1 to 1 microns for efficient transfer; thus it is comparable with the mean free path and we should investigate the heat transfer for the case of particles smaller than the mean free path.

Apparently, nothing has been done for such small particles at normal pressures but a clue to the method of attack can be obtained from the consideration of loss of heat from fine wires in rarefied gases. From the work of Knudsen (Reference 9) and Smoluchowski (Reference 10) on heat loss from wires of radii small compared to the mean free path of the molecules, it can be shown that

$$h_{pg} = 1.3\ ak/\lambda , \quad (57)$$

where k is the observed conductivity at densities such that the mean free path is small compared with the vessel radius and a is the accommodation coefficient of air molecules on the material of the smoke.

The constant 1.3 contains all factors relating to geometry, relation between rotational and translational energy and correction for difference between observed and theoretical heat conductivity. This relation should hold when r is vanishingly small compared to λ. On the other hand, relation (54) should hold when r is very large compared to λ. The form of the function for intermediate values may be guessed from some work by Smoluchowski (Reference 10) on concentric cylinders. It appears that an empirical relation of the form

$$h_{pg} = \frac{1.3\ ak}{\lambda + 1.3\ ar^*} \quad (58)$$

would be appropriate and would fit the end points satisfactorily.

The value of the accommodation coefficient for smoke substances which could be used in our application is uncertain. It is known well only for clean metal surfaces. It is surmised that for small particles of C or MgO it may be close to unity.

Simple kinetic theory indicates that the conductivity k varies as the square root of T. This is not verified by experiment and the discrepancy has been elucidated, at least partially, by Sutherland. However, it is well to use actual data for our present purpose. Table I presents values for air (Reference 11).

TABLE I. THERMAL CONDUCTIVITY OF AIR

T, degrees R.	k $\frac{\text{BTU}}{\text{hr.ft.}^2\text{ deg. R.}}$
400	0.0118
600	.0168
800	.0213
1000	.0260
1200	.0300
1400	.0350
1600	.0390
1800	.0430
2000	.0460
2200	.0500
2400	.0530

On plotting these data it is found that k is closely proportional to T. In our units the relation is

$$k = bT \cong 2.6 \times 10^{-5}\ T \tag{59}$$

Thus we can write

$$h_{pg} = \frac{1.3\ abT}{\lambda + 1.3\ ar^*} \tag{60}$$

Assuming that 1.3a equals 1, this becomes,

$$h_{pg} = \frac{bT}{\lambda + r^*} \tag{60a}$$

The rate of change of gas temperature along the tube is

$$\frac{dT}{dL} = \frac{q_{pg}}{\rho vc} = \frac{h_{pg}\ 4\pi Nr^2\ (T_p - T)}{\rho MC_s\ c} \tag{61}$$

$$= \beta\ T^{\frac{1}{2}}\ (T_p - T),$$

where, using (60a) we have put

$$= \frac{3\ b\ (S/A)\ T_d^{\frac{1}{2}}}{\rho_p\ M_d\ C_s\ c\ r^*\ (\lambda + r^*)}$$

To avoid having to know T_p as a function of L, we assume that $T_p - T = \delta$, a constant. The basic is that the radiation flux from the wall at 5000°R to the particles is constant within about 20%, in spite of the fact that the gas temperature changes from 700°R to 3500°R as we pass along the tube. This yields

$$dT/dL = \delta\beta\sqrt{T_g} \tag{62}$$

$$\text{or } \delta\beta L = 2\ (T_c^{\frac{1}{2}} - T_d^{\frac{1}{2}}).$$

Let us use the following assumed values:

$R_{as} = 100$	$M_d = 0.2$
$L = 10$	$C_s = 3.5 \times 10^6$
$c = 0.25$	$T_c = 3500°R$
$\rho_p = 80$	$T_d = 720°R$

The result is

$$r^*\ (\bar{\lambda} + r^*) = \frac{\delta}{380} \tag{63}$$

Thus if r^* is between 0.1 and 1 micron and $\bar{\lambda} \approx$ 0.25 microns, the temperature difference between particle and gas is only about 20°. Under the conditions assumed the heat transfer is very effective; and the heating of the smoke is an unimportant factor, because of the large value of R_{as}.

PART C. PROPULSION ANALYSIS

By A. C. Beer and A. W. Lemmon, Jr.

1. General Equations.

It is a relatively simple matter to predict the performance of a nuclear-energy ram-jet when the temperature of the exit gas stream is specified. Such problems as burning efficiency and chemical equilibrium as a function of temperature do not occur. Once the exit gas stream temperature is calculated by means of the heat transfer relationships derived in section B of this chapter, the propulsion performance can be predicted in the manner outlined below.

Following the same methods as used in reference 3 and subsequent analyses of ram-jet performance it is determined that for the case of no mass-flow spillover and straight tail configuration, the thrust coefficient is given by:

$$\frac{1}{2} C_t = \frac{A_1}{A_2}\left(\frac{C_i}{M_o} - 1\right) - \frac{1}{\gamma M_o^2} \qquad (1)$$

$$\text{where } C_i = \frac{gS_a}{c_o} = \text{impulse coefficient} \qquad (2)$$

A = duct cross-sectional area

M = Mach number

C_t = thrust coefficient (based on gas stream cross-sectional area at reactor)

g = conversion constant to gravitational units

c = local speed of sound

γ = ratio of specific heats

and the subscripts 0, 1, 2, etc. refer, respectively, to free stream, intake, diffuser, etc. In this equation the air specific impulse, defined at the location where the stream Mach number is unity, is given by

$$S_a = \frac{\gamma+1}{g} \sqrt{\frac{RT}{\gamma}} \tag{3}$$

where T = absolute temperature, °R

R = gas constant (1715 ft-lbs/slug-deg.F, for air)

γ = ratio of specific heats of the gas at constant pressure and volume respectively, a function of T.

This quantity, which is the stream force per unit mass flow, depends in the present case only on the air stream temperature at exit. The function is plotted in Fig. 5-1.

The intake-diffuser area ratio, A_1/A_2, is so chosen that (for the case of the conventional diffuser, i.e. subsonic pressure recovery after normal shock) the normal shock occurs at intake. This configuration therefore varies with the air specific impulse, S_a, and hence must be calculated for each exit gas temperature. Such a calculation involves computation of Mach numbers along the duct in the manner indicated below:

For given values of air specific impulse, input stagnation temperature, and reactor drag coefficient, the Mach number at the entrance to a reactor stage operated at choking is given by the following equation (cf. equations 13 and A-11 of ref. 3):

$$M_2^2 = \frac{2\epsilon \frac{R}{g^2} T_2^{(s)} - S_a^2 \left[1 - \sqrt{1 + 4\frac{R}{g^2}\frac{T_2^{(s)}}{\gamma S_a^2}(\beta - \epsilon\gamma)}\right]}{2\left[\beta S_a^2 - \epsilon^2 \gamma \frac{R}{g^2} T_2^{(s)}\right]} \tag{4}$$

where $\beta = \frac{\gamma - 1}{2}$ (all γ's being evaluated at temperature T_2) and $\epsilon = 1 - \frac{1}{2} C_{d_b}$ where C_{d_b} is the drag coefficient due to the reactor (see equations 11 and following). The stagnation temperature $T^{(s)}$ is defined in terms of the static temperature by the following relation:

$$T^{(s)} = T \left(1 + \frac{\gamma - 1}{2} M^2\right) \tag{5}$$

Now because of the conservation of energy, $T_2^{(s)} = T_o^{(s)}$, and so:

$$T_2^{(s)} = T_o^{(s)} = T_o \left(1 + \frac{\gamma - 1}{2} M_o^2\right) \tag{6}$$

Hence for a given set of flight conditions the Mach number M_2 at the end of the subsonic diffuser (entrance to reactor) is uniquely determined for each value of exit gas temperature.

Now the Mach number M_1 at the entrance to the diffuser, for our case of normal shock at intake, is obtained with the use of the well-known relation connecting the Mach numbers across normal shock, viz:

$$M_s^2 = \frac{M_a^2 + 5}{7M_a^2 - 1} \quad , \text{ for } \gamma = 1.4 \tag{7}$$

where M_s is the Mach number after the shock and M_a is the approach Mach number. For our case, of course:

$$M_s = M_1 \quad \text{and } M_a = M_o \tag{8}$$

where M_o is the flight Mach number.

With the knowledge of M_1 and M_2, the Mach numbers at entrance and exit of the subsonic diffuser, the ratio A_1/A_2 follows at once from the relationship between Mach number and duct cross-sectional area, namely:

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}\left(\frac{M_1^{\,2}+5}{M_2^{\,2}+5}\right)^{3.5\eta \;-\; 0.5} \qquad \text{(for } \gamma = 1.4\text{)} \qquad (9)$$

where M_1 and M_2 are the Mach numbers at cross-sectional areas A_1 and A_2 respectively and η is the efficiency of the diffuser (ref. 3, equation 20).

Equation (9) is commonly written in the form:

$$\frac{A_1}{A_2} = \frac{\chi(M_1)_\eta}{\chi(M_2)_\eta} \qquad (10)$$

where $\chi(M)_\eta = 1/M \left(\frac{5+M^2}{6}\right)^{3.5\eta \;-\; 0.5}$ for $\gamma = 1.4$

2. <u>Determination of Reactor Drag Coefficient.</u>

For the flow of air through long tubes the friction drag coefficient is given by

$$C_{d_b} = 4\ fL/D \qquad (11)$$

where L is the length of tube, D its diameter, f is the friction force per unit sidewall area per unit value of $\rho v^2/2$ and the subscript b refers to the burner, or reactor tube.

