## The GraphBLAS C API Specification $^{\dagger}:$

Version 2.0.0

- Benjamin Brock, Aydın Buluç, Timothy Mattson, Scott McMillan, José Moreira
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## $_{\scriptscriptstyle 12}$ Chapter 1

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## $_{\scriptscriptstyle 13}$ Introduction

The GraphBLAS standard defines a set of matrix and vector operations based on semiring algebraic structures. These operations can be used to express a wide range of graph algorithms. This document defines the C binding to the GraphBLAS standard. We refer to this as the GraphBLAS CAPI (Application Programming Interface).

The GraphBLAS C API is built on a collection of objects exposed to the C programmer as opaque data types. Functions that manipulate these objects are referred to as *methods*. These methods fully define the interface to GraphBLAS objects to create or destroy them, modify their contents, and copy the contents of opaque objects into non-opaque objects; the contents of which are under direct control of the programmer.

The GraphBLAS C API is designed to work with C99 (ISO/IEC 9899:199) extended with *static* type-based and number of parameters-based function polymorphism, and language extensions on par with the \_Generic construct from C11 (ISO/IEC 9899:2011). Furthermore, the standard assumes programs using the GraphBLAS C API will execute on hardware that supports floating point arithmetic such as that defined by the IEEE 754 (IEEE 754-2008) standard.

The GraphBLAS C API assumes programs will run on a system that supports acquire-release memory orders. This is needed to support the memory models required for multithreaded execution as described in section 2.5.2.

Implementations of the GraphBLAS C API will target a wide range of platforms. We expect cases will arise where it will be prohibitive for a platform to support a particular type or a specific parameter for a method defined by the GraphBLAS C API. We want to encourage implementors to support the GraphBLAS C API even when such cases arise. Hence, an implementation may still call itself "conformant" as long as the following conditions hold.

- Every method and operation from chapter 4 is supported for the vast majority of cases.
- Any cases not supported must be documented as an implementation-defined feature of the GraphBLAS implementation. Unsupported cases must be caught as an API error (section 2.6) with the parameter GrB\_NOT\_IMPLEMENTED returned by the associated method call.
- It is permissible to omit the corresponding nonpolymorphic methods from chapter 5 when it

is not possible to express the signature of that method.

The number of allowed omitted cases is vague by design. We cannot anticipate the features of target platforms, on the market today or in the future, that might cause problems for the GraphBLAS specification. It is our expectation, however, that such omitted cases would be a minuscule fraction of the total combination of methods, types, and parameters defined by the GraphBLAS C API specification.

The remainder of this document is organized as follows:

- Chapter 2: Basic Concepts
- Chapter 3: Objects
- Chapter 4: Methods
- Chapter 5: Nonpolymorphic interface
- Appendix A: Revision history
- Appendix B: Non-opaque data format definitions
- Appendix C: Examples

## <sup>285</sup> Chapter 2

# Basic concepts

- The GraphBLAS C API is used to construct graph algorithms expressed "in the language of linear
- <sup>288</sup> algebra." Graphs are expressed as matrices, and the operations over these matrices are generalized
- through the use of a semiring algebraic structure.
- In this chapter, we will define the basic concepts used to define the GraphBLAS C API. We provide
- 291 the following elements:
- Glossary of terms and notation used in this document.
- Algebraic structures and associated arithmetic foundations of the API.
- Functions that appear in the GraphBLAS algebraic structures and how they are managed.
- Domains of elements in the GraphBLAS.
- Indices, index arrays, scalar arrays, and external matrix formats used to expose the contents of GraphBLAS objects.
- The GraphBLAS opaque objects.
- The execution and error models implied by the GraphBLAS C specification.
- Enumerations used by the API and their values.

## $_{\scriptscriptstyle 301}$ 2.1 Glossary

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### 302 2.1.1 GraphBLAS API basic definitions

- application: A program that calls methods from the GraphBLAS C API to solve a problem.
- GraphBLAS C API: The application programming interface that fully defines the types, objects, literals, and other elements of the C binding to the GraphBLAS.

- function: Refers to a named group of statements in the C programming language. Methods, operators, and user-defined functions are typically implemented as C functions. When referring to the code programmers write, as opposed to the role of functions as an element of the GraphBLAS, they may be referred to as such.
  - method: A function defined in the GraphBLAS C API that manipulates GraphBLAS objects or other opaque features of the implementation of the GraphBLAS API.
- operator: A function that performs an operation on the elements stored in GraphBLAS matrices and vectors.
  - GraphBLAS operation: A mathematical operation defined in the GraphBLAS mathematical specification. These operations (not to be confused with operators) typically act on matrices and vectors with elements defined in terms of an algebraic semiring.

### 2.1.2 GraphBLAS objects and their structure

- non-opaque datatype: Any datatype that exposes its internal structure and can be manipulated directly by the user.
- opaque datatype: Any datatype that hides its internal structure and can be manipulated only through an API.
- GraphBLAS object: An instance of an opaque datatype defined by the GraphBLAS C API that is manipulated only through the GraphBLAS API. There are four kinds of GraphBLAS opaque objects: domains (i.e., types), algebraic objects (operators, monoids and semirings), collections (scalars, vectors, matrices and masks), and descriptors.
  - handle: A variable that holds a reference to an instance of one of the GraphBLAS opaque objects. The value of this variable holds a reference to a GraphBLAS object but not the contents of the object itself. Hence, assigning a value to another variable copies the reference to the GraphBLAS object of one handle but not the contents of the object.
- domain: The set of valid values for the elements stored in a GraphBLAS collection or operated on by a GraphBLAS operator. Note that some GraphBLAS objects involve functions that map values from one or more input domains onto values in an output domain. These GraphBLAS objects would have multiple domains.
  - collection: An opaque GraphBLAS object that holds a number of elements from a specified domain. Because these objects are based on an opaque datatype, an implementation of the GraphBLAS C API has the flexibility to optimize the data structures for a particular platform. GraphBLAS objects are often implemented as sparse data structures, meaning only the subset of the elements that have values are stored.
  - *implied zero*: Any element that has a valid index (or indices) in a GraphBLAS vector or matrix but is not explicitly identified in the list of elements of that vector or matrix. From a mathematical perspective, an *implied zero* is treated as having the value of the zero element of the relevant monoid or semiring. However, GraphBLAS operations are purposefully defined

- using set notation in such a way that it makes it unnecessary to reason about implied zeros.

  Therefore, this concept is not used in the definition of GraphBLAS methods and operators.
  - mask: An internal GraphBLAS object used to control how values are stored in a method's output object. The mask exists only inside a method; hence, it is called an *internal opaque object*. A mask is formed from the elements of a collection object (vector or matrix) input as a mask parameter to a method. GraphBLAS allows two types of masks:
    - 1. In the default case, an element of the mask exists for each element that exists in the input collection object when the value of that element, when cast to a Boolean type, evaluates to true.
    - 2. In the *structure only* case, masks have structure but no values. The input collection describes a structure whereby an element of the mask exists for each element stored in the input collection regardless of its value.
  - complement: The complement of a GraphBLAS mask, M, is another mask, M', where the elements of M' are those elements from M that do not exist.

### 2.1.3 Algebraic structures used in the GraphBLAS

- associative operator: In an expression where a binary operator is used two or more times consecutively, that operator is associative if the result does not change regardless of the way operations are grouped (without changing their order). In other words, in a sequence of binary operations using the same associative operator, the legal placement of parenthesis does not change the value resulting from the sequence operations. Operators that are associative over infinitely precise numbers (e.g., real numbers) are not strictly associative when applied to numbers with finite precision (e.g., floating point numbers). Such non-associativity results, for example, from roundoff errors or from the fact some numbers can not be represented exactly as floating point numbers. In the GraphBLAS specification, as is common practice in computing, we refer to operators as associative when their mathematical definition over infinitely precise numbers is associative even when they are only approximately associative when applied to finite precision numbers.
  - No GraphBLAS method will imply a predefined grouping over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.
- commutative operator: In an expression where a binary operator is used (usually two or more times consecutively), that operator is commutative if the result does not change regardless of the order the inputs are operated on.
  - No GraphBLAS method will imply a predefined ordering over any commutative operators. Implementations of the GraphBLAS are encouraged to exploit commutativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the commutative operations.

• GraphBLAS operators: Binary or unary operators that act on elements of GraphBLAS objects. GraphBLAS operators are used to express algebraic structures used in the GraphBLAS such as monoids and semirings. They are also used as arguments to several GraphBLAS methods. There are two types of GraphBLAS operators: (1) predefined operators found in Table 3.5 and (2) user-defined operators created using GrB\_UnaryOp\_new() or GrB\_BinaryOp\_new() (see Section 4.2.1).

- monoid: An algebraic structure consisting of one domain, an associative binary operator, and the identity of that operator. There are two types of GraphBLAS monoids: (1) predefined monoids found in Table 3.7 and (2) user-defined monoids created using GrB\_Monoid\_new() (see Section 4.2.1).
  - semiring: An algebraic structure consisting of a set of allowed values (the domain), a commutative and associative binary operator called addition, a binary operator called multiplication (where multiplication distributes over addition), and identities over addition ( $\theta$ ) and multiplication (1). The additive identity is an annihilator over multiplication.
- GraphBLAS semiring: is allowed to diverge from the mathematically rigorous definition of a semiring since certain combinations of domains, operators, and identity elements are useful in graph algorithms even when they do not strictly match the mathematical definition of a semiring. There are two types of GraphBLAS semirings: (1) predefined semirings found in Tables 3.8 and 3.9, and (2) user-defined semirings created using GrB\_Semiring\_new() (see Section 4.2.1).
- index unary operator: A variation of the unary operator that operates on elements of GraphBLAS vectors and matrices along with the index values representing their location in the objects. There are predefined index unary operators found in Table 3.6), and user-defined operators created using GrB\_IndexUnaryOp\_new (see Section 4.2.1).

## 5 2.1.4 The execution of an application using the GraphBLAS C API

- program order: The order of the GraphBLAS method calls in a thread, as defined by the text of the program.
  - host programming environment: The GraphBLAS specification defines an API. The functions from the API appear in a program. This program is written using a programming language and execution environment defined outside of the GraphBLAS. We refer to this programming environment as the "host programming environment".
- execution time: time expended while executing instructions defined by a program. This term is specifically used in this specification in the context of computations carried out on behalf of a call to a GraphBLAS method.
  - sequence: A GraphBLAS application uniquely defines a directed acyclic graph (DAG) of GraphBLAS method calls based on their program order. At any point in a program, the state of any GraphBLAS object is defined by a subgraph of that DAG. An ordered collection of GraphBLAS method calls in program order that defines that subgraph for a particular object is the sequence for that object.

• complete: A GraphBLAS object is complete when it can be used in a happens-before relationship with a method call that reads the variable on another thread. This concept is used when reasoning about memory orders in multithreaded programs. A GraphBLAS object defined on one thread that is complete can be safely used as an IN or INOUT argument in a method-call on a second thread assuming the method calls are correctly synchronized so the definition on the first thread happens-before it is used on the second thread. In blocking-mode, an object is complete after a GraphBLAS method call that writes to that object returns. In nonblocking-mode, an object is complete after a call to the GrB\_wait() method with the GrB\_COMPLETE parameter.

- materialize: A GraphBLAS object is materialized when it is (1) complete, (2) the computations defined by the sequence that define the object have finished (either fully or stopped at an error) and will not consume any additional computational resources, and (3) any errors associated with that sequence are available to be read according to the GraphBLAS error model. A GraphBLAS object that is never loaded into a non-opaque data structure may potentially never be materialized. This might happen, for example, if the operations associated with the object are fused or otherwise changed by the runtime system that supports the implementation of the GraphBLAS C API. An object can be materialized by a call to the materialize mode of the GrB\_wait() method.
- context: An instance of the GraphBLAS C API implementation as seen by an application. An application can have only one context between the start and end of the application. A context begins with the first thread that calls GrB\_init() and ends with the first thread to call GrB\_finalize(). It is an error for GrB\_init() or GrB\_finalize() to be called more than one time within an application. The context is used to constrain the behavior of an instance of the GraphBLAS C API implementation and support various execution strategies. Currently, the only supported constraints on a context pertain to the mode of program execution.
- program execution mode: Defines how a GraphBLAS sequence executes, and is associated with the context of a GraphBLAS C API implementation. It is set by an application with its call to GrB\_init() to one of two possible states. In blocking mode, GraphBLAS methods return after the computations complete and any output objects have been materialized. In nonblocking mode, a method may return once the arguments are tested as consistent with the method (i.e., there are no API errors), and potentially before any computation has taken place.

#### 2.1.5 GraphBLAS methods: behaviors and error conditions

- *implementation-defined behavior*: Behavior that must be documented by the implementation and is allowed to vary among different compliant implementations.
- undefined behavior: Behavior that is not specified by the GraphBLAS C API. A conforming implementation is free to choose results delivered from a method whose behavior is undefined.
  - thread-safe: Consider a function called from multiple threads with arguments that do not overlap in memory (i.e. the argument lists do not share memory). If the function is thread-safe

then it will behave the same when executed concurrently by multiple threads or sequentially on a single thread.

- dimension compatible: GraphBLAS objects (matrices and vectors) that are passed as parameters to a GraphBLAS method are dimension (or shape) compatible if they have the correct number of dimensions and sizes for each dimension to satisfy the rules of the mathematical definition of the operation associated with the method. If any dimension compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB\_DIMENSION\_MISMATCH error is returned.
- domain compatible: Two domains for which values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other, and a domain from a user-defined type is only compatible with itself. If any domain compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB\_DOMAIN\_MISMATCH error is returned.

## **2.2** Notation

Notation	Description
$D_{out}, D_{in}, D_{in_1}, D_{in_2}$	Refers to output and input domains of various GraphBLAS operators.
$\mathbf{D}_{out}(*), \mathbf{D}_{in}(*),$	Evaluates to output and input domains of GraphBLAS operators (usually
$\mathbf{D}_{in_1}(*), \mathbf{D}_{in_2}(*)$	a unary or binary operator, or semiring).
$\mathbf{D}(*)$	Evaluates to the (only) domain of a GraphBLAS object (usually a monoi
, ,	vector, or matrix).
f	An arbitrary unary function, usually a component of a unary operator.
$\mathbf{f}(F_u)$	Evaluates to the unary function contained in the unary operator given the argument.
$\odot$	An arbitrary binary function, usually a component of a binary operator.
⊙(*)	Evaluates to the binary function contained in the binary operator or mono
O(1)	given as the argument.
$\otimes$	Multiplicative binary operator of a semiring.
	Additive binary operator of a semiring.
$\oplus$	Evaluates to the multiplicative binary operator of the semiring given as the
$\bigotimes(S)$	
$\Phi(c)$	argument.
$\bigoplus(S)$	Evaluates to the additive binary operator of the semiring given as the arg
0(.)	ment.
0(*)	The identity of a monoid, or the additive identity of a GraphBLAS semirin
$\mathbf{L}(*)$	The contents (all stored values) of the vector or matrix GraphBLAS object
	For a vector, it is the set of (index, value) pairs, and for a matrix it is t
(1)	set of (row, col, value) triples.
$\mathbf{v}(i)$ or $v_i$	The $i^{th}$ element of the vector $\mathbf{v}$ .
size(v)	The size of the vector $\mathbf{v}$ .
$ind(\mathbf{v})$	The set of indices corresponding to the stored values of the vector $\mathbf{v}$ .
$\mathbf{nrows}(\mathbf{A})$	The number of rows in the $\mathbf{A}$ .
$\mathbf{ncols}(\mathbf{A})$	The number of columns in the $\mathbf{A}$ .
indrow(A)	The set of row indices corresponding to rows in <b>A</b> that have stored value
$\mathbf{indcol}(\mathbf{A})$	The set of column indices corresponding to columns in <b>A</b> that have stor
	values.
$\mathbf{ind}(\mathbf{A})$	The set of $(i, j)$ indices corresponding to the stored values of the matrix.
$\mathbf{A}(i,j)$ or $A_{ij}$	The element of <b>A</b> with row index $i$ and column index $j$ .
$\mathbf{A}(:,j)$	The $j^{th}$ column of matrix <b>A</b> .
$\mathbf{A}(i,:)$	The $i^{th}$ row of matrix <b>A</b> .
$\mathbf{A}^T$	The transpose of matrix $\mathbf{A}$ .
$\neg \mathbf{M}$	The complement of $M$ .
$s(\mathbf{M})$	The structure of $M$ .
$rac{\mathrm{s}(\mathbf{M})}{\widetilde{\mathbf{t}}}$	A temporary object created by the GraphBLAS implementation.
< type >	A method argument type that is void * or one of the types from Table 3.
GrB_ALL	A method argument literal to indicate that all indices of an input arrangement
_	should be used.
GrB_Type	A method argument type that is either a user defined type or one of the
_ 7F -	types from Table 3.2.
	• -
GrB_Object	A method argument type referencing any of the GraphBLAS object types

### 2.3 Mathematical foundations

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Graphs can be represented in terms of matrices. The values stored in these matrices correspond to attributes (often weights) of edges in the graph. Likewise, information about vertices in a graph are stored in vectors. The set of valid values that can be stored in either matrices or vectors is referred to as their domain. Matrices are usually sparse because the lack of an edge between two vertices means that nothing is stored at the corresponding location in the matrix. Vectors may be sparse or dense, or they may start out sparse and become dense as algorithms traverse the graphs.

Operations defined by the GraphBLAS C API specification operate on these matrices and vectors to carry out graph algorithms. These GraphBLAS operations are defined in terms of GraphBLAS semiring algebraic structures. Modifying the underlying semiring changes the result of an operation to support a wide range of graph algorithms. Inside a given algorithm, it is often beneficial to change the GraphBLAS semiring that applies to an operation on a matrix. This has two implications for the C binding of the GraphBLAS API.

First, it means that we define a separate object for the semiring to pass into methods. Since in many cases the full semiring is not required, we also support passing monoids or even binary operators, which means the semiring is implied rather than explicitly stated.

Second, the ability to change semirings impacts the meaning of the *implied zero* in a sparse rep-490 resentation of a matrix or vector. This element in real arithmetic is zero, which is the identity of 491 the addition operator and the annihilator of the multiplication operator. As the semiring changes, 492 this implied zero changes to the identity of the addition operator and the annihilator (if present) 493 of the *multiplication* operator for the new semiring. Nothing changes regarding what is stored in 494 the sparse matrix or vector, but the implied zeros within them change with respect to a particular 495 operation. In all cases, the nature of the implied zero does not matter since the GraphBLAS C 496 API requires that implementations treat them as nonexistent elements of the matrix or vector. 497

As with matrices and vectors, GraphBLAS semirings have domains associated with their inputs and outputs. The semirings in the GraphBLAS C API are defined with two domains associated with the input operands and one domain associated with output. When used in the GraphBLAS C API these domains may not match the domains of the matrices and vectors supplied in the operations.

In this case, only valid domain compatible casting is supported by the API.

The mathematical formalism for graph operations in the language of linear algebra often assumes that we can operate in the field of real numbers. However, the GraphBLAS C binding is designed for implementation on computers, which by necessity have a finite number of bits to represent numbers. Therefore, we require a conforming implementation to use floating point numbers such as those defined by the IEEE-754 standard (both single- and double-precision) wherever real numbers need to be represented. The practical implications of these finite precision numbers is that the result of a sequence of computations may vary from one execution to the next as the grouping of operands (because of associativity) within the operations changes. While techniques are known to reduce these effects, we do not require or even expect an implementation to use them as they may add

<sup>&</sup>lt;sup>1</sup>More information on the mathematical foundations can be found in the following paper: J. Kepner, P. Aaltonen, D. Bader, A. Buluç, F. Franchetti, J. Gilbert, D. Hutchison, M. Kumar, A. Lumsdaine, H. Meyerhenke, S. McMillan, J. Moreira, J. Owens, C. Yang, M. Zalewski, and T. Mattson. 2016, September. Mathematical foundations of the GraphBLAS. In 2016 IEEE High Performance Extreme Computing Conference (HPEC) (pp. 1-9). IEEE.

Table 2.1:	Types of	GraphBLAS	opaque	objects.

GrB_Object types	Description
GrB_Type	Scalar type.
GrB_UnaryOp	Unary operator.
$GrB\_IndexUnaryOp$	Unary operator, that operates on a single value and its location index values.
GrB_BinaryOp	Binary operator.
GrB_Monoid	Monoid algebraic structure.
GrB_Semiring	A GraphBLAS semiring algebraic structure.
GrB_Scalar	One element; could be empty.
GrB_Vector	One-dimensional collection of elements; can be sparse.
GrB_Matrix	Two-dimensional collection of elements; typically sparse.
GrB_Descriptor	Descriptor object, used to modify behavior of methods (specifically
	GraphBLAS operations).
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considerable overhead. In most cases, these roundoff errors are not significant. When they are significant, the problem itself is ill-conditioned and needs to be reformulated.

## 514 2.4 GraphBLAS opaque objects

Objects defined in the GraphBLAS standard include types (the domains of elements), collections of elements (matrices, vectors, and scalars), operators on those elements (unary, index unary, and binary operators), algebraic structures (semirings and monoids), and descriptors. GraphBLAS objects are defined as opaque types; that is, they are managed, manipulated, and accessed solely through the GraphBLAS application programming interface. This gives an implementation of the GraphBLAS C specification flexibility to optimize objects for different scenarios or to meet the needs of different hardware platforms.

A GraphBLAS opaque object is accessed through its *handle*. A handle is a variable that references an instance of one of the types from Table 2.1. An implementation of the GraphBLAS specification has a great deal of flexibility in how these handles are implemented. All that is required is that the handle corresponds to a type defined in the C language that supports assignment and comparison for equality. The GraphBLAS specification defines a literal GrB\_INVALID\_HANDLE that is valid for each type. Using the logical equality operator from C, it must be possible to compare a handle to GrB\_INVALID\_HANDLE to verify that a handle is valid.

Every GraphBLAS object has a *lifetime*, which consists of the sequence of instructions executed in program order between the *creation* and the *destruction* of the object. The GraphBLAS C API predefines a number of these objects which are created when the GraphBLAS context is initialized by a call to GrB\_init and are destroyed when the GraphBLAS context is terminated by a call to GrB\_finalize.

An application using the GraphBLAS API can create additional objects by declaring variables of the appropriate type from Table 2.1 for the objects it will use. Before use, the object must be initialized

with a call call to one of the object's respective *constructor* methods. Each kind of object has at least one explicit constructor method of the form GrB\_\*\_new where '\*' is replaced with the type of object (e.g., GrB\_Semiring\_new). Note that some objects, especially collections, have additional constructor methods such as duplication, import, or descrialization. Objects explicitly created by a call to a constructor should be destroyed by a call to GrB\_free. The behavior of a program that calls GrB\_free on a pre-defined object is undefined.

These constructor and destructor methods are the only methods that change the value of a handle.

Hence, objects changed by these methods are passed into the method as pointers. In all other
cases, handles are not changed by the method and are passed by value. For example, even when
multiplying matrices, while the contents of the output product matrix changes, the handle for that
matrix is unchanged.

Several GraphBLAS constructor methods take other objects as input arguments and use these objects to create a new object. For all these methods, the lifetime of the created object must end strictly before the lifetime of any dependent input objects. For example, a vector constructor GrB\_Vector\_new takes a GrB\_Type object as input. That type object must not be destroyed until after the created vector is destroyed. Similarly, a GrB\_Semiring\_new method takes a monoid and a binary operator as inputs. Neither of these can be destroyed until after the created semiring is destroyed.

Note that some constructor methods like GrB\_Vector\_dup and GrB\_Matrix\_dup behave differently.

In these cases, the input vector or matrix can be destroyed as soon as the call returns. However,
the original type object used to create the input vector or matrix cannot be destroyed until after
the vector or matrix created by GrB\_Vector\_dup or GrB\_Matrix\_dup is destroyed. This behavior
must hold for any chain of duplicating constructors.

Programmers using GraphBLAS handles must be careful to distinguish between a handle and the object manipulated through a handle. For example, a program may declare two GraphBLAS objects of the same type, initialize one, and then assign it to the other variable. That assignment, however, only assigns the handle to the variable. It does not create a copy of that variable (to do that, one would need to use the appropriate duplication method). If later the object is freed by calling GrB\_free with the first variable, the object is destroyed and the second variable is left referencing an object that no longer exists (a so-called "dangling handle").

In addition to opaque objects manipulated through handles, the GraphBLAS C API defines an additional opaque object as an internal object; that is, the object is never exposed as a variable within an application. This opaque object is the mask used to control which computed values can be stored in the output operand of a *GraphBLAS operation*. Masks are described in Section 3.5.4.

## $_{\circ}$ 2.5 Execution model

A program using the GraphBLAS C API is called a GraphBLAS application. The application constructs GraphBLAS objects, manipulates them to implement a graph algorithm, and then extracts values from the GraphBLAS objects to produce the results for that algorithm. Functions defined within the GraphBLAS C API that manipulate GraphBLAS objects are called *methods*. If the method corresponds to one of the operations defined in the GraphBLAS mathematical specifica-

576 tion, we refer to the method as an operation.

The GraphBLAS application specifies an ordered collection of GraphBLAS method calls defined by the order they appear in the text of the program (the *program order*). These define a directed acyclic graph (DAG) where nodes are GraphBLAS method calls and edges are dependencies between method calls.

Each method call in the DAG uniquely and unambiguously defines the output GraphBLAS objects as long as there are no execution errors that put objects in an invalid state (see Section 2.6). An ordered collection of method calls, a subgraph of the overall DAG for an application, defines the state of a GraphBLAS object at any point in a program. This ordered collection is the *sequence* for that object.

Since the GraphBLAS execution is defined in terms of a DAG and the GraphBLAS objects are opaque, the semantics of the GraphBLAS specification affords an implementation considerable flexibility to optimize performance. A GraphBLAS implementation can defer execution of nodes in the DAG, fuse nodes, or even replace whole subgraphs within the DAG to optimize performance. We discuss this topic further in section 2.5.1 when we describe *blocking* and *non-blocking* execution modes.

A correct GraphBLAS application must be *race-free*. This means that the DAG produced by an application and the results produced by execution of that DAG must be the same regardless of how the threads are scheduled for execution. It is the application programmer's responsibility to control memory orders and establish the required synchronized-with relationships to assure race-free execution of a multi-threaded GraphBLAS application. Writing race-free GraphBLAS applications is discussed further in Section 2.5.2.

#### 598 2.5.1 Execution modes

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The execution of the DAG defined by a GraphBLAS application depends on the execution mode of the GraphBLAS program. There are two modes: blocking and nonblocking.

- blocking: In blocking mode, each method finishes the GraphBLAS operation defined by the method and all output GraphBLAS objects are materialized before proceeding to the next statement. Even mechanisms that break the opaqueness of the GraphBLAS objects (e.g., performance monitors, debuggers, memory dumps) will observe that the operation has finished.
- nonblocking: In nonblocking mode, each method may return once the input arguments have been inspected and verified to define a well formed GraphBLAS operation. (That is, there are no API errors; see Section 2.6.) The GraphBLAS method may not have finished, but the output object is ready to be used by the next GraphBLAS method call. If needed, a call to GrB\_wait with GrB\_COMPLETE or GrB\_MATERIALIZE can be used to force the sequence for a GraphBLAS object (obj) to finish its execution.

The execution mode is defined in the GraphBLAS C API when the context of the library invocation is defined. This occurs once before any GraphBLAS methods are called with a call to the

GrB\_init() function. This function takes a single argument of type GrB\_Mode with values shown in Table 3.1(a).

An application executing in nonblocking mode is not required to return immediately after input arguments have been verified. A conforming implementation of the GraphBLAS C API running in nonblocking mode may choose to execute as if in blocking mode. A sequence of operations in nonblocking mode where every GraphBLAS operation with output object obj is followed by a GrB\_wait(obj, GrB\_MATERIALIZE) call is equivalent to the same sequence in blocking mode with GrB\_wait(obj, GrB\_MATERIALIZE) calls removed.

Nonblocking mode allows for any execution strategy that satisfies the mathematical definition of the sequence. The methods can be placed into a queue and deferred. They can be chained together and fused (e.g., replacing a chained pair of matrix products with a matrix triple product). Lazy evaluation, greedy evaluation, and asynchronous execution are all valid as long as the final result agrees with the mathematical definition provided by the sequence of GraphBLAS method calls appearing in program order.

Blocking mode forces an implementation to carry out precisely the GraphBLAS operations defined by the methods and to complete each and every method call individually. It is valuable for debugging or in cases where an external tool such as a debugger needs to evaluate the state of memory during a sequence of operations.

In a sequence of operations free of execution errors, and with input objects that are well-conditioned, the results from blocking and nonblocking modes should be identical outside of effects due to roundoff errors associated with floating point arithmetic. Due to the great flexibility afforded to an implementation when using nonblocking mode, we expect execution of a sequence in nonblocking mode to potentially complete execution in less time.

It is important to note that, processing of nonopaque objects is never deferred in GraphBLAS.
That is, methods that consume nonopaque objects (e.g., GrB\_Matrix\_build(), Section 4.2.4.9) and
methods that produce nonopaque objects (e.g., GrB\_Matrix\_extractTuples(), Section 4.2.4.13) always finish consuming or producing those nonopaque objects before returning regardless of the
execution mode.

Finally, after all GraphBLAS method calls have been made, the context is terminated with a call to GrB\_finalize(). In the current version of the GraphBLAS C API, the context can be set only once in the execution of a program. That is, after GrB\_finalize() is called, a subsequent call to GrB\_init() is not allowed.

#### <sub>6</sub> 2.5.2 Multi-threaded execution

The GraphBLAS C API is designed to work with applications that utilize multiple threads executing within a shared address space. This specification does not define how threads are created, managed and synchronized. We expect the host programming environment to provide those services.

A conformant implementation of the GraphBLAS must be *thread safe*. A GraphBLAS library is thread safe when independent method calls (i.e., GraphBLAS objects are not shared between method calls) from multiple threads in a race-free program return the same results as would follow

from their sequential execution in some interleaved order. This is a common requirement in software libraries.

Thread safety applies to the behavior of multiple independent threads. In the more general case for multithreading, threads are not independent; they share variables and mix read and write operations to those variables across threads. A memory consistency model defines which values can be returned when reading an object shared between two or more threads. The GraphBLAS specification does not define its own memory consistency model. Instead the specification defines what must be done by a programmer calling GraphBLAS methods and by the implementor of a GraphBLAS library so an implementation of the GraphBLAS specification can work correctly with the memory consistency model for the host environment.

A memory consistency model is defined in terms of happens-before relations between methods in different threads. The defining case is a method that writes to an object on one thread that is read (i.e., used as an IN or INOUT argument) in a GraphBLAS method on a different thread. The following steps must occur between the different threads.

• A sequence of GraphBLAS methods results in the definition of the GraphBLAS object.

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- The GraphBLAS object is put into a state of completion by a call to GrB\_wait() with the GrB\_COMPLETE parameter (see Table 3.1(b)). A GraphBLAS object is said to be *complete* when it can be safely used as an IN or INOUT argument in a GraphBLAS method call from a different thread.
- Completion happens before a synchronized-with relation that executes with *at least* a release memory order.
  - A synchronized-with relation on the other thread executes with at least an acquire memory order.
- This synchronized-with relation happens-before the GraphBLAS method that reads the graph-BLAS object.

We use the phrase at least when talking about the memory orders to indicate that a stronger memory order such as sequential consistency can be used in place of the acquire-release order.

A program that violates these rules contains a data race. That is, its reads and writes are unordered across threads making the final value of a variable undefined. A program that contains a data race is invalid and the results of that program are undefined. We note that multi-threaded execution is compatible with both blocking and non-blocking modes of execution.

Completion is the central concept that allows GraphBLAS objects to be used in happens-before relations between threads. In earlier versions of GraphBLAS (1.X) completion was implied by any operation that produced non-opaque values from a GraphBLAS object. These operations are summarized in Table 2.2). In GraphBLAS 2.0, these methods no longer imply completion. This change was made since there are cases where the non-opaque value is needed but the object from which it is computed is not. We want implementations of the GraphBLAS to be able to exploit this case and not form the opaque object when that object is not needed.

Table 2.2: Methods that extract values from a GraphBLAS object that forcing completion of the operations contributing to that particular object in GraphBLAS 1.X. In GraphBLAS 2.0, these methods *do not* force completion.

Method	Section
GrB_Vector_nvals	4.2.3.6
GrB_Vector_extractElement	4.2.3.10
GrB_Vector_extractTuples	4.2.3.11
GrB_Matrix_nvals	4.2.4.8
GrB_Matrix_extractElement	4.2.4.12
$GrB$ _ $Matrix$ _ $extract$ $Tuples$	4.2.4.13
GrB_reduce (vector-scalar value variant)	4.3.10.2
${\sf GrB\_reduce}~({\rm matrix\text{-}scalar}~{\rm value}~{\rm variant})$	4.3.10.3

### $_{\scriptscriptstyle{99}}$ 2.6 Error $\operatorname{model}$

All GraphBLAS methods return a value of type GrB\_Info (an enum) to provide information available to the system at the time the method returns. The returned value will be one of the defined values shown in Table 3.13. The return values fall into three groups: informational, API errors, and execution errors. While API and execution errors take on negative values, informational return values listed in Table 3.13(a) are non-negative and include GrB\_SUCCESS (a value of 0) and GrB\_NO\_VALUE.

An API error (listed in Table 3.13(b)) means that a GraphBLAS method was called with parameters that violate the rules for that method. These errors are restricted to those that can be determined by inspecting the dimensions and domains of GraphBLAS objects, GraphBLAS operators, or the values of scalar parameters fixed at the time a method is called. API errors are deterministic and consistent across platforms and implementations. API errors are never deferred, even in nonblocking mode. That is, if a method is called in a manner that would generate an API error, it always returns with the appropriate API error value. If a GraphBLAS method returns with an API error, it is guaranteed that none of the arguments to the method (or any other program data) have been modified. The informational return value, GrB\_NO\_VALUE, is also deterministic and never deferred in nonblocking mode.

Execution errors (listed in Table 3.13(c)) indicate that something went wrong during the execution of a legal GraphBLAS method invocation. Their occurrence may depend on specifics of the execution environment and data values being manipulated. This does not mean that execution errors are the fault of the GraphBLAS implementation. For example, a memory leak could arise from an error in an application's source code (a "program error"), but it may manifest itself in different points of a program's execution (or not at all) depending on the platform, problem size, or what else is running at that time. Index out-of-bounds errors, for example, always indicate a program error.

If a GraphBLAS method returns with any execution error other than GrB\_PANIC, it is guaranteed that the state of any argument used as input-only is unmodified. Output arguments may be left in an invalid state, and their use downstream in the program flow may cause additional errors. If a

GraphBLAS method returns with a GrB\_PANIC execution error, no guarantees can be made about the state of any program data.

In nonblocking mode, execution errors can be deferred. A return value of GrB\_SUCCESS only guarantees that there are no API errors in the method invocation. If an execution error value is returned by a method with output object obj in nonblocking mode, it indicates that an error was found during execution of any of the pending operations on obj, up to and including the GrB\_wait() method (Section 4.2.7) call that completes those pending operations. When possible, that return value will provide information concerning the cause of the error.

As discussed in Section 4.2.7, a GrB\_wait(obj) on a specific GraphBLAS object obj completes all pending operations on that object. No additional errors on the methods that precede the call to GrB\_wait and have obj as an OUT or INOUT argument can be reported. From a GraphBLAS perspective, those methods are *complete*. Details on the guaranteed state of objects after a call to GrB\_wait can be found in Section 4.2.7.

After a call to any GraphBLAS method that modifies an opaque object, the program can re-732 trieve additional error information (beyond the error code returned by the method) though a call 733 to the function GrB\_error(), passing the method's output object as described in Section 4.2.8. 734 The function returns a pointer to a NULL-terminated string, and the contents of that string are 735 implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error 736 string. GrB error() is a thread-safe function, in the sense that multiple threads can call it simul-737 taneously and each will get its own error string back, referring to the object passed as an input 738 argument. 739

## $_{\scriptscriptstyle{740}}$ Chapter 3

# Objects

In this chapter, all of the enumerations, literals, data types, and predefined opaque objects defined in the GraphBLAS API are presented. Enumeration literals in GraphBLAS are assigned specific 743 values to ensure compatibility between different runtime library implementations. The chapter starts by defining the enumerations that are used by the init() and wait() methods. Then a num-745 ber of transparent (i.e., non-opaque) types that are used for interfacing with external data are 746 defined. Sections that follow describe the various types of opaque objects in GraphBLAS: types 747 (or domains), algebraic objects, collections and descriptors. Each of these sections also lists the 748 predefined instances of each opaque type that are required by the API. This chapter concludes with 749 a section on the definition for GrB Info enumeration that is used as the return type of all methods. 750

## $_{\scriptscriptstyle{751}}$ ${f 3.1}$ ${f Enumerations for init() and wait()}$

Table 3.1 lists the enumerations and the corresponding values used in the GrB\_init() method to set the execution mode and in the GrB\_wait() method for completing or materializing opaque objects.

## <sup>754</sup> 3.2 Indices, index arrays, and scalar arrays

In order to interface with third-party software (i.e., software other than an implementation of the GraphBLAS), operations such as GrB\_Matrix\_build (Section 4.2.4.9) and GrB\_Matrix\_extractTuples (Section 4.2.4.13) must specify how the data should be laid out in non-opaque data structures. To this end we explicitly define the types for indices and the arrays used by these operations.

For indices a typedef is used to give a GraphBLAS name to a concrete type. We define it as follows:

typedef uint64\_t GrB\_Index;

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The range of valid values for a variable of type GrB\_Index is [0, GrB\_INDEX\_MAX] where the largest index value permissible is defined with a macro, GrB\_INDEX\_MAX. For example:

An implementation is required to define and document this value.

An index array is a pointer to a set of GrB Index values that are stored in a contiguous block of 765 memory (i.e., GrB\_Index\*). Likewise, a scalar array is a pointer to a contiguous block of memory 766 storing a number of scalar values as specified by the user. Some GraphBLAS operations (e.g., 767 GrB assign) include an input parameter with the type of an index array. This input index array 768 selects a subset of elements from a GraphBLAS vector or matrix object to be used in the operation. 769 In these cases, the literal GrB\_ALL can be used in place of the index array input parameter to 770 indicate that all indices of the associated GraphBLAS vector or matrix object should be used. An 771 implementation of the GraphBLAS C API has considerable freedom in terms of how GrB\_ALL 772 is defined. Since GrB\_ALL is used as an argument for an array parameter, it must use a type 773 consistent with a pointer. GrB\_ALL must also have a non-null value to distinguish it from the 774 erroneous case of passing a NULL pointer as an array. 775

## 776 3.3 Types (domains)

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In GraphBLAS, domains correspond to the valid values for types from the host language (in our case, the C programming language). GraphBLAS defines a number of operators that take elements from one or more domains and produce elements of a (possibly) different domain. GraphBLAS also defines three kinds of collections: matrices, vectors and scalars. For any given collection, the elements of the collection belong to a *domain*, which is the set of valid values for the elements. For any variable or object V in GraphBLAS we denote as  $\mathbf{D}(V)$  the domain of V, that is, the set of possible values that elements of V can take.

Table 3.1: Enumeration literals and corresponding values input to various GraphBLAS methods.

(a) GrB\_Mode execution modes for the GrB\_init method.

Symbol	Value	Description
GrB_NONBLOCKING	0	Specifies the nonblocking mode context.
GrB_BLOCKING	1	Specifies the blocking mode context.

#### (b) GrB\_WaitMode wait modes for the GrB\_wait method.

Symbol	Value	Description
GrB_COMPLETE	0	The object is in a state where it can be used in a happens-
		before relation so that multithreaded programs can be properly synchronized.
GrB_MATERIALIZE	1	The object is <i>complete</i> , and in addition, all computation of the object is finished and any error information is available.

Table 3.2: Predefined  $GrB\_Type$  values, and the corresponding GraphBLAS domain suffixes, C type (for scalar parameters), and domains for GraphBLAS. The domain suffixes are used in place of I, F, and T in Tables 3.5, 3.6, 3.7, 3.8, and 3.9).

GrB_Type	Suffix	C type	Domain
GrB_BOOL	BOOL	bool	{false, true}
GrB_INT8	INT8	int8_t	$\mathbb{Z} \cap [-2^7, 2^7)$
GrB_UINT8	UINT8	uint8_t	$\mathbb{Z}\cap[0,2^8)$
GrB_INT16	INT16	int16_t	$\mathbb{Z} \cap [-2^{15}, 2^{15})$
GrB_UINT16	UINT16	uint16_t	$\mathbb{Z} \cap [0, 2^{16})$
GrB_INT32	INT32	int32_t	$\mathbb{Z} \cap [-2^{31}, 2^{31})$
GrB_UINT32	UINT32	uint32_t	$\mathbb{Z} \cap [0, 2^{32})$
GrB_INT64	INT64	int64_t	$\mathbb{Z} \cap [-2^{63}, 2^{63})$
GrB_UINT64	UINT64	uint64_t	$\mathbb{Z} \cap [0, 2^{64})$
GrB_FP32	FP32	float	IEEE 754 binary32
GrB_FP64	FP64	double	IEEE 754 binary64

The domains for elements that can be stored in collections and operated on through GraphBLAS methods are defined by GraphBLAS objects called GrB\_Type. The predefined types and corresponding domains used in the GraphBLAS C API are shown in Table 3.2. The Boolean type (bool) is defined in stdbool.h, the integral types (int8\_t, uint8\_t, int16\_t, uint16\_t, int32\_t, uint32\_t, int64\_t, uint64\_t) are defined in stdint.h, and the floating-point types (float, double) are native to the language and platform and in most cases defined by the IEEE-754 standard.

## <sup>1</sup> 3.4 Algebraic objects, operators and associated functions

GraphBLAS operators operate on elements stored in GraphBLAS collections. A binary operator is a function that maps two input values to one output value. A unary operator is a function that maps one input value to one output value. Binary operators are defined over two input domains and produce an output from a (possibly different) third domain. Unary operators are specified over one input domain and produce an output from a (possibly different) second domain.

In addition to the operators that operate on stored values, GraphBLAS also supports index unary operators that maps a stored value and the indices of its position in the matrix or vector to an output value. That output value can be used in the index unary operator variants of apply (§ 4.3.8) to compute a new stored value, or be used in the select operation (§ 4.3.9) to determine if the stored input value should be kept or annihilated.

Some GraphBLAS operations require a monoid or semiring. A monoid contains an associative binary operator where the input and output domains are the same. The monoid also includes an identity value of the operator. The semiring consists of a binary operator – referred to as the "times" operator – with up to three different domains (two inputs and one output) and a monoid

Table 3.3: Operator input for relevant GraphBLAS operations. The semiring add and times are shown if applicable.

Operation	Operator input
mxm, mxv, vxm	semiring
eWiseAdd	binary operator
	monoid
	semiring (add)
eWiseMult	binary operator
	monoid
	semiring (times)
reduce (to vector or GrB_Scalar)	binary operator
	monoid
reduce (to scalar value)	monoid
apply	unary operator
	binary operator with scalar
	index unary operator
select	index unary operator
kronecker	binary operator
	monoid
	semiring
dup argument (build methods)	binary operator
accum argument (various methods)	binary operator

- referred to as the "plus" operator – that is also commutative. Furthermore, the domain of the monoid must be the same as the output domain of the "times" operator.

The GraphBLAS algebraic objects operators, monoids, and semirings are presented in this section.
These objects can be used as input arguments to various GraphBLAS operations, as shown in
Table 3.3. The specific rules for each algebraic object are explained in the respective sections of
those objects. A summary of the properties and recipes for building these GraphBLAS algebraic

objects is presented in Table 3.4.

A number of predefined operators are specified by the GraphBLAS C API. They are presented in tables in their respective subsections below. Each of these operators is defined to operate on specific GraphBLAS types and therefore, this type is built into the name of the object as a suffix. These suffixes and the corresponding predefined GrB\_Type objects that are listed in Table 3.2.

#### 3.4.1 Operators

A GraphBLAS unary operator  $F_u = \langle D_{out}, D_{in}, f \rangle$  is defined by two domains,  $D_{out}$  and  $D_{in}$ , and an operation  $f: D_{in} \to D_{out}$ . For a given GraphBLAS unary operator  $F_u = \langle D_{out}, D_{in}, f \rangle$ , we define  $\mathbf{D}_{out}(F_u) = D_{out}$ ,  $\mathbf{D}_{in}(F_u) = D_{in}$ , and  $\mathbf{f}(F_u) = f$ .

A GraphBLAS binary operator  $F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$  is defined by three domains,  $D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$ 

Table 3.4: Properties and recipes for building GraphBLAS algebraic objects: unary operator, binary operator, monoid, and semiring (composed of operations *add* and *times*).

#### (a) Properties of algebraic objects.

Object	Must be	Must be	Identity	Number
	commutative	associative	must exist	of domains
Unary operator	n/a	n/a	n/a	2
Binary operator	no	no	no	3
Monoid	no	yes	yes	1
Reduction add	yes	yes	yes (see Note 1)	1
Semiring add	yes	yes	yes	1
Semiring times	no	no	no	3 (see Note 2)

### (b) Recipes for algebraic objects.

Object	Recipe	Number of domains
Unary operator	Function pointer	2
Binary operator	Function pointer	3
Monoid	Associative binary operator with identity	1
Semiring	Commutative monoid + binary operator	3

Note 1: Some high-performance GraphBLAS implementations may require an identity to perform reductions to sparse objects like GraphBLAS vectors and scalars. According to the descriptions of the corresponding GraphBLAS operations, however, this identity is mathematically not necessary. There are API signatures to support both. Note 2: The output domain of the semiring times must be same as the domain of the semiring's add monoid. This

ensures three domains for a semiring rather than four.

