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Variational Analysis in Optimization and Control: Theoretical and Computational Aspects with Practical Applications Applications

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Boris Mordukhovich

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14. ABSTRACT
The main objectives include developing advanced tools of vari-ational analysis and their applications to theoretical and numer-ical aspects of constrained optimization and optimal control of new classes of controlled dynamical of systems. Furthermore, a major goal is to apply the developed theoretical results and techniques to the modeling and solving of practical problems.

15. SUBJECT TERMS

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Q1. Award Number (Federal Award Identification Number XXXXXX-XX-X-XXXX)
FA9550-16-1-0178

Q3.
Our system shows you have the following report due:
Please confirm the appropriate report below.

- Final Performance

Q4. Principal Investigator
Boris Mordukhovich

Q5. Principal Investigator Email
boris@math.wayne.edu

Q6. Principal Investigator Phone
(734)580-2048

Q7. Project Title
Variational Analysis in Optimization and Control: Theoretical and Computational Aspects with Practical Applications

Q8. Recipient Organization
Wayne State University

Q310. Business Office Email
SPA@wayne.edu

Q9. Report Due Date
07/30/2021

Q10. Report Period Start Date
05/01/2016

Q11. Report Period End Date
04/30/2021

Q297. Current Program Officer
Dr. Fariba Fahroo

Q298. Please list any other Co-Program Officers (if applicable)
N/A

Q395. Please confirm the report type you are submitting is: Final Performance
- Yes, that is correct.

Q12. Is this survey being submitted by someone other than the Principal Investigator?
- No

Q407. How many participants worked on the grant during this period of performance?
This number includes all PIs and each person who worked, and was funded by the project during this reporting period.

You will be asked to provide the following information for: (1) PDs/PIs; and (2) each person who worked, and was funded by the project, during this reporting period. Please note that such reporting does not constitute a formal institutional report of effort on the project, but rather is used by agency program staff to evaluate the progress of the project during a given reporting period.
(Max 20 participants)
10

Q401#1. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Name

	Last Name, First Name
Participant 1	Mordukhovich, Boris
Participant 2	Sarabi, Ebrahin
Participant 3	Nguyen, Dao
Participant 4	Mohammadi, Ashkan
Participant 5	Nguyen, Hang
Participant 6	Can , Tao
Participant 7	Vo, Phat
Participant 8	Nguyen, Trang
Participant 9	Bajaj, Anuj
Participant 10	Do, Hong
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

Q401#2. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Months Worked

	#
Participant 1	10
Participant 2	3
Participant 3	3
Participant 4	3
Participant 5	3
Participant 6	3
Participant 7	2
Participant 8	2
Participant 9	2
Participant 10	2
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

Q401#3. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Describe briefly how this person contributed to the project

Participant 1	leadership and active participation in all aspects of the research
Participant 2	developing the theory of critical multipliers in variational analysis with applications to generalized Newton algorithms of nonsmooth optimization
Participant 3	research on controlled sweeping processes and applications to robotics
Participant 4	developments on second-order variational analysis and applications to extended sequential programming methods in nonsmooth constrained optimization
Participant 5	applications of second-order variational analysis to conic programming and related topics
Participant 6	discrete approximations and optimality conditions for controlled sweeping processes with applications to crowd motion models
Participant 7	development of second-order algorithms to solve subgradient systems
Participant 8	optimal control of sweeping process with applications to traffic equilbria
Participant 9	developments of new algorithms for solving multifacility local problems
Participant 10	critical multipliers in extended nonlinear programming
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

Q401#4. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Project Role

Participant 1	Principal Investigator
Participant 2	Grad Student (Research Assistant)
Participant 3	Grad Student (Research Assistant)
Participant 4	Grad Student (Research Assistant)
Participant 5	Grad Student (Research Assistant)
Participant 6	Grad Student (Research Assistant)
Participant 7	Grad Student (Research Assistant)
Participant 8	Grad Student (Research Assistant)
Participant 9	Grad Student (Research Assistant)
Participant 10	Grad Student (Research Assistant)

Q401#5. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- International Business during Reporting Period. 1) Did the individual collaborate with individuals located in a foreign country? 2) Did this individual travel to a foreign country as part of the collaboration?

Participant 1	International Collaborations?, International Travel?