The value of f for long tubes has been given by McAdams (ref. 17) in terms of the Reynolds number of the flow, either by the empirical equations

$$f = \frac{.046}{(R_e)^{0.20}}$$

or $$f = 0.0014 + 0.125/(R_e)^{0.30} \qquad (12a)$$

where R_e is the Reynolds number, D $\rho_d V_d/\mu$;

or by the theoretical equation of Von Karman (ref. 18)

$$1/f^{1/2} = 4 \log \frac{Re}{f^{1/2}} - 4.0 \qquad (12b)$$

For the range of values of Re found in the ram-jets here considered all three equations give approximately the same value of f, but equation 12 was chosen because of its simplicity. Thus

$$C_{d_b} = \frac{.184}{(Re)^{0.2}} \frac{L}{D} \qquad (13)$$

This is the equation used to calculate C_{d_b} and hence the value of ϵ appearing in equation 4.

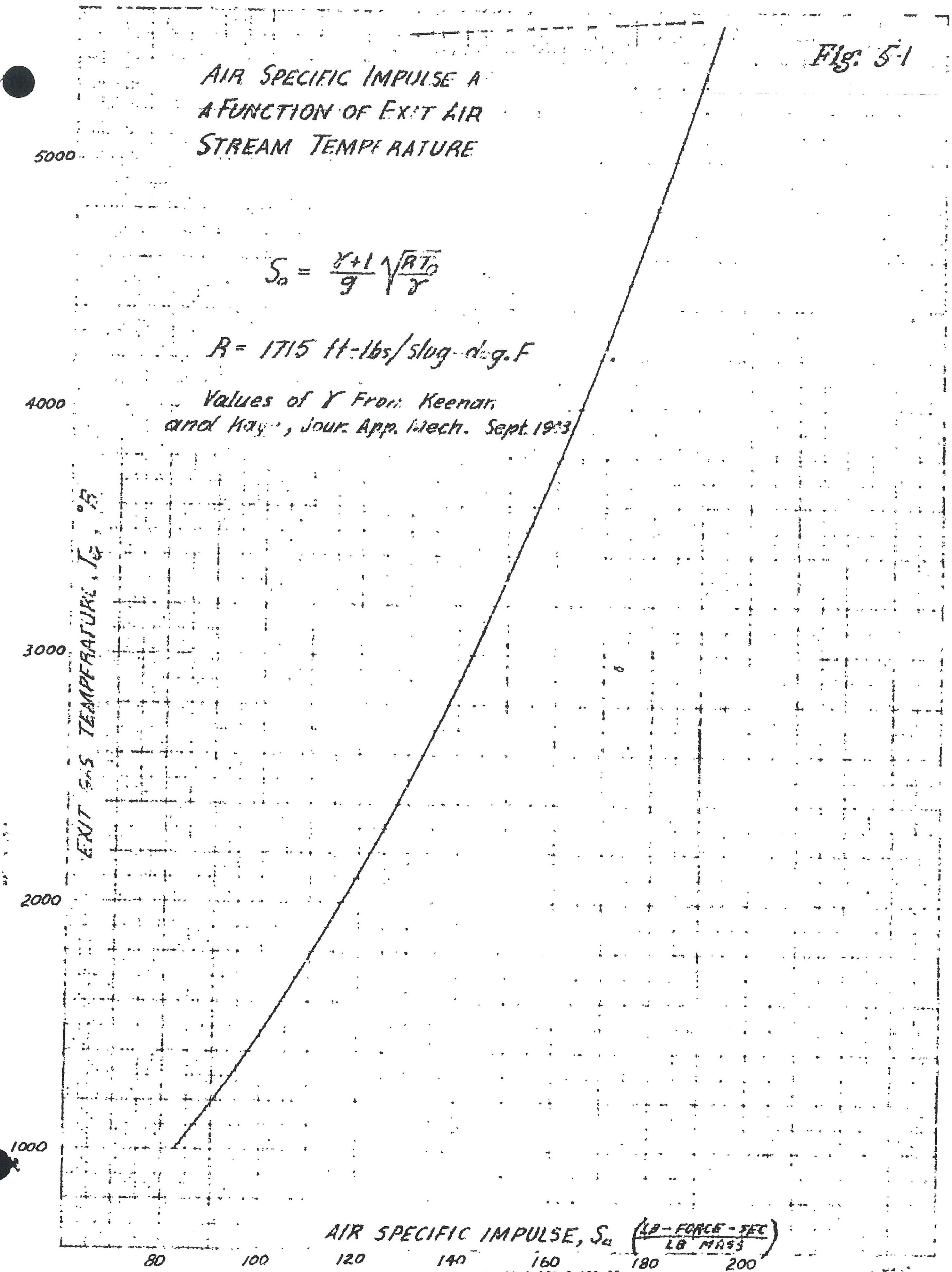


Fig. 5-1
AIR SPECIFIC IMPULSE A
A FUNCTION OF EXIT AIR
STREAM TEMPERATURE
$S_a = \frac{\gamma+1}{g}\sqrt{\frac{RT_0}{\gamma}}$
R = 1715 ft-lbs/slug-deg.F
Values of γ From Keenan and Kaye, Jour. App. Mech. Sept. 1943
EXIT GAS TEMPERATURE, T_G, °F
5000
4000
3000
2000
1000
AIR SPECIFIC IMPULSE, S_a $\left(\frac{LB-FORCE-SEC}{LB\ MASS}\right)$
80
100
120
140
160
180
200

PART D. AERODYNAMIC ANALYSIS

By R. J. Vicars

A missile suitable for nuclear-powered flight is shown schematically in Fig. 5-2. The basic size and shape of the fuselage is initially a function of the volume and space requirements of the power plant - the reactor - and of the payload - arbitrarily established as a shape six feet in diameter and twelve thousand pounds in weight. Secondary aerodynamic considerations refine the basic shape into a vehicle suitable for flight, and analyses based on this shape will supply knowledge of the missile performance.

For a missile to be in non-accelerated flight (dynamic equilibrium), two conditions must be fulfilled, namely

(a) Lift = Weight (1)

(b) Thrust = Drag (2)

Accordingly an analysis was made to determine the lift, weight, thrust, and drag of the proposed missile.

For the purposes of this analysis, arbitrary values of flight altitude and Mach number were selected as 50,000 feet and 2.0 respectively. Power plant calculations indicated that a diffuser ratio (maximum body cross-sectional area/intake area) of 3.33 would permit a suitable pressure recovery. The reactor was established as a perforated cylinder eight feet in diameter and approximately seven feet in length with a free area of 55.6% of the cross-section.

The total external drag of the missile is made up of the nose form drag, the friction drag, the tail drag, and the wing drag.

(a) Nose form drag $= q\, A_N\, C_{D_N}$ (3)

where q = dynamic head

A_N = projected frontal area

C_{D_N} = nose form drag coefficient (ref. APL/JHU Aerodynamics Handbook)

Nose form drag = 680 x 35.2 x .032 = 776 lbs.

(b) Friction drag = $q\ A_f\ C_f$ (4)

where A_f = external wetted area of missile

C_f = friction drag coefficient (Ref. APL/JHU Aerodynamics Handbook)

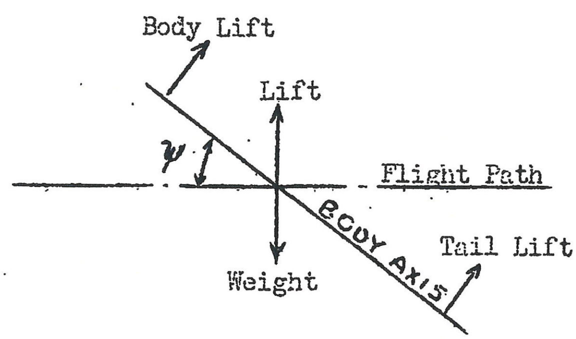
Friction drag = 680 x 541 x 0.00264 = 970 lbs.

(c) Tail drag = $q\ A_T\ C_{D_T}$ (5)

where A_T = tail planform area (total)

C_{D_T} = drag coefficient of the tail based on the tail planform area. (ref. APL/JHU Aerodynamics Handbook)

It was arbitrarily established that the tail area should be twice that required for neutral stability thus ensuring adequate dynamic stability throughout flight. An examination of the loads (Fig. 5-3) on the missile when at a slight angle of yaw, (ψ)



(Normal forces assumed equal to lift forces at small angles considered)

Fig. 5-3

indicates that, to be neutrally stable, a summation of moments about the center of gravity must equal zero, that is

Body lift x moment arm = tail lift x moment arm

or $$q\, A_T\, C_{L_T}\, \eta\, l_T = q\, A_B\, C_{L_B}\, l_B \qquad (6)$$

where A_T = tail planform area (in plane perpendicular to body lift)

C_{L_T} = tail lift coefficient (ref. APL/JHU-CM-321)

η = tail efficiency factor (0.75)

l_T = moment arm of tail lift force

A_B = body cross-sectional area

C_{L_B} = body lift coefficient (ref. APL/JHY-CF-166)

l_B = moment arm of body lift force

From equation (6)

$$A_T = \frac{q\, A_B\, C_{L_B}\, l_B}{q\, C_{L_T}\, \eta\, l_T} = \frac{l_B\, C_{LB}}{l_T\, \eta\, C_{L_T}}\, A_B \qquad (7)$$

In order to establish the various moment arms, it was assumed that

(1). The center of gravity of the missile is on the longitudinal axis at a point 60% of the body length aft of the forward tip of the nose.

(2). The wing lift force acts at the missile c.g.

(3). The tail lift force acts at a point 1/2 diameter forward of the aft end of the missile.

(4). The body lift acts at a point one fourth of the conical nose length aft of the forward tip of the nose.

These assumptions then allow the calculation of the required tail area. If

$(\frac{dC_L}{d\alpha})_B$ = 2.6/radian and $(\frac{dC_L}{d\alpha})_T$ = 2.64/radian

then

$$A_T = 99.\ ft^2$$

This is the area in one plane necessary to give neutral stability. Since the missile must be stabilized in two planes the area must be doubled. Another factor of two must be used in order to meet the previous requirement that twice the area needed for neutral stability be used.