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822 D_{in_2}, and an operation \odot: D_{in_1} \times D_{in_2} \to D_{out}. For a given GraphBLAS binary operator F_b =
823 \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle, we define \mathbf{D}_{out}(F_b) = D_{out}, \mathbf{D}_{in_1}(F_b) = D_{in_1}, \mathbf{D}_{in_2}(F_b) = D_{in_2}, and \mathbf{O}(F_b) =
824 \mathbf{O}. Note that \mathbf{O} could be used in place of either \mathbf{O} or \mathbf{O} in other methods and operations.
```

A GraphBLAS index unary operator  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$  is defined by three domains,  $D_{out}, D_{in_1}, D_{in_2}$ , the domain of GraphBLAS indices, and an operation  $f_i : D_{in_1} \times I_{U64}^2 \times D_{in_2} \to D_{out}$  (where  $I_{U64}$  corresponds to the domain of a GrB\_Index). For a given GraphBLAS index operator  $F_i$ , we define  $\mathbf{D}_{out}(F_i) = D_{out}$ ,  $\mathbf{D}_{in_1}(F_i) = D_{in_1}$ ,  $\mathbf{D}_{in_2}(F_i) = D_{in_2}$ , and  $\mathbf{f}(F_i) = f_i$ .

User-defined operators can be created with calls to GrB UnaryOp new, GrB BinaryOp new, and 829 GrB\_IndexUnaryOp\_new, respectively. See Section 4.2.1 for information on these methods. The 830 GraphBLAS C API predefines a number of these operators. These are listed in Tables 3.5 and 3.6. 831 Note that most entries in these tables represent a "family" of predefined operators for a set of 832 different types represented by the T, I, or F in their names. For example, the multiplicative 833 inverse (GrB\_MINV\_F) function is only defined for floating-point types (F = FP32 or FP64). The 834 division (GrB\_DIV\_T) function is defined for all types, but only if  $y \neq 0$  for integral and floating 835 point types and  $y \neq$  false for the Boolean type. 836

Table 3.5: Predefined unary and binary operators for GraphBLAS in C. The T can be any suffix from Table 3.2, I can be any integer suffix from Table 3.2, and F can be any floating-point suffix from Table 3.2.

Operator	GraphBLAS			
type	identifier	Domains	Description	
GrB_UnaryOp	$GrB\_IDENTITY\_T$	$T \to T$	f(x) = x,	identity
$GrB\_UnaryOp$	$GrB\_ABS\_T$	$T \to T$	f(x) =  x ,	absolute value
$GrB\_UnaryOp$	$GrB\_AINV\_T$	$T \to T$	f(x) = -x,	additive inverse
$GrB\_UnaryOp$	$GrB\_MINV\_F$	$F \to F$	$f(x) = \frac{1}{x},$	multiplicative inverse
$GrB\_UnaryOp$	GrB_LNOT	$\texttt{bool} \to \texttt{bool}$	$f(x) = \neg x,$	logical inverse
GrB_UnaryOp	GrB_BNOT_I	I  o I	$\int f(x) = \tilde{x},$	bitwise complement
GrB_BinaryOp	GrB_LOR	$oxed{bool  imes bool  o bool}$	$f(x,y) = x \vee y,$	logical OR
$GrB\_BinaryOp$	GrB_LAND	$ exttt{bool}  imes  exttt{bool}  o  exttt{bool}$	$f(x,y) = x \wedge y,$	logical AND
$GrB\_BinaryOp$	GrB_LXOR	$ exttt{bool}  imes  exttt{bool}  o  exttt{bool}$	$f(x,y) = x \oplus y,$	logical XOR
GrB_BinaryOp	GrB_LXNOR	$ exttt{bool}  imes  exttt{bool}  o  exttt{bool}$	$f(x,y) = \overline{x \oplus y},$	logical XNOR
GrB_BinaryOp	GrB_BOR_ <i>I</i>	$I \times I \to I$	$f(x,y) = x \mid y,$	bitwise OR
GrB_BinaryOp	GrB_BAND_ <i>I</i>	$I \times I \to I$	f(x,y) = x & y,	bitwise AND
GrB_BinaryOp	GrB_BXOR_ <i>I</i>	$I \times I \to I$	$f(x,y) = x \hat{y},$	bitwise XOR
GrB_BinaryOp	GrB_BXNOR_ <i>I</i>	$I \times I \to I$	$f(x,y) = \overline{x \hat{y}},$	bitwise XNOR
GrB_BinaryOp	$GrB \underline{\mathsf{L}} \mathsf{E} \mathsf{Q} \underline{\mathsf{T}}$	$T  imes T  o  exttt{bool}$	f(x,y) = (x == y)	equal
GrB_BinaryOp	$GrB \_NE \_ T$	$T  imes T  o  exttt{bool}$	$f(x,y) = (x \neq y)$	not equal
GrB_BinaryOp	$GrB\_GT\_T$	$T  imes T  o  exttt{bool}$	f(x,y) = (x > y)	greater than
GrB_BinaryOp	GrB L T L T	$T  imes T  o  exttt{bool}$	f(x,y) = (x < y)	less than
GrB_BinaryOp	$GrB\_GE\_T$	$T  imes T  o  exttt{bool}$	$f(x,y) = (x \ge y)$	greater than or equal
GrB_BinaryOp	GrB LE L T	$T  imes T  o  exttt{bool}$	$f(x,y) = (x \le y)$	less than or equal
GrB_BinaryOp	GrB_ONEB_T	$T \times T \to T$	f(x,y) = 1,	1  (cast to  T)
GrB_BinaryOp	$GrB_FIRST_T$	$T \times T \to T$	f(x,y) = x,	first argument
GrB_BinaryOp	GrB_SECOND_T	$T \times T \to T$	f(x,y) = y,	second argument
GrB_BinaryOp	$GrB_MIN_T$	$T \times T \to T$	f(x,y) = (x < y) ? x : y,	minimum
GrB_BinaryOp	GrB_MAX_T	$T \times T \to T$	f(x,y) = (x > y) ? x : y,	maximum
GrB_BinaryOp	$GrB\_PLUS\_T$	$T \times T \to T$	f(x,y) = x + y,	addition
GrB_BinaryOp	$GrB_MINUS_T$	$T \times T \to T$	f(x,y) = x - y,	subtraction
GrB_BinaryOp	$GrB\_TIMES\_T$	$T \times T \to T$	f(x,y) = xy,	multiplication
GrB_BinaryOp	GrB_DIV_T	$T \times T \to T$	$f(x,y) = \frac{x}{y},$	division

Table 3.6: Predefined index unary operators for GraphBLAS in C. The T can be any suffix from Table 3.2.  $I_{U64}$  refers to the unsigned 64-bit, GrB\_Index, integer type,  $I_{32}$  refers to the signed, 32-bit integer type, and  $I_{64}$  refers to signed, 64-bit integer type. The parameters,  $u_i$  or  $A_{ij}$ , are the stored values from the containers where the i and j parameters are set to the row and column indices corresponding to the location of the stored value. When operating on vectors, j will be passed with a zero value. Finally, s is an additional scalar value used in the operators. The expressions in the "Description" column are to be treated as mathematical specifications. That is, for the index arithmetic functions in the first two groups below, each one of i, j, and s is interpreted as an integer number in the set  $\mathbb{Z}$ . Functions are evaluated using arithmetic in  $\mathbb{Z}$ , producing a result value that is also in  $\mathbb{Z}$ . The result value is converted to the output type according to the rules of the C language. In particular, if the value cannot be represented as a signed 32- or 64-bit integer type, the output is implementation defined. Any deviations from this ideal behavior, including limitations on the values of i, j, and s, or possible overflow and underflow conditions, must be defined by the implementation.

Operator type	GraphBLAS	Dom	ains (-	is don'	t care)			Des	scription
Type	Name	A, u	i, j	s	result				
GrB_IndexUnaryOp	GrB_ROWINDEX_ $I_{32/64}$	_	$I_{U64}$	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	(i+s),	replace with its row index (+ s)
	,	_	$I_{U64}$	$I_{32/64}$	$I_{32/64}$	$f(u_i, i, 0, s)$	=	(i+s)	
GrB_IndexUnaryOp	GrB_COLINDEX $I_{32/64}$	_	$I_{U64}$	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	(j+s)	replace with its column index $(+ s)$
GrB_IndexUnaryOp	$GrB\_DIAGINDEX\_I_{32/64}$	_	$I_{U64}$	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	(j-i+s)	replace with its diagonal index $(+ s)$
GrB_IndexUnaryOp	GrB_TRIL	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	$(j \le i + s)$	triangle on or below diagonal s
ωGrB_IndexUnaryOp	GrB_TRIU	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	$(j \ge i + s)$	triangle on or above diagonal s
$^{\infty}$ GrB_IndexUnaryOp	GrB_DIAG	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	(j == i + s)	diagonal s
$GrB\_IndexUnaryOp$	GrB_OFFDIAG	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	$(j \neq i + s)$	all but diagonal s
$GrB\_IndexUnaryOp$	GrB_COLLE	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	$(j \le s)$	columns less or equal to s
GrB_IndexUnaryOp	GrB_COLGT	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	(j>s)	columns greater than s
$GrB\_IndexUnaryOp$	GrB_ROWLE	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	$(i \leq s),$	rows less or equal to s
		_	$I_{U64}$	$I_{64}$	bool	$f(u_i, i, 0, s)$	=	$(i \le s)$	
GrB_IndexUnaryOp	GrB_ROWGT	_	$I_{U64}$	$I_{64}$	bool	$f(A_{ij}, i, j, s)$	=	(i>s),	rows greater than s
		_	$I_{U64}$	$I_{64}$	bool	$f(u_i, i, 0, s)$	=	(i > s)	
GrB_IndexUnaryOp	$GrB\_VALUEEQ\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} == s),$	elements equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i == s)$	
GrB_IndexUnaryOp	$GrB\_VALUENE\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \neq s),$	elements not equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \neq s)$	
GrB_IndexUnaryOp	$GrB\_VALUELT\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} < s),$	elements less than value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i < s)$	
GrB_IndexUnaryOp	$GrB\_VALUELE\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \leq s),$	elements less or equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \leq s)$	
GrB_IndexUnaryOp	$GrB\_VALUEGT\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} > s),$	elements greater than value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i > s)$	
$GrB\_IndexUnaryOp$	$GrB\_VALUEGE\_T$	T	_	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \geq s),$	elements greater or equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \ge s)$	

#### 3.4.2 Monoids

- A GraphBLAS monoid  $M = \langle D, \odot, 0 \rangle$  is defined by a single domain D, an  $associative^1$  operation  $\odot: D \times D \to D$ , and an identity element  $0 \in D$ . For a given GraphBLAS monoid  $M = \langle D, \odot, 0 \rangle$  we define  $\mathbf{D}(M) = D$ ,  $\odot(M) = \odot$ , and  $\mathbf{0}(M) = 0$ . A GraphBLAS monoid is equivalent to the conventional monoid algebraic structure.
- Let  $F = \langle D, D, D, \odot \rangle$  be an associative GraphBLAS binary operator with identity element  $0 \in D$ . Then  $M = \langle F, 0 \rangle = \langle D, \odot, 0 \rangle$  is a GraphBLAS monoid. If  $\odot$  is commutative, then M is said to be a commutative monoid. If a monoid M is created using an operator  $\odot$  that is not associative, the outcome of GraphBLAS operations using such a monoid is undefined.
- User-defined monoids can be created with calls to GrB\_Monoid\_new (see Section 4.2.1). The
  GraphBLAS C API predefines a number of monoids that are listed in Table 3.7. Predefined
  monoids are named GrB\_op\_MONOID\_T, where op is the name of the predefined GraphBLAS
  operator used as the associative binary operation of the monoid and T is the domain (type) of the
  monoid.

# 851 3.4.3 Semirings

- A GraphBLAS semiring  $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  is defined by three domains  $D_{out}, D_{in_1}$ , and  $D_{in_2}$ ; an associative<sup>1</sup> and commutative additive operation  $\oplus : D_{out} \times D_{out} \to D_{out}$ ; a multiplicative operation  $\otimes : D_{in_1} \times D_{in_2} \to D_{out}$ ; and an identity element  $0 \in D_{out}$ . For a given GraphBLAS semiring  $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  we define  $\mathbf{D}_{in_1}(S) = D_{in_1}, \mathbf{D}_{in_2}(S) = D_{in_2}, \mathbf{D}_{out}(S) = D_{out}, \oplus (S) = \oplus, \otimes(S) = \otimes, \text{ and } \mathbf{0}(S) = 0.$
- Let  $F = \langle D_{out}, D_{in_1}, D_{in_2}, \otimes \rangle$  be an operator and let  $A = \langle D_{out}, \oplus, 0 \rangle$  be a commutative monoid, then  $S = \langle A, F \rangle = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  is a semiring.
- In a GraphBLAS semiring, the multiplicative operator does not have to distribute over the additive operator. This is unlike the conventional *semiring* algebraic structure.
- Note: There must be one GraphBLAS monoid in every semiring which serves as the semiring's additive operator and specifies the same domain for its inputs and output parameters. If this monoid is not a commutative monoid, the outcome of GraphBLAS operations using the semiring is undefined.
- A UML diagram of the conceptual hierarchy of object classes in GraphBLAS algebra (binary operators, monoids, and semirings) is shown in Figure 3.1.
- User-defined semirings can be created with calls to GrB\_Semiring\_new (see Section 4.2.1). A list of predefined true semirings and convenience semirings can be found in Tables 3.8 and 3.9, respectively.

  Predefined semirings are named GrB\_add\_mul\_SEMIRING\_T, where add is the semiring additive operation, mul is the semiring multiplicative operation and T is the domain (type) of the semiring.

<sup>&</sup>lt;sup>1</sup>It is expected that implementations of the GraphBLAS will utilize floating point arithmetic such as that defined in the IEEE-754 standard even though floating point arithmetic is not strictly associative.

Table 3.7: Predefined monoids for GraphBLAS in C. Maximum and minimum values for the various integral types are defined in  $\mathtt{stdint.h.}$  Floating-point infinities are defined in  $\mathtt{math.h.}$  The x in  $\mathsf{UINT}x$  or  $\mathsf{INT}x$  can be one of 8, 16, 32, or 64; whereas in  $\mathsf{FP}x$ , it can be 32 or 64.

$\operatorname{GraphBLAS}$	Domains, $T$		
identifier	$(T \times T \to T)$	Identity	Description
GrB_PLUS_MONOID_T	UINTx	0	addition
	INTx	0	
	FPx	0	
$GrB\_TIMES\_MONOID\_T$	UINTx	1	multiplication
	INTx	1	
	FPx	1	
$GrB \_MIN \_MONOID \_T$	UINTx	${\tt UINT}x\_{\tt MAX}$	minimum
	INTx	${\sf INT}x\_{\sf MAX}$	
	FPx	INFINITY	
$GrB\_MAX\_MONOID\_T$	UINTx	0	maximum
	INTx	$ $ INT $x$ _MIN	
	FPx	-INFINITY	
GrB_LOR_MONOID_BOOL	BOOL	false	logical OR
GrB_LAND_MONOID_BOOL	BOOL	true	logical AND
GrB_LXOR_MONOID_BOOL	BOOL	false	logical XOR (not equal)
GrB_LXNOR_MONOID_BOOL	BOOL	true	logical XNOR (equal)

Table 3.8: Predefined true semirings for GraphBLAS in C where the additive identity is the multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

	Domains, $T$	+ identity	
GraphBLAS identifier	$(T \times T \to T)$	$\times$ annihilator	Description
GrB_PLUS_TIMES_SEMIRING_T	UINTx	0	arithmetic semiring
	INTx	0	
	FPx	0	
$GrB \_MIN \_PLUS \_SEMIRING \_T$	UINTx	$\mathtt{UINT}x\_\mathtt{MAX}$	min-plus semiring
	INTx	$\mathtt{INT}x\mathtt{\_MAX}$	
	FPx	INFINITY	
$GrB\_MAX\_PLUS\_SEMIRING\_T$	INTx	$\mathtt{INT}x\mathtt{\_MIN}$	max-plus semiring
	FPx	-INFINITY	
$GrB \_MIN \_TIMES \_SEMIRING \_T$	UINTx	$\mathtt{UINT}x\_\mathtt{MAX}$	min-times semiring
$GrB \_MIN \_MAX \_SEMIRING \_T$	UINTx	$\mathtt{UINT}x\_\mathtt{MAX}$	min-max semiring
	INTx	$\mathtt{INT}x\mathtt{\_MAX}$	
	FPx	INFINITY	
$GrB\_MAX\_MIN\_SEMIRING\_T$	UINTx	0	max-min semiring
	INTx	$\mathtt{INT}x\mathtt{\_MIN}$	
	FPx	-INFINITY	
$GrB\_MAX\_TIMES\_SEMIRING\_T$	UINTx	0	max-times semiring
$GrB\_PLUS\_MIN\_SEMIRING\_T$	UINTx	0	plus-min semiring
GrB_LOR_LAND_SEMIRING_BOOL	BOOL	false	Logical semiring
GrB_LAND_LOR_SEMIRING_BOOL	BOOL	true	"and-or" semiring
GrB_LXOR_LAND_SEMIRING_BOOL	BOOL	false	same as NE_LAND
GrB_LXNOR_LOR_SEMIRING_BOOL	BOOL	true	same as EQ_LOR

Table 3.9: Other useful predefined semirings for GraphBLAS in C that don't have a multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

	Domains, $T$		
GraphBLAS identifier	$(T \times T \to T)$	+ identity	Description
GrB_MAX_PLUS_SEMIRING_T	UINTx	0	max-plus semiring
$GrB \_MIN \_TIMES \_SEMIRING \_T$	INTx	$\mathtt{INT}x\mathtt{\_MAX}$	min-times semiring
	FPx	INFINITY	
$GrB\_MAX\_TIMES\_SEMIRING\_T$	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	max-times semiring
	FPx	-INFINITY	
$GrB\_PLUS\_MIN\_SEMIRING\_T$	INTx	0	plus-min semiring
	FPx	0	
$GrB \_MIN \_FIRST \_SEMIRING \_T$	UINTx	$\mathtt{UINT}x\_\mathtt{MAX}$	min-select first semiring
	INTx	$\mathtt{INT}x\mathtt{\_MAX}$	
	FPx	INFINITY	
$GrB \_MIN \_SECOND \_SEMIRING \_T$	UINTx	$\mathtt{UINT}x\_\mathtt{MAX}$	min-select second semiring
	INTx	$\mathtt{INT}x\mathtt{\_MAX}$	
	FPx	INFINITY	
$GrB\_MAX\_FIRST\_SEMIRING\_T$	UINTx	0	max-select first semiring
	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	
	FPx	-INFINITY	
$GrB\_MAX\_SECOND\_SEMIRING\_T$	UINTx	0	max-select second semiring
	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	
	FPx	-INFINITY	

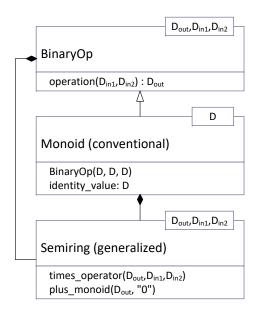


Figure 3.1: Hierarchy of algebraic object classes in GraphBLAS. GraphBLAS semirings consist of a conventional monoid with one domain for the addition function, and a binary operator with three domains for the multiplication function.

# $_{871}$ 3.5 Collections

## 872 **3.5.1** Scalars

A GraphBLAS scalar,  $s = \langle D, \{\sigma\} \rangle$ , is defined by a domain D, and a set of zero or one scalar value,  $\sigma$ , where  $\sigma \in D$ . We define  $\mathbf{size}(s) = 1$  (constant), and  $\mathbf{L}(s) = \{\sigma\}$ . The set  $\mathbf{L}(s)$  is called the contents of the GraphBLAS scalar s. We also define  $\mathbf{D}(s) = D$ . Finally,  $\mathbf{val}(s)$  is a reference to the scalar value,  $\sigma$ , if the GraphBLAS scalar is not empty, and is undefined otherwise.

# 877 **3.5.2** Vectors

A vector  $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$  is defined by a domain D, a size N > 0, and a set of tuples  $(i, v_i)$ where  $0 \le i < N$  and  $v_i \in D$ . A particular value of i can appear at most once in  $\mathbf{v}$ . We define size( $\mathbf{v}$ ) = N and  $\mathbf{L}(\mathbf{v}) = \{(i, v_i)\}$ . The set  $\mathbf{L}(\mathbf{v})$  is called the *content* of vector  $\mathbf{v}$ . We also define the set  $\mathbf{ind}(\mathbf{v}) = \{i : (i, v_i) \in \mathbf{L}(\mathbf{v})\}$  (called the *structure* of  $\mathbf{v}$ ), and  $\mathbf{D}(\mathbf{v}) = D$ . For a vector  $\mathbf{v}$ ,  $\mathbf{v}(i)$  is a reference to  $v_i$  if  $(i, v_i) \in \mathbf{L}(\mathbf{v})$  and is undefined otherwise.

#### 3.5.3 Matrices

A matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$  is defined by a domain D, its number of rows M > 0, its 884 number of columns N > 0, and a set of tuples  $(i, j, A_{ij})$  where  $0 \le i < M$ ,  $0 \le j < N$ , and 885  $A_{ij} \in D$ . A particular pair of values i, j can appear at most once in **A**. We define  $\mathbf{ncols}(\mathbf{A}) = N$ , 886  $\mathbf{nrows}(\mathbf{A}) = M$ , and  $\mathbf{L}(\mathbf{A}) = \{(i, j, A_{ij})\}$ . The set  $\mathbf{L}(\mathbf{A})$  is called the *content* of matrix  $\mathbf{A}$ . We also 887 define the sets  $indrow(\mathbf{A}) = \{i : \exists (i, j, A_{ij}) \in \mathbf{A}\}$  and  $indcol(\mathbf{A}) = \{j : \exists (i, j, A_{ij}) \in \mathbf{A}\}$ . (These 888 are the sets of nonempty rows and columns of A, respectively.) The structure of matrix A is the set  $ind(A) = \{(i,j): (i,j,A_{ij}) \in \mathbf{L}(A)\}, \text{ and } \mathbf{D}(A) = D.$  For a matrix A, A(i,j) is a reference to  $A_{ij}$  if  $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})$  and is undefined otherwise. 891 If **A** is a matrix and  $0 \leq j < N$ , then  $\mathbf{A}(:,j) = \langle D, M, \{(i,A_{ij}) : (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$  is a 892 vector called the j-th column of A. Correspondingly, if A is a matrix and  $0 \le i < M$ , then 893  $\mathbf{A}(i,:) = \langle D, N, \{(j,A_{ij}): (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$  is a vector called the *i*-th row of  $\mathbf{A}$ . Given a matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$ , its transpose is another matrix  $\mathbf{A}^T = \langle D, N, M, \{(j, i, A_{ij}) : A_{ij} :$ 895  $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A}) \} \rangle.$ 

#### of 3.5.3.1 External matrix formats

The specification also supports the export and import of matrices to/from a number of commonly used formats, such as COO, CSR, and CSC formats. When importing or exporting a matrix to or from a GraphBLAS object using GrB\_Matrix\_import (§ 4.2.4.17) or GrB\_Matrix\_export (§ 4.2.4.16), it is necessary to specify the data format for the matrix data external to GraphBLAS, which is being imported from or exported to. This non-opaque data format is specified using an argument of enumeration type GrB\_Format that is used to indicate one of a number of predefined formats. The predefined values of GrB\_Format are specified in Table 3.10. A precise definition of the non-opaque data formats can be found in Appendix B.

Table 3.10: GrB\_Format enumeration literals and corresponding values for matrix import and export methods.

Symbol	Value	Description
GrB_CSR_FORMAT	0	Specifies the compressed sparse row matrix format.
GrB_CSC_FORMAT	1	Specifies the compressed sparse column matrix format.
GrB_COO_FORMAT	2	Specifies the sparse coordinate matrix format.

#### 3.5.4 Masks

The GraphBLAS C API defines an opaque object called a *mask*. The mask is used to control how computed values are stored in the output from a method. The mask is an *internal* opaque object; that is, it is never exposed as a variable within an application.

The mask is formed from input objects to the method that uses the mask. For example, a Graph-BLAS method may be called with a matrix as the mask parameter. The internal mask object is constructed from the input matrix in one of two ways. In the default case, an element of the mask is created for each tuple that exists in the matrix for which the value of the tuple cast to Boolean evaluates to true. Alternatively, the user can specify *structure*-only behavior where an element of the mask is created for each tuple that exists in the matrix *regardless* of the value stored in the input matrix.

The internal mask object can be either a one- or a two-dimensional construct. One- and twodimensional masks, described more formally below, are similar to vectors and matrices, respectively,
except that they have structure (indices) but no values. When needed, a value is implied for the
elements of a mask with an implied value of true for elements that exist and an implied value
of false for elements that do not exist (i.e., the locations of the mask that do not have a stored
value imply a value of false). Hence, even though a mask does not contain any values, it can be
considered to imply values from a Boolean domain.

A one-dimensional mask  $\mathbf{m} = \langle N, \{i\} \rangle$  is defined by its number of elements N > 0, and a set  $\mathbf{ind}(\mathbf{m})$  of indices  $\{i\}$  where  $0 \le i < N$ . A particular value of i can appear at most once in  $\mathbf{m}$ . We define  $\mathbf{size}(\mathbf{m}) = N$ . The set  $\mathbf{ind}(\mathbf{m})$  is called the *structure* of mask  $\mathbf{m}$ .

A two-dimensional mask  $\mathbf{M} = \langle M, N, \{(i,j)\} \rangle$  is defined by its number of rows M > 0, its number of columns N > 0, and a set  $\mathbf{ind}(\mathbf{M})$  of tuples (i,j) where  $0 \le i < M$ ,  $0 \le j < N$ . A particular pair of values i,j can appear at most once in  $\mathbf{M}$ . We define  $\mathbf{ncols}(\mathbf{M}) = N$ , and  $\mathbf{nrows}(\mathbf{M}) = M$ . We also define the sets  $\mathbf{indrow}(\mathbf{M}) = \{i : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$  and  $\mathbf{indcol}(\mathbf{M}) = \{j : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$ . These are the sets of nonempty rows and columns of  $\mathbf{M}$ , respectively. The set  $\mathbf{ind}(\mathbf{M})$  is called the structure of mask  $\mathbf{M}$ .

One common operation on masks is the *complement*. For a one-dimensional mask  $\mathbf{m}$  this is denoted as  $\neg \mathbf{m}$ . For a two-dimensional mask  $\mathbf{M}$ , this is denoted as  $\neg \mathbf{M}$ . The complement of a one-dimensional mask  $\mathbf{m}$  is defined as  $\operatorname{ind}(\neg \mathbf{m}) = \{i : 0 \le i < N, i \notin \operatorname{ind}(\mathbf{m})\}$ . It is the set of all possible indices that do not appear in  $\mathbf{m}$ . The complement of a two-dimensional mask  $\mathbf{M}$  is defined as the set  $\operatorname{ind}(\neg \mathbf{M}) = \{(i,j) : 0 \le i < M, 0 \le j < N, (i,j) \notin \operatorname{ind}(\mathbf{M})\}$ . It is the set of all possible indices that do not appear in  $\mathbf{M}$ .

# 939 3.6 Descriptors

Descriptors are used to modify the behavior of a GraphBLAS method. When present in the signature of a method, they appear as the last argument in the method. Descriptors specify how the other input arguments corresponding to GraphBLAS collections – vectors, matrices, and masks – should be processed (modified) before the main operation of a method is performed. A complete list of what descriptors are capable of are presented in this section.

The descriptor is a lightweight object. It is composed of (*field*, *value*) pairs where the *field* selects one of the GraphBLAS objects from the argument list of a method and the *value* defines the indicated modification associated with that object. For example, a descriptor may specify that a particular input matrix needs to be transposed or that a mask needs to be complemented (defined in Section 3.5.4) before using it in the operation.

For the purpose of constructing descriptors, the arguments of a method that can be modified

are identified by specific field names. The output parameter (typically the first parameter in a 951 GraphBLAS method) is indicated by the field name, GrB\_OUTP. The mask is indicated by the 952 GrB\_MASK field name. The input parameters corresponding to the input vectors and matrices are 953 indicated by GrB INP0 and GrB INP1 in the order they appear in the signature of the GraphBLAS 954 method. The descriptor is an opaque object and hence we do not define how objects of this type 955 should be implemented. When referring to (field, value) pairs for a descriptor, however, we often use 956 the informal notation desc[GrB Desc Field]. GrB Desc Value without implying that a descriptor is 957 to be implemented as an array of structures (in fact, field values can be used in conjunction with 958 multiple values that are composable). We summarize all types, field names, and values used with 959 descriptors in Table 3.11. 960

In the definitions of the GraphBLAS methods, we often refer to the *default behavior* of a method with respect to the action of a descriptor. If a descriptor is not provided or if the value associated with a particular field in a descriptor is not set, the default behavior of a GraphBLAS method is defined as follows:

• Input matrices are not transposed.

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- The mask is used, as is, without complementing, and stored values are examined to determine whether they evaluate to true or false.
- Values of the output object that are not directly modified by the operation are preserved.

GraphBLAS specifies all of the valid combinations of (field, value) pairs as predefined descriptors.
Their identifiers and the corresponding set of (field, value) pairs for that identifier are shown in
Table 3.12.

# $_{72}$ 3.7 GrB Info return values

All GraphBLAS methods return a GrB\_Info enumeration value. The three types of return codes (informational, API error, and execution error) and their corresponding values are listed in Table 3.13.

Table 3.11: Descriptors are GraphBLAS objects passed as arguments to GraphBLAS operations to modify other GraphBLAS objects in the operation's argument list. A descriptor, desc, has one or more (*field*, *value*) pairs indicated as desc[GrB\_Desc\_Field].GrB\_Desc\_Value. In this table, we define all types and literals used with descriptors.

# (a) Types used with GraphBLAS descriptors.

Type	Description
GrB_Descriptor	Type of a GraphBLAS descriptor object.
GrB_Desc_Field	The descriptor field enumeration.
GrB_Desc_Value	The descriptor value enumeration.

(b) Descriptor field names of type GrB\_Desc\_Field enumeration and corresponding values.

Field Name	Value	Description
GrB_OUTP	0	Field name for the output GraphBLAS object.
GrB_MASK	1	Field name for the mask GraphBLAS object.
GrB_INP0	2	Field name for the first input GraphBLAS object.
GrB_INP1	3	Field name for the second input GraphBLAS object.

(c) Descriptor field values of type GrB\_Desc\_Value enumeration and corresponding values.

Value Name	Value	Description
(reserved)	0	Unused
GrB_REPLACE	1	Clear the output object before assigning computed values.
GrB_COMP	2	Use the complement of the associated object. When combined with GrB_STRUCTURE, the complement of the structure of the associated object is used without evaluating the values stored.
GrB_TRAN	3	Use the transpose of the associated object.
GrB_STRUCTURE	4	The write mask is constructed from the structure (pattern of stored values) of the associated object. The stored values are not examined.

Table 3.12: Predefined GraphBLAS descriptors. The list includes all possible descriptors, according to the current standard. Columns list the possible fields and entries list the value(s) associated with those fields for a given descriptor.

Identifier	GrB_OUTP	GrB_MASK	GrB_INP0	GrB_INP1
GrB_NULL	_	_	_	_
GrB_DESC_T1	_	_	_	GrB_TRAN
GrB_DESC_T0	_	_	$GrB\_TRAN$	_
GrB_DESC_T0T1	_	_	$GrB\_TRAN$	GrB_TRAN
GrB_DESC_C	_	GrB_COMP	_	_
GrB_DESC_S	_	GrB_STRUCTURE	_	_
GrB_DESC_CT1	_	GrB_COMP	_	GrB_TRAN
GrB_DESC_ST1	_	GrB_STRUCTURE	_	GrB_TRAN
GrB_DESC_CT0	_	GrB_COMP	$GrB\_TRAN$	_
GrB_DESC_ST0	_	GrB_STRUCTURE	$GrB\_TRAN$	_
GrB_DESC_CT0T1	_	GrB_COMP	$GrB \_TRAN$	GrB_TRAN
GrB_DESC_ST0T1	_	GrB_STRUCTURE	$GrB\_TRAN$	$GrB\_TRAN$
GrB_DESC_SC	_	GrB_STRUCTURE, GrB_COMP	_	_
GrB_DESC_SCT1	_	GrB_STRUCTURE, GrB_COMP	_	GrB_TRAN
GrB_DESC_SCT0	_	GrB_STRUCTURE, GrB_COMP	$GrB\_TRAN$	_
GrB_DESC_SCT0T1	_	GrB_STRUCTURE, GrB_COMP	$GrB\_TRAN$	$GrB\_TRAN$
GrB_DESC_R	GrB_REPLACE	_	_	_
GrB_DESC_RT1	GrB_REPLACE	_	_	$GrB\_TRAN$
GrB_DESC_RT0	GrB_REPLACE	_	$GrB\_TRAN$	_
GrB_DESC_RT0T1	GrB_REPLACE	_	$GrB \_TRAN$	GrB_TRAN
GrB_DESC_RC	GrB_REPLACE	GrB_COMP	_	_
GrB_DESC_RS	GrB_REPLACE	GrB_STRUCTURE	_	_
GrB_DESC_RCT1	GrB_REPLACE	GrB_COMP	_	$GrB\_TRAN$
GrB_DESC_RST1	GrB_REPLACE	GrB_STRUCTURE	_	GrB_TRAN
GrB_DESC_RCT0	GrB_REPLACE	GrB_COMP	$GrB \_TRAN$	_
GrB_DESC_RST0	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	_
GrB_DESC_RCT0T1	GrB_REPLACE	GrB_COMP	$GrB \_TRAN$	GrB_TRAN
GrB_DESC_RST0T1	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	GrB_TRAN
GrB_DESC_RSC	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	_	_
GrB_DESC_RSCT1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	_	GrB_TRAN
GrB_DESC_RSCT0	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	_
GrB_DESC_RSCT0T1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	$GrB\_TRAN$
	,			

Table 3.13: Enumeration literals and corresponding values returned by GraphBLAS methods and operations.

# (a) Informational return values

Symbol	Value	Description
GrB_SUCCESS	0	The method/operation completed successfully (blocking mode), or
		encountered no API errors (non-blocking mode).
GrB_NO_VALUE	1	A location in a matrix or vector is being accessed that has no stored
		value at the specified location.

# (b) API errors

Symbol	Value	Description
GrB_UNINITIALIZED_OBJECT	-1	A GraphBLAS object is passed to a method before
		new was called on it.
GrB_NULL_POINTER	-2	A NULL is passed for a pointer parameter.
GrB_INVALID_VALUE	-3	Miscellaneous incorrect values.
GrB_INVALID_INDEX	-4	Indices passed are larger than dimensions of the ma-
		trix or vector being accessed.
GrB_DOMAIN_MISMATCH	-5	A mismatch between domains of collections and op-
		erations when user-defined domains are in use.
GrB_DIMENSION_MISMATCH	-6	Operations on matrices and vectors with incompati-
		ble dimensions.
GrB_OUTPUT_NOT_EMPTY	-7	An attempt was made to build a matrix or vector
		using an output object that already contains valid
		tuples (elements).
GrB_NOT_IMPLEMENTED	-8	An attempt was made to call a GraphBLAS method
		for a combination of input parameters that is not
		supported by a particular implementation.

# (c) Execution errors

Symbol	Value	Description
GrB_PANIC	-101	Unknown internal error.
GrB_OUT_OF_MEMORY	-102	Not enough memory for operations.
GrB_INSUFFICIENT_SPACE	-103	The array provided is not large enough to hold out-
GrB_INVALID_OBJECT	-104	put. One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error.
GrB_INDEX_OUT_OF_BOUNDS	-105	Reference to a vector or matrix element that is outside the defined dimensions of the object.
GrB_EMPTY_OBJECT	-106	One of the opaque GraphBLAS objects does not
		have a stored value.

# Chapter 4

# $_{\scriptscriptstyle 77}$ ${f Methods}$

- This chapter defines the behavior of all the methods in the GraphBLAS C API. All methods can be declared for use in programs by including the GraphBLAS.h header file.
- We would like to emphasize that no GraphBLAS method will imply a predefined order over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.

# 984 4.1 Context methods

The methods in this section set up and tear down the GraphBLAS context within which all Graph-BLAS methods must be executed. The initialization of this context also includes the specification of which execution mode is to be used.

# 988 4.1.1 init: Initialize a GraphBLAS context

Creates and initializes a GraphBLAS C API context.

### 990 C Syntax

grB\_Info GrB\_init(GrB\_Mode mode);

#### Parameters

993

mode Mode for the GraphBLAS context. Must be either GrB\_BLOCKING or GrB\_NONBLOCKING.

#### 994 Return Values

995 GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_INVALID\_VALUE invalid mode specified, or method called multiple times.

#### 998 Description

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The init method creates and initializes a GraphBLAS C API context. The argument to GrB\_init defines the mode for the context. The two available modes are:

- GrB\_BLOCKING: In this mode, each method in a sequence returns after its computations have completed and output arguments are available to subsequent statements in an application. When executing in GrB\_BLOCKING mode, the methods execute in program order.
- GrB\_NONBLOCKING: In this mode, methods in a sequence may return after arguments in the method have been tested for dimension and domain compatibility within the method but potentially before their computations complete. Output arguments are available to subsequent GraphBLAS methods in an application. When executing in GrB\_NONBLOCKING mode, the methods in a sequence may execute in any order that preserves the mathematical result defined by the sequence.

An application can only create one context per execution instance. An application may only call GrB\_Init once. Calling GrB\_Init more than once results in undefined behavior.

# 1012 4.1.2 finalize: Finalize a GraphBLAS context

Terminates and frees any internal resources created to support the GraphBLAS C API context.

### 1014 C Syntax

```
GrB_Info GrB_finalize();
```

# 1016 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

### 1019 Description

The finalize method terminates and frees any internal resources created to support the GraphBLAS C API context. GrB\_finalize may only be called after a context has been initialized by calling GrB\_init, or else undefined behavior occurs. After GrB\_finalize has been called to finalize a Graph-BLAS context, calls to any GraphBLAS methods, including GrB\_finalize, will result in undefined behavior.

# 1025 4.1.3 getVersion: Get the version number of the standard.

1026 Query the library for the version number of the standard that this library implements.

# 1027 C Syntax

```
GrB_Info GrB_getVersion(unsigned int *version, unsigned int *subversion);
```

#### 1030 Parameters

version (OUT) On successful return will hold the value of the major version number.

version (OUT) On successful return will hold the value of the subversion number.

#### 1033 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

#### 1036 Description

1042

The getVersion method is used to query the major and minor version number of the GraphBLAS C API specification that the library implements at runtime. To support compile time queries the following two macros shall also be defined by the library.

```
#define GRB_VERSION 2
#define GRB_SUBVERSION 0
```

# 4.2 Object methods

This section describes methods that setup and operate on GraphBLAS opaque objects but are not part of the the GraphBLAS math specification.

# 1045 4.2.1 Algebra methods

# 1046 4.2.1.1 Type\_new: Construct a new GraphBLAS (user-defined) type

Creates a new user-defined GraphBLAS type. This type can then be used to create new operators, monoids, semirings, vectors and matrices.

# 1049 C Syntax

```
GrB_Info GrB_Type_new(GrB_Type *utype,
size_t sizeof(ctype));
```

#### 1052 Parameters

1055

utype (INOUT) On successful return, contains a handle to the newly created user-defined
GraphBLAS type object.

ctype (IN) A C type that defines the new GraphBLAS user-defined type.

### Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

Grb NULL POINTER utype pointer is NULL.

## 1061 Description

Given a C type ctype, the Type\_new method returns in utype a handle to a new GraphBLAS type
that is equivalent to the C type. Variables of this ctype must be a struct, union, or fixed-size array.
In particular, given two variables, src and dst, of type ctype, the following operation must be a
valid way to copy the contents of src to dst:

```
memcpy(&dst, &src, sizeof(ctype))
```

A new, user-defined type utype should be destroyed with a call to GrB\_free(utype) when no longer needed.

1069 It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1071 4.2.1.2 UnaryOp\_new: Construct a new GraphBLAS unary operator

Initializes a new GraphBLAS unary operator with a specified user-defined function and its types (domains).

# 1074 C Syntax

```
GrB_Info GrB_UnaryOp_new(GrB_UnaryOp *unary_op,
void (*unary_func)(void*, const void*),
GrB_Type d_out,
GrB_Type d_in);
```

#### 1079 Parameters

- unary\_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS unary operator object.
- unary\_func (IN) a pointer to a user-defined function that takes one input parameter of d\_in's type and returns a value of d\_out's type, both passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void *out, const void *in);
```

1086

- d\_out (IN) The GrB\_Type of the return value of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- d\_in (IN) The GrB\_Type of the input argument of the unary operator being created.
  Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

#### 1093 Return Values

- GrB\_SUCCESS operation completed successfully.
- GrB\_PANIC unknown internal error.
- GrB\_OUT\_OF\_MEMORY not enough memory available for operation.
- GrB\_UNINITIALIZED\_OBJECT any GrB\_Type parameter (for user-defined types) has not been initialized by a call to GrB\_Type\_new.
- GrB\_NULL\_POINTER unary\_op or unary\_func pointers are NULL.

### 1100 Description

The UnaryOp\_new method creates a new GraphBLAS unary operator

```
f_u = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in}), \mathsf{unary\_func} \rangle
```

and returns a handle to it in unary\_op.

The implementation of unary\_func must be such that it works even if the d\_out and d\_in arguments are aliased. In other words, for all invocations of the function:

```
unary_func(out,in);
```

the value of out must be the same as if the following code was executed:

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1114 4.2.1.3 BinaryOp\_new: Construct a new GraphBLAS binary operator

Initializes a new GraphBLAS binary operator with a specified user-defined function and its types (domains).

#### 1117 C Syntax

```
GrB_Info GrB_BinaryOp_new(GrB_BinaryOp *binary_op,
1118
                                                          (*binary_func)(void*,
                                            void
                                                                           const void*,
1120
                                                                           const void*),
1121
                                            GrB_Type
                                                            d_out,
1122
                                           GrB_Type
                                                            d_in1,
1123
                                           GrB_Type
                                                            d_in2);
1124
```

#### Parameters

1125

binary\_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS binary operator object.

```
binary_func (IN) A pointer to a user-defined function that takes two input parameters of types
d_in1 and d_in2 and returns a value of type d_out, all passed as void pointers.

Specifically the signature of the function is expected to be of the form:
```

void func(void \*out, const void \*in1, const void \*in2);

1131 1132

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1155

- d\_out (IN) The GrB\_Type of the return value of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined Graph-BLAS type.
- d\_in1 (IN) The GrB\_Type of the left hand argument of the binary operator being created.

  Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
  - d\_in2 (IN) The GrB\_Type of the right hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

#### 1142 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

the GrB\_UNINITIALIZED\_OBJECT the GrB\_Type (for user-defined types) has not been initialized by a call to GrB\_Type\_new.

GrB\_NULL\_POINTER binary\_op or binary\_func pointer is NULL.

#### 1149 Description

1150 The BinaryOp\_new methods creates a new GraphBLAS binary operator

```
f_b = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{binary\_func} \rangle
```

and returns a handle to it in binary\_op.

The implementation of binary\_func must be such that it works even if any of the d\_out, d\_in1, and d\_in2 arguments are aliased to each other. In other words, for all invocations of the function:

```
binary_func(out,in1,in2);
```

the value of out must be the same as if the following code was executed:

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1166 4.2.1.4 Monoid\_new: Construct a new GraphBLAS monoid

1167 Creates a new monoid with specified binary operator and identity value.

# 1168 C Syntax

```
GrB_Info GrB_Monoid_new(GrB_Monoid *monoid,
GrB_BinaryOp binary_op,
<type> identity);
```

#### $_{12}$ Parameters

- monoid (INOUT) On successful return, contains a handle to the newly created GraphBLAS monoid object.
- binary\_op (IN) An existing GraphBLAS associative binary operator whose input and output types are the same.
- identity (IN) The value of the identity element of the monoid. Must be the same type as
  the type used by the binary\_op operator.

## 1179 Return Values

1182

```
GrB_SUCCESS operation completed successfully.
```

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT the GrB\_BinaryOp (for user-defined operators) has not been initialized by a call to GrB\_BinaryOp\_new.

GrB\_NULL\_POINTER monoid pointer is NULL.

GrB\_DOMAIN\_MISMATCH all three argument types of the binary operator and the type of the identity value are not the same.

### 1188 Description

The Monoid\_new method creates a new monoid  $M = \langle \mathbf{D}(\mathsf{binary\_op}), \mathsf{binary\_op}, \mathsf{identity} \rangle$  and returns a handle to it in monoid.

If binary\_op is not associative, the results of GraphBLAS operations that require associativity of this monoid will be undefined.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1195 4.2.1.5 Semiring\_new: Construct a new GraphBLAS semiring

1196 Creates a new semiring with specified domain, operators, and elements.

# 1197 C Syntax

```
GrB_Info GrB_Semiring_new(GrB_Semiring *semiring, 1199 GrB_Monoid add_op, 1200 GrB_BinaryOp mul_op);
```

#### 1201 Parameters

1204

1205

1211

semiring (INOUT) On successful return, contains a handle to the newly created GraphBLAS semiring.

add\_op (IN) An existing GraphBLAS commutative monoid that specifies the addition operator and its identity.

mul\_op (IN) An existing GraphBLAS binary operator that specifies the semiring's multiplication operator. In addition, mul\_op's output domain,  $\mathbf{D}_{out}(\mathsf{mul\_op})$ , must be the same as the add\_op's domain  $\mathbf{D}(\mathsf{add\_op})$ .

#### 1209 Return Values

1210 GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for this method to complete.

1213 GrB\_UNINITIALIZED\_OBJECT the add\_op (for user-define monoids) object has not been initialized with a call to GrB\_Monoid\_new or the mul\_op (for user-defined operators) object has not been initialized by a call to GrB\_BinaryOp\_new.

GrB\_NULL\_POINTER semiring pointer is NULL.

GrB\_DOMAIN\_MISMATCH the output domain of mul\_op does not match the domain of the add\_op monoid.

# 1220 Description

1217

1218

1219

The Semiring\_new method creates a new semiring:

```
S = \langle \mathbf{D}_{out}(\mathsf{mul\_op}), \mathbf{D}_{in_1}(\mathsf{mul\_op}), \mathbf{D}_{in_2}(\mathsf{mul\_op}), \mathsf{add\_op}, \mathsf{mul\_op}, \mathbf{0}(\mathsf{add\_op}) \rangle
```

and returns a handle to it in semiring. Note that  $\mathbf{D}_{out}(\mathsf{mul\_op})$  must be the same as  $\mathbf{D}(\mathsf{add\_op})$ .

1224 If add\_op is not commutative, then GraphBLAS operations using this semiring will be undefined.

1225 It is not an error to call this method more than once on the same variable; however, the handle to 1226 the previously created object will be overwritten.

## 1227 4.2.1.6 IndexUnaryOp\_new: Construct a new GraphBLAS index unary operator

Initializes a new GraphBLAS index unary operator with a specified user-defined function and its types (domains).

