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Correction for Range Restriction: Lessons from 20 Scenarios

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14. ABSTRACT
Data are often available only for a preselected range-restricted sample in many applied settings. This creates the potential for drawing incorrect inferences and making poor decisions. This is because most inferences and decisions concern the population from which the sample was drawn. Despite these problems, researchers must try to determine statistical values as if the sample were not range-restricted. Although methods for correcting the effects of range restriction have been available for more than a century, often they are not applied or applied incorrectly. Technical psychometric discussions of range restriction have been insufficient in improving the practices of researchers. As an alternative, realistic scenarios are presented to illustrate and explain the consequences of (1) failing to correct, (2) using the wrong correction formula. (3) correcting when information about correlations, previous selection variables is unavailable, (4) using an inappropriate unrestricted sample, (5) incorrectly computing the confidence interval for the corrected correlation and (6) interpretation of results of some common statistical methods in applied psychology. Although there are situations under which correction has little effect, in most instances it provides better estimates of the relations among variables and improves theoretical understanding and interpretation of real-world applications.

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1.0 INTRODUCTION

Frequently correlational studies are conducted with samples of those already selected for training or with job incumbents. Although we want the value of our estimated correlations to equal the population parameters, in these studies the values of the correlations are biased and do not necessarily equal the parameter values. The use of selected samples can result in substantial bias in the apparent magnitude and sign of the correlations (Ree, Carretta, Earles, & Albert, 1994; Thorndike, 1949). All of these biases are the consequences of the methods of sample selection. Most selection methods reduce variability to create differences between the standard deviations of the unrestricted normative or applicant populations and the standard deviations of prior-selected samples. In certain instances, the selection method can increase the variability of the sample compared to the unrestricted population. This is called range enhancement. The bias created by using selected samples can cause the estimated correlations to be either higher, range enhancement, or lower, range restriction, than if the correlations were computed in an unselected unrestricted group (Johnson, Deary, & Bouchard, 2018, Levin, 1972). Methods to correct correlations for range restriction have been available for more than a century (Aitken, 1934; Lawley, 1943; Le, Oh, Schmidt, & Wooldridge, 2016; Pearson, 1903; Thorndike, 1949). Technical psychometric presentations of range restriction found in the literature have been inadequate in improving statistical practices. As an alternative, scenarios based on realistic situations faced by researchers are presented. Each scenario is aimed at teaching a lesson about the appropriate use of range-restriction corrections by pointing out common errors and providing appropriate actions as well as how the application of the corrections would change the results and interpretation.

The best-known methods of correcting for range restriction are the Pearson-Thorndike (Pearson, 1903; Thorndike, 1949) Cases 1, 2, and 3. Cases 1 and 2 are appropriate for direct selection, Case 3 is appropriate for indirect selection. Direct selection occurs when decisions for hiring, training, or academic admission are made on the one variable of interest. Indirect range restriction occurs when measures are administered, but not used for selection, and applicants are directly selected on scores from other measures or when antecedent variables are not recognized. Multivariate correction (Aitken, 1934; Lawley, 1943) is appropriate when more than one variable is used for selection. The multivariate procedure corrects simultaneously for direct and indirect selection. Bryant and Gokhal (1972) have provided a method to correct correlations if the selection variables are unknown. Additionally, the recent development of Case IV and Case V is presented. Little attention is paid to Case IV due to a potentially unmeetable mediation assumption and work on Case IV has all but stopped. See Table 1 for appropriate and inappropriate uses of the correction formulas and the Appendix for the formulas and how to use them. The Appendix shows the equations for Cases 1, 2, and 3 that can be computed by hand, programmed in MathCad©, Excel©, or the programming language R (Dahkle & Wiernik, 2018, 2019b). The programming language R also provides programs to compute Cases 1, 2, 3, IV, V, and the multivariate method.

Table 1. ***Procedures for Range Restriction Correction.***

Procedure	References	Intended Use	Appropriate Situations	Inappropriate Situations
Case 1	Pearson, (1903) Thorndike (1949)	Direct truncation due to selection on one variable	Used to correct the correlation between 2 variables, a and *b*, when range restriction occurs on variable *b*, the observed correlation between *a* and *b* is known, and the standard deviation of *b* is known in the observed sample and population	Indirect selection occurred; multivariate selection occurred, non-linear relationships
Case 2	Pearson, (1903) Thorndike (1949)	Direct truncation due to selection on one variable	Estimates the correlation between 2 variables of interest, *a* and *y,* when the variances for variable *a* are known in the restricted sample and population, but the correlation between *a* and *y* is available only in the restricted sample that has been directly selected on *x*	Indirect selection occurred; multivariate selection, non-linear relationships
Case 3	Pearson, (1903) Thorndike (1949)	Indirect restriction on variable *y* is produced by direct restriction on	Assumes direct top-down selection has been based entirely on *a* and has not been affected by any other information	Direct selection, violation of assumptions, non-linear relationships

		a third known variable (x)	and r_{xz}, r_{ya}, r_{xy}, and Sx are known	
Case IV	Hunter & Schmidt (2004) Le et al. (2016) Schmidt, Oh, & Le (2006)	Indirect selection	Used as an alternative to Case 3 (relaxes conditions (a) and (b) required for Case 3). Work on Case IV has largely stopped in favor of Case V due to Case IV mediation requirement	Multivariate selection, non-linear relationships
Case V	Dahlke, & Wiernik (2018) Le et al. (2016)	Indirect selection	Used for correction for indirect range restriction between variables X and Y without knowledge about selection on a third variable, Z.	Multivariate selection, non-linear relationships, when correlations are not substantially different from zero. In that situation use Case 3.
Multivariate	Aitken (1934) Lawley (1943)	Selection using multiple variables	Selection on two or more variables; correction for both direct and indirect selection; know the inter-correlations of the independent variables in both the restricted sample and unrestricted population	Insufficient information available, non-linear relationships

Note: All the above procedures can be computed in the R programming language

1.1 Importance of Correcting for Range Restriction or Range Enhancement

Recognizing and correcting for range restriction or range enhancement is especially important if the goal is to estimate the relationships between scores or between constructs. Often only selected samples, such as accepted students, hired applicants or job incumbents are available. Correlations computed in selected samples will be biased. Bryant and Gokhale (1972) have observed that "…to infer beyond the sample a correction for restriction in range is necessary" (p. 305). The same is true of range enhancement (Levin, 1972; Zimmerman & Williams, 2000). There is also the issue of cumulative knowledge about relationships between scores and between constructs. Lack of cumulative knowledge can lead to unnecessary confusion about well-understood relationships. Without applying the correction for range restriction, the information in studies does not contribute to cumulative knowledge. In both primary meta-analyses and secondary meta-analyses, proper methods must be employed including corrections for range restriction.

Correction for range restriction should be employed in meta-analyses even if the magnitude of the correction is small. The weighted mean of the corrected correlations provides a better estimate of the true correlation between scores or between constructs. Additionally, corrected correlations contribute not only to the true correlation but also to computing the variance around the mean corrected correlation and the credibility interval. The credibility interval can be computed on correlations only corrected for sampling error but for best estimates, these correlations should also be corrected for range restriction.