The tail drag $= q\ A_T\ C_{D_T} = 680 \times 396 \times .0081 = 2740$ lbs.

(d) Wing drag $= \frac{L}{L/D} = \frac{W}{L/D}$ (3)

where L = wing lift

W = total missile weight

L/D = wing lift to drag ratio

In order to calculate the wing drag it is necessary to have some knowledge of the total missle weight. Since ram-jets of this order are still only a designer's dream, no accurate weight data is available. The following are, at best, reasonable estimates.

Item	Weight
Reactor	31,000 lbs.
Payload	12,000
Tail (400 ft^2 @ 4 lb/ft^2)	1,500
Wing (760 ft^2 @ 6.5 lb/ft^2)	4,900
Fuselage	25,000
Total weight	74,400 lbs.

To verify the selection of 760 ft^2 for wing area

$$L = W = q A_W C_L \quad (9)$$

where A_W = wing area

and C_L = wing lift coefficient (0.145) (ref. APL/JHU Aerodynamics Handbook)

L = 680 x 760 x 0.145 = 75,000 lbs. which is reasonably close to the originally assumed value.

$$\text{Wing drag} = \frac{W}{L/D} = \frac{74,400}{9} = 8250 \text{ lbs.}$$

The value of 9.0 used as the L/D ratio of the wing would appear attainable for a biconvex wing of 3% thickness ratio, at an angle of attack of about 3° (ref. APL/JHU Aerodynamics Handbook). The total drag of the missile then is:

Nose form drag	=	766 lbs.
Friction drag	=	970
Tail drag	=	2,740
Wing drag	=	8,250
Total drag	=	12,726 lbs.

These total lift and drag figures indicate an overall lift to drag ratio for the missile of about 5.9. The drag coefficient based on the maximum body cross-sectional area is

$$C_D = \frac{\text{Total drag}}{q A} \quad (10)$$

$$C_D = \frac{12,726}{680 \times 50.3} = .371$$

By analysis it has been established that a vehicle designed within the previously established limits will meet the initial requirement that the lift equal the weight and will have a drag coefficient equal to 0.371. This means that the missile will be sustained in flight at an altitude of 50,000 ft. and a Mach number of 2.0 when a thrust coefficient of 0.371 is maintained by the power source.

The schematic figure chosen for analysis seemed to be fairly representative of conventional designs. However, due to the fact that the heaviest single item in the missile, the reactor, must be located in the aft section of the missile, thus placing the center of gravity far towards the rear, it seems probable that a more practical design may be possible using the Canard type configuration. This type design places the main supporting surfaces at the rear. However, no detailed study of the Canard type will be made at this time because the purposes of the analysis have been served by the conventional design chosen.

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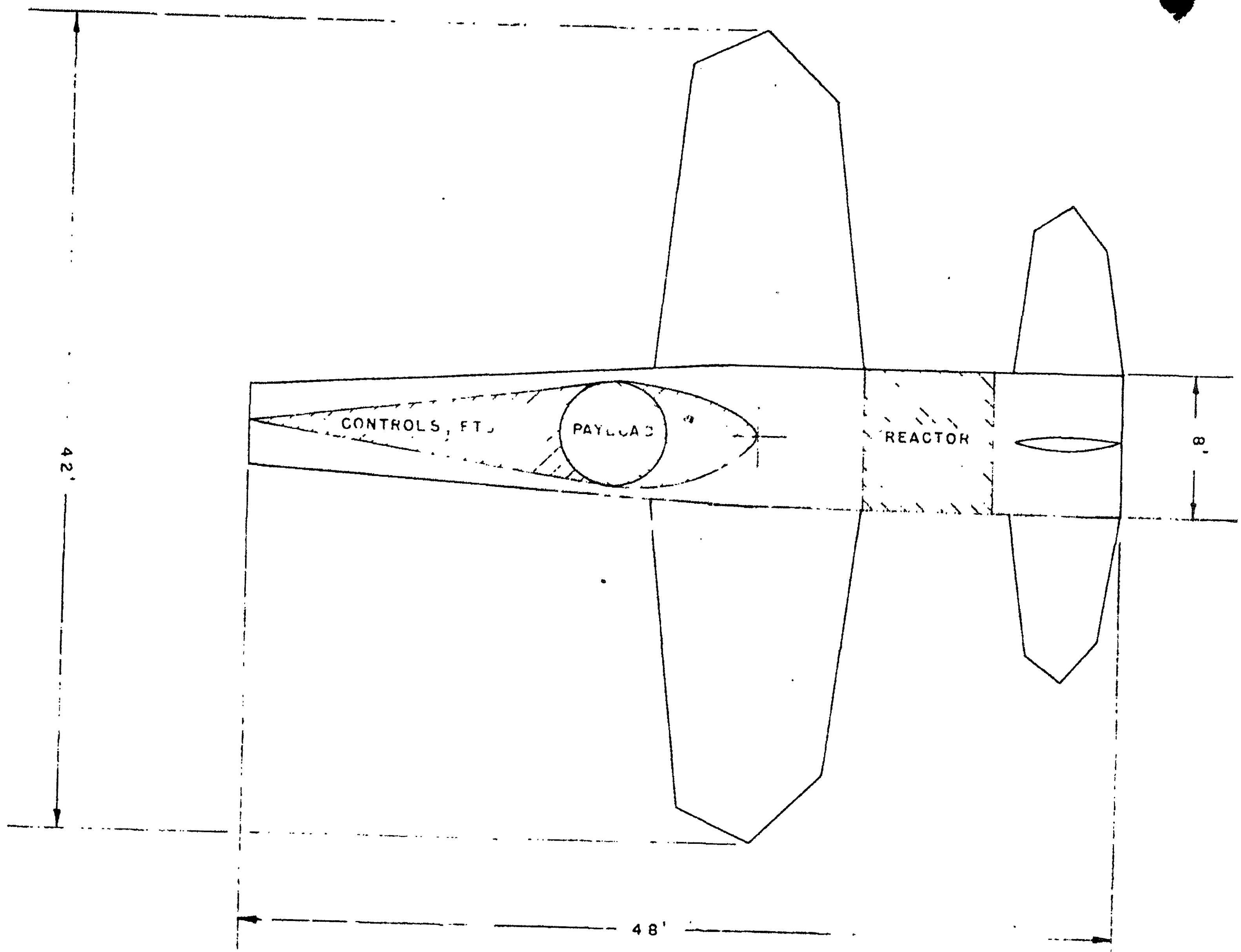

Fig 5-2
SCHEMATIC DIAGRAM
NUCLEAR POWERED RAM JET

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PART E. DESIGN REQUIREMENTS FOR A CONVECTIVE HEATED RAM-JET

A. C. Beer and C. F. Meyer

1. Introduction.

The analysis of parts B, C. and D now enables us to proceed systematically with numerical design. The application of constraints imposed by such considerations as, optimum critical nuclear reactor size, heat transfer conditions, maximum wall temperatures, flight altitude, etc., to ram-jet theory lead to a design capable of flight at a Mach number 2 at altitude about 50,000 ft. and having essentially unlimited range in the absence of material difficulties. The calculation of thrust coefficients for this vehicle is similar to that employed in the case of the conventional fuel ram-jet (ref. 3), except that as a result of the additional constraints imposed by the conditions enumerated above, certain parameters are no longer arbitrary. The equations leading to the design described above will now be presented.

2. Reactor Requirements.

The nuclear reactor design equations give the following critical sizes for the reactor having the optimum concentration of uranium. (These are based on an average number of 2.1 neutrons emitted per fission and are approximately correct for both carbon and BeO reactors.)

$$r_c = \frac{1.78}{1-\Gamma} \text{ ft.}$$

$$L_c = \frac{3.25}{1-\Gamma} \text{ ft.} \qquad (1)$$

where Γ is the ratio of the gas stream free cross-sectional area to the total cross-section of the reactor. We shall assume that the diameter of the reactor equals the diameter of the vehicle, which was taken as 8 ft. to accomodate the desired payload. Using this in equation 1 we obtain:

$$\rho = .556$$
$$1 - \rho = .444 \qquad (1a)$$
$$L_c = 7.32 \text{ ft.}$$

Hence the gas stream cross-sectional area must be

$$A_2 = \rho \frac{\pi D_B^2}{4} = 28.0 \text{ sq. ft.}$$

The total weight of the reactor is given by

$$W_R = (1 - \rho) \frac{\pi D_B^2}{4} L_C \rho_R \qquad (2)$$

where ρ_R is the density of the reactor moderator. This gives for carbon and BeO reactors

$$W_R \text{ (carbon)} = 22,600 \text{ lb.} = 11.3 \text{ tons}$$
$$W_R \text{ (BeO)} = 31,000 \text{ lb.} = 15.5 \text{ tons}$$

3. Heat Transfer Relations.

For flight at 50,000 ft. altitude and at Mach number of 2.0 the mass current density at intake is

$$\rho_1 V_1 = 22.5 \frac{\text{lb.}}{\text{ft}^2 \text{ sec.}}$$

and in the reactor it is given by

$$\rho_2 V_2 = 22.5 \frac{A_1}{A_2} \qquad (3)$$

For the purposes of estimating length-diameter ratios we shall assume that $A_1/A_2 = 2.66$, which will be seen later to be a fair average of calculated area ratios. Thus we have

$$\rho_2 V_2 = 8.45 \frac{\text{lb.}}{\text{ft}^2\text{sec}} = 30{,}400 \frac{\text{lb.}}{\text{ft}^2\text{hr.}}$$

The equation governing the heat transfer from the reactor walls to gas stream when integrated is (equation 5, part B)

$$\ln \frac{T_w - T_d}{T_w - T_e} = \frac{.0604}{(\rho_2 V_2)^{0.2}} \frac{L}{d^{1.2}} \qquad (4)$$

where L and d are the length and diameter of an individual tube of the reactor.