#### 1230 C Syntax

```
GrB_Info GrB_IndexUnaryOp_new(GrB_IndexUnaryOp
                                                                  *index_unary_op,
1231
                                            void (*index_unary_func)(void*,
1232
                                                                          const void*,
1233
                                                                          GrB_Index,
1234
                                                                          GrB_Index,
1235
                                                                          const void*),
1236
                                            GrB_Type
                                                                    d_out,
1237
                                            GrB_Type
                                                                    d_in1,
1238
                                            GrB_Type
                                                                    d_in2);
1239
```

## 1240 Parameters

index\_unary\_op (INOUT) On successful return, contains a handle to the newly created Graph-BLAS index unary operator object.

index\_unary\_func (IN) A pointer to a user-defined function that takes input parameters of types d\_in1, GrB\_Index, GrB\_Index and d\_in2 and returns a value of type d\_out. Except for the GrB\_Index parameters, all are passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void
                                                         *out,
1247
                                            const void *in1,
1248
                                           GrB_Index
                                                           row_index,
1249
                                           GrB_Index
                                                           col_index,
1250
                                            const void *in2);
1251
1252
                d out (IN) The GrB Type of the return value of the index unary operator being created.
1253
                       Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined
1254
                       GraphBLAS type.
1255
                d_in1 (IN) The GrB_Type of the first input argument of the index unary operator being
1256
                       created and corresponds to the stored values of the GrB_Vector or GrB_Matrix
1257
                       being operated on. Should be one of the predefined GraphBLAS types in Ta-
1258
                       ble 3.2, or a user-defined GraphBLAS type.
1259
                d_in2 (IN) The GrB_Type of the last input argument of the index unary operator be-
1260
                       ing created and corresponds to a scalar provided by the GraphBLAS operation
1261
                       that uses this operator. Should be one of the predefined GraphBLAS types in
1262
                       Table 3.2, or a user-defined GraphBLAS type.
1263
    Return Values
1264
                     GrB_SUCCESS operation completed successfully.
1265
                        GrB PANIC unknown internal error.
1266
         GrB_OUT_OF_MEMORY not enough memory available for operation.
1267
   GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a
1268
                                      call to GrB_Type_new.
1269
             GrB_NULL_POINTER index_unary_op or index_unary_func pointer is NULL.
1270
    Description
1271
    The IndexUnaryOp_new methods creates a new GraphBLAS index unary operator
1272
          f_i = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{index\_unary\_func} \rangle
1273
    and returns a handle to it in index_unary_op.
1274
     The implementation of index_unary_func must be such that it works even if any of the d_out,
1275
    d_in1, and d_in2 arguments are aliased to each other. In other words, for all invocations of the
1276
    function:
1277
           index_unary_func(out,in1,row_index,col_index,n,in2);
1278
```

the value of out must be the same as if the following code was executed (shown here for matrices):

```
GrB_Index row_index = ...;
1280
         GrB_Index col_index = ...;
1281
         D(d in1) *tmp1 = malloc(sizeof(D(d in1)));
1282
         D(d_{in2}) *tmp2 = malloc(sizeof(D(d_{in2})));
1283
         memcpy(tmp1,in1,sizeof(D(d_in1));
1284
         memcpy(tmp2,in2,sizeof(D(d_in2)));
1285
         index_unary_func(out,tmp1,row_index,col_index,tmp2);
1286
         free(tmp2);
1287
         free(tmp1);
1288
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

### 1291 4.2.2 Scalar methods

# 1292 4.2.2.1 Scalar\_new: Construct a new scalar

1293 Creates a new empty scalar with specified domain.

# 1294 C Syntax

```
GrB_Info GrB_Scalar_new(GrB_Scalar *s, GrB_Type d);
```

#### 1297 Parameters

- s (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.
- d (IN) The type corresponding to the domain of the scalar being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

#### Return Values

1303

1308

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar s is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1309 GraphBLAS objects (input or output) is in an invalid state caused 1310 by a previous execution error. Call GrB\_error() to access any error 1311 messages generated by the implementation. 1312 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 1313 GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new 1314 (needed for user-defined types). 1315 GrB NULL POINTER The s pointer is NULL. 1316

#### 7 Description

Creates a new GraphBLAS scalar s of domain  $\mathbf{D}(\mathsf{d})$  and empty  $\mathbf{L}(s)$ . The method returns a handle to the new scalar in s.

1320 It is not an error to call this method more than once on the same variable; however, the handle to 1321 the previously created object will be overwritten.

# 4.2.2.2 Scalar\_dup: Construct a copy of a GraphBLAS scalar

1323 Creates a new scalar with the same domain and contents as another scalar.

# 1324 C Syntax

```
GrB_Info GrB_Scalar_dup(GrB_Scalar *t, const GrB_Scalar s);
```

#### 1327 Parameters

1330

1331

1336

t (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.

s (IN) The GraphBLAS scalar to be duplicated.

#### Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar t is ready to be used in the next method of the sequence.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1337 GraphBLAS objects (input or output) is in an invalid state caused 1338 by a previous execution error. Call GrB\_error() to access any error 1339 messages generated by the implementation. 1340 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 1341 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to 1342 Scalar\_new or Scalar\_dup. 1343 GrB\_NULL\_POINTER The t pointer is NULL. 1344 Description 1345 Creates a new scalar t of domain  $\mathbf{D}(s)$  and contents  $\mathbf{L}(s)$ . The method returns a handle to the new 1346 scalar in t. 1347 It is not an error to call this method more than once with the same output variable; however, the 1348 handle to the previously created object will be overwritten. 1340 4.2.2.3 Scalar\_clear: Clear/remove a stored value from a scalar Removes the stored value from a scalar. 135 C Syntax 1352 1353 GrB\_Info GrB\_Scalar\_clear(GrB\_Scalar s); **Parameters** s (INOUT) An existing GraphBLAS scalar to clear. 1355 Return Values 1356 GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-1357 blocking mode, this indicates that the API checks for the input 1358 arguments passed successfully. Either way, output scalar s is ready 1359 to be used in the next method of the sequence. 1360 GrB\_PANIC Unknown internal error. 1361 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1362 GraphBLAS objects (input or output) is in an invalid state caused 1363 by a previous execution error. Call GrB\_error() to access any error 1364 messages generated by the implementation. 1365

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

# 1369 Description

Removes the stored value from an existing scalar. After the call, L(s) is empty. The size of the scalar does not change.

## 1372 4.2.2.4 Scalar\_nvals: Number of stored elements in a scalar

Retrieve the number of stored elements in a scalar (either zero or one).

# 1374 C Syntax

```
GrB_Info GrB_Scalar_nvals(GrB_Index *nvals, const GrB_Scalar s);
```

#### 1377 Parameters

1380

nvals (OUT) On successful return, this is set to the number of stored elements in the scalar (zero or one).

s (IN) An existing GraphBLAS scalar being queried.

#### 1381 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

GrB\_NULL\_POINTER The nvals pointer is NULL.

### 1393 Description

Return nvals(s) in nvals. This is the number of stored elements in scalar s, which is the size of L(s), and can only be either zero or one (see Section 3.5.1).

# 1396 4.2.2.5 Scalar\_setElement: Set the single element in a scalar

1397 Set the single element of a scalar to a given value.

# 1398 C Syntax

```
GrB_Info GrB_Scalar_setElement(GrB_Scalar s, <br/>
<type> val);
```

#### 1401 Parameters

1403

1410

1415

1418

s (INOUT) An existing GraphBLAS scalar for which the element is to be assigned.

val (IN) Scalar value to assign. The type must be compatible with the domain of s.

#### 1404 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output scalar s is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

GrB\_DOMAIN\_MISMATCH The domains of s and val are incompatible.

### 1419 Description

First, val and output GraphBLAS scalar are tested for domain compatibility as follows: **D**(val) must be compatible with **D**(s). Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_Scalar\_setElement ends and the domain mismatch error listed above is returned.

We are now ready to carry out the assignment val; that is:

$$\mathsf{s}(0) = \mathsf{val}$$

1428 If s already had a stored value, it will be overwritten; otherwise, the new value is stored in s.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of s is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of scalar s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 4.2.2.6 Scalar\_extractElement: Extract a single element from a scalar.

Assign a non-opaque scalar with the value of the element stored in a GraphBLAS scalar.

# 1435 C Syntax

```
GrB_Info GrB_Scalar_extractElement(<type> *val,
const GrB_Scalar s);
```

#### 38 Parameters

val (INOUT) Pointer to a non-opaque scalar of type that is compatible with the domain of scalar s. On successful return, val holds the result of the operation, and any previous value in val is overwritten.

s (IN) The GraphBLAS scalar from which an element is extracted.

#### 1443 Return Values

1442

1444

1445

1446

1447

1448

1449

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val, has been computed and is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1450 GraphBLAS objects (input or output) is in an invalid state caused 1451 by a previous execution error. Call GrB\_error() to access any error 1452 messages generated by the implementation. 1453 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 1454 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to 1455 Scalar\_new or Scalar\_dup. 1456 GrB\_NULL\_POINTER val pointer is NULL. 1457

GrB\_DOMAIN\_MISMATCH The domains of the scalar or scalar are incompatible. 1458

GrB NO VALUE There is no stored value in the scalar.

#### Description 1460

1459

First, val and input GraphBLAS scalar are tested for domain compatibility as follows: D(val) 1461 must be compatible with  $\mathbf{D}(s)$ . Two domains are compatible with each other if values from 1462 one domain can be cast to values in the other domain as per the rules of the C language. In 1463 particular, domains from Table 3.2 are all compatible with each other. A domain from a user-1464 defined type is only compatible with itself. If any compatibility rule above is violated, execution of 1465 GrB\_Scalar\_extractElement ends and the domain mismatch error listed above is returned. 1466

Then, if no value is currently stored in the GraphBLAS scalar, the method returns GrB\_NO\_VALUE 1467 and val remains unchanged. 1468

Finally the extract into the output argument, val can be performed; that is: 1469

$$val = s(0)$$

In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value 1471 GrB\_SUCCESS, the new contents of val are as defined above.

#### 4.2.3 Vector methods 1473

#### 4.2.3.1Vector\_new: Construct new vector 1474

Creates a new vector with specified domain and size. 1475

#### C Syntax 1476

```
GrB_Info GrB_Vector_new(GrB_Vector *v,
1477
                                         GrB_Type
                                                       d,
1478
                                         GrB Index
                                                       nsize);
1479
```

#### 1480 Parameters

- v (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.
- d (IN) The type corresponding to the domain of the vector being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nsize (IN) The size of the vector being created.

#### 487 Return Values

1492

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector v is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new (needed for user-defined types).

GrB\_NULL\_POINTER The v pointer is NULL.

Grb INVALID VALUE nsize is zero or outside the range of the type Grb Index.

#### 1502 Description

Creates a new vector  $\mathbf{v}$  of domain  $\mathbf{D}(\mathsf{d})$ , size nsize, and empty  $\mathbf{L}(\mathbf{v})$ . The method returns a handle to the new vector in  $\mathbf{v}$ .

1505 It is not an error to call this method more than once on the same variable; however, the handle to 1506 the previously created object will be overwritten.

# 4.2.3.2 Vector\_dup: Construct a copy of a GraphBLAS vector

1508 Creates a new vector with the same domain, size, and contents as another vector.

# 1509 C Syntax

```
GrB_Info GrB_Vector_dup(GrB_Vector *w, const GrB_Vector u);
```

#### 1512 Parameters

1515

w (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.

u (IN) The GraphBLAS vector to be duplicated.

#### 1516 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

1526 GrB OUT OF MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, u, has not been initialized by a call to Vector new or Vector dup.

GrB\_NULL\_POINTER The w pointer is NULL.

#### $\mathbf{Description}$

1535

Creates a new vector  $\mathbf{w}$  of domain  $\mathbf{D}(u)$ , size  $\mathbf{size}(u)$ , and contents  $\mathbf{L}(u)$ . The method returns a handle to the new vector in  $\mathbf{w}$ .

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

#### 4.2.3.3 Vector\_resize: Resize a vector

1536 Changes the size of an existing vector.

# C Syntax

1537

1542

1548

1555

```
GrB_Info GrB_Vector_resize(GrB_Vector w,
GrB_Index nsize);
```

#### 1540 Parameters

w (INOUT) An existing Vector object that is being resized.

nsize (IN) The new size of the vector. It can be smaller or larger than the current size.

#### 1543 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

Grb Out of Memory Not enough memory available for operation.

GrB\_NULL\_POINTER The w pointer is NULL.

GrB\_INVALID\_VALUE nsize is zero or outside the range of the type GrB\_Index.

#### 1556 Description

Changes the size of w to nsize. The domain  $\mathbf{D}(w)$  of vector w remains the same. The contents  $\mathbf{L}(w)$  are modified as described below.

Let  $w = \langle \mathbf{D}(w), N, \mathbf{L}(w) \rangle$  when the method is called. When the method returns,  $w = \langle \mathbf{D}(w), \text{nsize}, \mathbf{L}'(w) \rangle$ where  $\mathbf{L}'(w) = \{(i, w_i) : (i, w_i) \in \mathbf{L}(w) \land (i < \text{nsize})\}$ . That is, all elements of w with index greater than or equal to the new vector size (nsize) are dropped.

# 1562 4.2.3.4 Vector\_clear: Clear a vector

Removes all the elements (tuples) from a vector.

```
C Syntax
              GrB_Info GrB_Vector_clear(GrB_Vector v);
1565
    Parameters
                  v (INOUT) An existing GraphBLAS vector to clear.
1567
    Return Values
1568
                    GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
1569
                                    blocking mode, this indicates that the API checks for the input
1570
                                    arguments passed successfully. Either way, output vector v is ready
1571
                                    to be used in the next method of the sequence.
1572
                       GrB PANIC Unknown internal error.
1573
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
1574
                                    GraphBLAS objects (input or output) is in an invalid state caused
1575
                                    by a previous execution error. Call GrB_error() to access any error
1576
                                    messages generated by the implementation.
1577
        GrB_OUT_OF_MEMORY Not enough memory available for operation.
1578
   GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to
1579
                                    Vector_new or Vector_dup.
1580
    Description
1581
    Removes all elements (tuples) from an existing vector. After the call to GrB_Vector_clear(v),
1582
    \mathbf{L}(\mathbf{v}) = \emptyset. The size of the vector does not change.
1583
    4.2.3.5
              Vector_size: Size of a vector
1584
    Retrieve the size of a vector.
    C Syntax
1586
```

const GrB\_Vector v);

\*nsize,

GrB\_Info GrB\_Vector\_size(GrB\_Index

1587

1588

#### Parameters 589

1591

nsize (OUT) On successful return, is set to the size of the vector.

v (IN) An existing GraphBLAS vector being queried.

#### 1592 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nsize has been set.

1595 GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_NULL\_POINTER nsize pointer is NULL.

# 1603 Description

1604 Return size(v) in nsize.

# 4.2.3.6 Vector\_nvals: Number of stored elements in a vector

Retrieve the number of stored elements (tuples) in a vector.

# 1607 C Syntax

```
GrB_Info GrB_Vector_nvals(GrB_Index *nvals, const GrB_Vector v);
```

#### Parameters

1610

nvals (OUT) On successful return, this is set to the number of stored elements (tuples) in the vector.

v (IN) An existing GraphBLAS vector being queried.

#### 1614 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_NULL\_POINTER The nvals pointer is NULL.

# 1626 Description

Return  $\mathbf{nvals}(\mathbf{v})$  in  $\mathbf{nvals}$ . This is the number of stored elements in vector  $\mathbf{v}$ , which is the size of  $\mathbf{L}(\mathbf{v})$  (see Section 3.5.2).

# 4.2.3.7 Vector\_build: Store elements from tuples into a vector

# 1630 C Syntax

```
GrB_Info GrB_Vector_build(GrB_Vector w,
const GrB_Index *indices,
const <type> *values,
GrB_Index n,
const GrB_BinaryOp dup);
```

# 1636 Parameters

1638

1641

w (INOUT) An existing Vector object to store the result.

indices (IN) Pointer to an array of indices.

values (IN) Pointer to an array of scalars of a type that is compatible with the domain of vector w.

n (IN) The number of entries contained in each array (the same for indices and values).

dup (IN) An associative and commutative binary operator to apply when duplicate values for the same location are present in the input arrays. All three domains of dup must be the same; hence  $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$ . If dup is GrB\_NULL, then duplicate locations will result in an error.

#### 6 Return Values

1642

1643

1644

1645

1651

1656

1660

1667

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT Either w has not been initialized by a call to by GrB\_Vector\_new or by GrB\_Vector\_dup, or dup has not been initialized by a call to by GrB\_BinaryOp\_new.

GrB NULL POINTER indices or values pointer is NULL.

1661 GrB INDEX OUT OF BOUNDS A value in indices is outside the allowed range for w.

GrB\_DOMAIN\_MISMATCH Either the domains of the GraphBLAS binary operator dup are not all the same, or the domains of values and w are incompatible with each other or  $D_{dup}$ .

GrB\_OUTPUT\_NOT\_EMPTY Output vector w already contains valid tuples (elements). In other words, GrB\_Vector\_nvals(C) returns a positive value.

GrB\_INVALID\_VALUE indices contains a duplicate location and dup is GrB\_NULL.

#### 1668 Description

If dup is not GrB\_NULL, an internal vector  $\widetilde{\mathbf{w}} = \langle D_{dup}, \mathbf{size}(\mathbf{w}), \emptyset \rangle$  is created, which only differs from w in its domain; otherwise,  $\widetilde{\mathbf{w}} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \emptyset \rangle$ .

Each tuple {indices[k], values[k]}, where  $0 \le k < n$ , is a contribution to the output in the form of

$$\widetilde{\mathbf{w}}(\mathsf{indices}[\mathsf{k}]) = \begin{cases} (D_{\mathit{dup}})\,\mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{w}))\,\mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB\_NULL, dup is used to reduce the values before assignment into  $\widetilde{\mathbf{w}}$  as follows:

$$\widetilde{\mathbf{w}}_i = \bigoplus_{k: \, \mathsf{indices[k]} = i} (D_{dup}) \, \mathsf{values[k]},$$

where  $\oplus$  is the dup binary operator. Finally, the resulting  $\widetilde{\mathbf{w}}$  is copied into  $\mathbf{w}$  via typecasting its values to  $\mathbf{D}(\mathbf{w})$  if necessary. If  $\oplus$  is not associative or not commutative, the result is undefined.

The nonopaque input arrays, indices and values, must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB\_Vector\_nvals(w) should evaluate to zero prior to calling this function.

After GrB\_Vector\_build returns, it is safe for a programmer to modify or delete the arrays indices or values.

# 1683 4.2.3.8 Vector\_setElement: Set a single element in a vector

Set one element of a vector to a given value.

# 1685 C Syntax

```
// scalar value
1686
             GrB_Info GrB_Vector_setElement(GrB_Vector
1687
                                                                      W,
                                                  <type>
                                                                      val,
1688
                                                 GrB_Index
                                                                      index);
1689
1690
              // GraphBLAS scalar
1691
             GrB_Info GrB_Vector_setElement(GrB_Vector
1692
                                                  const GrB_Scalar
1693
                                                 GrB_Index
                                                                      index);
1694
```

# 1695 Parameters

1697

1698

1700

1701

1702

w (INOUT) An existing GraphBLAS vector for which an element is to be assigned.

val or s (IN) Scalar assign. Its domain (type) must be compatible with the domain of w.

index (IN) The location of the element to be assigned.

#### 1699 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed suc-

cessfully. Either way, the output vector **w** is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, w, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

GrB\_DOMAIN\_MISMATCH The domains of the vector and the scalar are incompatible.

# Description

1714

First, the scalar and output vector are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{w})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_Vector\_setElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

If this condition is violated, execution of GrB\_Vector\_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$w(\mathsf{index}) = \begin{cases} \mathbf{L}(\mathsf{s}), & \operatorname{GraphBLAS\ scalar}.\\ \mathsf{val}, & \operatorname{otherwise}. \end{cases}$$

In the case of a transparent scalar or if  $\mathbf{L}(s)$  is not empty, then a value will be stored at the specified location in  $\mathbf{w}$ , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar, if  $\mathbf{L}(s)$  is empty, then any value stored at the specified location in  $\mathbf{w}$  will be removed.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 1734 4.2.3.9 Vector\_removeElement: Remove an element from a vector

1735 Remove (annihilate) one stored element from a vector.

# 1736 C Syntax

GrB\_Info GrB\_Vector\_removeElement(GrB\_Vector w,
GrB\_Index index);

#### 1739 Parameters

w (INOUT) An existing GraphBLAS vector from which an element is to be removed.

index (IN) The location of the element to be removed.

#### 1742 Return Values

1748

1757

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, w, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

#### Description

First, the index parameter is checked for a valid value where the following condition must hold:

$$0 < \mathsf{index} < \mathsf{size}(\mathsf{w})$$

If this condition is violated, execution of GrB\_Vector\_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by index. If a value does not exist at the specified location in w, no error is reported and the operation has no effect on the state of w. In either case, the following will be true on return from the method: index  $\notin$  ind(w).

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 4.2.3.10 Vector\_extractElement: Extract a single element from a vector.

Extract one element of a vector into a scalar.

# 1772 C Syntax

```
// scalar value
1773
             GrB_Info GrB_Vector_extractElement(<type>
                                                                          *val,
1774
                                                      const GrB_Vector
                                                                           u,
1775
                                                      GrB_Index
                                                                           index);
1776
1777
             // GraphBLAS scalar
1778
             GrB_Info GrB_Vector_extractElement(GrB_Scalar
                                                                           s,
1779
                                                      const GrB Vector
1780
                                                                           u,
                                                      GrB_Index
                                                                           index);
1781
```

#### 1782 Parameters

1786

1787

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1793

val or s (INOUT) An existing scalar of whose domain is compatible with the domain of vector
u. On successful return, this scalar holds the result of the extract. Any previous
value stored in val or s is overwritten.

u (IN) The GraphBLAS vector from which an element is extracted.

index (IN) The location in u to extract.

#### 1788 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val or s, has been computed and is ready to be used in the next method of the sequence.

GrB\_NO\_VALUE When using the transparent scalar, val, this is returned when there is no stored value at specified location.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB OUT OF MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, u, or scalar, s, has not been initialized by a call to a corresponding constructor.

1804 GrB NULL POINTER val pointer is NULL.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

GrB DOMAIN MISMATCH The domains of the vector and scalar are incompatible.

# 1807 Description

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First, the scalar and input vector are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{u})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of  $\mathsf{GrB\_Vector\_extractElement}$  ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 \leq \mathsf{index} < \mathbf{size}(\mathsf{u})$$

If this condition is violated, execution of GrB\_Vector\_extractElement ends and the invalid index error listed above is returned.

We are now ready to carry out the extract into the output scalar; that is:

$$\left. \begin{array}{c} \mathbf{L}(\mathsf{s}) \\ \mathsf{val} \end{array} \right\} = \mathsf{u}(\mathsf{index})$$

If index  $\in$  ind(u), then the corresponding value from u is copied into s or val with casting as necessary. If index  $\notin$  ind(u), then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB\_SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB\_NO\_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB\_BLOCKING mode GrB\_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB\_SUCCESS or GrB\_NO\_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB\_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB\_BLOCKING mode.

In GrB\_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 4.2.3.11 Vector\_extractTuples: Extract tuples from a vector

Extract the contents of a GraphBLAS vector into non-opaque data structures.

# 1831 C Syntax

1832 1833 1834 1835 1836	<pre>GrB_Info GrB_Vector_extractTuples(GrB_Index *indices,</pre>
1837 1838	indices (OUT) Pointer to an array of indices that is large enough to hold all of the stored values' indices.
1839 1840	values (OUT) Pointer to an array of scalars of a type that is large enough to hold all of the stored values whose type is compatible with $\mathbf{D}(\mathbf{v})$ .
1841 1842 1843	n (INOUT) Pointer to a value indicating (on input) the number of elements the values and indices arrays can hold. Upon return, it will contain the number of values written to the arrays.
1844	v (IN) An existing GraphBLAS vector.

#### Return Values

1846	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed suc-
1847		cessfully. This indicates that the compatibility tests on the input
1848		argument passed successfully, and the output arrays, indices and
1849		values, have been computed.
1850	GrB_PANIC	Unknown internal error.
1851	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
1852		GraphBLAS objects (input or output) is in an invalid state caused
1853		by a previous execution error. Call GrB_error() to access any error
1854		messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_INSUFFICIENT\_SPACE Not enough space in indices and values (as indicated by the n parameter) to hold all of the tuples that will be extacted.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_NULL\_POINTER indices, values, or n pointer is NULL.

GrB\_DOMAIN\_MISMATCH The domains of the v vector or values array are incompatible with one another.

#### 1863 Description

This method will extract all the tuples from the GraphBLAS vector v. The values associated with those tuples are placed in the values array and the indices are placed in the indices array.

Both indices and values must be pre-allocated by the user to have enough space to hold at least GrB Vector nvals(v) elements before calling this function.

Upon return of this function, n will be set to the number of values (and indices) copied. Also, the entries of indices are unique, but not necessarily sorted. Each tuple  $(i, v_i)$  in v is unzipped and copied into a distinct kth location in output vectors:

$$\{\mathsf{indices}[\mathsf{k}], \mathsf{values}[\mathsf{k}]\} \leftarrow (i, v_i),$$

where  $0 \le k < GrB\_Vector\_nvals(v)$ . No gaps in output vectors are allowed; that is, if indices[k] and values[k] exist upon return, so does indices[j] and values[j] for all j such that  $0 \le j < k$ .

Note that if the value in n on input is less than the number of values contained in the vector v, then a GrB\_INSUFFICIENT\_SPACE error is returned because it is undefined which subset of values would be extracted otherwise.

In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value GrB\_SUCCESS, the new contents of the arrays indices and values are as defined above.

# 1878 4.2.4 Matrix methods

#### 1879 4.2.4.1 Matrix new: Construct new matrix

1880 Creates a new matrix with specified domain and dimensions.

# 1881 C Syntax

1884	${\tt GrB\_Index}$	nrows,
1885	GrB_Index	ncols);

#### 1886 Parameters

- A (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.
- d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nrows (IN) The number of rows of the matrix being created.
- ncols (IN) The number of columns of the matrix being created.

#### 1894 Return Values

1895	GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
1896	blocking mode, this indicates that the API checks for the input ar-
1897	guments passed successfully. Either way, output matrix A is ready
1898	to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new (needed for user-defined types).

1907 Grb NULL POINTER The A pointer is NULL.

GrB\_INVALID\_VALUE nrows or ncols is zero or outside the range of the type GrB\_Index.

# 1909 Description

- Creates a new matrix  $\mathbf{A}$  of domain  $\mathbf{D}(\mathsf{d})$ , size nrows  $\times$  ncols, and empty  $\mathbf{L}(\mathbf{A})$ . The method returns a handle to the new matrix in  $\mathbf{A}$ .
- 1912 It is not an error to call this method more than once on the same variable; however, the handle to 1913 the previously created object will be overwritten.

# 1914 4.2.4.2 Matrix\_dup: Construct a copy of a GraphBLAS matrix

1915 Creates a new matrix with the same domain, dimensions, and contents as another matrix.

# 1916 C Syntax

```
GrB_Info GrB_Matrix_dup(GrB_Matrix *C,
const GrB Matrix A);
```

# 1919 Parameters

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1925

1926

1927

1928

1936

C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.

A (IN) The GraphBLAS matrix to be duplicated.

#### 1923 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER The C pointer is NULL.

# 1937 Description

Creates a new matrix C of domain D(A), size  $\mathbf{nrows}(A) \times \mathbf{ncols}(A)$ , and contents L(A). It returns a handle to it in C.

1940 It is not an error to call this method more than once on the same variable; however, the handle to 1941 the previously created object will be overwritten.

# 1942 4.2.4.3 Matrix\_diag: Construct a diagonal GraphBLAS matrix

Creates a new matrix with the same domain and contents as a GrB\_Vector, and square dimensions appropriate for placing the contents of the vector along the specified diagonal of the matrix.

# 1945 C Syntax

```
GrB_Info GrB_Matrix_diag(GrB_Matrix *C,
const GrB_Vector v,
int64_t k);
```

#### 1949 Parameters

- C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix. The matrix is square with each dimension equal to  $\operatorname{size}(\mathsf{v}) + |k|$ .
- v (IN) The GraphBLAS vector whose contents will be copied to the diagonal of the matrix.
- k (IN) The diagonal to which the vector is assigned. k=0 represents the main diagonal, k>0 is above the main diagonal, and k<0 is below.

#### 1956 Return Values

1961

1957 GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-1958 blocking mode, this indicates that the API checks for the input 1959 arguments passed successfully. Either way, output matrix C is ready 1960 to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

1969 GrB\_NULL\_POINTER The C pointer is NULL.

1971 Creates a new matrix C of domain D(v), size  $(size(v) + |k|) \times (size(v) + |k|)$ , and contents

1972 
$$\mathbf{L}(\mathsf{C}) = \{(i, i + k, v_i) : (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k \ge 0 \text{ or}$$
1973 
$$\mathbf{L}(\mathsf{C}) = \{(i - k, i, v_i) : (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k \ge 0.$$

It returns a handle to it in C. It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1976 4.2.4.4 Matrix\_resize: Resize a matrix

1977 Changes the dimensions of an existing matrix.

# 1978 C Syntax

```
GrB_Info GrB_Matrix_resize(GrB_Matrix C,
GrB_Index nrows,
GrB_Index ncols);
```

#### 1982 Parameters

1983

1984

1985

1986

1987

1993

1998

C (INOUT) An existing Matrix object that is being resized.

nrows (IN) The new number of rows of the matrix. It can be smaller or larger than the current number of rows.

ncols (IN) The new number of columns of the matrix. It can be smaller or larger than the current number of columns.

#### 1988 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

```
GrB_NULL_POINTER The C pointer is NULL.
1999
             GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.
2000
    Description
2001
     Changes the number of rows and columns of C to nrows and ncols, respectively. The domain \mathbf{D}(\mathsf{C})
2002
     of matrix C remains the same. The contents L(C) are modified as described below.
2003
     Let C = \langle \mathbf{D}(C), M, N, \mathbf{L}(C) \rangle when the method is called. When the method returns C is modified
2004
     to C = \langle D(C), \text{nrows}, \text{ncols}, L'(C) \rangle where L'(C) = \{(i, j, C_{ij}) : (i, j, C_{ij}) \in L(C) \land (i < \text{nrows}) \land (j < i) \}
2005
     ncols). That is, all elements of C with row index greater than or equal to nrows or column index
2006
     greater than or equal to ncols are dropped.
2007
     4.2.4.5
               Matrix_clear: Clear a matrix
2008
     Removes all elements (tuples) from a matrix.
2009
     C Syntax
2010
               GrB_Info GrB_Matrix_clear(GrB_Matrix A);
2011
     Parameters
2012
                   A (IN) An exising GraphBLAS matrix to clear.
2013
     Return Values
2014
                     GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
2015
                                      blocking mode, this indicates that the API checks for the input ar-
2016
                                      guments passed successfully. Either way, output matrix A is ready
2017
                                      to be used in the next method of the sequence.
2018
                         GrB PANIC Unknown internal error.
2019
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2020
                                      GraphBLAS objects (input or output) is in an invalid state caused
2021
                                      by a previous execution error. Call GrB_error() to access any error
2022
                                      messages generated by the implementation.
2023
```

2025 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

Grb Out of Memory Not enough memory available for operation.

2024

Removes all elements (tuples) from an existing matrix. After the call to  $GrB\_Matrix\_clear(A)$ ,  $L(A) = \emptyset$ . The dimensions of the matrix do not change.

# 2030 4.2.4.6 Matrix\_nrows: Number of rows in a matrix

2031 Retrieve the number of rows in a matrix.

# 2032 C Syntax

```
GrB_Info GrB_Matrix_nrows(GrB_Index *nrows, const GrB_Matrix A);
```

#### 2035 Parameters

2037

2041

nrows (OUT) On successful return, contains the number of rows in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

#### 2038 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nrows has been set.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

2046 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

2048 GrB\_NULL\_POINTER nrows pointer is NULL.

# 2049 Description

2050 Return **nrows**(A) in **nrows** (the number of rows).

## 2051 4.2.4.7 Matrix ncols: Number of columns in a matrix

2052 Retrieve the number of columns in a matrix.

```
C Syntax
2053
              GrB_Info GrB_Matrix_ncols(GrB_Index
                                                                *ncols,
2054
                                            const GrB_Matrix
                                                                A);
2055
    Parameters
               ncols (OUT) On successful return, contains the number of columns in the matrix.
2057
                  A (IN) An existing GraphBLAS matrix being queried.
2058
    Return Values
                   GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2060
                                   cessfully and the value of ncols has been set.
2061
                       GrB_PANIC Unknown internal error.
2062
          GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2063
                                   GraphBLAS objects (input or output) is in an invalid state caused
2064
                                   by a previous execution error. Call GrB_error() to access any error
2065
                                   messages generated by the implementation.
2066
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to
2067
                                   any matrix constructor.
2068
            GrB_NULL_POINTER ncols pointer is NULL.
2069
    Description
2070
    Return ncols(A) in ncols (the number of columns).
2071
    4.2.4.8
              Matrix_nvals: Number of stored elements in a matrix
2072
    Retrieve the number of stored elements (tuples) in a matrix.
2073
    C Syntax
2074
              GrB_Info GrB_Matrix_nvals(GrB_Index
                                                                *nvals,
2075
```

2076

const GrB\_Matrix A);

#### 2077 Parameters

nvals (OUT) On successful return, contains the number of stored elements (tuples) in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

#### 2081 Return Values

2080

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

<sup>2090</sup> GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

2092 GrB\_NULL\_POINTER The nvals pointer is NULL.

# 2093 Description

Return nvals(A) in nvals. This is the number of tuples stored in matrix A, which is the size of L(A) (see Section 3.5.3).

# 2096 4.2.4.9 Matrix\_build: Store elements from tuples into a matrix

# 2097 C Syntax

## 98 Parameters

2099

C (INOUT) An existing Matrix object to store the result.

row\_indices (IN) Pointer to an array of row indices. 2100 col\_indices (IN) Pointer to an array of column indices. 2101 values (IN) Pointer to an array of scalars of a type that is compatible with the domain of 2102 matrix, C. 2103 n (IN) The number of entries contained in each array (the same for row indices, 2104 col indices, and values). 2105 dup (IN) An associative and commutative binary operator to apply when duplicate 2106 values for the same location are present in the input arrays. All three domains of 2107 dup must be the same; hence  $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$ . If dup is GrB\_NULL, 2108 then duplicate locations will result in an error. 2109 Return Values 2110 GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-2111 blocking mode, this indicates that the API checks for the input 2112 arguments passed successfully. Either way, output matrix C is 2113 ready to be used in the next method of the sequence. 2114 GrB\_PANIC Unknown internal error. 2115 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the 2116 opaque GraphBLAS objects (input or output) is in an invalid 2117 state caused by a previous execution error. Call GrB error() to 2118 access any error messages generated by the implementation. 2119 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 2120 GrB UNINITIALIZED OBJECT Either C has not been initialized by a call to any matrix construc-2121 tor, or dup has not been initialized by a call to by GrB BinaryOp new. 2122 GrB\_NULL\_POINTER row\_indices, col\_indices or values pointer is NULL. 2123 GrB\_INDEX\_OUT\_OF\_BOUNDS A value in row\_indices or col\_indices is outside the allowed range 2124 for C. 2125 Grb\_DOMAIN\_MISMATCH Either the domains of the GraphBLAS binary operator dup are 2126 not all the same, or the domains of values and C are incompatible 2127 with each other or  $D_{dup}$ . 2128 Grb Output NOT EMPTY Output matrix C already contains valid tuples (elements). In 2129 other words, GrB\_Matrix\_nvals(C) returns a positive value.

2131

GrB\_INVALID\_VALUE indices contains a duplicate location and dup is GrB\_NULL.

If dup is not GrB\_NULL, an internal matrix  $\widetilde{\mathbf{C}} = \langle D_{dup}, \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$  is created, which only differs from C in its domain; otherwise,  $\widetilde{\mathbf{C}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$ .

Each tuple {row\_indices[k], col\_indices[k], values[k]}, where  $0 \le k < n$ , is a contribution to the output in the form of

$$\widetilde{\mathbf{C}}(\mathsf{row\_indices}[\mathsf{k}],\mathsf{col\_indices}[\mathsf{k}]) = \begin{cases} (D_{dup})\,\mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{C}))\,\mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB\_NULL, dup is used to reduce the values before assignment into  $\tilde{\mathbf{C}}$  as follows:

$$\widetilde{\mathbf{C}}_{ij} = \bigoplus_{k:\, \mathsf{row\_indices}[\mathtt{k}] = i \, \land \, \mathsf{col\_indices}[\mathtt{k}] = j} (D_{dup}) \, \mathsf{values}[\mathtt{k}],$$

where  $\oplus$  is the dup binary operator. Finally, the resulting  $\widetilde{\mathbf{C}}$  is copied into C via typecasting its values to  $\mathbf{D}(C)$  if necessary. If  $\oplus$  is not associative or not commutative, the result is undefined.

The nonopaque input arrays row\_indices, col\_indices, and values must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB\_Matrix\_nvals(C) should evaluate to zero prior to calling this function.

After GrB\_Matrix\_build returns, it is safe for a programmer to modify or delete the arrays row\_indices, col\_indices, or values.

#### 4.2.4.10 Matrix\_setElement: Set a single element in matrix

2149 Set one element of a matrix to a given value.

#### 2150 C Syntax

```
// scalar value
2151
             GrB Info GrB Matrix setElement(GrB Matrix
                                                                        C,
2152
                                                  <type>
                                                                        val,
2153
                                                  GrB_Index
                                                                        row_index,
2154
                                                  GrB Index
                                                                        col_index);
2155
2156
              // GraphBLAS scalar
2157
             GrB_Info GrB_Matrix_setElement(GrB_Matrix
                                                                        С,
2158
                                                  const GrB_Scalar
2159
                                                  GrB_Index
                                                                        row_index,
2160
                                                  GrB Index
                                                                        col index);
2161
```

#### 2162 Parameters

C (INOUT) An existing GraphBLAS matrix for which an element is to be assigned.

val or s (IN) Scalar to assign. Its domain (type) must be compatible with the domain of
C.

row\_index (IN) Row index of element to be assigned

# Return Values

2167

2168

2174

2184

2193

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

col\_index (IN) Column index of element to be assigned

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

2180 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB\_INVALID\_INDEX row\_index or col\_index is outside the allowable range (i.e., not less than  $\mathbf{nrows}(C)$  or  $\mathbf{ncols}(C)$ , respectively).

Grb\_DOMAIN\_MISMATCH The domains of the matrix and the scalar are incompatible.

# 2185 Description

First, the scalar and output matrix are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{C})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of  $\mathsf{GrB}_\mathsf{Matrix\_setElement}$  ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

$$0 \le \text{row\_index} < \mathbf{nrows}(C),$$
  
 $0 \le \text{col\_index} < \mathbf{ncols}(C)$ 

If either of these conditions is violated, execution of GrB\_Matrix\_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$C(row\_index, col\_index) = \begin{cases} \mathbf{L}(s), & GraphBLAS \ scalar. \\ val, & otherwise. \end{cases}$$

In the case of a transparent scalar or if  $\mathbf{L}(s)$  is not empty, then a value will be stored at the specified location in  $\mathsf{C}$ , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar and if  $\mathbf{L}(s)$  is empty, then any value stored at the specified location in  $\mathsf{C}$  will be removed.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 2204 4.2.4.11 Matrix\_removeElement: Remove an element from a matrix

2205 Remove (annihilate) one stored element from a matrix.

# 2206 C Syntax

```
GrB_Info GrB_Matrix_removeElement(GrB_Matrix C,

GrB_Index row_index,

GrB_Index col_index);
```

#### Parameters

2213

2214

2215

2216

2217

2218

2219

2220

<sup>2211</sup> C (INOUT) An existing GraphBLAS matrix from which an element is to be removed.

row\_index (IN) Row index of element to be removed

col index (IN) Column index of element to be removed

# Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

```
GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.
```

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

2226 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, C, has not been initialized by a call to any matrix constructor.

GrB\_INVALID\_INDEX row\_index or col\_index is outside the allowable range (i.e., not less than nrows(C) or ncols(C), respectively).

## 2230 Description

2225

First, both index parameters are checked for valid values where following conditions must hold:

```
0 \le \text{row\_index} < \mathbf{nrows}(\mathsf{C}),0 \le \text{col index} < \mathbf{ncols}(\mathsf{C})
```

If either of these conditions is violated, execution of GrB\_Matrix\_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by (row\_index, col\_index). If a value does not exist at the specified location in C, no error is reported and the operation has no effect on the state of C. In either case, the following will be true on return from this method: (row\_index, col\_index)  $\notin$  ind(C)

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

#### 2243 4.2.4.12 Matrix\_extractElement: Extract a single element from a matrix

Extract one element of a matrix into a scalar.

# 2245 C Syntax

```
// scalar value
2246
             GrB_Info GrB_Matrix_extractElement(<type>
                                                                          *val,
2247
2248
                                                      const GrB_Matrix
                                                      GrB_Index
                                                                           row_index,
2249
                                                      GrB_Index
                                                                           col_index);
2250
2251
              // GraphBLAS scalar
2252
```

```
GrB_Info GrB_Matrix_extractElement(GrB_Scalar
                                                                             s,
2253
                                                        const GrB_Matrix
                                                                             Α,
2254
                                                        GrB_Index
                                                                             row_index,
2255
                                                        GrB_Index
                                                                             col_index);
2256
2257
    Parameters
2258
           val or s (INOUT) An existing scalar whose domain is compatible with the domain of matrix
2259
                   A. On successful return, this scalar holds the result of the extract. Any previous
2260
                   value stored in val or s is overwritten.
2261
                 A (IN) The GraphBLAS matrix from which an element is extracted.
2262
        row index (IN) The row index of location in A to extract.
2263
         col_index (IN) The column index of location in A to extract.
2264
    Return Values
2265
                    GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2266
                                    cessfully. This indicates that the compatibility tests on dimensions
2267
                                    and domains for the input arguments passed successfully, and the
2268
                                    output scalar, val or s, has been computed and is ready to be used
2269
                                    in the next method of the sequence.
2270
                  GrB_NO_VALUE When using the transparent scalar, val, this is returned when there
2271
                                    is no stored value at specified location.
2272
                       GrB_PANIC Unknown internal error.
2273
           Grb INVALID OBJECT This is returned in any execution mode whenever one of the opaque
2274
                                    GraphBLAS objects (input or output) is in an invalid state caused
2275
                                    by a previous execution error. Call GrB_error() to access any error
2276
                                    messages generated by the implementation.
2277
        GrB OUT OF MEMORY Not enough memory available for operation.
2278
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, or scalar, s, has not been initialized by
2279
                                    a call to a corresponding constructor.
2280
            GrB_NULL_POINTER val pointer is NULL.
2281
            GrB INVALID INDEX row index or col index is outside the allowable range (i.e. less than
2282
```

GrB\_DOMAIN\_MISMATCH The domains of the matrix and scalar are incompatible.

tively).

2283

2284

2285

zero or greater than or equal to  $\mathbf{nrows}(A)$  or  $\mathbf{ncols}(A)$ , respec-

2294

2300

2301

First, the scalar and input matrix are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of  $\mathsf{GrB}_{\mathsf{Matrix}\_\mathsf{extractElement}}$  ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

$$0 \le \text{row\_index} < \mathbf{nrows}(A),$$
  
 $0 < \text{col index} < \mathbf{ncols}(A)$ 

If either condition is violated, execution of GrB\_Matrix\_extractElement ends and the invalid index error listed above is returned.

We are now ready to carry out the extract into the output scalar; that is,

$$\left\{ egin{array}{l} \mathbf{L}(s) \\ \text{val} \end{array} 
ight\} = \mathsf{A}(\mathsf{row\_index}, \mathsf{col\_index})$$

If  $(row\_index, col\_index) \in ind(A)$ , then the corresponding value from A is copied into s or values with casting as necessary. If  $(row\_index, col\_index) \notin ind(A)$ , then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB\_NO\_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB\_BLOCKING mode GrB\_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB\_SUCCESS or GrB\_NO\_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB\_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB\_BLOCKING mode.

In GrB\_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

#### 2309 4.2.4.13 Matrix extractTuples: Extract tuples from a matrix

2310 Extract the contents of a GraphBLAS matrix into non-opaque data structures.

# 2311 C Syntax

2314 2315 2316		<type> *values,  GrB_Index *n,  const GrB_Matrix A);</type>
2317	Parameters	
2318 2319	row_indices (OUT) Pointer row indices.	to an array of row indices that is large enough to hold all of the
2320 2321	col_indices (OUT) Pointer column indices	to an array of column indices that is large enough to hold all of the s.
2322 2323	` ,	to an array of scalars of a type that is large enough to hold all of ses whose type is compatible with $\mathbf{D}(\mathbf{A})$ .
2324 2325 2326	row_indices, a	ter to a value indicating (in input) the number of elements the values, and col_indices arrays can hold. Upon return, it will contain the nes written to the arrays.
2327	A (IN) An existing	ng GraphBLAS matrix.
2328	Return Values	
2329 2330 2331 2332	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indices and values, have been computed.
2333	GrB_PANIC	Unknown internal error.
2334 2335 2336 2337	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
2338	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
2339 2340 2341	GrB_INSUFFICIENT_SPACE	Not enough space in $row\_indices$ , $col\_indices$ , and values (as indicated by the $n$ parameter) to hold all of the tuples that will be extacted.
2342 2343	GrB_UNINITIALIZED_OBJECT	The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
2344	GrB_NULL_POINTER	row_indices, col_indices, values or n pointer is NULL.
2345 2346	GrB_DOMAIN_MISMATCH	The domains of the ${\sf A}$ matrix and ${\sf values}$ array are incompatible with one another.

This method will extract all the tuples from the GraphBLAS matrix A. The values associated with those tuples are placed in the values array, the column indices are placed in the col\_indices array, and the row indices are placed in the row\_indices array. These output arrays are pre-allocated by the user before calling this function such that each output array has enough space to hold at least GrB\_Matrix\_nvals(A) elements.

Upon return of this function, a pair of  $\{\text{row\_indices}[k], \text{col\_indices}[k]\}$  are unique for every valid k, but they are not required to be sorted in any particular order. Each tuple  $(i, j, A_{ij})$  in A is unzipped and copied into a distinct kth location in output vectors:

 $\{\text{row\_indices}[k], \text{col\_indices}[k], \text{values}[k]\} \leftarrow (i, j, A_{ij}),$ 

where  $0 \le k < GrB\_Matrix\_nvals(v)$ . No gaps in output vectors are allowed; that is, if row\_indices[k], col\_indices[k] and values[k] exist upon return, so does row\_indices[j], col\_indices[j] and values[j] for all j such that  $0 \le j < k$ .

