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Report On

Scale Model Tests for High Pressure High Temperature Steam Piping

Third Partial Report

NAVAL RESEARCH LABORATORY
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Prepared by:

Irwin Vigness, Associate Physicist

Eugene Pardue, Contract Employee

Reviewed by:

Ross Gunn, Superintendent Mechanics and Electricity Division

Approved by:

H. G. Bowen, Read Admiral, USN, Director

Distribution: Bureau of Ships (25)

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ABSTRACT

Apparatus has been constructed by which the end reactions, caused by given end displacements of model pipes that contain branches, can be determined. Two model pipe systems are studied. The accuracy and usability of the apparatus are discussed. Changes of end reactions caused by small rotations of the pipe ends, are shown. Displacements of any point along the pipe axis, caused by thermal expansion absorbed between fixed ends, can be easily measured.

INTRODUCTION

a. Authorization

1. This problem was authorized by Bureau of Engineering Project Order 69/40 of 15 September 1939.

b. Statement of Problem

2. A previous report has demonstrated the validity and practicability of model pipe methods for the determination of end reactions of pipe systems containing no branches.

The problems included in this report are:

(a) Refinement of apparatus previously described [1] for end reactions.
(b) Extension of the methods used [1, 7] to pipe systems containing more than two ends. (branched pipe systems)
(c) Effects of small rotations of an end of a pipe on end reactions.
(d) Measurements of the displacement of a pipe axis caused by thermal expansion absorbed between fixed ends of the pipe.

c. Known Facts Bearing on the Problem

3. It has been shown in previous reports [1, 2] that the use of model pipe systems with no branches for the determination of end reactions is practical, and, for models of reasonable stiffness, the reactions can be measured to within 15 percent of the calculated values.

4. Present theory has been shown to be adequate for pipe systems contained in a single plane, but to be incorrect for bending perpendicular to a plane containing a pipe bend. For most piping systems the flexibility contributions of the incorrect mathmetical components are but a small part of the total flexibility of the system and will therefore not greatly effect the end reactions. The correct theory can be applied with no additional difficulty in such methods of calculation as described in a recent [3] publication on the design of piping systems. The correct theory should always be used for stress determinations because of stress multuplication factors introduced. When tubular model pipes of proper scale are used the accuracy of the results depend only on the accuracy of measurement.

5. Theoretical means[3] of determining end reactions have progressed so that solutions of systems that were impractical a few years ago may be now said to be almost routine. Graphical methods [4,5] have been developed to speed the solution of the more simple cases. Approximate "square corner" solutions have been advocated[6] as of sufficient accuracy in all but exceptional cases. Therefore, it is to be expected that the application of model tests will be applied especially for the more complicated piping systems or

in cases where the stresses and end reactions are of more than usual importance.

6. Model piping systems should be constructed of tubing wherever possible. If the piping system is inherently rigid, or if a section of piping considered contains mostly straight sections, it may be necessary or convenient to use rods rather than tubes for all or part of the model system. The advantage of rods is that the scale factor between the model and full scale system is independent of the diameter of the rod and therefore a model system constructed of rods can be made as flexible as desired. The disadvantage is that the measured end reactions are too high. In branched piping systems different sections may consist of pipes of different sizes. It may be difficult to obtain tubing of such wall thicknesses that the same scale factor can apply throughout the system. In such cases it may be necessary to use rods, which can be turned to any desired diameter, in sections that contain but a small proportion of their lengths in bends, or in sections that contribute little to the flexibility properties of the system.

APPARATUS

7. The function of the apparatus is to hold the ends of a model pipe system rigidly at any desired point in space, to allow any desired displacment of the pipe ends, and to permit measurements of the reactions necessary to hold the pipe. Plates 1 and 2 show such an assembly.

8. An essential feature of this apparatus is rigidity. When tubing is used for model pipes its minimum diameter must usually be between 1/2 and 3/4 inches in order that the ends of the tubing may be conveniently far apart. This is the result of the scale factor being dependent on the tube wall thickness. Under such conditions the model pipe systems may be stiff enough to render bending of the supporting apparatus an important factor unless adequate precautions are taken.

9. The apparatus (see plate 1) consists of four essential parts:

(a) A bed-plate, 4 x 6 square feet in area.
(b) Right-angle supports, 36 inches high. The right angle supports are of welded construction, and possess good torsional and bending rigidity. They can be mounted in any position on the bed-plate with one face perpendicular to the bed plate.
(c) Lathe slide rest. The slide rest is part of a compound lathe rest and can be mounted at any point along the face of the right angle support, and can be turned so as to allow a motion in any direction contained in the plane of the right angle support face.
(d) Measuring head. The measuring head is similar to that described in the first partial report,[1,7], except for the following changes. (See plate 4). (1) Hardened steel inserts are used for the bearing points of the pin supports.

[2]Sleeves are of bronze and have a smooth bearing surface extension to keep the reaction force indicator aligned. [3]The reaction force indicator has an internal bearing surface extension to fit over the extension of the sleeve. [4]A linkage system is used so that the force acting parallel to the bracket axis is brought out the side as is shown in Plate 3. (The bracket axis is identical with the z' axis of the measuring head coordinate system). [5]Adjustment screws and pins (Plate 5) are provided to quickly change the clearance of the pins supporting the bracket (Plate 4). [6]Pipe clamp extensions are used to hold the ends of a model pipe at any orientation with respect to the measuring head. This is constructed so that the pipe ends are always fixed on the axis of the measuring head bracket.

METHOD

(a) Construction and Mounting of the Model Pipe.

10. Certain approximations may be necessary in the construction of a model pipe system. These are: (1) Use of rods rather than tubes if the tubular model is too rigid. (2) Use of rods in sections where tubing of the desired cross-sectional moment of inertia cannot be obtained. This is necessary only when several sizes of pipes are used in the piping system. The rods should be placed where their additional rigidity at the bends will have the least effect on the system. (3) Tube turns of short radius of curvature must be substituted by miter turns or square corners in the model. (4) Pipe junctions and manifolds in the model are usually constructed of rigid sections. All of the approximations above are on the side of safety and for most systems, will not greatly change the measured end reactions.

11. The model system should be constructed accurately enough so that the positions of its ends can determine the positions of the measuring heads. The measuring heads should then be mounted in the proper orientation and the pipe ends clamped so as to give small initial stresses in the model pipe. The measuring head is properly oriented when the face of the right angle support (Plate 1) is parallel to a plane containing the desired direction of motion of the end of the pipe; and when the lathe rest is at such an angle on the right angle support as to allow a motion of the end of the pipe in the desired direction.

(b) Measurements and Adjustment of Pins

12. The total clearance between the bracket pin supports contained in any given line is made equal to one mil by means of adjustment screws (See plates 3 and 5). An adjustment screw pin (plate 5) is made to support its component of pipe reaction and to keep a fixed clearance between the bracket and the pin on the opposite side of the bracket. (plate 4). If the force changes direction because of pipe end displacement, the adjustment screw is placed on the opposite bronze sleeve and caused to move the bracket one mil. Then the bracket is in the same position as before. When a reaction is to be measured, an adjustment screw is removed and the bracket is forced to the

same position that it held when the adjustment screw was in place by the calibrated spring of the force indicator (Plate 4).

13. If the pins supporting the bracket are in the perfect alignment and if their points are reasonably sharp, forces from 5 lbs. to 200 lbs. can be measured with an accuracy of plus or minus one per cent. Forces higher than this should not be encountered. Small forces can be measured within an accuracy of plus or minus one ounce. The reaction force indicator should measure very nearly the same value whether it is approaching the balance point from above or below. If there is back lash present it is most likely caused by poor alignment and dullness of the supporting pins.

14. If the pins are not aligned (perpendicular to the bracket or bracket arm) there will be a false component of force perpendicular to the proper direction. This false component can be a considerable part of a minor force and can lead to a systematic error that is a large percentage of the minor force. It will be shown later (paragraph 34) that this type error may lead to poor results when moments are transferred long distances perpendicular to their minor forces. Great care must be taken to be sure that the supporting pins are correctly aligned.

15. The displacement of the end of the pipe is measured by a dial indicator placed on an extension of the bracket, or on the pipe clamp extension (Plate 5).

(c) Coordinate Systems

16. In general, reactions measured at a measuring head will not be in the direction of the principal axes of a coordinate system given for a piping arrangement. It is necessary therefore to rotate and translate the measured forces and moments to the piping coordinate system and to the end of the pipe. The translation is only necessary if a pipe clamp extension (Plate 5) is used. Equations necessary for this transformation will be given. A complete example is given in appendix 1.

17. A primed set of coordinates, plate 3, is determined by a measuring head. The sign and coordinate reaction of each reaction point is stamped near each bronze sleeve supporting that reaction. When the reaction force indicator is applied to any bronze sleeve the sign and coordinate description of the reaction is copied from that stamped near the sleeve. There is, therefore, no loss of time and little opportunity of error in determining the sign and coordinate reaction measured.

18. The construction of the apparatus is such that when the measuring head is mounted on a right angle support the bracket, or z' axis, is always in a horizontal plane, and the y' axis is always parallel to the direction of displacement of the end of the pipe. This displacement may be in either the positive or negative y' direction. As only a right handed system of coordinates is used the position of the x' axis is determined by the location of the other two coordinate directions.