Q401#6. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Add'l Funding Source(s)

Participant 1	No
Participant 2	No
Participant 3	No
Participant 4	No
Participant 5	No
Participant 6	No
Participant 7	No
Participant 8	No
Participant 9	No
Participant 10	No

Q399#1. International Business - Countries of International Collaborators

Participant 1:	Austria, Germany, Italy
Participant 2:	N/A
Participant 3:	N/A
Participant 4:	N/A
Participant 5:	N/A
Participant 6:	N/A
Participant 7:	N/A
Participant 8:	N/A
Participant 9:	N/A
Participant 10:	N/A
Participant 11:	N/A
Participant 12:	N/A
Participant 13:	N/A
Participant 14:	N/A
Participant 15:	N/A
Participant 16:	N/A
Participant 17:	N/A
Participant 18:	N/A
Participant 19:	N/A
Participant 20:	N/A

Q399#2. International Business - International Travel Related to Grant

	Number of Days in each Country (Country, # Days)	Total International Travel Duration (Days)
Participant 1:	Austria (15 Days), Germany (15 Days), Italy (30 Days)	60
Participant 2:	N/A	N/A
Participant 3:	N/A	N/A
Participant 4:	N/A	N/A
Participant 5:	N/A	N/A
Participant 6:	N/A	N/A
Participant 7:	N/A	N/A
Participant 8:	N/A	N/A
Participant 9:	N/A	N/A
Participant 10:	N/A	N/A
Participant 11:	N/A	N/A
Participant 12:	N/A	N/A
Participant 13:	N/A	N/A
Participant 14:	N/A	N/A
Participant 15:	N/A	N/A
Participant 16:	N/A	N/A
Participant 17:	N/A	N/A
Participant 18:	N/A	N/A
Participant 19:	N/A	N/A
Participant 20:	N/A	N/A

Q403. Please confirm whether any of your participants had any international business associated with this grant during this reporting period.
(If you input any information on the above International Business question, you should select yes.)

- Yes, at least one of the participants had international business

Q353. Archival Publications (published) during reporting period: State "Nothing to Report" if nothing to report

Mordukhovich, B.S.
Variational Analysis and Applications, Springer Monographs in Mathematics, XIX+622 pp., Springer, Cham, Switzerland 2018.
Hosseini, S.; Mordukhovich, B.S.; Uschmajew, A. (eds.)
``Nonsmooth Optimization and Applications,'' Birkhauser, Cham, Switzerland, 2019, 127 pp.
Colombo, G.; Henrion, R.; Hoang, N.D.; Mordukhovich, B.S.
Optimal control of the sweeping process over polyhedral
controlled sets, J. Diff. Eqs. 260 (2016), 3397-3447.
Mordukhovich, B.S.; Sarabi, M.E.
Second-order analysis of piecewise linear functions with
applications to optimization and stability, J. Optim. Theory Appl. 171 (2016), 504--526.
Cao, T.H.; Mordukhovich, B.S., Optimal control of a perturbed sweeping process via discrete approximations, Disc. Cont. Dyn. Syst. 21 (2016), 3331-3358.
Cao, T.H.; Mordukhovich, B.S.
Optimality conditions for sweeping process with applications
to the crowd motion model, Disc. Cont. Dyn. Syst. 22 (2017), 267-306.
Gfrerer, H.; Mordukhovich, B.S.
Robinson stability of parametric constraint systems via variational analysis, SIAM J. Optim. 27 (2017), 438-465.