Note that if the value in n on input is less than the number of values contained in the matrix A, then a GrB\_INSUFFICIENT\_SPACE error is returned since it is undefined which subset of values would be extracted.

In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value GrB\_SUCCESS, the new contents of the arrays row\_indices, col\_indices and values are as defined above.

2365 4.2.4.14 Matrix\_exportHint: Provide a hint as to which storage format might be most efficient for exporting a matrix

2367 C Syntax

#### 2368 Parameters

2369

2370

2374

hint (OUT) Pointer to a value of type GrB Format.

A (IN) A GraphBLAS matrix object.

# 2371 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of hint has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER hint is NULL.

GrB\_NO\_VALUE If the implementation does not have a preferred format, it may return the value GrB\_NO\_VALUE.

#### 2385 Description

2379

2380

2381

2382

2383

2384

Given a GraphBLAS matrix A, provide a hint as to which format might be most efficient for exporting the matrix A. GraphBLAS implementations might return the current storage format of the matrix, or the format to which it could most efficiently be exported. However, implementations are free to return any value for format defined in Section 3.5.3.1. Note that an implementation is free to refuse to provide a format hint, returning GrB\_NO\_VALUE.

# 2391 **4.2.4.15** Matrix\_exportSize: Return the array sizes necessary to export a GraphBLAS matrix object

#### 2393 C Syntax

```
GrB_Info GrB_Matrix_exportSize(GrB_Index *n_indptr,
GrB_Index *n_indices,
GrB_Index *n_values,
GrB_Format format,
GrB_Matrix A);
```

#### 2394 Parameters

2396

2397

2400

n\_indptr (OUT) Pointer to a value of type GrB\_Index.

n\_indices (OUT) Pointer to a value of type GrB\_Index.

n\_values (OUT) Pointer to a value of type GrB\_Index.

format (IN) a value indicating the format in which the matrix will be exported, as defined in Section 3.5.3.1.

A (IN) A GraphBLAS matrix object.

#### 2401 Return Values

GrB\_SUCCESS In blocking mode or non-blocking mode, the operation com-2402 pleted successfully. This indicates that the API checks for the 2403 input arguments passed successfully, and the number of elements 2404 necessary for the export buffers have been written to n\_indptr, 2405 n\_indices, and n\_values, respectively. 2406 GrB PANIC Unknown internal error. 2407 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the 2408 opaque GraphBLAS objects (input or output) is in an invalid 2409 state caused by a previous execution error. Call GrB\_error() to 2410

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS Matrix, A, has not been initialized by a call to any matrix constructor.

access any error messages generated by the implementation.

GrB\_NULL\_POINTER n\_indptr, n\_indices, or n\_values is NULL.

#### 2416 Description

Given a matrix **A**, returns the required capacities of arrays values, indptr, and indices necessary to export the matrix in the format specified by format. The output values n\_values, n\_indptr, and indices will contain the corresponding sizes of the arrays (in number of elements) that must be allocated to hold the exported matrix. The argument format can be chosen arbitrarily by the user as one of the values defined in Section 3.5.3.1.

#### 4.2.4.16 Matrix\_export: Export a GraphBLAS matrix to a pre-defined format

# 2423 C Syntax

```
GrB_Info GrB_Matrix_export(GrB_Index
                                                    *indptr,
                             GrB_Index
                                                    *indices,
                             <type>
                                                    *values,
                             GrB_Index
                                                    *n_indptr,
                             GrB_Index
                                                    *n_indices,
                             GrB Index
                                                    *n_values,
                             GrB_Format
                                                     format,
                             GrB_Matrix
                                                     A);
```

#### 424 Parameters

indptr (INOUT) Pointer to an array that will hold row or column offsets, or row in-2425 dices, depending on the value of format. It must be large enough to hold at 2426 least n indptr elements of type GrB Index, where n indices was returned from 2427 GrB\_Matrix\_exportSize() method. 2428 indices (INOUT) Pointer to an array that will hold row or column indices of the elements 2429 in values, depending on the value of format. It must be large enough to hold at 2430 least n\_indices elements of type GrB\_Index, where n\_indices was returned from 2431 GrB Matrix exportSize() method. 2432 values (INOUT) Pointer to an array that will hold stored values. The type of ele-2433 ment must match the type of the values stored in A. It must be large enough 2434 to hold at least n\_values elements of that type, where n\_values was returned from 2435 GrB\_Matrix\_exportSize. 2436 n\_indptr (INOUT) Pointer to a value indicating (on input) the number of elements the indptr 2437 array can hold. Upon return, it will contain the number of elements written to the 2438 array. 2439 n\_indices (INOUT) Pointer to a value indicating (on input) the number of elements the indices 2440 array can hold. Upon return, it will contain the number of elements written to the 2441 array. 2442 n\_values (INOUT) Pointer to a value indicating (on input) the number of elements the values 2443 array can hold. Upon return, it will contain the number of elements written to the 2444 array. 2445 format (IN) a value indicating the format in which the matrix will be exported, as defined 2446 in Section 3.5.3.1. 2447 A (IN) A GraphBLAS matrix object. 2448

#### Return Values

2450	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed suc-
2451		cessfully. This indicates that the compatibility tests on the input
2452		argument passed successfully, and the output arrays, indptr, in-
2453		dices and values, have been computed.
2454	GrB_PANIC	Unknown internal error.
2455	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
2456		opaque GraphBLAS objects (input or output) is in an invalid
2457		state caused by a previous execution error. Call GrB_error() to
2458		access any error messages generated by the implementation.
2459	GrB OUT OF MEMORY	Not enough memory available for operation.

GrB\_INSUFFICIENT\_SPACE Not enough space in indptr, indices, and/or values (as indicated 2460 by the corresponding  $n_*$  parameter) to hold all of the corre-2461 sponding elements that will be extacted. 2462 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to 2463 any matrix constructor. 2464 GrB\_NULL\_POINTER indptr, indices, values n\_indptr, n\_indices, n\_values pointer is 2465 NULL. 2466 GrB\_DOMAIN\_MISMATCH The domain of A does not match with the type of values. 2467

# 2468 Description

Given a matrix **A**, this method exports the contents of the matrix into one of the pre-defined GrB\_Format formats from Section 3.5.3.1. The user-allocated arrays pointed to by indptr, indices, and values must be at least large enough to hold the corresponding number of elements returned by calling GrB\_Matrix\_exportSize. The value of format can be chosen arbitrarily, but a call to GrB\_Matrix\_exportHint may suggest a format that results in the most efficient export. Details of the contents of indptr, indices, and values corresponding to each supported format is given in Appendix B.

# 4.2.4.17 Matrix\_import: Import a matrix into a GraphBLAS object

#### 2477 C Syntax

```
GrB_Info GrB_Matrix_import(GrB_Matrix
                                                    *A,
                                                     d,
                             GrB_Type
                             GrB_Index
                                                     nrows,
                                                     ncols
                             GrB_Index
                             const GrB_Index
                                                    *indptr,
                             const GrB Index
                                                    *indices,
                             const <type>
                                                    *values,
                             GrB Index
                                                     n indptr,
                             GrB_Index
                                                     n indices,
                             GrB_Index
                                                     n_values,
                             GrB_Format
                                                     format);
```

#### Parameters Parameters

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- A (INOUT) On a successful return, contains a handle to the newly created Graph-BLAS matrix.
  - d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

nrows (IN) Integer value holding the number of rows in the matrix. 2484 ncols (IN) Integer value holding the number of columns in the matrix. 2485 indptr (IN) Pointer to an array of row or column offsets, or row indices, depending on the 2486 value of format. 2487 indices (IN) Pointer to an array row or column indices of the elements in values, depending 2488 on the value of format. 2489 values (IN) Pointer to an array of values. Type must match the type of d. 2490 n indptr (IN) Integer value holding the number of elements in the array pointed to by indptr. 2491 n\_indices (IN) Integer value holding the number of elements in the array pointed to by indices. 2492 n values (IN) Integer value holding the number of elements in the array pointed to by values. 2493 format (IN) a value indicating the format of the matrix being imported, as defined in 2494 Section 3.5.3.1. 2495 Return Values 2496 GrB SUCCESS In blocking mode, the operation completed successfully. In non-2497 blocking mode, this indicates that the API checks for the input 2498 arguments passed successfully and the input arrays have been 2499 consumed. Either way, output matrix A is ready to be used in 2500 the next method of the sequence. 2501 GrB PANIC Unknown internal error. 2502 Grb Out of Memory Not enough memory available for operation. 2503 GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new 2504 (needed for user-defined types). 2505 GrB\_NULL\_POINTER A, indptr, indices or values pointer is NULL. 2506 GrB\_INDEX\_OUT\_OF\_BOUNDS A value in indptr or indices is outside the allowed range for indices 2507 in A and or the size of values, n\_values, depending on the value 2508 of format. 2509 GrB\_INVALID\_VALUE nrows or ncols is zero or outside the range of the type GrB\_Index. 2510 GrB\_DOMAIN\_MISMATCH The domain given in parameter d does not match the element 2511 type of values.

2512

Creates a new matrix **A** of domain **D**(d) and dimension nrows × ncols. The new GraphBLAS matrix will be filled with the contents of the matrix pointed to by indptr, and indices, and values.

The method returns a handle to the new matrix in **A**. The structure of the data being imported is defined by format, which must be equal to one of the values defined in Section 3.5.3.1. Details of the contents of indptr, indices and values for each supported format is given in Appendix B.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

#### 2521 4.2.4.18 Matrix\_serializeSize: Compute the serialize buffer size

<sup>2522</sup> Compute the buffer size (in bytes) necessary to serialize a GrB\_Matrix using GrB\_Matrix\_serialize.

# 2523 C Syntax

#### 2524 Parameters

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size (OUT) Pointer to GrB\_Index value where size in bytes of serialized object will be written.

A (IN) A GraphBLAS matrix object.

#### 2528 Return Values

GrB\_SUCCESS The operation completed successfully and the value pointed to by \*size has been computed and is ready to use.

Grb Panic Unknown internal error.

Grb\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_NULL\_POINTER size is NULL.

# Description

Returns the size in bytes of the data buffer necessary to serialize the GraphBLAS matrix object A.
Users may then allocate a buffer of size bytes to pass as a parameter to GrB\_Matrix\_serialize.

# 2537 4.2.4.19 Matrix\_serialize: Serialize a GraphBLAS matrix.

<sup>2538</sup> Serialize a GraphBLAS Matrix object into an opaque stream of bytes.

# 2539 C Syntax

#### 2540 Parameters

serialized\_data (INOUT) Pointer to the preallocated buffer where the serialized matrix will be written.

serialized\_size (INOUT) On input, the size in bytes of the buffer pointed to by serialized\_data.

On output, the number of bytes written to serialized\_data.

A (IN) A GraphBLAS matrix object.

#### 2546 Return Values

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2558

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output buffer serialized\_data and serialized\_size, have been computed and are ready to use.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

2557 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_NULL\_POINTER serialized\_data or serialize\_size is NULL.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_INSUFFICIENT\_SPACE The size of the buffer serialized\_data (provided as an input serialized\_size) was not large enough.

Serializes a GraphBLAS matrix object to an opaque buffer. To guarantee successful execution, 2564 the size of the buffer pointed to by serialized\_data, provided as an input by serialized\_size, must 2565 be of at least the number of bytes returned from GrB Matrix serializeSize. The actual size of the 2566 serialized matrix written to serialized\_data is provided upon completion as an output written to 2567 serialized size. 2568

The contents of the serialized buffer are implementation defined. Thus, a serialized matrix created 2569 with one library implementation is not necessarily valid for describilization with another implemen-2570 tation. 2571

#### Matrix deserialize: Deserialize a GraphBLAS matrix. 4.2.4.202572

Construct a new GraphBLAS matrix from a serialized object. 2573

#### C Syntax 2574

```
GrB_Info GrB_Matrix_deserialize(GrB_Matrix
                                 GrB Type
                                               d,
                                 const void
                                              *serialized_data,
                                 GrB_Index
                                               serialized_size);
```

#### **Parameters**

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A (INOUT) On a successful return, contains a handle to the newly created Graph-2576 BLAS matrix. 2577

d (IN) the type of the matrix that was serialized in serialized\_data.

serialized data (IN) a pointer to a serialized GraphBLAS matrix created with GrB Matrix serialize. 2579 serialized size (IN) the size of the buffer pointed to by serialized data in bytes.

# Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-2582 blocking mode, this indicates that the API checks for the input 2583 arguments passed successfully. Either way, output matrix A is 2584 ready to be used in the next method of the sequence. 2585

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned if serialized\_data is invalid or corrupted. 2587

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new (needed for user-defined types).

GrB\_NULL\_POINTER serialized\_data or A is NULL.

GrB\_DOMAIN\_MISMATCH The type given in d does not match the type of the matrix serialized in serialized\_data.

#### Description

2594

Creates a new matrix **A** using the serialized matrix object pointed to by serialized\_data. The object pointed to by serialized\_data must have been created using the method GrB\_Matrix\_serialize. The domain of the matrix is given as an input in d, which must match the domain of the matrix serialized in serialized\_data. Note that for user-defined types, only the size of the type will be checked.

Since the format of a serialized matrix is implementation-defined, it is not guaranteed that a matrix serialized in one library implementation can be describilized by another.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

# 2603 4.2.5 Descriptor methods

The methods in this section create and set values in descriptors. A descriptor is an opaque Graph-BLAS object the values of which are used to modify the behavior of GraphBLAS operations.

#### 2606 4.2.5.1 Descriptor new: Create new descriptor

<sup>2607</sup> Creates a new (empty or default) descriptor.

# 2608 C Syntax

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GrB\_Info GrB\_Descriptor\_new(GrB\_Descriptor \*desc);

# Parameters

desc (INOUT) On successful return, contains a handle to the newly created GraphBLAS descriptor.

# 2613 Return Value

GrB\_SUCCESS The method completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

GrB\_NULL\_POINTER desc pointer is NULL.

### 2618 Description

2617

Creates a new descriptor object and returns a handle to it in desc. A newly created descriptor can be populated by calls to Descriptor set.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

## 2623 4.2.5.2 Descriptor\_set: Set content of descriptor

2624 Sets the content for a field for an existing descriptor.

# 2625 C Syntax

```
GrB_Info GrB_Descriptor_set(GrB_Descriptor desc,

GrB_Desc_Field field,

GrB_Desc_Value val);
```

### 2629 Parameters

desc (IN) An existing GraphBLAS descriptor to be modified.

field (IN) The field being set.

val (IN) New value for the field being set.

## 2633 Return Values

GrB\_SUCCESS operation completed successfully.

2635 GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

2637 GrB\_UNINITIALIZED\_OBJECT the desc parameter has not been initialized by a call to new.

GrB\_INVALID\_VALUE invalid value set on the field, or invalid field.

### 2639 Description

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For a given descriptor, the GrB\_Descriptor\_set method can be called for each field in the descriptor to set the value associated with that field. Valid values for the field parameter include the following:

Grb OUTP refers to the output parameter (result) of the operation.

GrB\_MASK refers to the mask parameter of the operation.

GrB\_INPO refers to the first input parameters of the operation (matrices and vectors).

GrB\_INP1 refers to the second input parameters of the operation (matrices and vectors).

Valid values for the val parameter are:

GrB\_STRUCTURE Use only the structure of the stored values of the corresponding mask (GrB\_MASK) parameter.

GrB\_COMP Use the complement of the corresponding mask (GrB\_MASK) parameter. When combined with GrB\_STRUCTURE, the complement of the structure of the mask is used without evaluating the values stored.

GrB\_TRAN Use the transpose of the corresponding matrix parameter (valid for input matrix parameters only).

GrB\_REPLACE When assigning the masked values to the output matrix or vector, clear the matrix first (or clear the non-masked entries). The default behavior is to leave non-masked locations unchanged. Valid for the GrB\_OUTP parameter only.

Descriptor values can only be set, and once set, cannot be cleared. As, in the case of GrB\_MASK, multiple values can be set and all will apply (for example, both GrB\_COMP and GrB\_STRUCTURE). A value for a given field may be set multiple times but will have no additional effect. Fields that have no values set result in their default behavior, as defined in Section 3.6.

### 4.2.6 free: Destroy an object and release its resources

Destroys a previously created GraphBLAS object and releases any resources associated with the object.

### 2665 C Syntax

#### 7 Parameters

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obj (INOUT) An existing GraphBLAS object to be destroyed. The object must have been created by an explicit call to a GraphBLAS constructor. It can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful completion of GrB\_free, obj behaves as an uninitialized object.

### 2673 Return Values

GrB\_SUCCESS operation completed successfully

GrB\_PANIC unknown internal error. If this return value is encountered when in nonblocking mode, the error responsible for the panic condition could be from any method involved in the computation of the input object. The GrB\_error() method should be called for additional information.

# Description

GraphBLAS objects consume memory and other resources managed by the GraphBLAS runtime system. A call to GrB\_free frees those resources so they are available for use by other GraphBLAS objects.

The parameter passed into GrB\_free is a handle referencing a GraphBLAS opaque object of a data type from table 2.1. The object must have been created by an explicit call to a GraphBLAS constructor. The behavior of a program that calls GrB\_free on a pre-defined object is implementation defined.

After the GrB\_free method returns, the object referenced by the input handle is destroyed and the handle has the value GrB\_INVALID\_HANDLE. The handle can be used in subsequent GraphBLAS methods but only after the handle has been reinitialized with a call the the appropriate \_new or \_dup method.

Note that unlike other GraphBLAS methods, calling GrB\_free with an object with an invalid handle is legal. The system may attempt to free resources that might be associated with that object, if possible, and return normally.

When using GrB\_free it is possible to create a dangling reference to an object. This would occur when a handle is assigned to a second variable of the same opaque type. This creates two handles that reference the same object. If GrB\_free is called with one of the variables, the object is destroyed and the handle associated with the other variable no longer references a valid object. This is not an error condition that the implementation of the GraphBLAS API can be expected to catch, hence programmers must take care to prevent this situation from occurring.

# 2701 4.2.7 wait: Return once an object is either complete or materialized

Wait until method calls in a sequence put an object into a state of completion or materialization.

## 2703 C Syntax

```
GrB_Info GrB_wait(GrB_Object obj, GrB_WaitMode mode);
```

### 2705 Parameters

- obj (INOUT) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful return of GrB\_wait, the obj can be safely read from another thread (completion) or all computing to produce obj by all GraphBLAS operations in its sequence have finished (materialization).
- mode (IN) Set's the mode for GrB\_wait for whether it is waiting for obj to be in the state of completion or materialization. Acceptable values are GrB\_COMPLETE or GrB\_MATERIALIZE.

### 715 Return values

GrB\_SUCCESS operation completed successfully.

GrB\_INDEX\_OUT\_OF\_BOUNDS an index out-of-bounds execution error happened during completion of pending operations.

GrB\_OUT\_OF\_MEMORY and out-of-memory execution error happened during completion of pending operations.

GrB\_UNINITIALIZED\_OBJECT object has not been initialized by a call to the respective \*\_new, or other constructor, method.

GrB\_PANIC unknown internal error.

GrB\_INVALID\_VALUE method called with a GrB\_WaitMode other than GrB\_COMPLETE GrB\_MATERIALIZE.

### Description

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On successful return from GrB\_wait(), the input object, obj is in one of two states depending on the mode of GrB\_wait:

- complete: obj can be used in a happens-before relation, so in a properly synchronized program 2729 it can be safely used as an IN or INOUT parameter in a GraphBLAS method call from another 2730 thread. This result occurs when the mode parameter is set to GrB\_COMPLETE. 2731
- materialized: obj is complete, but in addition, no further computing will be carried out on 2732 behalf of obj and error information is available. This result occurs when the mode parameter 2733 is set to GrB MATERIALIZE. 2734
- Since in blocking mode OUT or INOUT parameters to any method call are materialized upon return, 2735 GrB wait(obj,mode) has no effect when called in blocking mode.
- In non-blocking mode, the status of any pending method calls, other than those associated with pro-2737 ducing the *complete* or *materialized* state of obj, are not impacted by the call to GrB\_wait(obj,mode). 2738
- Methods in the sequence for obj, however, most likely would be impacted by a call to GrB\_wait(obj,mode); 2739
- especially in the case of the materialized mode for which any computing on behalf of obj must be
- finished prior to the return from GrB\_wait(obj,mode). 2741

#### 4.2.8 error: Retrieve an error string 2742

Retrieve an error-message about any errors encountered during the processing associated with an 2743 object. 2744

#### C Syntax 2745

```
GrB_Info GrB_error(const char
2746
                                                          **error,
                                   const GrB_Object
                                                            obj);
2747
```

#### **Parameters** 2748

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- error (OUT) A pointer to a null-terminated string. The contents of the string are implementation defined.
- obj (IN) An existing GraphBLAS object. The object must have been created by an 2751 explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS 2752 objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, 2753 or type. 2754

### Return value

GrB SUCCESS operation completed successfully. 2756

GrB\_UNINITIALIZED\_OBJECT object has not been initialized by a call to the respective \*\_new, 2757 or other constructor, method. 2758

GrB PANIC unknown internal error.

### 2760 Description

This method retrieves a message related to any errors that were encountered during the last Graph-276 BLAS method that had the opaque GraphBLAS object, obj, as an OUT or INOUT parameter. 2762 The function returns a pointer to a null-terminated string and the contents of that string are 2763 implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error 2764 string. The string that is returned is owned by obj and will be valid until the next time obj is 2765 used as an OUT or INOUT parameter or the object is freed by a call to GrB\_free(obj). This is a 2766 thread-safe function. It can be safely called by multiple threads for the same object in a race-free 2767 program. 2768

# 2769 4.3 GraphBLAS operations

The GraphBLAS operations are defined in the GraphBLAS math specification and summarized in Table 4.1. In addition to methods that implement these fundamental GraphBLAS operations, we support a number of variants that have been found to be especially useful in algorithm development. A flowchart of the overall behavior of a GraphBLAS operation is shown in Figure 4.1.

### 2774 Domains and Casting

A GraphBLAS operation is only valid when the domains of the GraphBLAS objects are mathemat-2775 ically consistent. The C programming language defines implicit casts between built-in data types. 2776 For example, floats, doubles, and ints can be freely mixed according to the rules defined for implicit 2777 casts. It is the responsibility of the user to assure that these casts are appropriate for the algorithm 2778 in question. For example, a cast to int implies truncation of a floating point type. Depending on 2779 the operation, this truncation error could lead to erroneous results. Furthermore, casting a wider 2780 type onto a narrower type can lead to overflow errors. The GraphBLAS operations do not attempt 2781 to protect a user from these sorts of errors. 2782

When user-define types are involved, however, GraphBLAS requires strict equivalence between types and no casting is supported. If GraphBLAS detects these mismatches, it will return a domain mismatch error.

### 2786 Dimensions and Transposes

GraphBLAS operations also make assumptions about the numbers of dimensions and the sizes of vectors and matrices in an operation. An operation will test these sizes and report an error if they are not *shape compatible*. For example, when multiplying two matrices,  $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ , the number of rows of  $\mathbf{C}$  must equal the number of rows of  $\mathbf{A}$ , the number of columns of  $\mathbf{A}$  must match the number of rows of  $\mathbf{B}$ , and the number of columns of  $\mathbf{C}$  must match the number of columns of  $\mathbf{B}$ . This is the behavior expected given the mathematical definition of the operations.

For most of the GraphBLAS operations involving matrices, an optional descriptor can modify the matrix associated with an input GraphBLAS matrix object. For example, if an input matrix is an

Table 4.1: A mathematical notation for the fundamental GraphBLAS operations supported in this specification. Input matrices  $\mathbf{A}$  and  $\mathbf{B}$  may be optionally transposed (not shown). Use of an optional accumulate with existing values in the output object is indicated with  $\odot$ . Use of optional write masks and replace flags are indicated as  $\mathbf{C}\langle\mathbf{M},r\rangle$  when applied to the output matrix,  $\mathbf{C}$ . The mask controls which values resulting from the operation on the right-hand side are written into the output object (complement and structure flags are not shown). The "replace" option, indicated by specifying the r flag, means that all values in the output object are removed prior to assignment. If "replace" is not specified, only the values/locations computed on the right-hand side and allowed by the mask will be written to the output ("merge" mode).

Operation Name	N	Iath	ematical No	otation
mxm	$\mathbf{C}\langle\mathbf{M},r angle$	=	<b>C</b> ⊙	$\mathbf{A} \oplus . \otimes \mathbf{B}$
mxv	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w} \odot$	
vxm	$\mathbf{w}^T \langle \mathbf{m}^T, r  angle$	=	$\mathbf{w}^T$ $\odot$	$\mathbf{u}^T \oplus . \otimes \mathbf{A}$
eWiseMult	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C}$ $\odot$	$\mathbf{A} \otimes \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w} \odot$	$\mathbf{u} \otimes \mathbf{v}$
eWiseAdd	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C}$ $\odot$	$\mathbf{A} \oplus \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w} \odot$	$\mathbf{u} \oplus \mathbf{v}$
extract	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C}$ $\odot$	$\mathbf{A}(m{i},m{j})$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w}$ $\odot$	$\mathbf{u}(m{i})$
assign	$\mathbf{C}\langle\mathbf{M},r angle(oldsymbol{i},oldsymbol{j})$	=	$\mathbf{C}(m{i},m{j})$ $\odot$	$\mathbf{A}$
	$\mathbf{w}\langle\mathbf{m},r\rangle(\boldsymbol{i})$	=	$\mathbf{w}(i)$ $\odot$	u
reduce (row)	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w} \odot$	$[\oplus_j \mathbf{A}(:,j)]$
reduce (scalar)	s	=	$s$ $\odot$	$[\oplus_{i,j} \mathbf{A}(i,j)]$
	s	=	$s$ $\odot$	$[\oplus_i \mathbf{u}(i)]$
apply	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C}$ $\odot$	$f_u(\mathbf{A})$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w}$ $\odot$	$f_u(\mathbf{u})$
apply(indexop)	$\mathbf{C}\langle\mathbf{M},r angle$	=	<b>C</b> ⊙	$f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s)$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w} \odot$	$f_i(\mathbf{u},\mathbf{ind}(\mathbf{u}),s)$
select	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C} \odot$	$\mathbf{A}\langle f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s) \rangle$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	$\mathbf{w}$ $\odot$	$\mathbf{u}\langle f_i(\mathbf{u},\mathbf{ind}(\mathbf{u}),s)\rangle$
transpose	$\mathbf{C}\langle\mathbf{M},r angle$	=	<b>C</b> ⊙	$\mathbf{A}^T$
kronecker	$\mathbf{C}\langle\mathbf{M},r angle$	=	$\mathbf{C}$ $\odot$	$\mathbf{A} \otimes \mathbf{B}$
	•			

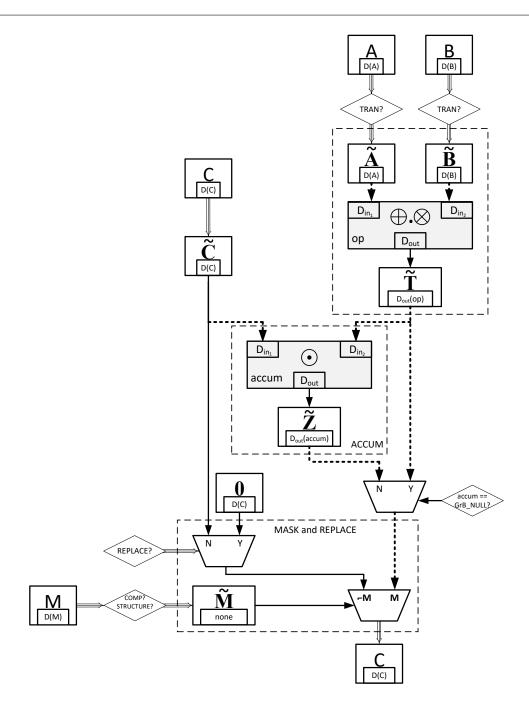


Figure 4.1: Flowchart for the GraphBLAS operations. Although shown specifically for the mxm operation, many elements are common to all operations: such as the "ACCUM" and "MASK and REPLACE" blocks. The triple arrows  $(\Rrightarrow)$  denote where "as if copy" takes place (including both collections and descriptor settings). The bold, dotted arrows indicate where casting may occur between different domains.

argument to a GraphBLAS operation and the associated descriptor indicates the transpose option, then the operation occurs as if on the transposed matrix. In this case, the relationships between the sizes in each dimension shift in the mathematically expected way.

### 2798 Masks: Structure-only, Complement, and Replace

When a GraphBLAS operation supports the use of an optional mask, that mask is specified through 2799 a GraphBLAS vector (for one-dimensional masks) or a GraphBLAS matrix (for two-dimensional 2800 masks). When a mask is used and the GTB\_STRUCTURE descriptor value is not set, it is applied 2801 to the result from the operation wherever the stored values in the mask evaluate to true. If the 2802 GrB\_STRUCTURE descriptor is set, the mask is applied to the result from the operation wherever the 2803 mask as a stored value (regardless of that value). Wherever the mask is applied, the result from 2804 the operation is either assigned to the provided output matrix/vector or, if a binary accumulation 2805 operation is provided, the result is accumulated into the corresponding elements of the provided 2806 output matrix/vector. 2807

Given a GraphBLAS vector  $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$ , a one-dimensional mask is derived for use in the operation as follows:

$$\mathbf{m} = \begin{cases} \langle N, \{ \mathbf{ind}(\mathbf{v}) \} \rangle, & \text{if GrB\_STRUCTURE is specified,} \\ \langle N, \{ i : (\mathsf{bool}) v_i = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool) $v_i$  denotes casting the value  $v_i$  to a Boolean value (true or false). Likewise, given a GraphBLAS matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$ , a two-dimensional mask is derived for use in the operation as follows:

$$\mathbf{M} = \begin{cases} \langle M, N, \{\mathbf{ind}(\mathbf{A})\} \rangle, & \text{if GrB\_STRUCTURE is specified,} \\ \langle M, N, \{(i,j) : (\mathsf{bool}) A_{ij} = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool)  $A_{ij}$  denotes casting the value  $A_{ij}$  to a Boolean value. (true or false)

In both the one- and two-dimensional cases, the mask may also have a subsequent complement operation applied (Section 3.5.4) as specified in the descriptor, before a final mask is generated for use in the operation.

When the descriptor of an operation with a mask has specified that the GrB\_REPLACE value is to be applied to the output (GrB\_OUTP), then anywhere the mask is not true, the corresponding location in the output is cleared.

### Invalid and uninitialized objects

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Upon entering a GraphBLAS operation, the first step is a check that all objects are valid and initialized. (Optional parameters can be set to GrB\_NULL, which always counts as a valid object.) An invalid object is one that could not be computed due to a previous execution error. An unitialized object is one that has not yet been created by a corresponding new or dup method. Appropriate error codes are returned if an object is not initialized (GrB\_UNINITIALIZED\_OBJECT) or invalid (GrB\_INVALID\_OBJECT).

To support the detection of as many cases of uninitialized objects as possible, it is strongly recommended to initialize all GraphBLAS objects to the predefined value GrB\_INVALID\_HANDLE at the point of their declaration, as shown in the following examples:

```
GrB_Type type = GrB_INVALID_HANDLE;

2833 GrB_Semiring semiring = GrB_INVALID_HANDLE;

2834 GrB_Matrix matrix = GrB_INVALID_HANDLE;
```

### 2835 Compliance

We follow a *prescriptive* approach to the definition of the semantics of GraphBLAS operations.

That is, for each operation we give a recipe for producing its outcome. Any implementation that produces the same outcome, and follows the GraphBLAS execution model (Section 2.5) and error model (Section 2.6) is a conforming implementation.

# 2840 4.3.1 mxm: Matrix-matrix multiply

Multiplies a matrix with another matrix on a semiring. The result is a matrix.

# 2842 C Syntax

```
GrB_Info GrB_mxm(GrB_Matrix
                                                             С,
2843
                                  const GrB_Matrix
                                                             Mask,
2844
                                  const GrB_BinaryOp
                                                             accum,
2845
                                  const GrB_Semiring
                                                             op,
2846
                                  const GrB_Matrix
                                                             Α,
2847
                                  const GrB Matrix
                                                             В,
2848
                                  const GrB_Descriptor
                                                             desc);
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```

#### 2850 Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the matrix product. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing C
  entries. If assignment rather than accumulation is desired, GrB\_NULL should be
  specified.
  - op (IN) The semiring used in the matrix-matrix multiply.
  - A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
    - B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
1				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
	В	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

#### 2 Return Values

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2873	GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
2874	blocking mode, this indicates that the compatibility tests on di-
2875	mensions and domains for the input arguments passed successfully.
2876	Either way, output matrix C is ready to be used in the next method
2877	of the sequence.
2878	GrB_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix\_dup for matrix parameters).

2886 GrB\_DIMENSION\_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

# 2891 Description

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GrB\_mxm computes the matrix product  $C = A \oplus . \otimes B$  or, if an optional binary accumulation operator ( $\odot$ ) is provided,  $C = C \odot (A \oplus . \otimes B)$  (where matrices A and B can be optionally transposed). Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 2897 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB\_mxm operation:
- 2900 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2901 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 2902 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4.  $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 2906 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask)
  2907 must be from one of the pre-defined types of Table 3.2.
- 2908 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

- If any compatibility rule above is violated, execution of GrB\_mxm ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :
- 1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 2922 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- 2923 (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\} \rangle$ .
- (b) If Mask  $\neq$  GrB NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
    - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. Matrix  $\tilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 2936 2.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{B}}).$
- 5.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{B}}).$
- If any compatibility rule above is violated, execution of GrB\_mxm ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix multiplication and any additional associated operations.
- 2945 We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the product of matrices  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
  - ullet  $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(: j, j)) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

$$T_{ij} = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(:,j))} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{B}}(k,j)),$$

where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a standard matrix accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{-}\operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 

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• If  $\mathbf{z}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $\operatorname{\mathsf{desc}}[\mathsf{GrB\_OUTP}].\mathsf{GrB\_REPLACE}$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

# 979 4.3.2 vxm: Vector-matrix multiply

Multiplies a (row) vector with a matrix on an semiring. The result is a vector.

### 2981 C Syntax

```
GrB_Info GrB_vxm(GrB_Vector
2982
                                                             W,
                                  const GrB_Vector
                                                             mask,
2983
                                  const GrB_BinaryOp
                                                             accum,
2984
                                  const GrB_Semiring
2985
                                                             op,
                                  const GrB_Vector
2986
                                                             u,
                                  const GrB Matrix
                                                             Α,
2987
                                  const GrB_Descriptor
                                                             desc);
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```

#### 2989 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the vector-matrix product. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) Semiring used in the vector-matrix multiply.
    - u (IN) The GraphBLAS vector holding the values for the left-hand vector in the multiplication.
      - A (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
    - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
Α	GrB INP1	GrB TRAN	Use transpose of A for the operation.

### Return Values

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3012 3013 3014 3015 3016	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence.
3017	GrB_PANIC	Unknown internal error.
3018 3019 3020 3021	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
3022	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
3023 3024	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).
3025	GrB_DIMENSION_MISMATCH	Mask, vector, and/or matrix dimensions are incompatible.
3026 3027 3028 3029	GrB_DOMAIN_MISMATCH	The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

# 3030 Description

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GrB\_vxm computes the vector-matrix product  $\mathbf{w}^T = \mathbf{u}^T \oplus . \otimes \mathsf{A}$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $\mathbf{w}^T = \mathbf{w}^T \odot \left( \mathbf{u}^T \oplus . \otimes \mathsf{A} \right)$  (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

**Setup** The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

3038 Up to four argument vectors or matrices are used in the GrB\_vxm operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

3040 2. 
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$$

3. 
$$\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$$

3042 4. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3047 2.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 30.48 3.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB\_vxm ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- 3066 (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

4. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$ .

The internal matrices and masks are checked for shape compatibility. The following conditions must hold:

- 3071 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}).$
- 3072 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$
- 3.  $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$

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If any compatibility rule above is violated, execution of GrB\_vxm ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the vector-matrix multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the product of vector  $\tilde{\mathbf{u}}^T$  and matrix  $\tilde{\mathbf{A}}$ .
  - $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{ncols}(\tilde{\mathbf{A}}), \{(j, t_j) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{A}}(:, j)) \neq \emptyset \} \rangle$  is created.

The value of each of its elements is computed by

$$t_j = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{A}}(:,j))} (\widetilde{\mathbf{u}}(k) \otimes \widetilde{\mathbf{A}}(k,j)),$$

where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 3113 4.3.3 mxv: Matrix-vector multiply

Multiplies a matrix by a vector on a semiring. The result is a vector.

### 3115 C Syntax

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```
GrB_Info GrB_mxv(GrB_Vector
                                                             W,
3116
                                  const GrB_Vector
                                                             mask,
3117
                                  const GrB_BinaryOp
                                                             accum,
3118
                                  const GrB Semiring
3119
                                                             op,
                                  const GrB_Matrix
                                                             Α,
3120
                                  const GrB Vector
3121
                                                             u,
                                  const GrB Descriptor
                                                             desc);
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```

### Parameters

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w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the matrix-vector product. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain

3130 3131 3132	of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
3133 3134 3135	eccum (IN) An optional binary operator used for accumulating entries into existing we entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
3136	op (IN) Semiring used in the vector-matrix multiply.
3137 3138	A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
3139 3140	u (IN) The GraphBLAS vector holding the values for the right-hand vector in the multiplication.
3141 3142 3143	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:
3143	Param Field Value Description

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

# 3145 Return Values

3	146	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3	147		blocking mode, this indicates that the compatibility tests on di-
3	148		mensions and domains for the input arguments passed successfully.
3	149		Either way, output vector w is ready to be used in the next method
3	150		of the sequence.
3	151	GrB_PANIC	Unknown internal error.
3	152	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3	153		GraphBLAS objects (input or output) is in an invalid state caused
3	154		by a previous execution error. Call GrB_error() to access any error
3	155		messages generated by the implementation.
3	156	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
	157 <b>GrB_</b> 158	_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).

3159 GrB\_DIMENSION\_MISMATCH Mask, vector, and/or matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

### 3164 Description

GrB\_mxv computes the matrix-vector product  $w = A \oplus . \otimes u$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (A \oplus . \otimes u)$  (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

- Setup The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.
- 3170 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 3172 Up to four argument vectors or matrices are used in the GrB\_mxv operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3174 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 31. A =  $\langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3176 4.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3181 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 3.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 3183 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB\_mxv ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$ .
- (b) If mask  $\neq$  GrB NULL,
  - i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool}) \mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- The internal matrices and masks are checked for shape compatibility. The following conditions must hold:
- 3205 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}}).$
- 3206 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- 3.  $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_mxv ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix-vector multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the product of matrix  $\tilde{\mathbf{A}}$  and vector  $\tilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(i, t_i) : \mathbf{ind}(\widetilde{\mathbf{A}}(i, :)) \cap \mathbf{ind}(\widetilde{\mathbf{u}}) \neq \emptyset \} \rangle$  is created.

  The value of each of its elements is computed by

$$t_i = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}})} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{u}}(k)),$$

where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{accum} = \operatorname{GrB} \ \operatorname{NULL}$ , then  $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$ .

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

# 4.3.4 eWiseMult: Element-wise multiplication

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

#### 3253 **4.3.4.1** eWiseMult: Vector variant

Perform element-wise (general) multiplication on the intersection of elements of two vectors, producing a third vector as result.

### 3256 C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Vector
                                                                    W,
3257
                                         const GrB_Vector
                                                                    mask,
3258
                                         const GrB_BinaryOp
                                                                    accum,
3259
                                         const GrB_Semiring
                                                                    op,
3260
                                         const GrB_Vector
3261
                                                                    u,
                                         const GrB_Vector
                                                                    v,
3262
                                         const GrB_Descriptor
                                                                    desc);
3263
3264
              GrB_Info GrB_eWiseMult(GrB_Vector
3265
                                                                    W,
                                         const GrB_Vector
                                                                    mask,
3266
                                         const GrB_BinaryOp
                                                                    accum,
3267
                                         const GrB_Monoid
                                                                    op,
3268
                                         const GrB Vector
                                                                    u,
3269
                                         const GrB Vector
                                                                    v,
3270
                                         const GrB_Descriptor
                                                                    desc);
3271
3272
              GrB_Info GrB_eWiseMult(GrB_Vector
3273
                                                                    W,
                                         const GrB_Vector
3274
                                                                    mask,
                                         const GrB_BinaryOp
3275
                                                                    accum,
                                         const GrB_BinaryOp
3276
                                                                    op,
                                         const GrB_Vector
                                                                    u,
3277
                                         const GrB_Vector
                                                                    v,
3278
                                         const GrB_Descriptor
                                                                    desc);
3279
```

#### 3280 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes(\mathsf{op}) \rangle$ ; the additive monoid is ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB MASK	GrB COMP	Use the complement of mask.

# Return Values

3310	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3311		blocking mode, this indicates that the compatibility tests on di-
3312		mensions and domains for the input arguments passed successfully.
3313		Either way, output vector <b>w</b> is ready to be used in the next method
3314		of the sequence.
3315	GrB_PANIC	Unknown internal error.
3316	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3317		GraphBLAS objects (input or output) is in an invalid state caused

GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

3321 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3323 GrB\_DIMENSION\_MISMATCH Mask or vector dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

### Description

- This variant of GrB\_eWiseMult computes the element-wise "product" of two GraphBLAS vectors:  $w = u \otimes v$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (u \otimes v)$ .

  Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3334 Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- Up to four argument vectors are used in the GrB\_eWiseMult operation:
- 3337 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3338 2.  $\operatorname{mask} = \langle \mathbf{D}(\operatorname{mask}), \operatorname{size}(\operatorname{mask}), \mathbf{L}(\operatorname{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3339 3.  $u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$
- 3340 4.  $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$
- The argument vectors, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3345 2.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3.  $\mathbf{D}(\mathbf{v})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 3360 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If  $mask \neq GrB\_NULL$ ,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- 3363 ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
  - (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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4. Vector  $\tilde{\mathbf{v}} \leftarrow \mathbf{v}$ .

The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:

- 3369 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}}) = \operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{size}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.
- We describe this in terms of two intermediate vectors:
  - $\widetilde{\mathbf{t}}$ : The vector holding the element-wise "product" of  $\widetilde{\mathbf{u}}$  and vector  $\widetilde{\mathbf{v}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by:

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$$t_i = (\widetilde{\mathbf{u}}(i) \otimes \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

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• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
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$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
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$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4.3.4.2 eWiseMult: Matrix variant

Perform element-wise (general) multiplication on the intersection of elements of two matrices, producing a third matrix as result.

### 3411 C Syntax

```
3412
             GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    С,
                                         const GrB_Matrix
                                                                   Mask,
3413
                                         const GrB_BinaryOp
                                                                    accum,
3414
                                         const GrB_Semiring
                                                                    op,
3415
                                         const GrB_Matrix
                                                                    Α,
3416
                                         const GrB_Matrix
                                                                    Β,
3417
                                         const GrB Descriptor
                                                                    desc);
3418
3419
             GrB Info GrB eWiseMult(GrB Matrix
                                                                    C,
3420
                                         const GrB Matrix
                                                                   Mask,
3421
                                         const GrB BinaryOp
                                                                    accum,
3422
                                         const GrB_Monoid
                                                                    op,
3423
                                         const GrB_Matrix
                                                                   Α,
3424
                                         const GrB_Matrix
                                                                   В,
3425
                                         const GrB_Descriptor
                                                                    desc);
3426
3427
             GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    C,
3428
                                         const GrB_Matrix
                                                                   Mask,
3429
                                         const GrB_BinaryOp
                                                                    accum,
3430
                                         const GrB_BinaryOp
                                                                    op,
3431
                                         const GrB_Matrix
                                                                    Α,
3432
                                         const GrB Matrix
3433
                                                                   В,
                                         const GrB_Descriptor
                                                                    desc);
3434
```

### 3435 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

3451		BinaryOp: $F_t$	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$(p), \mathbf{D}_{in_2}(op), \bigcirc(op) \rangle.$
3452		Monoid: $F_t$	$\mathbf{D}_{o} = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{D}(op) \rangle$	$O(op), \bigcirc(op)$ ; the identity element is ig-
3453		no	ored.	
3454		Semiring: $F_t$	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$(p), \mathbf{D}_{in_2}(op), \otimes (op)$ ; the additive monoid
3455		is	ignored.	
3456 3457	` '	The GraphBI ation.	LAS matrix holding t	the values for the left-hand matrix in the
0.0.	-			
3458	` ,	-	AS matrix holding the	he values for the right-hand matrix in the
3459	oper	ation.		
3460	desc (IN)	An optional op	peration descriptor. If	a default descriptor is desired, GrB_NULL
3460 3461	` /		•	a default descriptor is desired, GrB_NULL alue pairs are listed as follows:
	shou	ld be specified	. Non-default field/va	alue pairs are listed as follows:
3461	` /		•	·
3461	shou	ld be specified	. Non-default field/va	alue pairs are listed as follows:
3461	shou	ld be specified Field GrB_OUTP	. Non-default field/va	Description  Output matrix C is cleared (all elements removed) before the result is stored in it.  The write mask is constructed from the
3461	Shou Param C	ld be specified Field GrB_OUTP	Value  GrB_REPLACE	Description Output matrix C is cleared (all elements removed) before the result is stored in it.

GrB\_COMP

GrB\_TRAN

GrB\_TRAN

Use the complement of  $\mathsf{Mask}.$ 

Use transpose of A for the operation.

Use transpose of B for the operation.

# Return Values

Mask

Α

В

 $\mathsf{GrB} \_\mathsf{MASK}$ 

GrB\_INP0

GrB\_INP1

3465	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3466		blocking mode, this indicates that the compatibility tests on di-
3467		mensions and domains for the input arguments passed successfully.
3468		Either way, output matrix C is ready to be used in the next method
3469		of the sequence.
3470	GrB_PANIC	Unknown internal error.
3471	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3472		GraphBLAS objects (input or output) is in an invalid state caused
3473		by a previous execution error. Call GrB_error() to access any error
3474		messages generated by the implementation.
3475	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
<sub>3476</sub> (	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by
3477		a call to $new$ (or Matrix_dup for matrix parameters).
3478	GrB DIMENSION MISMATCH	Mask and/or matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

# 3483 Description

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This variant of GrB\_eWiseMult computes the element-wise "product" of two GraphBLAS matrices:  $C = A \otimes B$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \otimes B)$ .

Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3489 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB\_eWiseMult operation:
- 3492 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3493 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3495 4.  $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3501 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any

- compatibility rule above is violated, execution of GrB\_eWiseMult ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :
- 3513 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- $\mathbf{M}$ , is computed from argument Mask as follows:
- 3515 (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - $$\begin{split} \text{ii. Otherwise, } \mathbf{M} &= \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ &\{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle. \end{split}$$
    - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3523 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}) = \operatorname{nrows}(\widetilde{\mathbf{C}}).$
- 3528 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.
- We describe this in terms of two intermediate matrices:
  - $\widetilde{\mathbf{T}}$ : The matrix holding the element-wise product of  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
  - ullet  $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \otimes \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
3549
3550
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4.3.5 eWiseAdd: Element-wise addition

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

#### 3573 4.3.5.1 eWiseAdd: Vector variant

Perform element-wise (general) addition on the elements of two vectors, producing a third vector as result.

### 3576 C Syntax