This equation can be solved for $\frac{L}{d^{1.2}}$ for various values of T_w and T_e. Also, since L is known from equation 1a, the value of d and L/d follow at once. We shall now assume that the efficiency of the heating process is such that the exit gas stream temperature (T_e) is 1000°R below the wall temperature. We then obtain Table 1.

TABLE 1

T_w	T_e	d	L/d	n	C_{D_B}
5000°R	4000°R	.045 ft.	162	17500	4.7
4600	3600	.048	153	15600	4.3
4200	3200	.051	143	13600	3.9
3800	2800	.056	131	11500	3.5
3400	2400	.062	118	9200	3.0
3000	2000	.072	102	6900	2.5
2800	1800	.079	92	5700	2.2
2600	1600	.089	82	4400	1.9

The value of n, the number of tubes required in the reactor is given by,

$$n\pi \frac{d^2}{4} = \Gamma \pi \frac{D_B^2}{4}$$

or

$$n = \Gamma \left(\frac{D_B}{d}\right)^2 \qquad (5)$$

The internal drag coefficient (C_{D_B}) in the reactor was calculated by equation 13 of part C. The Reynolds number used in that equation was obtained by using an average value of the viscosity (μ) of air for each case.

Using equations 4, 9, and 1 of part C the values of ϵ, M_2, A_1/A_2 and C_t where calculated for each wall temperature. The values of S_A were taken from Fig. 5-1. The values are shown in Table 2.

TABLE 2

T	ϵ	M_2	A_1/A_2	C_t	$C_t'(\Gamma C_t)$
5000	-1.35	.166	.331	.784	.436
4600	-1.15	.176	.349	.743	.413
4200	-0.95	.187	.371	.701	.390
3800	-0.75	.200	.395	.656	.365
3400	-0.50	.218	.429	.599	.333
3000	-0.25	.241	.472	.524	.292
2800	-0.10	.257	.502	.480	.267
2600	+0.05	.278	.540	.419	.233

The calculated thrust coefficients C_t are referred to the free area ΓA_B, so that if we wish to refer them to the total ram-jet cross-sectional area, A_B, the values should be multiplied by Γ. These results, C_t', shown in the last column of Table 2.

Once the thrust coefficients are ascertained, we can apply the equations of section D to calculate the total vehicle weight W_b that can be maintained in flight at selected altitude, namely

$$W_b = (\text{Lift/Drag}) \quad q \quad A_B C_t'$$

For an 8 ft. diameter vehicle flying at 50,000 ft. at a Mach number of 2.0

$$q = 680 \text{ lb/ft}^2$$

$$A_B = 50.3 \text{ ft}^2$$

Now from the study of conventional fuel ram-jets and from the analysis in part D it is reasonable to expect lift-drag ratios of the order of 6. Thus we have

$$W_b = 205{,}000 \; C_t'$$

If we subtract from this the weight of the reactor and that of the payload (12,000 lbs.), the remaining weight is allowable for structure, controls, etc. In Table 3 are shown the total weights and structure weights for both carbon and BeO moderated reactors as a function of wall temperature.

TABLE 3

T_w, deg. R.	W_b, lbs.	Carbon Reactor W_s, lbs.	BeO Reactor W_s, lbs.
5000	89,380	54,780	46,380
4600	84,670	50,070	41,670
4200	79,950	45,350	36,950
3800	74,830	40,230	31,830
3400	68,280	33,670	25,270
3000	59,800	25,260	16,860
2800	54,740	20,140	11,740
2600	47,770	13,170	4,770

Fig. 5-4 shows this weight data as a function of T_W, while Fig. 5-5 gives a similar plot vs wall temperature of the weight available for structure, etc., expressed as percentage of the total weight. It appears possible, in view of these data, to build a ram-jet capable of flight at altitudes of 50,000 ft. and Mach number 2.0 providing that wall temperatures can be maintained above 3600°R. The above figures have been based on true densities of graphite and BeO. Realistic bulk densities obtained in present practice are much lower so that it is possible that actual designs will be larger and heavier than the ones presented.

The design analyzed above was a straight tail configuration, i.e. with exhaust at Mach number unity at the reactor area ΓA_B. A slight increase in thrust could be obtained by expanding the exit gas stream by means of a nozzle to a supersonic velocity. The amount of useful expansion is limited, however, due to the fact that the thrust coefficient reaches its maximum when the exhaust pressure is equal to that of the flight medium. With the straight tail (no expansion) the exit pressure is only a factor of two or three times the ambient. Hence, it follows that only a restricted degree of expansion is permissible. Therefore, the gain in thrust which can be obtained by this mechanism is quite limited. Since estimates indicated gains of the order of only 5% or so, the more involved calculations were not carried out.

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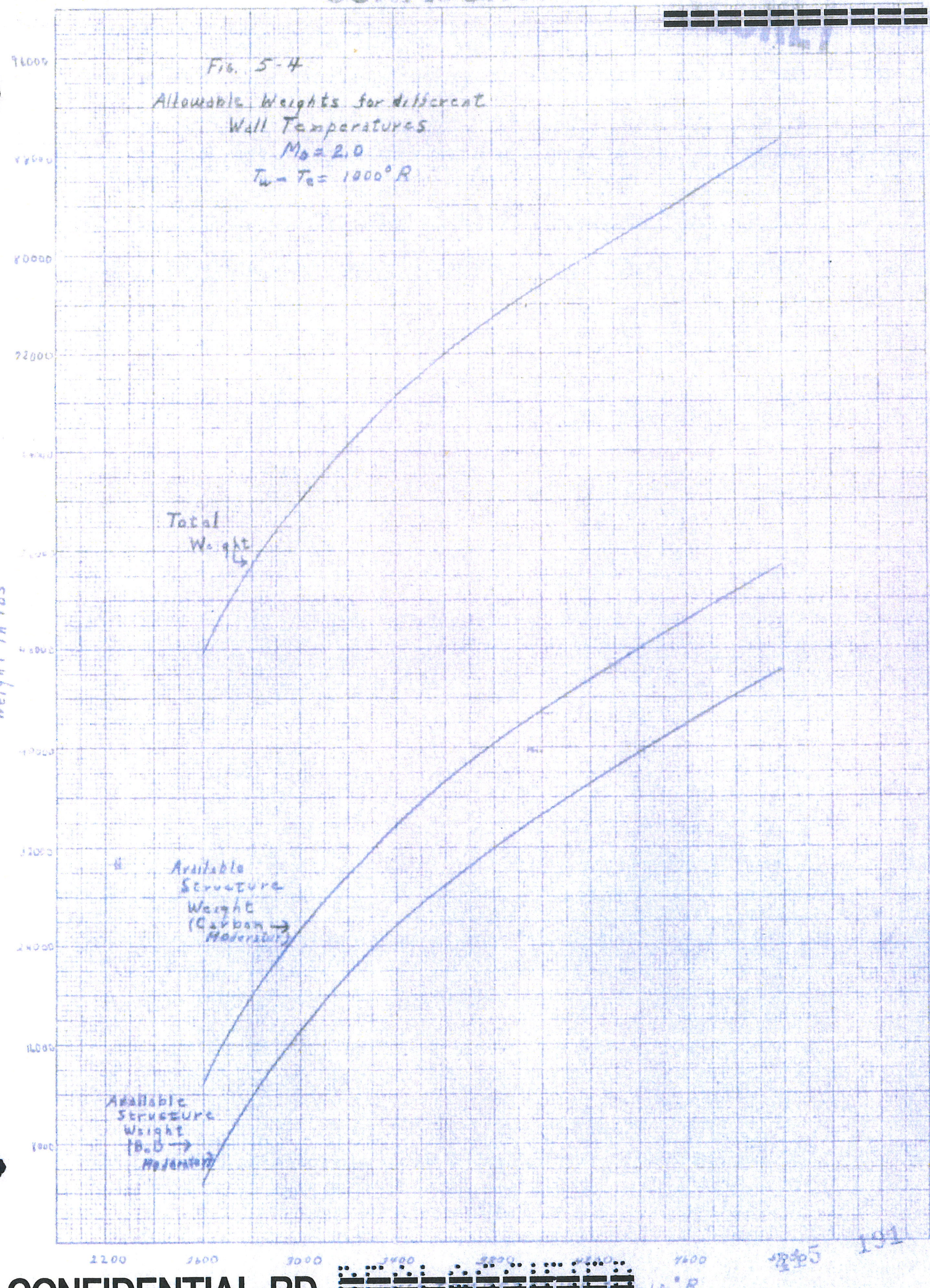


Fig. 5-4
Allowable Weights for different Wall Temperatures
$M_0 = 2.0$
$T_w - T_0 = 1000°R$

[redacted]