The consequence of failing to correct for range restriction is biased correlations that will not represent the normative or applicant populations. Failing to correct for range restriction can lead to inappropriate conclusions (i.e., over or underestimating the relations between scores) and actions (Alexander, Bennett, Alliger, & Carson, 1986; Hunter, Schmidt, & Le, 2006; Linn, 1968; Ree et al., 1994; Sackett & Yang, 2000). However, there are situations in which the correction for range restriction results in small changes, and conclusions drawn from the uncorrected correlation will not change. One example is when there is a high selection ratio and nearly all applicants are accepted. Another is when direct selection on one variable has little effect on the standard deviations of one or more variables that underwent indirect selection. Since there was little range restriction on the indirectly selected variables, correction for range restriction may have only a small impact. The conclusions drawn from the uncorrected and corrected correlation would be the same.

A third example comes from a meta-analysis of meta-analyses. Wilmont and Ones (2019) declined to correct their random effects second-order meta-analysis for range restriction. They argued that sporadic reporting of the needed information threatened to provide biased estimates. The purpose of a random-effects second-order meta-analysis is to estimate the between-meta-analysis variance. They could have obtained the raw data from the various authors and conducted a first-order ('omnibus') meta-analysis. This is unrealistic in many cases. Given the situation, they must present their results, particularly between meta-analysis variance with caveats.

Considering this a missing data problem, multiple imputations might provide additional values or might not be depending on the extent of missing data and how the data were missing (e.g., missing at random, missing completely at random, or missing not at random).

1.1.1. Range Restriction or Range Enhancement? It should be noted that prior selection will not always decrease the magnitude of the correlations. Under unusual circumstances when there is "range enhancement" meaning increased variance in either the independent or the dependent variable due to selection, increased correlations would be observed (Levin, 1972). Also, Zimmerman and Williams (2000) noted that distributions with extreme outliers can cause downwardly biased correlations or under some situations increase the variance of the variables and produce upwardly biased correlations. They provided a statistical method to correct correlations in these situations. Some studies compare extreme groups, eliminating the middle of the distribution. This may cause "range enhancement" leading to overestimation of correlations.

How well do correction formulas work under this circumstance? Preacher, Rucker, MacCallum, and Nicewander (2005) referencing Pearson (1903) and Wherry (1984) observed that the range restriction correction formulas can be used to correct for overestimation caused by the use of extreme groups. However, Preacher et al. stated that they found no examples of corrections applied to correlational analyses of extreme groups.

1.2 Considering Range Restriction Correction

Range restriction correction formulas have been known for more than a century (Lawley, 1943; Pearson, 1903; Thorndike, 1949) and their utility has been well documented. Nonetheless, some question their use when there is not complete truncation (i.e., complete truncation would be having no scores below a specified numeric value). Campbell (1976, p. 218) concluded about corrections for range restriction that "… the safest recourse is to not use them." Damos (1996) referred to range restriction as a "red herring" in explanation for low predictive validities observed in commercial and military pilot selection batteries. She argued that commercial air carriers are not going "to administer tests to a completely unrestricted population: some type of selection based on the candidate's background and experiences will occur before any testing is conducted" (p. 202). Damos concluded that the uncorrected correlations provide the most accurate estimates of the predictive validity of a test in most cases. She acknowledged that some type of selection has occurred, but does not suggest correcting for that selection. Testing a random sample of the population is not advocated. The appropriate unrestricted sample for pilot selection is applicants who present themselves for testing.

The argument for the use of range-restriction-correction formulas is that they provide a more accurate estimate of correlations (Hunter & Schmidt, 2004; Linn, Harnish, & Dunbar, 1981; Ree et al., 1994). Linn et al. (1981) analyzed more than 700 validity studies and concluded "Thus it seems desirable to routinely compute and report corrected correlations along with their uncorrected counterparts. Though still conservative, the corrected values will generally provide a better indication of predictive validity and be less misleading than uncorrected correlations alone" (p. 662).

Hunter and Schmidt (2004) concluded that in educational and employment selection, the predictive validity of cognitive ability has been underestimated considerably due to the failure to correct for range restriction. Held and Foley (1994) provided an empirical example. They calculated the predictive validity of aptitude scores while varying the selection ratio from 1.00 (unrestricted group) to .10 (highly restricted group). The validity (r) of the aptitude scores steadily decreased as selection became more restrictive. In all instances, the corrected validities were closer to the unrestricted validities than were the uncorrected validities.

A series of realistic scenarios follows with information on which correction should be applied. They illustrate how failure to apply the appropriate correction can affect the interpretation of the relations between variables and lead to incorrect decisions. These scenarios provide examples, explanations for commonly occurring situations, and appropriate advice about correcting for range restriction.

2.0 SCENARIOS

2.1 Early Correction Models

Scenario 1: Why Correct for Range Restriction?
Situation
To determine if five variables were predictive of first-semester grade point average of college applicants a psychologist has completed a validity study. The sample was collected from volunteer undergraduate college students to earn class credit. Two of the five potential predictors were statistically significant and surprisingly one was negative. A colleague suggested correcting the variables for range restriction. The psychologist replied, "Why should I, two of the variables are significant and that's all I need." No corrections were applied and the three nonsignificant potential predictors were removed from consideration.

Problem
The sample-of-undergraduates differs from college applicants by their interest in applying for admission and by their selection using the admissions standards. This produced range restriction and biased correlations. The psychologist would have erroneous beliefs about the relationships of the variables leading to incorrect actions by dismissing variables that were not statistically significant. The corrected values would be more appropriate and lead to correct decisions about which variables to use.

R. L. Thorndike (1949) reported a study conducted during World War II in which 1,036 soldiers were allowed to enter Army Air Corps pilot training without regard to scores on aptitude tests. After the trainees completed training or dropped out, correlations were computed on those who would have been selected for pilot training if the aptitude standards in place at the end of the war had been applied. In this subset of individuals, not all of the correlations for the tests were statically significant and one test showed an unexpected negative correlation. A second analysis using all subjects showed all test validities to be significant and the one test that had a negative correlation showed a positive correlation. If the researchers only considered the correlations from the range-restricted sample of those eligible for pilot training they would have been wrong and led others to false conclusions. The appropriate interpretation was that all tests were valid and that the negative correlation was a selection artifact.

Since the psychologist in the scenario was examining the predictive validity of multiple (5) variables, he should have applied the multivariate procedure to correct the correlations using the applicants as the unrestricted group. Correction will provide a better estimate of the correlations and may reverse the sign of the unexpected negative correlation. Additionally, failure to correct for range restriction reduces the ability to accumulate knowledge.

Scenario 2: Range Restriction Assumptions

Situation

In a research committee meeting, an organizational psychologist suggested to an experimental psychologist that correlations should be corrected for range restriction. The experimental psychologist replied, "I cannot meet the assumptions for correction and corrections over-correct the correlations." No corrections were made.

Problem

The assumptions for all the correction procedures are linearity and homoscedasticity. These are two of the three assumptions for correlation. The third assumption for correlation is normality, but it is not needed for range restriction correction. If you are confident that you can compute a correlation, you can apply the correction.