```
GrB_Info GrB_eWiseAdd(GrB_Vector
3577
                                                                   W,
                                       const GrB_Vector
                                                                   mask,
3578
                                        const GrB_BinaryOp
                                                                   accum,
3579
                                        const GrB_Semiring
                                                                   op,
3580
                                        const GrB_Vector
3581
                                                                   u,
                                        const GrB_Vector
                                                                   v,
3582
                                        const GrB_Descriptor
                                                                   desc);
3583
3584
              GrB_Info GrB_eWiseAdd(GrB_Vector
3585
                                                                   w,
                                       const GrB_Vector
                                                                   mask,
3586
                                       const GrB BinaryOp
                                                                   accum,
3587
                                       const GrB_Monoid
3588
                                                                   op,
                                        const GrB Vector
                                                                   u,
3589
                                       const GrB Vector
                                                                   v,
3590
                                        const GrB_Descriptor
                                                                   desc);
3591
3592
              GrB_Info GrB_eWiseAdd(GrB_Vector
3593
                                                                   W,
                                       const GrB_Vector
                                                                   mask,
3594
                                       const GrB_BinaryOp
3595
                                                                   accum,
                                        const GrB_BinaryOp
                                                                   op,
3596
                                        const GrB_Vector
                                                                   u,
3597
                                        const GrB_Vector
3598
                                                                   v,
                                       const GrB_Descriptor
                                                                   desc);
3599
```

#### 3600 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$ , used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigoplus (\mathsf{op}) \rangle$ ; the multiplicative binary op and additive identity are ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB MASK	GrB COMP	Use the complement of mask.

#### Return Values

3630	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3631		blocking mode, this indicates that the compatibility tests on di-
3632		mensions and domains for the input arguments passed successfully.
3633		Either way, output vector w is ready to be used in the next method
3634		of the sequence.
3635	GrB_PANIC	Unknown internal error.
3636	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3637		GraphBLAS objects (input or output) is in an invalid state caused
3638		by a previous execution error. Call GrB_error() to access any error

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

messages generated by the implementation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3643 GrB\_DIMENSION\_MISMATCH Mask or vector dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

## $\mathbf{Description}$

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This variant of GrB\_eWiseAdd computes the element-wise "sum" of two GraphBLAS vectors:  $w = u \oplus v$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (u \oplus v)$ . Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3654 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB\_eWiseAdd operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

- 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3659 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- 3660 4.  $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3665 2.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3.  $\mathbf{D}(\mathbf{v})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 3667 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5.  $\mathbf{D}(\mathbf{u})$  and  $\mathbf{D}(\mathbf{v})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 6. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
  - (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. Vector  $\widetilde{\mathbf{v}} \leftarrow \mathbf{v}$ .
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 3690 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.
  We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the element-wise "sum" of  $\widetilde{\mathbf{u}}$  and vector  $\widetilde{\mathbf{v}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cup \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \oplus \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$
3702
$$t_i = \widetilde{\mathbf{u}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

3704
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$$t_i = \widetilde{\mathbf{v}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{v}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a *standard vector accumulate*:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP]$ .GrB\\_REPLACE is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.5.2 eWiseAdd: Matrix variant

Perform element-wise (general) addition on the elements of two matrices, producing a third matrix as result.

## 3737 C Syntax

```
3738
             GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                  С,
                                       const GrB_Matrix
                                                                  Mask,
3739
                                       const GrB_BinaryOp
                                                                  accum,
3740
                                       const GrB_Semiring
                                                                  op,
3741
                                       const GrB_Matrix
                                                                  Α,
3742
                                       const GrB_Matrix
                                                                  Β,
3743
                                       const GrB_Descriptor
                                                                  desc);
3744
3745
             GrB Info GrB eWiseAdd(GrB Matrix
                                                                  С,
3746
                                       const GrB Matrix
                                                                  Mask,
3747
                                       const GrB_BinaryOp
                                                                  accum,
3748
                                       const GrB_Monoid
                                                                  op,
3749
                                       const GrB_Matrix
                                                                  Α,
3750
                                       const GrB_Matrix
                                                                  В,
3751
                                       const GrB_Descriptor
                                                                  desc);
3752
3753
             GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                  С,
3754
                                       const GrB_Matrix
                                                                  Mask,
3755
                                       const GrB_BinaryOp
                                                                  accum,
3756
                                       const GrB_BinaryOp
                                                                  op,
3757
                                       const GrB_Matrix
                                                                  Α,
3758
                                       const GrB Matrix
                                                                  В,
3759
                                       const GrB_Descriptor
                                                                  desc);
3760
```

## 3761 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$ , used:

3777	BinaryOp: $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op), \mathbf{D}_{in_2}(op), \bigcirc(op) \rangle$ .
3778	Monoid: $F_b = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{D}(op), \bigcirc(op) \rangle$ ; the identity element is ig-
3779	nored.
3780	Semiring: $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op), \mathbf{D}_{in_2}(op), \bigoplus (op) \rangle$ ; the multiplicative bi-
3781	nary op and additive identity are ignored.
3782 3783	A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the operation.
3784	B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the
3785	operation.
3786	$desc$ (IN) An optional operation descriptor. If a $\mathit{default}$ descriptor is desired, $GrB\_NULL$
3787	should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
			removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input Mask matrix. The stored values are
			not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
В	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

3791 3792 3793 3794 3795	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
3796	GrB_PANIC	Unknown internal error.
3797 3798 3799 3800	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
3801	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
3802 <b>(</b>	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
3804	GrB_DIMENSION_MISMATCH	Mask and/or matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

# 3809 Description

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This variant of  $GrB_eWiseAdd$  computes the element-wise "sum" of two GraphBLAS matrices:  $C = A \oplus B$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \oplus B)$ . Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB\_eWiseAdd operation:
- 3818 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3819 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3820 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3821 4.  $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 3826 2.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3827 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 3828 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5.  $\mathbf{D}(\mathsf{A})$  and  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 6. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- 3844 (b) If  $Mask \neq GrB\_NULL$ ,
- i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 3854 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}).$
- 3855 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.
  We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the element-wise sum of  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
  - $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cup \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

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$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \oplus \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$
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$$T_{ij} = \widetilde{\mathbf{A}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$
3870 
$$T_{ij} = \widetilde{\mathbf{B}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{B}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

• If  $accum = GrB\_NULL$ , then  $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$ .

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{C}}$  and  $\tilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\dot{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 3899 4.3.6 extract: Selecting sub-graphs

3900 Extract a subset of a matrix or vector.

### 3901 4.3.6.1 extract: Standard vector variant

Extract a sub-vector from a larger vector as specified by a set of indices. The result is a vector whose size is equal to the number of indices.

## 3904 C Syntax

```
GrB_Info GrB_extract(GrB_Vector
3905
                                                                  W,
                                      const GrB_Vector
                                                                  mask,
3906
                                      const GrB_BinaryOp
                                                                  accum,
3907
                                      const GrB_Vector
                                                                  u,
3908
                                      const GrB_Index
                                                                 *indices,
3909
                                      GrB_Index
                                                                  nindices,
3910
                                      const GrB_Descriptor
                                                                  desc);
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```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - u (IN) The GraphBLAS vector from which the subset is extracted.
- indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations of elements from u that are extracted. If all elements of u are to be extracted in order from 0 to nindices 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.
- nindices (IN) The number of values in indices array. Must be equal to size(w).

2	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL
3	should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
3935				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

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3937 3938 3939 3940 3941	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
3942	GrB_PANIC	Unknown internal error.
3943 3944 3945 3946	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
3947	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
3948 3949	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
3950 3951	GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to $\mathbf{size}(u).$ In non-blocking mode, this error can be deferred.
3952	GrB_DIMENSION_MISMATCH	$mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nindices \neq \mathbf{size}(w).$
3953 3954 3955 3956	GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
3957	GrB_NULL_POINTER	Argument row_indices is a NULL pointer.

# Description

This variant of  $GrB_{extract}$  computes the result of extracting a subset of locations from a Graph-BLAS vector in a specific order: w = u(indices); or, if an optional binary accumulation operator 3961 ( $\odot$ ) is provided,  $w = w \odot u$ (indices). More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \mathsf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices}, \ \ \mathsf{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices} \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3966 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB\_extract operation:

```
3969 1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

3970 2. 
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$$

3971 3. 
$$\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$$

The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 3976 2.  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}(u)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:

(a) If mask = GrB\_NULL, then 
$$\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$$
.

- 3990 (b) If mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- 3992 ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 3995 4. The internal index array,  $\widetilde{I}$ , is computed from argument indices as follows:
- (a) If indices = GrB\_ALL, then  $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nindices}$ .
- (b) Otherwise,  $\tilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 4000 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 2. nindices =  $\mathbf{size}(\widetilde{\mathbf{w}})$ .
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- $\frac{4006}{1000}$  We are now ready to carry out the extract and any additional associated operations. We describe  $\frac{4007}{10000}$  this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the extraction from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{w}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- 4010 The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, \widetilde{\mathbf{u}}(\widetilde{\boldsymbol{I}}[i])) \ \forall \ i, 0 \leq i < \mathsf{nindices} : \widetilde{\boldsymbol{I}}[i] \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

- At this point, if any value in  $\tilde{I}$  is not in the valid range of indices for vector  $\tilde{\mathbf{u}}$ , the execution of GrB\_extract ends and the index-out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 4.3.6.2 extract: Standard matrix variant

Extract a sub-matrix from a larger matrix as specified by a set of row indices and a set of column indices. The result is a matrix whose size is equal to size of the sets of indices.

### 4046 C Syntax

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```
C,
             GrB_Info GrB_extract(GrB_Matrix
4047
                                      const GrB_Matrix
                                                               Mask,
4048
                                      const GrB_BinaryOp
                                                               accum,
4049
                                      const GrB_Matrix
                                                               Α,
4050
                                      const GrB_Index
                                                               *row_indices,
4051
                                      GrB_Index
                                                               nrows,
4052
                                      const GrB_Index
                                                               *col_indices,
4053
                                      GrB_Index
                                                               ncols,
4054
                                      const GrB Descriptor
                                                               desc);
4055
```

# 4056 Parameters

4057 4058 4059	С	that	may be accum		rix. On input, the matrix provides values it of the extract operation. On output, the n.
4060 4061 4062 4063 4064	Mask	store matr of th in Ta	ed into the out ix C. If the Gr e Mask matrix able 3.2. If the	Eput matrix C. The r B_STRUCTURE desc must be of type book e default mask is des	trols which results from this operation are mask dimensions must match those of the criptor is <i>not</i> set for the mask, the domain of or any of the predefined "built-in" types sired (i.e., a mask that is all true with the
4065 4066 4067 4068	accum	(IN)	An optional less. If assignment		for accumulating entries into existing C mulation is desired, GrB_NULL should be
4069	А	(IN)	The GraphBL	AS matrix from which	ch the subset is extracted.
4070 4071 4072 4073 4074	row_indices	from in or value	which element der, GrB_ALL e, this array n	ts are extracted. If el should be specified. may be manipulated	of indices corresponding to the rows of A ements in all rows of A are to be extracted Regardless of execution mode and return by the caller after this operation returns tions for this operation.
4075	nrows	(IN)	The number o	of values in the row_i	ndices array. Must be equal to $\mathbf{nrows}(C)$ .
4076 4077 4078 4079 4080	col_indices	of A be ex mode	from which extracted in order and return v	lements are extracted ler, then GrB_ALL showalue, this array may	of indices corresponding to the columns d. If elements in all columns of A are to nould be specified. Regardless of execution y be manipulated by the caller after this deferred computations for this operation.
4081	ncols	(IN)	The number o	of values in the col_in	ndices array. Must be equal to $\mathbf{ncols}(C).$
4082 4083 4084	desc	, ,		=	f a default descriptor is desired, GrB_NULL alue pairs are listed as follows:
	Pa	ram	Field	Value	Description
	С		GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
4085	Ma	ask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.

Use the complement of  $\mathsf{Mask}.$ 

Use transpose of A for the operation.

GrB\_MASK GrB\_COMP

 $\mathsf{GrB} \mathsf{\_TRAN}$ 

GrB\_INP0

Mask

4087 4088 4089 4090 4091	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
4092	GrB_PANIC	Unknown internal error.
4093 4094 4095 4096	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4097	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
4098 4099	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
4100 4101 4102	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$ , or a value in $\mathbf{col\_indices}$ is greater than or equal to $\mathbf{ncols}(A)$ . In non-blocking mode, this error can be deferred.
4103 4104	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows $\neq$ $\mathbf{nrows}(C)$ , or $\mathbf{ncols} \neq \mathbf{ncols}(C)$ .
4105 4106 4107 4108	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4109 4110	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

### 4111 Description

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This variant of GrB\_extract computes the result of extracting a subset of locations from specified rows and columns of a GraphBLAS matrix in a specific order: C = A(row\_indices, col\_indices); or, if an optional binary accumulation operator (\odot) is provided, C = C \odot A(row\_indices, col\_indices). More explicitly (not accounting for an optional transpose of A):
```

```
\mathsf{C}(i,j) = \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols}, \ \mathsf{or} \\ \mathsf{C}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \\ \mathsf{ncols}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{ncols}(i), \mathsf{ncols}(i), \mathsf{
```

Logically, this operation occurs in three steps:

**Setup** The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

- 4120 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4122 Up to three argument matrices are used in the GrB\_extract operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2. Mask =  $\langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .
- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
- 4148 ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\operatorname{\mathsf{desc}}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$

- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- (a) If row\_indices = GrB\_ALL, then  $\tilde{I}[i] = i, \forall i : 0 \le i < \text{nrows}$ .
- (b) Otherwise,  $\tilde{I}[i] = \text{row\_indices}[i], \forall i : 0 \le i < \text{nrows}.$
- 5. The internal column index array,  $\tilde{J}$ , is computed from argument col\_indices as follows:
- (a) If col\_indices = GrB\_ALL, then  $\widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathsf{nrows}.$
- 4.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathsf{ncols}.$
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the extraction from  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i, j, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j])) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j]) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

- At this point, if any value in the  $\widetilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$  or any value in the  $\widetilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\widetilde{\mathbf{A}}))$ , the execution of  $\mathsf{GrB\_extract}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.
- The intermediate matrix  $\widetilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:
- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathsf{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \ \mathrm{if} \ (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.6.3 extract: Column (and row) variant

Extract from one column of a matrix into a vector. Note that with the transpose descriptor for the source matrix, elements of an arbitrary row of the matrix can be extracted with this function as well.

## 4210 C Syntax

4211	<pre>GrB_Info GrB_extract(GrB_Vector</pre>	w ,
4212	const GrB_Vector	mask,
4213	const GrB_BinaryOp	accum,
4214	const GrB_Matrix	Α,
4215	const GrB_Index	*row_indices,
4216	${\tt GrB\_Index}$	nrows,
4217	${\tt GrB\_Index}$	<pre>col_index,</pre>
4218	const GrB_Descriptor	desc);

### 4219 Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - A (IN) The GraphBLAS matrix from which the column subset is extracted.
- row\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations within the specified column of A from which elements are extracted. If elements in all rows of A are to be extracted in order, GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.
  - nrows (IN) The number of indices in the row indices array. Must be equal to size(w).
- col\_index (IN) The index of the column of A from which to extract values. It must be in the range [0, ncols(A)).
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4245				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

4247 4248 4249 4250 4251	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence.
4252	GrB_PANIC	Unknown internal error.
4253 4254 4255 4256	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4257	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4258 4259	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
4260	GrB_INVALID_INDEX	${\tt col\_index} \ {\rm is} \ {\rm outside} \ {\rm the} \ {\rm allowable} \ {\rm range} \ ({\rm i.e.}, {\rm greater} \ {\rm than} \ {\bf ncols}(A)).$
4261 4262	GrB_INDEX_OUT_OF_BOUNDS	A value in $row\_indices$ is greater than or equal to $nrows(A)$ . In non-blocking mode, this error can be deferred.
4263	GrB_DIMENSION_MISMATCH	$mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nrows \neq \mathbf{size}(w).$
4264 4265 4266 4267	GrB_DOMAIN_MISMATCH	The domains of the vector or matrix are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4268	GrB_NULL_POINTER	Argument row_indices is a NULL pointer.

# 4269 Description

This variant of  $GrB\_extract$  computes the result of extracting a subset of locations (in a specific order) from a specified column of a GraphBLAS matrix:  $w = A(:,col\_index)(row\_indices)$ ; or, if

an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot A(:,col\_index)(row\_indices)$ .

More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows}, \; \; \mathsf{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows} \end{aligned}$$

- Logically, this operation occurs in three steps:
- Setup The internal matrices, vectors, and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4278 Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4280 Up to three argument vectors and matrices are used in this GrB\_extract operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$

- 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument vectors, matrix and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 4288 2.  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}(A)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vector, matrix, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .

- 4302 (b) If  $mask \neq GrB\_NULL$ ,
- i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE} \text{ is } \mathsf{set}, \text{ then } \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i: i \in \mathbf{ind}(\mathsf{mask})\} \rangle,$
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- 4308 (a) If indices = GrB\_ALL, then  $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i : 0 \leq i < \text{nrows}$ .
- (b) Otherwise,  $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$
- The internal vector, mask, and index array are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- $2. \mathbf{size}(\widetilde{\mathbf{w}}) = \mathsf{nrows}.$
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- The col\_index parameter is checked for a valid value. The following condition must hold:
- $1. 0 \leq \text{col\_index} < \text{ncols}(A)$
- If the rule above is violated, execution of GrB\_extract ends and the invalid index error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the extraction from a column of  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{A}), \mathsf{nrows}, \{(i, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index})) \ \forall \ i, 0 \leq i < \mathsf{nrows} : (\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index}) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle.$$

- 4328 At this point, if any value in  $\widetilde{I}$  is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$ , the execution of  $\mathsf{GrB}$ \_extract
- ends and the index-out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode,
- the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.
- 4331 Vector, w, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

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• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\text{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.7 assign: Modifying sub-graphs

4360 Assign the contents of a subset of a matrix or vector.

## 4.3.7.1 assign: Standard vector variant

Assign values from one GraphBLAS vector to a subset of a vector as specified by a set of indices.

The size of the input vector is the same size as the index array provided.

## 4364 C Syntax

```
GrB_Info GrB_assign(GrB_Vector
                                                               W,
4365
                                     const GrB Vector
                                                              mask,
4366
                                     const GrB_BinaryOp
                                                               accum,
4367
                                     const GrB Vector
                                                               u,
4368
                                     const GrB_Index
                                                              *indices.
4369
                                     GrB_Index
                                                              nindices,
4370
                                     const GrB_Descriptor
                                                              desc);
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```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - u (IN) The GraphBLAS vector whose contents are assigned to a subset of w.
  - indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0 to nindices 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
    - nindices (IN) The number of values in indices array. Must be equal to size(u).
      - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
4207	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4397				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

4399 4400 4401 4402 4403	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
4404	GrB_PANIC	Unknown internal error.
4405 4406 4407 4408	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4409	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4410 4411	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
4412 4413	GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to $\mathbf{size}(w).$ In non-blocking mode, this can be reported as an execution error.
4414	GrB_DIMENSION_MISMATCH	$mask$ and $w$ dimensions are incompatible, or $nindices \neq \mathbf{size}(u).$
4415 4416 4417 4418	GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4419	GrB_NULL_POINTER	Argument indices is a NULL pointer.

## Description

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This variant of  $GrB_assign$  computes the result of assigning elements from a source GraphBLAS vector to a destination GraphBLAS vector in a specific order: w(indices) = u; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w(indices) = w(indices) \odot u$ . More explicitly:

- Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4428 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4430 Up to three argument vectors are used in the GrB\_assign operation:
- 4431 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 4432 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4438 2.  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}(u)$ .
- 3. If accum is not  $GrB_NULL$ , then D(w) must be compatible with  $D_{in_1}(accum)$  and  $D_{out}(accum)$  of the accumulation operator and D(u) must be compatible with  $D_{in_2}(accum)$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask and index array used in the computation are formed (← denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathsf{w}) \} \rangle$ .
- 4452 (b) If  $mask \neq GrB\_NULL$ ,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .

3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. The internal index array,  $\widetilde{I}$ , is computed from argument indices as follows:
  - (a) If indices = GrB\_ALL, then  $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nindices}$ .
  - (b) Otherwise,  $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4463 2. nindices =  $\mathbf{size}(\widetilde{\mathbf{u}})$ .

If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{w}}$ .
- $\bullet$   $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\mathbf{u}}(i)) \forall i, 0 \leq i < \mathsf{nindices} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

At this point, if any value of  $\tilde{I}[i]$  is outside the valid range of indices for vector  $\tilde{\mathbf{w}}$ , computation ends and the method returns the index-out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector,  $\mathbf{w}$ , is invalid from this point forward in the sequence.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB \ NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector  $\tilde{\mathbf{z}}$  as follows: We start with the structure of  $\tilde{\mathbf{w}}$  ( $\mathbf{ind}(\tilde{\mathbf{w}})$ ) and remove from it all the indices of  $\tilde{\mathbf{w}}$  that are in the set of indices being assigned ( $\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{w}})$ ). Finally, we add the structure of  $\tilde{\mathbf{t}}$  ( $\mathbf{ind}(\tilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
  
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.7.2 assign: Standard matrix variant

Assign values from one GraphBLAS matrix to a subset of a matrix as specified by a set of indices.

The dimensions of the input matrix are the same size as the row and column index arrays provided.

## C Syntax

```
4519 GrB_Info GrB_assign(GrB_Matrix C,
4520 const GrB_Matrix Mask,
4521 const GrB_BinaryOp accum,
4522 const GrB_Matrix A,
```

4523	const GrB_Index	*row_indices,
4524	<pre>GrB_Index</pre>	nrows,
4525	const GrB_Index	$*col_indices,$
4526	<pre>GrB_Index</pre>	ncols,
4527	<pre>const GrB_Descriptor</pre>	desc);

### 4528 Parameters

- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - A (IN) The GraphBLAS matrix whose contents are assigned to a subset of C.
- row\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C that are assigned. If all rows of C are to be assigned in order from 0 to nrows 1, then GrB\_ALL can be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.
  - nrows (IN) The number of values in the row\_indices array. Must be equal to **nrows**(A) if A is not transposed, or equal to **ncols**(A) if A is transposed.
  - col\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C that are assigned. If all columns of C are to be assigned in order from 0 to ncols 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.
    - ncols (IN) The number of values in col\_indices array. Must be equal to  $\mathbf{ncols}(A)$  if A is not transposed, or equal to  $\mathbf{nrows}(A)$  if A is transposed.

desc (	(IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL
S	should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4563				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

4565 4566 4567 4568 4569	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
4570	GrB_PANIC	Unknown internal error.
4571 4572 4573 4574	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4575	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
4576 4577	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to <code>new</code> (or <code>Matrix_dup</code> for matrix parameters).
4578 4579 4580	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(C)$ , or a value in $\mathbf{col\_indices}$ is greater than or equal to $\mathbf{ncols}(C)$ . In non-blocking mode, this can be reported as an execution error.
4581 4582	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, $nrows \neq nrows(A)$ , or $ncols \neq ncols(A)$ .
4583 4584 4585 4586	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4587 4588	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

## 4589 Description

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This variant of GrB\_assign computes the result of assigning the contents of A to a subset of rows and columns in C in a specified order:  $C(row\_indices, col\_indices) = A$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices) \odot$ A. More explicitly (not accounting for an optional transpose of A):

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

4600 Up to three argument matrices are used in the GrB\_assign operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2.  $Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$

The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 4608 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathbf{C}$ .

- 2. Two-dimensional mask M is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If  $Mask \neq GrB\_NULL$ ,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
- 4626 ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$ 
  - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$ .
- (b) Otherwise,  $\tilde{I}[i] = \text{row\_indices}[i], \forall i : 0 \le i < \text{nrows}.$
- 5. The internal column index array,  $\tilde{J}$ , is computed from argument col\_indices as follows:
- 4634 (a) If col\_indices = GrB\_ALL, then  $\widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \text{col\_indices}[j], \ \forall \ j: 0 \leq j < \text{ncols.}$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 4639 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathsf{nrows}.$
- 4641 4.  $\mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathsf{ncols}.$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{T}}$ : The matrix holding the contents from  $\widetilde{\mathbf{A}}$  in their destination locations relative to  $\widetilde{\mathbf{C}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \widetilde{\mathbf{A}}(i,j)) \ \forall \ (i,j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

At this point, if any value in the  $\widetilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{C}}))$  or any value in the  $\widetilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\widetilde{\mathbf{C}}))$ , the execution of  $\mathsf{GrB\_assign}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows:

• If  $accum = GrB\_NULL$ , then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ \{(i, j, Z_{ij}) \forall (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The above expression defines the structure of matrix  $\widetilde{\mathbf{Z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{C}}$  ( $\mathbf{ind}(\widetilde{\mathbf{C}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{C}}$  that are in the set of indices being assigned ( $\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{T}}$  ( $\mathbf{ind}(\widetilde{\mathbf{T}})$ ).

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 4.3.7.3 assign: Column variant

Assign the contents a vector to a subset of elements in one column of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a row of a matrix.

## 4699 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
4700
                                     const GrB Vector
                                                               mask,
4701
                                     const GrB BinaryOp
4702
                                                               accum,
                                     const GrB Vector
                                                               u,
4703
                                     const GrB_Index
                                                              *row_indices,
4704
                                     GrB Index
                                                               nrows,
4705
                                     GrB Index
                                                               col_index,
4706
                                     const GrB Descriptor
                                                               desc);
4707
```

### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified column of the output matrix C. The mask dimensions must match those of a single column of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type

4716 4717 4718		is de	•	mask that is all true	' types in Table 3.2. If the default mask with the dimensions of a column of C),
4719 4720 4721	accum	` ′	les. If assignm		for accumulating entries into existing C mulation is desired, GrB_NULL should be
4722 4723	u	(IN) of C.	-	AS vector whose cont	ents are assigned to (a subset of) a column
4724 4725 4726 4727 4728 4729 4730	row_indices	the sin C be sp man ferre impl	specified colum are to be assign pecified. Regatipulated by the d computation	on of C that are to be gned in order from in ardless of execution made caller after this op- areas for this operation.	of indices corresponding to the locations in the assigned. If all elements of the column dex $0$ to $nrows-1$ , then $GrB\_ALL$ should node and return value, this array may be deration returns without affecting any description of the same location which leads to
4732	nrows	(IN)	The number of	of values in row_indice	es array. Must be equal to $\mathbf{size}(u).$
4733	col_index	(IN)	The index of t	the column in C to as	ssign. Must be in the range $[0, \mathbf{ncols}(C))$ .
4734 4735 4736	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:				
4737	m.	aram_	Field  GrB_OUTP  GrB_MASK	Value  GrB_REPLACE  GrB_STRUCTURE	Description  Output column in C is cleared (all elements removed) before result is stored in it.  The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
	m	ask	GrB_MASK	GrB_COMP	Use the complement of mask.

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4739	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
4740		blocking mode, this indicates that the compatibility tests on
4741		dimensions and domains for the input arguments passed suc-
4742		cessfully. Either way, output matrix C is ready to be used in the
4743		next method of the sequence.

GrB\_PANIC Unknown internal error.

```
GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the
4745
                                     opaque GraphBLAS objects (input or output) is in an invalid
4746
                                     state caused by a previous execution error. Call GrB_error() to
4747
                                     access any error messages generated by the implementation.
4748
           GrB_OUT_OF_MEMORY Not enough memory available for operation.
4749
      Grb_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized
4750
                                     by a call to new (or dup for vector or matrix parameters).
               GrB INVALID INDEX col index is outside the allowable range (i.e., greater than ncols(C)).
4752
    GrB_INDEX_OUT_OF_BOUNDS A value in row_indices is greater than or equal to nrows(C). In
4753
                                     non-blocking mode, this can be reported as an execution error.
4754
      GrB_DIMENSION_MISMATCH mask size and number of rows in C are not the same, or nrows \neq
4755
                                     size(u).
4756
         Grb DOMAIN MISMATCH The domains of the matrix and vector are incompatible with
4757
                                     each other or the corresponding domains of the accumulation
4758
                                     operator, or the mask's domain is not compatible with bool (in
4759
                                     the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
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```

### 4762 Description

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```
This variant of GrB_assign computes the result of assigning a subset of locations in a column of a GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector:

C(:, col_index) = u; or, if an optional binary accumulation operator (\odot) is provided, C(:, col_index) = C(:, col_index) \odot u. Taking order of row_indices into account, it is more explicitly written as:
```

C(row\_indices[
$$i$$
], col\_index) = u( $i$ ),  $\forall i : 0 \le i < \text{nrows}$ , or C(row\_indices[ $i$ ], col\_index) = C(row\_indices[ $i$ ], col\_index)  $\odot$  u( $i$ ),  $\forall i : 0 \le i < \text{nrows}$ .

GrB\_NULL\_POINTER Argument row\_indices is a NULL pointer.

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

4771 Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

4773 Up to three argument vectors and matrices are used in this GrB\_assign operation:

```
1. C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle
```

4775 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$ 

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{u})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

The col\_index parameter is checked for a valid value. The following condition must hold:

```
1. 0 \leq \text{col\_index} < \mathbf{ncols}(C)
```

If the rule above is violated, execution of GrB\_assign ends and the invalid index error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):

1. The vector,  $\tilde{\mathbf{c}}$ , is extracted from a column of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \{(i, C_{ij}) \ \forall \ i: 0 \leq i < \mathbf{nrows}(\mathsf{C}), j = \mathsf{col\_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
  - (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{nrows}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4805 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- 4806 (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \ \forall \ i : 0 \leq i < \text{nrows}.$

(b) Otherwise,  $\widetilde{I}[i] = \text{row\_indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$ 

The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:

- 4810 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4811 2. nrows =  $\mathbf{size}(\widetilde{\mathbf{u}})$ .

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- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
  - $\tilde{\mathbf{t}}$ : The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{c}}$ .
    - $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(\widetilde{I}[i], \widetilde{\mathbf{u}}(i)) \ \forall \ i, \ 0 \le i < \mathsf{nrows} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

- At this point, if any value of  $\tilde{I}[i]$  is outside the valid range of indices for vector  $\tilde{\mathbf{c}}$ , computation ends and the method returns the index out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:
  - If  $accum = GrB\_NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

- The above expression defines the structure of vector  $\tilde{\mathbf{z}}$  as follows: We start with the structure of  $\tilde{\mathbf{c}}$  ( $\mathbf{ind}(\tilde{\mathbf{c}})$ ) and remove from it all the indices of  $\tilde{\mathbf{c}}$  that are in the set of indices being assigned ( $\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{c}})$ ). Finally, we add the structure of  $\tilde{\mathbf{t}}$  ( $\mathbf{ind}(\tilde{\mathbf{t}})$ ).
- The values of the elements of  $\widetilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{c}}$  and  $\widetilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{c}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$
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$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
4845
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up the  $\tilde{\mathbf{z}}$  vector are written into the column of the final result matrix,  $C(:, col\_index)$ . This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C(:, col\_index) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : j \neq \mathsf{col\_index}\} \cup \{(i,\mathsf{col\_index},z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the column of the final result matrix,  $C(:,col\_index)$ , and elements of this column that fall outside the set indicated by the mask are unchanged:

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$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}): j \neq \mathsf{col\_index}\} \cup \\ \{(i,\mathsf{col\_index},\widetilde{\mathbf{c}}(i)): i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ \{(i,\mathsf{col\_index},z_i): i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 4.3.7.4 assign: Row variant

Assign the contents a vector to a subset of elements in one row of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a column of a matrix.

## 4868 C Syntax

```
GrB_Info GrB_assign(GrB_Matrix
                                                              С,
4869
                                     const GrB_Vector
                                                              mask,
4870
                                     const GrB_BinaryOp
                                                              accum.
4871
                                     const GrB_Vector
                                                              u,
4872
                                     GrB_Index
                                                              row_index,
4873
                                     const GrB_Index
                                                             *col_indices,
4874
                                                              ncols,
                                     GrB Index
4875
                                     const GrB_Descriptor
                                                              desc);
4876
```

#### 4877 Parameters

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- C (INOUT) An existing GraphBLAS Matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified row of the output matrix C. The mask dimensions must match those of a single row of the matrix C. If the GrB\_STRUCTURE descriptor is not set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a row of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - u (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a row of C.
- row\_index (IN) The index of the row in C to assign. Must be in the range [0, nrows(C)).
- col\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified row of C that are to be assigned. If all elements of the row in C are to be assigned in order from index 0 to ncols 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
  - ncols (IN) The number of values in col\_indices array. Must be equal to size(u).
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
4905	С	GrB_OUTP	GrB_REPLACE	Output row in C is cleared (all elements
				removed) before result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

# Return Values

4907 4908 4909 4910 4911	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
4912	GrB_PANIC	Unknown internal error.
4913 4914 4915 4916	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4917	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4918 4919	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
4920	GrB_INVALID_INDEX	${\sf row\_index} \ {\rm is} \ {\rm outside} \ {\rm the} \ {\rm allowable} \ {\rm range} \ ({\rm i.e., greater} \ {\rm than} \ {\bf nrows}(C)).$
4921 4922	GrB_INDEX_OUT_OF_BOUNDS	A value in $col\_indices$ is greater than or equal to $ncols(C)$ . In non-blocking mode, this can be reported as an execution error.
4923 4924	GrB_DIMENSION_MISMATCH	mask size and number of columns in $C$ are not the same, or $n\text{cols} \neq \mathbf{size}(u).$
4925 4926 4927 4928	GrB_DOMAIN_MISMATCH	The domains of the matrix and vector are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4929	GrB_NULL_POINTER	Argument col_indices is a NULL pointer.

#### Description 4930

This variant of GrB\_assign computes the result of assigning a subset of locations in a row of a 4931 GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector: 4932

 $C(row\_index,:) = u;$  or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C(row\_index,:) = C(row\_index,:) \odot u$ . Taking order of col\_indices into account it is more explicitly written as:

4936 Logically, this operation occurs in three steps:

- Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4939 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4941 Up to three argument vectors and matrices are used in this GrB\_assign operation:
- 4942 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 4943 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4949 2.  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}(u)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- 4958 The row\_index parameter is checked for a valid value. The following condition must hold:
- $1. 0 \leq \text{row\_index} < \text{nrows}(C)$
- If the rule above is violated, execution of GrB\_assign ends and the invalid index error listed above is returned.
- From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):

1. The vector,  $\tilde{\mathbf{c}}$ , is extracted from a row of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(j, C_{ij}) \ \forall \ j : 0 \le j < \mathbf{ncols}(\mathsf{C}), i = \mathsf{row\_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
  - (a) If  $mask = GrB\_NULL$ , then  $\widetilde{\mathbf{m}} = \langle \mathbf{ncols}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- 4970 ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- 4971 (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. The internal column index array,  $\widetilde{J}$ , is computed from argument col\_indices as follows:
- 4974 (a) If col\_indices = GrB\_ALL, then  $\widetilde{J}[j] = j, \ \forall \ j: 0 \leq j < \text{ncols.}$
- (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \ \forall \ j: 0 \leq j < \mathsf{ncols}.$
- The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:
- 4978 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4979 2.  $\operatorname{ncols} = \operatorname{size}(\widetilde{\mathbf{u}}).$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
  - t: The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{c}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{ (\widetilde{J}[j], \widetilde{\mathbf{u}}(j)) \ \forall \ j, \ 0 \le j < \mathsf{ncols} : j \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle.$$

- At this point, if any value of  $\widetilde{J}[j]$  is outside the valid range of indices for vector  $\widetilde{\mathbf{c}}$ , computation ends and the method returns the index out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB \ \ NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector  $\widetilde{\mathbf{z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{c}}$  ( $\mathbf{ind}(\widetilde{\mathbf{c}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{c}}$  that are in the set of indices being assigned ( $\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{t}}$  ( $\mathbf{ind}(\widetilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{c}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
  
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$ 

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(j, z_i) \ \forall \ j \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_j = \widetilde{\mathbf{c}}(j) \odot \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$

$$z_j = \widetilde{\mathbf{c}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_j = \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up the  $\tilde{\mathbf{z}}$  vector are written into the column of the final result matrix,  $C(row\_index,:)$ . This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C(row\_index,:) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : i \neq \mathsf{row\_index}\} \cup \{(\mathsf{row\_index},j,z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{z}}$  indicated by the mask are copied into the column of the final result matrix,  $C(row\_index,:)$ , and elements of this column that fall outside the set indicated by the mask are unchanged:

$$\begin{split} \mathbf{L}(\mathsf{C}) &= \{(i,j,C_{ij}) : i \neq \mathsf{row\_index}\} \cup \\ &\{(\mathsf{row\_index},j,\widetilde{\mathbf{c}}(j)) : j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ &\{(\mathsf{row\_index},j,z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}. \end{split}$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.7.5 assign: Constant vector variant

Assign the same value to a specified subset of vector elements. With the use of GrB\_ALL, the entire destination vector can be filled with the constant.

## 5035 C Syntax

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```
GrB_Info GrB_assign(GrB_Vector
                                                               w,
5036
                                     const GrB_Vector
5037
                                                               mask,
                                     const GrB BinaryOp
                                                               accum,
5038
                                     <type>
                                                               val,
5039
                                     const GrB_Index
                                                              *indices.
5040
                                     GrB_Index
                                                               nindices,
5041
                                     const GrB_Descriptor
                                                               desc);
5042
             GrB_Info GrB_assign(GrB_Vector
                                                               W,
5043
                                     const GrB_Vector
                                                               mask,
5044
                                     const GrB_BinaryOp
                                                               accum,
5045
                                     const GrB_Scalar
5046
                                                               s,
                                                              *indices,
                                     const GrB_Index
5047
                                     GrB Index
                                                               nindices,
5048
                                     const GrB_Descriptor
                                                               desc);
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```

## Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
    - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - val (IN) Scalar value to assign to (a subset of) w.
- s (IN) Scalar value to assign to (a subset of) w.
- indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0

to nindices – 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. In this variant, the specific order of the values in the array has no effect on the result. Unlike other variants, if there are duplicated values in this array the result is still defined.

nindices (IN) The number of values in indices array. Must be in the range: [0, size(w)]. If nindices is zero, the operation becomes a NO-OP.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
5078	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

## Return Values

5080	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
5081		blocking mode, this indicates that the compatibility tests on
5082		dimensions and domains for the input arguments passed suc-
5083		cessfully. Either way, output vector w is ready to be used in the
5084		next method of the sequence.
5085	GrB_PANIC	Unknown internal error.
5086	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
5087		opaque GraphBLAS objects (input or output) is in an invalid
5088		state caused by a previous execution error. Call GrB_error() to
5089		access any error messages generated by the implementation.
5090	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
5091	GrB LININITIALIZED ORIECT	One or more of the GraphBLAS objects has not been initialized
	GID_GINITIALIZED_GBJECT	by a call to new (or dup for vector parameters).
5092		by a can to new (or dup for vector parameters).
5093	GrB INDEX OUT OF BOUNDS	A value in indices is greater than or equal to size(w). In non-
5094		blocking mode, this can be reported as an execution error.
		•
5095	GrB_DIMENSION_MISMATCH	mask and w dimensions are incompatible, or nindices is not less
5096		than $size(w)$ .

GrB\_DOMAIN\_MISMATCH The domains of the vector and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB NULL POINTER Argument indices is a NULL pointer.

## 5102 Description

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This variant of GrB\_assign computes the result of assigning a constant scalar value – either val or s – to locations in a destination GraphBLAS vector. Either w(indices) = val or w(indices) = s is performed. If an optional binary accumulation operator  $(\odot)$  is provided, then either  $w(indices) = w(indices) \odot val$  or  $w(indices) = w(indices) \odot s$  is performed. More explicitly, if a non-opaque value val is provided:

 $\mathsf{w}(\mathsf{indices}[i]) = \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}, \ \text{ or } \\ \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}.$ 

5109 Correspondingly, if a GrB\_Scalar s is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{s}, \ \forall \ i: 0 \le i < \mathsf{nindices}, \ \mathrm{or}$$
  
 $\mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{s}, \ \forall \ i: 0 \le i < \mathsf{nindices}.$ 

5111 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5114 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

5116 Up to two argument vectors are used in the GrB\_assign operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)

The argument scalar, vectors, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5123 2.  $\mathbf{D}(w)$  must be compatible with either  $\mathbf{D}(val)$  or  $\mathbf{D}(s)$ , depending on the signature of the method.
  - 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator.

- 4. If accum is not GrB\_NULL, then either  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask and index array used in the computation are formed  $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 5138 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Scalar  $\tilde{s} \leftarrow s$  (GrB Scalar version only).
- 4. The internal index array,  $\tilde{I}$ , is computed from argument indices as follows:
- (a) If indices = GrB\_ALL, then  $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i: 0 \leq i < \text{nindices}$ .
- 5146 (b) Otherwise,  $\tilde{I}[i] = \mathsf{indices}[i], \ \forall \ i: 0 \leq i < \mathsf{nindices}.$
- The internal vector and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- $2. 0 < \text{nindices} < \text{size}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the copies of the scalar, either val or  $\widetilde{s}$ , in their destination locations relative to  $\widetilde{\mathbf{w}}$ .

•  $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows. If a non-opaque scalar val is provided:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{val}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{I}[i], \mathsf{val}) \ \forall \ i, \ 0 \le i < \mathsf{nindices} \} \rangle.$$

Correspondingly, if a non-empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 1$ ):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{I}[i], \mathbf{val}(\widetilde{s})) \ \forall \ i, \ 0 \le i < \mathsf{nindices}\} \rangle.$$

Finally, if an empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 0$ ):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \emptyset \rangle.$$

If  $\tilde{I}$  is empty, this operation results in an empty vector,  $\tilde{\mathbf{t}}$ . Otherwise, if any value in the  $\tilde{I}$  array is not in the range  $[0, \mathbf{size}(\tilde{\mathbf{w}}))$ , the execution of GrB\_assign ends and the index out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector,  $\mathbf{w}$ , is invalid from this point forward in the sequence.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB \ NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector  $\widetilde{\mathbf{z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{w}}$  ( $\mathbf{ind}(\widetilde{\mathbf{w}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{w}}$  that are in the set of indices being assigned ( $\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{t}}$  ( $\mathbf{ind}(\widetilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
  
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$ 

where the difference operator refers to set difference. We note that in this case of assigning a constant,  $\{\widetilde{I}[k], \forall k\}$  and  $\operatorname{ind}(\widetilde{\mathbf{t}})$  are identical.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\text{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 5209 4.3.7.6 assign: Constant matrix variant

Assign the same value to a specified subset of matrix elements. With the use of GrB\_ALL, the entire destination matrix can be filled with the constant.

## 5212 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
5213
                                     const GrB_Matrix
                                                               Mask,
5214
                                     const GrB BinaryOp
                                                               accum,
5215
                                     <type>
                                                               val,
5216
                                     const GrB_Index
                                                              *row_indices,
5217
                                     GrB_Index
                                                               nrows,
5218
                                     const GrB_Index
                                                              *col_indices,
5219
                                     GrB_Index
                                                               ncols,
5220
                                     const GrB_Descriptor
                                                               desc);
5221
                                                               С,
             GrB_Info GrB_assign(GrB_Matrix
5222
                                     const GrB_Matrix
                                                               Mask,
5223
                                     const GrB_BinaryOp
                                                               accum,
5224
                                     const GrB_Scalar
5225
                                                               s,
                                     const GrB_Index
                                                              *row_indices,
5226
                                     GrB_Index
                                                               nrows,
5227
```

5228		<pre>const GrB_Index *col_indices,</pre>
5229		GrB_Index ncols,
5230		<pre>const GrB_Descriptor desc);</pre>
5231	Parameters	
5232	С	(INOUT) An existing GraphBLAS matrix. On input, the matrix provides values
5233		that may be accumulated with the result of the assign operation. On output, the
5234		matrix holds the results of the operation.
	Mode	(IN) An antional "write" mode that controls which regults from this apprection are
5235	iviask	(IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the
5236		matrix C. If the GrB_STRUCTURE descriptor is <i>not</i> set for the mask, the domain
5237 5238		of the Mask matrix must be of type bool or any of the predefined "built-in" types
5239		in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the
5240		dimensions of C), GrB_NULL should be specified.
		,,
5241	accum	(IN) An optional binary operator used for accumulating entries into existing C
5242		entries. If assignment rather than accumulation is desired, GrB_NULL should be
5243		specified.
5244	val	(IN) Scalar value to assign to (a subset of) $C.$
5245	S	(IN) Scalar value to assign to (a subset of) $C.$
5246	row_indices	(IN) Pointer to the ordered set (array) of indices corresponding to the rows of C
5247		that are assigned. If all rows of $C$ are to be assigned in order from 0 to nrows $-1$ ,
5248		then GrB_ALL can be specified. Regardless of execution mode and return value,
5249		this array may be manipulated by the caller after this operation returns without
5250		affecting any deferred computations for this operation. Unlike other variants, if
5251		there are duplicated values in this array the result is still defined.
5252	nrows	(IN) The number of values in row_indices array. Must be in the range: $[0, \mathbf{nrows}(C)]$ .
5253		If nrows is zero, the operation becomes a NO-OP.
		,
5254	col_indices	(IN) Pointer to the ordered set (array) of indices corresponding to the columns of C
5255		that are assigned. If all columns of C are to be assigned in order from 0 to ncols – 1,
5256		then GrB_ALL should be specified. Regardless of execution mode and return value,
5257		this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. Unlike other variants, if
5258		there are duplicated values in this array the result is still defined.
5259		there are dupheated values in this array the result is still defined.
5260	ncols	(IN) The number of values in $col\_indices$ array. Must be in the range: $[0, \mathbf{ncols}(C)]$ .
5261		If ncols is zero, the operation becomes a NO-OP.
5262	desc	(IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL

 should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
5265	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.

