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Error Analysis of Heat Transfer Measurements Made About a Cylinder in Crossflow

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PREFACE

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14. ABSTRACT

An uncertainty analysis was applied to the heat transfer coefficient data measured about a right circular cylinder that was mounted in a crossflow configuration. The cylinder was 12 inches in diameter, and the wind tunnel speeds ranged from 10 m/s to 50 m/s yielding Reynolds' number range of 2×10^5 to 1×10^6. The data is described in reference 1. The Nusselt Number ranged from low of 200 to a high of 1,920. The results showed the uncertainty in the Nusselt number ranges from a low of 15% to a high of 25%. The minimum uncertainty occurs at the highest wind tunnel speed and the maximum uncertainty occurs at the lowest speed. The low values of the Nusselt number have slightly more uncertainty than the higher values

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1. INTRODUCTION

It is desired to estimate the uncertainty in the non-dimensional heat transfer coefficient, the Nusselt number, Nu, given by

$$Nu = \frac{hD}{k}\,, \tag{1-1}$$

where h is the convective heat transfer coefficient, D is the characteristic dimension taken as the outside diameter of the cylinder, and k is the thermal conductivity of the free stream fluid (air in this case). The uncertainty in Nu is

$$(\delta_{Nu})^2 = \left(\frac{d}{dh}Nu\right)^2 (\delta_h)^2 + \left(\frac{d}{dD}Nu\right)^2 (\delta_D)^2 + \left(\frac{d}{dk}Nu\right)^2 (\delta_k)^2\,. \tag{1-2}$$

The partial derivatives are

$$\left(\frac{d}{dh}Nu\right)^2 (\delta_h)^2 = \left(\frac{D}{k}\right)^2 (\delta_h)^2 = Nu^2\left(\frac{\delta_h}{h}\right)^2\,, \tag{1-3}$$

$$\left(\frac{d}{dD}Nu\right)^2 (\delta_D)^2 = \left(\frac{h}{k}\right)^2 (\delta_D)^2 = Nu^2\left(\frac{\delta_D}{D}\right)^2\,, \tag{1-4}$$

$$\left(\frac{d}{dk}Nu\right)^2 (\delta_h)^2 = \left(-\frac{hD}{k^2}\right)^2 (\delta_k)^2 = Nu^2\left(\frac{\delta_k}{k}\right)^2\,. \tag{1-5}$$

Therefore,

$$\left(\frac{\delta_{Nu}}{Nu}\right)^2 = \left(\frac{\delta_h}{h}\right)^2 + \left(\frac{\delta_D}{D}\right)^2 + \left(\frac{\delta_k}{k}\right)^2\,. \tag{1-6}$$

The convective heat transfer coefficient is found from the measured heat flux and temperature difference between the heated surface and the free stream by

$$h = \frac{\dot{Q}_{conv}}{A\,(T - T_0)}\,. \tag{1-7}$$

where $\dot{Q}_{conv}$ is the heat transfer rate from the surface, A is the area over which the heat transfer is being considered, T is the surface temperature, and T_0 is the free stream temperature. The surface area is given by

$$A = wl_{unit} \text{ ,} \tag{1-8}$$

where w is the width of the ribbon and l_{unit} is a unit length of ribbon.

$$h = \frac{\dot{Q}_{conv}}{wl_{unit}\,(T - T_0)} \text{ .} \tag{1-9}$$

The uncertainty in h is given by

$$(\delta_h)^2 = \left(\frac{d}{d\dot{Q}_{conv}} h\right)^2 \left(\delta_{\dot{Q}_{conv}}\right)^2 + \left(\frac{d}{dw} h\right)^2 (\delta_w)^2 \tag{1-10}$$

$$+ \left(\frac{d}{dl_{unit}} h\right)^2 \left(\delta_{l_{unit}}\right)^2 + \left(\frac{d}{dT} h\right)^2 (\delta_T)^2 + \left(\frac{d}{dT_0} h\right)^2 \left(\delta_{T_0}\right)^2 \text{ .}$$

The partial derivatives are

$$\left(\frac{d}{d\dot{Q}_{conv}} h\right)^2 \left(\delta_{\dot{Q}_{conv}}\right)^2 = \left(\frac{1}{wl_{unit}\,(T - T_0)}\right)^2 \left(\delta_{\dot{Q}_{conv}}\right)^2 = h^2 \left(\frac{\delta_{\dot{Q}_{conv}}}{\dot{Q}_{conv}}\right)^2 \text{ ,} \tag{1-11}$$

$$\left(\frac{d}{dw} h\right)^2 (\delta_w)^2 = \left(-\frac{\dot{Q}}{w^2 (T - T_0)}\right)^2 (\delta_w)^2 = h^2 \left(\frac{\delta_w}{w}\right)^2 \text{ ,} \tag{1-12}$$

$$\left(\frac{d}{dl_{unit}} h\right)^2 \left(\delta_{l_{unit}}\right)^2 = \left(-\frac{\dot{Q}}{{l_{unit}}^2 (T - T_0)}\right)^2 \left(\delta_{l_{unit}}\right)^2 = h^2 \left(\frac{\delta_{l_{unit}}}{l_{unit}}\right)^2 \text{ ,} \tag{1-13}$$

$$\left(\frac{d}{dT} h\right)^2 (\delta_T)^2 = \left(-\frac{\dot{Q}}{{l_{unit}}^2 (T - T_0)^2}\right)^2 (\delta_T)^2 = h^2 \left(\frac{\delta_T}{T - T_0}\right)^2 \text{ ,} \tag{1-14}$$

$$\left(\frac{d}{dT_0} h\right)^2 \left(\delta_{T_0}\right)^2 = \left(\frac{\dot{Q}}{w{l_{unit}}^2 (T - T_0)^2}\right)^2 \left(\delta_{T_0}\right)^2 = h^2 \left(\frac{\delta_{T_0}}{T - T_0}\right)^2 \text{ .} \tag{1-15}$$

Therefore,

$$\left(\frac{\delta_h}{h}\right)^2 = \left(\frac{\delta_{\dot{Q}_{conv}}}{\dot{Q}_{conv}}\right)^2 + \left(\frac{\delta_w}{w}\right)^2 + \left(\frac{\delta_{l_{unit}}}{l_{unit}}\right)^2 + \left(\frac{\delta_T}{T - T_0}\right)^2 + \left(\frac{\delta_{T_0}}{T - T_0}\right)^2 \text{ .} \tag{1-16}$$

The values for measured temperatures T and T_0 are between 70°F and 130°F with an uncertainty of 0.5°F. The values $\dot{Q}_{conv}$, A, and their uncertainty are discussed in the following sections.

2. HEAT TRANSFER RATE FROM THE RIBBON

This section examines the value and uncertainty in the heat transfer rate from the ribbon. The heat transfer surface is made of a ribbon of NiChrome 1-inch wide, 0.002-inches thick, and wrapped around the outside of the cylinder 22 times, forming a helix. The heat is supplied by the Joule heating of the ribbon. The heat generated can be conducted into the cylinder along the ribbon, convected into the free stream, and radiated into the environment. The following is an attempt to quantify each of these values and to place an uncertainty on the estimated values.

The heat generated per unit area for the normal surface area of the ribbon, $\dot{q}''_{ribbon}$, is given by

$$\dot{q}''_{ribbon} = \frac{i^2 R}{w L_{ribbon}}, \tag{2-1}$$

where i the electrical current flowing through the ribbon, R is the end-to-end electrical resistance of the ribbon, w is the width of the ribbon, and L_{ribbon} is the end-to-end length of the ribbon. The heat generated by a segment of the ribbon of l_{unit} length in the direction of the helical wrapping is given by

$$\dot{Q}_{ribbon} = \dot{q}''_{ribbon} w l_{unit}, \tag{2-2}$$

Figure 2-1 shows an energy balance for a segment of ribbon of length, l_{unit}. The heat generated by the electrical current flowing through the ribbon, $\dot{Q}_{ribbon}$, plus the heat being conducted into the segment due to the temperature gradient in the ribbon is balanced by the heat being convected into the air plus the heat being radiated into the environment plus the heat being conducted into the cylinder substrate plus the heat being conducted out of the segment due to the temperature gradient in the ribbon, as follows,

$$\dot{Q}_{ribbon} + \dot{Q}_{in} = \dot{Q}_{conv} + \dot{Q}_{out} + \dot{Q}_{rad} + \dot{Q}_{cond}, \tag{2-3}$$

where $\dot{Q}_{ribbon}$ is the heat being generated in the segment due to the electrical current (see equation (2-2)), $\dot{Q}_{in}$ is the heat being conducted into the segment due to the temperature gradient in the ribbon, $\dot{Q}_{conv}$ is the heat being convected into the air, $\dot{Q}_{out}$ is the heat being conducted out of the segment due to the temperature gradient in the ribbon, $\dot{Q}_{rad}$ is the heat being radiated into the environment, and $\dot{Q}_{cond}$ is the heat being conducted into the cylinder substrate.

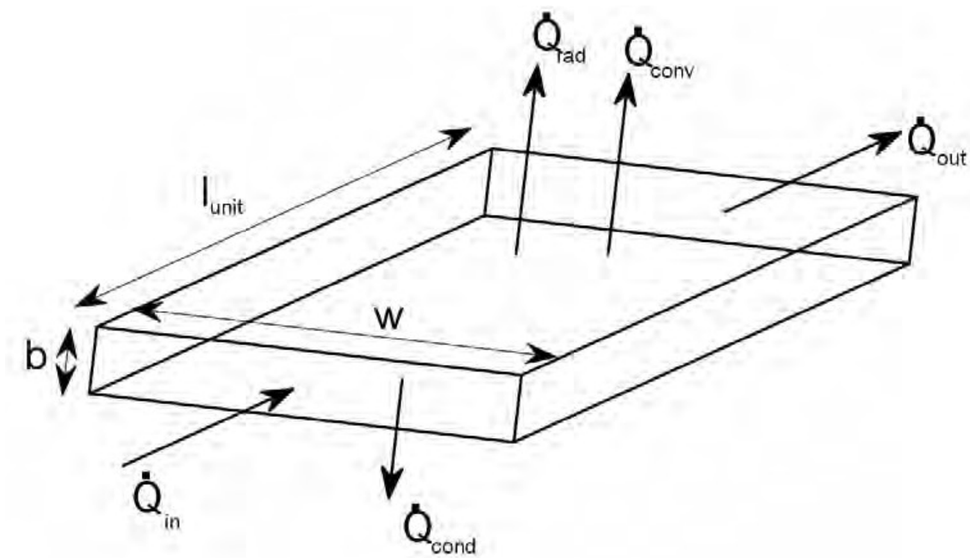

Figure 2-1. Heat Balance on a Unit Length of Ribbon

Equation (2-3) can be rewritten as

$$\dot{Q}_{conv} = \dot{Q}_{ribbon} + \left(\dot{Q}_{in} - \dot{Q}_{out}\right) - \dot{Q}_{rad} - \dot{Q}_{cond}\ . \tag{2-4}$$

Therefore, the uncertainty in the convective heat flux is

$$\begin{aligned}\left(\delta_{\dot{Q}_{conv}}\right)^2 = &\left(\frac{d}{d\dot{Q}_{ribbon}}\dot{Q}_{conv}\right)^2 \left(\delta_{\dot{Q}_{ribbon}}\right)^2 \\ &+ \left(\frac{d}{d\left(\dot{Q}_{in} - \dot{Q}_{out}\right)}\dot{Q}_{conv}\right)^2 \left(\delta_{(\dot{Q}_{in}-\dot{Q}_{out})}\right)^2 \\ &+ \left(\frac{d}{d\dot{Q}_{rad}}\dot{Q}_{conv}\right)^2 \left(\delta_{\dot{Q}_{rad}}\right)^2 \\ &+ \left(\frac{d}{d\dot{Q}_{cond}}\dot{Q}_{conv}\right)^2 \left(\delta_{\dot{Q}_{cond}}\right)^2 .\end{aligned} \tag{2-5}$$

and, after evaluating the derivatives,

$$\left(\delta_{\dot{Q}_{conv}}\right)^2 = \left(\delta_{\dot{Q}_{ribbon}}\right)^2 + \left(\delta_{(\dot{Q}_{in}-\dot{Q}_{out})}\right)^2 + \left(\delta_{\dot{Q}_{rad}}\right)^2 + \left(\delta_{\dot{Q}_{cond}}\right)^2 . \tag{2-6}$$

