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Wireless Network Topology Control: Supporting Link Cost Constraints and Resiliency

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14. ABSTRACT
In this report, we research topology control approaches to aid in reducing the required transmission power within wireless ad hoc networks. Using simulation and graph theory, we evaluate the efficiency and resiliency of the k resilient XTC protocol against other basic fundamental topology control approaches. We conclude that the kXTC protocol provides a basic design framework for ad hoc wireless topology control by balancing the minimization of maximum transmission power cost across the network against competing resiliency design requirements in ad hoc networks.

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EXECUTIVE SUMMARY

Ad hoc network topology control is a fundamental problem in wireless communication networks and is especially useful for constructing and maintaining networks with desirable properties such as reduced transmission power requirements. Much of the previous foundational work in wireless sensor networks (WSNs) relies on simplified geometric or linear distance assumptions, which produce ineffective solutions to resolve effects of more complex signal propagation, external interference and detection, and heterogeneous nodal characteristics. In this paper, we present related experiments to demonstrate an approach with desirable design features including: generic network link cost support, adaptive resiliency, distributed operation, and position independence.

A weakness of directly applying optimal theoretical solutions to topology control, such as the minimum spanning tree (MST), is that they often result large network diameters and sparse connections within the resulting network. Such characteristics lead to significant congestion, delay, and fragility to dynamics in actual real world ad hoc wireless networks. To address these issues, we demonstrate an approach that balances the formation of resilient structures against the competing requirement of reducing network transmission power requirements. We confirm the initial results of resiliency adaptation through a series of network link failure experiments.

We also present a working prototype of a software modeling environment developed to assist in the overall analysis of ad hoc network topology control with cost constraints representing more accurate wireless environments. An example, is the use of International Telecommunication Union (ITU) wireless propagation models to develop more accurate communication link cost estimations that do not necessarily strongly correlate with distance as in the free space propagation case.

Finally, we demonstrate an approach to adjust the overall network transmission power profile against more complex constraints related to an external receiver or location. We demonstrate that the network can adapt transmission power across the network relative to these external constraints. This ability can be used in future wireless network operations to assist in mitigating network interference and/or detection in spectral aware environments.

We conclude with a discussion of ongoing challenges and future planned work, such as optimizing the data throughput of these networks and addressing various forms of heterogeneity, including directional wireless assets within the network.

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WIRELESS NETWORK TOPOLOGY CONTROL: SUPPORTING LINK COST CONSTRAINTS AND RESILIENCY

1. OVERVIEW

1.1 Problem Background

Communication network topology control is a process to establish and maintain a subset of connections from a set of possible communication links. Topology control design goals can be multi-varied and often include: managing network energy and transmission characteristics, reducing wireless channel contention, reducing network traffic congestion, or increasing network resiliency. In many wireless network deployments it is often beneficial, or even necessary, to have the ability to perform topology control in a decentralized manner. Balancing these multi-varied network design goals proves non-trivial, and there does not exist a single approach to topology control suitable for all networking scenarios and applications. In this paper we explore fundamental designs for ad hoc network topology control focusing on reducing communication link costs within multi-hop wireless networks. While generic link costs will be supported by the approach we investigate, our overall objective focus in this report will be achieving a reduction in the maximum transmission power required at each node within the network, so in this case, cost is represented as the maximum transmission power needed by each node to maintain local network communication links or edges. In general, performing optimal topology control to minimize network edge costs without other considerations results in sparse network topologies with large diameters. Extremely sparse wireless network topologies have the detrimental side effect of fragility to dynamics and increased data congestion along longer or more common routes within the network. Therefore, we will also address and investigate resiliency extensions to topology control. It is important to construct redundancy via controlled methods in order to maximize network resiliency while simultaneously controlling for cost constraints. In addition to balancing these competing design constraints, we are targeting topology control solutions that can be further adapted for distributed operations.

We will also be investigating the use of network topology control in more complex wireless propagation environments. Past research in fundamental wireless-based topology control often modeled wireless transmission cost (or edge cost) as proportional to the distance between nodes raised to some exponent, $d^{\varepsilon}, \varepsilon = [2,4]$. This model supports only a coarse view of electromagnetic frequency propagation characteristics where the transmission power to communicate reliably between two wireless network nodes u and v grows *at least* quadratically with the distance between them. Such limited models do not provide a sufficient framework for real world environments, since solutions designed only for these types of propagation models assume homogeneous communication characteristics for the entire network. Wireless communication networks operating in real world heterogeneous environments (e.g., terrain, environments, nodal characteristics, multiple channels) experience something much different. In the basic example of terrain propagation, it is often the case that some longer distance links prove more reliable with lower transmission cost than do some sets of shorter distance links. Such real world conditions result in optimum topology control solutions that need to be distance independent and often non-planar to address proper cost minimization. By way of