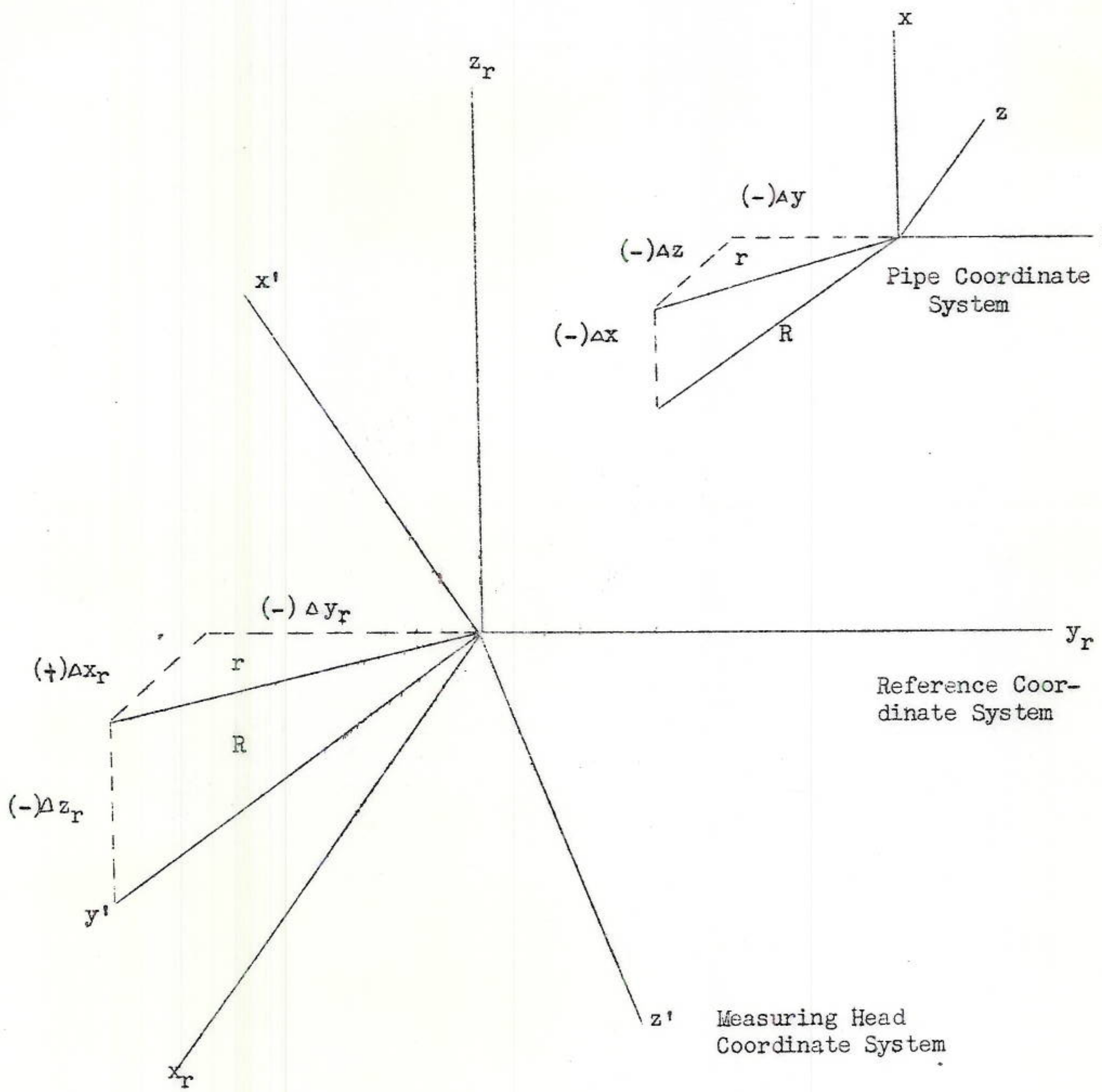

Figure 1 - Coordinate systems and pipe end displacements. Case (c) is illustrated.

19. Let x, y, z be coordinates given for a piping system. Let x_r, y_r, z_r, be a reference coordinate system with the xr, yr plane horizontal and with z_r positive upwards. The pipe system should be mounted so that one of its principal coordinate planes is horizontal. The reference coordinate system should then be aligned so that each of its axes is in the same (or opposite) direction as an axis of the pipe coordinate system. One arrangement is shown on figure 1.

20. Let Δx, Δy, Δz, be the given displacements required of a pipe end in pipe coordinates, and let Δx_r, Δy_r, Δz_r, be the same displacements in reference coordinates. These latter values can be obtained from the former given values by inspection of figure 1. The resultant displacement is

$$R = +\sqrt{\Delta x_r^2 + \Delta y_r^2 + \Delta z_r^2}$$

and the resultant displacement in the horizontal plane is

$$r = +\sqrt{\Delta x_r^2 + \Delta y_r^2}$$

(d) Rotation of Coordinates, Four Cases

21. If V is any vector, with components V_x', V_y', V_z' along the measuring head axes, its components along the reference axes are given below for the four possible arrangements of the two axes with respect to each other.

Case (a) The y' axis is in a negative R direction and x' has a positive component upwards.

$$V_{x_r} = -V_x' \frac{\Delta x_r \, \Delta z_r}{rR} - V_y' \frac{\Delta x_r}{R} + V_z' \frac{\Delta y_r}{r}$$

$$V_{y_r} = -V_x' \frac{\Delta y_r \, \Delta z_r}{rR} - V_y' \frac{\Delta y_r}{R} - V_z' \frac{\Delta x_r}{r}$$

$$V_{z_r} = +V_x' \frac{r}{R} - V_y' \frac{\Delta z_r}{R}$$

Case (b) The y' axis is in a negative R direction and x' has a positive component downwards.

$$V_{x_r} = +V_x' \frac{\Delta x_r \, \Delta z_r}{rR} - V_y' \frac{\Delta x_r}{R} - V_z' \frac{\Delta y_r}{r}$$

$$V_{y_r} = +V_x' \frac{\Delta y_r \, \Delta z_r}{rR} - V_y' \frac{\Delta y_r}{R} + V_z' \frac{\Delta x_r}{r}$$

$$V_{z_r} = -V_x' \frac{r}{R} - V_y' \frac{\Delta z_r}{R}$$

Case (c) The y' axis is in a positive R direction and x' has a positive component upward.

$$V_{x_r} = - V_x' \frac{\Delta x_r \Delta z_r}{rR} + V_y' \frac{\Delta x_r}{R} - V_z' \frac{\Delta y_r}{r}$$

$$V_{y_r} = - V_x' \frac{\Delta y_r \Delta z_r}{rR} + V_y' \frac{\Delta y_r}{R} + V_z' \frac{\Delta x_r}{r}$$

$$V_{z_r} = + V_x' \frac{r}{R} + V_y' \frac{\Delta z_r}{R}$$

Case (d) The y' axis is in a positive R direction and x' has a positive component downward.

$$V_{x_r} = + V_x' \frac{\Delta x_r \Delta z_r}{rR} + V_y' \frac{\Delta x_r}{R} + V_z' \frac{\Delta y_r}{r}$$

$$V_{y_r} = + V_x' \frac{\Delta y_r \Delta z_r}{rR} + V_y' \frac{\Delta y_r}{R} - V_z' \frac{\Delta x_r}{r}$$

$$V_{z_r} = - V_x' \frac{r}{R} + V_y' \frac{\Delta z_r}{R}$$

22. The case appropriate for the orientation of the measuring head coordinates and the reference coordinates can be seen by inspection of a diagram similar to figure 1. Forces and moments in terms of the reference coordinates are determined by applying the proper case. By inspection the forces and moments are converted to the pipe coordinates and then multiplied by scale factors[1] to convert to full scale values in pipe coordinates. An illustrative example is given in Appendix 1.

DATA OBTAINED

(a) Pipe Systems Studied

23. Two branched pipe systems are here reported. The first, shown on plate 1 and figure 2, was initially studied by Hill[8] and calculated values were later given by Rossheim, Markl, and Andrews[9]. The reactions given by Hill were obtained from a model constructed of rods, and as the purpose of this first test was to obtain data that could be compared with others, a rod model was constructed at this Laboratory. The main section of this piping model could have been constructed of tubing with no added difficulty. The second branched pipe system is a Naval installation. A branched section is shown in Plate 2 and diagrams of the complete section studied are shown in Plates 6 and 7. In addition to end reactions caused by translation of the ends of a pipe, end reaction changes caused by small rotations of pipe ends are illustrated with the Navy pipe. These end reactions have been determined in order that the importance of similar possible rotations on board ship might be evaluated.

(b) The Hill Pipe

24. Data on the full scale and model pipe are given in Tables 1 and 2. Linear dimensions can be had from Figure 2. Reference should be made to Figure 2 for location of sections referred to in the Table. Thermal expansion

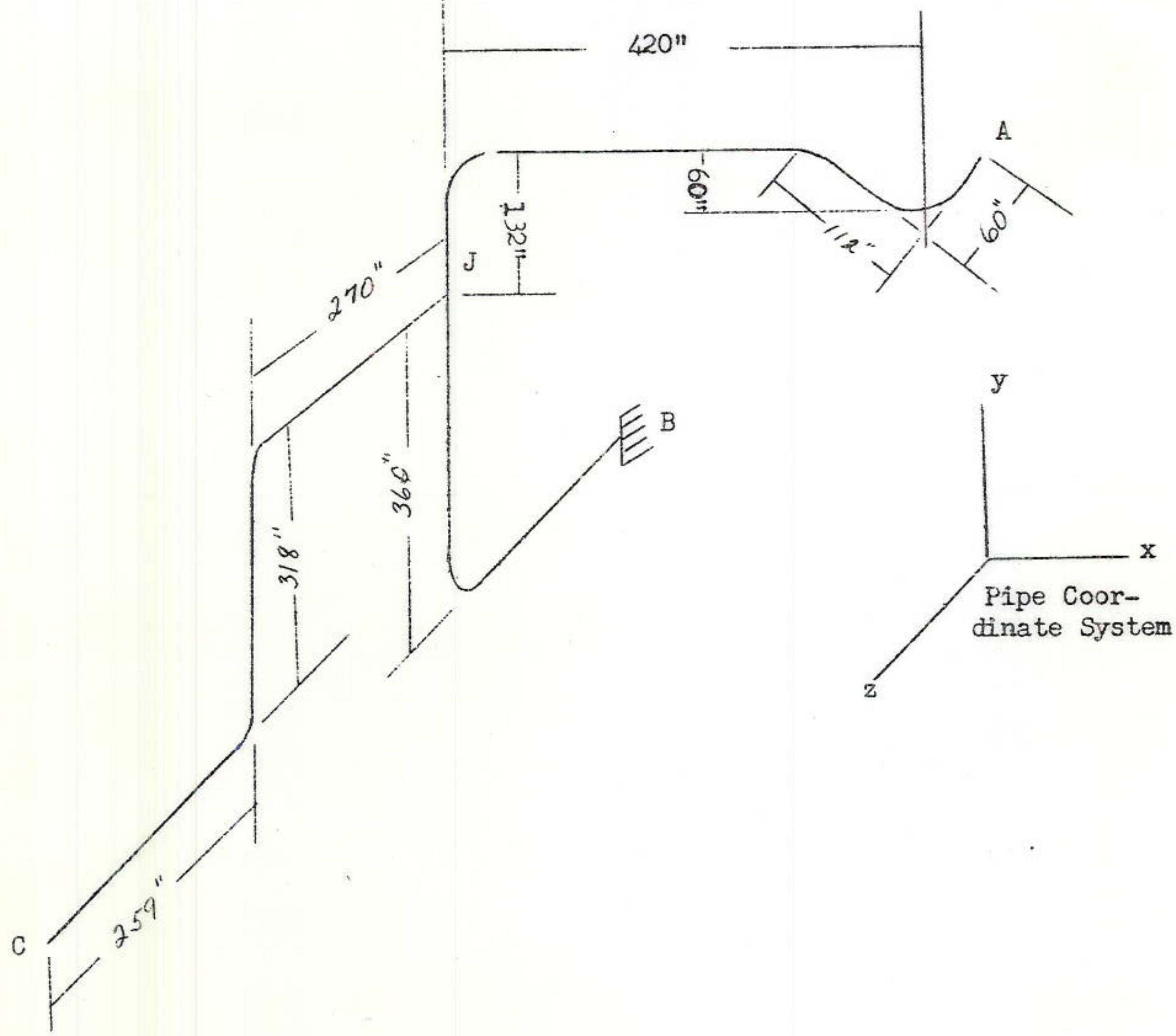

Figure 2

Diagram of the "Hill" pipe. Dimensions are for the full scale pipe. The radius of curvature of bends is 60 inches. To get the dimensions of the model pipe divide the full scale values by 20, which is the scale factor.

of the pipe is 7.2 inches per 100 feet over the temperature range considered.