3. JOULE HEATING OF THE RIBBON

When the electrical current flows through the ribbon, the ribbon generates heat due to Joule heating. The amount of heat produced, $\dot{Q}_{ribbon}$, is given by equation (2-2). Substituting equation (2-1) into equation (2-2) yields

$$\dot{Q}_{ribbon} = \dot{q}''_{ribbon} w l_{unit} = \frac{i^2 R}{w L_{ribbon}} w l_{unit} = \frac{i^2 R l_{unit}}{L_{ribbon}}, \tag{3-1}$$

The uncertainty in the amount of heat produced is given by

$$\begin{aligned}\left(\delta_{\dot{Q}_{ribbon}}\right)^2 = \left(\frac{d}{di}\dot{Q}_{ribbon}\right)^2 (\delta_i)^2 + \left(\frac{d}{dR}\dot{Q}_{ribbon}\right)^2 (\delta_R)^2 \\ + \left(\frac{d}{dl_{unit}}\dot{Q}_{ribbon}\right)^2 \left(\delta_{l_{unit}}\right)^2 + \left(\frac{d}{dL_{ribbon}}\dot{Q}_{ribbon}\right)^2 \left(\delta_{L_{ribbon}}\right)^2,\end{aligned} \tag{3-2}$$

$$\frac{d}{di}\dot{Q}_{ribbon} = \frac{2iRl_{unit}}{L_{ribbon}} = \frac{2\dot{Q}_{ribbon}}{i}, \tag{3-3}$$

$$\frac{d}{dR}\dot{Q}_{ribbon} = \frac{i^2 l_{unit}}{L_{ribbon}} = \frac{\dot{Q}_{ribbon}}{R}, \tag{3-4}$$

$$\frac{d}{dl_{unit}}\dot{Q}_{ribbon} = \frac{i^2 R}{L_{ribbon}} = \frac{\dot{Q}_{ribbon}}{l_{unit}}, \tag{3-5}$$

$$\frac{d}{dL_{ribbon}}\dot{Q}_{ribbon} = \frac{i^2 R l_{unit}}{L_{ribbon}^2} = \frac{\dot{Q}_{ribbon}}{L_{ribbon}}. \tag{3-6}$$

Substituting equations (3-4) through (3-6) into equation (3-2) yields

$$\left(\frac{\delta_{\dot{Q}_{ribbon}}}{\dot{Q}_{ribbon}}\right)^2 = (2)^2\left(\frac{\delta_i}{i}\right)^2 + \left(\frac{\delta_R}{R}\right)^2 + \left(\frac{\delta_{l_{unit}}}{l_{unit}}\right)^2 + \left(\frac{\delta_{L_{ribbon}}}{L_{ribbon}}\right)^2. \tag{3-7}$$

3.1 HEAT CONDUCTION ALONG THE LENGTH OF THE RIBBON

The heat transfer rate along an infinitesimal length of the ribbon due to the temperature gradient in the ribbon is given by

$$\dot{Q} = k_{NiCr} b w \frac{dT}{ds}, \tag{3-8}$$

where k_{NiCr} is the thermal conductivity of the NiChrome ribbon, b is the ribbon thickness, w is the width of the ribbon and, $\frac{dT}{ds}$ is the temperature gradient in the ribbon. And for a finite segment,

$$\Delta\dot{Q} = \dot{Q}_{out} - \dot{Q}_{in} \; . \tag{3-9}$$

Writing a Taylor series for the heat transfer rate out of the segment in terms of the heat transfer rate into the segment yields,

$$\dot{Q}_{out} = \dot{Q}_{in} + \Delta s \frac{d\dot{Q}}{ds} + \frac{\Delta s^2}{2} \frac{d^2\dot{Q}}{ds^2} \, , \tag{3-10}$$

where $\Delta s = l_{unit}$ is the length of the segment.

The rate of change of the heat transfer is found by differentiation of equation (3-8),

$$\frac{d\dot{Q}}{ds} = k_{NiCr} bw \frac{d^2T}{ds^2} \; . \tag{3-11}$$

Substituting equation (3-11) into equation (3-10) and neglecting the second order term yields

$$\dot{Q}_{out} = \dot{Q}_{in} + k_{NiCr} bw \frac{d^2T}{ds^2} \Delta s \, , \tag{3-12}$$

or rearranged and letting $\Delta\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out}$ yields

$$\Delta\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out} = -k_{NiCr} bw \frac{d^2T}{ds^2} \Delta s \; . \tag{3-13}$$

The uncertainty in this heat flux is

$$\begin{aligned} \left(\delta_{\Delta\dot{Q}}\right)^2 &= \left(\frac{d}{dk_{NiCr}}\Delta\dot{Q}\right)^2 \left(\delta_{k_{NiCr}}\right)^2 \\ &+ \left(\frac{d}{db}\Delta\dot{Q}\right)^2 \left(\delta_b\right)^2 + \left(\frac{d}{dw}\Delta\dot{Q}\right)^2 \left(\delta_w\right)^2 \\ &+ \left(\frac{d}{d\frac{d^2T}{ds^2}}\Delta\dot{Q}\right)^2 \left(\delta_{\frac{d^2T}{ds^2}}\right)^2 \\ &+ \left(\frac{d}{d\Delta s}\Delta\dot{Q}\right)^2 \left(\delta_{\Delta s}\right)^2 \end{aligned} \tag{3-14}$$

.

Carrying out the differentiation

$$\left(\frac{\delta_{\Delta\dot{Q}}}{\Delta\dot{Q}}\right)^2 = \left(\frac{\delta_{k_{NiCr}}}{k_{NiCr}}\right)^2 + \left(\frac{\delta_b}{b}\right)^2 + \left(\frac{\delta_w}{w}\right)^2 + \left(\frac{\delta_{\frac{d^2T}{ds^2}}}{\frac{d^2T}{ds^2}}\right)^2 + \left(\frac{\delta_{\Delta s}}{\Delta s}\right)^2. \tag{3-15}$$

The second derivative of the temperature distribution along the ribbon was found by using a Taylor's series expansion about the point in question.

$$T_{i+1} = T_i + \Delta s \left.\frac{dT}{ds}\right|_i + \frac{\Delta s^2}{2!}\left.\frac{d^2T}{ds^2}\right|_i + \ldots \tag{3-16}$$

,

$$T_{i-1} = T_i - \Delta s \left.\frac{dT}{ds}\right|_i + \frac{\Delta s^2}{2!}\left.\frac{d^2T}{ds^2}\right|_i + \ldots \tag{3-17}$$

.

adding these equations and neglecting all terms higher than second order yields

$$\left.\frac{d^2T}{ds^2}\right|_i = \frac{T_{i-1} + T_{i+1} - 2\,T_i}{\Delta s^2} \tag{3-18}$$

.

The distance Δs can be rewritten in terms of arc angle as

$$\Delta s = r\,\Delta\theta = l_{unit}\,. \tag{3-19}$$

where r is the outside radius of the cylinder, and $\Delta\theta$ is the angular displacement. For the cylinder, the thermocouples were placed $10° \pm 0.1°$ apart, which gave a position uncertainty of $\delta_{\Delta s} = 0.01\ inch$.

The uncertainty in equation (3-18) is given as

$$\left(\delta_{\left.\frac{d^2T}{ds^2}\right|_i}\right)^2 = \frac{1}{\Delta s^4}\left[\left(\delta_{T_{i-1}}\right)^2 + \left(\delta_{T_{i+1}}\right)^2 + 4\left(\delta_{T_i}\right)^2\right] + \left(\frac{T_{i-1} + T_{i+1} - 2T_i}{2\Delta s^3}\right)^2 \left(\delta_{\Delta s}\right)^2 . \tag{3-20}$$

3.2 HEAT LOSS THROUGH THE CYLINDER

If the system were in steady state, the heat transfer rate into the cylinder is given by

$$\dot{Q}_{cond} = UA(T - T_{int}) , \tag{3-21}$$

where UA is the overall heat transfer coefficient, and T and T_{int} are the temperatures of the ribbon and the interior of the cylinder. The overall heat transfer coefficient is given by

$$UA = \frac{1}{R_{Internal\ Convection} + R_{pvc} + R_{contact}} , \tag{3-22}$$

where $R_{Internal\ Convection}$ is the thermal resistance due to convective heat transfer to the interior of the cylinder, R_{pvc} is the thermal resistance of the PVC cylinder, and $R_{contact}$ is the thermal resistance between the ribbon and the outside of the cylinder, and was neglected,

$$R_{Internal\ Convection} = \frac{1}{\frac{r_i}{r_o} w l_{unit}\, h_{int}} , \tag{3-23}$$

$$R_{pvc} = \frac{r_o \ln(r_o/r_i)}{k_{pvc}\, w l_{unit}} . \tag{3-24}$$

Substituting equations (3-22), (3-23), and (3-24) into equation (3-21) yields

$$\dot{Q}_{cond} = \frac{(T - T_{int})}{\frac{r_o}{r_i\, w\, l_{unit}\, h_{int}} + \frac{r_o \ln(r_o/r_i)}{k_{pvc}\, w\, l_{unit}}} . \tag{3-25}$$

This can be simplified to

$$\dot{Q}_{cond} = \frac{h_{int}\, k_{pvc}\, l_{unit}\, r_i\, w\, \left(T - T_{int}\right)}{r_o\left(k_{pvc} + h_{int}\, r_i\, \ln\left(r_o/r_i\right)\right)} \,. \tag{3-26}$$

The uncertainty in this heat flux is

$$\begin{aligned}
\left(\delta_{\dot{Q}_{cond}}\right)^2 &= \left(\frac{d}{dT}\dot{Q}_{cond}\right)^2 \left(\delta_T\right)^2 \\
&+ \left(\frac{d}{dT_{int}}\dot{Q}_{cond}\right)^2 \left(\delta_{T_{int}}\right)^2 \\
&+ \left(\frac{d}{dr_o}\dot{Q}_{cond}\right)^2 \left(\delta_{r_o}\right)^2 \\
&+ \left(\frac{d}{dr_i}\dot{Q}_{cond}\right)^2 \left(\delta_{r_i}\right)^2 \\
&+ \left(\frac{d}{dw}\dot{Q}_{cond}\right)^2 \left(\delta_w\right)^2 \\
&+ \left(\frac{d}{dl_{unit}}\dot{Q}_{cond}\right)^2 \left(\delta_{l_{unit}}\right)^2 \\
&+ \left(\frac{d}{dh_{int}}\dot{Q}_{cond}\right)^2 \left(\delta_{h_{int}}\right)^2 \\
&+ \left(\frac{d}{dk_{pvc}}\dot{Q}_{cond}\right)^2 \left(\delta_{k_{pvc}}\right)^2
\end{aligned} \,. \tag{3-27}$$

Carrying out the differentiations

$$\frac{d}{dT}\dot{Q}_{cond} = \frac{h_{int}\,k_{pvc}\,l_{unit}\,r_i\,w}{r_o\left(k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right)} = \frac{\dot{Q}_{cond}}{T - T_{int}}, \tag{3-28}$$

$$\frac{d}{dT_{int}}\dot{Q}_{cond} = -\frac{h_{int}\,k_{pvc}\,l_{unit}\,r_i\,w}{r_o\left(k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right)} = -\frac{\dot{Q}_{cond}}{T - T_{int}}, \tag{3-29}$$

$$\begin{aligned}\frac{d}{dr_o}\dot{Q}_{cond} &= \frac{h_{int}\,k_{pvc}\,l_{unit}\,r_i\,w\,\left(T - T_{int}\right)\left[k_{pvc} + h_{int}\,r_i\,\left(1 + \ln\left(r_o/r_i\right)\right)\right]}{r_o^2\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]^2} \\ &= \frac{\dot{Q}_{cond}}{r_o}\left[1 + \frac{h_{int}\,r_i}{k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)}\right],\end{aligned} \tag{3-30}$$

$$\begin{aligned}\frac{d}{dr_i}\dot{Q}_{cond} &= \frac{h_{int}\,k_{pvc}\,l_{unit}\,w\,\left(T - T_{int}\right)\left[k_{pvc} + h_{int}\,r_i\right]}{r_o\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]^2} \\ &= \frac{\dot{Q}_{cond}}{r_i}\,\frac{k_{pvc} + h_{int}\,r_i}{k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)},\end{aligned} \tag{3-31}$$

$$\frac{d}{dw}\dot{Q}_{cond} = \frac{h_{int}\,k_{pvc}\,l_{unit}\,r_i\,\left(T - T_{int}\right)}{r_o\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]} = \frac{\dot{Q}_{cond}}{w}, \tag{3-32}$$

$$\frac{d}{dl_{unit}}\dot{Q}_{cond} = \frac{h_{int}\,k_{pvc}\,r_i\,w\,\left(T - T_{int}\right)}{r_o\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]} = \frac{\dot{Q}_{cond}}{l_{unit}}, \tag{3-33}$$

$$\begin{aligned}\frac{d}{dh_{int}}\dot{Q}_{cond} &= \frac{k_{pvc}^2\,l_{unit}\,r_i\,w\,\left(T - T_{int}\right)}{r_o\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]^2} \\ &= \frac{\dot{Q}_{cond}}{h_{int}}\,\frac{k_{pvc}}{k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)},\end{aligned} \tag{3-34}$$

$$\begin{aligned}\frac{d}{dk_{pvc}}\dot{Q}_{cond} &= \frac{h_{int}^2\,l_{unit}\,r_i^2\,w\,\ln\left(r_o/r_i\right)\,\left(T - T_{int}\right)}{r_o\left[k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)\right]^2} \\ &= \dot{Q}_{cond}\left(\frac{1}{k_{pvc}} - \frac{1}{k_{pvc} + h_{int}\,r_i\,\ln\left(r_o/r_i\right)}\right).\end{aligned} \tag{3-35}$$

