AFRL-AFOSR-VA-TR-2021-0066

Analysis and Design of Large-Scale Multi-Agent Systems via Continuum Spatial Approximations

Sonia Martinez Diaz
UNIVERSITY OF CALIFORNIA, SAN DIEGO
9500 GILMAN DR
LA JOLLA, CA, 92093
USA

07/06/2021
Final Technical Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
06-07-2021	Final	15 Mar 2018 - 14 Mar 2021

4. TITLE AND SUBTITLE
Analysis and Design of Large-Scale Multi-Agent Systems via Continuum Spatial Approximations

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-18-1-0158

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
Sonia Martinez Diaz

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UNIVERSITY OF CALIFORNIA, SAN DIEGO
9500 GILMAN DR
LA JOLLA, CA 92093
USA

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF Office of Scientific Research
875 N. Randolph St. Room 3112
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)
AFRL/AFOSR RTA2

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-AFOSR-VA-TR-2021-0066

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
This research effort has been led by Dr. Martinez at UC San Diego. The work has had particu-lar impact on the development of a science of autonomy, by focusing on the analysis and design of algorithms for the coordination of very large-scale (spatial) networks. The development of ef-ficient algorithms for large-scale systems remains to be challenging and makes the operation of large networked systems often intractable. Two of the main approaches considered in the liter-ature to deal with large-scale systems have been: a) the direct adoption of algorithms originally developed for small scale multi-agent systems to large ensembles, or b) the discretization of al-gorithms developed for large scale systems for implementation of small-scale groups. Neither of these translations is optimal, as by missing the relationship between the two we may obtain algorithms that are i) unnecessarily conservative, ii) do not capture or exploit large-scale effects (single agents are negligible) and their impact on performance, and iii) do not capture the under-lying geometric properties (or structure) of interest that characterize large dynamic systems.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			FREDERICK LEVE
U	U	U	UU	6	**19b. TELEPHONE NUMBER** *(Include area code)* 696-9730

Standard Form 298 (Rev.8/98)
Prescribed by ANSI Std. Z39.18

Analysis and design of large-scale multi-agent systems via continuum spatial approximations

AFOSR - FA9550-18-1-0158

Start date: 03/15/19

Progress report for period 03/15/20 - 03/15/21

Sonia Martinez (UC San Diego)

AFOSR Program Officer: Dr. Frederick Leve, frederick.leve@us.af.mil.

Point of contact: Dr. Sonia Martínez, (858) 822 4243, soniamd@eng.ucsd.edu

Project summary and main objectives

This research effort has been led by Dr. Martinez at UC San Diego. The work has had particular impact on the development of a science of autonomy, by focusing on the analysis and design of algorithms for the coordination of very large-scale (spatial) networks. The development of efficient algorithms for large-scale systems remains to be challenging and makes the operation of large networked systems often intractable. Two of the main approaches considered in the literature to deal with large-scale systems have been: a) the direct adoption of algorithms originally developed for small scale multi-agent systems to large ensembles, or b) the discretization of algorithms developed for large scale systems for implementation of small-scale groups. Neither of these translations is optimal, as by missing the relationship between the two we may obtain algorithms that are i) unnecessarily conservative, ii) do not capture or exploit large-scale effects (single agents are negligible) and their impact on performance, and iii) do not capture the underlying geometric properties (or structure) of interest that characterize large dynamic systems. On the other hand, algorithms developed as in b) usually i) do not account for small network constraints given by various types of multi-agent distributed interactions or other limitations such as coordinate-free algorithms or collision constraints. Because of this, we have sought to study multi-scale algorithms that are both probably correct at each scale ($N \approx \infty$ and N small) respectively, and can be translated correctly from each other. A particular result of interest has been a novel understanding of the relationship between the classical optimal transport problems and coverage control algorithms developed for multi-agent systems. By doing this, we have been able to interpret how different coverage control metrics for small-scale groups translate into optimal transport objectives for large-scale swarms. Counter to intuition, the right coordination problem which translates correctly into a meaningful large-scale version is given by the "equitable" coverage control problem formulation.

Recognitions received by the PIs during the duration of this award

1. Elected Program director of the SIAM Activity Group on Control and Systems Theory (and program chair and conference chair of the SIAM CT21),
2. Named Editor in Chief of a new CSS archival publication the *IEEE Open Journal on Control Systems*, 2021,
3. Plenary speaker at the 2020 American Control Conference,
4. Named Jacobs Faculy Scholar, July 2020, a 5 year chair position given by the Jacobs School at UC San Diego,
5. 2018 IEEE Fellow.