Table 1 Data for Hill Full Scale Pipe

Pipe Section	O. D. Inches	Wall Thickness	Moment of Inertia	Modulus of Elasticity	Rigidity Factor K
AJ & BJ	12.75	0.843	561.75	$23x10^6$	0.739
CJ	17.75	0.500	361.48	$23x10^6$	0.500

Table 2 Data for Hill Model Pipe (Rod)

Pipe Section	O. D. Inches	Wall Thickness	Moment of Inertia	Modulus of Elasticity
AJ or BJ	.373"		$9.527x10^{-4}$	$29x10^6$
CJ	.334"		$6.102x10^{-4}$	$29x10^6$

25. If the end B of the full scale pipe is assumed fixed, the expansion to be absorbed in C-B is 4.243" and the expansion to be absorbed between A-B is 3.696 inches.

26. In the model system (plate 1) reactions are measured at B and C. End B is held fixed and the forces and moments at end B and C are determined by measurements. The reactions at A can then be calculated. Tables 3, 4, 5 give values of end reactions for A, B, and C as obtained by calculation[9] and as obtained by model test. Reactions are given according to accepted pipe theory according to "rod" theory, according to model test results of Hill[8], and according to model test results of the Naval Research Laboratory. All significantly large forces and moments are in remarkably good agreement.

27. It is interesting to note the differences of the reactions as calculated by pipe theory and rod theory (flattening effect neglected). In general it is expected that the reaction calculated by tube (or pipe) theory would be less than those calculated by rod theory, but in branched pipe systems there may be a considerable redistribution of stresses so that many of the reactions may be increased in value. The differences of the reactions, as determined by the two theories, are small, but as their magnitudes cannot be foretold because of the redistribution, it is therefore advisable to use tubular models whenever possible.

(c) The Navy Pipe

28. The Navy pipe[11] studied is shown in part on Plate 2, and in full on Plates 6 and 7. The junction between pipes D and E is assumed rigidly fixed. The thermal expansion of the pipes together with motions of the ends of the pipes caused by displacement of the terminals to which they are attached are given by Δx, Δy, and Δz. The ends of pipes A and C opposite J are attached to boilers and the displacements of the pipe terminals caused by boiler expansion are accounted for in the given values of Δx, Δy, and Δz.

Table 3 - Reaction at end A of Hill pipe. Comparison of end reactions as determined by calculation[9] and by model test[8]. The model test values should be compared with rod theory calculations. Forces are in pounds and moments are in inch-pounds.

Reaction	Calculated Values		Model Test with Rods by	
	Pipe Theory	Rod Theory	Hill	N.R.L.
Fx	-2474	-2477	-2326	-2611
Fy	-3597	-3379	-3118	-3410
Fz	+ 632	+1142	+1009	+1120
Resultant	4411	4343	4019	4340
% Deviation of Resultant from Rod Theory	+ 1.5	0	- 7.5	- 0.1
Mx	+288000	+323000	+300000	+261000
My	+ 19000	+168000	+123000	+163000
Mz	+861000	+779000	+688000	+747000
Resultant	908000	860000	761000	808000
% Deviation of Resultant from Rod Theory	+ 5.6	0	- 11.6	- 6.0

Table 4 - Reactions at End B of Hill pipe. Comparison of end reactions as determined by calculation [9] and by model test[8]. The model test values should be compared with rod theory. Forces are in pounds, and moments are in inch-pounds.

Reaction	Calculated Values		Model Test with Rods by	
	Pipe Theory	Rod Theory	Hill	N.R.L.
Fx	+ 1975	+ 1891	+ 1885	+ 1990
Fy	+ 2478	+ 1712	+ 1380	+ 1770
Fz	+ 1593	+ 1906	+ 1660	+ 2050
Resultant	3547	3184	2866	3361
% Deviation of Resultant from Rod Theory	+ 11.4	0	- 10.0	+ 5.6
Mx	- 71000	+ 99000	+ 195000	+ 98000
My	+ 311000	+ 326000	+ 373000	+ 322000
Mz	- 299000	- 292000	- 255000	- 302000
Resultant	437000	448000	492000	457000
% Deviation of Resultant from Rod Theory	- 2.5	0	+ 9.8	+ 2.2

Table 5 - Reactions at end C of Hill pipe. Comparison of end reactions as determined by calculation[9] and by model test[8]. The model test values should be compared with rod theory. Forces are in pounds and moments are in inch-pounds.

Reaction	Calculated Values		Test with Model Constructed of Rods	
	Pipe Theory	Rod Theory	Hill	N.R.L.
F_x	+ 500	+586	+441	+621
F_y	+1119	+1667	+1137	+1640
F_z	-2225	-3048	-2669	-3170
Resultant	2540	3523	3215	3575
% Deviation of resultant from Rod Theory	-28.0	0	-8.7	+1.5
M_x	-30000	-10000	+41000	+46000
M_y	-127000	-138000	-112000	-143000
M_z	-107000	-122000	-120000	-125000
Resultant	169000	184000	170000	195000
% Deviation of resultant from Rod Theory	- 8.2	0	- 7.6	+ 6.0

29. Measuring heads were placed at all terminals of branches A, B, C, and D. The reactions for pipe E were measured separately as there was a fixed anchor between it and the rest of the system. Checks may be obtained on the measured reactions when measuring heads are used at all of the terminals by use of the conditions for static equilibrium, namely: (a) The sum of the forces in any given direction is equal to zero. (b) The sum of the moments in a given plane about any point is equal to zero.

30. Two models were constructed. One model was constructed entirely of rods and one was constructed of tubes except for branches D and B. Branch D is a short and almost straight section of piping which would be equally elastic for either a tube or rod model. Pipe B has a much smaller cross section than other branches of the system and its reactions are but a small part of the principal reactions of the system. The reactions at end B may be between 10 and 20 per cent too high. This error, as reflected on the rest of the system, will be within a few per cent. Scale factors and other information concerning the Navy pipe are given in Appendix 1 where it is used as an illustrative example for pipe end reaction measurements.

31. Results of the model tests are given in Tables 6 and 7. Checks of the results are shown in Tables 8 through 11.

are in pounds and moments are in inch-pounds. Reactions are those exerted <u>on</u> the pipe

Reaction	HEAD A			HEAD B*			HEAD C			HEAD D		
	Model Values		Ship-Builders Calc	Model Values		Ship-Builders Calc	Model Values		Ship-Builders Calc	Model Values		Ship-Builders Calc
	Rod	Tube		Rod	Tube		Rod	Tube		Rod	Tube	
F_x	-365	-332	-467		-238		-21	-127	-76	+595	+710	
F_y	-344	-237	+121		-182		-1775	-1709	-1745	+2340	+1828	
F_z	-980	-1164	-1640		-8		-757	-532	-875	+1700	+1697	
Resultant	1100	1234	1710		299		1930	1790	1950	2950	2590	
% Deviation of Resultants from Tube Value	-10.9	0	+38.6				+7.8	0	+8.9	+13.9	0	
M_x	-68300	-72400			-2450		-174700	-171000		+241500	+240000	
M_y	+46600	+67800			-168		-76200	-67000		-71500	-100000	
M_z	+7370	+26500			-19500		+196400	+202000		-36100	-52600	
Resultant	82600	103000			19700		273500	27300		252000	265000	
% Deviation of Resultants from Tube Value	-19.8	0					0	0		-4.9	0	

* The moment of inertia of the model used for branch B was about 15 per cent too high in order that a stock diameter rod might be used. This would cause the reactions for branch B to be about 15% too high (exclusive of an error caused by using a rod rather than a tube). These errors will have little effect on the other heads. The errors have not been corrected in order that the check equations might be easily illustrated.

Table 7 - End Reactions for Section E of Navy Pipe. Forces and moments are in pounds or inch-pounds. Reactions are those exerted on the pipe.

Reaction	End of pipe near					
	Strainer			Bulkhead		
	Model Values		Ship-* Builders Calc.	Model Values		Ship-* Builders Calc.
	Rod	Tube		Rod	Tube	
F_x	-2815	-2062	1912	+3285	+2135	1912
F_y	-1620	-1206	1150	+1678	+1195	1150
F_z	-1176	- 761	783	+1087	+ 784	783
Resultant	3452	2509	2363	3840	2570	2363
% Deviation of Resultants from Tube Value	+37.5	0	-5.8	+53.3	0	-8.1
M_x	-31500	-18700		+60000	+41700	
M_y	+49900	+31200		+29400	+19300	
M_z	-15500	- 7100		-197300	-139300	
Resultant	61000	37100		208000	146600	
% Deviation of Resultants from Tube Value	+ 64.5	0		+ 42.0	0	

* Shipbuilders calculated values are taken from pipe #14307 of BuShips Plan#CV9-S4810-64 Alt. 1

32. The difference between the r actions of the rod model and tube model for branch E are large. However, branch E consists mostly of curved sections and the results are as expected. The agreement between the ship builders' calculations and the model tests is closer than might ordinarily be expected and speaks well for the judgment and accuracy of the piping designers in this respect. Variations up to 50 per cent might easily have been expected.

(d) Check of Pipe End Reactions

33. The check given below is a check of systematic and probable errors that enter into the measurements of end reactions. The check does not include errors caused by bending of the measuring apparatus. With rigid measuring apparatus this error is small for flexible pipe systems of the type under test. This check is the same as that mentioned in paragraph 29.

Check (a) - The sum of positive forces in a given direction is equal numerically to the sum of the negative forces in that direction.
Check (b) - The sum of all the moments in a given plane acting about a given point is equal numerically to the measured moment in that plane at that point.

For end reactions of the tube model of the Navy pipe the checks given in Tables 8 through 11 are obtained.