Substituting these partial derivatives into equation (3-27) yields the uncertainty in this heat flux as

$$
\begin{aligned}
\left(\dot{Q}_{cond}\right)^2 &= \left(\frac{\dot{Q}_{cond}}{T-T_{int}}\right)^2 (\delta_T)^2 + \left(\frac{\dot{Q}_{cond}}{T-T_{int}}\right)^2 (\delta_{T_{int}})^2 \\
&+ \left(\frac{\dot{Q}_{cond}}{r_o}\left(1+\frac{h_{int}\, r_i}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)\right)^2 (\delta_{r_o})^2 \\
&+ \left(\frac{\dot{Q}_{cond}}{r_i}\frac{k_{pvc}+h_{int}\, r_i}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)^2 (\delta_{r_i})^2 \\
&+ \left(\frac{\dot{Q}_{cond}}{w}\right)^2 (\delta_w)^2 + \left(\frac{\dot{Q}_{cond}}{l_{unit}}\right)^2 (\delta_{l_{unit}})^2 \\
&+ \left(\frac{\dot{Q}_{cond}}{h_{int}}\left(\frac{k_{pvc}}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)\right)^2 (\delta_{h_{int}})^2 \\
&+ \left(\frac{\dot{Q}_{cond}}{k_{pvc}}\left(1-\frac{1}{1+\frac{h_{int}}{k_{pvc}}\, r_i\, \ln r_o/r_i}\right)\right)^2 (\delta_{k_{pvc}})^2 ,
\end{aligned}
\tag{3-36}
$$

or

$$
\begin{aligned}
\left(\frac{\delta \dot{Q}_{cond}}{\dot{Q}_{cond}}\right)^2 &= \left(\frac{\delta_T}{T-T_{int}}\right)^2 + \left(\frac{\delta_{T_{int}}}{T-T_{int}}\right)^2 \\
&+ \left(1+\frac{h_{int}\, r_i}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)^2 \left(\frac{\delta_{r_o}}{r_o}\right)^2 \\
&+ \left(\frac{k_{pvc}+h_{int}\, r_i}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)^2 \left(\frac{\delta_{r_i}}{r_i}\right)^2 \\
&+ \left(\frac{\delta_w}{w}\right)^2 + \left(\frac{\delta_{l_{unit}}}{l_{unit}}\right)^2 \\
&+ \left(\frac{k_{pvc}}{k_{pvc}+h_{int}\, r_i\, \ln r_o/r_i}\right)^2 \left(\frac{\delta_{h_{int}}}{h_{int}}\right)^2 \\
&+ \left(1-\frac{1}{1+\frac{h_{int}}{k_{pvc}}\, r_i\, \ln r_o/r_i}\right)^2 \left(\frac{\delta_{\delta_{k_{pvc}}}}{\delta_{k_{pvc}}}\right)^2 ,
\end{aligned}
\tag{3-37}
$$

4. INTERNAL NATURAL CONVECTION

The heat transfer on the internal surface of the cylinder was assumed to be by natural convection. The natural convection heat coefficient, h_{int}, is given by

$$h_{int} = \frac{Nu_{int} k}{2r_i}, \tag{4-1}$$

where Nu_{int} is the Nusselt number for the flow within the cylinder, k is the thermal conductivity of the air within the cylinder, and r_i is the inside radius of the cylinder. The natural convection Nusselt number for an external heated cylinder[2]

$$Nu_{int} = 0.15 Ra^{0.22}, \tag{4-2}$$

where Ra is the Rayleigh number given by

$$Ra = \frac{g\beta\,(T_{surface} - T_{int})(2r_i)^3}{v\alpha}, \tag{4-3}$$

where g is the acceleration due to gravity, β is the volumetric expansion coefficient of air and is equal to the reciprocal of the absolute temperature, and v and α are the kinematic viscosity and the thermal diffusivity of air, respectively. $T_{surface}$ is the surface temperature of the inside of the cylinder and was assumed to be uniform. T_{int} is the temperature of the air within the cylinder and was also assumed to be uniform.

The uncertainty in the internal convective heat transfer coefficient is

$$\begin{aligned}(\delta_{h_{int}})^2 &= \left(\frac{d}{dNu_{int}} h_{int}\right)^2 (\delta_{Nu_{int}})^2 \\ &+ \left(\frac{d}{dk} h_{int}\right)^2 (\delta_k)^2 + \left(\frac{d}{dr_i} h_{int}\right)^2 (\delta_{r_i})^2 .\end{aligned} \tag{4-4}$$

Carrying out the differentiations,

$$\frac{d}{dNu_{int}} h_{int} = \frac{k}{2r_i} = \frac{h_{int}}{Nu_{int}}, \tag{4-5}$$

$$\frac{d}{dk} h_{int} = \frac{Nu_{int} k}{2r_i} = \frac{h_{int}}{k}, \tag{4-6}$$

$$\frac{d}{dr_i} h_{int} = -\frac{Nu_{int}}{2r_i^2} = -\frac{h_{int}}{2r_i}, \tag{4-7}$$

$$\left(\frac{\delta_{h_{int}}}{h_{int}}\right)^2 = \left(\frac{\delta_{Nu_{int}}}{Nu_{int}}\right)^2 + \left(\frac{\delta_k}{k}\right)^2 + \left(\frac{\delta_{r_i}}{r_i}\right)^2 . \tag{4-8}$$

The uncertainty in the Nusselt number is given by

$$(\delta_{Nu_{int}})^2 = \left(\frac{d}{dRa} Nu_{int}\right)^2 (\delta_{Ra})^2 = \left(0.033\, Ra^{-0.78}\right)^2 (\delta_{Ra})^2 , \tag{4-9}$$

and the uncertainty in the Rayleigh number is given by

$$\begin{aligned} (\delta_{Ra})^2 &= \left(\frac{d}{dg} Ra\right)^2 (\delta_g)^2 \\ &+ \left(\frac{d}{d\beta} Ra\right)^2 (\delta_\beta)^2 \\ &+ \left(\frac{d}{dT_{surface}} Ra\right)^2 (\delta_{T_{surface}})^2 \\ &+ \left(\frac{d}{dT_{int}} Ra\right)^2 (\delta_{T_{int}})^2 \\ &+ \left(\frac{d}{dr_i} Ra\right)^2 (\delta_{r_i})^2 \\ &+ \left(\frac{d}{d\nu} Ra\right)^2 (\delta_\nu)^2 \\ &+ \left(\frac{d}{d\alpha} Ra\right)^2 (\delta_\alpha)^2 , \end{aligned} \tag{4-10}$$

Carrying out the differentiations,

$$\frac{d}{dg} Ra = \frac{\beta \left(T_{surface} - T_{int}\right) (2\, r_i)^3}{\nu\, \alpha} = \frac{Ra}{g} , \tag{4-11}$$

$$\frac{d}{d\beta}Ra = \frac{g\left(T_{surface}-T_{int}\right)(2\,r_i)^3}{\nu\,\alpha} = \frac{Ra}{\beta}\,, \tag{4-12}$$

$$\frac{d}{dT_{surface}}Ra = \frac{g\,\beta\,(2\,r_i)^3}{\nu\,\alpha} = \frac{Ra}{(T_{surface}-T_{int})}\,, \tag{4-13}$$

$$\frac{d}{dT_{int}}Ra = -\frac{g\,\beta\,(2\,r_i)^3}{\nu\,\alpha} = -\frac{Ra}{(T_{surface}-T_{int})}\,, \tag{4-14}$$

$$\frac{d}{dr_i}Ra = \frac{g\,\beta\,(T_{surface}-T_{int})\,24\,r_i^2}{\nu\,\alpha} = \frac{3\,Ra}{r_i}\,, \tag{4-15}$$

$$\frac{d}{d\nu}Ra = -\frac{g\,\beta\,(T_{surface}-T_{int})\,(2\,r_i)^3}{\nu^2\,\alpha} = -\frac{Ra}{\nu}\,, \tag{4-16}$$

$$\frac{d}{d\alpha}Ra = -\frac{g\,\beta\,(T_{surface}-T_{int})\,(2\,r_i)^3}{\nu\,\alpha^2} = -\frac{Ra}{\alpha}\,. \tag{4-17}$$

Substituting back into equation (4-10) yields

$$\begin{aligned}\left(\frac{\delta_{Ra}}{Ra}\right)^2 &= \left(\frac{\delta_g}{g}\right)^2 + \left(\frac{\delta_\beta}{\beta}\right)^2 \\ &+ \left(\frac{\delta_{T_{surface}}}{T_{surface}-T_{int}}\right)^2 + \left(\frac{\delta_{T_{int}}}{T_{surface}-T_{int}}\right)^2 \\ &+ (3)^2\left(\frac{\delta_{r_i}}{r_i}\right)^2 + \left(\frac{\delta_\nu}{\nu}\right)^2 + \left(\frac{\delta_\alpha}{\alpha}\right)^2 \,.\end{aligned} \tag{4-18}$$

4.1 HEAT LOSS DUE TO RADIATION TO THE AMBIENT

The heat being radiated into the environment is

$$\dot{Q}_{rad} = \epsilon\,\sigma\,w\,l_{unit}(T^4 - T_0^4)\,. \tag{4-19}$$

The uncertainty in this heat flux is

$$\begin{aligned}
\left(\delta_{\dot{Q}_{rad}}\right)^2 &= \left(\frac{d}{d\epsilon}\dot{Q}_{rad}\right)^2 (\delta_\epsilon)^2 + \left(\frac{d}{d\sigma}\dot{Q}_{rad}\right)^2 (\delta_\sigma)^2 \\
&+ \left(\frac{d}{dw}\dot{Q}_{rad}\right)^2 (\delta_w)^2 + \left(\frac{d}{dl_{unit}}\dot{Q}_{rad}\right)^2 (\delta_{l_{unit}})^2 \\
&+ \left(\frac{d}{dT}\dot{Q}_{rad}\right)^2 (\delta_T)^2 + \left(\frac{d}{dT_0}\dot{Q}_{rad}\right)^2 (\delta_{T_0})^2 \quad .
\end{aligned} \tag{4-20}$$

Carrying out the differentiation,

$$\begin{aligned}
\left(\frac{\delta_{\dot{Q}_{rad}}}{\dot{Q}_{rad}}\right)^2 &= \left(\frac{\delta_\epsilon}{\epsilon}\right)^2 + \left(\frac{\delta_\sigma}{\sigma}\right)^2 + \left(\frac{\delta_w}{w}\right)^2 + \left(\frac{\delta_{l_{unit}}}{l_{unit}}\right)^2 \\
&+ \left(\frac{4\,T^4}{T^4 - T_0^4}\right)^2 \left(\frac{\delta_T}{T}\right)^2 + \left(\frac{4\,T_0^4}{T^4 - T_0^4}\right)^2 \left(\frac{\delta_{T_0}}{T_0}\right)^2 \quad .
\end{aligned} \tag{4-21}$$

5. RESULTS

The uncertainty analysis was applied to the data described in reference 1 using the parameters given in table 5-1. The results showed the uncertainty in the Nusselt number ranges from a low of 15% to a high of 25%, as shown in table 5-2. For each wind tunnel speed, the table presents the maximum and minimum values of the Nusselt number, the angle where it occurred, and the uncertainty in those values. The minimum uncertainty occurs at the highest wind tunnel speed, and the maximum occurs at the lowest speed. The low values of the Nusselt number have slightly more uncertainty than the higher values.