Personnel supported by this project and collaborations

During the duration of this project, the project has provided support for

- Scott Brown, PhD student (new student)

- Shenyu Liu, postdoc (new student)
- Parth Paritosh, PhD student
- Dimitris Boskos, postdoc
- Vishaal Krishnan, PhD student
- Aamodh Suresh, PhD student
- Sonia Martinez, professor

The PIs, and the students, have interacted approximately weekly. After COVID, interactions were reduced to virtual ones. In particular, Vishaal Krishan wrote his dissertation on fully on the topics of the project. In addition, we have performed a collaboration on the global controllability tests for hybrid systems [1], with Profs. Barbero, Martin de Diego, and Munoz in Spain, Prof. Tartakovsky at Stanford University, as well as Prof. Cortes at UCSD.

Summary of research activities and findings during the award period

Some details about the research problems we have worked on during this project are the following:

1. In [2, 3], we have looked into the coordinate-free deployment of very large robotic swarms. We have studied the design distributed control laws for the deployment of multi-agent swarms in 1D and 2D spatial domains. Since individual agents in a swarm are not themselves of interest and we are concerned only with a macroscopic objective, the network of agents in the swarm is viewed as a discrete approximation of a continuous medium and we design control laws to shape the density distribution of this continuous manifold. The key feature of this work is that the agents in the swarm do not have access to position information. Each individual agent is capable of measuring the current local density of agents and can communicate with its spatial neighbors. The network of agents implement a Laplacian-based distributed algorithm, which we call pseudo-localization, to localize themselves in a new coordinate frame, and a distributed control law to converge to the desired spatial density distribution. In other words, agents can interchange local density measurements in order to learn how they are placed with respect the boundary of the swarm. This is akin to learning a âĂŸdensity-basedâĂŹ range measurement with respect to the boundary of the swarm. Under certain geometric conditions, we provide sufficient conditions on the algorithm to correspond to the distributed computation of a diffeomorphism or density-based coordinate transformation.

2. The characterization of most critical nodes of a network with respect to a given objective has gathered significant attention, with several centrality measures being considered in the literature. With respect to dynamic consensus, an important metric is the second smallest eigenvalue of the Laplacian matrix, which governs the convergence rate of the dynamics. For small networks, critical nodes correspond to the smallest values of the Fieldler eigenvector, and it is not apparent how those nodes are "central" in a geometry way. When going to the large scale domain, and for Random Geometric Graphs, the analogous situation is to consider the second smallest eigenvalue of a (appropriately weighted) Laplace operator. In

this context, we design novel in [4, 5] gradient flows to converge to such critical points, and derive a characterization of such nodes in terms of the nodal set of the second smallest eigenvalue. This corresponds to a geometric center, effectively relating the domain where the network is deployed with the dynamics the nodes implement with the critical location of the most important nodes.

3. In [6], we investigate the design of a scalable, distributed iterative algorithms for large-scale optimal transport of collectives of autonomous agents. Algorithms for multi-agent transport are available, but not a way of calculating distributed solutions for these algorithms. We formulate the problem as one of steering the collective towards a target probability measure while minimizing the total cost of transport, with the additional constraint of distributed implementation imposed by a range-limited network topology. Working within the framework of optimal transport theory, we have proposed a solution as an iterative transport based on a saddle point dynamics (but of PDEs). At each stage of the transport, the agents implement an online, distributed primal-dual algorithm to obtain local estimates of the Kantorovich potential for optimal transport from the current distribution of the collective to the target distribution. Using these estimates as their local objective functions, the agents can then implement the transport by a proximal point algorithm. This two-step process is carried out recursively by the collective to converge asymptotically to the target distribution. At the moment, we are working on the rigorous analysis of the behavior of the algorithm via a candidate system of feedback interconnected PDEs for the continuous time and N infinity limit, and establish the asymptotic stability of this system of PDEs. This work is under revision in *SICON* and a preliminary journal version can be found in arXiv.

4. As a culmination of this effort, we have aimed to establish a unifying framework for the analysis and design of multi-agent systems, with two main goals in mind: (i) the formalization of meaningful coordination objectives for very large groups of multi-agent systems, which can account for their particular features, (ii) the systematic design of implementable, and provable correct algorithms for multi-agent systems that can show scale consistency between the large ("macroscopic") and the small ("microscopic") scales. In particular, we have sought to get a better understanding of the behavior of the widely studied coverage control algorithms (based on Voronoi partitions and the Lloyd algorithm) as the number of agents goes to infinity. To do address (i) and (ii), we define a macroscopic objective for a very large swarm as a functional on a space of probability distributions, which represent (infinite-sized) multi-agent system configurations. Based on the theory of gradient flows for functionals in this space, we define a coordination algorithm as a proximal-gradient algorithm for a given functional. We then obtain implementable, agent-level algorithms via the discretization of the functional, which leads to a new class of âĂIJvariationalâĂİ (gradient-type) of algorithms. It can be seen how this class of algorithms subsumes previously defined coverage control algorithms based on distortion metrics and equitable space partitions. This allows for the connection between macroscopic and microscopic multi-agent coverage control algorithms, and presents a unified theory for these systems. The setting can also open the way for the design of new consistent algorithms at multiple scales. A preliminary version of these results is available in the PhD thesis [7], and has been widely presented in multiple works and conferences. During the final year of this project, we have expanded and formalized this idea in the manuscript [8], which is now under revision in *Automatica*.