Table 8 - Check (a) End Reactions of Navy Pipe, Branches A, B, C, and D. Determined from a tubular model. Values are taken from Table 6.

Force	Negative Values	Positive Values	Difference	
			Numerical	Percent
F_x	697 lbs.	710	13	1.9
F_y	2128	1828	300	16.4
F_z	1704	1697	7	0.4
Resultant	2810	2590	220	8.5

Table 9 - Check (b) End Reaction of the Navy Pipe, Branches A, B, C, and D, determined from a tubular Model. Values are taken from Table 6.

Moment at D	Value of Moment		Difference,	
	By Transfer to D	By measurement at D	Numerical	Percent
M_x	208000 in.-lbs.	240000	22000	9
M_y	109000	100000	9000	9
M_z	96000	52600	43400	83
Resultant	254000	265000	11000	4

Table 10. Check (a) End Reactions of Navy Pipe, Branch E, determined from a tubular Model. Values are taken from table 7.

Force	Negative Values	Positive Values	Difference Numerical	Difference Percent
F_x	2062 lbs.	2135	73	3.5
F_y	1206	1195	11	0.9
F_z	761	784	23	3.0
Resultant	2509	2570	61	2.5

Table 11. Check (b) End Reactions of Navy Pipe, Branch E, determined from a tubular Model. Values are taken from table 7.

Moment	Moment at pipe end near Bulkhead		Difference	
	By Transfer	By Measurement	Numerical	Percent
M_x	+38,400 in.-lbs	+41,700	3.300	8.0
M_y	+31,800	+19,300	12,500	64.7
Mz	-147,600	-139,300	8,300	4.3
Resultant	155,700	146,600	9,100	6.2

34. A consideration of errors makes it improbable that the moments obtained by measurements be off by more than 15 per cent in the above cases. The moments obtained by transfer may be in error by a considerable amount as they are usually relatively small differences of large numbers. Moreover some of these large numbers are the products of small forces and large distances. The percentage error of a small force is usually high.

35. In order that small components of a reaction be not over emphasized it has been recommended by Rossheim, Markl, and Andrews[9] that "the resultant forces and moments be used as a basis of comparison rather than the components which avoids placing undue emphasis on the smaller components which, in many cases, may actually reverse sign without effecting the utility of the results." However, it is suggested that as small components may be in considerable error, but as they may be important in the calculation of moments at a far end of the pipe, that the reactions be measured at every pipe end where thrusts and moments are important, and that moments be transferred from the nearest end for the calculation of pipe stresses. More experience with the apparatus is necessary before checks of the above type may be dropped.

(e) End Reaction Changes Caused by Small Rotations of the Pipe Ends

36. By changing the position of the bracket, by adjustments of the supporting pins, the pipe end can be made to rotate a small amount in any given plane. The "fixed" ends of a pipe on board ship may rotate with respect to each other because of ship motion. The determination of end reactions caused by a given rotation will enable persons, acquainted with the magnitude of such motion on board ship, to tell whether or not the reactions are important.

37. The "fixed" end of branch D was rotated in the yz plane (See plates 6 and 7), and the end reaction changes were determined. A similar procedure was followed for other points and planes but as the reaction changes were usually less for the same rotation only the results shown in Table 12 are reported. The pipe ends were rotated about 7 minutes of arc. This could be measured within a ± 5 per cent error. As the end reaction changes were small, they may be in considerable (50 per cent) error.

Table 12. Effects of rotating the "fixed" end D of the Navy Pipe in the yz plane on the end reactions. Units are pounds or inch-pounds per degree of rotation.

Reaction	Head A	Head C	Head D
F_x	13	2	-19
F_y	-26	5	39
F_z	115	104	-160
M_x	554	4840	-76700
M_y	-2410	25000	14000
M_z	301	-3700	6800

38. Table 12 shows that one degree rotation of the end of pipe D in the yz plane makes little difference in the reaction forces (about 10%), but quite an appreciable change in the reaction moments (about 30% max.). From information of this type and from knowledge of relative motions of the ends of a pipe caused by twisting of a ship's structure, ship designers should be able to estimate the importance of such motions on piping structures.

(f) <u>Measurements of the Thermal Displacements of a Pipe</u>

39. Consider a pipe AB, fixed at end A and free at end B (See figure 3) At temperature T_o it occupies position AB_o and at higher temperature T, it occupies position AB_t. The distance of any given point from the fixed end is L_o ΔL is the displacement caused by thermal expansion of any given point of the pipe contained on the line L.

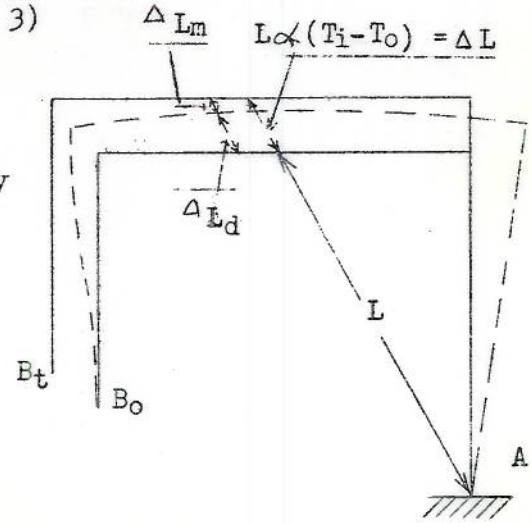

Figure 3. Displacment of a pipe equivalent to that caused by temperature expansion.

40. If a model pipe (at room temperature) is assumed to initially occupy the position AB_t, and if by mechanical means, the end B_t is moved to the position B_o the pipe will occupy a position as shown by the dotted lines of Figure 3. ΔL_m is the amount of free displacement prevented when the pipe expands thermally and the ends of the pipe are fixed at points A and B_o.

$\Delta L_d = \Delta L - \Delta L_m$ is then the displacement or distortion of the pipe axis, caused by thermal expansion, when the ends of the pipe are fixed. ΔL can be written as

$$\Delta L = L\alpha(Ti - To)$$

and

$$\Delta L_d = L\alpha(T_i - T_o) - \Delta L_m$$

where α is the mean coefficient of expansion over the temperature range T_o to T_i. If the end B moves, because of expansion of the apparatus to which it is attached, no trouble need be anticipated. ΔL_d will be found to have some value different from zero to end B.

41. The value ΔL_m can be measured with a dial indicator and therefore the distortion, or displacement, ΔL_d, can be determined. The above reasoning holds for any component of L and ΔL.

42. As ΔL_m is considered measured from the AB_t pipe position, and as ΔL_d is considered in terms of the cold pipe position, in order to be consistent with signs the above equation is

$$\Delta L_d = L\alpha (T_i - T_o) + \Delta L_m$$

or in terms of components this equation becomes:

$$\Delta L_{dx} = L_x \alpha (T_i - T_o) + \Delta L_{mx} \qquad \text{etc.}$$

For the pipe considered $\alpha (T_i - T_o) = 0.0000075 \times (850-70) = 0.00585$.

43. To obtain full scale values of the distortion of the pipe axis from model measurements, determine the model measured displacement, L_m, per unit deflection of any given end of the pipe system. Multiply these per unit displacements by the total given displacement of the above given pipe end of the full scale pipe. This gives the full scale value of L_m. Displacements obtained in this manner are given in Table 13. The values are all given in terms of the full scale pipe. The coordinates of the points considered are shown in the Table and the approximate locations of the points are given by the circled numbers 1 through 8 on Plate 6 and 7.

Table 13. Thermal Distortion of the axis of the Navy Pipe.

Pipe Location	Distance from Anchor Point			Displacement of Pipe Axis from Cold Pipe Position		
	L_x	L_y	L_z	ΔL_{dx}	ΔL_{dy}	ΔL_{dz}
1	+51.8	+461	+78	-	+0.61	-0.58
2	-42.8	+461	+78	-	+1.29	-0.28
3	-102.0	+461	+36	-1.04	+2.01	-
4	0	+415	-30	-0.34	-	-0.86
5	0	+301	-30	-0.14	-	-0.71
6	0	+145	-30	-0.03	-	-0.35
7	+154	+ 77	- 6	+0.75	-	-0.98
8	+128	+ 50	+53	-	-0.11	-0.34

CONCLUSIONS AND RECOMMENDATIONS

(a) Facts Established

44. End reactions for two systems of branched pipes have been measured. The results, by checks and by comparison with calculated results, are within limits of accuracy necessary for piping design. Opportunity for error in obtaining the model test results are few and checks are available to determine the accuracy of the results. Systematic errors, caused by lack of rigidity of the supports of the measuring apparatus are not included in the checks.

45. Small components of force may be in considerable error (see paragraph 35). This error is not important for stress or thrust determinations at or near the end of a pipe at which the measurements were made. The error may, however, cause considerable difference in moment determinations when the moments are determined indirectly by summing up about a given point, and when the point is a long distance, perpendicularly, from the small force. It is suggested, this in important cases, the end reactions be measured at every end of a pipe system, and that bending moments be obtained by transferring from the nearest ends.

46. End reactions of complicated piping systems can be measured as easily as geometrically simple systems of the same number of ends.

47. Reactions of complicated piping systems can be determined more quickly and with less chance of error by model test than by calculation. Piping systems contained in a single plane and geometrically simple three dimensional systems probable can be solved more quickly by calculation.

48. Measurements of the displacement of a pipe's axis, caused by thermal expansion between fixed points have proved to involve little additional labor. The fixed points need not be the ends of the pipe.

49. End reaction changes caused by small rotations of an end of a pipe are easily measurable and may cause significant changes, especially in the moment reactions.

(b) Opinions and Recommendations

50. Model pipe tests provide a reasonably accurate method of end reaction determinations with little opportunity for error. Because of the time necessary to fabricate and mount the model, the reactions may be determined more quickly for simple pipe systems by theoretical methods.

51. Approximate solutions relieve mathematical difficulties more than those of model construction. In many cases where the approximations can be justified, mathematical solutions can be obtained more easily than model solutions. Crocker[6] believes that square corner short solutions of three-dimensional problems are quicker and of sufficient accuracy to displace model tests except in exceptional cases. Crocker refers only to pipe systems with no branches. Conrad[10] of Westinghouse states "that even on less complicated three-dimensional systems model tests have been found more satisfactory than calculations. Possibility of error is far less in model testing than in any method of calculation and is preferred for that reason." Model testing should supplement calculations as a means of determining end reactions.

52. The model testing apparatus at the Naval Research Laboratory has accomplished much of its original purpose of determining its accuracy, practicability, and of checking and correcting theoretical equations used in bending. Work is planned on direct experimental stress measurements and on flexibility measurements of typical piping configurations with a view

toward design simplification. In addition it is suggested that the Bureau submit specific practical problems in piping design that are useful of solution. It is also suggested that the Laboratory make end reaction determinations on specific piping systems of interest to the Bureau.