Table 5-1. Uncertainty Values

	Symbol	Minimum	Maximum	Mean	Uncertainty	Units
Cylinder Diameter	D	32.192	32.580	32.385	0.191	cm
Cylinder Inside Radius	r_i	15.065	15.258	15.161	0.097	cm
Cylinder Outside Radius	r_o	16.096	16.290	16.193	0.097	cm
Ribbon Thickness	b	0.005	0.006	0.005	0.0005	cm
Ribbon Width	w	2.527	2.553	2.540	0.013	cm
Segment Length	l_{unit}	2.814	2.840	2.827	0.013	cm
Segment Length	Δs	2.814	2.840	2.827	0.013	cm
Ribbon Temperature	T	25	54	40	0.5	°C
Ambient Temperature	T_o	17	24	20	0.5	°C
Cylinder Inside Surface Temperature	$T_{surface}$	17	54	35.5	20.0	°C
cylinder Inside Air Temperature	T_{int}	17	54	35.5	20.0	°C
Current	i	5.00	6.55	6.00	0.01	Amperes
Ribbon Resistance	R	20.50	20.55	20.53	0.025	Ω
Air Thermal Conductivity	k			0.027	0.0027	watt/m K
NiChrome Thermal Conductivity	k_{NiCr}			11.3	2.1	watt/m K
PVC Thermal Conductivity	k_{pvc}			0.19	0.01	watt/m K
Stefan-Boltzmann Constant	σ	-	-	5.670400×10^{-8}	-	watt/m^{2}K^4
Emissivity of Ribbon	E			0.2	0.4	-
Number of Wraps	N_{wraps}			22	0.5	-
Coefficient of Thermal Expansion Air	β			3.2×10^{-3}	3.2×10^{-4}	$\frac{1}{K}$
Kinematic Viscosity Air	ν			1.68×10^{-5}	1.68×10^{-6}	m^2/s
Thermal Diffusivity Air	α			2.38×10^{-5}	2.38×10^{-6}	m^2/s

Table 5-2. Nusselt Number Values and Uncertainty

Tunnel Speed (m/s)	Minimum Nu				Maximum Nu			
	Angle	Value	Uncertainty	% Uncertainty	Angle	Value	Uncertainty	% Uncertainty
10	−74	206	51	24.90%	−20	463	93	20.13%
20	−100	301	66	21.91%	−114	1404	287	20.48%
30	−100	388	68	17.54%	−110	1534	255	16.61%
40	−90	523	85	16.18%	−102	1691	275	16.27%
50	−90	603	92	15.30%	−100	1922	303	15.76%

Figures 5-1 through 5-5 show the Nusselt Number as a function of angle from the stagnation point for each of the five wind tunnel speeds. The plots contain the error bars indicating the uncertainty of each measurement.

