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Nonlinear Optics in Parity-time-symmetric and Quasi-parity-time-symmetric Systems

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14. ABSTRACT
The objective of this proposal is to investigate novel behaviors of nonlinear optics in parity-time (PT) symmetric and quasi-PT symmetric systems and to explore their practical applications in single-mode lasers. Nonlinear optics in PT-symmetric complex media, i.e., optical media with spatially balanced gain and loss, is currently one of the research frontiers in optics and applied mathematics. PT optics exhibits a wide array of novel phenomena, which are inspiring new optical applications such as PT-symmetric single-mode lasers and unidirectional reflectionless optical devices. Thus, deep understanding of nonlinear light behaviors in PT-symmetric and PT-related optical media is important for both theoretical and practical reasons. In this project, a number of outstanding theoretical and practical problems on nonlinear optics in PT-symmetric and quasi-PT-symmetric complex media will be analyzed. Specifically, theoretical questions such as the general forms of one- and multi-dimensional PT-symmetric potentials which admit symmetry breaking of solitons, general forms of non-PT-symmetric complex potentials which admit soliton families, stability properties of these solitons, and extension of PT symmetry to multi-light-frequency systems will be investigated. In addition, practical questions such as the nonlinear operation of realistic PT-symmetric single-mode lasers will be theoretically analyzed. The amount of output powers for different geometries of PT-laser cavities will be computed, and the geometry which produces the most power will be identified. These problems will be investigated through a combination of mathematical modeling, asymptotic analysis, numerical computations, and comparison with real experiments. These studies will not only reveal and explain new physical phenomena in PT-related optical media, but also provide guidance to physical applications such as PT-symmetric single-mode lasers.

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Program Manager: Dr. Arje Nachman

I. Summary of the P.I.'s activities in this period

In this funding period, the P.I. investigated linear and nonlinear light propagation in parity-time (PT) symmetric and non-PT-symmetric systems. These systems are open systems (with gain and loss), and they exhibit novel properties that are inspiring new applications such as PT lasers and ultra-sensitive sensors. The focus of the P.I.'s work on this subject was to discover and analytically explain new properties of these systems.

In this funding period, the P.I. also investigated nonlinear light behaviors in a topological photonic lattice, in collaboration with an experimentalist in San Francisco State University. Topological photonics is interesting because of its potential to realize robust optical circuitry against disorder. Our focus on this subject was to explore how nonlinearity affects topological properties of these photonic systems.

In addition, in this funding period, the P.I. investigated rogue wave phenomena in the PT-symmetric nonlinear Schrodinger (NLS) equation, the classical NLS equation, and several other mathematical models which govern optical and other physical systems. Rogue waves are spontaneous large wave excitation that appears with little warning and can cause extreme damage, and the focus of our work was to analytically explain and predict rogue events in various physical systems.

During this period, the P.I. published 9 refereed journal articles in leading optics and applied mathematics journals, including one article in Optics Letters (2019) which was designated as "Editor's Pick" for its novelty and significance.

The following list of people were also involved in this project:

Bo Yang (University of Vermont, USA, postdoc)
Zhigang Chen (San Francesco State University, USA, experimental collaborator)
Bijan Bagchi (Shiv Nadar University, India, theoretical collaborator)
Junchao Chen (Lishui University, China, theoretical collaborator)

II. Summary of the P.I.'s research results in this period

1. Investigation of linear and nonlinear light propagation in PT-symmetric and non-PT-symmetric systems (3 papers)

   On this subject, the P.I. published 3 papers, and results of these papers are summarized below.

   (a) In a Opt. Lett. (2019) paper, the P.I. discovered a new type of symmetry-breaking bifurcation of solitons in optical systems with PT-symmetric potentials. In this bifurcation, the two bifurcated branches of asymmetric solitons exhibit opposite stability, which contrasts all previous symmetry-breaking bifurcations in conservative and non-conservative systems. We showed that this novel symmetry-breaking bifurcation can be exploited to achieve unidirectional propagation of high-intensity light beams in PT-symmetric potentials.
   (b) In a J. Math. Phys. (2020) paper (jointly with B. Bagchi), we derived new families of non-PT-symmetric complex potentials with all-real spectra by the supersymmetry method and the pseudo-Hermiticity method. Different from previous supersymmetry derivations of potentials with real spectra, our derivation does not utilize discrete eigenmodes of base potentials. As a result, our new potentials feature explicit analytical expressions, which contain free functions. With the pseudo-Hermiticity method, we derived a new class of non-parity-time-symmetric complex potentials with free functions and constants, whose spectra can be all-real. Tuning these free functions and constants, phase transition can also be induced.
   (c) In a Stud. Appl. Math. (2021) paper, the P.I. theoretically explained a mysterious phenomenon, namely, the existence of soliton families in a certain type of non-PT-symmetric optical systems. By utilizing the conservation law of the underlying non-Hamiltonian wave system, we convert the complex soliton equation into a new real system. For this new real system, we perturbatively construct a continuous family of low-amplitude solitons bifurcating from a linear eigenmode to all orders of the small soliton amplitude. Hence, the emergence of soliton families in these non-PT-symmetric complex potentials is analytically explained.