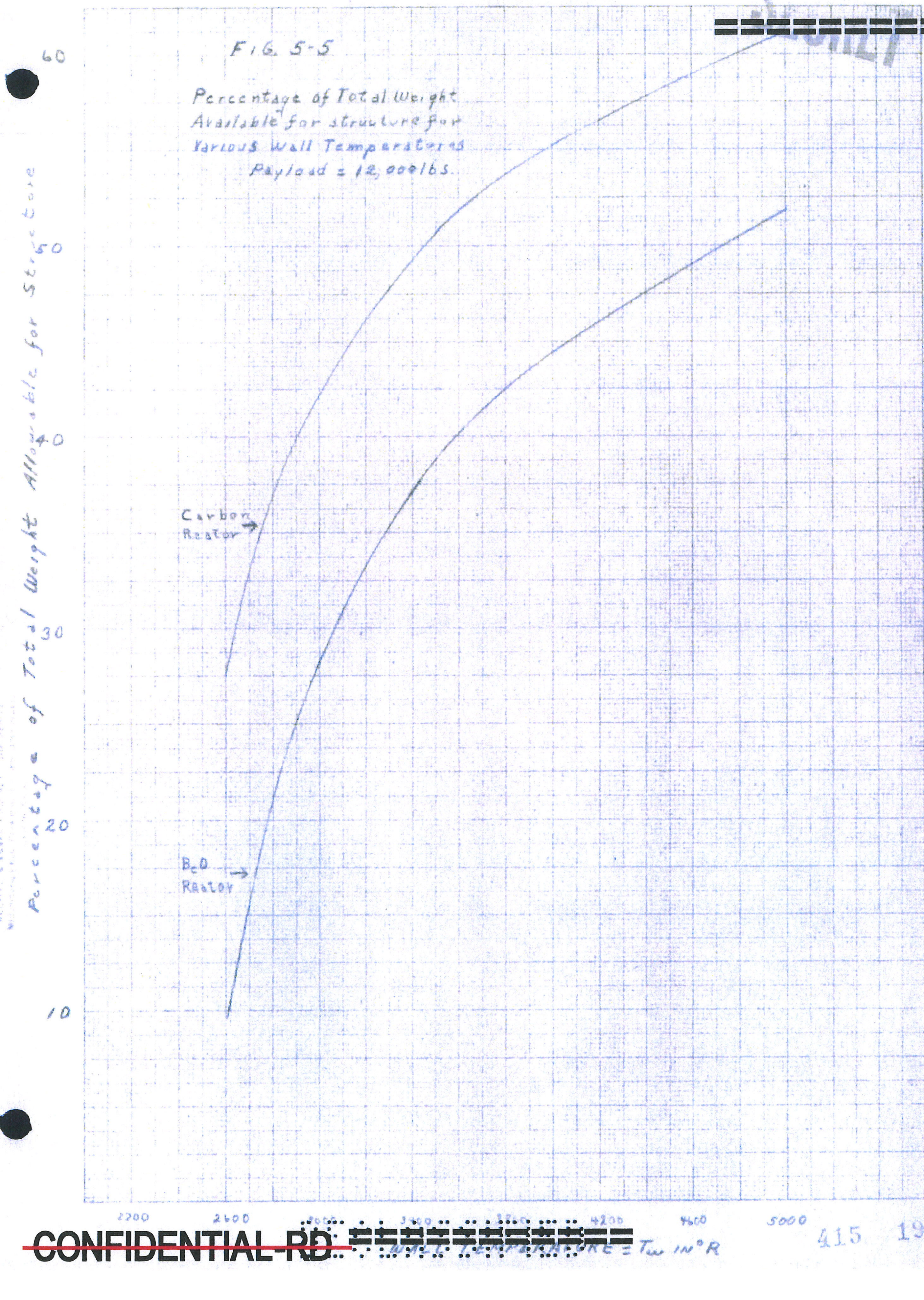


FIG. 5-5
Percentage of Total Weight Available for structure for Various Wall Temperatures Payload = 12,000 lbs.
Carbon Reator
B_eO Reator
60
50
40
30
20
10
Percentage of Total Weight Allowable for Structure
2200
2600
3400
4200
4600
5000
WALL TEMPERATURE = T_w IN °R

PART F. DESIGN REQUIREMENTS FOR A RADIATIVE-HEATED RAM-JET

By C. F. Meyer and A. C. Beer

Considerations of nuclear design, heat transfer, and aerodynamic conditions lead to the following arguments against the feasibility of using a reasonable sized radiative-heated single tube annular reactor surrounding the gas stream to propel a ram-jet. These considerations are:

1. The exit gas temperatures must be of the order of 4000°R leading to wall temperatures of the order of 5000°R.
2. The large heat flow at the high wall temperatures required necessitates a great difference of temperature through the body of the reactor in order to obtain adequate heat transfer.
3. The volume and weight of smoke material required for a 5000-mile flight results in the necessity for further increased sizes.
4. The length-diameter ratio of the reactor tube must be of the order of 10 and the radiation absorption path should be 6-ft. or longer. This requires a very large vehicle.
5. The aerodynamic conditions necessary for flight at reasonable altitudes postulates a large gas stream free area and consequently a large reactor.
6. The uranium requirements of the reactor are fantastic.

A study of the possibility of using long solid cylinder reactors in various combinations, as radiators, indicates that the heat transfer is less efficient than that for a single-tube design and that there is no gain over the annular reactor design insofar as size is concerned.

The sum total of these considerations suggests that a radiative-heated ram-jet is inoperable unless it is of very large size. It, therefore, seems feasible only for some very speculative future use.

PART G. PRELIMINARY ANALYSIS OF IDEALIZED NUCLEAR-POWERED SUPERSONIC TURBO-JET VEHICLE

By A. C. Beer and R. J. Vicars

1. Introduction.

It is well known that a considerable increase in the thrust of a ram-jet can be obtained at the low supersonic Mach numbers if mechanical compression as well as ram-pressure recovery is utilized. The disadvantages are, of course, the added weight and structure complications caused by the introduction of the turbine and compressor units. In a nuclear supersonic turbo-jet, the indications are that two reactors should be used, the first having the primary purpose of supplying to the turbine the energy necessary to drive the compressor, while the second is used to raise the gas stream temperature to the desired value at exit.

In order to estimate the feasibility of a nuclear-powered turbo-jet with exhaust heating, analysis of an idealized model is carried out in the following sections. It is shown that a configuration is possible which is consistent with the constraints imposed by the design conditions, such as flight Mach number of 1.4; optimum nuclear reactor design; reasonable values of mechanical compression ratio, gas temperatures, and reactor wall temperatures. From the calculated value of the thrust coefficient the total weight which can be supported in flight at a Mach number of 1.4 at 50,000 ft. is ascertained. Calculation of the payload and reactor weights and estimation of the weights of the turbine-compressor unit, structure, etc., provides data from which the practicability of constructing such a vehicle may be considered. Although calculations are carried through only for the carbon-moderated reactor, from the data given in Chapter 2 a parallel calculation could be made for a beryllia reactor. With carbon, of course, the surface must be coated by a protecting material in order to prevent oxidation.

Nomenclature.

C_{d_b} – reactor drag coefficient based on gas stream cross-sectional area.

C_i – impulse coefficient, $C_i = \frac{gS_a}{c_o}$; c_o = speed of sound in flight medium.

c_p – specific heat of gas at constant pressure, BTU/lb, deg. F.

C_t – thrust coefficient based on reactor gas stream cross-sectional area.

C_t' – thrust coefficient based on overall reactor cross-sectional area.

d – diameter of individual tube through reactor, ft.

g – conversion factor, absolute to gravitational units.

m – mass-flow, lb/sec.

R – gas constant, $\frac{\text{ft} - \text{lbs}}{\text{slug deg. F}}$.

Re – Reynolds number.

L – length of reactor, ft.

n – number of tubes in reactor

r – radius of reactor, ft.

ΔW – work done on a unit mass of gas.

S_a – specific impulse of gas, stream force/air mass flow, i.e., lb force sec/lb mass.

[illegible] – ratio of cross-sectional area available to gas stream to total cross-sectional area in the reactor.

ϵ – dimensionless quantity defined by $\epsilon = 1 - \frac{1}{2} C_{d_b}$.

η – subsonic diffuser efficiency.

η_{c_t} – compressor-turbine efficiency.

$\chi(M)_\eta$ – function defined by equation 4.

$\chi(M)$ – function defined by equation

μ – coefficient of viscosity.

The following symbols are usually written with subscripts to indicate location. The subscripts 0, 1, 2, 3, ..., 6, refer to locations indicated in Fig. 5-6 and w refers to reactor wall.

A – gas stream cross-sectional area, sq. ft.

c – speed of sound, ft/sec.

F – stream thrust or momentum flow, lbs.

M – Mach number.

p – pressure (absolute), lb/ft^2.

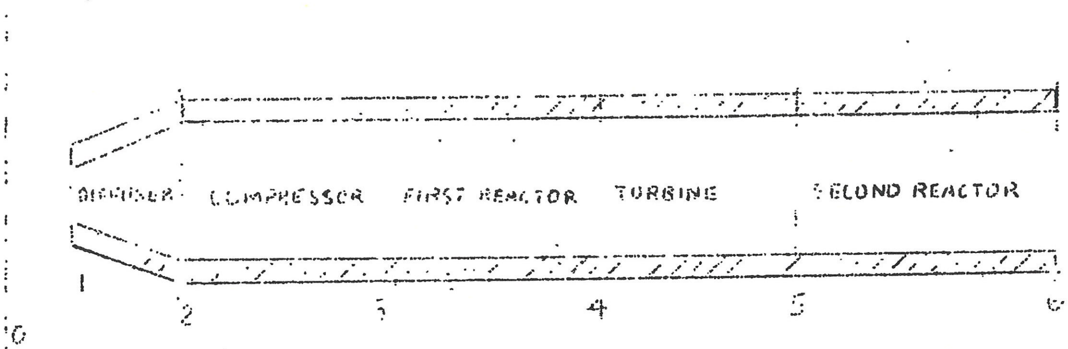
T – static temperature, deg. Rankine

$T^{(s)}$ – stagnation temperature, deg. Rankine

V – gas stream velocity, ft/sec.

γ – ratio of specific heats.

ρ – density, lb/ft^3 or $slugs/ft^3$ as indicated.



As is shown in the drawing the gas stream cross-sectional areas at locations 2, ..., 6, i.e., A_2, ... A_6 are assumed to be the same for reasons of simplicity.

Fig. 5-6

Description of Processes in Idealized Nuclear Turbo-Jet.

Air enters area A_1 (see Fig. 5-6), negotiates a normal shock at intake and passes through the subsonic diffuser of cross-sectional area ratio A_2/A_1 and efficiency of η. The air then flows to the compressor where isentropic compression of ratio p_3/p_2 is assumed. This stage is followed by the first reactor, the design of which is determined from considerations of optimum nuclear reactors subject to the condition that the exit gas stagnation temperature be 2000°R, – the maximum value permitted at the present stage of gas turbine design. In the subsequent stage the gas turbine removes from the stream a quantity of energy equal to the work done on the compressor divided by an efficiency factor η_{ct}. A second reactor stage then heats the air stream until it is exhausted at choking (Mach number of unity).