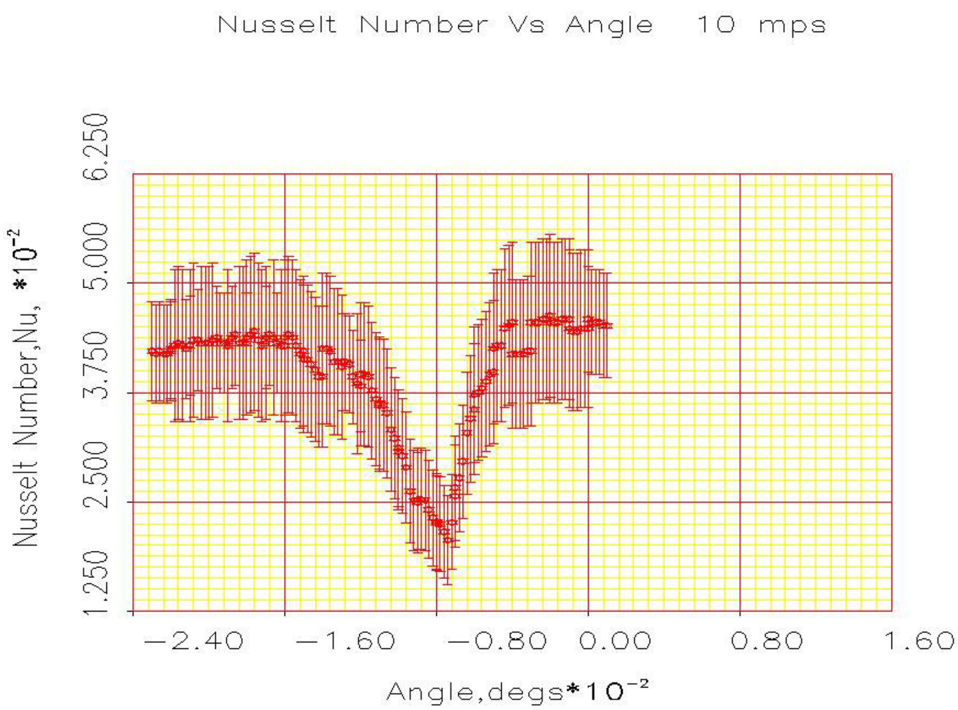

Figure 5-1. Nusselt Number 10 m/s

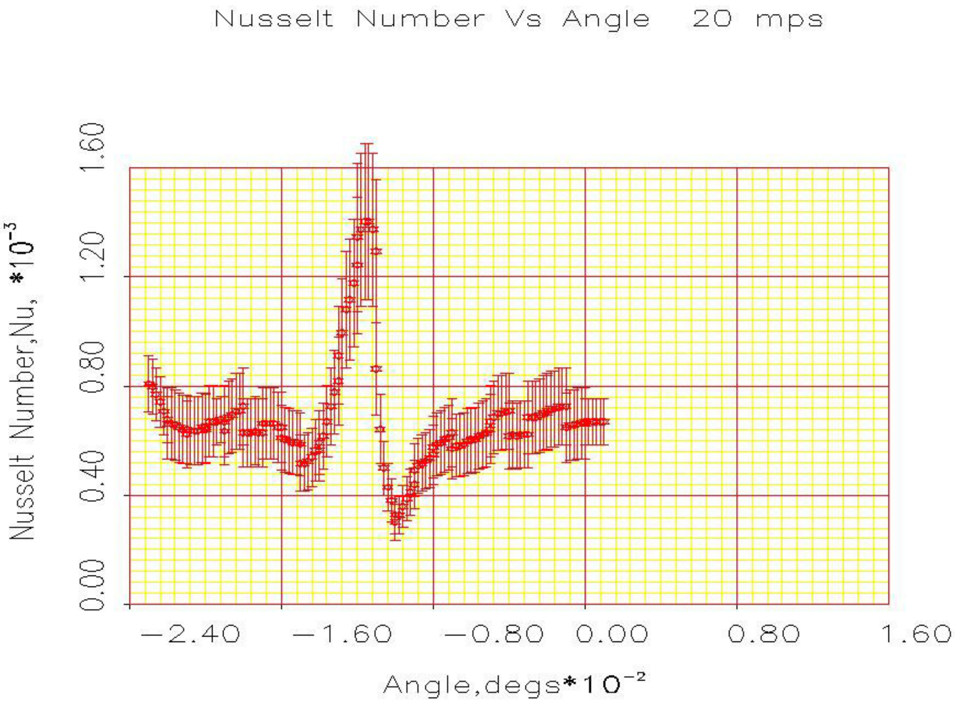

Figure 5-2. Nusselt Number 20 m/s

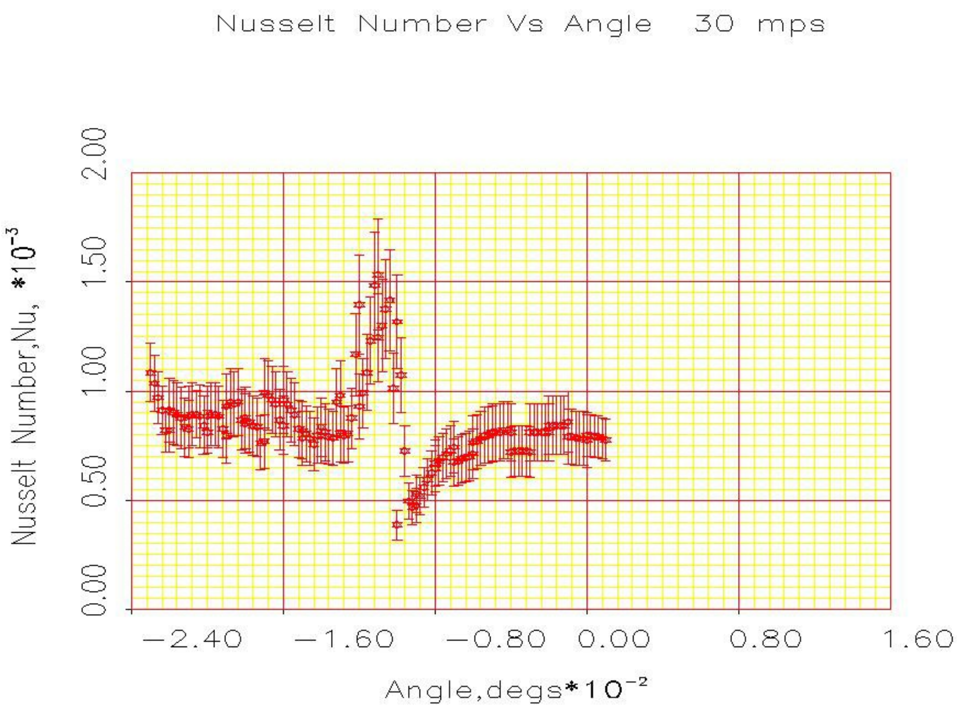

Figure 5-3. Nusselt Number 30 m/s

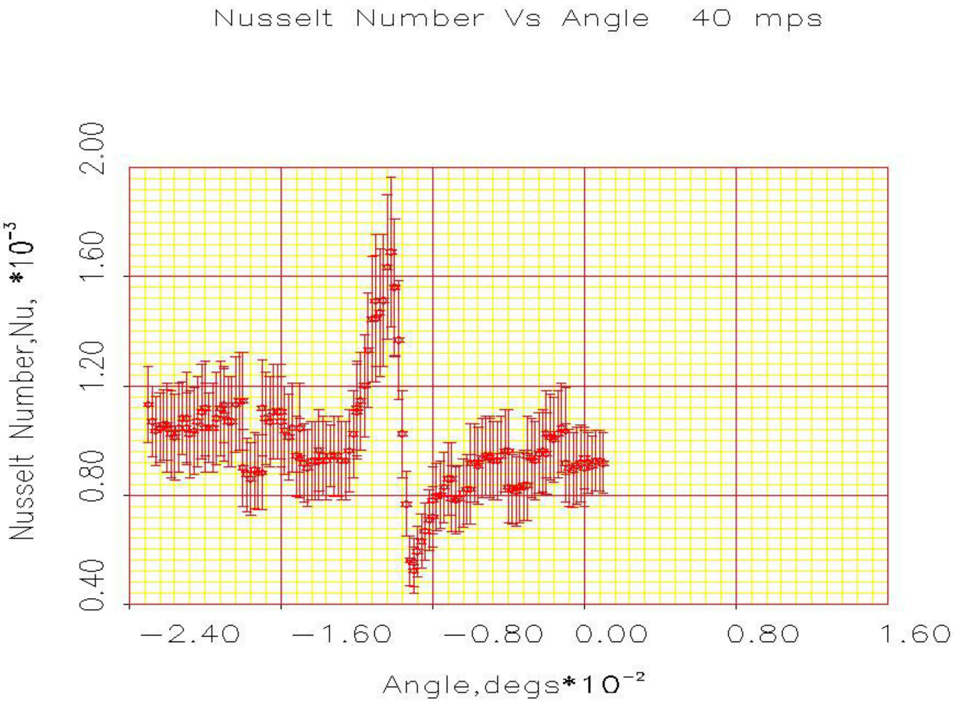

Figure 5-4. Nusselt Number 40 m/s

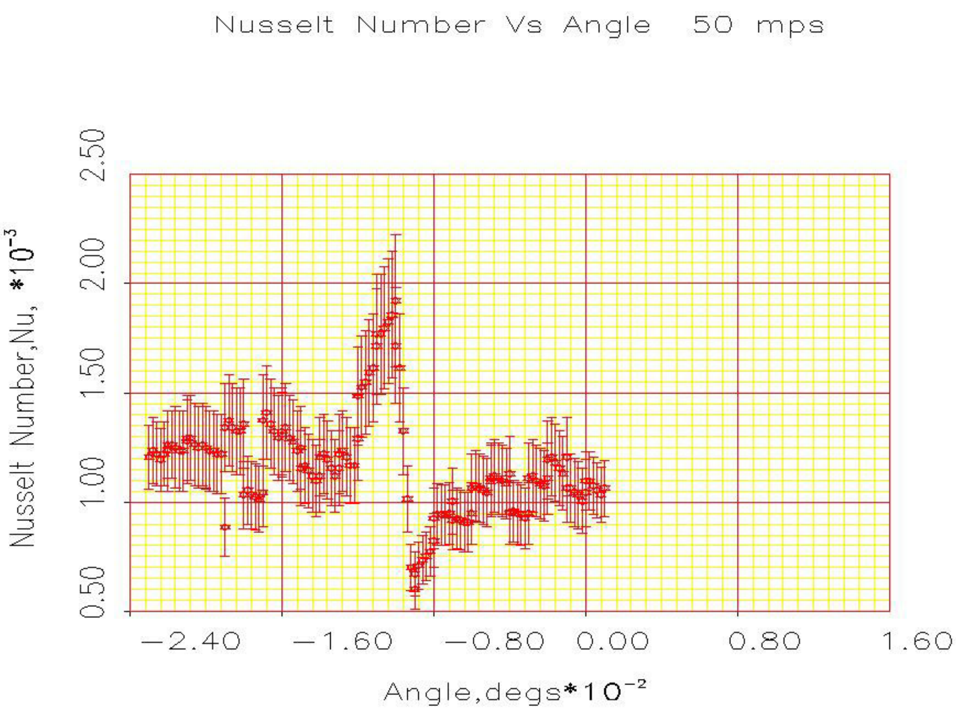

Figure 5-5. Nusselt Number 50 m/s

In order to understand the major contributors to uncertainty in the Nusselt number, it is useful to examine the components of the energy balance given by equation (2-3). Figures 5-6 through 5-10 show the percentage of the heat transfer by external convection from the ribbon of the heat generated by the ribbon. The figures show that depending on the angle, for 10 m/s the heat transfer due to external convection ranged from 90% to 95.4% of the power generated by the ribbon. At 50 m/s, the percentages increased to 96% to 99%.

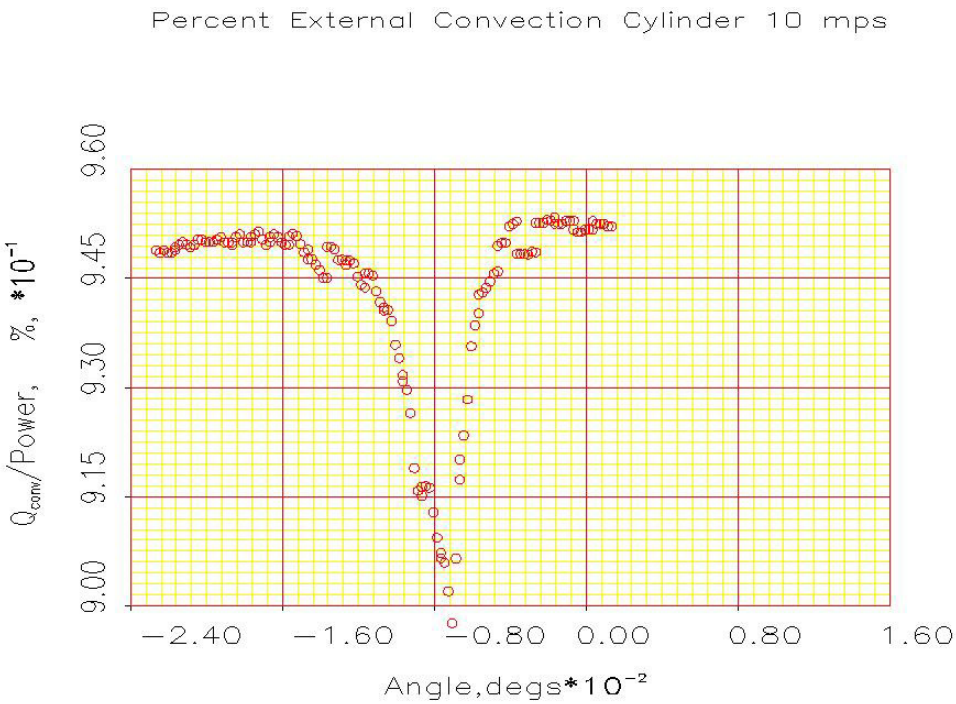

Figure 5-6. Percent External Convection Heat Transfer Rate 10 m/s

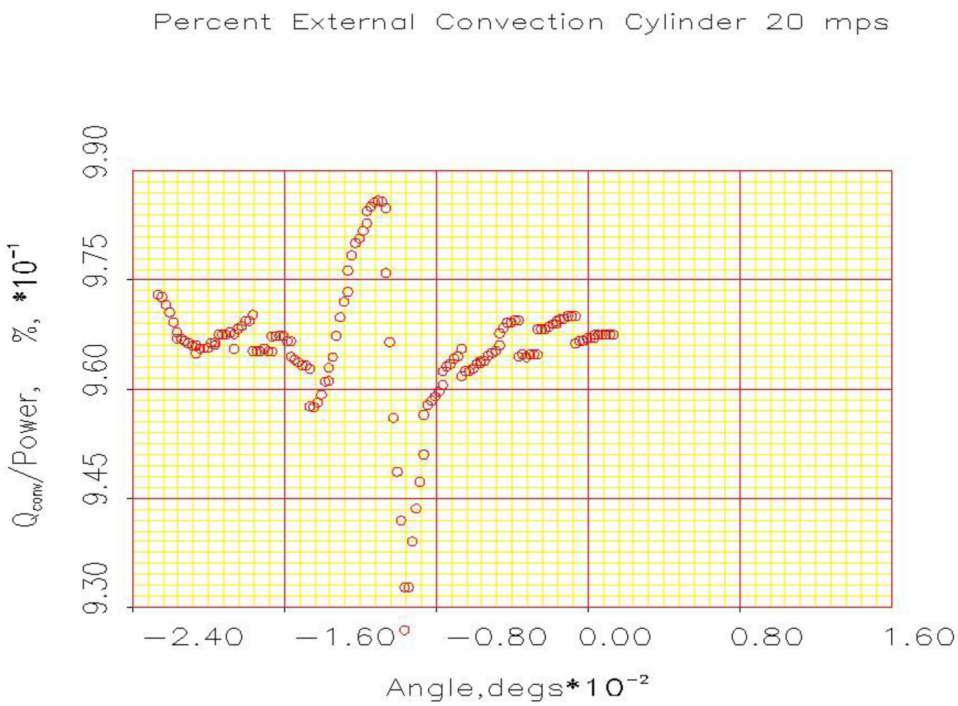

Figure 5-7. Percent External Convection Heat Transfer Rate 20 m/s

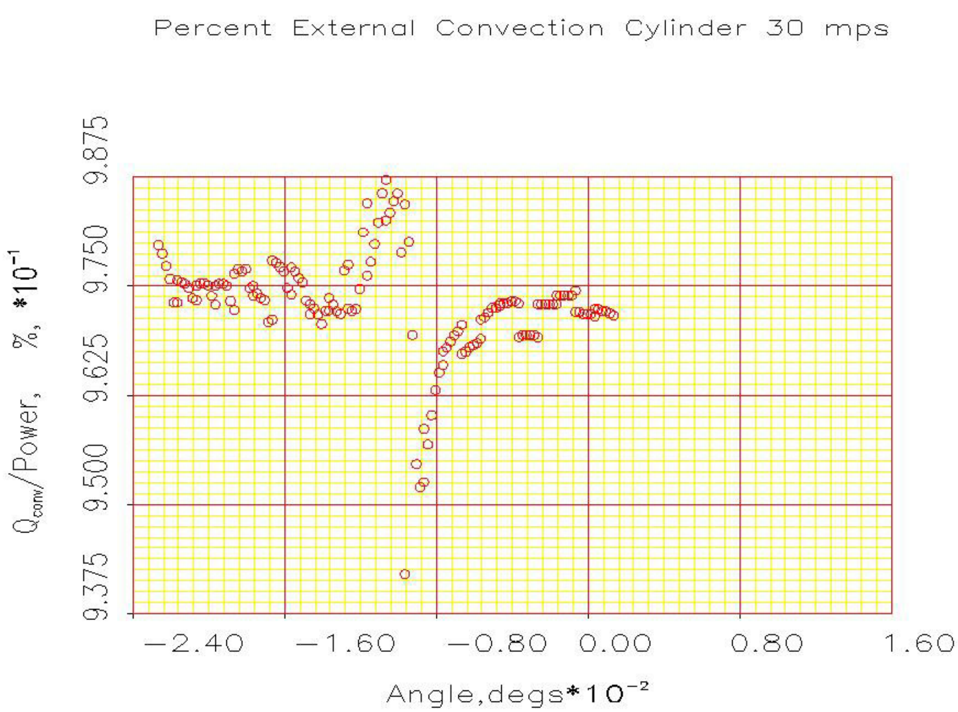

Figure 5-8. Percent External Convection Heat Transfer Rate 30 m/s

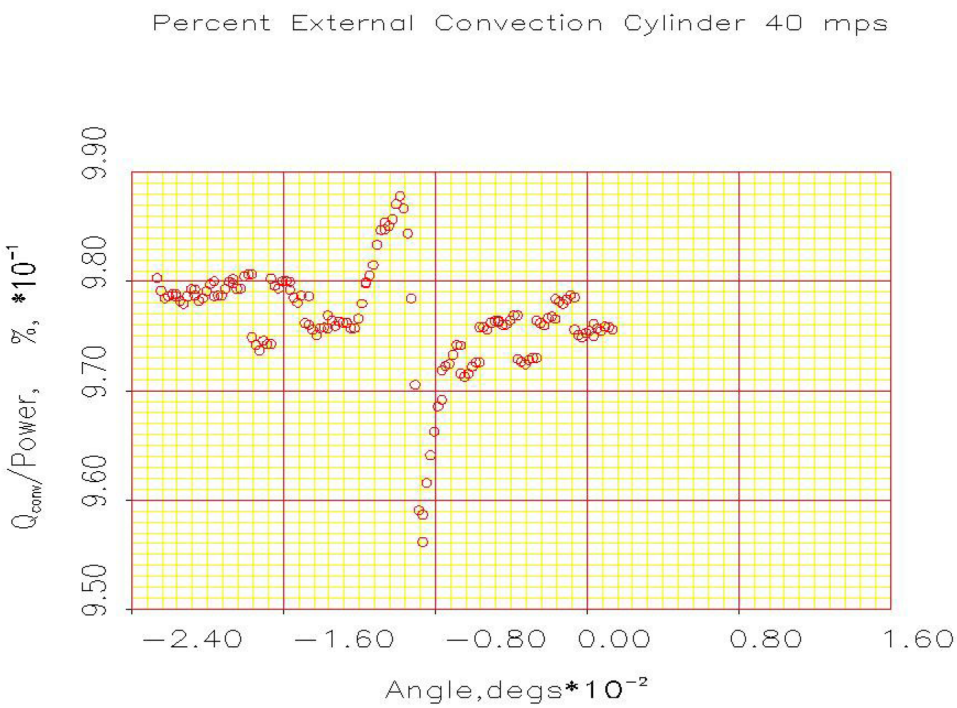

Figure 5-9. Percent External Convection Heat Transfer Rate 40 m/s

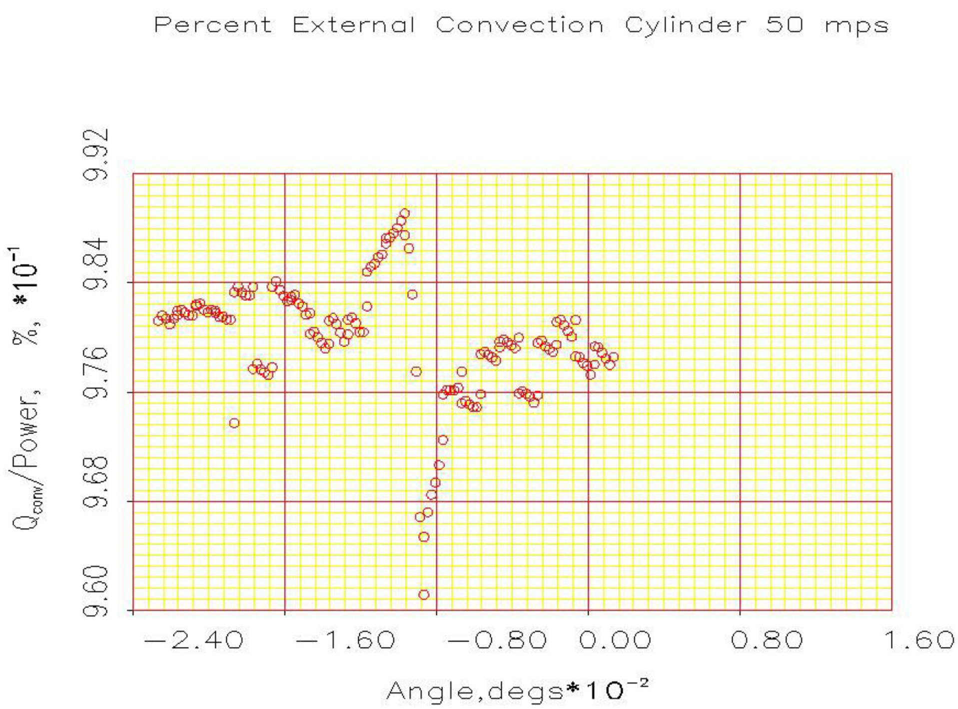

Figure 5-10. Percent External Convection Heat Transfer Rate 50 m/s

Figures 5-11 through 5-15 present the percentage of the heat generated by the ribbon that is conducted into the cylinder. The maximum percentage of the heat conducted into the cylinder is less than 3.2% and can be safely neglected.

