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From High-Level Task Specifications to Geometric Control via Lyapunov Abstractions

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1 Proposal Summary

The goal of this research project is to narrow the existing gap between high-level discrete task planning and low-level continuous control in complex multi-agent missions within a control-theoretic framework. We develop techniques that enable consistent mappings between high-level specifications and low-level control commands. We propose novel forms of Lyapunov-like barrier functions that capture high-level spatiotemporal specifications and interactions among agents, and low-level geometric flows capturing feasible system trajectories. The main idea lies on the pairing of a Lyapunov-like barrier function and a geometric flow using notions and tools from geometric control and dynamical systems theory. The proposed method offers a reactive motion planning, decision-making and control design mechanism that is scalable with the number of agents and tasks, and thus applicable to large-scale systems involving hundreds of agents. The expected research outcomes will advance knowledge and the state-of-the-art in real-time planning, decision making and control in situations involving multiple autonomous and semi-autonomous agents. The proposed methods will allow safety-critical and time-critical missions to be carried out with minimal human supervision and minimal effort on planning and coordinating the mission. This will increase the situational awareness and readiness of the Air Force personnel and will provide better means to implement strategies and tactics in unknown, uncertain environments.

2 Overview

In recent years, there has been a lot of work on encoding high-level task specifications for mission synthesis using Linear Temporal Logic (LTL). More recently, tasks that further include time specifications, e.g., *reach to place A before* T_1 *units while avoiding region B and then reach to place C before* T_f, have been expressed via Signal Temporal Logic (STL).

In this research we investigate the design of low-level controllers that accomplish given spatiotemporal high-level mission specifications with certain, provable guarantees. The overview of the envisioned research tasks is given in Figure 1.
We consider multi-agent systems of the form:

$$\dot{x}_j = f_j(x_j, u_j),\ x_j(t) \in \mathcal{X}_s,\ u_j(t) \in U_j \tag{1}$$

where $x_j \in \mathbb{R}^n$ and $u_j \in \mathbb{R}^m$ are the state and the control vectors of the j-th agent, respectively, $j \in \{1, \dots, N\}$, $f_j : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the vector field governing the evolution of the agent states,