Linn, Harnisch, and Dunbar (1981) demonstrated that in most situations the corrected correlations are underestimates. Additionally, Millsap (1989) confirming Linn et al. showed that on average correcting correlations under-corrects. However, under extreme selection ratios, correction is not as accurate as under less extreme conditions. Held and Foley (1994) demonstrated this with a large sample (n = 147,288) of US Navy enlistment test scores for Case 2, Case 3, and the multivariate method while varying the selection ratio from 0.10 (highly selective) to 1.00 (all applicants). The multivariate correction was generally more accurate than the univariate corrections, even when the directly selected variable was negatively skewed and failed the assumptions of linearity and homoscedasticity did not offset one another. Under extreme selection ratios the Case 2, Case 3, and multivariate method corrections were not as accurate as with less extreme selection ratios. Considering the use of correction for range restriction even with extreme selection ratios, they concluded "All corrected validities were more accurate than the respective uncorrected validities." (p. 361). Research (Millsap, 1989) consistently shows that, on average, correcting correlations under corrects. Greater accuracy will be obtained if correlations are corrected for range restriction.

Scenario 3: Case 1 - Direct Selection on the Criterion

Situation

An inexperienced personnel selection psychologist has been tasked with finding and validating an experimental test to select shipping clerks. This organization has employed shipping clerks for 30 years and has evaluated their job performance with a standardized work-sample test with norms based on U. S Department of Labor data since the job was established. After a job analysis and review of journal articles, he decided to test whether a computerized test of Attention to Detail was valid for predicting the job performance of shipping clerks. The Attention to Detail test was administered and scored for 29 job incumbents who had been employed for at least 5 years. From the organization's job performance records collected over the last 30 years, the Attention to Detail test scores and job performance ratings for their first 5 years on the job were obtained. The analysis consisted of computing the correlation of the Attention to Detail test and job performance and the mean and standard deviations of both variables. The psychologist chose not to correct the correlation for range restriction because he did not have data for shipping-clerk applicants on the experimental test. Results showed the correlation was not significant and the personnel psychologist concluded that the Attention to Detail test lacked predictive validity.

Problem

There are two problems with the conclusions. First, 29 subjects is a small sample making statistical significance less likely to be found even when the relationship was statistically significant. Second, the personnel psychologist said he did not correct for range restriction because he did not have applicant data on the experimental test. He did however have normative criterion data from the Department of Labor. In this situation, it is appropriate to consider the participants to have been directly selected for the study. The participants were included because they had criterion data. Because of the effect of range restriction, the Attention to Detail test was deemed invalid. Range restriction occurred on the job performance criterion. The psychologist should have applied the Case 1 correction formula to obtain the corrected validity of the test which would lead to the interpretation that the Attention to Detail test was predictive of job performance.

Scenario 4: Case 2 - Direct Selection on the Independent Variable

Situation

To test the validity of a college entrance test, an admissions counselor gave it to a sample (n = 61) of physics majors and an equally sized sample of sociology majors. Statistical testing of the correlation between the entrance test score and first-year grade point average (GPA) showed a nonsignificant correlation for the physics majors, (r = .05), but a statistically significant correlation for the sociology majors (r = .27). The admissions counselor had confidence that the college entrance test was useful for selecting sociology majors but not physics majors.

Problem

His confidence is misplaced. The standard deviation on the college entrance test among physics majors is only 25% as great as for the sociology majors. Students are not assigned to college majors randomly. Individual choice plays a large role. Part of the choice is related to the ability and interests of the student. Frequently, students sort themselves by ability. The sociology department receives students with a wide variance of ability while the physics department receives students with a narrower variance of ability. Correlation is dependent on variability and the lower variability in the college entrance test scores for physics majors restricts the magnitude of the correlation. As computed, the two correlations cannot be compared. Differential range restriction for the physics and sociology majors has contributed to the apparent validity difference for the two groups. The belief of the admissions counselor is erroneous.

The Case 2 correction for range restriction for direct selection should be applied. In this instance, students were selected only by scores on the college entrance test. Each department sets unique minimum selection scores. Students who scored below departmental minimums were not admitted.

The college entrance test manual specifies that in a random sample of college applicants the standard deviation of the college entrance test is 20. This value of 20 was used in each of the Case 2 corrections along with the standard deviation found in each sample. After Case 2 correction the correlation was $r = .40$ for the sociology students and $r = .41$ for the physics students. Differential range restriction caused the original correlations to seem different when they were about the same. The admissions counselor should conclude that the college entrance test is valid for both departments.

Scenario 5: Case 3 – Indirect Selection

Situation

An industrial/organizational psychologist was consulted to improve the validity of a selection system for entry-level management jobs. The organization currently used a test of general mental ability (X). After reading the management selection literature, she identified a candidate test of leadership. Department supervisors were asked to use a structured rating scale to measure the performance of the management trainees selected last year. These ratings were collected by the psychologist. Next, she administered the leadership test (Z) to all 81 of last year's management trainees. The test of general mental ability correlated with job performance (Y) ratings at $r = .29$, while the leadership test correlated with the job performance ratings at $r = .35$. She concluded that the test of leadership was more valid than the test of general mental ability.

Problem

The problem is the failure to recognize a situation of direct selection on the test of general mental ability and indirect selection on the test of leadership. Evaluation of the uncorrected correlations leads to the inappropriate conclusion that the leadership test is more valid than the general mental ability test. Corrections require only three correlations r_{xy}, r_{zy}, and r_{zx}, all of which were available. After application of the Case 3 correction for indirect range restriction on the leadership test and Case 2 correction for direct range restriction on the general mental ability test, the correlations with job performance were .46 for general mental ability and .40 for the test of leadership. Her wrong conclusion leads to an erroneous interpretation of the validities. Case 2 and Case 3 corrections should have been applied.

Scenario 6: Multivariate Selection

Situation

A researcher was studying the relationship of math, spatial reasoning, spatial orientation, and mechanical comprehension to the safe operation of forklifts in a warehouse measured by safety-related accidents such as hitting people, shelves, doors, walls, and other forklifts. All participants were selected on scores on the math and spatial orientation tests. The other two tests, spatial reasoning, and mechanical comprehension were experimental and not used for selection. A group of 194 forklift operators participated in the study. Accelerometers were installed on all forklifts to measure the number of times it struck objects, as the criterion. Scores on the four tests were retrieved from company records and after study completion, the correlations were examined. Spatial reasoning (r = .44) and mechanical comprehension (r = .56) showed strong correlations with the number of times a forklift struck objects, while the math and spatial orientation tests had only weak relations. The researcher concluded that math ability (r = .12) and spatial orientation (r = .10) were not meaningfully related to the criterion, but spatial reasoning and mechanical comprehension were.

Problem

The conclusions of the researcher are wrong because the test scores were subject to different types and amounts of range restriction and were not corrected. The math and spatial orientation tests were used for selection and hiring, the other tests were not. Given the four independent variables, the multivariate correction for range restriction developed by Lawley (1943) is appropriate. In this scenario, the multivariate correction would include the four independent variables and the criterion. At least one unrestricted correlation such as between math and spatial reasoning is needed. In addition to the correlations of the four independent variables with the criterion, the corrected correlations of the variables with each other are provided after applying the multivariate correcting. The resultant matrix of corrected correlations is more accurate than the uncorrected correlations or the individual correlations corrected separately (Held & Foley, 1994). The multivariate method uses more information than the univariate methods to correct the correlations. It should be used in situations when two or more variables are used for selection.