53. It is difficult at this time to estimate the time required to construct and carry through a model test of piping as the method and work have not been developed into a routine. An estimate of the actual time necessary to make a test of the Navy pipe sections A,B, C, and D would be two weeks' work of one person. This test would include time necessary to fabricate the model. The model would probably occupy the measuring apparatus about three days, so with sufficient personnel (five or six persons) and with testing on a routine bases, one pipe system similar to the Navy pipe could be analyzed every three days, although it would still require about two weeks to construct and analyze a single pipe system. Pipes containing no branches could have their end reactions determined much more rapidly.

SUMMARY

54. It has been shown that a study of end reactions of piping systems by means of scale models is sufficiently accurate, is subject to little probability of error, and is capable of solving simple or branched piping systems. It is probable that for two-dimensional systems and for the more simple three-dimensional systems, the model pipe method has no advantage over mathematical method except less opportunity for error. For these simple cases the model pipe method may require more time than calculations. In more complicated piping systems little or no added difficulty is encountered in model tests although additional branches entail more work. Mathematical solutions of complicated piping systems become extremely long and often impractical. It is believed that mathematical and model test methods are supple mentary, and that model test methods should be used in the more complicated piping arrangements, and in other piping arrangements where the end reactions are especially important. In this latter case both methods may be used.

55. Model test methods can give information, with little additional work, on the displacement of a pipe axis caused by thermal expansion absorbed between fixed points.

56. In addition to work scheduled at this Laboratory it is suggested that the model pipe apparatus be put to practical use on problems supplied by, and of interest to, the Bureau. The performance of this work should help in the development of routine methods and may suggest changes to improve the convenience and usability of the apparatus.

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APPENDIX 1.

MODEL PIPE TEST OF A BRANCHED PIPE SYSTEM

1. A complete numerical example of the end reaction determinations for the Navy branched pipe system containing pipes A, B, C, and D (Plates 2, 6, and 7) is given below.

2. In order that the model be of convenient size a scale factor of about 1/10 was selected. This is the ratio between the linear dimensions of the model and full scale pipe. Details of full scale pipe diameter and wall thickness are given on Plate 6. The scale factor for tube models[1] must satisfy the relation.

$$S = \left(\frac{r_m}{r_a}\right)^2 \frac{t_a}{t_m}$$

Where r_a, r_m, t_a and t_m refer to the mean radius and wall thickness of the full scale pipe and model pipe respectively. With tubing available it was possible to select a size having a mean radius $r_m = 0.3275"$, and wall thickness $t_m = 0.034"$. The scale factor was determined as $S = 1/10.8$.
Model pipe D was constructed from a rod. In using a rod as model any convenient scale factor can be chosen. In this case the value of 1/10.8 had already been selected.

3. The ratio of the moments of inertia of the full scale and model system must be a constant throughout the system.

$$\frac{I_a}{I_m} = \text{Constant} = C$$

For full scale pipes A and C the values of $I_a = 104.0$ inch.^{4}
For model pipes A and C the value of $I_m = 0.00375$ inch.^{4}

so: $$C = \frac{I_a}{I_m} = \frac{104}{0.00375} = 27740$$

For full scale pipe D the value of $I_a = 258.5$ inch.^{4}

So for model D
$$I_m = \frac{I_a}{C} = \frac{258.5}{27740} = 0.00932 \text{ inch.}^4$$

For full scale pipe B the value of $I_a = 4.71$ inch.^{4}
For model B
$$I_m = \frac{4.71}{27740} = 0.000170 \text{ inch.}^4$$

4. A rod was used for model pipe B as its effect on the rest of the pipe system was small. If more accurate results for the end reactions and stresses of pipe B were required a separate tube model, consisting only of pipe B, would be constructed. The displacements of the pipe ends of this

new model would be as obtained from temperature expansion plus displacements of junction J. The displacement of J would be determined from direct measurements on the branched model.

5. With moments of inertia given as above the dimensions of the model are:

Section	Axial dimensions	Outside Diameter	Wall Thickness
A and C	Full scale x 1/10.8	0.689"	0.034"
B	Full scale x 1/10.8	0.2425	- - -
D	Full scale x 1/10.8	0.660	- - -

Section A and C were made from stock size seamless steel tubing. Section B was made from 1/4" dia. cold rolled steel rod. No correction has been made for the slight difference in diameter. Section D was very short and was constructed of a rod turned to the desired diameter.

6. In model construction junctions and valves were assumed constructed of rigid materials. Welding elbows were approximated by square corners.

7. In mounting the model system the lathe slide rest should first be adjusted to the correct angle on the right angle support. This angle, whose sine is $\frac{r}{R}$, (see figure 1 of report body), can be read directly on the scale, as shown on Plate 5. The lathe slide rest should then be fixed at approximately the correct vertical position on the right angle support, and the support moved into the position determined by the end of the model. The right angle support is then clamped to the bed plate so that the junction of its face and the bed plate makes a line parallel to r (fig. 1 of report body). The measuring head is constructed so that a force acting in the direction of the bracket axis can be measured whether the force is positive or negative. However, as some adjustments are required to change from measurements of the positive forces to measurements of the negative forces, it has been found more convenient to measure only positive values and, when necessary, to use rubber bands between the pipe clamp extension and a bronze sleeve to make this force positive.

Data

8. In all data coordinates used will be consistent with figure 1 of this report body. Reactions are arbitrarily determined per inch displacement of Head C.

Head A — Measuring Head Reactions

$R_{x'} = -24.4$ lbs/inch

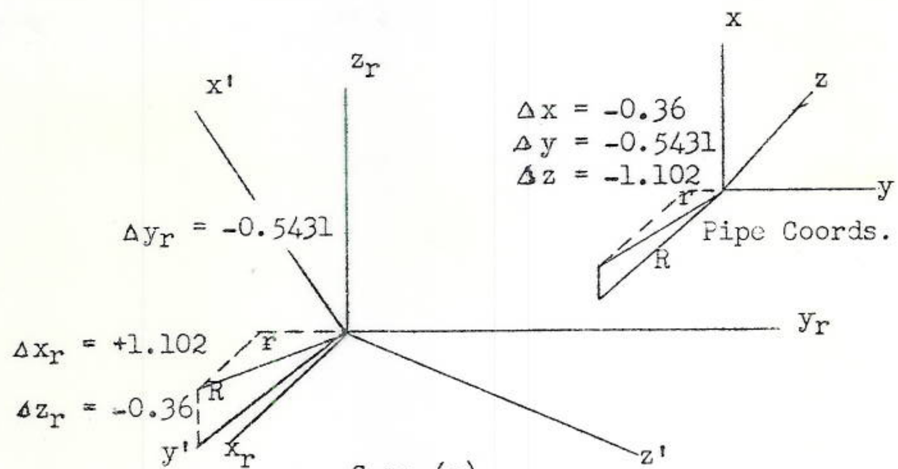
$R_{y'} = +60.0$

$R_{z'} = +5.7$

$R_{x'y'} = +14.97$

$R_{y'z'} = -37.45$

$R_{z'x'} = -9.51$



Case (c)