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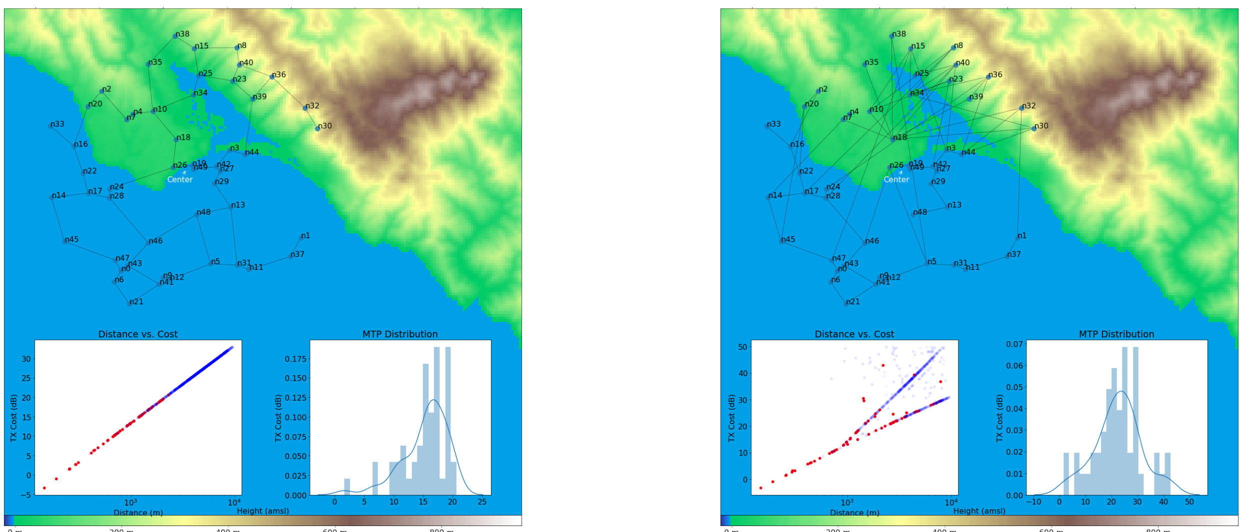
illustration, we present Fig. 1(a) showing a topology control solution and its cost to distance relationship graph for every possible link (lognormal x scale, topology links in red) using free space propagation loss as the determining factor for transmission cost. A topology control algorithm using these costs produced a well structured planar topology driven by the directly proportional cost to distance relationship. When using terrain affected loss, however, topology control becomes much more complicated, as evidenced in Fig. 1(b). Cost and distance retain some proportionality, but the costs can vary widely, especially at longer distances. Accurate power optimization, therefore, requires a cost function considering more than just distance or simple geometric positional relationships.



(a) Free Space Loss

(b) Terrain Loss

Fig. 1—Free Space Loss vs Terrain Loss

Since many previous topology control approaches use only distance-based or geometric models, they produce results in planar geometric graph topologies (non-intersecting edges). These planar topologies possess graph theoretic assurances in ideal environments, but by simply modeling cost more accurately as a complex function, the assurances of traditional approaches dissipate [1]. In addition, we will outline an approach to support heterogeneous characteristics (e.g., antenna gains, multiple RF technologies) and other factors influencing optimization constraints shown to be increasingly important for many scenario use cases.

1.2 Graph-based Cost Model

In our analytic studies we will be representing the communication network using a graph-based model $G(V,E,w)$. V is the set of vertices, or wireless nodes; E represents the potential edges, or communication links between neighbors; and w represents a set of costs associated with each edge. The edge cost w can be a simple metric such as distance to a neighbor or it could also represent a more complex minimal cost needed to establish a reliable neighbor link. Given an initial graph with all possible communication links and costs represented as $G(V,E,w)$, the topology goal is to find a subset of the network $G^{'}(V,E^{'})$ such that the use of topology links $E^{'} \subseteq E$ targets a minimization of the desired cost function often with some additional bounded constraints (e.g., maximum transmit power per node or the minimum sum power of a path). In the basic model, E can also represent a set of directed edges with differing unidirectional costs. In the case where a graph-based algorithm needs an undirected network model to operate correctly, a bidirectional cost may need to be formed from the set of directed neighbor edges.

Two main subproblems of topology control in ad hoc networks are *topology construction* and *topology maintenance*. This paper will primarily consider aspects of *topology construction*, but aspects of resilient maintenance and use of the topology under dynamics and link failure will be briefly examined and covered since the approach presented lends itself to distributed maintenance and updates.