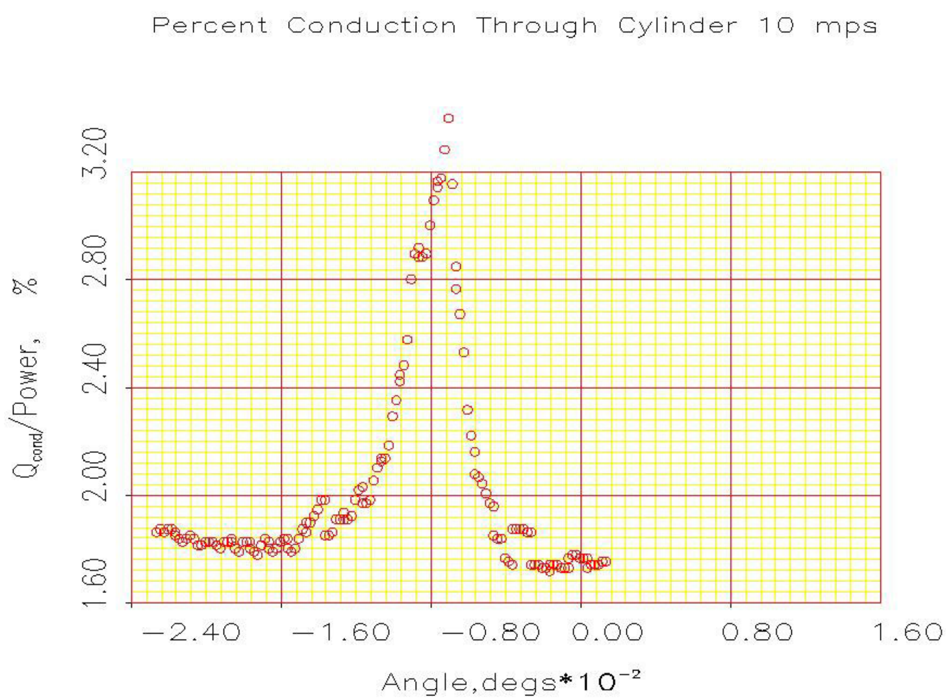

Figure 5-11. Percent Conduction Heat Transfer Rate through Cylinder 10 m/s

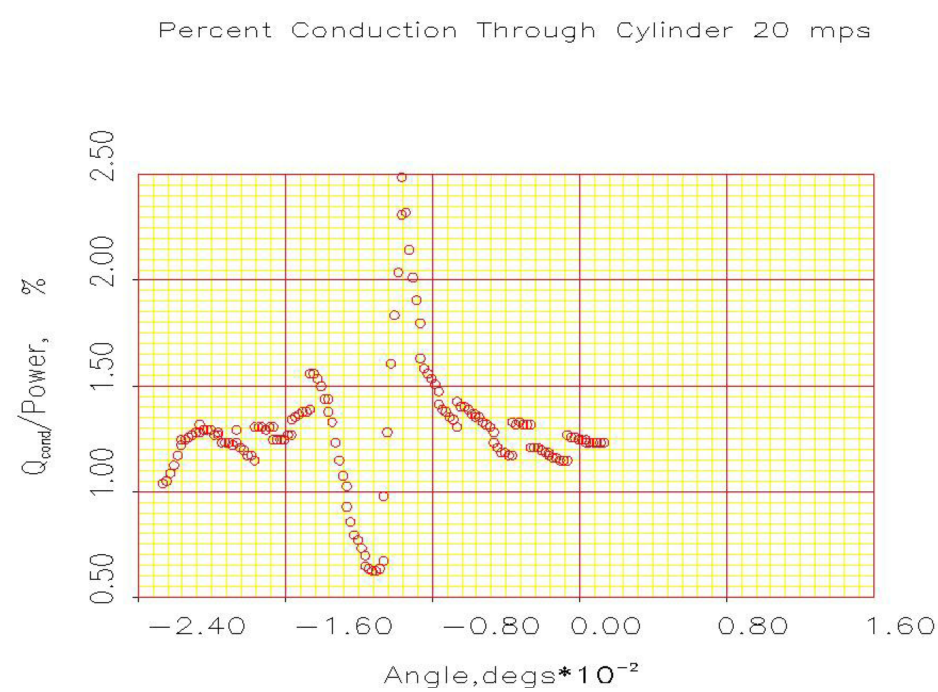

Figure 5-12. Percent Conduction Heat Transfer Rate through Cylinder 20 m/s

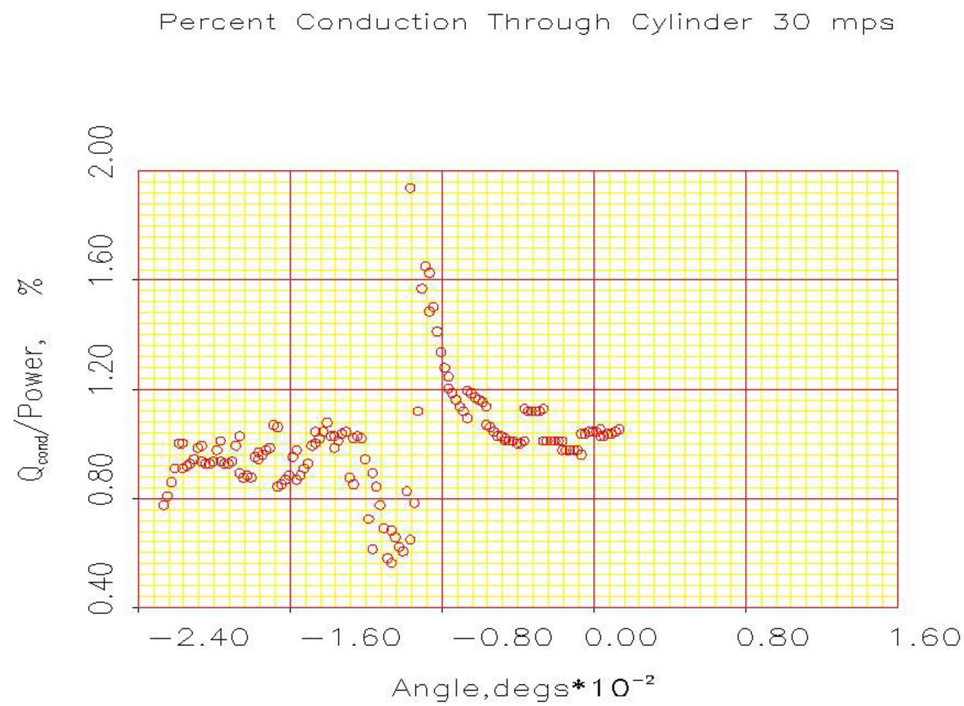

Figure 5-13. Percent Conduction Heat Transfer Rate through Cylinder 30 m/s

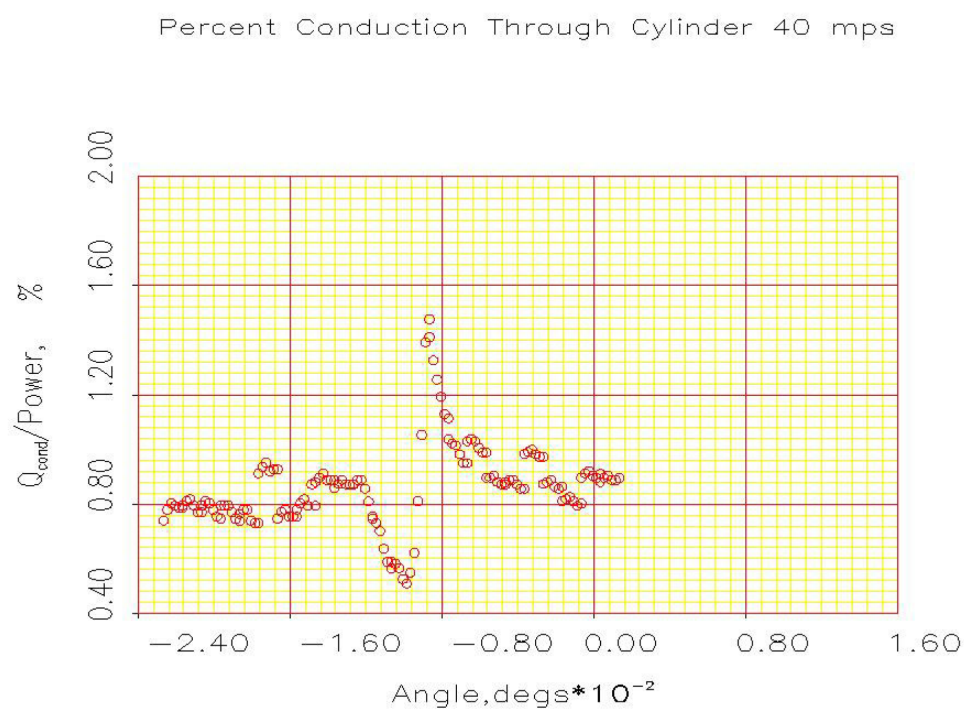

Figure 5-14. Percent Conduction Heat Transfer Rate through Cylinder 40 m/s

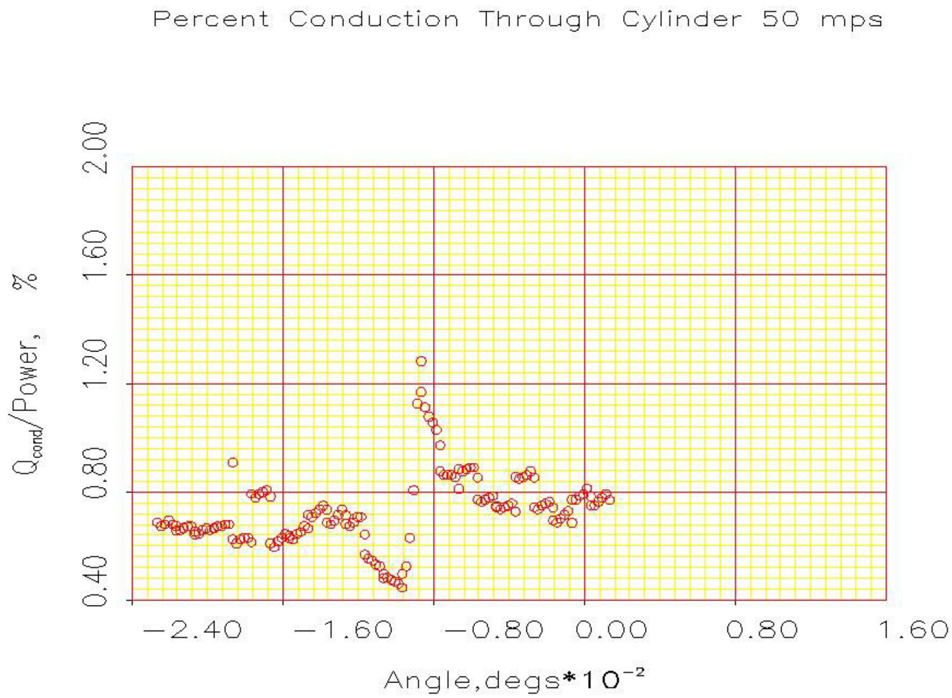

Figure 5-15. Percent Conduction Heat Transfer Rate through Cylinder 50 m/s

Figures 5-16 through 5-20 show the heat conducted along the ribbon as a percentage of the heat generated. The heat conducted along the ribbon is less than 0.1% of that generated at any wind tunnel speed.