2.2 Effects

Scenario 7: Consequence of Failing to Correct for Range Restriction

Situation

A researcher was presented with data on verbal and mathematical aptitude and five domain scores for a Big Five personality measure. The participants (n = 217) were college-graduate applicants for an entry-level job with a large car rental company. She computed a 7 by 7 correlation matrix and evaluated individual correlations and noted that the lowest correlation was between the verbal and mathematical aptitude scores. Additionally, the correlation between the personality domain score for conscientiousness with the other four domains scores was low. The correlations of the verbal and mathematical scores with the five personality domain scores also were low. She concluded that the test scores are relatively independent.

Problem
The researcher has failed to identify range restriction as an influence on the magnitude of the correlations of the variables. If she consulted the user's manual for the tests, it would have been noted that the standard deviations of the variables were all much higher than in her sample. Because multiple variables were involved, the multivariate correction for range restriction should have been used. After application of the multivariate correction, it would be noticed that the corrected correlation matrix shows moderate to strong correlations changing her conclusion.

Scenario 8: Determining the Correct Unrestricted Group
Situation
A law school selection board was interested in examining the predictive validity of law school admissions test scores for predicting first-year law school GPA. Data from archival records for the last 5 years of those accepted to the law school were used to calculate the correlation between the law school admissions test scores and first-year law school GPA. Understanding that the variance of the test scores was restricted due to prior selection the board decided to use the law school admissions test national norms as the unrestricted data for range correction to provide a better estimate of the validity. The Case 2 correction for direct selection was used.

Problem
National norms are not the correct unrestricted group in this situation. Using national norms would likely overcorrect the correlations. The appropriate unrestricted group is the group about which inferences are to be made. Since the selection board is interested in the relation of the law school admissions test scores to first-year GPA, the appropriate unrestricted group is applicants to this law school.

Scenario 9: Extreme Groups and Range Enhancement
Situation
A graduate student in organizational leadership was interested in the relationship between personality and leader-member exchange (*LMX*). She hypothesized a positive relationship between agreeableness and LMX ratings. She administers an on-line survey to collect data from 253 employees who rated their immediate supervisors on agreeableness and leader-member exchange. She then selected only the highest and lowest 20% of job incumbents based on their agreeableness scores. The resulting correlation between agreeableness and leader-member exchange was $r = .53$, confirming her hypothesis.

Problem

An examination of the standard deviation for the agreeableness test indicated a higher value in the extreme-groups sample than the normative population. The use of extreme groups has increased the variability of the scores creating range enhancement (Johnson, Deary, & Bouchard, 2018; Levin, 1972) and an increase in the correlation between agreeableness and leader-member exchange. The graduate student should correct the correlation for direct (Case 2) range restriction to provide a more accurate estimate. The corrected correlation between agreeableness and leader-member exchange was $r = .27$. The use of extreme groups created range enhancement.

Scenario 10: Univariate versus Multivariate Correction

Situation

A large metropolitan police department used a measure of general mental ability (GMA) to screen job applicants for over a decade. Although the measure of GMA had consistently demonstrated validity for supervisor ratings of first-year performance, the Chief of Police is concerned about occurrences of counterproductive behavior. In response, the human resources department introduced the three tests of the Dark Triad, Machiavellianism, neuroticism, and psychopathy into the selection battery. Correlations in a normative sample are available for these three tests. Although the Dark Triad tests were administered, their scores were not used in the selection process. After a large sample of new hires completed their first-year on the job, the human resources manager calculated the correlations between the measures of GMA, Machiavellianism, neuroticism, psychopathy, and supervisory ratings of counterproductive work behavior. All tests had statistically significant correlations with supervisor ratings in the expected direction. He was aware of the effects of range restriction and corrected the correlation to estimate the correlations in the applicant sample. The GMA measure was corrected for direct selection (Case 2) and the validities of the measures of the Dark Triad were corrected for indirect range restriction (Case 3). As expected, all of the validities increased in magnitude, with a larger increase for the measure of GMA than for the measures of personality.

Problem

The application of the Case 2 and Case 3 correction formulas is commendable as they show recognition of the problem of range restriction. Case 2 and Case 3 corrections provide better estimates than the uncorrected correlations. However, the multivariate method (Lawley, 1943) should have been applied to produce more accurate results (Held & Foley, 1994). The multivariate correction makes more accurate corrections because it uses more information than either Case 2 or Case 3. Additionally, the multivariate correction also provides corrected correlations of the tests and criteria as well as the tests with one another. This facilitates the use of the matrix of corrected correlations in other analyses such as multiple regression. For example, the multivariate correction allows the researcher to examine the incremental validity of the Machiavellianism, neuroticism, and psychopathy measures over the measure of GMA.

2.3 Analysis

Scenario 11: Sign Changes When Correcting for Range Restriction

Situation

A psychologist was interested in evaluating the predictive validity of scores from a multiple-aptitude test battery for grades in an advanced electronics training course. The test battery was administered to a large group of applicants. The selection process was competitive and only the top 25% of applicants were selected based on a composite of the test scores from the battery. The psychologist computed the correlations between the test scores and training grades noting they were low with one unexpected negative correlation. He applied the multivariate correction and noted that the magnitudes of all of the correlations increased and the test with the negative correlation was now positive. After observing the change in sign, he concluded that something went wrong with the multivariate correction and applied the Case 2 correction (direct selection) instead. The Case 2 corrections lead to increased correlations for all test scores including a larger negative value for the uncorrected negative correlation.

Problem

Under some selection conditions, sign changes can occur (Thorndike, 1947). These can be corrected using Case 3 (indirect univariate) or the multivariate method (Ree et al., 1994). Case 1 and Case 2 cannot correct changes in sign due to range restriction. The experimental psychologist was correct the first time when he applied the multivariate correction. The corrected correlations were more accurate estimates leading to correct decisions.

Scenario 12: Reliability and Range Restriction

Situation

A researcher at a highly selective university was investigating the theory that student performance on a standardized test of advanced algebra was negatively related to class size as measured by the number of registered students. After a correlation was computed, the researcher first corrected it for unreliability, then corrected for direct range restriction to estimate the true score correlation (r_t) of the variables. This was accomplished by dividing the observed correlation (r_{xy} = -.57) by the product of the square roots of the reliabilities (R_{xx} and R_{yy}). The researcher assumed that the student count was perfectly reliable and found the reliability of .8 in the manual for the test of advanced algebra. The correction for unreliability is

$$r_t = rxy/\sqrt{Rxx}\sqrt{Ryy}$$

where r_t is the true-score correlation, *rxy* is the observed correlation, and *Rxx* and *Ryy* are the reliabilities of the measures. The reported result after correction for unreliability and direct range restriction was r_t = -.63 giving a 10% change in the magnitude of the correlation.