# Return Values

5267 5268 5269 5270 5271	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
5272	GrB_PANIC	Unknown internal error.
5273 5274 5275 5276	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5277	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
5278 5279	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
5280 5281 5282	GrB_INDEX_OUT_OF_BOUNDS	A value in $row\_indices$ is greater than or equal to $nrows(C)$ , or a value in $col\_indices$ is greater than or equal to $ncols(C)$ . In non-blocking mode, this can be reported as an execution error.
5283 5284	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows is not less than $\mathbf{nrows}(C)$ , or ncols is not less than $\mathbf{ncols}(C)$ .
5285 5286 5287 5288	GrB_DOMAIN_MISMATCH	The domains of the matrix and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5289 5290	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

# Description

This variant of  $GrB_assign$  computes the result of assigning a constant scalar value – either val or s – to locations in a destination GraphBLAS matrix: Either  $C(row_indices, col_indices) = val$ 

or  $C(row\_indices, col\_indices) = s$  is performed. If an optional binary accumulation operator  $(\odot)$  is provided, then either  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices)$  or  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices)$  or  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices)$  or  $C(row\_indices, col\_indices)$  or  $C(row\_indices$ 

5299 Correspondingly, if a GrB Scalar s is provided:

5301 Logically, this operation occurs in three steps:

- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 5304 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 5306 Up to two argument matrices are used in the GrB\_assign operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 5308 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- The argument scalar, matrices, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(C)$  must be compatible with either  $\mathbf{D}(val)$  or  $\mathbf{D}(val)$ , depending on the signature of the method.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator.
- 4. If accum is not GrB\_NULL, then either  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, index arrays, and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 2. Two-dimensional mask  $\widetilde{\mathbf{M}}$  is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
      - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
      - (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Scalar  $\tilde{s} \leftarrow s$  (GrB\_Scalar version only).
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$ .
- 5339 (b) Otherwise,  $\widetilde{I}[i] = \text{row\_indices}[i], \forall i : 0 \leq i < \text{nrows}.$
- 5. The internal column index array,  $\widetilde{J}$ , is computed from argument col\_indices as follows:
- (a) If col\_indices = GrB\_ALL, then  $\widetilde{m{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- 5342 (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrix and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 5346 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $0 \le \operatorname{nrows}(\widetilde{\mathbf{C}})$
- 4.  $0 \le \operatorname{ncols} \le \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the copies of the scalar, either val or  $\widetilde{s}$ , in their destination locations relative to  $\widetilde{\mathbf{C}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\tilde{\mathbf{T}}$ , is created as follows. If a non-opaque scalar val is provided:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{val}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathsf{val}) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Correspondingly, if a non-empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 1$ ):

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathbf{val}(\widetilde{s})) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Finally, if an empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 0$ ):

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \emptyset \rangle.$$

If either  $\tilde{I}$  or  $\tilde{J}$  is empty, this operation results in an empty matrix,  $\tilde{\mathbf{T}}$ . Otherwise, if any value in the  $\tilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\tilde{\mathbf{C}}))$  or any value in the  $\tilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\tilde{\mathbf{C}}))$ , the execution of  $\mathsf{GrB\_assign}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.

The intermediate matrix  $\hat{\mathbf{Z}}$  is created as follows:

• If  $accum = GrB\_NULL$ , then  $\widetilde{\mathbf{Z}}$  is defined as

$$\begin{split} \widetilde{\mathbf{Z}} &= \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i, j, Z_{ij}) \forall (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle. \end{split}$$

The above expression defines the structure of matrix  $\widetilde{\mathbf{Z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{C}}$  ( $\mathbf{ind}(\widetilde{\mathbf{C}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{C}}$  that are in the set of indices being assigned ( $\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{T}}$  ( $\mathbf{ind}(\widetilde{\mathbf{T}})$ ).

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference. We note that, in this particular case of assigning a constant to a matrix, the sets  $\{(\tilde{I}[k], \tilde{J}[l]), \forall k, l\}$  and  $\mathbf{ind}(\tilde{\mathbf{T}})$  are identical.

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\dot{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.8 apply: Apply a function to the elements of an object

Computes the transformation of the values of the elements of a vector or a matrix using a unary function, or a binary function where one argument is bound to a scalar.

## $_{5412}$ **4.3.8.1** apply: Vector variant

5413 Computes the transformation of the values of the elements of a vector using a unary function.

## 5414 C Syntax

5415	<pre>GrB_Info GrB_apply(GrB_Vector</pre>	W,
5416	const GrB_Vector	mask,
5417	const GrB_BinaryOp	accum,
5418	const GrB_UnaryOp	op,
5419	const GrB_Vector	u,
5420	const GrB Descriptor	desc);

### Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) A unary operator applied to each element of input vector u.
    - u (IN) The GraphBLAS vector to which the unary function is applied.
    - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
39	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

### Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

5446 GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

5454 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or unary function, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

## 5459 Description

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This variant of GrB\_apply computes the result of applying a unary function to the elements of a GraphBLAS vector:  $\mathbf{w} = f(\mathbf{u})$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided, w = w  $\odot$   $f(\mathbf{u})$ .

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5466 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

5468 Up to three argument vectors are used in this GrB\_apply operation:

- 5469 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5470 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5476 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathbf{op})$  of the unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 5480 4.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in}(\mathsf{op})$ .

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 5488 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 5489 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathsf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 5498 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 5499 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the result from applying the unary operator to the input vector  $\tilde{\mathbf{u}}$ .
- $\bullet$   $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

5510 where f = f(op).

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a *standard vector accumulate*:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ orall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} 
angle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$L(w) = \{(i, z_i) : i \in (ind(\widetilde{z}) \cap ind(\widetilde{m}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 5538 4.3.8.2 apply: Matrix variant

5539 Computes the transformation of the values of the elements of a matrix using a unary function.

# 5540 C Syntax

```
GrB_Info GrB_apply(GrB_Matrix
                                                             С,
5541
                                    const GrB_Matrix
                                                             Mask,
5542
                                    const GrB_BinaryOp
                                                             accum,
5543
                                    const GrB_UnaryOp
5544
                                                             op,
                                    const GrB_Matrix
                                                             Α,
5545
                                    const GrB_Descriptor
                                                             desc);
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```

#### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) A unary operator applied to each element of input matrix A.
  - A (IN) The GraphBLAS matrix to which the unary function is applied.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
5565				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

#### Return Values

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Grb SUCCESS In blocking mode, the operation completed successfully. In non-5567 blocking mode, this indicates that the compatibility tests on 5568 dimensions and domains for the input arguments passed suc-5569 cessfully. Either way, output matrix C is ready to be used in the 5570 next method of the sequence. 5571 5572

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB error() to access any error messages generated by the implementation.

GrB OUT\_OF\_MEMORY Not enough memory available for the operation.

Grb Uninitialized Object One or more of the GraphBLAS objects has not been initialized 5578 by a call to new (or Matrix\_dup for matrix parameters). 5579

GrB\_DIMENSION\_MISMATCH Mask and C dimensions are incompatible,  $nrows \neq nrows(C)$ , or  $ncols \neq ncols(C)$ .

5582 Grb DOMAIN MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or unary 5583 function, or the mask's domain is not compatible with bool (in 5584 the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set). 5585

#### Description 5586

This variant of GrB\_apply computes the result of applying a unary function to the elements of a 5587 GraphBLAS matrix: C = f(A); or, if an optional binary accumulation operator  $(\odot)$  is provided, 5588  $C = C \odot f(A)$ . 5589

Logically, this operation occurs in three steps: 5590

**Setup** The internal matrices and mask used in the computation are formed and their domains 5591 and dimensions are tested for compatibility. 5592

**Compute** The indicated computations are carried out. 5593

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB\_apply operation: 5595

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$ 5596
  - 2.  $Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$  (optional)

```
3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument matrices, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 5603 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 4.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in}(\mathsf{op})$  of the unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 5616 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 5629 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

4. 
$$\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$$

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If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\tilde{\mathbf{T}}$ , is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

where f = f(op).

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a standard matrix accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{C}}$  and  $\tilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.
```

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.8.3 apply: Vector-BinaryOp variants

Computes the transformation of the values of the stored elements of a vector using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the vector are passed as the second argument. In the *bind-second* variant, the elements of the vector are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

## 5677 C Syntax

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```
// bind-first + scalar value
5678
             GrB_Info GrB_apply(GrB_Vector
5679
                                                              W,
                                    const GrB Vector
                                                              mask,
5680
                                    const GrB_BinaryOp
                                                              accum,
5681
                                    const GrB_BinaryOp
5682
                                                              op,
                                    <type>
                                                              val,
5683
                                    const GrB_Vector
5684
                                                              u,
                                    const GrB_Descriptor
                                                              desc);
5685
             // bind-first + GraphBLAS scalar
5686
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5687
                                    const GrB Vector
                                                              mask,
5688
                                    const GrB_BinaryOp
                                                              accum,
5689
                                    const GrB_BinaryOp
5690
                                                              op,
                                    const GrB_Scalar
                                                              s,
5691
                                    const GrB_Vector
5692
                                                              u,
                                    const GrB_Descriptor
5693
                                                              desc);
              // bind-second + scalar value
5694
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5695
                                    const GrB_Vector
                                                              mask,
5696
```

```
const GrB_BinaryOp
                                                               accum,
5697
                                    const GrB_BinaryOp
5698
                                                               op,
                                    const GrB_Vector
5699
                                                               u,
                                    <type>
                                                               val,
5700
                                    const GrB Descriptor
                                                               desc);
5701
             // bind-second + GraphBLAS scalar
5702
             GrB_Info GrB_apply(GrB_Vector
5703
                                                               w,
                                    const GrB Vector
                                                              mask,
5704
                                    const GrB_BinaryOp
                                                               accum,
5705
                                    const GrB_BinaryOp
5706
                                                               op,
                                    const GrB_Vector
                                                               u,
5707
                                    const GrB_Scalar
5708
                                                               s,
                                    const GrB_Descriptor
                                                               desc);
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```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) A binary operator applied to each element of input vector, u, and the scalar value, val.
  - u (IN) The GraphBLAS vector whose elements are passed to the binary operator as the right-hand (second) argument in the *bind-first* variant, or the left-hand (first) argument in the *bind-second* variant.
  - val (IN) Scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant.
    - s (IN) A GraphBLAS scalar that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant. It must not be empty.

5734	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL
5735	should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
5737	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

## 5738 Return Values

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5739 5740 5741 5742 5743	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
5744	GrB_PANIC	Unknown internal error.
5745 5746 5747 5748	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5749	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
5750 5751	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
5752	GrB_DIMENSION_MISMATCH	mask,w and/or $u$ dimensions are incompatible.
5753 5754 5755 5756	GrB_DOMAIN_MISMATCH	The domains of the various vectors and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5757 5758	GrB_EMPTY_OBJECT	The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the binary operator.

# 5759 **Description**

This variant of GrB\_apply computes the result of applying a binary operator to the elements of a GraphBLAS vector each composed with a scalar constant, either val or s:

bind-first: w = f(val, u) or w = f(s, u)

bind-second: w = f(u, val) or w = f(u, s),

or if an optional binary accumulation operator (⊙) is provided:

bind-first:  $w = w \odot f(val, u)$  or  $w = w \odot f(s, u)$ 

bind-second:  $w = w \odot f(u, val)$  or  $w = w \odot f(u, s)$ .

5767 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5770 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB\_apply operation:

- 5773 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5774 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5780 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- 5785 5. If bind-first:

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- (a)  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.

6. If bind-second:

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- (a)  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\text{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
  - (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 5805 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- size(w),  $\{i, \forall i : 0 \le i < \mathbf{size}(w)\}$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
  - (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 5812 4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 5815 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 5816 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB Scalar  $\tilde{s}$  is provided (**nvals**( $\tilde{s}$ ) = 0), the method returns with code GrB EMPTY OBJECT.

If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable val with the same domain as  $\tilde{s}$  and set  $\mathbf{val} = \mathbf{val}(\tilde{s})$ .

We are now ready to carry out the apply and any additional associated operations. We describe

this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the result from applying the binary operator to the input vector  $\tilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as one of the following:

```
bind-first: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\mathsf{val}, \widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
bind-second: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i), \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
```

where f = f(op).

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The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a *standard vector accumulate*:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 
  - If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ orall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} 
angle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 5859 4.3.8.4 apply: Matrix-BinaryOp variants

Computes the transformation of the values of the stored elements of a matrix using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the matrix are passed as the second argument. In the *bind-second* variant, the elements of the matrix are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

## 5866 C Syntax

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```
// bind-first + scalar value
5867
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
5868
                                    const GrB_Matrix
                                                             Mask,
5869
                                    const GrB_BinaryOp
                                                             accum,
5870
                                    const GrB_BinaryOp
5871
                                                             op,
                                    <type>
                                                             val,
5872
                                    const GrB_Matrix
                                                             Α,
5873
                                    const GrB_Descriptor
                                                             desc);
5874
5875
             // bind-first + GraphBLAS scalar
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
5876
                                    const GrB Matrix
                                                             Mask,
5877
                                    const GrB_BinaryOp
                                                             accum,
5878
                                    const GrB_BinaryOp
5879
                                                             op,
                                    const GrB_Scalar
                                                             s,
5880
                                    const GrB Matrix
                                                             Α,
5881
                                    const GrB_Descriptor
                                                             desc);
5882
             // bind-second + scalar value
5883
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
5884
                                    const GrB_Matrix
                                                             Mask,
5885
                                    const GrB BinaryOp
5886
                                                             accum,
                                    const GrB_BinaryOp
                                                             op,
                                    const GrB_Matrix
                                                             Α,
5888
                                    <type>
                                                             val.
5889
                                    const GrB_Descriptor
                                                             desc);
5890
             // bind-second + GraphBLAS scalar
5891
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
5892
                                    const GrB_Matrix
                                                             Mask,
5893
                                    const GrB_BinaryOp
                                                             accum,
5894
                                    const GrB_BinaryOp
                                                             op,
5895
                                    const GrB_Matrix
                                                             Α,
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```

5898		<pre>const GrB_Descriptor desc);</pre>				
5899	Parameters					
5900	С	(INOUT) An existing GraphBLAS matrix. On input, the matrix provides values				
5901		that may be accumulated with the result of the apply operation. On output, the				
5902		matrix holds the results of the operation.				
5903	Mask	(IN) An optional "write" mask that controls which results from this operation are				
5904		stored into the output matrix C. The mask dimensions must match those of the				
5905		matrix C. If the GrB_STRUCTURE descriptor is <i>not</i> set for the mask, the domain				
5906		of the Mask matrix must be of type bool or any of the predefined "built-in" types				
5907		in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the				
5908		dimensions of C), GrB_NULL should be specified.				
5909	accum	(IN) An optional binary operator used for accumulating entries into existing C				
5910		entries. If assignment rather than accumulation is desired, GrB_NULL should be				
5911		specified.				
5912	ор	(IN) A binary operator applied to each element of input matrix, A, with the element				
5913		of the input matrix used as the left-hand argument, and the scalar value, val, used				
5914		as the right-hand argument.				
5915	А	(IN) The GraphBLAS matrix whose elements are passed to the binary operator as				
5916		the right-hand (second) argument in the bind-first variant, or the left-hand (first)				
5917		argument in the bind-second variant.				
5918	val	(IN) Scalar value that is passed to the binary operator as the left-hand (first)				
5919		argument in the bind-first variant, or the right-hand (second) argument in the				
5920		bind-second variant.				
5921	s	(IN) GraphBLAS scalar value that is passed to the binary operator as the left-hand				
5922	J	(first) argument in the bind-first variant, or the right-hand (second) argument in				
5923		the bind-second variant. It must not be empty.				

const GrB\_Scalar

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5925 5926 s,

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL

should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
5927				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation
				(bind-second variant only).
	Α	GrB_INP1	GrB_TRAN	Use transpose of A for the operation
				(bind-first variant only).

# 5928 Return Values

5929 5930 5931 5932 5933	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
5934	GrB_PANIC	Unknown internal error.
5935 5936 5937 5938	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5939	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
5940 5941	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
5942 5943 5944	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$ , or a value in $\mathbf{col\_indices}$ is greater than or equal to $\mathbf{ncols}(A)$ . In non-blocking mode, this can be reported as an execution error.
5945 5946	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows $\neq$ $\mathbf{nrows}(C)$ , or $\mathbf{ncols} \neq \mathbf{ncols}(C)$ .
5947 5948 5949 5950 5951	GrB_DOMAIN_MISMATCH	The domains of the various matrices and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5952 5953	GrB_EMPTY_OBJECT	The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the binary operator.

# 5954 Description

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This variant of GrB\_apply computes the result of applying a binary operator to the elements of a GraphBLAS matrix each composed with a scalar constant, val or s:

bind-first: 
$$C = f(val, A) \text{ or } C = f(s, A)$$

bind-second: 
$$C = f(A, val)$$
 or  $C = f(A, s)$ ,

or if an optional binary accumulation operator (①) is provided:

bind-first: 
$$C = C \odot f(val, A) \text{ or } C = C \odot f(s, A)$$

bind-second: 
$$C = C \odot f(A, val)$$
 or  $C = C \odot f(A, s)$ .

5962 Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5965 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

5967 Up to three argument matrices are used in the GrB\_apply operation:

5968 1. 
$$C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$$

2. 
$$\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$$
 (optional)

5970 3. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument scalar, matrices, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 5975 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- 5980 5. If bind-first:
  - (a)  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

- 5982 (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.

#### 6. If bind-second:

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- (a)  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed (← denotes copy):

- 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .
  - 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
  - (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
    - (b) If Mask  $\neq$  GrB\_NULL,
      - i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
      - $$\begin{split} \text{ii. Otherwise, } \widetilde{\mathbf{M}} &= \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ &\{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle. \end{split}$$
      - (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}}$  is computed from argument A as follows:

bind-first:  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$ bind-second:  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$ 

4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$ 

- 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$
- 6019 If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch 6020 error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GD22 GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB\_Scalar  $\tilde{s}$  is provided (nvals( $\tilde{s}$ ) = 0), the method returns with code GrB\_EMPTY\_OBJECT.
- If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable
- val with the same domain as  $\tilde{s}$  and set val = val( $\tilde{s}$ ).
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the binary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- 6030 The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as one of the following:

bind-first: 
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\mathsf{val}, \widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

bind-second: 
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j), \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

where  $f = \mathbf{f}(\mathsf{op})$ .

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- The intermediate matrix  $\widetilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:
- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathsf{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij}=\widetilde{\mathbf{T}}(i,j), ext{ if } (i,j)\in (\mathbf{ind}(\widetilde{\mathbf{T}})-(\mathbf{ind}(\widetilde{\mathbf{T}})\cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \odot(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a standard matrix mask and replace. This is carried out under control of the 6047 mask which acts as a "write mask". 6048

• If desc[GrB OUTP].GrB REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB NONBLOCKING mode, the method 6057 exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but 6058 may not be fully computed. However, it can be used in the next GraphBLAS method call in a 6059 sequence. 6060

#### apply: Vector index unary operator variant 4.3.8.5

Computes the transformation of the values of the stored elements of a vector using an index unary 6062 operator that is a function of the stored value, its location indices, and an user provided scalar 6063 value. The scalar can be passed either as a non-opaque variable or as a GrB Scalar object. 6064

#### C Syntax 6065

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```
GrB_Info GrB_apply(GrB_Vector
                                                                W,
6066
                                    const GrB_Vector
6067
                                                                mask,
                                    const GrB_BinaryOp
                                                                accum,
6068
                                    const GrB_IndexUnaryOp
6069
                                                                op,
                                    const GrB_Vector
                                                                u,
6070
                                    <type>
                                                                val,
6071
                                    const GrB_Descriptor
                                                                desc);
6072
             GrB_Info GrB_apply(GrB_Vector
6073
                                                                W,
                                    const GrB_Vector
                                                                mask,
6074
                                    const GrB_BinaryOp
6075
                                                                accum,
                                    const GrB_IndexUnaryOp
                                                                op,
6076
                                    const GrB_Vector
                                                                u,
6077
                                    const GrB_Scalar
6078
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
6079
```

#### 6080 Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values 6081 that may be accumulated with the result of the apply operation. On output, this 6082 vector holds the results of the operation. 6083 mask (IN) An optional "write" mask that controls which results from this operation are 6084 stored into the output vector w. The mask dimensions must match those of the 6085 vector w. If the GrB STRUCTURE descriptor is not set for the mask, the domain 6086 of the mask vector must be of type bool or any of the predefined "built-in" types 6087 in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the 6088 dimensions of w), GrB\_NULL should be specified. 6089 accum (IN) An optional binary operator used for accumulating entries into existing w 6090 entries. If assignment rather than accumulation is desired, GrB\_NULL should be 6091 specified. 6092 op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied 6093 to each element stored in the input vector, u. It is a function of the stored element's 6094 value, its location index, and a user supplied scalar value (either s or val). 6095 u (IN) The GraphBLAS vector whose elements are passed to the index unary oper-6096 ator. 6097 val (IN) An additional scalar value that is passed to the index unary operator. 6098 s (IN) An additional GraphBLAS scalar that is passed to the index unary operator. 6099 It must not be empty. 6100 desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL 6101 should be specified. Non-default field/value pairs are listed as follows: 6102 6103 Param Value Field Description GrB OUTP **GrB\_REPLACE** Output vector w is cleared (all elements removed) before the result is stored in it. mask GrB\_MASK GrB\_STRUCTURE The write mask is constructed from the 6104 structure (pattern of stored values) of the input mask vector. The stored values are

### Return Values

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mask

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

not examined.

Use the complement of mask.

GrB\_MASK GrB\_COMP

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

6119 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB\_EMPTY\_OBJECT The GrB\_Scalar s used in the call is empty ( $\mathbf{nvals}(s) = 0$ ) and therefore a value cannot be passed to the index unary operator.

## 6126 Description

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This variant of GrB\_apply computes the result of applying an index unary operator to the elements of a GraphBLAS vector each composed with the element's index and a scalar constant, val or s:

w = 
$$f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathbf{val})$$
 or  $\mathbf{w} = f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathbf{s}),$ 

or if an optional binary accumulation operator  $(\odot)$  is provided:

w = w 
$$\odot f_i(u, \mathbf{ind}(u), 0, \mathsf{val})$$
 or w = w  $\odot f_i(u, \mathbf{ind}(u), 0, \mathsf{s})$ .

6132 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6135 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

6137 Up to three argument vectors are used in this GrB\_apply operation:

- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 6139 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 5. If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- 6. If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool}) \mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 6169 4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6173 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB\_Scalar  $\tilde{s}$  is provided (nvals( $\tilde{s}$ ) = 0), the method returns with code GrB\_EMPTY\_OBJECT.
- If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (**nvals**( $\tilde{s}$ ) = 1), we then create an internal variable val
- with the same domain as  $\tilde{s}$  and set  $val = val(\tilde{s})$ .
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the result from applying the index unary operator to the input vector  $\widetilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{ (i, f_i(\widetilde{\mathbf{u}}(i), [i], 0, \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

where  $f_i = f(op)$ .

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- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
 $z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$ 
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$ 

where  $\odot = \bigcirc(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.
```

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6216 4.3.8.6 apply: Matrix index unary operator variant

Computes the transformation of the values of the stored elements of a matrix using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

# 6220 C Syntax

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```
GrB_Info GrB_apply(GrB_Matrix
                                                                С,
6221
                                    const GrB_Matrix
                                                                Mask,
6222
                                    const GrB_BinaryOp
                                                                accum,
6223
                                    const GrB_IndexUnaryOp
                                                                op,
6224
                                    const GrB_Matrix
                                                                Α,
6225
                                                                val,
                                    <type>
6226
                                    const GrB_Descriptor
                                                                desc);
6227
             GrB_Info GrB_apply(GrB_Matrix
                                                                C,
6228
                                    const GrB_Matrix
                                                                Mask,
6229
                                    const GrB_BinaryOp
                                                                accum,
6230
                                    const GrB_IndexUnaryOp
                                                                op,
6231
                                    const GrB_Matrix
                                                                Α,
6232
                                    const GrB_Scalar
6233
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
6234
```

#### Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
    - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
    - val (IN) An additional scalar value that is passed to the index unary operator.
      - s (IN) An additional GraphBLAS scalar that is passed to the index unary operator.
    - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
			removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input Mask matrix. The stored values are
			not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

#### 6259 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output matrix C is ready to be used in the next method of the sequence.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

6271 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

6273 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB\_EMPTY\_OBJECT The GrB\_Scalar s used in the call is empty  $(\mathbf{nvals}(s) = 0)$  and therefore a value cannot be passed to the index unary operator.

#### 6280 Description

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This variant of GrB\_apply computes the result of applying a index unary operator to the elements of a GraphBLAS matrix each composed with the elements row and column indices, and a scalar constant, val or s:

$$C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{sol}(\mathbf{ind}(A)), \mathsf{sol}(A))$$

or if an optional binary accumulation operator (①) is provided:

$$C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{s}).$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

6289 Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6292 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

6294 Up to three argument matrices are used in the GrB\_apply operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 6296 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)

```
3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 5. If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- 6. If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- 6322 (b) If Mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}}$  is computed from argument A as follows:

$$\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}$$

6330 4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$

6337 If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch 6338 error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB\_Scalar  $\tilde{s}$  is provided (nvals( $\tilde{s}$ ) = 0), the method returns with code GrB\_EMPTY\_OBJECT.

If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable

val with the same domain as  $\tilde{s}$  and set val = val( $\tilde{s}$ ).

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the index unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
  - $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, T, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

where  $f_i = \mathbf{f}(\mathsf{op})$ .

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The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a  $standard\ matrix\ accumulate$ :

- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathrm{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) orall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} 
angle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If desc[GrB\_OUTP].GrB\_REPLACE is not set, the elements of  $\hat{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6379 **4.3.9** select:

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Apply a select operator to the stored elements of an object to determine whether or not to keep them.

#### 6382 4.3.9.1 select: Vector variant

Apply a select operator (an index unary operator) to the elements of a vector.

## 6384 C Syntax

```
// scalar value variant
6385
             GrB Info GrB select(GrB Vector
                                                                 W,
6386
                                     const GrB Vector
                                                                 mask,
6387
                                     const GrB_BinaryOp
                                                                 accum.
6388
                                     const GrB_IndexUnaryOp
                                                                 op,
6389
                                     const GrB_Vector
                                                                 u,
6390
                                     <type>
                                                                 val,
6391
                                     const GrB_Descriptor
                                                                 desc);
6392
6393
              // GraphBLAS scalar variant
6394
             GrB_Info GrB_select(GrB_Vector
6395
                                                                 W,
                                     const GrB_Vector
                                                                 mask.
6396
```

_	
const GrB_IndexUnaryOp op,	
const GrB_Vector u,	
const GrB_Scalar s,	
const GrB_Descriptor des	sc);

### 6403 Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the select operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input vector, u. It is a function of the stored element's value, its location index, and a user supplied scalar value (either s or val).
  - u (IN) The GraphBLAS vector whose elements are passed to the index unary operator.
  - val (IN) An additional scalar value that is passed to the index unary operator.
    - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
6427				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

## 6428 Return Values

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GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.

6442 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB\_EMPTY\_OBJECT The GrB\_Scalar s used in the call is empty (nvals(s) = 0) and therefore a value cannot be passed to the index unary operator.

#### 6449 Description

This variant of GrB\_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS vector. The operator takes, as input, the value of each stored element, along with the element's index and a scalar constant – either val or s. The corresponding element of the input vector is selected (kept) if the function evaluates to true when cast to bool.

This acts like a functional mask on the input vector as follows:

$$\mathbf{w} = \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle,$$
 
$$\mathbf{w} = \mathbf{w} \odot \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle.$$

6457 Correspondingly, if a GrB\_Scalar, s, is provided:

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$$\mathsf{w} = \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle,$$
 
$$\mathsf{w} = \mathsf{w} \odot \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle.$$

- 6460 Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6463 Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 6465 Up to three argument vectors are used in this GrB\_select operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 6467 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 6473 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}(\mathbf{u})$ .
- 3. If accum is not  $GrB_NULL$ , then D(w) must be compatible with  $D_{in_1}(accum)$  and  $D_{out}(accum)$  of the accumulation operator and D(u) must be compatible with  $D_{in_2}(accum)$  of the accumulation operator.
- 4.  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
- 5.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 6.  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_select ends and the domain mismatch error listed above is returned.
- From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:

```
(a) If mask = GrB_NULL, then \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle.
```

- (b) If mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 4. Scalar  $\widetilde{s} \leftarrow s$  (GrB Scalar version only).
- The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6501 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_select ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with G505 GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty  $GrB\_Scalar\ \widetilde{s}$  is provided (i.e.,  $nvals(\widetilde{s}) = 0$ ), the method returns with code  $GrB\_EMPTY\_OBJECT$ .
- If a non-empty  $GrB\_Scalar$ ,  $\widetilde{s}$ , is provided (i.e.,  $nvals(\widetilde{s}) = 1$ ), we then create an internal variable val with the same domain as  $\widetilde{s}$  and set  $val = val(\widetilde{s})$ .
- We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the result from applying the index unary operator to the input vector  $\widetilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathsf{u}}), \{(i, \widetilde{\mathsf{u}}(i), : i \in \mathbf{ind}(\widetilde{\mathsf{u}}) \land (\mathsf{bool}) f_i(\widetilde{\mathsf{u}}(i), i, 0, \mathsf{val}) = \mathsf{true} \} \rangle,$$

6516 where  $f_i = \mathbf{f}(\mathsf{op})$ .

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}$ , then  $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$ .
- If accum is a binary operator, then  $\widetilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
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$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
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$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6544 4.3.9.2 select: Matrix variant

Apply a select operator (an index unary operator) to the elements of a matrix.

#### 6546 C Syntax

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```
// scalar value variant
6547
             GrB_Info GrB_select(GrB_Matrix
                                                                 С,
6548
                                     const GrB_Matrix
                                                                 Mask,
6549
                                     const GrB_BinaryOp
                                                                 accum,
6550
                                     const GrB_IndexUnaryOp
6551
                                                                 op,
                                     const GrB_Matrix
                                                                 Α,
6552
                                     <type>
                                                                 val,
6553
                                     const GrB_Descriptor
                                                                 desc);
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```

```
// GraphBLAS scalar variant
6556
             GrB_Info GrB_select(GrB_Matrix
                                                                С,
6557
                                     const GrB_Matrix
                                                                Mask,
6558
                                     const GrB_BinaryOp
                                                                accum,
6559
                                     const GrB IndexUnaryOp
                                                                op,
6560
                                     const GrB Matrix
                                                                Α,
6561
                                     const GrB Scalar
                                                                s,
6562
                                     const GrB_Descriptor
                                                                desc);
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```

#### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the select operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
  - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
  - val (IN) An additional scalar value that is passed to the index unary operator.
    - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
6588				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

## 89 Return Values

6590 6591 6592 6593 6594	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output mattrix C is ready to be used in the next method of the sequence.
6595	GrB_PANIC	Unknown internal error.
6596 6597 6598 6599	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
6600	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
6601 6602	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.
6603	GrB_DIMENSION_MISMATCH	Mask,C and/or $A$ dimensions are incompatible.
6604 6605 6606 6607	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
6608 6609	GrB_EMPTY_OBJECT	The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s) = 0)$ and therefore a value cannot be passed to the index unary operator.

# 6610 Description

This variant of GrB\_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS matrix. The operator takes, as input, the value of each stored element, along with the element's row and column indices and a scalar constant – from either val or s. The corresponding element of the input matrix is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input matrix as follows when specifying a transparent scalar value:

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$$\mathsf{C} = \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle, \text{ or}$$

$$\mathsf{C} = \mathsf{C} \odot \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle.$$

6619 Correspondingly, if a GrB\_Scalar, s, is provided:

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$$C = A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle, \text{ or}$$

$$C = C \odot A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle.$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

6624 Logically, this operation occurs in three steps:

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**Setup** The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6627 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

6629 Up to three argument matrices are used in the GrB\_select operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$ 
  - 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$

The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6637 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not  $GrB_NULL$ , then D(C) must be compatible with  $D_{in_1}(accum)$  and  $D_{out}(accum)$  of the accumulation operator and D(A) must be compatible with  $D_{in_2}(accum)$  of the accumulation operator.
- 4.  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
- 5.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 66.  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_select ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed (← denotes copy):

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- $_{6654}$  2. Two-dimensional mask,  $\dot{\mathbf{M}}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB\_NULL,
  - i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}}$  is computed from argument A as follows:  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN}$ ? A
- 4. Scalar  $\tilde{s} \leftarrow s$  (GrB\_Scalar version only).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 6668 2.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}})$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB\_select ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

If an empty  $GrB\_Scalar \tilde{s}$  is provided (i.e.,  $nvals(\tilde{s}) = 0$ ), the method returns with code  $GrB\_EMPTY\_OBJECT$ .

If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $nvals(\tilde{s}) = 1$ ), we then create an internal variable

val with the same domain as  $\tilde{s}$  and set  $val = val(\tilde{s})$ .

We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the index unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
  - $ilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\tilde{\mathbf{T}}$ , is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \\ \{(i, j, \widetilde{\mathbf{A}}(i, j) : i, j \in \mathbf{ind}(\widetilde{\mathbf{A}}) \land (\mathsf{bool}) f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val}) = \mathsf{true} \} \rangle,$$

where  $f_i = f(op)$ .

The intermediate matrix  $\hat{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

# 6713 4.3.10 reduce: Perform a reduction across the elements of an object

6714 Computes the reduction of the values of the elements of a vector or matrix.

#### 6715 4.3.10.1 reduce: Standard matrix to vector variant

This performs a reduction across rows of a matrix to produce a vector. If reduction down columns is desired, the input matrix should be transposed using the descriptor.

# 6718 C Syntax

```
GrB_Info GrB_reduce(GrB_Vector
                                                                W,
6719
                                     const GrB_Vector
6720
                                                                mask,
                                     const GrB_BinaryOp
                                                                accum
6721
                                     const GrB Monoid
                                                                op,
6722
                                     const GrB_Matrix
                                                                Α,
6723
                                     const GrB Descriptor
                                                                desc);
6724
6725
             GrB_Info GrB_reduce(GrB_Vector
6726
                                                                W,
                                     const GrB_Vector
                                                                mask,
6727
                                     const GrB_BinaryOp
                                                                accum,
6728
                                     const GrB BinaryOp
6729
                                                                op,
                                     const GrB_Matrix
6730
                                                                Α,
                                     const GrB Descriptor
                                                                desc);
6731
```

# Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the reduction operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The monoid or binary operator used in the element-wise reduction operation. Depending on which type is passed, the following defines the binary operator with one domain,  $F_b = \langle D, D, D, \oplus \rangle$ , that is used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ , the identity element of the monoid is ignored.

If op is a GrB\_BinaryOp, then all its domains must be the same. Furthermore, in both cases  $\bigcirc(op)$  must be commutative and associative. Otherwise, the outcome of the operation is undefined.

A (IN) The GraphBLAS matrix on which reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

## Return Values

6760 6761 6762 6763 6764	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence.
6765	GrB_PANIC	Unknown internal error.
6766 6767 6768 6769	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
6770	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
6771 6772	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
6773	GrB_DIMENSION_MISMATCH	mask,w and/or $u$ dimensions are incompatible.
6774 6775 6776	GrB_DOMAIN_MISMATCH	Either the domains of the various vectors and matrices are incompatible with the corresponding domains of the accumulation operator or reduce function, or the domains of the GraphBLAS binary

operator op are not all the same, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

# 6780 Description

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This variant of GrB\_reduce computes the result of performing a reduction across each of the rows of an input matrix:  $w(i) = \bigoplus A(i,:) \forall i$ ; or, if an optional binary accumulation operator is provided,  $w(i) = w(i) \odot (\bigoplus A(i,:)) \forall i$ , where  $\bigoplus = \bigodot (F_b)$  and  $\odot = \bigodot (accum)$ .

6784 Logically, this operation occurs in three steps:

Setup The internal vector, matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6787 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

6789 Up to two vector and one matrix argument are used in this GrB\_reduce operation:

```
6790 1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

optional) 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)

3. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument vector, matrix, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(w)$  must be compatible with the domain of the reduction binary operator,  $\mathbf{D}(F_b)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(F_b)$ , must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 4.  $\mathbf{D}(A)$  must be compatible with the domain of the binary reduction operator,  $\mathbf{D}(F_b)$ .

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_reduce ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathsf{w}) \} \rangle$ .
- (b) If  $mask \neq GrB\_NULL$ ,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6820 2.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_reduce ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We carry out the reduce and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the result from reducing along the rows of input matrix  $\tilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, t_i) : \mathbf{ind}(A(i, :)) \neq \emptyset \} \rangle.$$

The value of each of its elements is computed by

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$$t_i = \bigoplus_{j \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:))} \widetilde{\mathbf{A}}(i,j),$$

where  $\bigoplus = \bigcirc(F_b)$ .

- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If accum = GrB NULL, then  $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$ .

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_{i} = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
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$$z_{i} = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
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$$z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 6861 4.3.10.2 reduce: Vector-scalar variant

Reduce all stored values into a single scalar.

# 6863 C Syntax

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```
// scalar value + monoid (only)

GrB_Info GrB_reduce(<type> *val,

const GrB_BinaryOp accum,

const GrB_Monoid op,

const GrB_Vector u,
```

```
const GrB_Descriptor
                                                              desc);
6869
6870
              // GraphBLAS Scalar + monoid
6871
             GrB_Info GrB_reduce(GrB_Scalar
6872
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
6873
                                     const GrB_Monoid
                                                              op,
6874
                                     const GrB_Vector
                                                              u,
6875
                                     const GrB_Descriptor
                                                              desc);
6876
6877
              // GraphBLAS Scalar + binary operator
6878
             GrB_Info GrB_reduce(GrB_Scalar
6879
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
6880
                                     const GrB_BinaryOp
                                                              op,
6881
                                     const GrB_Vector
                                                              u,
6882
                                     const GrB_Descriptor
                                                              desc);
6883
```

#### 6884 Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.
  - accum (IN) An optional binary operator used for accumulating entries into an existing scalar (s or val) value. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) The monoid  $(M = \langle D, \oplus, 0 \rangle)$  or binary operator  $(F_b = \langle D, D, D, \oplus \rangle)$  used in the reduction operation. The  $\oplus$  operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
      - u (IN) The GraphBLAS vector on which reduction will be performed.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

# Param Field Value Description

Note: This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

#### Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

6912 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

6914 GrB\_NULL\_POINTER val pointer is NULL.

GrB\_DOMAIN\_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

### 6918 Description

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This variant of GrB\_reduce computes the result of performing a reduction across all of the stored elements of an input vector storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} \; = \left\{ \begin{aligned} &\bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i), \quad \text{or} \\ &\mathsf{val} \; \odot \; \left[ \bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i) \right], \; \text{if the the optional accumulator is specified.} \end{aligned} \right.$$

where  $\bigoplus = \bigcirc(\mathsf{op})$  and  $\odot = \bigcirc(\mathsf{accum})$ .

6920 Logically, this operation occurs in three steps:

Setup The internal vector used in the computation is formed and its domain is tested for compatibility.

6923 Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One vector argument is used in this GrB\_reduce operation:

6926 1. 
$$u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$$

The output scalar, argument vector, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If accum is GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

- 2. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator, and  $\mathbf{D}(\mathsf{op})$  from M (or  $\mathbf{D}_{out}(\mathsf{op})$  from  $F_b$ ) must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 3.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

Two domains are compatible with each other if values from one domain can be cast to values in 6935 the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If 6937 any compatibility rule above is violated, execution of GrB reduce ends and the domain mismatch 6938 error listed above is returned. 6939

The number of values stored in the input, u, is checked. If there are no stored values in u, then one 6940 of the following occurs depending on the output variant: 6941

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \text{GrB\_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$
 or 
$$\text{val} = \begin{cases} \mathbf{0}(\text{op}), & \text{(cleared) if accum} = \text{GrB\_NULL}, \\ \\ \text{val} \odot \mathbf{0}(\text{op}), & \text{otherwise}, \end{cases}$$

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where  $\mathbf{0}(\mathsf{op})$  is the identity of the monoid. The operation returns immediately with  $\mathsf{GrB}$  SUCCESS. 6945

For all other cases, the internal vector and scalar used in the computation is formed ( $\leftarrow$  denotes 6946 copy): 6947

1. Vector  $\tilde{\mathbf{u}} \leftarrow \mathbf{u}$ . 6948

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2. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case). 6949

We are now ready to carry out the reduction and any additional associated operations. An inter-6950 mediate scalar result t is computed as follows: 6951

$$t = \bigoplus_{i \in \mathbf{ind}(\widetilde{\mathbf{u}})} \widetilde{\mathbf{u}}(i),$$

where  $\oplus = \bigcirc(\mathsf{op})$ . 6953

The final reduction value is computed as follows: 6954

or val 
$$\leftarrow \begin{cases} t, & \text{when accum} = \mathsf{GrB\_NULL}, \text{ or} \\ \mathsf{val} \ \odot \ t, & \text{otherwise}; \end{cases}$$

In both GrB\_BLOCKING and GrB\_NONBLOCKING modes, the method exits with return value GrB\_SUCCESS and the new contents of the output scalar is as defined above.

#### 6960 4.3.10.3 reduce: Matrix-scalar variant

6961 Reduce all stored values into a single scalar.

# 6962 C Syntax

```
// scalar value + monoid (only)
6963
             GrB_Info GrB_reduce(<type>
                                                             *val,
6964
                                    const GrB_BinaryOp
                                                              accum,
6965
                                    const GrB_Monoid
6966
                                                              op,
                                     const GrB_Matrix
                                                              Α,
6967
                                     const GrB_Descriptor
                                                              desc);
6968
6969
             // GraphBLAS Scalar + monoid
6970
             GrB_Info GrB_reduce(GrB_Scalar
6971
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
6972
6973
                                    const GrB_Monoid
                                                              op,
                                     const GrB_Matrix
                                                              Α,
6974
                                     const GrB Descriptor
                                                              desc);
6975
6976
             // GraphBLAS Scalar + binary operator
6977
             GrB_Info GrB_reduce(GrB_Scalar
6978
                                                              s,
                                    const GrB_BinaryOp
                                                              accum,
6979
                                    const GrB_BinaryOp
6980
                                                              op,
                                     const GrB_Matrix
                                                              Α,
6981
                                     const GrB_Descriptor
                                                              desc);
6982
```

## 6983 Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.
  - accum (IN) An optional binary operator used for accumulating entries into existing (s or val) value. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) The monoid  $(M = \langle D, \oplus, 0 \rangle)$  or binary operator  $(F_b = \langle D, D, D, \oplus \rangle)$  used in the reduction operation. The  $\oplus$  operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
    - A (IN) The GraphBLAS matrix on which the reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

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Param Field Value Description

*Note:* This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

#### 7001 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

7011 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB\_NULL\_POINTER val pointer is NULL.

GrB\_DOMAIN\_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

#### 7017 Description

This variant of GrB\_reduce computes the result of performing a reduction across all of the stored elements of an input matrix storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} \; = \left\{ \begin{aligned} &\bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j), \quad \text{or} \\ & \\ \mathsf{val} \; \odot \; \left[ \bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j) \right], \; \text{if the the optional accumulator is specified.} \end{aligned} \right.$$

where  $\bigoplus = \bigcirc(\mathsf{op})$  and  $\odot = \bigcirc(\mathsf{accum})$ .

7019 Logically, this operation occurs in three steps:

**Setup** The internal matrix used in the computation is formed and its domain is tested for compatibility.

7022 Compute The indicated computations are carried out.

Output The result is written into the output scalar.

7024 One matrix argument is used in this GrB\_reduce operation:

1. 
$$A = \langle \mathbf{D}(A), \mathbf{size}(A), \mathbf{L}(A) = \{(i, j, A_{i,j})\} \rangle$$

The output scalar, argument matrix, reduction operator and accumulation operator (if provided)
are tested for domain compatibility as follows:

- 1. If accum is GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).
  - 2. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator, and  $\mathbf{D}(\mathsf{op})$  from M (or  $\mathbf{D}_{out}(\mathsf{op})$  from  $F_b$ ) must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 3.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, A, is checked. If there are no stored values in A, then one of the following occurs depending on the output variant:

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \mathsf{GrB\_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$

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where  $\mathbf{0}(\mathsf{op})$  is the identity of the monoid. The operation returns immediately with  $\mathsf{GrB\_SUCCESS}$ .

For all other cases, the internal matrix and scalar used in the computation is formed ( $\leftarrow$  denotes copy):

- 1. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{A}$ .
- 2. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

We are now ready to carry out the reduce and any additional associated operations. An intermediate scalar result t is computed as follows:

$$t = \bigoplus_{(i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})} \widetilde{\mathbf{A}}(i,j),$$

where  $\oplus = \bigcirc(\mathsf{op})$ .

7053 The final reduction value is computed as follows:

$$\mathbf{L}(\mathsf{s}) \leftarrow \begin{cases} \{t\}, & \text{when accum} = \mathsf{GrB\_NULL} \text{ or } \tilde{s} \text{ is empty, or} \\ \{\mathbf{val}(\tilde{s}) \odot t\}, & \text{otherwise;} \end{cases}$$

In both GrB\_BLOCKING and GrB\_NONBLOCKING modes, the method exits with return value GrB\_SUCCESS and the new contents of the output scalar is as defined above.

# 7059 4.3.11 transpose: Transpose rows and columns of a matrix

7060 This version computes a new matrix that is the transpose of the source matrix.

## 7061 C Syntax

```
GrB_Info GrB_transpose(GrB_Matrix C,
const GrB_Matrix Mask,
const GrB_BinaryOp accum,
const GrB_Matrix A,
const GrB_Descriptor desc);
```

#### Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the transpose operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.

7077	accum (IN) An optional binary operator used for accumulating entries into existing C
7078	entries. If assignment rather than accumulation is desired, GrB_NULL should be
7079	specified.

A (IN) The GraphBLAS matrix on which transposition will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

### 85 Return Values

7086 7087 7088 7089 7090	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
7091	GrB_PANIC	Unknown internal error.
7092 7093 7094 7095		This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
7096	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
7097 7098	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
7099	GrB_DIMENSION_MISMATCH	mask,Cand/orAdimensionsareincompatible.
7100 7101 7102 7103		The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCT is not set).

#### 7104 Description

- GrB\_transpose computes the result of performing a transpose of the input matrix:  $C = A^T$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot A^T$ . We note that the input matrix A can itself be optionally transposed before the operation, which would cause either an assignment from A to C or an accumulation of A into C.
- 7109 Logically, this operation occurs in three steps:
- Setup The internal matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 7112 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 7114 Up to three matrix arguments are used in this GrB\_transpose operation:
- 7115 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 7116 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 7122 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$  of the input matrix.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  of the input matrix must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_transpose ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed  $\leftarrow$  denotes copy):
- 7133 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:

```
7135 (a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
```

(b) If Mask  $\neq$  GrB\_NULL,

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- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 7147 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB\_transpose ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix transposition and any additional associated operations.