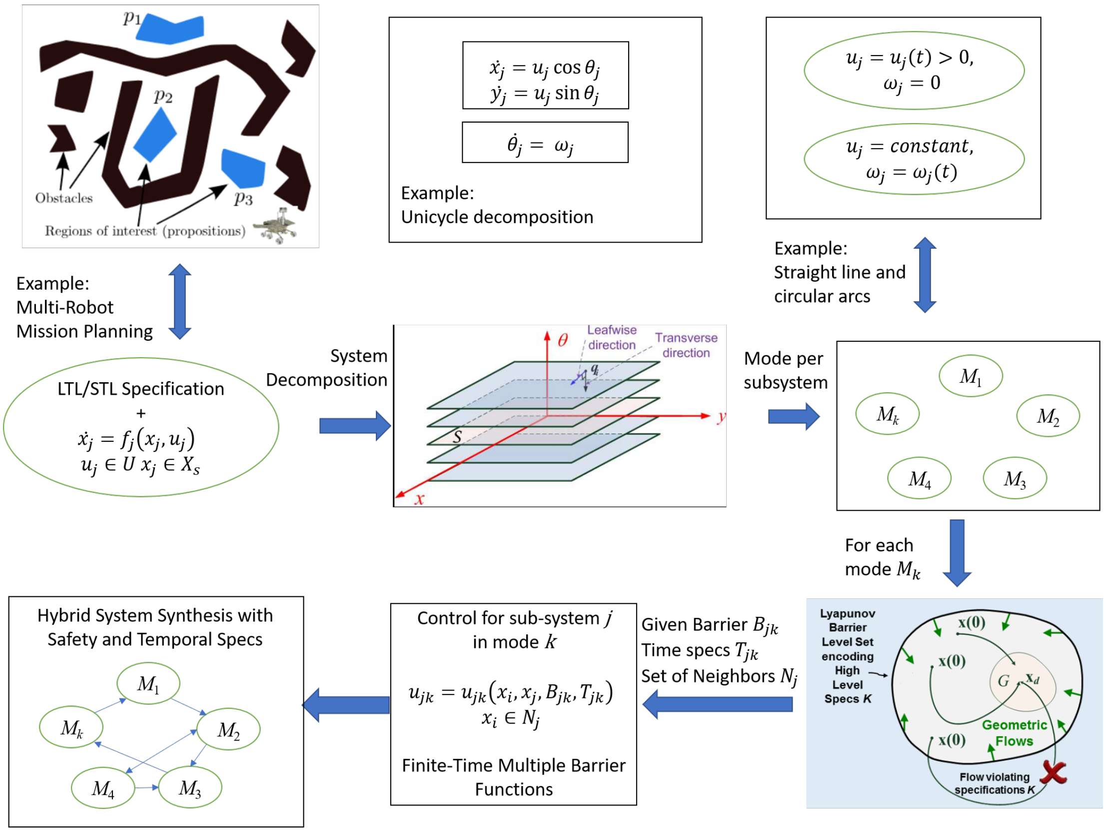

Figure 1: Overview of the proposed research tasks

$\mathcal{X}_s$ is the safe set for the agent trajectories, U_j is the set or permissible control inputs. Given the system (1) and a set of high-level specifications encoding the mission, we are investigating how to design the low-level controllers u_j, $\forall j$, so that the mission is accomplished. As an illustrative example, consider the multi-robot motion planning in dynamic environments, where each robot has to accomplish a set of spatiotemporal tasks. The mission description ($\mathcal{M}_1$) may have specifications such as: maintain safe distance d_s from each other, avoid unsafe regions $\mathcal{X}_{us}$ of the state-space, maintain a formation given by relative positions x_{ij}, and reach to destination region $\mathcal{X}_g$ within a given period of time T_f. The motion of each robot can is modeled via linear dynamics (single or double integrator), or via nonholonomic unicycle kinematics. The control input is required to satisfy constraints of the form $\|u_j(t)\| \leq u_{max}$, for all $t \geq 0$.

We propose the following framework:

S1: Given the system dynamics, we decompose the *configuration* space into *leafwise* and *transverse* directions to identify basic motion primitives for the original system. To illustrate the basic idea of this decomposition, consider the n-dimensional drift-free system:

$$\dot{q} = \sum_{i=1}^{m} g_i(q) u_i, \tag{2}$$

where $q \in \mathbb{R}^n$, and for each i, $g_i : \mathbb{R}^n \to \mathbb{R}^n$ is the control vector field associated with the control input $u_i \in \mathbb{R}$, subject to non-integrable constraints of the Pfaffian form:

$$A(q)\dot{q} = 0, \tag{3}$$

where $A(q) \in \mathbb{R}^{k \times n}$. Hence, the objective of the design of the geometric flow F to serve as a velocity reference for (2) includes satisfaction of the constrained equation:

$$A(q)F = 0, \tag{4}$$

for all q. We say in this case that F satisfies, or is consistent with, the constraints at q. Now, if $A(q)$ contains a zero column $\begin{bmatrix} a_{1j} & a_{2j} & \cdots & a_{kj} \end{bmatrix}^T$ for some $j \in \{1, 2, \cdots, n\}$, then the corresponding j-th component of the geometric flow, F_j does not play a role in the satisfaction of the consistency condition (4). Hence, if the matrix $A(q)$ has n_0 zero columns, the associated n_0 states are termed as transverse states, while the rest of $N = n - n_0$ states are referred to as leafwise states.

Example. The unicycle model given by

$$\dot{x} = u \cos\theta, \tag{5}$$

$$\dot{y} = u \sin\theta, \tag{6}$$

$$\dot{\theta} = \omega, \tag{7}$$

with (x, y) the position coordinates, θ the orientation or the heading of the vehicle, u the linear speed, ω the angular speed, has the constraint matrix $A(q) = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \end{bmatrix}$. Hence, the unicycle model has its orientation θ in the tranverse direction while the position vector $[x \ \ , y]$ lies in the leafwise direction. This simplifies the control design since the vector field F and the corresponding controllers for the two sub-configuration spaces can be designed independently.

S2: Once we have identified the motion primitives, we define control modes on the basis of the specifications and geometry of the system, which can live in the individual subspaces as well as in the complete configuration space. Take the example of the mission $\mathcal{M}_1$ explained above: We define a set of modes that render the safe set $\mathcal{X}_s$ controlled invariant, as well as a globally convergent mode to reach the goal set $\mathcal{X}_g$. This way, the specifications, geometry and constraints of the agents dictate how various modes can be defined and synthesized in a hybrid system.