Method of Calculating Thrust Coefficients.

In the case of the ram-jet the procedure was (1) to determine the diffuser Mach number from the air specific impulse of the burner, (2) to proceed upstream, determining the diffuser-intake area ratio necessary to place normal shock at intake, and (3) to use the general thrust equation. Unfortunately, due to the implicit form of the equations in the present case of mechanical compression, it is necessary to assume a set of intake-diffuser area ratios and then proceed downstream, calculating Mach number and stagnation temperature at each stage and ending with the Mach number preceding the final reactor. That configuration is then chosen which leads to results consistent with practical reactor design, such as optimum critical nuclear reactor size, maximum wall temperature of 4000°R, exit gas stream Mach number of one, etc. In particular, calculations have shown that for a flight Mach number of 1.4, a diffuser-intake cross-sectional area ratio of 2 and mechanical compression ratio of 8 are not incompatible with the above conditions and do lead to a high value of thrust coefficient. This statement will now be verified by evaluating the flow equations for each stage (see Fig. 5-6):

(a) <u>Intake-Diffuser Stage.</u>

The Mach number, M_s, is immediately following the normal shock at intake is given by

$$M_s^2 = \frac{M_a^2 + 5}{7M_a^2 - 1} \qquad (\text{for } \gamma = 1.4) \tag{1}$$

where M_a is the approach Mach number. Since in the present example the flight speed, and hence M_a, is 1.4 this equation gives

$$M_s = .740 \tag{2}$$

The relationship of Mach numbers after a change in duct cross-sectional area is usually given in the following form:

$$\frac{A_i}{A_j} = \frac{\chi(M_i)_\eta}{\chi(M_j)_\eta} . \tag{3}$$

M_i and M_j are the Mach numbers at cross-sectional areas A_i and A_j respectively and η is the efficiency of the process (ref. 3, equation 20); also,

$$\chi(M)_\eta = \frac{1}{M}\left(\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2}}\right)^{\frac{(2\eta - 1)\gamma + 1}{2(\gamma - 1)}} \tag{4}$$

$$= \frac{1}{M}\left(\frac{5 + M^2}{6}\right)^{3.5\eta - 0.5} \qquad (\text{for } \gamma = 1.4) \tag{5}$$

In the present example with $A_1/A_2 = \frac{1}{2}$ and with η assumed to be 0.8 the above equations yield:

$$M_2 = .304 \tag{6}$$

Since no energy is added to the gas stream the stagnation temperature is unchanged, and hence

$$T_2^{(s)} = T_o^{(s)} = T_o \left[1 + \frac{\gamma - 1}{2} M_o^2 \right] = 547^oR \tag{7}$$

where the static temperature at 50,000 ft, is taken to be 393^oR.

(b) Compressor.

Under the assumption of adiabatic compression a comparatively simple equation connects the Mach numbers across this stage, namely (equation 15, ref. 4):

$$M_3/M_2 = (p_2/p_3)^{\frac{\gamma+1}{2\gamma}} \tag{8}$$

Since $\gamma = 1.4$ and a compression ratio (p_3/p_2) of 8 is considered:

$$M_3/M_2 = (1/8)^{6/7} = .16824 \; . \tag{9}$$

Hence, $$M_3 = .0512 \; . \tag{10}$$

For the change in stagnation temperature across the compressor we have (equation 17, loc.cit.):

$$\frac{T_3^{(s)}}{T_2^{(s)}} = (p_3/p_2)^{\frac{\gamma-1}{\gamma}} \left(\frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\gamma-1}{2} M_2^2} \right) \tag{11}$$

$$= 8^{2/7} \left(\frac{5 + M_3^2}{5 + M_2^2} \right) = 1.811 \left(\frac{5 + M_3^2}{5 + M_2^2} \right) \tag{12}$$

It follows that:

$$T_3^{(s)} = 974°R. \tag{13}$$

Finally, the work done by the compressor is given by equation 19, loc. cit.:

$$\Delta W = 6000\ (T_3^{(s)} - T_2^{(s)}) \quad \text{ft. - lbs./slug} \tag{14}$$

$$= 2.56 \times 10^6 \quad \text{ft. - lbs./slug} \tag{15}$$

(c) First Reactor Stage.

Nuclear reactor design considerations give the following critical sizes for optimum reactor design for an average number of 2.1 neutrons emitted per fission (Table 2, Chapter 2):

$$r_c = \frac{1.78}{1-\Gamma}\ \text{ft.}, \quad L_c = \frac{3.25}{1-\Gamma}\ \text{ft.} \tag{16}$$

where Γ is the ratio of the gas stream free cross-sectional area to the total cross-section in the reactor. Since the diameter of the vehicle is to be 8 ft., we get at once

$$\Gamma = .556, \quad r = 4.\ \text{ft.}, \quad L = 7.32\ \text{ft.} \tag{17}$$

Hence the stream cross-sectional area must be:

$$A_2 = A_3 = \Gamma \pi 4^2 = 27.95\ \text{sq. ft.} \tag{18}$$

Assuming a carbon moderator of density 138 lbs/cu.ft., the mass of the reactor will be

$$\text{Wt.} = 138(1 - \Gamma)\ 7.32\pi 4^2 = 22{,}500 \text{ lbs.} \tag{19}$$

For flight at 50,000 ft. at a Mach number of 1.4 ($V = 1362$ ft/sec) the mass current density at intake is

$$\rho_1 V_1 = 15.77 \text{ lb/ft}^2 \text{ sec} \tag{20}$$

and in the reactor,

$$\rho_3 V_3 = \rho_1 V_1 \frac{A_1}{A_3} = 7.885 \text{ lb/ft}^2 \text{ sec} \tag{21}$$

The equation governing the heat transfer in the reactor when integrated is (equation 5, part B):

$$\log \frac{T_w - T_3^{(s)}}{T_w - T_4^{(s)}} = \frac{.0604}{(\rho_3 V_3)^{0.2}} \frac{L}{d^{1.2}} \tag{22}$$

where L, d are the length and diameter of an individual tube of the convective heater, and the mass current density $\rho_3 V_3$ is in lb/ft^2 hr. The stagnation temperature at input, $T_3^{(s)}$, was previously found to be 974°R, while that at the output, due to limitations imposed by gas turbine design, was stated to be 2000°R. Assuming a wall temperature of 4000°R in the reactor, we get:

$$\frac{L}{d^{1.2}} = \frac{(28390)^{0.2}}{.0604} \log \frac{4000 - 974}{4000 - 2000} \tag{23}$$

$$\frac{L}{d^{1.2}} = 53.29 \tag{24}$$

and since L = 7.32 ft., it follows that

$$d = .191 \text{ ft.} = 2.29 \text{ in.} \tag{25}$$

With the gas stream cross-sectional area of 27.95 sq. ft. (equation 18), it can be seen that the number of tubes through the reactor is:

$$n = \frac{27.95}{\pi/4(.193)^2} = 976. \tag{26}$$

In calculating the Mach number at the exit of the reactor, the drag per unit cross-sectional area is taken as one dynamic head - a value arrived at with the use of the following equation (equation 13, part C):

$$C_{d_b} = \frac{.184}{Re^{0.2}} L/d \tag{27}$$

The Reynolds number, $d\rho V/\mu$, is approximately 63,000 and L/d is 38.3 (from equation 25), so that

$$C_{d_b} = \frac{.184}{(63000)^{0.2}} (38.3) = .8 \sim 1. \tag{28}$$

This is a convenient and sufficiently accurate approximation.

The relationship between the Mach numbers across the reactor stage is given by the following equation which is derived in the appendix:

$$\frac{1 + \epsilon \gamma_3 M_3^2}{\sqrt{\gamma_3}\, M_3 \left(1 + \frac{\gamma_3 - 1}{2} M_3^2\right)^{1/2}} = \sqrt{\frac{T_4^{(s)}}{T_3^{(s)}}}\; \frac{1 + \gamma_4 M_4^2}{\sqrt{\gamma_4}\, M_4 \left(1 + \frac{\gamma_4 - 1}{2} M_4^2\right)^{1/2}} \tag{29}$$

where $\epsilon \equiv 1 - \frac{1}{2} C_{d_b} = \frac{1}{2}$, in the present case.

Since M_4 is less than 0.1 this equation is readily soluble by successive approximations by writing it in the form:

$$M_4 = \frac{1 + \gamma_4 M_4'^2}{\sqrt{\gamma_4}\sqrt{1 + \frac{\gamma_4 - 1}{2} M_4'^2}} \quad \frac{1}{F(M_3)} \tag{30}$$

where

$$F(M_3) = \sqrt{\frac{T_3^{(s)}}{T_4^{(s)}}} \; \frac{1 + \epsilon \gamma_3 M_3^2}{\sqrt{\gamma_3}\, M_3 \sqrt{1 + \frac{\gamma_3 - 1}{2} M_3^2}} \tag{31}$$

Substitution of the values previously found, with $\gamma_3 = 1.4$, yields:

$$F(M_3) = 11.54 \tag{32}$$

and finally

$$M_4 = .0757 \tag{33}$$

In this case γ_4 was taken to be 1.328 (ref. 11) since the temperature $T_4^{(s)}$ is 2000°R.

(d) Turbine Stage.

Application of the energy equation to this process yields

$$T_5^{(s)} = T_4^{(s)} - \frac{\Delta W}{C_p \eta_{ct}} \tag{34}$$

where ΔW is the energy expended by the compressor which is driven by the gas turbine, and η_{ct} is the efficiency of the compressor-turbine combination (equation 29, reference 4).