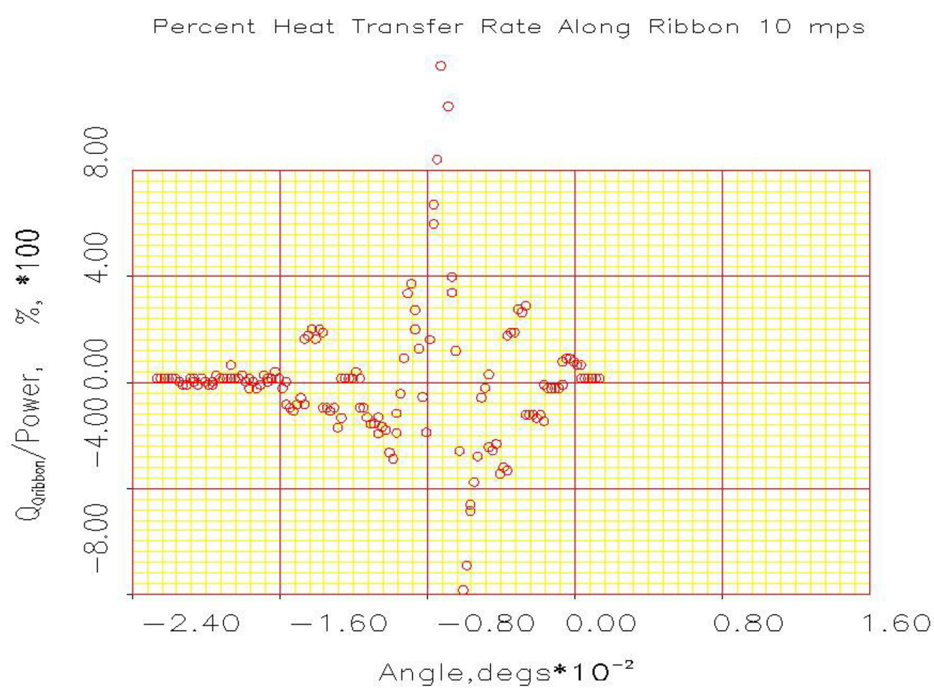

Figure 5-16. Percent Heat Transfer Rate Along Ribbon 10 m/s

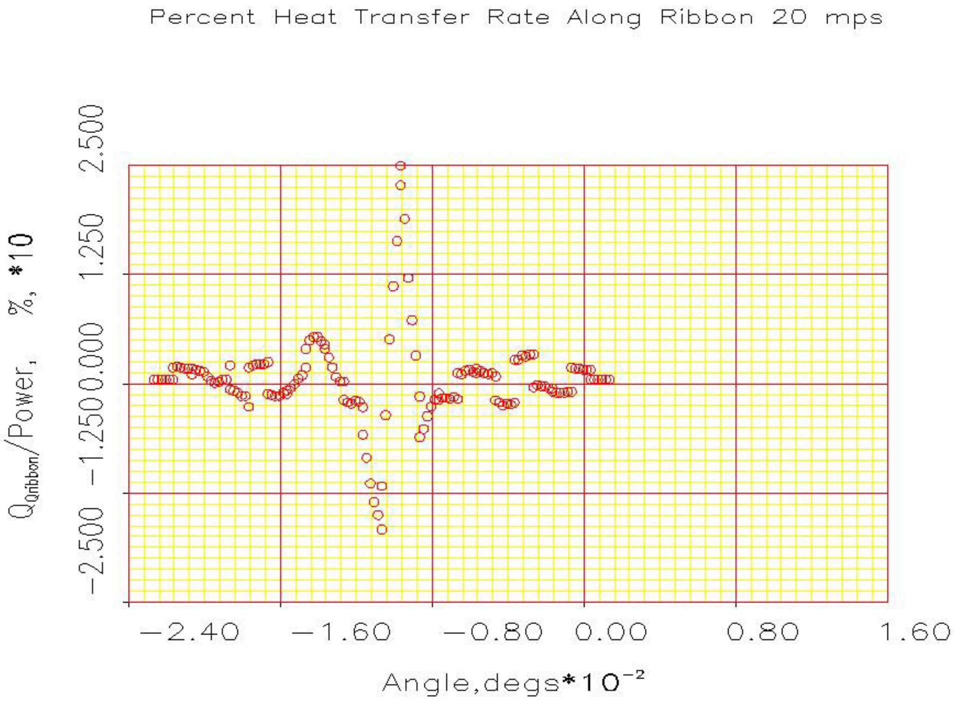

Figure 5-17. Percent Heat Transfer Rate Along Ribbon 20 m/s

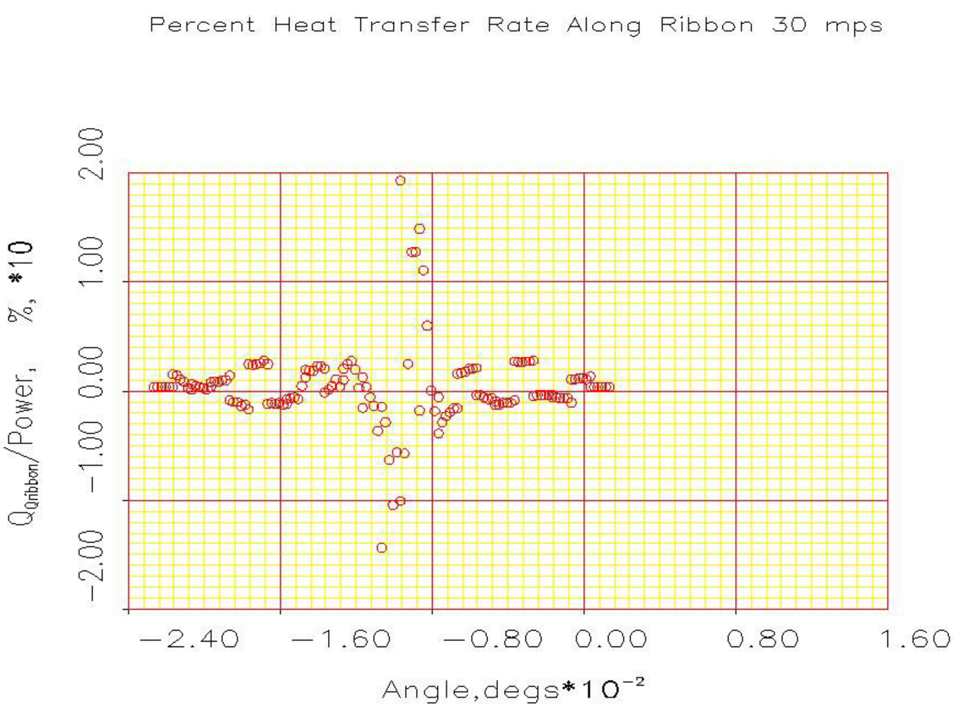

Figure 5-18. Percent Heat Transfer Rate Along Ribbon 30 m/s

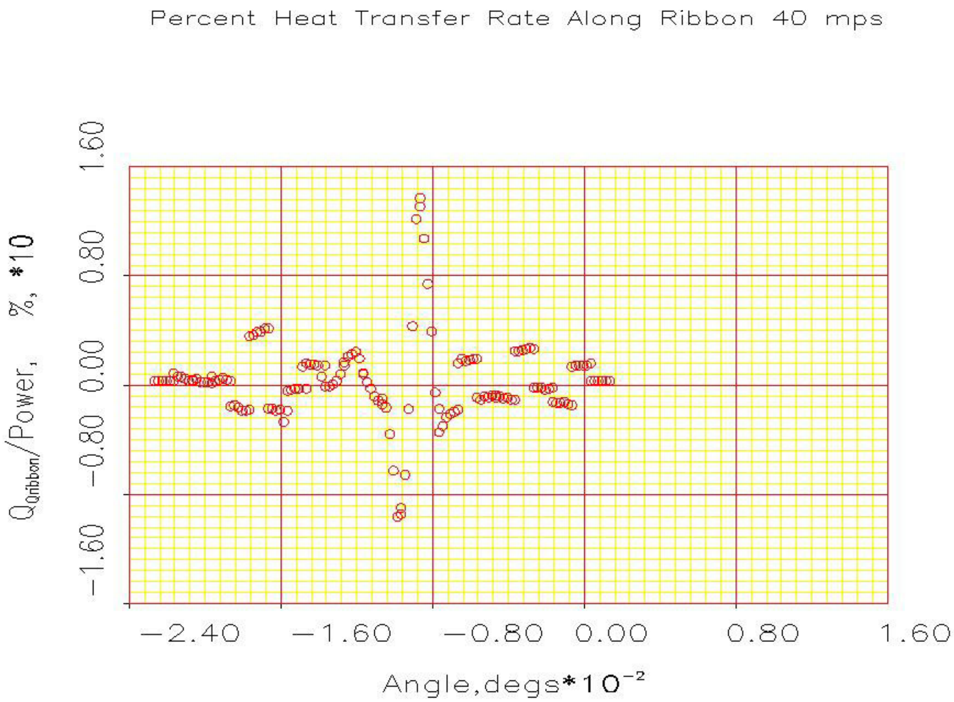

Figure 5-19. Percent Heat Transfer Rate Along Ribbon 40 m/s

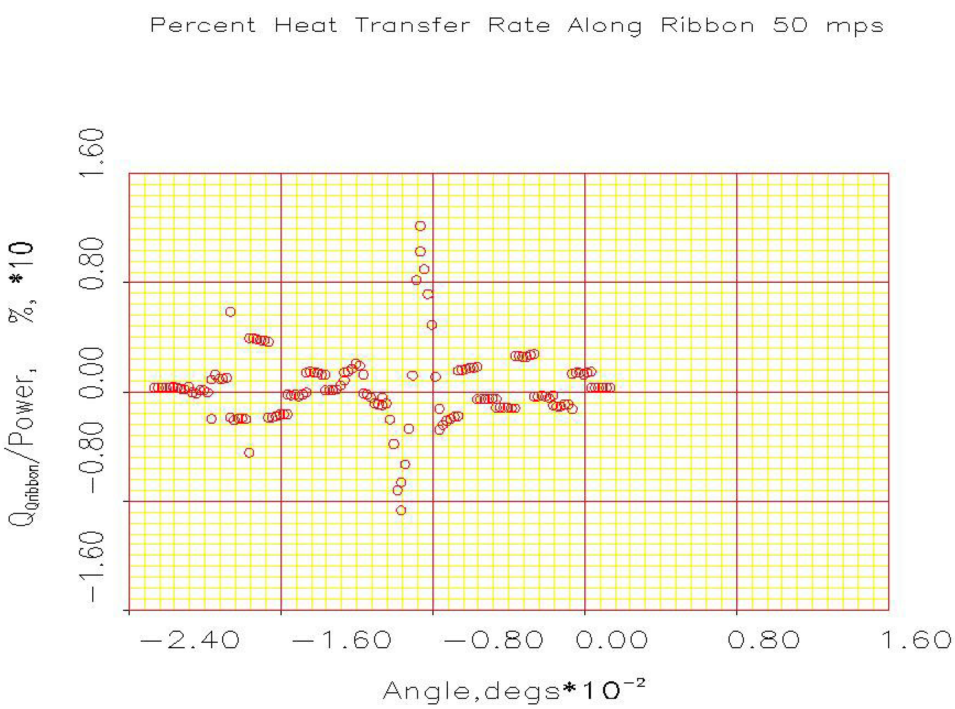

Figure 5-20. Percent Heat Transfer Rate Along Ribbon 50 m/s

Figures 5-21 through 5-25 show the percentage of the generated heat transferred by radiation to the environment. The peak percentage of heat transfer due to radiation occurs at 10 m/s and totals about 7% of the heat generated. Examining figure 5-6 shows almost all the heat not going to external convection is lost through radiation. At 50 m/s, the peak percentage of heat transfer due to radiation has dropped to about 2.5%. These plots show radiation is a small factor in the heat transfer rate and only at the lowest speed and the lowest local convection heat transfer.