Example: In [1], we consider fixed-wing type of aircraft, modeled as constrained unicycles. For moving along the leafwise subspace, we defined a mode $Change - U$ with the controller taking the form $u = u(t) > 0$, $\omega(t) = 0$. For moving along the transverse direction, the controllers in the mode $Go - Round$ can be restricted as $u = constant$ and $\omega = \omega(t)$. Conflicts between two vehicles are resolved by switching between the two modes depending upon their relative positions. We also define three more modes which live in the complete configuration space, as they need maneuvers in both the leafwise and transverse directions. For more than two vehicles in conflict, we define a mode $Follow - Leader$, while for the case where there is no conflict, we define a mode $Go - Towards - Goal$ that drives each vehicle towards its goal location. Lastly, the mode *Loiter* is used once a vehicle is near its desired location.

S3: The control design is based on Lyapunov-like Barrier Functions (Barriers) encoding the high-level specifications. The candidate Barriers shall depend both upon spatial and temporal specifications and the system dynamics. The problem reduces to finding a controller that makes certain level sets of the barrier controlled invariant. More specifically, the safety requirement, given as the system trajectories not entering the set $\mathcal{X}_{us} = \{(x, u) \mid h_{us}((x, u) \leq 0\}$ and the convergence requirement, given as reaching the set $\mathcal{X}_g = \{(x, u) \mid h_g((x, u) \geq 0\}$ can be encoded as keeping the trajectories outside the zero-level set of h_{us}, while deriving it towards the zero-level set of h_g. In addition to these *spatial* constraints or specifications, the barrier should also encode the *temporal* constraints. In the illustrative mission $\mathcal{M}_1$, it is needed that the formation reaches the goal/desired region $\mathcal{X}_g$ within a given time period T_f units.

Example: For spatiotemporal specifications in multi-task problems such as: "reach a given goal set in a given prescribed time, while remaining in a given safe set at all times", we develop a Quadratic Programming (QP) based approach in [2] (see also [3]). More details on our contribution and ongoing work on barriers that can additionally impose temporal specifications are discussed in Section 3.5.

S4: Given the individual modes M_k and their respective controllers u_{jk}, the last step is to synthesize a switching law among the various modes that is consistent with the mission statement. One major challenge is to ensure that the resulting trajectories do not exhibit Zeno or chattering behavior. While asymptotic stability of switched systems has been extensively addressed, the design of switched systems with finite-time convergence guarantees has not been considered in the literature. This is one of the objectives of this project.

Example: In our recent work [4], we present conditions in terms of multiple generalized Lyapunov functions for finite-time stability of switched systems, and a method of designing the switching law so that finite-time convergence is guaranteed. We extend these results to a class of hybrid systems in [5].

3 Contributions

The current contributions to the subtasks described in Section 1 are outlined as follows:

1. Results on finite-time stability for switched and hybrid systems, which enable the development of controllers that can achieve tasks with time specifications.
2. A systematic method of finding an optimal barrier function for multi-task specifications.
3. Results on safe motion planning in the presence of uncertainty using finite-time controllers.
4. Results on control synthesis under spatiotemporal specifications *and* input constraints:
   - QP approach: We formulate a QP-based optimization problem using Fixed-Time CLFs and CBFs, and discuss its feasibility.
   - CLF approach: We design a prescribed-time convergent control law using Sontag's formula and time-reparametrization.
5. Robust control synthesis under spatiotemporal and input constraints in the presence of disturbances.

3.1 Results on Finite-Time and Fixed-Time Stability

As discussed in Section 2, the agents may have temporal specifications of reaching some region $A \subset \mathcal{X}_s$ in time T_1, and then reach the region $C \subset \mathcal{X}_s$ before time T_f. This requires the set or point A to be finite-time stable for the closed-loop trajectories. More specifically, we need the trajectories to have the property that for all $x(0) \in \mathcal{X}_s$, $x(t) \in A$ for some $t \in [0, T_1]$ and for all $x(0) \in A$, $x(t) \in C$ for some $t \in [0, T_f - T_1]$. To be able to fulfill these requirements, especially for the cases when the set A and/or C are singleton locations, asymptotic controllers would not work; instead, the closed-loop trajectories shall converge in finite time.

In [4], we studied finite-time stability of switched systems. We presented sufficient conditions in terms of multiple generalized Lyapunov functions for the origin of the switched system to be finite-time stable. The main assumption is the presence of a finite-time convergent mode that is active for a sufficiently long time. We show that even if the value of the generalized Lyapunov function increases between consecutive switches, finite-time stability can still be guaranteed if the finite-time convergent mode is active long enough. We also present a method of designing a finite-time stabilizing switching law. As a case study, we use the developed schemes to design a finite-time stable state observer for linear switched systems for the case when only one of the modes is observable.