In addition to the above, the project has served to support synergistic activities that are of interest for a science of autonomy, which have been i) the dynamic quantification of Wasserstein ambiguity sets via data-driven methods [9,10], ii) the use of concentration-of-measure results (also based on the Wasserstein metric). Distributed estimation algorithms for multi-agent systems [11] On a separated note, we have designed novel iii) human-in-the loop multi-agent control algorithms [12, 13].

Dissemination of the results

The results of this project have been widely disseminated and support from AFOSR has been acknowledge in doing so. Presentations include personal invited seminars (at Yale University, UC Riverside, UC Berkeley, Georgia Tech, the Institute of Mathematics in Madrid, Spain, U of Texas San Antonio, and at MAE at Princeton University), as well as workshops (two CDC 2019 workshops on "Resiliency and controllability of large-scale systems: a network theoretic approach", "Learning, Decision and Control over Networks"), and conferences (2020 Plenary Talk at the American Control Conference), and regular conference presentations.

Conclusions

While much work remains to be done on the analysis and design of large-scale networks, the research performed under this project has allowed us to revisit tools, establish connections, and unveil some of the impact that large scales impose on multi-agent systems via underlying geometric properties. There are several avenues for future work. One of the main issues that remains unaddressed is the derivation of dynamic models for large swarms subject to nontrivial dynamics, different communication delays, and performing collision avoidance tasks. Given the current trend on data-driven methods, a potential approach is to actively learn such models directly from data by exciting agent behaviors. In particular, it should be interesting to understand how a non-zero mass volume of agent excitatory motions can be used to identify such systems, and how can this be done from separated experiments.

References

[1] M. Barbero, J. Cortés, D. Martín de Diego, and M. C. Muñoz-Lecanda. Global controllability tests for geometric hybrid control systems. *Nonlinear Analysis: Hybrid Systems*, 2020. To appear.

[2] V. Krishnan and S. Martínez. Distributed control for spatial self-organization of multi-agent swarms. *SIAM Journal on Control and Optimization*, 56(5):3642–3667, 2018.

[3] V. Krishnan and S. Martínez. Self-organization in multi-agent swarms via distributed computation of diffeomorphisms. In *Mathematical Theory of Networks and Systems*, Minneapolis, MN, USA, July 2016.

[4] V. Krishnan and S. Martínez. Identification of critical node clusters for consensus in large-scale spatial networks. In *IFAC World Congress*, pages 14721–14726, Tolouse, France, July 2017.

[5] V. Krishnan and S. Martínez. Identification of critical nodes in large-scale spatial networks. *IEEE Transactions on Control of Network Systems*, 6(2):842–851, 2019.

[6] V. Krishnan and S. Martínez. Distributed optimal transport for the deployment of swarms. In *IEEE Int. Conf. on Decision and Control*, pages 4583–4588, Miami Beach, FL, USA, December 2018.

[7] V. Krishnan. *Large-scale, multi-agent transport: theory, algorithm, and analysis*. PhD thesis, UC San Diego, 2019.

[8] V. Krishnan and S. Martínez. A multiscale analysis of multi-agent coverage control algorithms. *arXiv preprint arXiv:2102.11411*, 2021.

[9] D. Boskos, J. Cortés, and S. Martínez. Data-driven ambiguity sets with probabilistic guarantees for dynamic processes. *IEEE Transactions on Automatic Control*, 2020. To appear.

[10] D. Boskos, J. Cortés, and S. Martínez. Tackling distributional uncertainty for time-varying linear dynamics with high confidence and data-driven ambiguity sets. 2020. Preprint.

[11] P. Paritosh, N. Atanasov, and S. Martínez. Hypothesis assignment and partial likelihood averaging for cooperative estimation. In *IEEE Int. Conf. on Decision and Control*, Nice, France, December 2019. To appear.

[12] A. Suresh and S. Martínez. Human-swarm interactions for formation control using interpreters. *International Journal of Control, Automation and Systems*, 2020. DOI: 1007/s12555-019-0497-3.

[13] A. Suresh and S. Martínez. Gesture-based human-swarm interactions for formation control using interpreters. In *IFAC Conf. on Cyber Physical and Human Systems*, volume 51, pages 83–88, Miami, FL, USA, December 2018.