Problem

The researcher has not reported a correct true-score correlation because she applied the corrections in the wrong order. When selection has occurred on an observed score the correction for range restriction should be applied first followed by the correction for unreliability. When selection has occurred on a latent variable such as self-selection the correction for unreliability should be done before the correction for range restriction. Stauffer and Mendoza (2001) have shown that the sequences for observed scores and latent variables apply to Cases 1, 2, 3, and V. Order of corrections with the multivariate procedure has not been studied. Correcting for range restriction yielded r = -.60 and after correction for unreliability r = -.75. This is a 32% change beyond the uncorrected correlation of -.57.

Scenario 13: Regression Weights and Range Restriction

Situation

A researcher uses multiple regressions to predict the number of hours to complete training in computer programming. Applicants were selected based on a test of logical reasoning and a test of non-verbal reasoning. An experimental test of the ability to follow complex procedures was administered, but not used in selection. For admission to the course, the reasoning tests had required minimum scores of the 50th percentile for logical reasoning and the 45th percentile for non-verbal reasoning. The number of hours to complete training was the criterion. Analyses were conducted both without and with the experimental test. The first multiple regression produced a statistically significant R = .390 with the following unstandardized regression weights: logical reasoning b = .372 and non-verbal reasoning b = .306. A second regression was conducted including adding the experimental test showing a significant R = .422 and the unstandardized weights were: logical reasoning b = .305, non-verbal reasoning b = .253, and ability to follow complex procedures b = .173. The new test of ability to follow complex procedures demonstrated incremental validity (.422 vs .394) and was recommended as an addition to the selection tests that the researcher suggested using the unstandardized regression coefficients as multiplicative weights for computing scores for use in selecting applicants.

Problem

The unstandardized b-weights are biased due to range restriction. There was direct selection on logical reasoning and non-verbal reasoning. The experimental test was subjected to indirect range restriction. Had the correlations been corrected for range restriction the results of the first multiple regression would have shown a statistically significant R = .662 with the following unstandardized regression weights: logical reasoning b = .366 and non-verbal reasoning b = .374. When the experimental test was included in the second regression the R = .692 and the unstandardized weights were b = .266 for logical reasoning, b = .269 for non-verbal reasoning, and b = .269 for the test of ability to follow complex procedures. After correction, the unstandardized weights were not biased and the test of ability to follow complex procedures showed less incremental validity. The unstandardized regression coefficients could be used for selection.

Mendoza and Mumford (1987) noted that while direct range restriction has no effect on regression slopes, indirect restriction leads to a reduction in the regression slope (Hunter et al. 2006; Mendoza & Mumford, 1987). If indirect range restriction has occurred, the unstandardized regression weights in the population and the range-restricted sample would be equal only if the independent variables were perfectly reliable.

Scenario 14: Factor Analysis and Range Restriction

Situation

Senior military leadership was concerned that while new enlistees have adequate verbal, math, and technical knowledge, they cannot solve novel or unique problems. They suggested additional measures be added to the enlistment test battery. A personnel research psychologist familiar with cognitive psychology theory recognizes that the distinction being made is between crystallized intelligence (Gc) and fluid intelligence (Gf). He noted that the current enlistment test consists entirely of measures of Gc. As a first step in investigating the utility of fluid intelligence in personnel selection, he administered a battery of Gf tests to new recruits and conducted a confirmatory factor analysis that includes both the Gc tests from the extant enlistment battery and the battery of Gf tests. Drawing on the published literature, he tested and confirmed a hierarchical factor structure for each of the test batteries and interpreted the higher-order factors as general crystallized intelligence and general fluid intelligence. He observed a correlation of .705 (r^2 = .497) between the two higher-order factors and concluded that while they are strongly correlated there is enough unique variance in the measures of crystallized and fluid intelligence that the Gf tests will have substantial incremental validity when used with the current test battery.

Problem

The confirmatory factor analysis was done with a sample of enlistees who had been directly selected based on their scores on the measures of Gc. The correlations involving the measures of fluid intelligence were affected by indirect selection due to their relation with the Gc enlistment test. The correlations between the tests for the enlistees (restricted sample) are lower than those for military applicants (unrestricted sample). Thus, the correlations among the factors also had been underestimated. The researcher overestimated the uniqueness of the Gf tests and their potential incremental validity. He applied the multivariate correction for range restriction then conducted another confirmatory factor analysis. Two higher-order factors correlated at r = .911 (r^2 = .829) indicating little unique variance and that Gf would have little or no incremental validity.

Given that both confirmatory factor analysis and exploratory factor analysis are based on correlations (or covariances), range restriction will affect the results of each method. Correction for range restriction is desirable for both confirmatory and exploratory factor analysis to provide a better estimate of the relations among the constructs (factors).

2.4 Application

Scenario 15: Correcting When Information about Previous Selection Variables is Unavailable

Situation

A researcher was testing a hypothesis from a theory of management. The hypothesis states that there is a strong positive relationship between situational judgment and verbal analogies. The 199 employee participants were previously selected for employment, but information on the variables used for selection was lost and unavailable. The researcher administered the Management Appraisal of Situational Judgement and Verbal Analogies Assessment tests. She computed a correlation between these scores ($r = .18$) and reported a weak relationship.

Problem

Frequently, a problem is the effect of factors or variables not accounted for because they are unknown or unavailable, but are influential in creating range restriction nonetheless (see Gross, 1990; Gross, & McGanney, 1987; Jackson & Ree, 1992; Olson & Becker, 1983). The researcher should have used the equation of Bryant and Gokhale (1972) who developed a method of correcting for range restriction when information about previous selection variables is unavailable. The data required by their formula are the restricted correlation and the unrestricted and restricted standard deviations for each variable. The result of their equation is a corrected correlation of the variables. Applying Bryant and Gohkale's formula to the data, a corrected correlation of $r = .36$.

There are times when the correlation of the constructs underlying the observed scores is of interest. When the relations among the constructs are the focus as in meta-analysis, it is necessary to also correct for the reliability of the measures. The Case V correction for range restriction (Le, Oh, Schmidt, & Wooldridge, 2016) is an important expansion of the Bryant and Gokhale (1972) formula that also corrects correlations for unreliability. Applying Case V to the data in this scenario, the fully corrected correlation was $r = .45$.

Scenario 16: Estimating the Utility of a New Selection Variable

Situation

The human resources manager of a regional airline was considering adding a structured interview to the pilot selection process. She conducted a study where all applicants during two years completed the interview and received ratings from the board of interviewers. Those hired received quarterly job performance ratings from crewmembers who worked with them daily. After a sufficiently large number of pilots had been hired and evaluated by their peers, the human resources manager analyzed the data to determine the utility of the structured interview when added to current selection procedures. She calculated the validity of the current procedure and the current procedure plus the interview to estimate the baseline predictive validity and incremental validity. Success on the job was defined as receiving a mean peer rating of 4 or higher on a scale ranging from 1 to 5. The success base rate was 30%. The correlation of the current system is $r = .30$ and when the structured interview was added $r = .36$. She then consulted the Taylor-Russel tables (Taylor & Russel, 1936) to estimate utility. The success rate of the current system is 30% and the system with the structured interview added has a success rate of 36% indicating a 20% increase (6/30 = 20%) over the base rate.