In [5], we extend these results to a class of hybrid systems. In contrast to earlier work where the Lyapunov functions are required to be decreasing during the continuous flows and non-increasing at the discrete jumps, we allow the generalized Lyapunov functions to increase *both* during the continuous flows and the discrete jumps. As thus, the derived stability results are less conservative compared to the related literature.

In [6], we presented novel controllers that yield finite-time stability for linear systems. We designed novel finite-time controllers based on vector fields and barrier functions to demonstrate the utility of this geometric condition. We also demonstrated how asymptotic stabilizing controllers can be modified to achieve finite-time convergence. The results have been utilized in finite-time resilient consensus [7] in the presence of misbehaving agents.

In [8] we studied the effect of control-input constraints on the domain of attraction of an FxTS equilibrium point. We first present a new result on FxTS, where we allow a positive term in the time derivative of the Lyapunov function. We provide analytical expressions for the domain of attraction and the settling time to the equilibrium in terms of the coefficients of the positive and negative terms that appear in the time derivative of the Lyapunov function. We show that this result serves as a robustness characterization of FxTS equilibria in the presence of additive, vanishing disturbances. We use the new FxTS result in formulating a provably feasible quadratic program (QP) that computes control inputs that drive the trajectories of a class of nonlinear, control-affine systems to a goal set, in the presence of control-input constraints.

3.2 Application of Fixed-Time Stability to Optimization Problems

As an application of our work on finite-time and fixed-time stability, we explored novel optimization schemes that achieve convergence to the optimal (minimum) point of the objective function in a fixed time. As a preliminary set of results, we have modified the Gradient-flow as well as the Newton's method for optimization problems (e.g., minimization, and min-max or saddle-point problems) with convergence guarantees to the global optimal value in a fixed time under certain regularity and convexity conditions. Given a generic optimization problem of the form:

$$\min \ f(x), \tag{8a}$$
$$g(x) = 0, \tag{8b}$$
$$h(x) \leq 0, \tag{8c}$$

with some regularity and convexity conditions on the functions involved, we investigated the differential equations of the form:

$$\dot{x} = F(x), \tag{9}$$

whose equilibrium is the optimizer of the problem (8), and in addition, the solutions of (9) converge to this optimizer x^* in a given time, i.e., $\lim_{t\to T} x(t) = x^*$ where $T < \infty$ is a user specified time. The results are presented in [9].

3.3 Multi-task Formation Control via Barrier Functions

The construction of a barrier function that meets certain spatiotemporal specifications remains an open problem in general. Most of the related work considers linear dynamics under convex constraints to formulate a convex optimization problem in order to find the barrier function. In our work [10–12] we consider both linear and nonlinear dynamics, as well as nonlinear, nonconvex constraints (such as limited communication radius and minimum safety distance), and design analytic expressions for barrier functions to ensure satisfaction of the given requirements. More specifically, in [10] and [11], we addressed robust multi-task formation control for multiple agents whose communication and measurements are disturbed by uncertain parameters. The control objectives include 1. achieving the desired configuration; 2. avoiding collisions; 3. preserving the connectivity of the uncertain topology. We designed distributed controllers under a new type of Lyapunov-like barrier functions, called parametric Lyapunov-like barrier functions, that account for uncertainties in communication and measurements.

Similarly, in [13] we considered the multi-task coordination problem under the following objectives: 1. collision avoidance; 2. connectivity maintenance; 3. convergence to desired destinations for multiple agents. We focused on computing the safety guaranteed region of multi-task coordination (SG-RMTC), i.e., the set of initial states from which all trajectories converge to the desired configuration, while at the same time avoid unsafe sets. The main underlying idea is to employ the sublevel sets of Lyapunov-like barrier functions to approximate the SG-RMTC. Rather than using fixed Lyapunov-like barrier functions, a systematic way is proposed to search an optimal Lyapunov-like barrier function such that the under-estimate of SG-RMTC is maximized. This work lays a foundation for a systematic way of searching for an optimal barrier function that can encode multiple tasks.