2. Investigation of nonlinear effects on topological photonic systems (1 paper)

   On this subject, the P.I. published one paper (Opt. Lett. 2020), in collaboration with the experimentalist Z. Chen and his research group. In this joint work, we studied both theoretically and experimentally the effect of nonlinearity on topologically protected linear interface modes in a photonic Su–Schrieffer–Heeger (SSH) lattice. We showed that under nonlinearity, the linear topological mode of the SSH lattice turns into a family of topological gap solitons. These solitons are stable. However, they exhibit only a low amplitude and power and are thus weakly nonlinear. Therefore, if the initial beam has modest or high power, it will either delocalize, or evolve into a soliton not belonging to the family of topological gap solitons. These theoretical predictions are observed in our experiments with optically induced SSH-type photorefractive lattices.

3. Investigation of rogue waves in various physical models (5 papers)

   Rogue waves generally "appear from nowhere and disappear with no trace". Thus, their peculiar wave dynamics has attracted the attention of the scientific community and has been heavily studied. On this subject, the P.I. has published six papers, in collaboration with my postdoc B. Yang and visitor J. Chen. Our results are summarized as follows.

   (a) In three papers (J. Math. Anal. Appl. 2020; J. Nonl. Sci. 2020 and IMA J. Appl. Math. 2021), we derived general rogue waves in physical models such as the nonlocal PT-symmetric NLS equation, the generalized derivative NLS equations and the three-wave resonant interaction system. Our derivation was achieved by improving the previous bilinear method. In all these models, we discovered novel rogue dynamics with distinctive new patterns.
   (b) In two papers [Physica D 2021(a, b)], we showed that universal rogue wave patterns exist in integrable systems. These rogue patterns comprise fundamental rogue waves arranged in shapes such as a triangle, pentagon, and heptagon, with a possible lower-order rogue wave at the center. These patterns appear when one of the internal parameters in bilinear expressions of rogue waves gets large. Analytically, these patterns are determined by root structures of the Yablonskii–Vorob'ev polynomial hierarchy through a linear transformation. As examples, these universal rogue patterns are explicitly determined and graphically illustrated for the NLS equation, the generalized derivative NLS equations, the Boussinesq equation, and the Manakov system.

III. List of publications in this period

Journal articles:

1. J. Yang, "Symmetry breaking with opposite stability between bifurcated asymmetric solitons in parity-time-symmetric potentials", Opt. Lett. 44, 2641-2644 (2019) [Editors` Pick].
2. B. Yang and J. Yang, "On general rogue waves in the parity-time-symmetric nonlinear Schroedinger equation", J. Math. Anal. Appl. 487, 124023 (2020).
3. B. Bagchi and J. Yang, "New families of non-parity-time-symmetric complex potentials with all-real spectra", J. Math. Phys. 61, 063506 (2020).
4. B. Yang, J. Chen and J. Yang, "Rogue waves in the generalized derivative nonlinear Schrodinger equations", J. Nonl. Sci. 30, 3027-3056 (2020).
5. M. Guo, S. Xia, N. Wang, D. Song, Z. Chen and J. Yang, "Weakly nonlinear topological gap solitons in Su-Schrieffer-Heeger photonic lattices", Opt. Lett. 45, 6466-6469 (2020).
6. B. Yang and J. Yang, "Rogue wave patterns in the nonlinear Schrodinger equation", Physica D 419, 132850 (2021).

7. B. Yang and J. Yang, "General rogue waves in the three-wave resonant interaction systems", IMA J. Appl. Math. 86, 378-425 (2021).
8. J. Yang, "Analytical construction of soliton families in one- and two-dimensional nonlinear Schrodinger equations with non-parity-time-symmetric complex potentials", Stud. Appl. Math. 147, 4-31 (2021).
9. B. Yang and J. Yang, "Universal rogue wave patterns associated with the Yablonskii-Vorob'ev polynomial hierarchy", Physica D 425, 132958 (2021).