Problem

Taylor and Russel (1936) showed that the proportion of successful employees after selection is a function of the selection ratio (i.e., the proportion of applicants selected from those who apply), the base rate, and the validity of the proposed selection test. Base rate is defined as the percentage of employees who would be successful under random selection. The accuracy of the results of utility (proportion successful) analysis (Naylor & Shine, 1965; Schmidt, Oh, & Shaffer, 2016; Taylor & Russel, 1936) depends on the numeric value of the correlation between the selection score and success on the job. The use of uncorrected correlations will frequently lead to underestimates, sometimes severely, of the utility of the predictor. In this example of utility analysis, the corrected proportion of success was 43% for the current system and the proportion successful with the addition of the structured interview was 48% when the correlations were corrected for range restriction. The 20% improvement over the baseline with uncorrected correlations was reduced to 12% (5/43 = 12%). Corrected correlations give a more accurate estimate of utility.

Scenario 17: More Accurate Meta-Analyses

Situation

A scientist at Drones R Us conducted a literature review regarding the relation of general mental ability and effectiveness of human-automation interaction (HAI). Among these, several independent small-scale studies included an assessment of video game experience (VGE). Participants in these studies had been selected based on scores on a general mental ability test; VGE was assessed as part of a background questionnaire, but not used for selection. The scientist conducted a meta-analysis to estimate the correlations among general mental ability, HAI, and VGE. Incremental validity of VGE for predicting HAI was based on the meta-analytic results. He corrects the validities involving general mental ability for direct range restriction (Case 2) and those involving VGE for indirect range restriction (Case 3).

Problem

Case 2, is appropriate in the scenario for correcting the validity due to direct selection; it could be improved to include correction for measurement error by use of the correction for unreliability. Both variables used in Case 2 would be corrected for unreliability using the formula shown in Scenario 12. The order of correction for range restriction and unreliability (Stauffer & Mendoza, 2001) is dependent on how the range restriction occurred (observed or latent variables) and is discussed in Scenario 12.

The accuracy of the Case 3, univariate indirect range restriction (UVIRR) correction for VGE could be improved by using Case V when appropriate data are available. The use of the Case V procedure (Le et al., 2016) provides a better estimate because it corrects the correlations for measurement error, indirect range restriction, and in meta-analyses, facilitates estimation of sampling error across studies. Dahlke and Wiernik (2019a) using the term bivariate indirect range restriction (BVIRR) to describe Case V, noted that Le et al.'s (2016) method did not take into account important impacts of the BVIRR correction of the sampling error of corrected correlations and noted the implications of applying the BVIRR correction in primary research and meta-analyses. Dahlke and Wiernik provided a generalized Case V (BVIRR) formula that can correct for either range restriction or range enhancement and substantially improved parameter and correlation estimates by reducing bias. They also described new methods to adjust for its impact on the sampling variance of correlations. Dahlke and Wiernik noted that the Case V (BVIRR) correction functions very differently than univariate direct range restriction (UVDRR or Case 2) or univariate indirect range restriction (UVIRR, Case 3 or Case IV) corrections or correction for measurement error alone when used in psychometric meta-analyses.

Scenario 18: Meta-Analysis and Indirect Selection

Situation

A meta-analysis was conducted to estimate the validity of two variables for a criterion of a written job knowledge test. The first variable was a psychomotor test that was used in each study for direct selection of employees and the second variable was a supervisory rating of cooperation for only the selected employees. Correlations were computed between psychomotor test scores, cooperation ratings, and job knowledge. There were 112 studies, each with the correlations of interest. Using off-the-shelf software the researcher corrected the correlations for range restriction using Case 2 (direct selection) as implemented by the software. The meta-analytic results show an average correlation corrected for range restriction of .53 for the psychomotor test and .33 for cooperation ratings with job knowledge scores.

Problem

The corrected correlation for the measure of cooperation is wrong. Given the situation, the Case 3 equation (indirect selection) should have been used, not the Case 2 equation (direct selection). The Case 2 correction does not correct for indirect selection. In this situation with the use of the wrong equation, the meta-analytic average corrected correlation for cooperativeness was incorrectly estimated (Hunter & Schmidt, 2004) causing the meta-analyst to report incorrect underestimated results. When the correction for indirect selection was used in the meta-analysis the fully corrected correlation between cooperation rating and job knowledge was $r = .44$.

Scenario 19: Confidence Intervals for Corrected Correlations

Situation

A study was conducted to find the correlation between scores on a standardized test of physical fitness and the number of injuries for firefighters. Job incumbents were selected based on achieving a score at or above the 50th percentile on the physical fitness test. The 36 participants in this study were from a local fire department. A correlation of $r = -.33$ was found. The principal investigator corrected the correlation for direct selection (Case 2) giving a corrected correlation of $r = -.50$. He also computed the confidence interval for the correlation both before and after range-restriction correction using the equation for the standard error of a correlation. He reported both uncorrected and corrected correlations and their 95% confidence intervals.

Problem

There are two mistakes with the statistics reported. First, Millsap (1989) demonstrated that the standard error of range-restricted correlation is underestimated using the usual standard error equation. This makes confidence intervals too small but the effect is frequently very small. Because no accepted adjustment exists, reports should include a caveat. The second mistake is using the standard error equation for the corrected correlation. Because the correlation has been corrected, the standard error has increased and the estimated confidence interval is biased. Confidence intervals for correlations corrected for range restriction are used routinely in psychometric meta-analyses (see Hunter & Schmidt, 2004, p. 108). The appropriate method is to compute the two endpoints using the standard error formula and then apply the correction used for the correlation. For example, if a Case 2 correction was used, correct the lower end and upper end points of the interval using the formula for Case 2.

Scenario 20: Making Point Value Predictions

Situation
To study the relationship of agreeableness with job satisfaction, a measure of each was administered to a large group of incumbent financial advisors. One goal of this study was to determine if there was a statically significant relationship between agreeableness and job satisfaction and to make an inference about future financial advisor job applicants. After scoring the measures a research psychologist computed means, standard deviations, and the correlation ($r = .17$) between agreeableness and job satisfaction. Using the formula for the standard error of a correlation, the psychologist tests the correlation of .17 and finds non-significance. A manual for the agreeableness measure gives a standard deviation for a nationally representative sample of adults greater than the value computed in the study sample. Applying the Case 2 correction for range restriction a corrected correlation of $r = .44$ results. The researcher reports that the corrected correlation in a future group of applicants for the job of financial advisor will be r = .44.

Problem
There are two problems. First, research has shown that the standard errors of range-restricted correlations are greater than the result of the textbook formula (Millsap, 1989). The probability that the corrected-for-range-restriction correlation in a new group of applicants will be .44 is extremely small. Second, while applying the Case 2 correction is admirable, using a nationally representative sample to provide the unrestricted standard deviation is inappropriate in this situation. The appropriate unrestricted group is a representative group of financial advisor applicants. As previously discussed in Scenario 8, the unrestricted group should always represent the group about which inferences are to be made, in this circumstance the financial advisor job applicants.

3.0 CONCLUSIONS AND RECOMMENDATIONS

It is better to correct for range restriction in almost all instances. Even if the range restriction is small or if the selection ratio is high, the correction will adjust the correlations for bias and provide more accurate values.