3.4 Safe motion planning in the presence of uncertainty

We considered the case where the multi-agent system encounters disturbances or uncertainties. These could mode external factors such as wind disturbance, noisy or imperfect communication packages, as well as internal factors such as modeling errors, sensor noises, unknown system parameters. We developed robust coordination schemes for multiple agents in the presence of dynamic obstacles [14, 15]. The dynamic obstacles can be thought of as either uncontrolled vehicles, or vehicles with higher priority (e.g., constrained vehicles such as fixed-wing aircraft) that do not deviate from their paths to resolve conflict or avoid collision. Our method accounts for a class of state disturbances that can be thought of as wind disturbance for aerial vehicles. We designed finite-time state estimators, and finite-time, state-feedback controllers, and showed that even with limited and erroneous sensing, agents are capable of avoiding collisions with moving obstacles and with each other, and reach their desired locations in finite time. We plan to continue building on this work and generalize it to robust, finite-time control against more general class of uncertainties/disturbances.

3.5 Control synthesis under spatiotemporal specifications and input constraints via Quadratic Programming

The construction of a barrier function that meets certain spatiotemporal specifications remains an open problem in general. Most of the related work considers linear dynamics under convex constraints to formulate a convex optimization problem in order to find the barrier function.

In our earlier work [10,11] we consider both linear and nonlinear dynamics, as well as nonlinear, nonconvex constraints (such as limited communication radius and minimum safety distance), and design analytic expressions for barrier functions to ensure satisfaction of the given requirements.

In our more recent work [2,3], we present a control framework for a general class of control-affine, nonlinear systems under spatiotemporal and input constraints. We consider control-affine nonlinear dynamics given as:

$$\dot{x} = f(x) + g(x)u, \tag{10}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$. The intermediate problems we considered are listed as follows:

- We first present a new result on fixed-time stability, i.e., convergence within a fixed time independently of the initial conditions, in terms of a Lyapunov function. We show robustness of the proposed conditions in terms of fixed-time stability guarantees in the presence of a class of additive disturbances.

- Then, we consider the problem of reaching a set S in a user-defined or prescribed time T, and investigate the existence of a controller for this problem in terms of Fixed-Time Control Barrier Functions (FxT-CBFs). Then, we formulate a quadratic program (QP) to compute a control input that satisfies these sufficient conditions.

With these results in hand, we finally consider control synthesis under spatiotemporal objectives given as: the closed-loop trajectories remain in a given set S_s at all times; and, remain in a specific set S_i during the time interval $[t_i, t_{i+1})$ for $i = 0, 1, \cdots, N$; and, reach the set S_{i+1} on or before $t = t_{i+1}$, written formally as:

Problem statement: *Assume $x(t_0) \in S_0 \bigcap S_s$. Design a control input $u(t) \in \mathcal{U} = \{u \mid A_u u \leq b_u\}$, so that the closed-loop trajectories satisfy the following for all $i \in \Sigma$:*

$$x(t) \in S_s \; \forall \, t \geq t_0, \tag{11a}$$

$$x(t) \in S_i \; \forall \, t \in [t_i, t_{i+1}). \tag{11b}$$

An instance of the simple example of the problem is depicted in Figure 2.

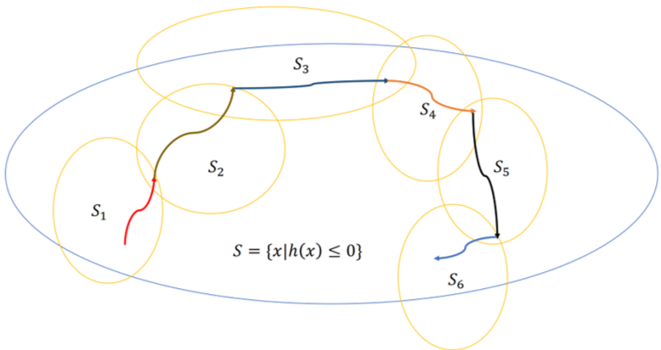

Figure 2: Motivating problem: The system trajectories need to visit the sets S_i, $i = 1, \ldots, 6$ (orange regions) in a given time sequence, while always remaining in the set S (blue region).

We show that such spatiotemporal specifications can be translated into temporal logic formulas. The STL specifications, given by formula ϕ include the following semantics:

- $(x, t) \models \phi \iff h(x(t)) \leq 0;$
- $(x, t) \models \neg\phi \iff h(x(t)) > 0;$
- $(x, t) \models \phi_1 \wedge \phi_2 \iff (x, t) \models \phi_1 \wedge (x, t) \models \phi_2;$
- $(x, t) \models G_{[a,b]}\phi \iff h(x(t)) \leq 0, \forall t \in [a, b];$
- $(x, t) \models F_{[a,b]}\phi \iff \exists t \in [a, b]$ such that $h(x(t)) \leq 0,$

where $\phi = $ true if $h(x) \leq 0$ and $\phi = $ false if $h(x) > 0$. So, the considered problem can be written in the STL semantics as follows.