Head B — Measuring Head Reactions

$R_{x'} = -4.58$ lbs/inch

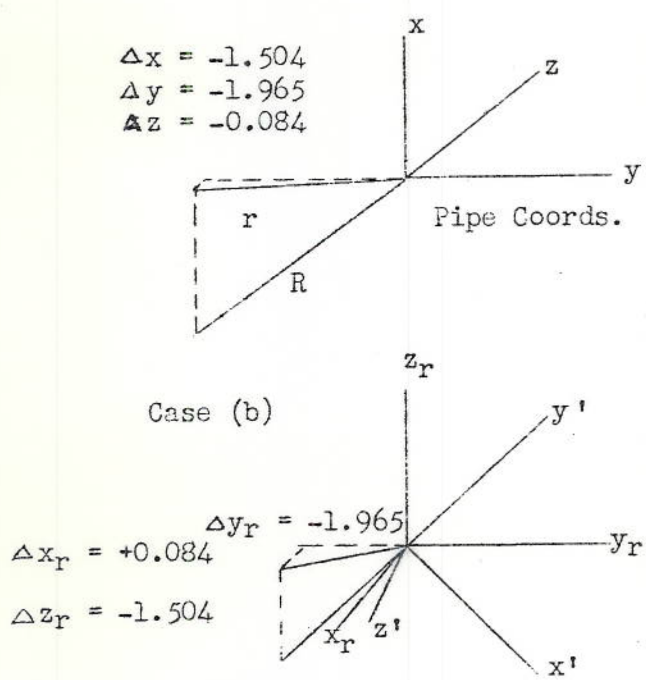
$R_{y'} = -8.61$

$R_{z'} = 0$

$R_{x'y'} = +5.92$

$R_{y'z'} = +3.23$

$R_{z'x'} = 0$



Head C

Measuring Head Reactions

$R_{x'} = +60.2$ lbs/inch

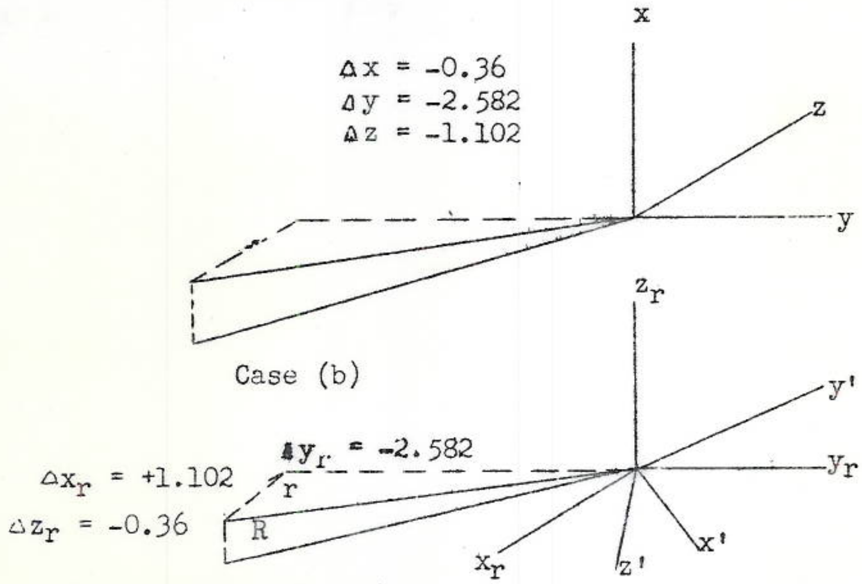
$R_{y'} = -103.5$

$R_{z'} = -3.41$

$R_{x'y'} = -64.4$

$R_{y'z'} = +70.0$

$R_{z'x'} = -2.33$



Head D

Measuring Head Reactions

$R_{x'} = -152.2$ lbs/inch

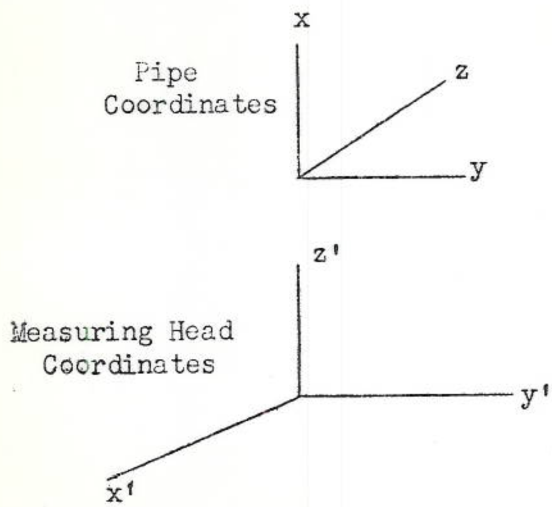
$R_{y'} = \pm 36.1$

$R_{z'} = +13.36$

$R_{x'y'} = +73.1$

$R_{y'z'} = -1.88$

$R_{z'y'} = -47.2$



9. The reaction forces are converted to forces and moments by [1] equations:

$$F_x' = R_x' + R_x'y' - R_z'x' \qquad M_x' = 5\ R_y'z'$$

$$F_y' = R_y' + R_y'z' \qquad M_y' = 5\ R_z'x'$$

$$F_z' = R_z' \qquad M_z' = 5\ R_x'y'$$

Reaction	Head			
	A	B	C	D
F_x'	+0.08 lbs/in	+1.34	-1.87	-31.9
F_y'	+22.55	-5.38	-33.5	+34.2
F_z'	+ 5.7	0	-3.41	+13.4
M_x'	-187.2 in-lbs/in	+16.15	+350.0	-9.4
M_y'	-47.55	0	-11.6	-236.0
M_z'	+74.85	+29.6	-32.20	+365.5

10. As the pipe clamp extension (Plate 5) was used on all of the heads the reactions must be transferred 2.625 inches along the bracket axis to the end of the pipe. The distance 2.625 inches is a constant of the apparatus. The forces are not changed. The moments are:

$$M_x'' = M_x' + 2.625\ F_y'$$

$$M_y'' = M_y' - 2.625\ F_x'$$

$$M_z'' = M_z'$$

Performing this step gives:

Reaction	Head			
	A	B	C	D
F_x'	+0.08 lbs/in	+1.34	-1.87	-31.9
F_y'	+22.55	-5.38	-33.5	+34.2
F_z'	+5.7	0.0	-3.41	+13.4
M_x''	-128.0 in-lbs/in.	+2.0	+264.0	+80.4
M_y''	-47.7	-3.5	-6.7	-152.2
M_z''	+74.8	+29.6	-322.0	+365.5

11. The above reactions are all in the coordinate systems of their respective measuring heads (primed coordinates). These can be rotated to the reference (r subscript) coordinate system by applying the formulae for the proper case. These are given in paragraph 21 of this report. The reactions in the reference coordinate system are given below.

Reaction	Head			
	A	B	C	D
F_{x_r}	+21.9 lbs/in	+0.15	+9.99	
F_{y_r}	-4.47	-3.43	-32.11	
F_{z_r}	-6.27	-4.30	-2.39	
M_{x_r}	-40.4 in-lbs/in	+29.7	-307.5	
M_{y_r}	+103.3	-0.3	-102.2	
M_{z_r}	-110.4	-3.7	-260.1	

12. By inspection of the coordinate diagrams, drawn for each head (see paragraph 8 of appendix), the forces and moments can be transferred to the pipe coordinate system. By multiplying the model forces by[1]

$$\frac{E_a\ I_a}{E_m\ I_m}\ s^3 x \text{ (displacement of C)}$$

the full scale forces are obtained. It is to be noted that previous values of reactions have been per unit displacement of end C. The full scale moments are obtained by multiplying the model values by

$$\frac{E_a\ I_a}{E_m\ I_m}\ s^2 x \text{ (displacement of C)}$$

Where E_a and E_m are the modulus of elasticity of the full scale and model pipes respectively and

$$E_a = 24.8 \times 10^6$$

$$E_m = 29.0 \times 10^6$$

I_a/I_m is the ratio of moments of pipe to model and is equal to 27740

s = scale factor = 1/10.8

13. Substitution of numerical values results in the following equations for converting scale model reactions to full scale reactions:

$$F_a = 53.25\ F_m$$

$$M_a = 657\ M_m$$

14. The values of full scale forces and moments in the pipe coordinate system are given below:

Reaction	Head			
	A	B	C	D
F_x	-332 lbs.	-238	-127	+710
F_y	-237	-182	-1709	+1828
F_z	-1164	-8	-532	+1697
M_x	-72400 in-lbs.	-2450	-171000	+240000
M_y	+67800	-168	-67000	-100000
M_z	+26500	-19700	+202000	-52600

15. When the reactions are measured at all points of support of the pipe system the results can be checked by observing if the conditions for static equilibrium hold. These checks have been performed in the report body (see paragraph 33 and Tables 8-11).