7155 We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the transpose of  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

7158 The intermediate matrix

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(j, i, A_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \}$$

7160 is created.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathrm{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

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$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\mathbf{C}$  and  $\mathbf{T}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
where  $\odot = \mathfrak{O}(\mathbf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, 7173 using what is called a standard matrix mask and replace. This is carried out under control of the 7174

• If desc[GrB OUTP].GrB REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In Grb BLOCKING mode, the method exits with return value Grb SUCCESS and the new content 7183 of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method 7184 exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but 7185 may not be fully computed. However, it can be used in the next GraphBLAS method call in a 7186 sequence. 7187

#### kronecker: Kronecker product of two matrices 4.3.127188

Computes the Kronecker product of two matrices. The result is a matrix. 7189

#### C Syntax 7190

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mask which acts as a "write mask".

```
GrB_Info GrB_kronecker(GrB_Matrix
                                                                    С,
7191
                                         const GrB_Matrix
                                                                    Mask,
7192
                                         const GrB_BinaryOp
                                                                    accum,
7193
                                         const GrB_Semiring
7194
                                                                    op,
                                         const GrB_Matrix
                                                                    Α,
7195
                                         const GrB Matrix
                                                                    В,
7196
                                         const GrB_Descriptor
                                                                    desc);
7197
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```

```
С,
              GrB_Info GrB_kronecker(GrB_Matrix
7199
                                         const GrB_Matrix
                                                                   Mask,
7200
                                         const GrB_BinaryOp
7201
                                                                    accum,
                                         const GrB_Monoid
                                                                    op,
7202
                                         const GrB Matrix
                                                                    Α,
7203
                                         const GrB Matrix
                                                                    В,
7204
                                         const GrB Descriptor
                                                                    desc);
7205
7206
              GrB_Info GrB_kronecker(GrB_Matrix
                                                                    C,
7207
                                         const GrB_Matrix
                                                                   Mask,
7208
                                         const GrB_BinaryOp
7209
                                                                    accum,
                                         const GrB_BinaryOp
                                                                    op,
7210
                                         const GrB_Matrix
                                                                    Α,
7211
                                         const GrB_Matrix
                                                                    Β,
7212
                                         const GrB_Descriptor
                                                                    desc);
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```

#### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the Kronecker product. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

```
BinaryOp: F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle.
```

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes(\mathsf{op}) \rangle$ ; the additive monoid is ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the product.

7237	B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the
7238	product.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
			removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input Mask matrix. The stored values are
			not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
В	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

### Return Values

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7244 7245 7246 7247 7248	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
7249	GrB_PANIC	Unknown internal error.
7250 7251 7252 7253	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
7254	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
7255 7256	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
7257	GrB_DIMENSION_MISMATCH	Mask and/or matrix dimensions are incompatible.
7258 7259 7260 7261	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

### Description

GrB\_kronecker computes the Kronecker product  $C = A \otimes B$  or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \otimes B)$  (where matrices A and B can be optionally transposed).

7265 The Kronecker product is defined as follows:

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$$\mathsf{C} = \mathsf{A} \ \otimes \ \mathsf{B} = \left[ \begin{array}{cccccc} A_{0,0} \otimes \mathsf{B} & A_{0,1} \otimes \mathsf{B} & \dots & A_{0,n_A-1} \otimes \mathsf{B} \\ A_{1,0} \otimes \mathsf{B} & A_{1,1} \otimes \mathsf{B} & \dots & A_{1,n_A-1} \otimes \mathsf{B} \\ \vdots & & \vdots & \ddots & & \vdots \\ A_{m_A-1,0} \otimes \mathsf{B} & A_{m_A-1,1} \otimes \mathsf{B} & \dots & A_{m_A-1,n_A-1} \otimes \mathsf{B} \end{array} \right]$$

where  $A: \mathbb{S}^{m_A \times n_A}$ ,  $B: \mathbb{S}^{m_B \times n_B}$ , and  $C: \mathbb{S}^{m_A m_B \times n_A n_B}$ . More explicitly, the elements of the Kronecker product are defined as

$$C(i_A m_B + i_B, j_A n_B + j_B) = A_{i_A, j_A} \otimes B_{i_B, j_B},$$

where  $\otimes$  is the multiplicative operator specified by the **op** parameter.

Logically, this operation occurs in three steps:

- 7273 **Setup** The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 7275 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.

7277 Up to four argument matrices are used in the GrB\_kronecker operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4.  $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$

The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 7286 2.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_kronecker ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed (\( \display \) denotes copy):

7299 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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7300 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:

```
7301 (a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
```

- (b) If Mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - $$\begin{split} \text{ii. Otherwise, } & \overline{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ & \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle. \end{split}$$
- 7308 (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 7313 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 7314 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}) \cdot \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{B}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) \cdot \operatorname{ncols}(\widetilde{\mathbf{B}}).$

If any compatibility rule above is violated, execution of GrB\_kronecker ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the Kronecker product and any additional associated operations.
We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the Kronecker product of matrices  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}) \times \mathbf{nrows}(\widetilde{\mathbf{B}}), \mathbf{ncols}(\widetilde{\mathbf{A}}) \times \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) \text{ where } i = i_A \cdot m_B + i_B, \ j = j_A \cdot n_B + j_B, \ \forall \ (i_A, j_A) = \mathbf{ind}(\widetilde{\mathbf{A}}), \ (i_B, j_B) = \mathbf{ind}(\widetilde{\mathbf{B}}) \rangle$  is created. The value of each of its elements is computed by

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$$T_{i_A \cdot m_B + i_B, \ j_A \cdot n_B + j_B} = \widetilde{\mathbf{A}}(i_A, j_A) \otimes \widetilde{\mathbf{B}}(i_B, j_B)),$$

where  $\otimes$  is the multiplicative operator specified by the **op** parameter.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{C}}$  and  $\tilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence. s

### Chapter 5

## Nonpolymorphic interface

Each polymorphic GraphBLAS method (those with multiple parameter signatures under the same name) has a corresponding set of long-name forms that are specific to each parameter signature.

That is show in Tables 5.1 through 5.11.

Table 5.1: Long-name, nonpolymorphic form of GraphBLAS methods.

Polymorphic signature	Nonpolymorphic signature
GrB_Monoid_new(GrB_Monoid*,,bool)	GrB_Monoid_new_BOOL(GrB_Monoid*,GrB_BinaryOp,bool)
$GrB\_Monoid\_new(GrB\_Monoid*,,int8\_t)$	GrB_Monoid_new_INT8(GrB_Monoid*,GrB_BinaryOp,int8_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,uint8\_t)$	GrB_Monoid_new_UINT8(GrB_Monoid*,GrB_BinaryOp,uint8_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,int16\_t)$	GrB_Monoid_new_INT16(GrB_Monoid*,GrB_BinaryOp,int16_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,uint16\_t)$	GrB_Monoid_new_UINT16(GrB_Monoid*,GrB_BinaryOp,uint16_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,int32\_t)$	GrB_Monoid_new_INT32(GrB_Monoid*,GrB_BinaryOp,int32_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,uint32\_t)$	GrB_Monoid_new_UINT32(GrB_Monoid*,GrB_BinaryOp,uint32_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,int64\_t)$	GrB_Monoid_new_INT64(GrB_Monoid*,GrB_BinaryOp,int64_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,uint64\_t)$	GrB_Monoid_new_UINT64(GrB_Monoid*,GrB_BinaryOp,uint64_t)
$GrB\_Monoid\_new(GrB\_Monoid*,,float)$	GrB_Monoid_new_FP32(GrB_Monoid*,GrB_BinaryOp,float)
$GrB\_Monoid\_new(GrB\_Monoid*,,double)$	GrB_Monoid_new_FP64(GrB_Monoid*,GrB_BinaryOp,double)
$GrB\_Monoid\_new(GrB\_Monoid*,,other)$	${\sf GrB\_Monoid\_new\_UDT(GrB\_Monoid*,GrB\_BinaryOp,void*)}$

Table 5.2: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_Scalar_setElement(, bool,)	GrB_Scalar_setElement_BOOL(, bool,)
$GrB\_Scalar\_setElement(, int8\_t,)$	$GrB\_Scalar\_setElement\_INT8(, int8\_t,)$
$GrB\_Scalar\_setElement(, uint8\_t,)$	$GrB\_Scalar\_setElement\_UINT8(, uint8\_t,)$
$GrB\_Scalar\_setElement(, int16\_t,)$	GrB_Scalar_setElement_INT16(, int16_t,)
$GrB\_Scalar\_setElement(, uint16\_t,)$	GrB_Scalar_setElement_UINT16(, uint16_t,)
$GrB\_Scalar\_setElement(, int32\_t,)$	GrB_Scalar_setElement_INT32(, int32_t,)
GrB_Scalar_setElement(, uint32_t,)	GrB_Scalar_setElement_UINT32(, uint32_t,)
$GrB\_Scalar\_setElement(, int64\_t,)$	GrB_Scalar_setElement_INT64(, int64_t,)
$GrB\_Scalar\_setElement(, uint64\_t,)$	GrB_Scalar_setElement_UINT64(, uint64_t,)
$GrB\_Scalar\_setElement(, float,)$	GrB_Scalar_setElement_FP32(, float,)
$GrB\_Scalar\_setElement(, double,)$	$GrB\_Scalar\_setElement\_FP64(, double,)$
$GrB\_Scalar\_setElement(,other,)$	GrB_Scalar_setElement_UDT(,const void*,)
GrB_Scalar_extractElement(bool*,)	GrB_Scalar_extractElement_BOOL(bool*,)
$GrB\_Scalar\_extractElement(int8\_t^*,)$	$GrB\_Scalar\_extractElement\_INT8(int8\_t*,)$
$GrB\_Scalar\_extractElement(uint8\_t*,)$	GrB_Scalar_extractElement_UINT8(uint8_t*,)
$GrB\_Scalar\_extractElement(int16\_t^*,)$	GrB_Scalar_extractElement_INT16(int16_t*,)
$GrB\_Scalar\_extractElement(uint16\_t^*,)$	GrB_Scalar_extractElement_UINT16(uint16_t*,)
$GrB\_Scalar\_extractElement(int32\_t^*,)$	GrB_Scalar_extractElement_INT32(int32_t*,)
$GrB\_Scalar\_extractElement(uint32\_t^*,)$	GrB_Scalar_extractElement_UINT32(uint32_t*,)
$GrB\_Scalar\_extractElement(int64\_t*,)$	GrB_Scalar_extractElement_INT64(int64_t*,)
$GrB\_Scalar\_extractElement(uint64\_t^*,)$	GrB_Scalar_extractElement_UINT64(uint64_t*,)
$GrB\_Scalar\_extractElement(float*,)$	GrB_Scalar_extractElement_FP32(float*,)
$GrB\_Scalar\_extractElement(double*,)$	GrB_Scalar_extractElement_FP64(double*,)
GrB_Scalar_extractElement(other*,)	GrB_Scalar_extractElement_UDT(void*,)

Table 5.3: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                 Nonpolymorphic signature
GrB_Vector_build(...,const bool*,...)
                                                 GrB\_Vector\_build\_BOOL(...,const bool*,...)
                                                 \label{eq:GrB_Vector_build_INT8(...,const int8\_t*,...)} $$\operatorname{GrB\_Vector\_build\_UINT8(...,const uint8\_t*,...)}$$
GrB_Vector_build(...,const int8_t*,...)
GrB_Vector_build(...,const uint8_t*,...)
                                                 GrB Vector_build_INT16(...,const int16_t*,...)
GrB_Vector_build(...,const int16_t*,...)
GrB_Vector_build(...,const uint16_t*,...)
                                                 GrB_Vector_build_UINT16(...,const uint16_t*,...)
\mathsf{GrB\_Vector\_build}(\dots, \mathsf{const\ int} 32\_t^*, \dots)
                                                 \mathsf{GrB}\_\mathsf{Vector}\_\mathsf{build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32}\_\mathsf{t*},\dots)
GrB_Vector_build(...,const uint32_t*,...)
                                                  GrB_Vector_build_UINT32(...,const_uint32_t*,...)
                                                 GrB_Vector_build_INT64(...,const int64_t*,...)
GrB_Vector_build(...,const int64_t*,...)
GrB_Vector_build(...,const uint64_t*,...)
                                                 GrB_Vector_build_UINT64(...,const uint64_t*,...)
GrB_Vector_build(...,const float*,...)
                                                 GrB_Vector_build_FP32(...,const float*,...)
GrB_Vector_build(...,const double*,...)
                                                 GrB_Vector_build_FP64(...,const double*,...)
GrB_Vector_build(...,const other*,...)
                                                  GrB_Vector_build_UDT(...,const void*,...)
                                                 GrB_Vector_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Vector\_setElement(...,GrB\_Scalar,...)
                                                 GrB Vector_setElement_BOOL(..., bool,...)
GrB Vector_setElement(...,bool,...)
GrB\_Vector\_setElement(...,int8\_t,...)
                                                 GrB_Vector_setElement_INT8(..., int8_t,...)
GrB\_Vector\_setElement(...,uint8\_t,...)
                                                 GrB\_Vector\_setElement\_UINT8(..., uint8\_t,...)
GrB_Vector_setElement(...,int16_t,...)
                                                  GrB\_Vector\_setElement\_INT16(..., int16\_t,...)
                                                 GrB_Vector_setElement_UINT16(..., uint16_t,...)
GrB\_Vector\_setElement(...,uint16\_t,...)
GrB_Vector_setElement(...,int32_t,...)
                                                 GrB_Vector_setElement_INT32(..., int32_t,...)
GrB_Vector_setElement(...,uint32_t,...)
                                                 GrB_Vector_setElement_UINT32(..., uint32_t,...)
GrB_Vector_setElement(...,int64_t,...)
                                                 GrB\_Vector\_setElement\_INT64(..., int64\_t,...)
                                                 GrB_Vector_setElement_UINT64(..., uint64_t,...)
GrB_Vector_setElement(...,uint64_t,...)
                                                 GrB_Vector_setElement_FP32(..., float,...)
GrB\_Vector\_setElement(...,float,...)
GrB\_Vector\_setElement(...,double,...)
                                                 GrB_Vector_setElement_FP64(..., double,...)
GrB_Vector_setElement(...,other,...)
                                                 GrB_Vector_setElement_UDT(...,const void*,...)
                                                 GrB\_Vector\_extractElement\_Scalar(GrB\_Scalar,...)
GrB\_Vector\_extractElement(GrB\_Scalar,...)
GrB_Vector_extractElement(bool*,...)
                                                  GrB_Vector_extractElement_BOOL(bool*,...)
                                                 GrB_Vector_extractElement_INT8(int8_t*,...)
GrB_Vector_extractElement(int8_t*,...)
GrB_Vector_extractElement(uint8_t*,...)
                                                 GrB_Vector_extractElement_UINT8(uint8_t*,...)
                                                 GrB\_Vector\_extractElement\_INT16(int16\_t^*,...)
GrB_Vector_extractElement(int16_t*,...)
GrB_Vector_extractElement(uint16_t*,...)
                                                 \label{lement_UINT16} GrB\_Vector\_extractElement\_UINT16(uint16\_t^*,\dots)
GrB_Vector_extractElement(int32_t*,...)
                                                 GrB_Vector_extractElement_INT32(int32_t*,...)
GrB_Vector_extractElement(uint32_t*,...)
                                                 GrB_Vector_extractElement_UINT32(uint32_t*,...)
GrB_Vector_extractElement(int64_t*,...)
                                                 GrB_Vector_extractElement_INT64(int64_t*,...)
GrB_Vector_extractElement(uint64_t*,...)
                                                 GrB_Vector_extractElement_UINT64(uint64_t*,...)
GrB_Vector_extractElement(float*,...)
                                                 GrB_Vector_extractElement_FP32(float*,...)
GrB_Vector_extractElement(double*,...)
                                                 GrB_Vector_extractElement_FP64(double*,...)
GrB_Vector_extractElement(other*,...)
                                                 GrB_Vector_extractElement_UDT(void*,...)
GrB\_Vector\_extractTuples(...,bool*,...)
                                                 GrB_Vector_extractTuples_BOOL(..., bool*,...)
GrB\_Vector\_extractTuples(...,int8\_t*,...)
                                                 GrB\_Vector\_extractTuples\_INT8(..., int8\_t*,...)
GrB_Vector_extractTuples(...,uint8_t*,...)
                                                 \label{linear_struct_toples} GrB\_Vector\_extractTuples\_UINT8(..., uint8\_t^*,...)
GrB\_Vector\_extractTuples(...,int16\_t*,...)
                                                 GrB_Vector_extractTuples_INT16(..., int16_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT16(..., uint16\_t*,...)
GrB\_Vector\_extractTuples(...,uint16\_t^*,...)
GrB\_Vector\_extractTuples(...,int32\_t^*,...)
                                                 GrB\_Vector\_extractTuples\_INT32(..., int32\_t*,...)
GrB_Vector_extractTuples(...,uint32_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT32(..., uint32\_t*,...)
GrB\_Vector\_extractTuples(...,int64\_t*,...)
                                                 \label{linear_continuity} GrB\_Vector\_extractTuples\_INT64(\dots,\ int64\_t^*,\dots)
GrB_Vector_extractTuples(...,uint64_t*,...)
                                                  GrB\_Vector\_extractTuples\_UINT64(..., uint64\_t*,...)
                                                 GrB\_Vector\_extractTuples\_FP32(..., float*,...)
GrB\_Vector\_extractTuples(...,float*,...)
                                                 GrB_Vector_extractTuples_FP64(..., double*,...)
GrB\_Vector\_extractTuples(...,double*,...)
GrB_Vector_extractTuples(...,other*,...)
                                                 GrB_Vector_extractTuples_UDT(..., void*,...)
```

Table 5.4: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Nonpolymorphic signature
Polymorphic signature
GrB_Matrix_build(...,const bool*,...)
                                                 GrB_Matrix_build_BOOL(...,const bool*,...)
                                                 GrB_Matrix_build_INT8(...,const int8_t*,...)
GrB_Matrix_build(...,const int8_t*,...)
GrB_Matrix_build(...,const uint8_t*,...)
                                                 GrB_Matrix_build_UINT8(...,const uint8_t*,...)
GrB_Matrix_build(...,const int16_t*,...)
                                                 GrB_Matrix_build_INT16(...,const int16_t*,...)
GrB_Matrix_build(...,const uint16_t*,...)
                                                 GrB_Matrix_build_UINT16(...,const uint16_t*,...)
                                                 \mathsf{GrB}\_\mathsf{Matrix\_build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32\_t*},\dots)
GrB_Matrix_build(...,const int32_t*,...)
GrB_Matrix_build(...,const uint32_t*,...)
                                                 GrB_Matrix_build_UINT32(...,const_uint32_t*,...)
                                                 \label{local_gradient} $\sf GrB\_Matrix\_build\_INT64(\dots,const\ int64\_t^*,\dots)$}
GrB_Matrix_build(...,const int64_t*,...)
GrB_Matrix_build(...,const uint64_t*,...)
                                                 GrB_Matrix_build_UINT64(...,const uint64_t*,...)
GrB_Matrix_build(...,const float*,...)
                                                 GrB_Matrix_build_FP32(...,const float*,...)
GrB_Matrix_build(...,const double*,...)
                                                 GrB_Matrix_build_FP64(...,const double*,...)
GrB_Matrix_build(...,const other*,...)
                                                 GrB_Matrix_build_UDT(...,const void*,...)
                                                 GrB_Matrix_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Matrix\_setElement(...,GrB\_Scalar,...)
GrB Matrix_setElement(...,bool,...)
                                                 GrB Matrix_setElement_BOOL(..., bool,...)
GrB\_Matrix\_setElement(...,int8\_t,...)
                                                 GrB_Matrix_setElement_INT8(..., int8_t,...)
GrB_Matrix_setElement(...,uint8_t,...)
                                                 \label{lement_UINT8} GrB\_Matrix\_setElement\_UINT8(..., uint8\_t,...)
GrB_Matrix_setElement(...,int16_t,...)
                                                 GrB\_Matrix\_setElement\_INT16(..., int16\_t,...)
GrB_Matrix_setElement(...,uint16_t,...)
                                                 GrB_Matrix_setElement_UINT16(..., uint16_t,...)
                                                 GrB_Matrix_setElement_INT32(..., int32_t,...)
GrB_Matrix_setElement(...,int32_t,...)
GrB_Matrix_setElement(...,uint32_t,...)
                                                 GrB_Matrix_setElement_UINT32(..., uint32_t,...)
                                                 GrB\_Matrix\_setElement\_INT64(..., int64\_t,...)
GrB_Matrix_setElement(...,int64_t,...)
GrB_Matrix_setElement(...,uint64_t,...)
                                                 GrB\_Matrix\_setElement\_UINT64(..., uint64\_t,...)
                                                 GrB_Matrix_setElement_FP32(..., float,...)
GrB\_Matrix\_setElement(...,float,...)
GrB_Matrix_setElement(...,double,...)
                                                 GrB_Matrix_setElement_FP64(..., double,...)
                                                 {\sf GrB\_Matrix\_setElement\_UDT}(\dots, {\sf const\ void*}, \dots)
GrB\_Matrix\_setElement(...,other,...)
GrB_Matrix_extractElement(GrB_Scalar,...)
                                                 GrB_Matrix_extractElement_Scalar(GrB_Scalar,...)
GrB_Matrix_extractElement(bool*,...)
                                                 GrB_Matrix_extractElement_BOOL(bool*,...)
                                                 GrB_Matrix_extractElement_INT8(int8_t*,...)
GrB_Matrix_extractElement(int8_t*,...)
GrB\_Matrix\_extractElement(uint8\_t^*,...)
                                                 GrB_Matrix_extractElement_UINT8(uint8_t*,...)
GrB_Matrix_extractElement(int16_t*,...)
                                                 GrB_Matrix_extractElement_INT16(int16_t*,...)
GrB_Matrix_extractElement(uint16_t*,...)
                                                 GrB_Matrix_extractElement_UINT16(uint16_t^*,...)
GrB_Matrix_extractElement(int32_t*,...)
                                                 GrB_Matrix_extractElement_INT32(int32_t*,...)
GrB_Matrix_extractElement(uint32_t*,...)
                                                 GrB_Matrix_extractElement_UINT32(uint32_t*,...)
GrB\_Matrix\_extractElement(int64\_t^*,...)
                                                 GrB_Matrix_extractElement_INT64(int64_t*,...)
GrB_Matrix_extractElement(uint64_t*,...)
                                                 GrB_Matrix_extractElement_UINT64(uint64_t*,...)
GrB_Matrix_extractElement(float*,...)
                                                 GrB_Matrix_extractElement_FP32(float*,...)
GrB_Matrix_extractElement(double*,...)
                                                 GrB_Matrix_extractElement_FP64(double*,...)
                                                 GrB_Matrix_extractElement_UDT(void*,...)
GrB_Matrix_extractElement(other,...)
GrB_Matrix_extractTuples(..., bool*,...)
                                                 GrB\_Matrix\_extractTuples\_BOOL(..., bool*,...)
GrB_Matrix_extractTuples(..., int8_t*,...)
                                                 GrB\_Matrix\_extractTuples\_INT8(..., int8\_t*,...)
                                                 GrB\_Matrix\_extractTuples\_UINT8(..., uint8\_t*,...)
GrB_Matrix_extractTuples(..., uint8_t*,...)
GrB_Matrix_extractTuples(..., int16_t*,...)
                                                 GrB_Matrix_extractTuples_INT16(..., int16_t*,...)
                                                 \label{linear_gradient} GrB\_Matrix\_extractTuples\_UINT16(\dots, uint16\_t^*,\dots)
GrB_Matrix_extractTuples(..., uint16_t*,...)
GrB_Matrix_extractTuples(..., int32_t*,...)
                                                 GrB_Matrix_extractTuples_INT32(..., int32_t*,...)
GrB_Matrix_extractTuples(..., uint32_t*,...)
                                                 GrB_Matrix_extractTuples_UINT32(..., uint32_t*,...)
GrB_Matrix_extractTuples(..., int64_t*,...)
                                                 GrB_Matrix_extractTuples_INT64(..., int64_t*,...)
GrB_Matrix_extractTuples(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_UINT64(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_FP32(..., float*,...)
GrB\_Matrix\_extractTuples(..., float*,...)
                                                 GrB_Matrix_extractTuples_FP64(..., double*,...)
GrB_Matrix_extractTuples(..., double*,...)
GrB_Matrix_extractTuples(...,other*,...)
                                                 GrB_Matrix_extractTuples_UDT(..., void*,...)
```

Table 5.5: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                   Nonpolymorphic signature
GrB_Matrix_import(...,const bool*,...)
                                                    GrB_Matrix_import_BOOL(...,const bool*,...)
GrB_Matrix_import(...,const int8_t*,...)
                                                    GrB_Matrix_import_INT8(...,const int8_t*,...)
GrB_Matrix_import(...,const uint8_t*,...)
                                                    GrB_Matrix_import_UINT8(...,const uint8_t*,...)
GrB_Matrix_import(...,const int16_t*,...)
                                                    GrB_Matrix_import_INT16(...,const int16_t*,...)
                                                    GrB_Matrix_import_UINT16(...,const uint16_t*,...)
GrB_Matrix_import(...,const uint16_t*,...)
                                                    GrB_Matrix_import_INT32(...,const int32_t*,...)
GrB_Matrix_import(...,const int32_t*,...)
GrB_Matrix_import(...,const uint32_t*,...)
                                                    GrB_Matrix_import_UINT32(...,const uint32_t*,...)
GrB_Matrix_import(...,const int64_t*,...)
                                                    \mathsf{GrB}\_\mathsf{Matrix}\_\mathsf{import}\_\mathsf{INT64}(\dots,\mathsf{const}\ \mathsf{int64}\_\mathsf{t*},\dots)
GrB_Matrix_import(...,const uint64_t*,...)
                                                    GrB_Matrix_import_UINT64(...,const uint64_t*,...)
                                                    GrB_Matrix_import_FP32(...,const float*,...)
GrB_Matrix_import(...,const float*,...)
GrB_Matrix_import(...,const double*,...)
                                                    GrB_Matrix_import_FP64(...,const double*,...)
                                                    GrB_Matrix_import_UDT(...,const void*,...)
GrB_Matrix_import(...,const other,...)
GrB_Matrix_export(...,bool*,...)
                                                    GrB\_Matrix\_export\_BOOL(...,bool*,...)
GrB_Matrix_export(...,int8_t*,...)
                                                    GrB_Matrix_export_INT8(...,int8_t*,...)
                                                   GrB_Matrix_export_UINT8(...,uint8_t*,...)
GrB_Matrix_export(...,uint8_t*,...)
GrB_Matrix_export(...,int16_t*,...)
                                                    GrB_Matrix_export_INT16(...,int16_t*,...)
GrB\_Matrix\_export(...,uint16\_t^*,...)
                                                    GrB\_Matrix\_export\_UINT16(...,uint16\_t*,...)
                                                    GrB\_Matrix\_export\_INT32(...,int32\_t*,...)
GrB_Matrix_export(...,int32_t*,...)
GrB_Matrix_export(...,uint32_t*,...)
                                                    GrB_Matrix_export_UINT32(...,uint32_t*,...)
                                                   GrB_Matrix_export_INT64(...,int64_t*,...)
GrB_Matrix_export(...,int64_t*,...)
GrB_Matrix_export(...,uint64_t*,...)
                                                    GrB_Matrix_export_UINT64(...,uint64_t*,...)
GrB_Matrix_export(...,float*,...)
                                                    GrB_Matrix_export_FP32(...,float*,...)
GrB\_Matrix\_export(...,double*,...)
                                                    GrB_Matrix_export_FP64(...,double*,...)
GrB_Matrix_export(...,other,...)
                                                    GrB_Matrix_export_UDT(...,void*,...)
GrB_free(GrB_Type*
                                                    GrB_Type_free(GrB_Type*)
GrB_free(GrB_UnaryOp*)
                                                    GrB_UnaryOp_free(GrB_UnaryOp*)
                                                   {\sf GrB\_IndexUnaryOp\_free}({\sf GrB\_IndexUnaryOp*})
GrB_free(GrB_IndexUnaryOp*)
GrB_free(GrB_BinaryOp*)
                                                    GrB_BinaryOp_free(GrB_BinaryOp*)
GrB_free(GrB_Monoid*)
                                                    GrB_Monoid_free(GrB_Monoid*)
                                                   GrB_Semiring_free(GrB_Semiring*)
GrB_free(GrB_Semiring*)
GrB_free(GrB_Scalar*)
                                                    GrB_Scalar_free(GrB_Scalar*)
                                                    GrB_Vector_free(GrB_Vector*)
GrB_free(GrB_Vector*)
GrB_free(GrB_Matrix*)
                                                    GrB_Matrix_free(GrB_Matrix*)
GrB_free(GrB_Descriptor*)
                                                    GrB_Descriptor_free(GrB_Descriptor*)
GrB_wait(GrB_Type, GrB_WaitMode)
                                                    GrB_Type_wait(GrB_Type, GrB_WaitMode)
GrB_wait(GrB_UnaryOp, GrB_WaitMode)
                                                    GrB_UnaryOp_wait(GrB_UnaryOp, GrB_WaitMode)
GrB_wait(GrB_IndexUnaryOp, GrB_WaitMode)
                                                    GrB_IndexUnaryOp_wait(GrB_IndexUnaryOp, GrB_WaitMode)
GrB_wait(GrB_BinaryOp, GrB_WaitMode)
                                                    GrB_BinaryOp_wait(GrB_BinaryOp, GrB_WaitMode)
GrB_wait(GrB_Monoid, GrB_WaitMode)
                                                    GrB_Monoid_wait(GrB_Monoid, GrB_WaitMode)
GrB_wait(GrB_Semiring, GrB_WaitMode)
                                                    GrB_Semiring_wait(GrB_Semiring, GrB_WaitMode)
GrB_wait(GrB_Scalar, GrB_WaitMode)
                                                    GrB_Scalar_wait(GrB_Scalar, GrB_WaitMode)
GrB_wait(GrB_Vector, GrB_WaitMode)
                                                    GrB_Vector_wait(GrB_Vector, GrB_WaitMode)
GrB_wait(GrB_Matrix, GrB_WaitMode)
                                                    GrB_Matrix_wait(GrB_Matrix, GrB_WaitMode)
GrB_wait(GrB_Descriptor, GrB_WaitMode)
                                                    GrB_Descriptor_wait(GrB_Descriptor, GrB_WaitMode)
GrB_error(const char**, const GrB_Type)
                                                    GrB_Type_error(const char**, const GrB_Type)
GrB_error(const char**, const GrB_UnaryOp)
                                                    GrB_UnaryOp_error(const char**, const GrB_UnaryOp)
GrB_error(const char**, const GrB_IndexUnaryOp)
                                                    GrB_IndexUnaryOp_error(const char**, const GrB_IndexUnaryOp)
GrB_error(const char**, const GrB_BinaryOp)
GrB_error(const char**, const GrB_Monoid)
                                                    GrB_BinaryOp_error(const char**, const GrB_BinaryOp)
                                                    GrB_Monoid_error(const char**, const GrB_Monoid)
                                                    GrB_Semiring_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Scalar)
                                                    GrB_Scalar_error(const char**, const GrB_Scalar)
                                                    GrB_Vector_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Matrix)
GrB_error(const char**, const GrB_Descriptor)
                                                    GrB_Matrix_error(const char**, const GrB_Matrix)
                                                    GrB_Descriptor_error(const char**, const GrB_Descriptor)
```

Table 5.6: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                                    Nonpolymorphic signature
GrB_eWiseMult(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseMult_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB\_Vector\_eWiseMult\_Monoid(GrB\_Vector,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_BinaryOp,...)
                                                                    GrB\_Vector\_eWiseMult\_BinaryOp(GrB\_Vector, ..., GrB\_BinaryOp, ...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB_Matrix_eWiseMult_Semiring(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseMult\_Monoid(GrB\_Matrix,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Monoid,...)
\mathsf{GrB\_eWiseMult}(\mathsf{GrB\_Matrix}, \ldots, \mathsf{GrB\_BinaryOp}, \ldots)
                                                                    GrB\_Matrix\_eWiseMult\_BinaryOp(GrB\_Matrix,...,GrB\_BinaryOp,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseAdd_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                     GrB\_Vector\_eWiseAdd\_Monoid(GrB\_Vector, \dots, GrB\_Monoid, \dots) 
GrB_eWiseAdd(GrB_Vector,...,GrB_Monoid,...)
                                                                    \label{lem:grb_vector_eWiseAdd_BinaryOp} GrB\_Vector, \dots, GrB\_BinaryOp, \dots)
GrB_eWiseAdd(GrB_Vector,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseAdd\_Semiring(GrB\_Matrix,...,GrB\_Semiring,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Monoid,...)
                                                                    GrB Matrix eWiseAdd Monoid(GrB Matrix,...,GrB Monoid,...)
GrB\_eWiseAdd(GrB\_Matrix,...,GrB\_BinaryOp,...)
                                                                    \label{linearyOp} GrB\_Matrix\_eWiseAdd\_BinaryOp(GrB\_Matrix, \ldots, GrB\_BinaryOp, \ldots)
GrB_extract(GrB_Vector,...,GrB_Vector,...
                                                                    GrB\_Vector\_extract(GrB\_Vector,...,GrB\_Vector,...)
GrB\_extract(GrB\_Matrix,...,GrB\_Matrix,...)
                                                                    GrB_Matrix_extract(GrB_Matrix,...,GrB_Matrix,...)
GrB_extract(GrB_Vector,...,GrB_Matrix,...)
                                                                    GrB\_Col\_extract(GrB\_Vector,...,GrB\_Matrix,...)
GrB_assign(GrB_Vector,...,GrB_Vector,...)
                                                                    GrB\_Vector\_assign(GrB\_Vector,...,GrB\_Vector,...)
GrB_assign(GrB_Matrix,...,GrB_Matrix,...)
                                                                    GrB_Matrix_assign(GrB_Matrix,...,GrB_Matrix,...)
\label{lem:grb_assign} $$\operatorname{\mathsf{GrB\_Matrix}},\ldots,\operatorname{\mathsf{GrB\_Vector}},\operatorname{\mathsf{const}} \ \operatorname{\mathsf{GrB\_Index}}^*,\ldots)$$
                                                                    {\sf GrB\_Col\_assign}({\sf GrB\_Matrix}, \ldots, {\sf GrB\_Vector}, {\sf const}\ {\sf GrB\_Index^*}, \ldots)
                                                                     \begin{array}{lll} & GrB\_Row\_assign(GrB\_Matrix, \ldots, GrB\_Vector, GrB\_Index, \ldots) \\ & GrB\_Vector\_assign\_Scalar(GrB\_Vector, \ldots, const \ GrB\_Scalar, \ldots) \end{array} 
GrB\_assign(GrB\_Matrix,...,GrB\_Vector,GrB\_Index,...)
GrB_assign(GrB_Vector,...,GrB_Scalar,...)
GrB_assign(GrB_Vector,...,bool,...)
                                                                    GrB_Vector_assign_BOOL(GrB_Vector,..., bool,...)
GrB_assign(GrB_Vector,...,int8_t,...)
                                                                    GrB_Vector_assign_INT8(GrB_Vector,..., int8_t,...)
GrB_assign(GrB_Vector,...,uint8_t,...)
                                                                    GrB_Vector_assign_UINT8(GrB_Vector,..., uint8_t,...)
GrB_assign(GrB_Vector,...,int16_t,...)
                                                                    GrB_Vector_assign_INT16(GrB_Vector,..., int16_t,...)
GrB_assign(GrB_Vector,...,uint16_t,...)
                                                                    GrB_Vector_assign_UINT16(GrB_Vector,..., uint16_t,...)
GrB_assign(GrB_Vector,...,int32_t,...)
                                                                    GrB_Vector_assign_INT32(GrB_Vector,..., int32_t,...)
GrB_assign(GrB_Vector,...,uint32_t,...)
                                                                    GrB_Vector_assign_UINT32(GrB_Vector,..., uint32_t,...)
GrB_assign(GrB_Vector,...,int64_t,...)
                                                                    GrB\_Vector\_assign\_INT64(GrB\_Vector,..., int64\_t,...)
GrB_assign(GrB_Vector,...,uint64_t,...)
                                                                    GrB_Vector_assign_UINT64(GrB_Vector,..., uint64_t,...)
GrB\_assign(GrB\_Vector,...,float,...)
                                                                    GrB_Vector_assign_FP32(GrB_Vector,..., float,...)
                                                                    GrB_Vector_assign_FP64(GrB_Vector,..., double,...)
GrB_assign(GrB_Vector,...,double,...)
GrB_assign(GrB_Vector,...,other,...)
                                                                    GrB_Vector_assign_UDT(GrB_Vector,...,const void*,...)
GrB_assign(GrB_Matrix,...,GrB_Scalar,...)
                                                                    GrB_Matrix_assign_Scalar(GrB_Matrix,...,const GrB_Scalar,...)
GrB_assign(GrB_Matrix,...,bool,...)
                                                                    GrB_Matrix_assign_BOOL(GrB_Matrix,..., bool,...)
                                                                    GrB\_Matrix\_assign\_INT8(GrB\_Matrix,..., int8\_t,...)
GrB_assign(GrB_Matrix,...,int8_t,...)
GrB_assign(GrB_Matrix,...,uint8_t,...)
                                                                    GrB_Matrix_assign_UINT8(GrB_Matrix,..., uint8_t,...)
GrB_assign(GrB_Matrix,...,int16_t,...)
                                                                    GrB_Matrix_assign_INT16(GrB_Matrix,..., int16_t,...)
GrB\_assign(GrB\_Matrix,...,uint16\_t,...)
                                                                    GrB\_Matrix\_assign\_UINT16(GrB\_Matrix,..., uint16\_t,...)
                                                                    GrB_Matrix_assign_INT32(GrB_Matrix,..., int32_t,...)
GrB_assign(GrB_Matrix,...,int32_t,...)
GrB_assign(GrB_Matrix,...,uint32_t,...)
                                                                    GrB_Matrix_assign_UINT32(GrB_Matrix,..., uint32_t,...)
GrB_assign(GrB_Matrix,...,int64_t,...)
                                                                    GrB_Matrix_assign_INT64(GrB_Matrix,..., int64_t,...)
GrB_assign(GrB_Matrix,...,uint64_t,...)
                                                                    GrB_Matrix_assign_UINT64(GrB_Matrix,..., uint64_t,...)
                                                                    {\sf GrB\_Matrix\_assign\_FP32}({\sf GrB\_Matrix}, \ldots, \ {\sf float}, \ldots)
GrB_assign(GrB_Matrix,...,float,...)
                                                                    GrB_Matrix_assign_FP64(GrB_Matrix,..., double,...)
GrB_assign(GrB_Matrix,...,double,...)
GrB_assign(GrB_Matrix,...,other,...)
                                                                    GrB_Matrix_assign_UDT(GrB_Matrix,...,const void*,...)
```

 ${\it Table 5.7: Long-name, nonpolymorphic form of GraphBLAS methods (continued)}.$ 

	Polymorphic signature	Nonpolymorphic signature
-	GrB_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)	GrB_Vector_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)
	$GrB_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)$	GrB_Matrix_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)
-	GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Scalar,GrB_Vector,)	GrB_Vector_apply_BinaryOp1st_Scalar(GrB_Vector,,GrB_BinaryOp,GrB_Scalar,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,bool,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_BOOL(GrB_Vector,,GrB_BinaryOp,bool,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int8\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT8(GrB_Vector,,GrB_BinaryOp,int8_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint8\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_UINT8(GrB_Vector,,GrB_BinaryOp,uint8_t,GrB_Vector,)
	GrB_apply(GrB_Vector,,GrB_BinaryOp,int16_t,GrB_Vector,)	GrB_Vector_apply_BinaryOp1st_INT16(GrB_Vector,,GrB_BinaryOp,int16_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint16\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_UINT16(GrB_Vector,,GrB_BinaryOp,uint16_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int32\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT32(GrB_Vector,,GrB_BinaryOp,int32_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint32\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_UINT32(GrB_Vector,,GrB_BinaryOp,uint32_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int64\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT64(GrB_Vector,,GrB_BinaryOp,int64_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint64\_t,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_UINT64(GrB_Vector,,GrB_BinaryOp,uint64_t,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,float,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_FP32(GrB_Vector,,GrB_BinaryOp,float,GrB_Vector,)
	$GrB\_apply(GrB\_Vector,, GrB\_BinaryOp, double, GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_FP64(GrB_Vector,,GrB_BinaryOp,double,GrB_Vector,)
_	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,other,GrB\_Vector,)$	GrB_Vector_apply_BinaryOp1st_UDT(GrB_Vector,,GrB_BinaryOp,const void*,GrB_Vector,)
	$GrB\_apply(GrB\_Vector, \dots, GrB\_BinaryOp, GrB\_Vector, GrB\_Scalar, \dots)$	GrB_Vector_apply_BinaryOp2nd_Scalar(GrB_Vector,,GrB_BinaryOp,GrB_Vector,GrB_Scalar,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,bool,)$	GrB_Vector_apply_BinaryOp2nd_BOOL(GrB_Vector,,GrB_BinaryOp,GrB_Vector,bool,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int8\_t,)$	GrB_Vector_apply_BinaryOp2nd_INT8(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int8_t,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint8\_t,)$	$\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_UINT8(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint8\_t,)$
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int16\_t,)$	GrB_Vector_apply_BinaryOp2nd_INT16(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int16_t,)
$\mathcal{C}_{0}$	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint16\_t,)$	GrB_Vector_apply_BinaryOp2nd_UINT16(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint16_t,)
<u> </u>	$GrB\_apply(GrB\_Vector, \dots, GrB\_BinaryOp, GrB\_Vector, int32\_t, \dots)$	GrB_Vector_apply_BinaryOp2nd_INT32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int32_t,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint32\_t,)$	GrB_Vector_apply_BinaryOp2nd_UINT32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint32_t,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int64\_t,)$	GrB_Vector_apply_BinaryOp2nd_INT64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int64_t,)
	$GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint64\_t,)$	$\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_UINT64\\ (GrB\_Vector,\dots,GrB\_BinaryOp,GrB\_Vector,uint64\_t,\dots)$
	$GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, float,)$	GrB_Vector_apply_BinaryOp2nd_FP32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,float,)
	$GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, double,)$	GrB_Vector_apply_BinaryOp2nd_FP64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,double,)
_	$GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, other,)$	GrB_Vector_apply_BinaryOp2nd_UDT(GrB_Vector,,GrB_BinaryOp,GrB_Vector,const void*,)

 ${\it Table 5.8: Long-name, nonpolymorphic form of GraphBLAS methods (continued).}$ 

	Polymorphic signature	Nonpolymorphic signature
-	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_Scalar(GrB_Matrix,,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,bool,GrB_Matrix,)	GrB_Matrix_apply_BinaryOp1st_BOOL(GrB_Matrix,,GrB_BinaryOp,bool,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int8\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT8(GrB_Matrix,,GrB_BinaryOp,int8_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint8\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT8(GrB_Matrix,,GrB_BinaryOp,uint8_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int16\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT16(GrB_Matrix,,GrB_BinaryOp,int16_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint16\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT16(GrB_Matrix,,GrB_BinaryOp,uint16_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int32\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT32(GrB_Matrix,,GrB_BinaryOp,int32_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint32\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT32(GrB_Matrix,,GrB_BinaryOp,uint32_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int64\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT64(GrB_Matrix,,GrB_BinaryOp,int64_t,GrB_Matrix,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint64\_t,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT64(GrB_Matrix,,GrB_BinaryOp,uint64_t,GrB_Matrix,)
2	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,float,GrB\_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_FP32(GrB_Matrix,,GrB_BinaryOp,float,GrB_Matrix,)
7	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,double,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_FP64(GrB_Matrix,,GrB_BinaryOp,double,GrB_Matrix,)
Ŭ _	GrB_apply(GrB_Matrix,,GrB_BinaryOp, <i>other</i> ,GrB_Matrix,)	GrB_Matrix_apply_BinaryOp1st_UDT(GrB_Matrix,,GrB_BinaryOp,const void*,GrB_Matrix,)
	$\label{lem:grb_apply} $$\operatorname{GrB\_Matrix},\ldots,\operatorname{GrB\_BinaryOp},\operatorname{GrB\_Matrix},\operatorname{GrB\_Scalar},\ldots)$$$	GrB_Matrix_apply_BinaryOp2nd_Scalar(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,GrB_Scalar,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,bool,)$	$\begin{tabular}{lll} GrB\_Matrix\_apply\_BinaryOp2nd\_BOOL(GrB\_Matrix,\dots,GrB\_BinaryOp,GrB\_Matrix,bool,\dots) \\ \hline \end{tabular}$
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int8_t,)	GrB_Matrix_apply_BinaryOp2nd_INT8(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int8_t,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,uint8\_t,)$	$\label{linear_gradient} GrB\_Matrix\_apply\_BinaryOp2nd\_UINT8 (GrB\_Matrix, \dots, GrB\_BinaryOp, GrB\_Matrix, uint8\_t, \dots)$
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int16_t,)	GrB_Matrix_apply_BinaryOp2nd_INT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int16_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint16_t,)	GrB_Matrix_apply_BinaryOp2nd_UINT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint16_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int32_t,)	GrB_Matrix_apply_BinaryOp2nd_INT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int32_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint32_t,)	GrB_Matrix_apply_BinaryOp2nd_UINT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint32_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int64_t,)	GrB_Matrix_apply_BinaryOp2nd_INT64(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int64_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint64_t,)	GrB_Matrix_apply_BinaryOp2nd_UINT64(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint64_t,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,float,)	GrB_Matrix_apply_BinaryOp2nd_FP32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,float,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,double,)	GrB_Matrix_apply_BinaryOp2nd_FP64(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,double,)
	$GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,other,)$	GrB_Matrix_apply_BinaryOp2nd_UDT(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,const void*,)