The unrestricted sample or population should represent the population or sample about which inferences are to be made. When applicants represent the sample about which inferences are to be made, use applicants for the unrestricted sample. When making inferences about the general population use normative data for the unrestricted sample. Correcting for range restriction can lead to a clearer interpretation of relationships among observed scores or constructs and scores and the statistical tests of these relationships.

In keeping with Linn et al. (1981), report both the uncorrected and corrected correlations. How the range restriction was created is the key to which correction formula to use. Apply the range correction formula that fits the restriction situation. Important questions to answer are: Was the selection direct or indirect or both? Was more than one variable used for selection? Are data available for selection variables not administered by the researcher? After these questions are answered proper correction procedures can be applied.

3.1 Determining the Appropriate Correction

1. If selection occurred on 1 variable use Case 1 or Case 2.
   Use Case 1 if selection occurred on the criterion.
   Use Case 2 if selection occurred on the predictor.

2. If there are 2 selection variables, one for direct selection, and one for indirect selection, use Case 3, Bryant and Gokhale (1972), or Case V.

3. Use Case 3 if direct selection occurred entirely of one variable and there is one variable subject to indirect selection.

4. Use Bryant and Gokhale if selection variables are unknown, unknowable, or unmeasurable, but the correlation of X and Y and unrestricted variances of X and Y are known. This is useful when there are selection variables not administered by the researcher.
When reliability information is available use Case V for meta-analyses or for a correlation corrected for both range restriction and measurement error.

5. If there are 2 or more variables used for selection, use the multivariate method. There is no need to specify direct or indirect selection as the multivariate method corrects for both simultaneously.

6. If a matrix of range-restricted correlations and a matrix of unrestricted correlations for a subset of those variables is available, use the Lawley (1943) multivariate procedure. The multivariate procedure is more accurate than Cases 1, 2, and 3 (Thorndike, 1949).

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LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

BVIRR	Bivariate indirect range restriction
Gc	Crystalized intelligence
Gf	Fluid intelligence
GPA	Grade point average
GMA	General mental ability
HAI	Human-automation interaction
LMX	Leader-Member Exchange
N	sample size
r	Correlation
R	Multiple correlation
r_t	True score correlation
r_{xy}	Correlation between x and y
r_{xx}, r_{yy}	Reliability of x and y
UVIRR	Univariate indirect range restriction
VGE	Video game experience
x	Variable

APPENDIX

Range Restriction Equations

Please note that all correction formulas assume linearity and homoscedasticity. The equations have been presented in the notation of the cited articles for ease of reference.

Case 1 Correction (direct range restriction)

The Case 1 correction applies when the correlation to be corrected is between two variables, *a* and *b*, direct selection occurred on variable *a*, and both restricted and unrestricted variances are known only for variable *b*. There is an indirect selection on variable *b*. The correlation between *a* and *b* has been estimated in the range-restricted sample of those selected on *a*. The Case 1 correction formula is:

$$r_c = \sqrt{1 - (sd_b^2)/(SD_b^2)\ (1 - r_{ab}{}^2)} \qquad (1)$$

where r_c is the correlation between variable *a* and variable *b* in the unrestricted population, r_{ab} is the correlation between *a* and *b* in the restricted sample, $sd_b$is the standard deviation (SD) of *b* in the restricted sample, and $SD_b$is the SD of *b* in the unrestricted population.

Case 2 Correction for direct range restriction

The Case 2 correction (direct selection) is well-known and frequently used. In Case 2, the variances for *a* are known in both the restricted sample and unrestricted population. However, the correlation between *a* and *y* is known only for the restricted sample that has been directly selected on *a*. The Case 2 correction equation is:

$$r_c = r_{aY}\left(\frac{SD_a}{sd_a}\right) / \sqrt{(1 - r_{ay}{}^2) + r_{ay}^2\left(\frac{SD_a{}^2}{sd_a^2}\right)} \qquad (2)$$

where r_c is the correlation between variable *x* and variable *y* in the unrestricted population, r_{xy} is the observed correlation between *a x* and *y* in the restricted sample, sd_a is the standard deviation of *x* in the restricted sample and SD_a is the standard deviation of *a* in the unrestricted population),

Case 3 Correction for indirect range restriction

Case 3 is also called correction for incidental selection. Suppose a test, *X*, was given to a group of applicants and top-down selection was based on their test *X* scores. The variance for test X is known for both the restricted sample and the unrestricted population. The correlation for an experimental test, *Z*, and the job performance criterion, *Y* are available only for the selected (restricted) sample. When all three correlations can be calculated the Case 3 correction equation is:

$$r_c = r_{xy} + r_{xz}\, r_{yz}\, ((S_z^2/s_z^2) - 1) / \sqrt{\left[1 + r_{xz}^2\left(\frac{S_z^2}{s_z^2}\right) - 1\right]\left[1 + r_{yz}^2\left(\frac{S_z^2}{s_z^2}\right) - 1\right]} \quad (3)$$

where S_x/s_x is the ratio of the unrestricted to the restricted standard deviation of test *x*.

Case IV Correction for indirect range restriction with true scores assuming complete mediation

The Case IV correction equation is presented below. As noted by Schmidt et al. (2006), an important difference between direct and indirect range restriction is that in the direct case, because observed scores are used to select, restriction occurs on the observed scores. Observed scores are not used in selection in the indirect case. In the indirect case, range restriction occurs on *true* scores rather than on observed scores (Hunter & Schmidt, 2004, Ch. 5; Hunter et al., 2006). In the Case IV correction for indirect range restriction, the restriction on the true scores t_x for the observed score *x* is estimated from the range restriction on the observed scores of *x*. It is assumed that *y* is fully mediated by the true score (*T*) of variable *X*. As in Case 2, the ratio of the restricted to the unrestricted observed *SD*s of *x* is defined as *sx/Sx*. The range restriction on the true scores *t* is estimated from observed scores *x* and is defined as *st/St*. The equation that gives *st/St* (Hunter & Schmidt, 2004, Equations 3.16 and 5.31; Hunter et al., 2006, Equation 22) is:

$$\left(\frac{s_t}{S_t}\right) = \frac{\sqrt{\left(\frac{s_X^2}{S_X^2}\right) - (1 - r_{XX})}}{r_{XX}} \quad (4)$$

where r_{XX} is the reliability of the predictor, *X*, in the unrestricted group (i.e., applicant group). The Case IV range correction equation is:

$$r_{XtYt} = \left[\left(\frac{1}{\frac{s_t}{S_t}}\right) r_{XtYt}\right] / \sqrt{\left[\left(\frac{1}{\frac{s_t^2}{S_t^2}}\right) - 1\right] r_{XtYt}^2 + 1} \quad (5)$$

In their discussion of various scenarios that can produce range restriction, Sackett and Yang (2000) introduced the little-known Case V correction for range restriction (Bryant & Gokhale, 1972) which was derived from the Case 3 equation. Le et al. (2016) conducted studies to improve the accuracy and reliability of the Case V formula for use in meta-analyses.

Case V Correction for indirect range restriction based on true scores with mediation not assumed

The Case V equation enables correction for indirect range restriction without requiring the full mediation assumption of Case IV.