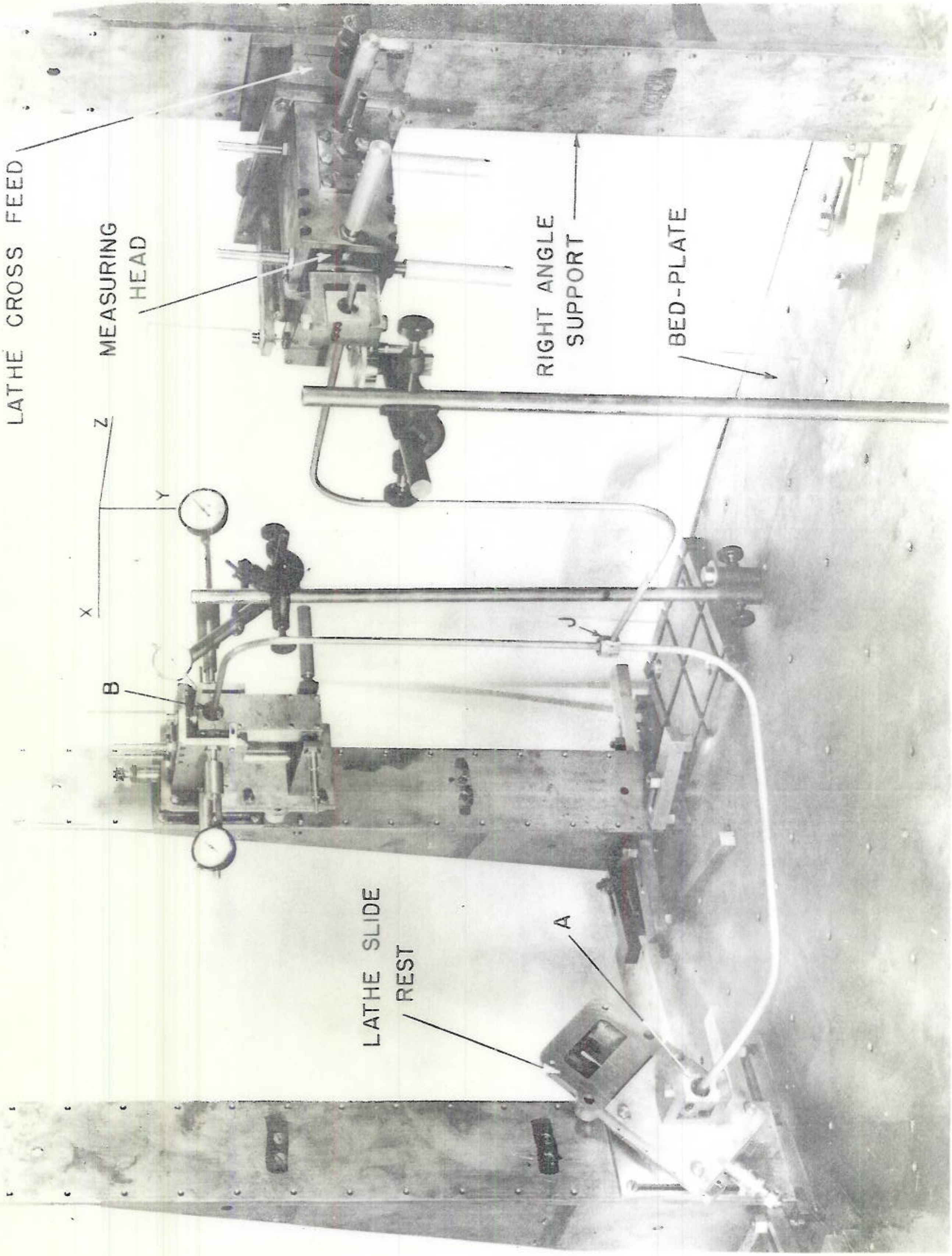

LATHE CROSS FEED
MEASURING HEAD
RIGHT ANGLE SUPPORT
BED-PLATE
Z
Y
X
B
J
A
LATHE SLIDE REST

PLATE I

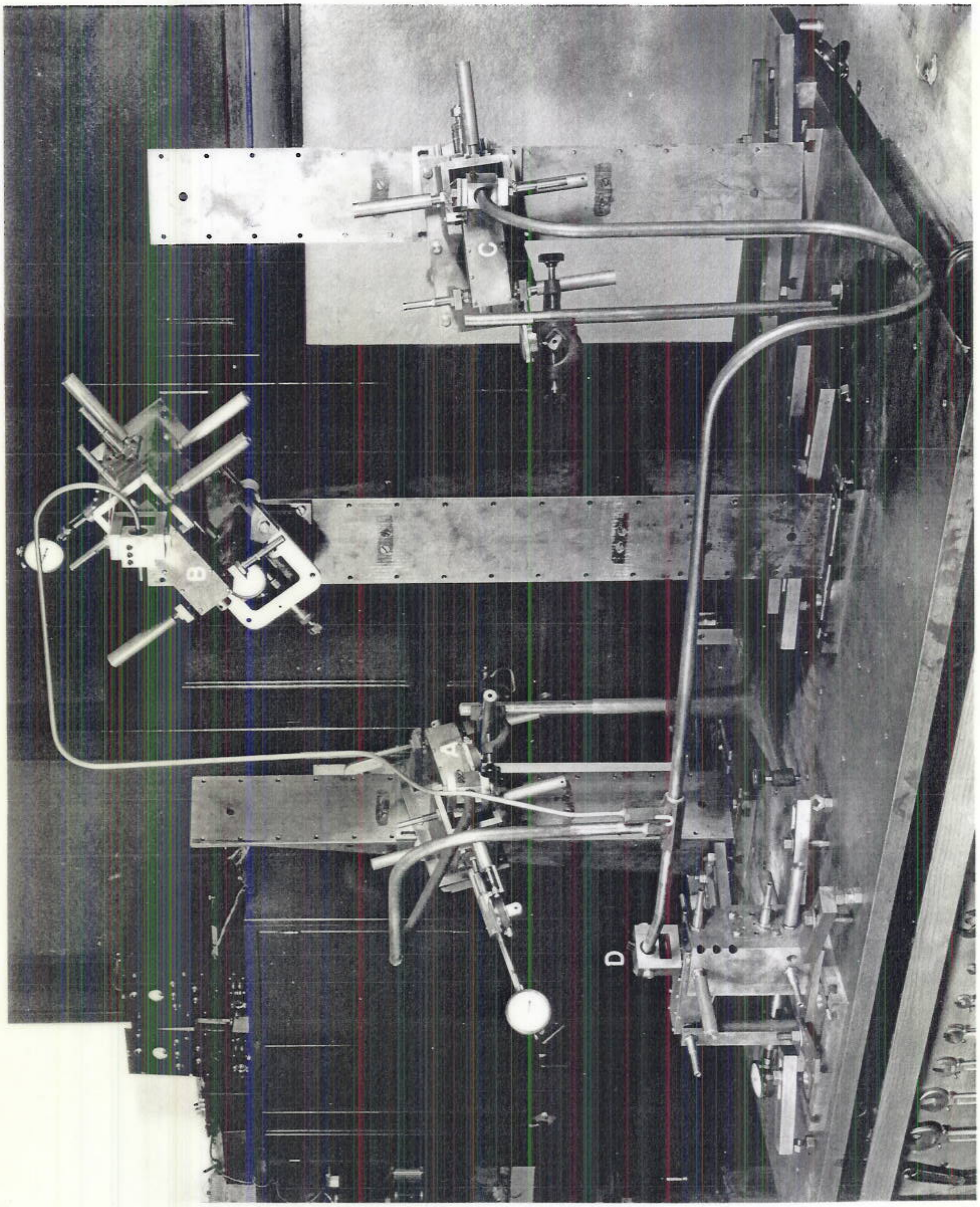


PLATE 2

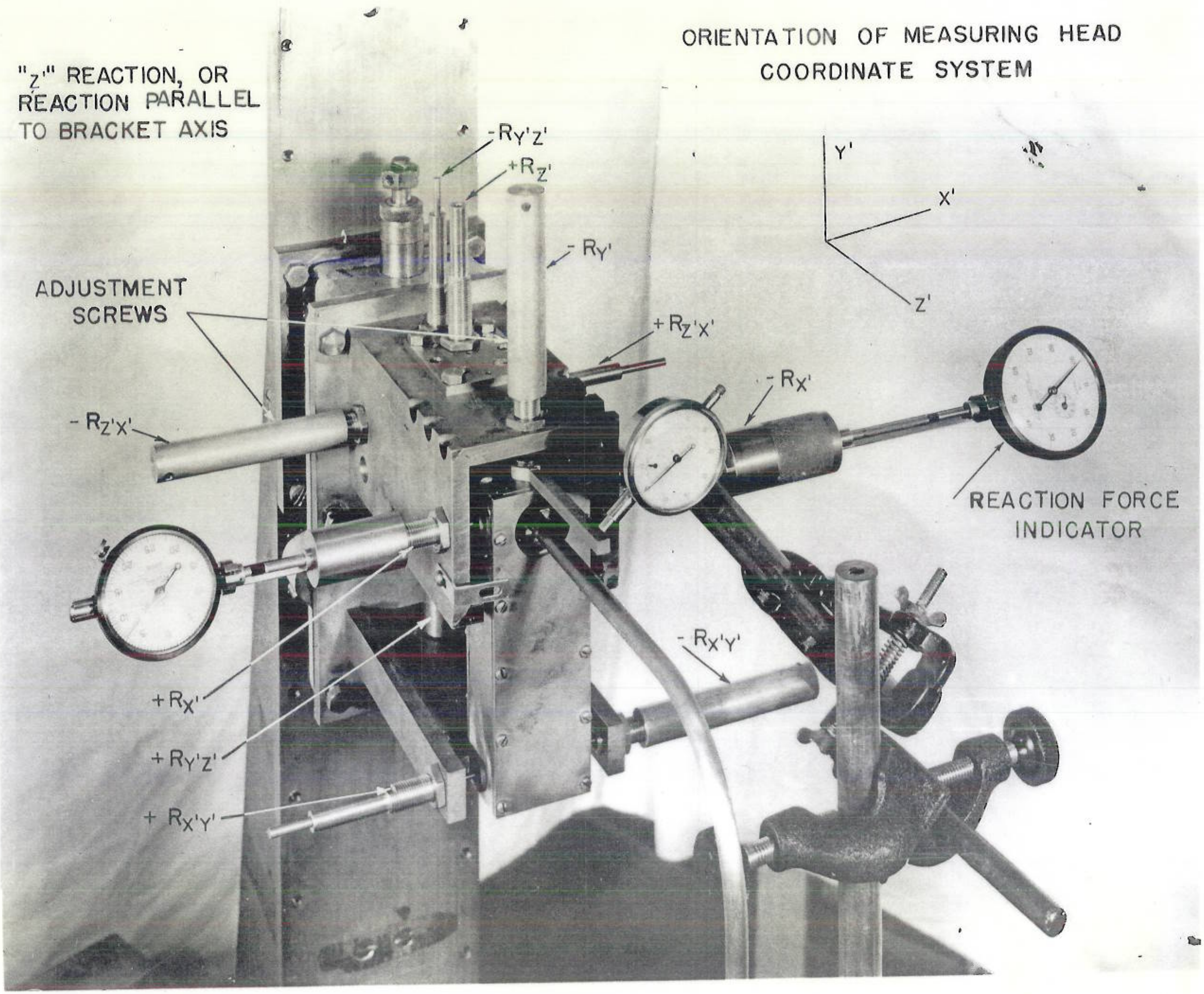
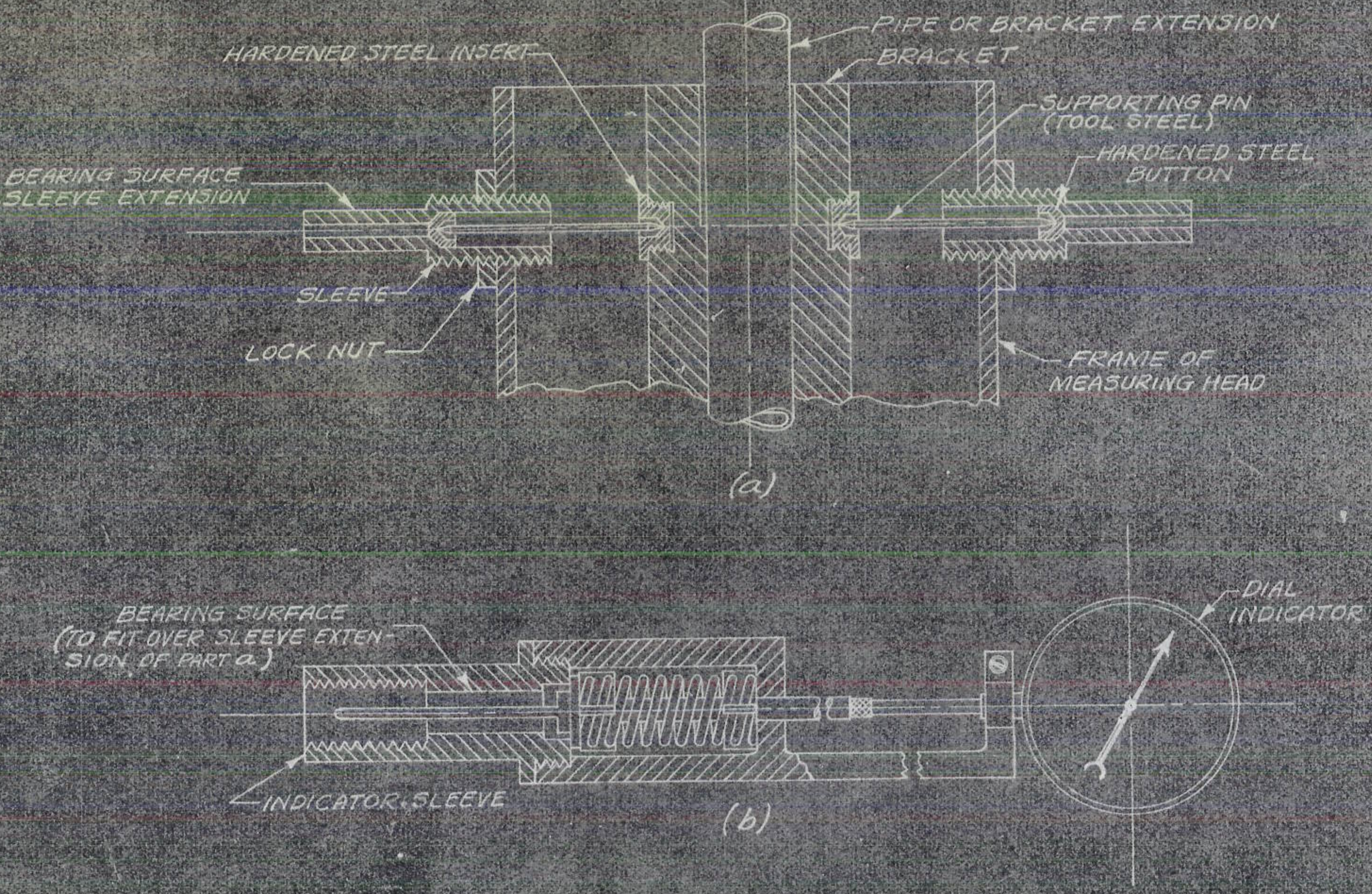

"Z'" REACTION, OR REACTION PARALLEL TO BRACKET AXIS
ORIENTATION OF MEASURING HEAD COORDINATE SYSTEM
Y'
X'
Z'
$-R_{Y'Z'}$
$+R_{Z'}$
$-R_{Y'}$
ADJUSTMENT SCREWS
$+R_{Z'X'}$
$-R_{X'}$
$-R_{Z'X'}$
REACTION FORCE INDICATOR
$-R_{X'Y'}$
$+R_{X'}$
$+R_{Y'Z'}$
$+R_{X'Y'}$