Assuming η_{ct} to be 70%, the above equation becomes:

$$T_5^{(s)} = T_4^{(s)} - \frac{\Delta W}{4200} = 2000 - \frac{2.56 \times 10^6}{4200} \qquad (35)$$

$$T_5^{(s)} = 1390°R. \qquad (36)$$

The relationship between the Mach numbers across the turbine is given by equation 31 of reference 4, namely:

$$\chi(M_5) = \left(\frac{T_5^{(s)}}{T_4^{(s)}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \chi(M_4) \qquad (37)$$

where the function $\chi(M)$ is now defined by

$$\chi(M) = 1/M \left(\frac{1 + \frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (38)$$

With a value of γ of 1.34, corresponding to an average temperature of 1700°R, substitution of preceding values yields

$$\chi(M_5) = 2.212$$

From which, by successive approximations, is obtained

$$M_5 = .274 \qquad (39)$$

(e) Final Reactor Stage.

In the analysis of this stage it is convenient to introduce a quantity which as been used extensively in ordinary ram-jet theory (see, for example, reference 3), namely, the air specific impulse of the exhaust gases. It is defined at the location where the Mach number is unity by the following equation:

$$S_a = \frac{\gamma+1}{g}\sqrt{\frac{RT}{\gamma}} \qquad (40)$$

It is therefore primarily a function of the temperature of the exhaust gases. Plots of S_a vs T are given in Fig. 5-1 of section d.

For given values of air specific impulse, input stagnation temperature, and reactor drag coefficient the Mach number at the entrance to a reactor stage operated at choking is given by the following equation (cf equations 13 and A-11 of ref. 3):

$$M_5^2 = \frac{2\epsilon \frac{R}{g^2} T_5^{(s)} - S_a^2\left[1 - \sqrt{1 + 4\frac{R}{g^2}\frac{T_5^{(s)}}{\gamma S_a^2}(\beta - \epsilon\gamma)}\right]}{2\left[\beta S_a^2 - \epsilon^2\gamma\frac{R}{g^2} T_5^{(s)}\right]} \qquad (41)$$

where $\beta = \frac{1}{2}(\gamma - 1)$. The Mach number M_5 is plotted as a function of S_a in Fig. 5-7 with the parameters having the following values:

$$T_5^{(s)} = 1395^\circ R \qquad R = 1715 \text{ ft-lbs/slug deg R}$$

$$g = 32.16 \text{ ft/sec}^2 \qquad \epsilon = -1/2 \qquad (42)$$

$$\gamma = 1.36 \text{ (evaluated at temperature } T_5\text{)}$$

The value of ϵ corresponds to a reactor drag per unit free cross-sectional area of 3 dynamic heads. It will be shown later that this choice is realistic.

Reference to Fig. 5-7 reveals that the condition expressed by equation 39 will be fulfilled if the following value of air specific impulse is obtained:

$$S_a = 142 \text{ lb. force sec/lb. mass} \qquad (43)$$

and this value will be obtained provided: (see Fig. 5-1).

$$T_6 = 2970°R. \qquad (41)$$

$$\text{Now } T_6^{(s)} = T_6 \left[1 + \frac{\gamma - 1}{2} M_6^2 \right] \qquad (42)$$

Since $M_6 = 1$ (exhaust at choking) and γ here is approximately 1.30, we obtain:

$$T_6^{(s)} = 3420°R. \qquad (43)$$

Now the optimum reactor design considerations lead to the same over-all reactor dimensions as given by equation 17, namely:

$$\Gamma = .556, \quad r = 4 \text{ ft.}, \quad L = 7.32$$

$$A_5 = \Gamma \pi 4^2 = 27.95 \text{ sq. ft.} \qquad (44)$$

$$\text{Total Wt.} = 22{,}500 \text{ lbs.} \qquad (45)$$

The dimensions of the individual convective tubes, however, will be different since they are determined by equation 22, namely:

$$\frac{L}{d^{1.2}} = \frac{(28390)^{0.2}}{.0604} \log \frac{T_w - T_5^{(s)}}{T_w - T_6^{(s)}} \qquad (46)$$

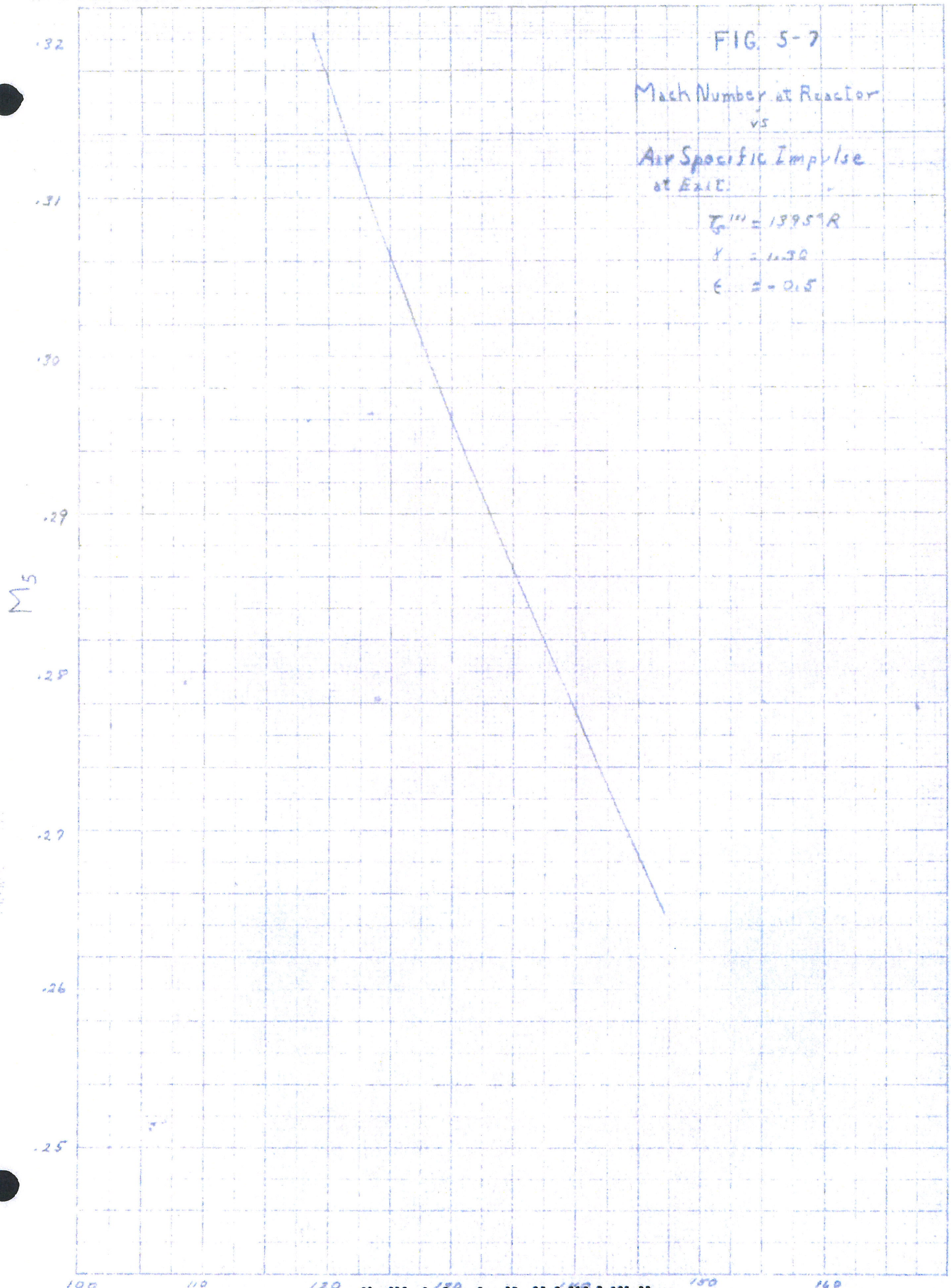

Mach Number at Reactor
vs
Air Specific Impulse
at Exit:
$T_5''' = 1395°R$
$\gamma = 1.30$
$\epsilon = -0.5$
.32
.31
.30
.29
.28
.27
.26
.25
M_5
100
110
120
130
140
150
160

FIG. 5-7

With $T_5^{(s)} = 1390$ and $T_6^{(s)} = 3420$ this becomes

$$\frac{L}{d^{1.2}} = 193.6 \qquad (47)$$

and therefore

$$d = .0653 \text{ ft.} = .856 \text{ in.} \qquad (48)$$

The number of tubes piercing the reactor will then be:

$$n = \frac{27.95}{\pi(.0659)^2/4} = 8194. \qquad (49)$$

The reactor design is therefore determined. The only additional point is to show that the choice of 3 as drag coefficient was a good one.

The Reynolds number at this location comes out

$$Re = \frac{d\rho V}{\mu} = 15,300 \qquad (50)$$

Hence equation 27 yields

$$C_{d_b} = \frac{.184}{Re^{0.2}} L/d = \frac{.184}{(15,300)^{0.2}} (112.1) = 3.00 \qquad (51)$$

(f) Thrust Coefficient.

Since the air specific impulse, S_a, and the configuration A_2/A_1 are now known and it has been shown that these values are consistent with the thermodynamic flow equations as well as with the reactor design requirements, the general thrust equation may now be used. (Part C, equations 1 and following), namely:

$$1/2\ C_t = A_1/A_2 \left(\frac{C_i}{M_o} - 1\right) - \frac{1}{\gamma_o M_o^2} \qquad (52)$$

where $C_i = \dfrac{gS_a}{c_o}$.

For flight at 50,000 ft. the value of c_o is 973 ft/sec. Evaluation of equation 52 then gives:

$$C_t = 1.62 \qquad (53)$$

Now this value of thrust coefficient is based on the diffuser gas stream cross-sectional area. For comparison with external drag, it is desirable to base it on the maximum body diameter. If we call the coefficient so determined C_t' we have:

$$C_t' = \Gamma C_t = 0.90 \qquad (54)$$

This is the value of the thrust coefficient which will be used in overall design calculations. Although a slight gain might be obtained by expanding the exhaust gases by a nozzle to exhaust at atmospheric pressure, calculations show that the increase in C amounts to only about 5% and hence is hardly worth the complication.