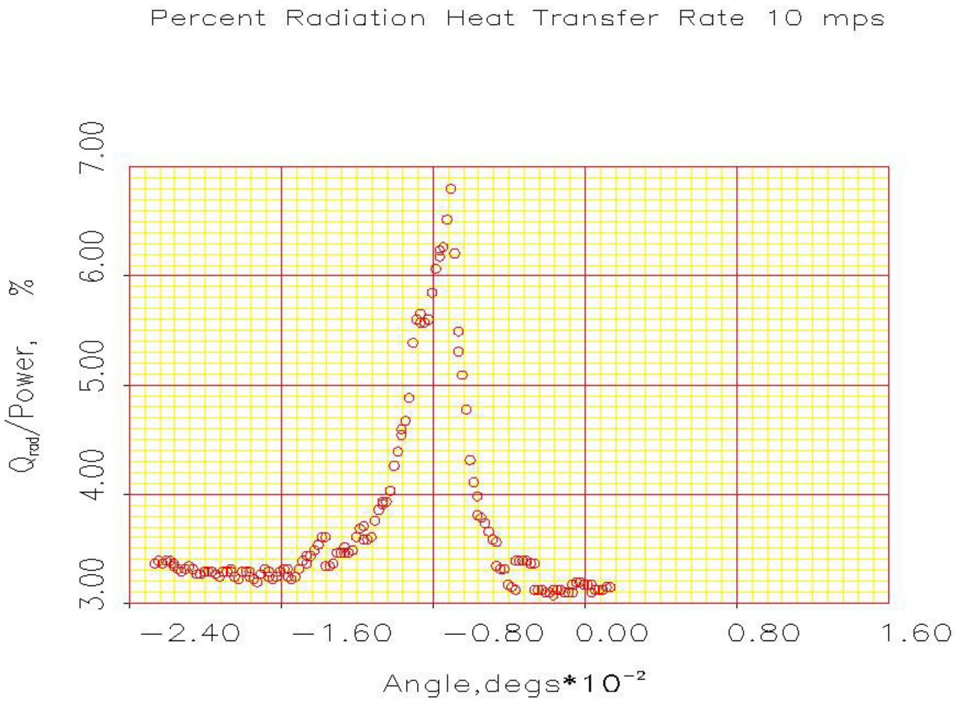

Figure 5-21. Percent Radiation Heat Transfer Rate 10 m/s

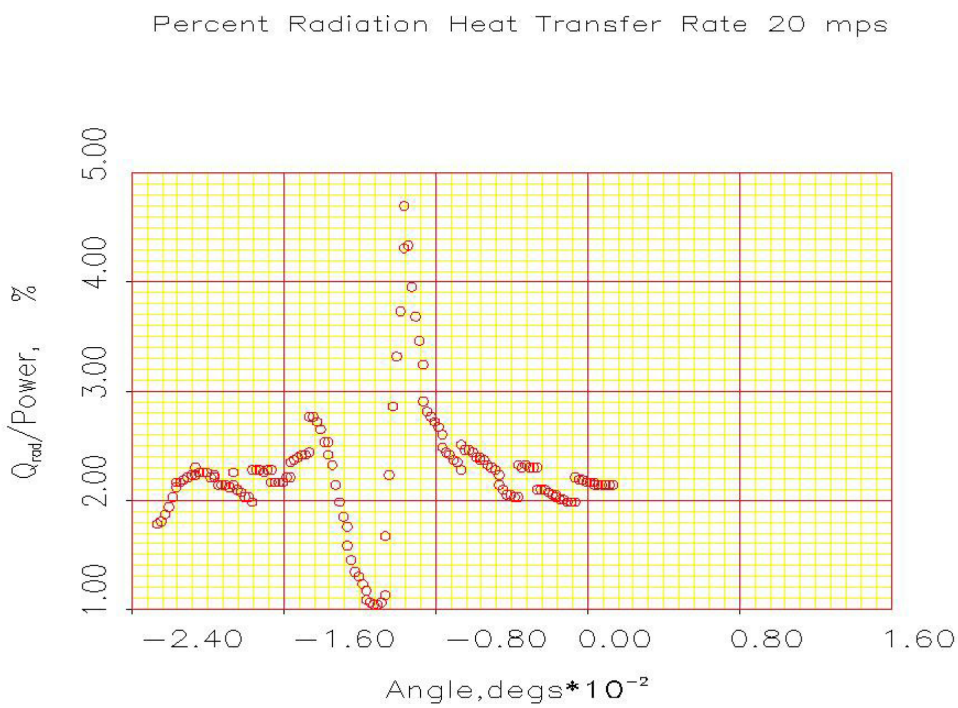

Figure 5-22. Percent Radiation Heat Transfer Rate 20 m/s

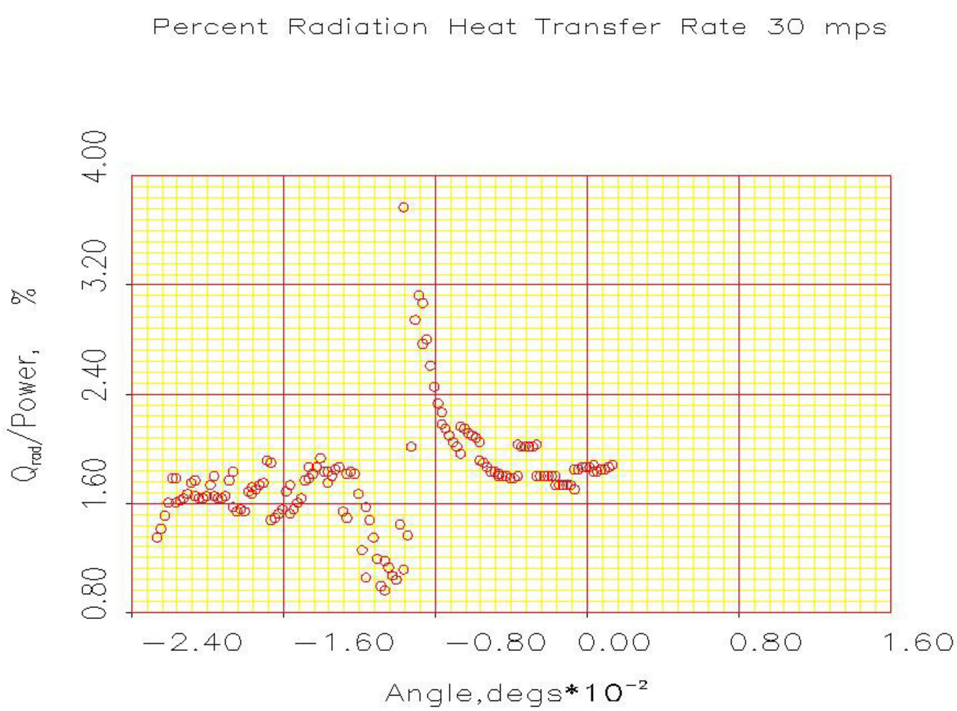

Figure 5-23. Percent Radiation Heat Transfer Rate 30 m/s

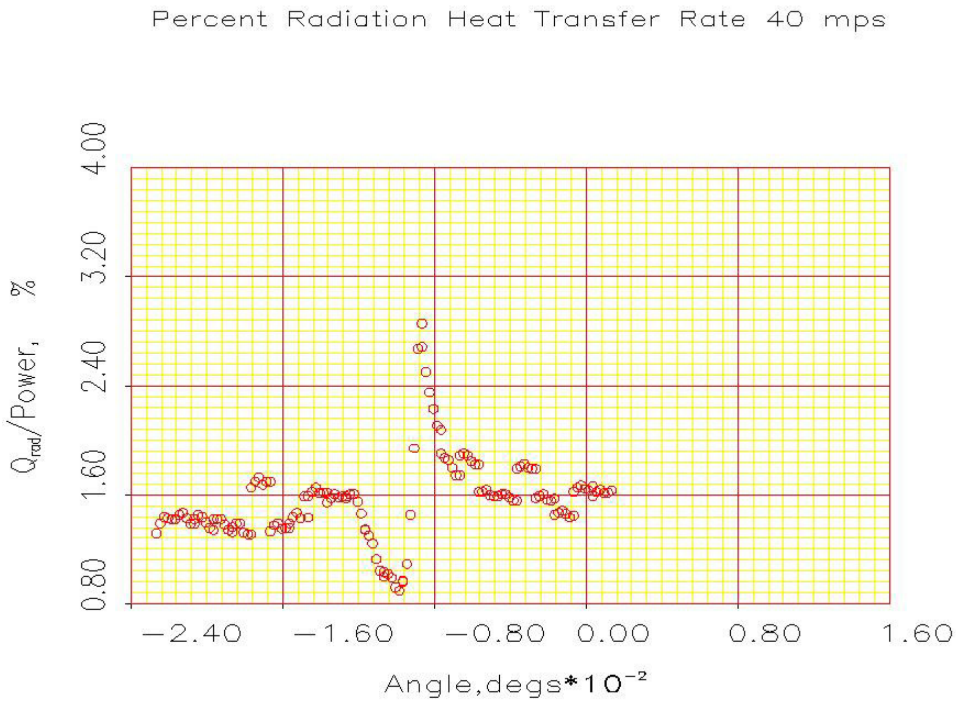

Figure 5-24. Percent Radiation Heat Transfer Rate 40 m/s

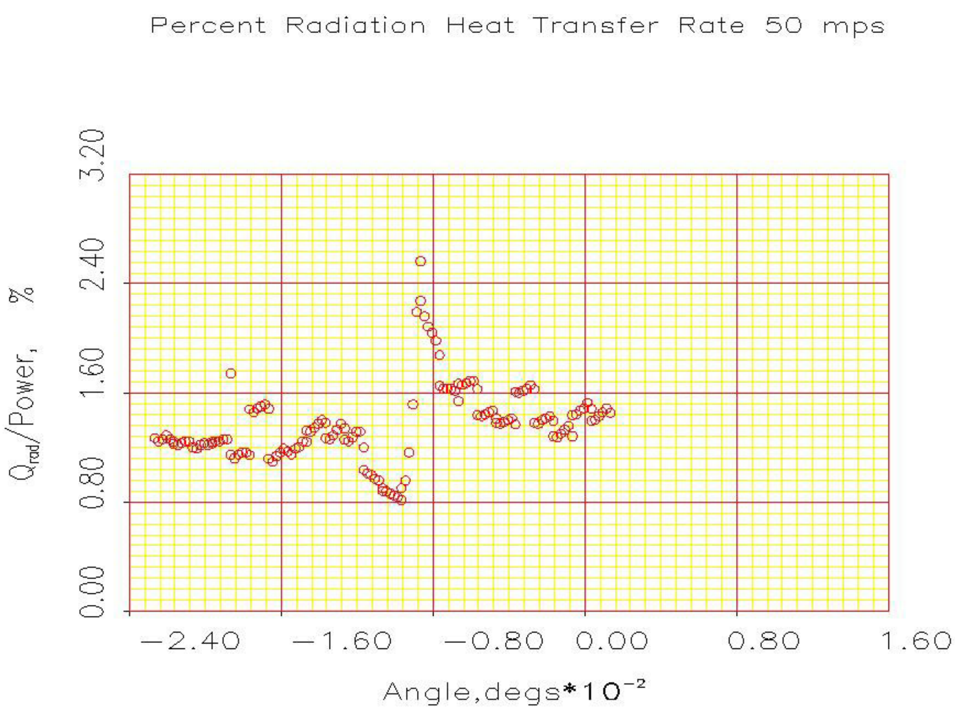

Figure 5-25. Percent Radiation Heat Transfer Rate 50 m/s

REFERENCES

1. S. Huyer and R. Roberts, "Comparison of Surface and Near Wake Flow Quantities Using Unsteady RANS and LES Simulations," NUWC-NPT Technical Report 12,353, Naval Undersea Warfare Center Division, Newport, RI, 1 February 2021.

2. D. Ludovisi and I. Garza, "Natural Convection Heat Transfer in Horizontal Cylindrical Cavities: A Computational Fluid Dynamics (CFD) Investigation," *Proceedings of the ASME 2013 Power Conference*, July 2013.

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