Mordukhovich, B.S.; Sarabi, M. E.
Critical multipliers in variational systems via second-order differentiation, Math. Program. 169 (2018), 605-648.
Gupta, R.; Jafari, F.; Kipka, R. J.; Mordukhovich, B.S.
Linear openness and feedback stabilization of nonlinear
control systems, Disc. Cont. Dynam. Syst., Ser. S., 11 (2018), 1103-1119.
Hang, N.T.V.; Mordukhovich, B.S.; Sarabi, M.E.
Second-order variational analysis in second-order cone programming, Math. Program. 180 (2020), 75-116.
Cao, T.H.; Mordukhovich, B.S.
Optimal control of a nonconvex perturbed sweeping process, J. Diff. Eqs. 266 (2019), 1003-1050.
Mordukhovich, B.S.
Variational analysis and optimization of sweeping processes
with controlled moving sets, Rev. Invest. 39 (2018), 281-300.
Gfrerer, H.; Mordukhovich, B.S.
Second-order variational analysis of parametric constraint and variational systems, SIAM J. Optim. 29 (2019), 423-453.
Cao, T.H.; Mordukhovich, B.S.
Applications of optimal control of a nonconvex sweeping process to optimization of the planar crowd motion model, Disc. Cont. Dyn. Syst.,Ser. B, 24 (2019), 4191-4216.
Hoang, N.D.; Mordukhovich, B.S.
Extended Euler-Lagrange and Hamiltonian formalisms in optimal control of sweeping processes with controlled moving sets, J. Optim. Theory Appl. 180 (2019), 256-289.
Mordukhovich, B.S.; Sarabi, M.E.
Criticality of Lagrange multipliers in variational systems, SIAM J. Optim. 29 (2019), 1524–1557.
Mordukhovich, B.S.; Soubeyran, A.
Variational analysis and variational rationality in behavioral sciences: local
traps, in: Variational Analysis and Set Optimization. Development and Applications in Decision Making (A. Khan et aleds.), pp. 1-25, Springer, Berlin, 2019
Colombo, G.; Mordukhovich, B.S.; Nguyen, D.
Optimization of a perturbed sweeping process by discontinuous controls, SIAM J. Control Optim. 58 (2020), 2678–2709.
Benko, M.; Gfrerer, H.; Mordukhovich, B.S.
Characterizations of tilt-stable minimizers in second-order cone programming, SIAM J. Optim. 29 (2019), 3100-3130.
Colombo, G.; Mordukhovich, B.S.; Nguyen, D.
Optimal control of sweeping processes in robotics and traffic flow models, J. Optim. Theory Appl. 182 (2019), 439-472.
Do, H.; Mordukhovich, B.S.; Sarabi, M.E.Criticality of Lagrange multipliers in extended nonlinear optimization, Optimization 70 (2021), 511-544.
Mordukhovich, B.S.
Avoiding critical multipliers and slow convergence of primal-dual methods for fully stable minimizers, J. Convex Nonlinear Anal. 20 (2019), 1475-1496.
Dempe S.; Mordukhovich, B.S.; Zemkoho, A.B.
Two-level value function approach to nonsmooth optimistic
and pessimistic programs, Optimization 68 (2019), 433-455.
Mordukhovich, B.S.
Optimal control of Lipschitzian and discontinuous differential inclusions with a variety of applications, Proc. Inst. Math. Mech. 45 (2019), 52-74.
Mohammadi, A.; Mordukhovich. B.S.; Sarabi, M.E.
Superlinear convergence of the sequential quadratic method in constrained optimization, J. Optim. Theory Appl. 186 (2020), 731-758.
Mohammadi, A.; Mordukhovich, B.S.; Sarabi, M.E.
Variational analysis of composite models with applications to continuous optimization, Math. Oper. Res. (2020); DOI: 10.1287/moor.2020.1074.
Bajaj, A.; Mordukhovich, B.S.; Nam, N.M.; Tran, T.
Solving a continuous multifacility location problem by DC algorithms, Optim. Methods Soft. (2020); DOI: 10.1080/10556788.2020.1771335.
Mohammadi, A.; Mordukhovich. B.S.; Sarabi, M.E.
Parabolic regularity in geometric variational analysis, Trans. Amer. Math. Soc. 374 (2021), 1711–1763.
Cao, T.H.; Colombo, G.; Mordukhovich, B.S.; Nguyen, D.
Optimization and discrete approximation of sweeping processes with controlled
moving sets and perturbations, J. Diff. Eqs. 274 (2021), 461-509.
Mordukhovich, B.S.; Sarabi, M.E.
Generalized Newton algorithms for tilt-stable minimizers in nonsmooth
optimization, SIAM J. Optim. 31 (2021), 1184–1214.
Mordukhovich, B.S.; Nguyen, D.
Discrete approximations and optimal control of nonsmooth perturbed sweeping

processes, J. Convex Anal. 28 (2021), 655-688.
Mohammadi, A.; Mordukhovich, B.S.
Variational analysis in normed spaces with applications to constrained optimization, SIAM J. Optim. 31 (2021), 569-603.
Cao, T.H.; Colombo, G.: Mordukhovich, B.S.; Nguyen, D.
Optimization of fully controlled sweeping processes, J. Diff. Eqs. 295 (2021), 138—186.

Q354. New discoveries, inventions, or patent disclosures to report for this period? This question is required.

- No

Q355. Changes in research objectives (if any):

no changes

Q356. Change in AFOSR Program Officer, if any:

For some period, Dr. Jean-Luc Cambier was the Program Officer for this project.

Q357. Extensions granted or milestones slipped, if any:

It was the NCE of the grant for one year due to the pandemic.

Q412. Abstract
Please submit your report abstract below.