Bryant and Gokhale's (1972) correction equation is:

$$R_{XY} = r_{XY}\left(\frac{sd_X}{SD_X}\right)\left(\frac{sd_Y}{SD_Y}\right) + \sqrt{(1-\frac{sd_X^2}{SD_X^2})(1-\frac{sd_Y^2}{SD_Y^2})} \quad (6)$$

where R_{XY} is the correlation between variables *X* and *Y* in the unrestricted population, r_{XY} is the observed correlation between *X* and *Y* in the restricted sample, the range restriction ratio of *X* is sd_X/SD_X, where sd_X is the standard deviation [SD] of *X* in the restricted sample and SD_X is the SD of X in the unrestricted population, and the range restriction ratio of *Y* is sd_Y/SD_Y, where sd_Y is the SD of *Y* in the restricted sample and SD_Y is the SD of *Y* in the unrestricted population. The Bryant and Gokhale (1972) correction equation allows for the correction of indirect range restriction on *X* and *Y* without knowledge about the third variable on which selection occurred. The Bryant and Gokhale equation can be used to correct individual study correlations. It also gave rise to Case V which is suggested for meta-analysis.

The Case V method (Le & Schmidt, 2006), building on Bryant and Gokhale (1972), first requires the estimation of true score means and true score correlations. Le et al. (2016) modified the Bryant and Gokhale formula to also correct for measurement error yielding true score means and correlations. They described a revised meta-analytic approach that incorporates the Case V correction. Le et al. provided the equations for the multi-step procedure for estimating true score elements and should be consulted for descriptions of the detailed steps.
Even though Case V has been applied in meta-analysis it is also appropriate for correction of a single correlation to estimate the true score correlation of *X* and *Y*. As noted by Le et al. the Case V method does not require any further assumptions beyond those of linearity and homoscedasticity, which underlie all existing range correction formulas. The steps below describe the process to perform correction meta-analysis based on Case V.

(1) Correct for measurement error in measure *Y*. For each study, the observed correlation, r_{XYi}, provides the best estimate for the restricted correlation, ρ_{XYi}.

$$\rho_{XYi} = r_{XYi} \quad (7)$$

Then using the equation below correct for measurement error in *Y*:

$$\rho_{XPi} = \rho_{XYi} \,/\sqrt{\rho_{YYi}} \quad (8)$$

where ρ_{XPi} is the reliability corrected correlation of variable *X* and its true score, the underlying construct, *P*. ρ_{XYi} is the observed correlation between *X* and *Y* and ρ_{YYi} is the reliability of *Y*.

(2) Correct for measurement error in measure *X*. Next, correct for measurement error in *X* using the reliability of measure *X* on the underlying construct *T*, in the restricted population, ρ_{XXi}.

$$\rho_{TPi}{=}\rho_{XPi} \ /\sqrt{\rho_{XXi}} \qquad (9)$$

where ρ_{TPI} is the correlation between construct *T* (the reliability corrected construct underlying variable *X*) and construct P underlying variable *Y* in the restricted population. ρ_{xpI} is the observed correlation between *X* and *Y* and ρ_{xx} is the reliability of *X*. Next compute μ_t and μ_p

(3) Correct for indirect range restriction. Le et al. (2016) adapted the Bryant and Gokhale (1972) equation (6) correcting the variables for measurement error. Le et al. replaced the range restriction ratio on *X* with the range restriction ratio on *T* and replaced the range restriction ratio on *Y* with the range restriction ratio on *P*.

Estimate u_T from $u_X\left(\frac{sd_X}{SD_X}\right)$ and ρ_{XXi}:

$$u_T = \sqrt{\frac{\rho_{XXi}u_X^2}{1{+}(\rho_{XXi}u_X^2 - u_X^2)}} \qquad (10)$$

Likewise, estimate u_P from $u_Y\left(\frac{sd_Y}{SD_Y}\right)$ and ρ_{YYi}:

$$u_P = \sqrt{\frac{\rho_{YYi}u_Y^2}{1{+}(\rho_{YYi}u_Y^2 - u_Y^2)}} \qquad (11)$$

Finally, estimate the correlation between *T* and P in the unrestricted population using the following formula:

$$\rho_{TPa}u_T u_P + \sqrt{(1-u_T^2)(1-u_P^2)} \qquad (12)$$

Multivariate Correction

Lawley (1943) developed the multivariate method that allows for correction for range restriction resulting from the selection on several variables. The procedure is explained in terms of variance-covariance matrices that can be converted to correlation matrices. The discussion that follows uses the notation of Birnbaum, Paulson, and Andrews (1950).

Suppose one has a current test battery of *p* variables for which population information is available. A sample is selected based on test scores on the current battery and suppose that *n-p* additional variables (perhaps a combination of new tests and criteria variables) are collected on the selected (or restricted) sample.
The variance-covariance matrix from the restricted sample is:

$$v^* = \begin{bmatrix} v^*_{p,p} & v^*_{p.n-p} \\ v^*_{n-p,p} & v^*_{n-p,n-p} \end{bmatrix}.$$

All of $v^*$is known. The comparable variance-covariance matrix from an unrestricted population is:

$$v = \begin{bmatrix} v_{p,p} & v_{p,n-p} \\ v_{n-p,p} & v_{n-p,n-p} \end{bmatrix},$$

where there is only knowledge of $v_{p,p}$. We want to estimate the other three parameters of *v*. $v_{n-p,n-p}$ gives estimates of unrestricted variances and covariances for variables of the new tests and criteria. $v_{p,n-p}$(or its transpose $v_{n-p,p}$)gives estimates of unrestricted covariances for variables of the current test battery with the new tests and criteria.

Lawley's (1943) procedure follows where x_i and x_j is any pair of variables.
The multivariate correction is expressed in matrix algebra notation and the assumptions are linearity and homoscedasticity.

Then the following equations use the known variances and covariances plus the variances and covariances from the restricted sample to provide corrected variances and covariances for all variables. These variances and covariances are then converted to correlations. The matrix equations follow:

$$v_{p,n-p} = v_{p,p} v^{*-1}_{p,p} \, (v^*_{p,n-p}) \text{ and}$$

$$v_{n-p,n-p} = v^*_{n-p,n-p} + \sqrt{v^*_{n-p,p} v^*_{p,p}} (v_{p,n-p} - v^*_{p,n-p})$$

Functions in R to Compute Corrections for Range Restriction

R is an increasingly popular free programming language and there is a vast library of useful programs or functions. The most popular archive is CRAN.R-project.org. There is no need to be a computer programmer to use it. Using R functions is very much like using a calculator.

These functions can be run using the free Rstudio© after loading the libraries listed below.

correct_r is found in the package "**psychmeta**." **correct_r** is very flexible and can compute Pearson Cases 1, 2, and 3. It can also compute Case V. In Case V if the reliabilities are set to 1.0 it computes the Bryant and Gokhale correction.
https://CRAN.R-project.org/package=psychmeta

lMvrrc in the package "**Iopsych**" computes the Lawley multivariate correction for range restriction. The input requires a matrix of the unrestricted correlations and a matrix of the restricted correlations. lMvrrc and Iopsych are copyrighted by Allen Goebl, Jeff Jones, and Adam Beatty, 2016https://CRAN.R-project.org/package=iopsych