Table 5.9: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

| Nonpolymorphic signature

Polymorphic signature	Nonpolymorphic signature
$\label{lem:grb_apply} $$\operatorname{GrB\_Vector}_{\ldots},\operatorname{GrB\_IndexUnaryOp}_{\operatorname{GrB\_Vector}_{\operatorname{GrB\_Scalar}_{\ldots}}}$$	GrB_Vector_apply_IndexOp_Scalar(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,)
$GrB_apply(GrB_Vector,, GrB_IndexUnaryOp, GrB_Vector, bool,)$	GrB_Vector_apply_IndexOp_BOOL(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,bool,)
$GrB\_apply(GrB\_Vector,, GrB\_IndexUnaryOp, GrB\_Vector, int8\_t,)$	GrB_Vector_apply_IndexOp_INT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)
$GrB\_apply(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint8\_t, \dots)$	$\begin{tabular}{ll} GrB\_Vector\_apply\_IndexOp\_UINT8(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint8\_t,) \end{tabular}$
$GrB\_apply(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, int16\_t, \ldots)$	GrB_Vector_apply_IndexOp_INT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int16_t,)
$\label{lem:grb_apply} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint16\_t, \dots)$	GrB_Vector_apply_IndexOp_UINT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint16_t,)
$GrB\_apply(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, int 32\_t, \ldots)$	GrB_Vector_apply_IndexOp_INT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int32_t,)
$\label{lem:grb_apply} GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, uint 32\_t, \ldots)$	GrB_Vector_apply_IndexOp_UINT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)
$GrB\_apply(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, int 64\_t, \ldots)$	$\label{local-control} GrB\_Vector\_apply\_IndexOp\_INT64(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, int64\_t, \dots)$
$GrB\_apply(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint64\_t, \dots)$	GrB_Vector_apply_IndexOp_UINT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint64_t,)
$GrB\_apply(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, float, \ldots)$	GrB_Vector_apply_IndexOp_FP32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,float,)
$GrB\_apply(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, double, \ldots)$	GrB_Vector_apply_IndexOp_FP64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,double,)
GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector, <i>other</i> ,)	GrB_Vector_apply_IndexOp_UDT(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,const void*,)
$GrB\_apply(GrB\_Matrix, \ldots, GrB\_IndexUnaryOp, GrB\_Matrix, GrB\_Scalar, \ldots)$	GrB_Matrix_apply_IndexOp_Scalar(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,)
$GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, bool, \dots)$	GrB_Matrix_apply_IndexOp_BOOL(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,bool,)
$GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int8\_t, \dots)$	GrB_Matrix_apply_IndexOp_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)
$GrB_apply(GrB_Matrix,, GrB_IndexUnaryOp, GrB_Matrix, uint8_t,)$	GrB_Matrix_apply_IndexOp_UINT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,)
$GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int16\_t, \dots)$	GrB_Matrix_apply_IndexOp_INT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int16_t,)
$GrB\_apply(GrB\_Matrix,, GrB\_IndexUnaryOp, GrB\_Matrix, uint16\_t,)$	GrB_Matrix_apply_IndexOp_UINT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)
$GrB_apply(GrB_Matrix,, GrB_IndexUnaryOp, GrB_Matrix, int 32_t,)$	GrB_Matrix_apply_IndexOp_INT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int32_t,)
$_{\sim}$ GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)	GrB_Matrix_apply_IndexOp_UINT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)
$\cdot$ GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)	GrB_Matrix_apply_IndexOp_INT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)
$\label{eq:GrB_apply} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint 64\_t, \dots)$	GrB_Matrix_apply_IndexOp_UINT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,)
GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)	GrB_Matrix_apply_IndexOp_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)
$GrB\_apply(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,double,)$	GrB_Matrix_apply_IndexOp_FP64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,double,)
$GrB\_apply(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,other,)$	GrB_Matrix_apply_IndexOp_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)

Table 5.10: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

	Polymorphic signature	Nonpolymorphic signature
_	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,GrB\_Scalar,)$	$ GrB\_Vector\_select\_Scalar(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, GrB\_Scalar, \ldots) $
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,bool,)$	$ GrB\_Vector\_select\_BOOL(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, bool, \ldots) $
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int8\_t,)$	$\label{lem:grb_vector_select_INT8} GrB\_Vector,, GrB\_IndexUnaryOp, GrB\_Vector, int8\_t,)$
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint8\_t,)$	GrB_Vector_select_UINT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint8_t,)
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int16\_t,)$	GrB_Vector_select_INT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int16_t,)
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint16\_t,)$	$\label{lem:grb_vector_select_UINT16} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint16\_t, \dots)$
	$GrB\_select(GrB\_Vector,, GrB\_IndexUnaryOp, GrB\_Vector, int32\_t,)$	$\label{lem:grb_vector_select_INT32} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, int 32\_t, \dots)$
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint32\_t,)$	GrB_Vector_select_UINT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int64\_t,)$	$\label{lem:grb_vector_select_INT64} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, int64\_t, \dots)$
	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint64\_t,)$	$\label{linear_gradient} GrB\_Vector\_select\_UINT64 (GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint64\_t,)$
2	$GrB\_select(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, float, \dots)$	$\label{lem:grb_vector_select_FP32} GrB\_Vector,, GrB\_IndexUnaryOp, GrB\_Vector, float,)$
72	$GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,double,)$	$GrB\_Vector\_select\_FP64 (GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, double, \dots)$
	GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,other,)	$\label{linear_gradient} GrB\_Vector\_select\_UDT(GrB\_Vector,\dots,GrB\_IndexUnaryOp,GrB\_Vector,const\ void^*,\dots)$
	$\label{lem:grb_select} GrB\_select \big(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, GrB\_Scalar, \dots \big)$	$\label{lem:grb_matrix} GrB\_Matrix\_select\_Scalar (GrB\_Matrix,\dots,GrB\_IndexUnaryOp,GrB\_Matrix,GrB\_Scalar,\dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,bool,)$	$\label{lem:grb_matrix} GrB\_Matrix\_select\_BOOL(GrB\_Matrix,\dots,GrB\_IndexUnaryOp,GrB\_Matrix,bool,\dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,int8\_t,)$	GrB_Matrix_select_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)
	$\label{lem:grb_select} GrB\_select(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint8\_t, \dots)$	$\label{linear_gradient} GrB\_Matrix\_select\_UINT8 (GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint8\_t, \dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,int16\_t,)$	$\label{lem:grb_matrix_select_INT16} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int16\_t, \dots)$
	$\label{lem:grb_select} GrB\_select \big(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint16\_t, \dots \big)$	$\label{lem:grb_matrix} Grb\_Matrix\_select\_UINT16 (Grb\_Matrix, \dots, Grb\_IndexUnaryOp, Grb\_Matrix, uint16\_t, \dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,int32\_t,)$	$\label{linear_gradient} GrB\_Matrix\_select\_INT32 (GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int32\_t, \dots)$
	$\label{lem:grb_select} GrB\_select(GrB\_Matrix,\dots,GrB\_IndexUnaryOp,GrB\_Matrix,uint32\_t,\dots)$	$\label{lem:grb_matrix} Grb\_Matrix\_select\_UINT32 (Grb\_Matrix,\dots,Grb\_IndexUnaryOp,Grb\_Matrix,uint32\_t,\dots)$
	$\label{lem:grb_select} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int 64\_t, \dots)$	$\label{lem:grb_matrix} GrB\_Matrix\_select\_INT64(GrB\_Matrix,\dots,GrB\_IndexUnaryOp,GrB\_Matrix,int64\_t,\dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,uint64\_t,)$	$\label{lem:grb_matrix} Grb\_Matrix\_select\_UINT64 (Grb\_Matrix,\dots,Grb\_IndexUnaryOp,Grb\_Matrix,uint64\_t,\dots)$
	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,float,)$	GrB_Matrix_select_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)
	$GrB\_select(GrB\_Matrix,, GrB\_IndexUnaryOp, GrB\_Matrix, double,)$	$\label{lem:grb_matrix_select_FP64} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, double, \dots)$
_	$GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,other,)$	GrB_Matrix_select_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)

 ${\it Table 5.11: Long-name, nonpolymorphic form of GraphBLAS methods (continued).}$ 

Polymorphic signature	Nonpolymorphic signature
GrB_reduce(GrB_Vector,,GrB_Monoid,)	GrB_Matrix_reduce_Monoid(GrB_Vector,,GrB_Monoid,)
$GrB\_reduce(GrB\_Vector,,GrB\_BinaryOp,)$	GrB_Matrix_reduce_BinaryOp(GrB_Vector,,GrB_BinaryOp,)
GrB_reduce(GrB_Scalar,,GrB_Monoid,GrB_Vector,)	GrB_Vector_reduce_Monoid_Scalar(GrB_Scalar,,GrB_Vector,)
GrB_reduce(GrB_Scalar,,GrB_BinaryOp,GrB_Vector,)	GrB_Vector_reduce_BinaryOp_Scalar(GrB_Scalar,,GrB_Vector,)
GrB_reduce(bool*,,GrB_Vector,)	GrB_Vector_reduce_BOOL(bool*,,GrB_Vector,)
$GrB\_reduce(int8\_t*,,GrB\_Vector,)$	GrB_Vector_reduce_INT8(int8_t*,,GrB_Vector,)
GrB_reduce(uint8_t*,,GrB_Vector,)	GrB_Vector_reduce_UINT8(uint8_t*,,GrB_Vector,)
GrB_reduce(int16_t*,,GrB_Vector,)	GrB_Vector_reduce_INT16(int16_t*,,GrB_Vector,)
GrB_reduce(uint16_t*,,GrB_Vector,)	GrB_Vector_reduce_UINT16(uint16_t*,,GrB_Vector,)
$GrB\_reduce(int32\_t^*,,GrB\_Vector,)$	GrB_Vector_reduce_INT32(int32_t*,,GrB_Vector,)
$GrB\_reduce(uint32\_t^*,,GrB\_Vector,)$	GrB_Vector_reduce_UINT32(uint32_t*,,GrB_Vector,)
$GrB\_reduce(int64\_t^*,,GrB\_Vector,)$	GrB_Vector_reduce_INT64(int64_t*,,GrB_Vector,)
GrB_reduce(uint64_t*,,GrB_Vector,)	GrB_Vector_reduce_UINT64(uint64_t*,,GrB_Vector,)
$GrB\_reduce(float*,\ldots,GrB\_Vector,\ldots)$	GrB_Vector_reduce_FP32(float*,,GrB_Vector,)
GrB_reduce(double*,,GrB_Vector,)	GrB_Vector_reduce_FP64(double*,,GrB_Vector,)
$GrB\_reduce(\mathit{other},\ldots,GrB\_Vector,\ldots)$	GrB_Vector_reduce_UDT(void*,,GrB_Vector,)
$GrB\_reduce(GrB\_Scalar,,GrB\_Monoid,GrB\_Matrix,)$	GrB_Matrix_reduce_Monoid_Scalar(GrB_Scalar,,GrB_Monoid,GrB_Matrix,)
$GrB\_reduce(GrB\_Scalar,,GrB\_BinaryOp,GrB\_Matrix,)$	GrB_Matrix_reduce_BinaryOp_Scalar(GrB_Scalar,,GrB_BinaryOp,GrB_Matrix,)
$GrB\_reduce(bool*,\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_BOOL(bool*,,GrB_Matrix,)
$GrB\_reduce(int8\_t^*,,GrB\_Matrix,)$	GrB_Matrix_reduce_INT8(int8_t*,,GrB_Matrix,)
$GrB\_reduce(uint8\_t^*,,GrB\_Matrix,)$	GrB_Matrix_reduce_UINT8(uint8_t*,,GrB_Matrix,)
$GrB\_reduce(int16\_t^*, \dots, GrB\_Matrix, \dots)$	GrB_Matrix_reduce_INT16(int16_t*,,GrB_Matrix,)
$GrB\_reduce(uint16\_t^*,\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_UINT16(uint16_t*,,GrB_Matrix,)
$GrB\_reduce(int32\_t^*, \dots, GrB\_Matrix, \dots)$	GrB_Matrix_reduce_INT32(int32_t*,,GrB_Matrix,)
$GrB\_reduce(uint32\_t^*,,GrB\_Matrix,)$	GrB_Matrix_reduce_UINT32(uint32_t*,,GrB_Matrix,)
$GrB\_reduce(int64\_t*,\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_INT64(int64_t*,,GrB_Matrix,)
$GrB\_reduce(uint64\_t^*,,GrB\_Matrix,)$	GrB_Matrix_reduce_UINT64(uint64_t*,,GrB_Matrix,)
$GrB\_reduce(float*,\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_FP32(float*,,GrB_Matrix,)
$GrB\_reduce(double*,\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_FP64(double*,,GrB_Matrix,)
$GrB\_reduce(\mathit{other},\ldots,GrB\_Matrix,\ldots)$	GrB_Matrix_reduce_UDT(void*,,GrB_Matrix,)
$GrB\_kronecker(GrB\_Matrix,,GrB\_Semiring,)$	GrB_Matrix_kronecker_Semiring(GrB_Matrix,,GrB_Semiring,)
$GrB\_kronecker(GrB\_Matrix,,GrB\_Monoid,)$	GrB_Matrix_kronecker_Monoid(GrB_Matrix,,GrB_Monoid,)
$GrB_kronecker(GrB_Matrix,,GrB_BinaryOp,)$	GrB_Matrix_kronecker_BinaryOp(GrB_Matrix,,GrB_BinaryOp,)

### Appendix A

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### Revision history

Changes in 2.0.0 (Released: ##-Xxxxx-2021:

- Reorganized Chapters 2 and 3: Chapter 2 contains prose regarding the basic concepts cap-7365 tured in the API; Chapter 3 presents all of the enumeratiosn, literals, data types, and prede-7366 fined objects required by the API. Made short captions for the List of Tables.
- (Issue BB-49, BB-50) Updated and corrected language regarding multithreading and comple-7368 tion, and requirements regarding acquire-release memory orders. Methods that used to force 7369 complete no longer do. 7370
- (Issue BB-74, BB-9) Assigned integer values to all return codes as well as all enumerations 7371 in the API to ensure run-time compatibility between libraries. 7372
- (Issues BB-70, BB-67) Changed semantics and signature of GrB wait(obj, mode). Added wait 7373 modes for 'complete' or 'materialize' and removed GrB\_wait(void). This breaks backward 7374 compatibility. 7375
- (Issue GH-51) Removed deprecated GrB\_SCMP literal from descriptor values. This breaks 7376 backward compatibility. 7377
- (Issues BB-8, BB-36) Added sparse GrB Scalar object and its use in additional variants of 7378 extract/setElement methods, and reduce, apply, assign and select operations. 7379
- (Issues BB-34, GH-33, GH-45) Added new select operation that uses an index unary operator. 7380 Added new variants of apply that take an index unary operator (matrix and vector variants). 7381
- (Issues BB-68, BB-51) Added serialize and descrialize methods for matrices to/from imple-7382 mentation defined formats. 7383
- (Issues BB-25, GH-42) Added import and export methods for matrices to/from API specified 7384 formats. Three formats have been specified: CSC, CSR, COO. Dense row and column formats 7385 have been deferred. 7386
  - (Issue BB-75) Added matrix constructor to build a diagonal GrB\_Matrix from a GrB\_Vector.

- (Issue BB-73) Allow GrB\_NULL for dup operator in matrix and vector build methods. Return error if duplicate locations encountered.
- (Issue BB-58) Added matrix and vector methods to remove (annihilate) elements.
- (Issue BB-17) Added GrB\_ABS\_T (absolute value) unary operator.
- (Issue GH-46) Adding GrB\_ONEB\_T binary operator that returns 1 cast to type T (not to be confused with the proposed unary operator).
- (Issue GH-53) Added language about what constitutes a "conformant" implementation. Added GrB\_NOT\_IMPLEMENTED return value (API error) for API any combinations of inputs to a method that is not supported by the implementation.
- Added GrB\_EMPTY\_OBJECT return value (execution error) that is used when an opaque object (currently only GrB\_Scalar) is passed as an input that cannot be empty.
- (Issue BB-45) Removed language about annihilators.
- (Issue BB-69) Made names/symbols containing underscores searchable in PDF.
- Updated a number algorithms in the appendix to use new operations and methods.
- Numerous additions (some changes) to the non-polymorphic interface to track changes to the specification.
- Typographical error in version macros was corrected. They are all caps: GRB\_VERSION and
   GRB\_SUBVERSION.
- Typographical change to eWiseAdd Description to be consistent in order of set intersections.
- Typographical errors in eWiseAdd: cut-and-paste errors from eWiseMult/set intersection fixed to read eWiseAdd/set union.
- Typographical error (NEQ  $\rightarrow$  NE) in Description of Table 3.8.
- 7410 Changes in 1.3.0 (Released: 25 September 2019):
- (Issue BB-50) Changed definition of completion and added GrB\_wait() that takes an opaque GraphBLAS object as an argument.
- (Issue BB-39) Added GrB kronecker operation.
- (Issue BB-40) Added variants of the GrB\_apply operation that take a binary function and a scalar.
- (Issue BB-59) Changed specification about how reductions to scalar (GrB\_reduce) are to be performed (to minimize dependence on monoid identity).
- (Issue BB-24) Added methods to resize matrices and vectors (GrB\_Matrix\_resize and GrB\_Vector\_resize).

- (Issue BB-47) Added methods to remove single elements from matrices and vectors (GrB\_Matrix\_removeElement).
- (Issue BB-41) Added GrB\_STRUCTURE descriptor flag for masks (consider only the structure of the mask and not the values).
- (Issue BB-64) Deprecated GrB\_SCMP in favor of new GrB\_COMP for descriptor values.
- (Issue BB-46) Added predefined descriptors covering all possible combinations of field, value pairs.
- Added unary operators: absolute value ( $GrB\_ABS\_T$ ) and bitwise complement of integers ( $GrB\_BNOT\_I$ ).
- (Issues BB-42, BB-62) Added binary operators: Added boolean exclusive-nor (GrB\_LXNOR)

  and bitwise logical operators on integers (GrB\_BOR\_I, GrB\_BAND\_I, GrB\_BXOR\_I, GrB\_BXNOR\_I).
- (Issue BB-11) Added a set of predefined monoids and semirings.
- (Issue BB-57) Updated all examples in the appendix to take advantage of new capabilities and predefined objects.
- (Issue BB-43) Added parent-BFS example.
- (Issue BB-1) Fixed bug in the non-batch betweenness centrality algorithm in Appendix C.4 where source nodes were incorrectly assigned path counts.
- (Issue BB-3) Added compile-time preprocessor defines and runtime method for querying the GraphBLAS API version being used.
- (Issue BB-10) Clarified GrB init() and GrB finalize() errors.
- (Issue BB-16) Clarified behavior of boolean and integer division. Note that GrB\_MINV for integer and boolean types was removed from this version of the spec.
  - (Issue BB-19) Clarified aliasing in user-defined operators.
- (Issue BB-20) Clarified language about behavior of GrB\_free() with predefined objects (implementation defined)
- (Issue BB-55) Clarified that multiplication does not have to distribute over addition in a GraphBLAS semiring.
- (Issue BB-45) Removed unnecessary language about annihilators.
- (Issue BB-61) Removed unnecessary language about implied zeros.
- (Issue BB-60) Added disclaimer against overspecification.
- Fixed miscellaneous typographical errors (such as  $\otimes . \oplus$ ).
- 7450 Changes in 1.2.0:

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• Removed "provisional" clause.

#### 7452 Changes in 1.1.0:

- Removed unnecessary const from nindices, nrows, and ncols parameters of both extract and assign operations.
- Signature of GrB\_UnaryOp\_new changed: order of input parameters changed.
- Signature of GrB\_BinaryOp\_new changed: order of input parameters changed.
- Signature of GrB\_Monoid\_new changed: removal of domain argument which is now inferred from the domains of the binary operator provided.
- Signature of GrB\_Vector\_extractTuples and GrB\_Matrix\_extractTuples to add an in/out argument, n, which indicates the size of the output arrays provided (in terms of number of elements, not number of bytes). Added new execution error, GrB\_INSUFFICIENT\_SPACE which is returned when the capacities of the output arrays are insufficient to hold all of the tuples.
- Changed GrB\_Column\_assign to GrB\_Col\_assign for consistency in non-polymorphic interface.
- Added replace flag (z) notation to Table 4.1.
- Updated the "Mathematical Description" of the assign operation in Table 4.1.
- Added triangle counting example.
- Added subsection headers for accumulate and mask/replace discussions in the Description sections of GraphBLAS operations when the respective text was the "standard" text (i.e., identical in a majority of the operations).
  - Fixed typographical errors.

#### 7473 Changes in 1.0.2:

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- Expanded the definitions of Vector\_build and Matrix\_build to conceptually use intermediate matrices and avoid casting issues in certain implementations.
- Fixed the bug in the GrB\_assign definition. Elements of the output object are no longer being erased outside the assigned area.
- Changes non-polymorphic interface:
  - Renamed GrB\_Row\_extract to GrB\_Col\_extract.
  - Renamed GrB\_Vector\_reduce\_BinaryOp to GrB\_Matrix\_reduce\_BinaryOp.
- 7481 Renamed GrB\_Vector\_reduce\_Monoid to GrB\_Matrix\_reduce\_Monoid.
  - Fixed the bugs with respect to isolated vertices in the Maximal Independent Set example.
- Fixed numerous typographical errors.

### Appendix B

## Non-opaque data format definitions

# Graph-

In this section, the non-opaque matrix formats specified by GrB\_Format and used in matrix import and export methods are defined.

### $^{7490}$ B.1.1 GrB\_CSR\_FORMAT

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The GrB\_CSR\_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse row (CSR) format. indptr is a pointer to an array of GrB\_Index of size nrows+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th row of the matrix. indices is a pointer to an array of number of stored elements (each a GrB\_Index), where each element contains the corresponding element's column index within a row of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each row are not required to be sorted by column index.

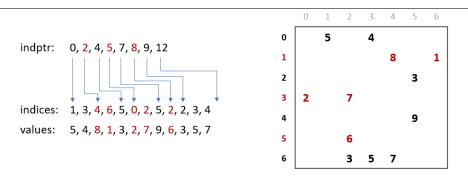


Figure B.1: Data layout for CSR format.

### 7499 B.1.2 GrB\_CSC\_FORMAT

The GrB\_CSC\_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse column (CSC) format. indptr is a pointer to an array of GrB\_Index of size ncols+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th column of the matrix. indices is a pointer to an array of number of stored elements (each a GrB\_Index), where each element contains the corresponding element's row index within a column of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each column are not required to be sorted by row index.

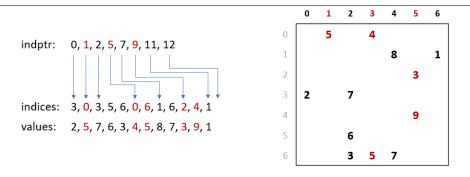


Figure B.2: Data layout for CSC format.

### B.1.3 GrB\_COO\_FORMAT

The GrB\_COO\_FORMAT format indicates that a matrix will be imported or exported using the coordinate list (COO) format. indptr is a pointer to an array of GrB\_Index of size number of stored elements, where each element contains the corresponding element's column index. indices will be a pointer to an array of GrB\_Index of size number of stored elements, where each element contains the corresponding element's row index. values will be a pointer to an array of size number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. Elements are not required to be sorted in any order.

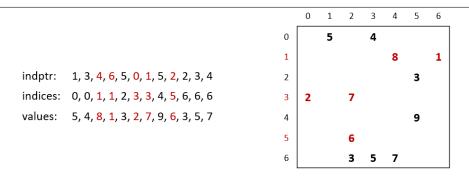


Figure B.3: Data layout for COO format.

 $^{7516}$  Appendix C

 $_{7517}$  Examples

### C.1 Example: Level breadth-first search (BFS) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reacheable from s, then v[i] = 0. (Vector v should be empty on input.)
10
11
    GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s)
13
      GrB_Index n;
14
                                                        // n = \# of rows of A
      GrB\_Matrix\_nrows(\&n,A);
15
16
                                                        // Vector < int32_t > v(n)
17
      GrB\_Vector\_new(v,GrB\_INT32,n);
18
19
      GrB_Vector q;
                                                        // vertices visited in each level
      GrB\_Vector\_new(\&q,GrB\_BOOL,n);
20
                                                        // Vector < bool > q(n)
21
      GrB_Vector_setElement(q,(bool)true,s);
                                                        // q[s] = true, false everywhere else
22
23
       * BFS traversal and label the vertices.
24
25
26
      int32 t d = 0;
                                                        // d = level in BFS traversal
27
      bool succ = false;
                                                        // succ == true when some successor found
28
      do {
29
                                                        // next level (start with 1)
30
        GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL);
                                                              // v[q] = d
31
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
                                                        // q[!v] = q ||.&& A; finds all the ||...| unvisited successors from current q
                 q, A, GrB\_DESC\_RC);
32
33
        GrB_reduce(&succ, GrB_NULL, GrB_LOR_MONOID_BOOL,
34
35
                    q, GrB_NULL);
                                                        // succ = //(q)
      } while (succ);
36
                                                        // if there is no successor in q, we are done.
37
                                                        // q vector no longer needed
38
      GrB_free(&q);
39
40
      return GrB SUCCESS;
41
```

### C.2 Example: Level BFS in GraphBLAS using apply

```
#include <stdlib.h>
   #include <stdio.h>
3 #include <stdint.h>
4 #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reachable from s, then v[i] does not have a stored element.
10
11
     * Vector v should be uninitialized on input.
12
   GrB_Info BFS(GrB_Vector *v, const GrB_Matrix A, GrB_Index s)
13
14
      GrB Index n;
15
                                                        // n = \# of rows of A
16
      GrB\_Matrix\_nrows(\&n,A);
17
18
      GrB_Vector_new(v,GrB_INT32,n);
                                                        // Vector < int32_t > v(n) = 0
19
                                                        // vertices visited in each level
20
      GrB_Vector q;
      GrB\_Vector\_new(&q,GrB\_BOOL,n);
                                                        // Vector < bool > q(n) = false
// q[s] = true, false everywhere else
21
      GrB_Vector_setElement(q,(bool)true,s);
22
23
^{24}
25
      * BFS traversal and label the vertices.
26
                                                        //\ level = depth\ in\ BFS\ traversal
27
      int32\_t level = 0;
28
      GrB_Index nvals;
29
      do {
30
        ++level;
                                                        // next level (start with 1)
        GrB_apply(*v,GrB_NULL,GrB_PLUS_INT32,
31
                   GrB\_SECOND\_INT32, q, level, GrB\_NULL); // v[q] = level
32
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
33
                                                        // q[!v] = q //.&&A; finds all the
34
                q, A, GrB\_DESC\_RC);
35
                                                         // unvisited successors from current q
36
        GrB_Vector_nvals(&nvals, q);
      } while (nvals);
37
                                                        // if there is no successor in q, we are done.
38
39
      GrB_free(&q);
                                                        // q vector no longer needed
40
41
      return GrB_SUCCESS;
42 }
```

### C.3 Example: Parent BFS in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
     * Given a binary n x n adjacency matrix A and a source vertex s, performs a BFS
8
     * traversal of the graph and sets parents[i] to the index of vertex i's parent.
     * The parent of the root vertex, s, will be set to itself (parents[s] == s). If * vertex i is not reachable from s, parents[i] will not contain a stored value.
10
11
12
    GrB\_Info\ BFS(GrB\_Vector\ *parents\ ,\ \textbf{const}\ GrB\_Matrix\ A,\ GrB\_Index\ s\ )
13
14
      GrB Index N;
15
                                                            //N = \# vertices
16
      GrB_Matrix_nrows(&N, A);
17
      GrB_Vector_new(parents, GrB_UINT64, N);
18
                                                            // parents[s] = s
      GrB_Vector_setElement(*parents, s, s);
20
21
      GrB Vector wavefront;
      GrB_Vector_new(&wavefront, GrB_UINT64, N);
22
23
      GrB_Vector_setElement(wavefront, 1UL, s);
                                                           // wavefront[s] = 1
^{24}
25
26
       * BFS traversal and label the vertices.
27
28
      GrB Index nvals;
29
      GrB_Vector_nvals(&nvals, wavefront);
30
31
      while (nvals > 0)
32
33
         // convert all stored values in wavefront to their 0-based index
        GrB_apply(wavefront, GrB_NULL, GrB_NULL, GrB_ROWINDEX_INT64,
34
35
                    wavefront , OUL, GrB_NULL);
36
        // "FIRST" because left-multiplying wavefront rows. Masking out the parent
37
         // list ensures wavefront values do not overwrite parents already stored.
38
        \label{eq:cont_state} GrB\_vxm(\,wavefront\,,\,\,*parents\,,\,\,GrB\_NULL,\,\,GrB\_MIN\_FIRST\_SEMIRING\_UINT64,
39
                  wavefront, A, GrB_DESC_RSC);
40
41
        //\ {\it Don't\ need\ to\ mask\ here\ since\ we\ did\ it\ in\ mxm.\ Merges\ new\ parents\ in}
42
         // current wavefront with existing parents: parents += wavefront
        GrB_apply(*parents, GrB_NULL, GrB_PLUS_UINT64,
44
45
                    GrB_IDENTITY_UINT64, wavefront, GrB_NULL);
46
        GrB_Vector_nvals(&nvals, wavefront);
47
48
49
50
      GrB free(&wavefront);
51
      return GrB_SUCCESS;
52
53
```

### C.4 Example: Betweenness centrality (BC) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
4
   #include <stdbool.h>
   #include "GraphBLAS.h"
7
8
     * Given a boolean n x n adjacency matrix A and a source vertex s,
9
     * compute the BC-metric vector delta, which should be empty on input.
10
    GrB_Info BC(GrB_Vector *delta, GrB_Matrix A, GrB_Index s)
11
12
13
      GrB_Index n;
      GrB\_Matrix\_nrows(\&n,A);
                                                           // n = \# of vertices in graph
14
15
      GrB Vector new(delta, GrB FP32, n);
                                                           // Vector < float > delta(n)
16
17
18
      GrB_Matrix sigma;
                                                           // Matrix < int32\_t > sigma(n,n)
      GrB_Matrix_new(&sigma, GrB_INT32, n, n);
                                                           // sigma [d,k] = \# shortest paths to node k at level d
19
20
21
      GrB_Vector q;
                                                           // Vector<int32_t> q(n) of path counts
22
      GrB_Vector_new(&q, GrB_INT32, n);
                                                           // q[s] = 1
23
      GrB_Vector_setElement(q,1,s);
24
                                                           //\ \ Vector < int 32\_t > p(n) \ \ shortest \ \ path \ \ counts \ \ so \ \ far
25
      GrB_Vector p;
      GrB\_Vector\_dup(\&p, q);
26
27
      GrB\_vxm(\,q\,,p\,,GrB\_NULL,GrB\_PLUS\_TIMES\_SEMIRING\_INT32\,,
28
                                                           // get the first set of out neighbors
29
               q, A, GrB\_DESC\_RC);
30
31
       * BFS phase
32
33
      GrB\_Index d = 0;
                                                           // BFS level number
                                                           // sum == 0 when BFS phase is complete
35
      int32\_t sum = 0;
36
37
         GrB\_assign\left(sigma,GrB\_NULL,GrB\_NULL,q,d,GrB\_ALL,n,GrB\_NULL\right); \qquad // \ sigma\left[d,:\right] = q 
38
         GrB_eWiseAdd(p,GrB_NULL,GrB_NULL,GrB_PLUS_INT32,p,q,GrB_NULL); // accum path counts on this level
39
40
        GrB_vxm(q,p,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
41
                  q, A, GrB\_DESC\_RC);
                                                                                  // q = \# paths to nodes reachable
42
                                                                                       from current level
        GrB reduce(&sum, GrB NULL, GrB PLUS MONOID INT32, q, GrB NULL);
                                                                                  // sum path counts at this level
43
44
        ++d;
45
      } while (sum);
46
47
48
       * BC computation phase
49
        * (t1, t2, t3, t4) are temporary vectors
50
      GrB_Vector t1; GrB_Vector_new(&t1,GrB_FP32,n);
51
       \begin{array}{ll} GrB\_Vector & t2 \ ; & GrB\_Vector\_new(\&t2 \ ,GrB\_FP32 \ ,n \ ) \ ; \end{array} 
52
      GrB_Vector t3; GrB_Vector_new(&t3,GrB_FP32,n);
53
54
      GrB_Vector t4; GrB_Vector_new(&t4,GrB_FP32,n);
55
      for (int i=d-1; i>0; i--)
56
57
         GrB assign(t1,GrB NULL,GrB NULL,1.0f,GrB ALL,n,GrB NULL);
                                                                                      // t1 = 1 + delta
58
        GrB_eWiseAdd(t1,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,t1,*delta,GrB_NULL);
59
        GrB_extract(t2,GrB_NULL,GrB_NULL,sigma,GrB_ALL,n,i,GrB_DESC_T0);
GrB_eWiseMult(t2,GrB_NULL,GrB_NULL,GrB_DIV_FP32,t1,t2,GrB_NULL);
60
                                                                                      // t2 = sigma[i,:]
                                                                                      // t2 = (1 + delta)/sigma[i,:]
61
        GrB_mxv(t3,GrB_NULL,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_FP32,
                                                                                      // add contributions made by
62
```

```
A, t2, GrB\_NULL); \\ GrB\_extract(t4, GrB\_NULL, GrB\_NULL, sigma, GrB\_ALL, n, i-1, GrB\_DESC\_T0); \\ // t4 = sigma[i-1,:] \\ GrB\_eWiseMult(t4, GrB\_NULL, GrB\_NULL, GrB\_TIMES\_FP32, t4, t3, GrB\_NULL); \\ // t4 = sigma[i-1,:]*t3 \\ GrB\_eWiseAdd(*delta, GrB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_NULL, GrB\_PLUS\_FP32, *delta, t4, GrB\_NULL); \\ // accumulate into delta \\ (frB\_PLUS\_FP32, *delta, t4, GrB\_PLUS\_FP32, *delta, t4, GrB\_PLUS
63
64
65
66
67
68
                                                         GrB_free(&sigma);
GrB_free(&q); GrB_free(&p);
69
70
71
                                                            \label{eq:GrB_free} $\operatorname{GrB\_free}(\&t1)$; $\operatorname{GrB\_free}(\&t2)$; $\operatorname{GrB\_free}(\&t3)$; $\operatorname{GrB\_free}(\&t4)$;}
72
73
                                                          return GrB_SUCCESS;
74
```

### C.5 Example: Batched BC in GraphBLAS

```
#include <stdlib.h>
   #include "GraphBLAS.h" // in addition to other required C headers
2
4
    /\!/ Compute partial BC metric for a subset of source vertices, s, in graph A
   GrB Info BC update(GrB Vector *delta, GrB Matrix A, GrB Index *s, GrB Index nsver)
5
6
7
     GrB_Index n;
     GrB_Matrix_nrows(&n, A);
8
                                                            // n = \# of vertices in graph
                                                             // // Vector < float > delta(n)
     GrB_Vector_new(delta,GrB_FP32,n);
9
10
     // index and value arrays needed to build numsp
11
12
     GrB_Index *i_nsver = (GrB_Index*) malloc(sizeof(GrB_Index)*nsver);
13
     int32\_t *ones = (int32\_t*) malloc(sizeof(int32\_t)*nsver);
     for(int i=0; i< nsver; ++i) {
14
15
       i_nsver[i] = i;
       ones [i] = 1;
16
17
18
     // numsp: structure holds the number of shortest paths for each node and starting vertex
19
20
      // discovered so far. Initialized to source vertices: numsp[s[i], i]=1, i=[0, nsver)
21
     GrB_Matrix numsp;
22
     GrB_Matrix_new(&numsp, GrB_INT32, n, nsver);
23
     GrB_Matrix_build(numsp,s,i_nsver,ones,nsver,GrB_PLUS_INT32);
24
     free(i_nsver); free(ones);
25
26
     // frontier: Holds the current frontier where values are path counts.
27
        Initialized to out vertices of each source node in s.
28
     GrB_Matrix frontier;
     GrB Matrix new(&frontier, GrB INT32, n, nsver);
30
     GrB_extract(frontier, numsp, GrB_NULL, A, GrB_ALL, n, s, nsver, GrB_DESC_RCT0);
31
     // sigma: stores frontier information for each level of BFS phase. The memory
32
     // for an entry in sigmas is only allocated within the do-while loop if needed.
33
      // n is an upper bound on diameter.
34
35
     GrB_Matrix *sigmas = (GrB_Matrix*) malloc(sizeof(GrB_Matrix)*n);
36
37
     int32 t d = 0;
                                                             // BFS level number
                                                             // nvals == 0 when BFS phase is complete
     GrB\_Index nvals = 0;
38
39
                          —— The BFS phase (forward sweep) —
40
41
     do {
        // sigmas [d](:,s) = d^{h} level frontier from source vertex s
42
       GrB_Matrix_new(&(sigmas[d]),GrB_BOOL,n,nsver);
43
44
       GrB\_apply(sigmas [d], GrB\_NULL, GrB\_NULL,
45
                  GrB_IDENTITY_BOOL, frontier ,GrB_NULL);
                                                            // sigmas[d](:,:) = (Boolean) frontier
46
       GrB\_eWiseAdd (numsp\,, GrB\_NULL, GrB\_NULL, GrB\_PLUS\_INT32\,,
47
48
                     numsp, frontier, GrB NULL);
                                                             // numsp += frontier (accum path counts)
       49
                                                            //\ f < !numsp > = A \ ' \ +.* \ f \ (update \ frontier)
                A, frontier, GrB_DESC_RCT0);
50
       GrB_Matrix_nvals(&nvals, frontier);
                                                             // number of nodes in frontier at this level
51
52
       d++:
53
     } while (nvals);
54
      // nspinv: the inverse of the number of shortest paths for each node and starting vertex.
55
     GrB_Matrix nspinv;
56
     GrB_Matrix_new(&nspinv,GrB_FP32,n,nsver);
57
     GrB_apply(nspinv,GrB_NULL,GrB_NULL,
58
                GrB_MINV_FP32, numsp ,GrB_NULL);
                                                            // nspinv = 1./numsp
59
60
61
      // bcu: BC updates for each vertex for each starting vertex in s
     GrB_Matrix bcu;
62
```

```
GrB_Matrix_new(&bcu,GrB_FP32,n,nsver);
63
64
      GrB assign (bcu , GrB NULL, GrB NULL,
                  1.0f, GrB_ALL, n, GrB_ALL, nsver, GrB_NULL); // filled with 1 to avoid sparsity issues
65
66
67
      GrB Matrix w;
                                                                 // temporary workspace matrix
68
      GrB_Matrix_new(&w, GrB_FP32, n, nsver);
69
70
                              - Tally phase (backward sweep) -
      for (int i=d-1; i>0; i--) {
71
        GrB\_eWiseMult (w, sigmas \cite{black} i \cite{black} i \cite{black}, GrB\_NULL,
72
73
                       74
         // add contributions by successors and mask with that BFS level's frontier
75
76
        GrB_mxm(w, sigmas[i-1], GrB_NULL, GrB_PLUS_TIMES_SEMIRING_FP32,
        \label{eq:continuous} $$ \prod_{x,w,\omega} (x,y); $$ $// w \leqslant igmas [i-1] > = (A +.* w) $$ GrB_eWiseMult(bcu,GrB_NULL,GrB_PLUS_FP32,GrB_TIMES_FP32, w,numsp.GrB_NULL).
77
78
79
                       w, numsp, GrB_NULL);
                                                                    // bcu += w .* numsp
80
      }
81
      // row reduce bcu and subtract "nsver" from every entry to account
82
83
      // for 1 extra value per bcu row element.
      GrB_reduce(*delta,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,bcu,GrB_NULL);
84
      GrB_apply(*delta,GrB_NULL,GrB_NULL,GrB_MINUS_FP32, *delta,(float)nsver,GrB_NULL);
85
86
87
      // Release resources
88
      for (int i=0; i < d; i++) {
89
        GrB\_free(\&(sigmas[i]));
90
91
      free (sigmas);
92
93
      GrB_free(&frontier);
                                  GrB_free(&numsp);
      GrB_free(&nspinv);
                                  GrB_free(&bcu);
94
                                                          GrB_free(&w);
95
96
      return GrB_SUCCESS;
97
  }
```

### C.6 Example: Maximal independent set (MIS) in GraphBLAS

```
1 #include <stdlib.h>
2 #include <stdio.h>
   #include <stdint.h>
4 #include <stdbool.h>
5 #include "GraphBLAS.h"
      Assign a random number to each element scaled by the inverse of the node's degree.
7
   // This will increase the probability that low degree nodes are selected and larger
   // sets are selected.
9
10
   void setRandom(void *out, const void *in)
11
12
      uint32\_t degree = *(uint32\_t*)in;
      *(float*)out = (0.0001f + random()/(1. + 2.*degree)); // add 1 to prevent divide by zero
13
   }
14
15
16
    * A variant of Luby's randomized algorithm [Luby 1985].
17
18
    * Given a numeric n x n adjacency matrix A of an unweighted and undirected graph (where
19
     * the value true represents an edge), compute a maximal set of independent vertices and
     * return it in a boolean n-vector, 'iset' where set[i] = true \ implies \ vertex \ i is a member
21
22
     * of the set (the iset vector should be uninitialized on input.)
23
24
   GrB_Info MIS(GrB_Vector *iset, const GrB_Matrix A)
25
26
     GrB Index n;
27
      GrB Matrix nrows(&n,A);
                                                      // n = \# of rows of A
28
                                                      // holds random probabilities for each node
29
      GrB Vector prob;
                                                      // holds value of max neighbor probability
30
      GrB_Vector neighbor_max;
31
      GrB_Vector new_members;
                                                      // holds set of new members to iset
                                                      // holds set of new neighbors to new iset mbrs.
      GrB_Vector new_neighbors;
32
      GrB_Vector candidates;
                                                      // candidate members to iset
33
      GrB_Vector_new(&prob, GrB_FP32, n);
35
36
      GrB_Vector_new(&neighbor_max, GrB_FP32, n);
37
      GrB_Vector_new(&new_members, GrB_BOOL, n);
38
      GrB_Vector_new(&new_neighbors,GrB_BOOL,n);
      GrB_Vector_new(&candidates, GrB_BOOL, n);
40
      GrB_Vector_new(iset ,GrB_BOOL, n);
                                                      // Initialize independent set vector, bool
41
42
      GrB_UnaryOp set_random;
      GrB\_UnaryOp\_new(\&set\_random\;, setRandom\;, GrB\_FP32\,, GrB\_UINT32\,)\;;
43
      // compute the degree of each vertex.
45
46
      GrB_Vector degrees;
      GrB\_Vector\_new(\&degrees, GrB\_FP64, n);
47
48
      GrB reduce(degrees, GrB NULL, GrB NULL, GrB PLUS FP64, A, GrB NULL);
49
50
      // Isolated vertices are not candidates: candidates[degrees !=0] = true
      GrB_assign(candidates, degrees, GrB_NULL, true, GrB_ALL, n, GrB_NULL);
51
52
      // add all singletons to iset: iset[degree == 0] = 1
53
54
      GrB_assign(*iset , degrees ,GrB_NULL, true ,GrB_ALL, n ,GrB_DESC_RC) ;
55
56
      // Iterate while there are candidates to check.
57
      GrB_Index nvals;
      GrB_Vector_nvals(&nvals, candidates);
58
59
      while (nvals > 0) {
        // compute a random probability scaled by inverse of degree
60
61
        GrB_apply(prob, candidates, GrB_NULL, set_random, degrees, GrB_DESC_R);
62
```

```
63
        // compute the max probability of all neighbors
64
        GrB mxv(neighbor max, candidates, GrB NULL, GrB MAX SECOND SEMIRING FP32, A, prob, GrB DESC R);
65
66
        //\ select\ vertex\ if\ its\ probability\ is\ larger\ than\ all\ its\ active\ neighbors\,,
        // and apply a "masked no-op" to remove stored falses
67
68
        GrB_eWiseAdd(new_members,GrB_NULL,GrB_NULL,GrB_GT_FP64,prob,neighbor_max,GrB_NULL);
69
        GrB_apply(new_members,new_members,GrB_NULL,GrB_IDENTITY_BOOL,new_members,GrB_DESC_R);
70
71
        // add new members to independent set.
        \label{eq:GrB_eWiseAdd} GrB\_eWiseAdd(*iset,GrB\_NULL,GrB\_NULL,GrB\_LOR,*iset,new\_members,GrB\_NULL);
72
73
74
        // remove new members from set of candidates c = c \mathcal{E} !new
        GrB_eWiseMult(candidates, new_members, GrB_NULL,
75
76
                       GrB_LAND, candidates, candidates, GrB_DESC_RC);
77
        GrB_Vector_nvals(&nvals, candidates);
78
79
        if (nvals == 0) { break; }
                                                         // early exit condition
80
        // Neighbors of new members can also be removed from candidates
81
        GrB_mxv(new_neighbors, candidates, GrB_NULL, GrB_LOR_LAND_SEMIRING_BOOL,
82
83
                 A, new_members, GrB_NULL);
        GrB\_eWiseMult(candidates, new\_neighbors, GrB\_NULL, GrB\_LAND,
84
                        candidates, candidates, GrB_DESC_RC);
85
86
87
        GrB\_Vector\_nvals(\&nvals\;,\; candidates\;)\;;
88
89
      GrB_free(&neighbor_max);
                                                        // free all objects "new'ed"
90
91
      GrB_free(&new_members);
      GrB_free(&new_neighbors);
92
93
      GrB_free(&prob);
      GrB_free(&candidates);
94
95
      GrB_free(&set_random);
96
      GrB_free(&degrees);
97
98
      return GrB_SUCCESS;
99
```

### C.7 Example: Counting triangles in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
 3 #include <stdint.h>
 4 #include <stdbool.h>
   #include "GraphBLAS.h"
 6
 7
     * Given an n x n boolean adjacency matrix, A, of an undirected graph, computes
 8
     * the number of triangles in the graph.
10
    uint64_t triangle_count(GrB_Matrix A)
11
12
      GrB_Index n;
13
14
      GrB_Matrix_nrows(&n, A);
                                                             // n = \# of vertices
15
      // L: NxN, lower-triangular, bool
16
      GrB_Matrix L;
17
18
      GrB_Matrix_new(&L, GrB_BOOL, n, n);
      \label{eq:conditional_grb_null} $\operatorname{GrB\_NULL}, \ \operatorname{GrB\_NULL}, \ \operatorname{GrB\_TRIL}, \ A, \ \operatorname{OUL}, \ \operatorname{GrB\_NULL});$
20
21
      GrB_Matrix C;
22
      GrB\_Matrix\_new(\&C, GrB\_UINT64, n, n);
23
24
      25
26
      uint64 t count;
      \label{eq:GrB_reduce} $$\operatorname{GrB\_NULL}, $\operatorname{GrB\_PLUS\_MONOID\_UINT64}, $\operatorname{C}, $\operatorname{GrB\_NULL})$;}
27
                                                                                        // 1-norm of C
28
29
      GrB_free(&C);
30
      GrB_free(&L);
31
32
      return count;
33 }
```