The project research during this annual period was mainly concentrated on the following topics:
---Investigating new classes of optimal control problems for sweeping processes governed by discontinuous differential inclusions with pointwise/hard control and state constraints. Developing numerically implemented approximation procedures for them and deriving necessary optimality conditions that involve, in particular, the maximization of extended Hamiltonians of the novel type.
---Applying the obtained theoretical and computational results for controlled sweeping processes to practical models of robotics, planar crowd motions, and traffic equilibria.
---Developing a criticality theory in general problems of constrained optimization with particular attention to problems of conic programming by using machinery of second-order variational analysis. Deriving in this way complete characterizations of critical and noncritical multipliers for such problems expressed entirely in terms of the problem data.
---Establishing efficient characterizations of fully stable local minimizers in problems of conic programming and proving that slow convergence of Newtonian algorithms is excluded by full stability of local minimizers in such problems.
---Developing a variational approach to local asymptotic and exponential stabilizability of nonlinear systems and deriving in this way new conditions for feedback control stabilization of such systems.
---Developing new applications of variational analysis to some fundamental problems of behavioral science via the variational rationality approach.
---Investigating major stability issues in both optimistic and pessimistic models of bilevel programming by implementing a novel two-level value function approach of variational analysis.

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Q394. Appendix Documents (Upload any additional documentation to support Appendix A and B as specified in AFOSR Instruction 61-7)
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VARIATIONAL ANALYSIS IN OPTIMIZATION AND CONTROL

AFOSR grant # 15RT0462
May 1, 2016–April 30, 2021

PI: BORIS MORDUKHOVICH
Distinguished Professor, Wayne State University

Main Objectives and Technical Approaches

The main objectives include developing advanced tools of variational analysis and their applications to theoretical and numerical aspects of constrained optimization and optimal control of new classes of controlled dynamical of systems. Furthermore, a major goal is to apply the developed theoretical results and techniques to the modeling and solving of practical problems.

The main technical approaches consist of developing and applying powerful tools and techniques of second-order variational analysis and generalized differentiation to complex constrained optimization problems, as well as the method of discrete approximations to the study and solving of optimal control problems with discontinuous dynamics.

More specific topics of the research and applications proposed in the project are listed in the Abstract of this report. All the proposed goals have been achieved in the course of the fulfillment of this project.

Summary of Advances

We developed powerful tools of second-order variational analysis and generalized differentiation important for their own sake

and needed for our applications to theoretical and numerical aspects of optimization, control, and practical modeling aimed at the proposed research. Our main achievements include careful investigations with complete characterizations of well-recognized tilt and full stability concepts for optimal solutions to polyhedral and nonpolyhedral problems of conic programming without restrictive nondegeneracy assumptions. The results obtained in this vein are largely applied to all other topics of this project.

Based on the constructions and techniques of second-order variational analysis, we developed a rather comprehensive criticality theory for multipliers in general problems of conic programming with deriving verifiable characterizations of critical and noncritical multipliers that entirely expressed via the given data. Besides being important for their own rate, the obtained

characterizations of critical multipliers and the above stability concepts for local minimizers in conic programming are crucial to find efficient conditions for excluding slow convergence of Newtonian and related primal-dual algorithms in polyhedral and nonpolyhedral problems of constrained optimization.

The new DC-type algorithms were developed for nonsmooth multifacility location problems highly important for applications. We applied these algorithms to solving some practical models arising in the airline industry, transportation, clustering, machine learning, etc.

A substantial part of this research was devoted to investigating novel classes of optimal control problems governed by dissipative differential inclusions known as the sweeping/Moreau processes. A systematic study and applications of controlled

processes of this type has been started rather recently by a group of researchers including the PI. Optimal control problems for such discontinuous dynamical systems with intrinsic pointwise/hard state constraints were realized to be extremely challenging. During the fulfillment of this project we considered, being largely motivated by demanding applications, some systems of this type with hard constraints on discontinuous controls. The major results obtained in this direction include justifying appropriate convergence of discrete-time optimal solutions and deriving new optimality conditions for controlled sweeping processes with both discrete and continuous time.

We developed efficient applications of the necessary optimal conditions and numerical procedures for controlled sweeping processes to some practical models formulated in this form. They include, in particular, systems with hysteresis, models in robotics, traffic equilibria, and planar crowd motions.

Novel variational approaches and obtained results of variational analysis were applied to feedback control stabilization (in local asymptotic and exponential senses) of nonlinear ODE systems. In this way, we were able to derive both necessary and sufficient conditions for feedback stabilization and solved, in particular, some long-standing problems in the area.

Another important achievement of this project concerns applications of variational analysis and the variational rationality approach to some models coming from behavioral sciences. The developed variational approach allowed us to use variational techniques and results in studying the so-called stationary traps that are significant to design effective algorithms to determine optimal strategies in such models.

Finally, we developed a new approach to the study both optimistic and pessimistic of bilevel programming, the major class

of problems in hierarchical optimization. Comprehensive stability results and optimality conditions were achieved by the implementing of the proposed two-level value function approach and the developed machinery of second-order variational analysis.