(g) <u>Turbo-Compressor Requirements</u>.

For purposes of weight estimations in the section to follow it is desirable to summarize the requirements placed on the turbo-compressor unit.

The mass flow rate is given by

$$m = \rho_1 V_1 A_1 = \rho_3 V_3 A_3 = 7.885 \times 27.51 = 216.9 \text{ lb/sec} \qquad (55)$$

$$= 6.744 \text{ slugs/sec.}$$

With the diffuser stagnation temperature of 547°R, the speed of sound at the compressor is

$$C_2 = 1146 \text{ ft/sec} \tag{56}$$

and since $M_2 = .3041$, we have

$$V_2 = 348.5 \text{ ft/sec} \tag{57}$$

Since $V_2 \rho_2 = 7.885 \text{ lb/ft}^2\text{/sec}$, it follows that

$$\rho_2 = 2.26 \times 10^{-2} \text{ lb/ft}^3$$

The capacity of the compressor must be

$$\text{Capacity} = m/\rho_2 = 9600 \text{ cu. ft/sec.} \tag{58}$$

The power used in compression is given by

$$\text{Power} = m \Delta W = 6.744 \times 2.56 \times 10^6 \text{ ft. lbs./sec.}$$

the value of ΔW being obtained from equation 15. Hence, Power $= 3.14 \times 10^4$ horsepower. (59)

Summarizing the above and expressing the results in round numbers we have:

TABLE

Turbo-Compressor Requirements

Air Mass Flow Rate, $\dot{m}$ = 217 lb/sec = 6.74 slugs/sec

Cross-sectional area of incoming gas stream, $A = 27.5 \text{ ft}^2$

Velocity of gas stream, $V = 350$ ft/sec

Density of air, $\rho = 2.26 \times 10^{-2} \text{ lb/ft}^3$

Capacity = 9600 cu.ft/sec

Power = 32,000 H.P.

Compression ratio = 8.

Gas temperature at turbine = 2000°R.

From experience gained in the analysis of many similar missiles of conventional design an overall lift to drag ratio of 6.0 appears attainable. Then for stable flight, the thrust must equal the drag and the lift must equal the weight, or, in equation form,

$$\text{Thrust} = \text{Drag} = qAC_T$$

$$\text{Lift} = \text{Weight} = (L/D)\ \text{Drag} = (L/D)\ \text{Thrust}$$

If flight is considered at 50,000 feet altitude, at a Mach number of 1.40, and with the previously established value of the thrust coefficient based on total body cross-sectional area, then

$$\text{Lift} = 335 \times 50.3 \times 0.90 \times 6 = 91{,}000\ \text{lb.}$$

$$\text{Thrust} = 335 \times 50.3 \times 0.90 = 15{,}150\ \text{lb.}$$

$$\text{Drag} = \frac{L}{L/D} = \frac{91{,}000}{6} = 15{,}150\ \text{lb.}$$

In an attempt to make a reasonable assessment of the weight of the turbo-compressor unit required, reference was made to several existing designs of small turbo-compressor engines. Preliminary observation seemed to indicate that a figure of .30 lb/HP would permit a valid calculation of the engine weight. Accordingly the following summary of weights was established for an 8 feet diameter by 80 feet long missile designed around a nuclear-powered turbo-jet engine.

Weight of turbine (32,000 HP)	9,600
Weight of reactors	45,000
Weight of carcass	24,000
Weight of warhead	12,000
Total gross weight	90,600

The conclusion may be drawn that sufficient thrust can be derived from the nuclear-powered engine to maintain stable flight of the missile previously described at an altitude of 50,000 feet and a Mach number of 1.4. The missile may be expected to continue in this stable flight until such time as the nuclear power source becomes inoperative or structural materials will no longer withstand operating loads.

Appendix 1 - Derivation of equation 29.

If we denote the stream forces, or momentum flows, at entrance and exit of the reactor by F_3 and F_4 respectively, and the drag force exerted by the reactor surfaces on the gas stream by F_d, we may write:

$$F_3 = F_4 + F_d \tag{A-1}$$

See for example equation 3, reference 3.

It is, however, convenient to express F_d in terms of a non-dimensional coefficient C_{d_b}, where

$$F_d = C_{d_b} \frac{1}{2} \rho_3 V_3^2 A_3. \tag{A-2}$$

Also, the stream thrust, F, may be written in terms of mass flow, Mach number, and stagnation temperature as follows (equation 22, ref. 2):

$$F = \frac{m}{gM}(1 + \gamma M^2) \left(\frac{R}{\gamma}\right)^{1/2} \left(\frac{T^{(s)}}{1 + \frac{\gamma-1}{2} M^2}\right)^{1/2} \tag{A-3}$$

Use of these equations together with the mass flow relation

$$m = g\rho VA \tag{A-4}$$

puts A-1 into the form:

$$\frac{1 + \gamma_3 M_3^2}{\sqrt{\gamma_3}\, M_3} \left(\frac{T_3^{(s)}}{1 + \frac{\gamma_3-1}{2} M_3^2}\right)^{1/2} = \frac{1 + \gamma_4 M_4^2}{\sqrt{\gamma_4}\, M_4} \left(\frac{T_4^{(s)}}{1 + \frac{\gamma_4-1}{2} M_4^2}\right) + \frac{1}{2} C_{d_b} V_3 \frac{1}{\sqrt{R}} \tag{A-5}$$

and we have:

$$\frac{1 + \gamma_3 M_3^2}{\sqrt{\gamma_3}\, M_3 (1 + \frac{\gamma_3-1}{2} M_3^2)^{1/2}} = \frac{1 + \gamma_4 M_4^2}{\sqrt{\gamma_4}\, M_4 (1 + \frac{\gamma_4-1}{2} M_4^2)^{1/2}} \left(\frac{T_4^{(s)}}{T_3^{(s)}}\right)^{1/2} + \frac{1}{2} C_{d_b} \frac{V_3 \sqrt{\gamma_3}}{C_3 (1 + \frac{\gamma_3-1}{2} M_3^2)^{1/2}} \tag{A-6}$$

This result simplifies at once to equation 29, namely:

$$\frac{1 + \epsilon \gamma_3 M_3^2}{\sqrt{\gamma_3}\, M_3 \left(1 + \frac{\gamma_3 - 1}{2} M_3^2\right)^{1/2}} = \sqrt{\frac{T_4^{(s)}}{T_3^{(s)}}}\; \frac{1 + \gamma_4 M_4^2}{\sqrt{\gamma_4}\, M_4 \left(1 + \frac{\gamma_4 - 1}{2} M_4^2\right)^{1/2}} \qquad \text{(A-7)}$$

CONCLUSIONS AND DISCUSSION

On the basis of computations presented in this chapter, it appears feasible to design either a ram-jet or a turbo-jet capable of flying at supersonic speeds at reasonable altitudes, powered by nuclear reactors and having its range limited only by failure of materials. The analysis which has been presented was carried out by means of certain simplifying assumptions. Certain of these were imposed because of presently unavailable information concerning certain nuclear processes while others were necessary in order to prevent this preliminary analysis from becoming hopelessly complicated and failing in its object of exploring the possibilities for further study.

The following assumptions and restrictions in particular are noted:

(a). Reactor.

1. Calculations were made only on the basis of optimum uranium concentrations and optimum reactor shape. It is possible that deviations from the optimum design of the reactor might lead to a better design for these types of vehicles in spite of increased uranium requirements.

2. The crystalline density figures for carbon and BeO were used in this analysis. It is well known that the bulk density of the commercial product is less than this, which would lead to larger and also heavier reactors than those considered here.

3. The average number of neutrons per fission, V, was taken to be 2.1. If this quantity is actually larger, the overall dimensions and weights of the reactors would be reduced.

4. Because of the oxidizing properties of the gas stream a carbon reactor would require a protective coating. The weight of this, as well as the weight of any necessary internal structural supports, has been neglected.

5. The possibility of surrounding the reactor by a neutron reflector, leading to somewhat smaller overall dimensions has not been explored.

(b). Propulsion.

1. No estimates of the internal reactor temperatures necessary to maintain the desired wall temperatures have been made because of lack of information on heat conductivities of reactor materials.

2. Calculations for both the ram-jet and the turbo-jet were made on designs employing conventional diffusers (i.e. subsonic pressure recovery after normal shock at intake). There is reason to believe that in the case of a ram-jet operating at a Mach number of 2 or higher a certain advantage would be obtained from the use of a special form of supersonic diffuser such as, for example, the Oswatitsch arrangement. This is a problem for further study.

3. An ordinary straight-tail exhaust configuration with exhaust at Mach number one was considered. It is clear that because of the fractional value of Γ, the exit gas stream could be expanded by use of a tail nozzle, giving an increase in thrust coefficient. Since, however, in this case, the maximum thrust is obtained when the exhaust pressure is equal to the pressure of the medium, the increase in thrust which can be realized by such a process is strictly limited.

4. Except for subsonic diffuser efficiency and normal shock losses, the processes in general have been considered from an idealized standpoint. It is not believed especially in the case of the ram-jet, that such treatment is too unrealistic to preclude acceptance of the overall conclusions. However, it would probably be beneficial to take additional factors into account when more involved analyses are made. This statement applies more particularly to the supersonic turbo-jet with exhaust heating because of its greater complexity.

(c). <u>Aerodynamics</u>.

1. The analysis was carried out only for a conventional configuration. The possibilities of using the Canard or the flying wing type has not been explored although they might be more practical for this type of power source.

2. The effects of aerodynamic interference between components of missiles were neglected.

3. The weights of structural components and controls were made to be consistent with the values obtained from previous analyses of conventional fueled vehicles because of the lack of adequate weight breakdowns of more advanced types.

4. Only the nuclear-powered flight of the ram-jet has been considered; no assessment of the launching problem has been made.

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