PLATE 3



(a) METHOD OF SUPPORTING END OF MODEL PIPE
(b) REACTION FORCE INDICATOR

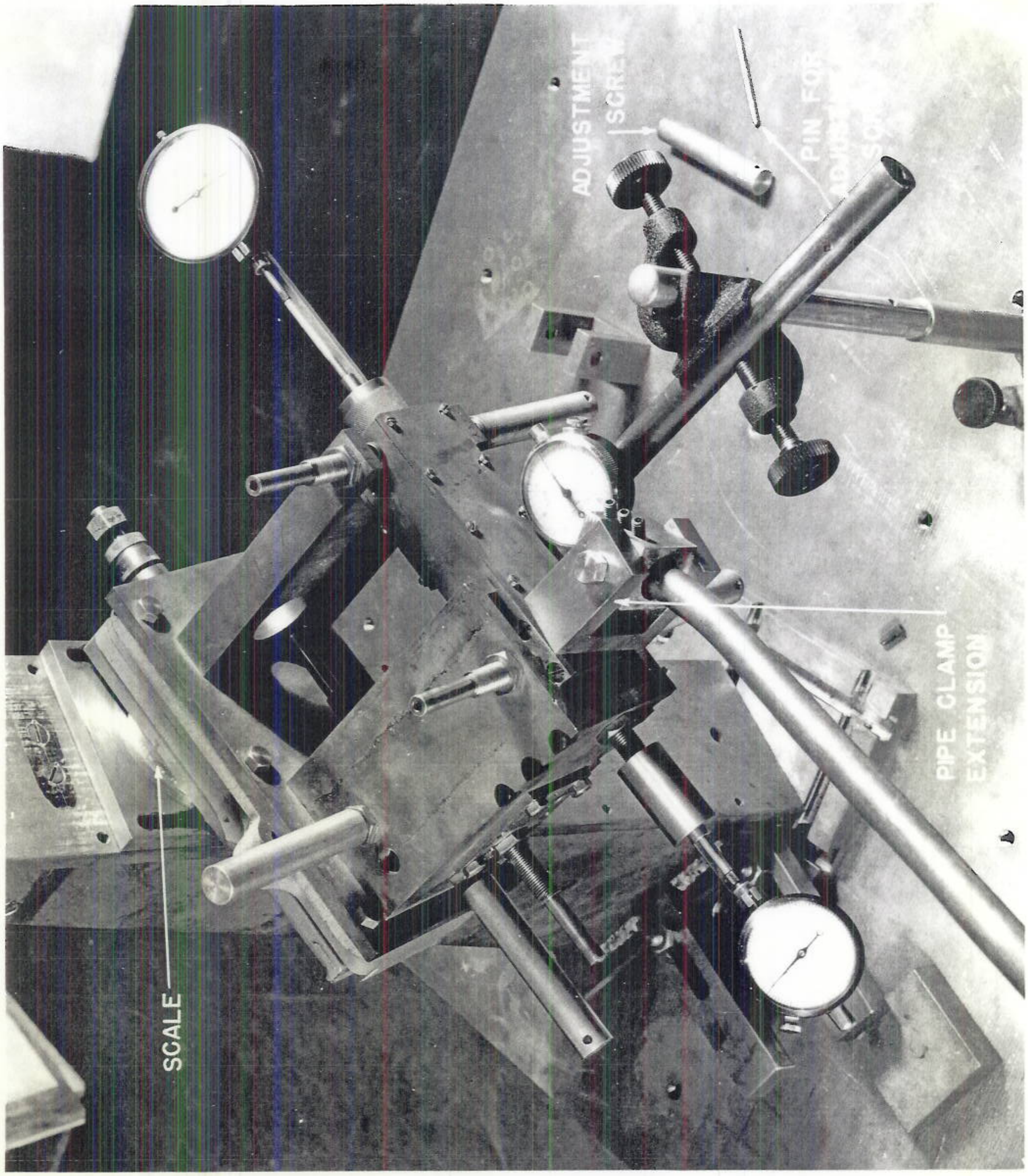
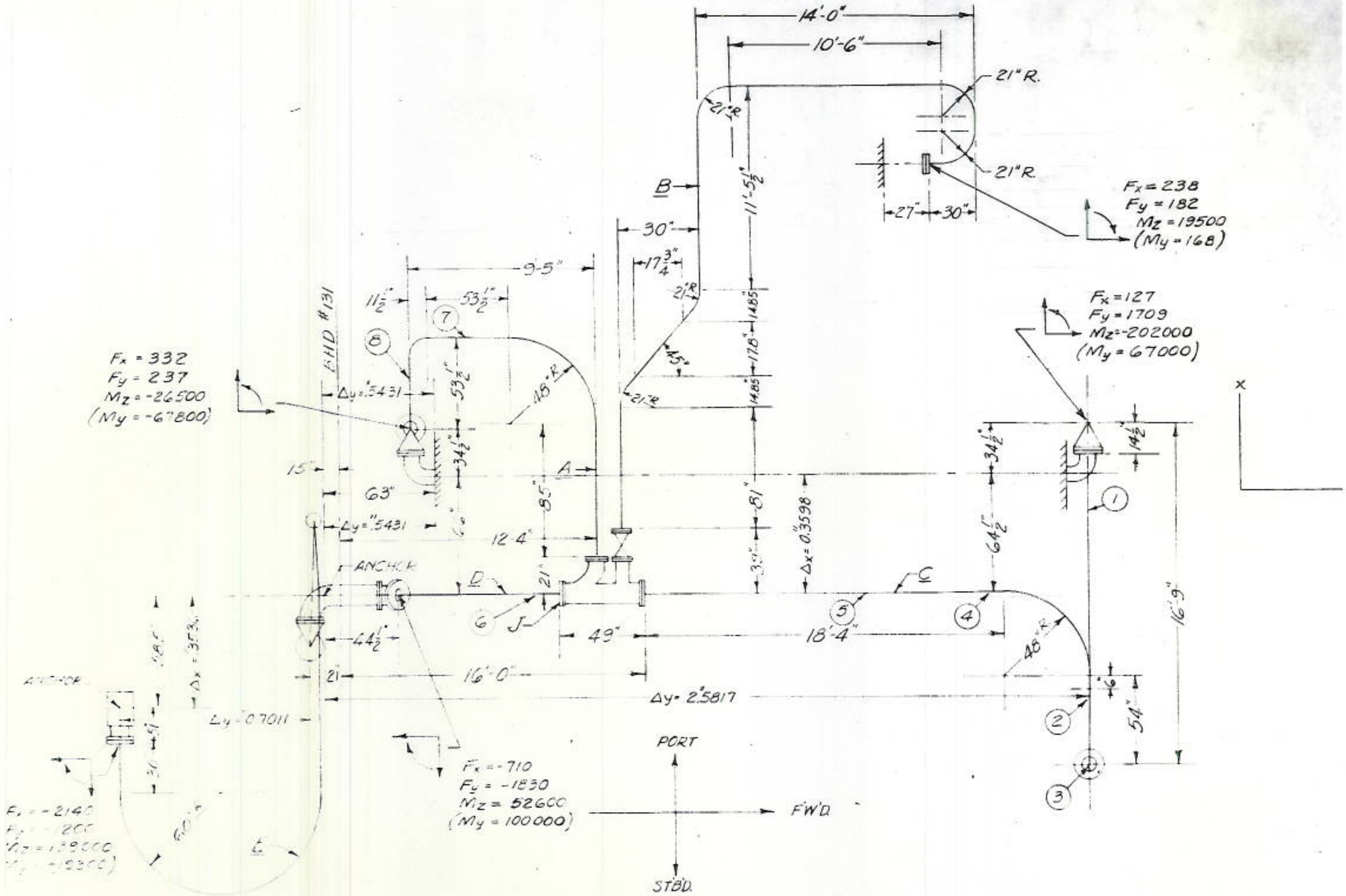
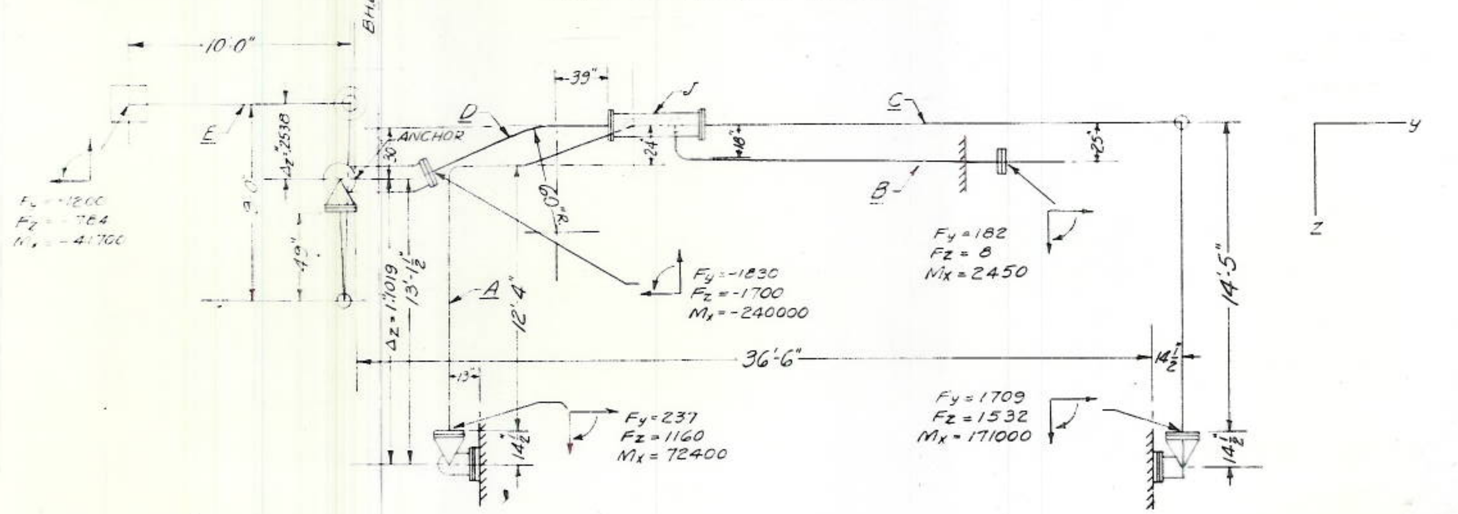


PLATE 5



PLAN

REACTIONS ARE THOSE EXERTED BY THE PIPE.
UNITS ARE IN LBS. OR IN.-LBS.



NAVY PIPE

ELEVATION

PLATE 6