Problem statement: *Design control input $u \in \mathcal{U}$ so that the closed-loop trajectories satisfy*

$$\begin{aligned}(x, t) \models & G_{[t_0,t_N]}\phi_s \wedge G_{[t_0,t_1]}\phi_0 \wedge F_{[t_0,t_1]}\phi_1 \wedge G_{[t_1,t_2]}\phi_1 \\ & \wedge F_{[t_1,t_2]}\phi_2 \wedge \cdots \wedge G_{[t_{N-1},t_N]}\phi_{N-1} \wedge F_{[t_{N-1},t_N]}\phi_N,\end{aligned} \tag{12}$$

where ϕ (respectively, ϕ_i) $=$ true if $h(x)$ (respectively, $h_i(x)$) ≤ 0, and false otherwise.

Then, we present the following quadratic program (QP) based formulation to compute the control input efficiently:

$$
\begin{aligned}
\min_{v,\alpha_1,\alpha_2,\delta_1,\delta_2} \quad & \frac{1}{2}\|v\|^2 + p\delta_1^2 && \text{(13a)}\\
\text{s.t.} \quad A_u v \leq & \, b_u, && \text{(13b)}\\
L_f h_g(x) + L_g h_g(x) v \leq & \, \delta_1 h_g(x) - \alpha_1 \max\{0, h_g(x)\}^{\gamma_1} \\
& - \alpha_2 \max\{0, h_g(x)\}^{\gamma_2} && \text{(13c)}\\
L_f h_s(x) + L_g h_s(x) v \leq & - \delta_2 h_s(x), && \text{(13d)}\\
\frac{\mu\pi}{2T} \leq & \, \alpha_1, && \text{(13e)}\\
\frac{\mu\pi}{2T} \leq & \, \alpha_2, && \text{(13f)}
\end{aligned}
$$

where $p > 0$ is some positive constant, $\gamma_1 = 1 + \frac{1}{\mu}$ and $\gamma_2 = 1 - \frac{1}{\mu}$ with $\mu > 1$. We show that the proposed QP is feasible, and discuss the cases when the solution of the QP solves the considered problem of control design. In contrast to prior work, we do not make any additional assumptions on existence of a Lyapunov or a Barrier function for the feasibility of the QP.

3.6 Prescribed-time convergence with input constraints: A control Lyapunov function based approach

For a class of nonlinear, control-affine systems, we investigate closed-form controllers for prescribed-time convergence to a given goal set in the presence of control input constraints. The proposed control architecture in [16] addresses the problem of reaching a given final set S in a prescribed (user-defined) time with bounded control inputs.

To this end, we utilize a time transformation technique to transform the system subject to temporal constraints into an equivalent form without temporal constraints. The transformation is defined so that asymptotic convergence in the transformed time scale results into prescribed-time convergence in the original time scale. To incorporate input constraints, we characterize a set of initial conditions D_M such that starting from this set, the closed-loop trajectories reach the set S within the prescribed time. We further show that starting from outside the set D_M, the system trajectories reach the set D_M in a finite time that depends upon the initial conditions and the control input bounds. We use a novel parameter μ in the controller, that controls the convergence-rate of the closed-loop trajectories and dictates the size of the set D_M.

3.7 Robust control synthesis under spatiotemporal and input constraints under bounded disturbances

We studied control synthesis for a general class of nonlinear, control-affine dynamical systems under additive disturbances and state-estimation errors [17]. We enforce forward invariance of static and dynamic safe sets and convergence to a given goal set within a user-defined time in the presence of input constraints. We use robust variants of control barrier functions (CBF) and fixed-time control Lyapunov functions (FxT-CLF) to incorporate a class of additive disturbances in the system dynamics, and state-estimation errors. To solve the underlying constrained control problem, we formulate a quadratic program and use the proposed robust CBF-FxT-CLF conditions to compute the control input. We showcase the efficacy of the proposed method on a numerical case study involving multiple underactuated marine vehicles.

4 Project Management

The research program resulted in 5 journal publications and 12 conference papers. It supported fully one PhD student, who graduated in April 2021, and partially three postdoctoral researchers.

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