1.3 Related Past Work

Topology control for both ad hoc communication networks and, more typically, wireless sensor networks (WSNs), has seen significant research activity over the past two decades. The emergence of low cost wireless sensor devices spurred the development of past approaches with the goal of optimizing network energy consumption while supporting distributed network traffic exchanges between sensor devices. The basic model often assumed the use of small, battery-powered wireless nodes where wireless data transmission, forwarding, and reception each consumed some amount of these finite energy resources. The bulk of the cost comes from transmission and forwarding, therefore, these solutions used *the network* as a means to conserve energy by balancing the use of shorter range links between neighbors to limit the impact of transmission and forwarding, thereby increasing network lifetime [2–4]. Solutions to route and exchange over these resulting network topologies were largely developed within both the mobile ad hoc network (MANET) and WSN technical communities. Actually, fundamental forms of past MANET routing work by the authors and others, Simplified Multicast Forwarding (SMF) and Optimum Link State Routing (OLSR), have used some forms of topology control to reduce overhead and congestion in MANET networks by applying distributed connected dominated set (CDS) graph theory concepts as an aid to both control and data forwarding in the network [5, 6]. In this paper, we focus more on fundamental topology control for the purposes of reducing required transmission power throughout the network.

Previous fundamental topology control solutions and concepts applied in wireless networking contexts borrow heavily from graph theory results, primarily: the minimum spanning tree (MST) [7], the Gabriel graph [8], and relative neighborhood graph (RNG) [8]. Enhancements and additional approaches were also developed that use positional geometry as a means to control topology via geometric constraints such as cone-based topology control (CBTC) [9] and the Yao graph [10]. Past solutions for WSN operation such as Common power (COMPOW) have also attempted to develop solutions where all nodes converge to a common transmit power level to ensure a balancing of energy consumption [11]. Additionally, as briefly mentioned, past work on clustering algorithms by the authors and others, such as CDS approaches, provide a means to generate an energy constrained network backbone to be used as a forwarding subgraph in support of a population of edge networks [5, 6].

While we review and borrow from some of these past efforts, our technical objectives differ in several fundamental ways. As mentioned, a large percentage of WSN research attempts to apply topology control to adjust network energy usage to maximize “network lifetime” as the chief technical goal. Our primary technical interest, rather, is to reduce the maximum transmit power required of each node in a generic wireless network. WSN techniques to reduce energy *consumption* throughout the life of the network do not necessarily meet this same requirement. In addition, previous WSN topology control designs often assumed homogeneity within a network of similar devices, to include both the broad range of wireless characteristics and operating conditions. The approach we outline and examine will support use in heterogeneous environments representative of real world wireless network operations, multiple technologies, and environmental conditions. In our terminology, heterogeneity may refer to non-uniformity in multiple aspects of the network system, including: differences in nodal characteristics (e.g., local power constraints, local directivity and link gains, multiple communication channels), environmental operating conditions (e.g., airborne vs. surface vs. space, antenna height, terrain propagation, other atmospheric effects), or networking characteristics (e.g., bandwidth, traffic load). In contrast, geometry and distance-based topology control mechanisms do not inherently cope with heterogeneity and will not represent the proper constraints. Algorithms such variants of MST and CDS do both have the ability to support generic cost, and we will contrast experimental results with MST later in the paper when we explore resiliency and network transmit power profiles.

1.4 Side Benefits of Improved Network Power Control Schemes

Reduction in required wireless network transmission power has secondary network benefits in areas such as spectrum reuse, interference avoidance, and contention reduction. For example, past simulation and empirical studies of ad hoc wireless power control demonstrate obtaining higher signal to interference noise ratio (SINR) and traffic throughput versus approaches with power control [12, 13]. These observed potential benefits are in part likely due to additional spatial reuse and contention reduction occurring throughout a network when reducing transmit power and overall wireless communication range. There also exist counter examples to the above argument, particularly when using extremely sparse topology control approaches such as MST. In these cases, network paths are longer and alternate paths may not exist, and therefore, the resultant topology can increase overall congestion and delay for the network traffic. The optimization tradeoff space between these approaches is non-trivial, given that throughput and contention issues are largely subject to lower level network designs and complex resolutions beyond topology control (e.g., media access layer and scheduling algorithms). Since many of these aspects are context specific, we point out their importance, but in this report we focus on examining fundamental characteristics of topology control in terms of adaptation to different constraints and flexibility in its resiliency.

1.5 Network Power Transmission Cost Models and Constraints

The traditional method of performing topology control is to favor some edges over others based on some notion of edge weight, or cost. These edge costs are then used to inform the generation of a network topology driven by meeting some cost objective. The goals of controlling topology can be many: cost minimization, load balancing, cost localization (focusing most of the cost in one area of the topology), or even maximizing connections for a given cost. The flexibility of the cost objective mandates that the underlying cost be well defined and reflect the nature by which connections between nodes are affected by some intrinsic characteristic, like distance, communication range, and link quality. Visually, this can be seen in Fig. 2:

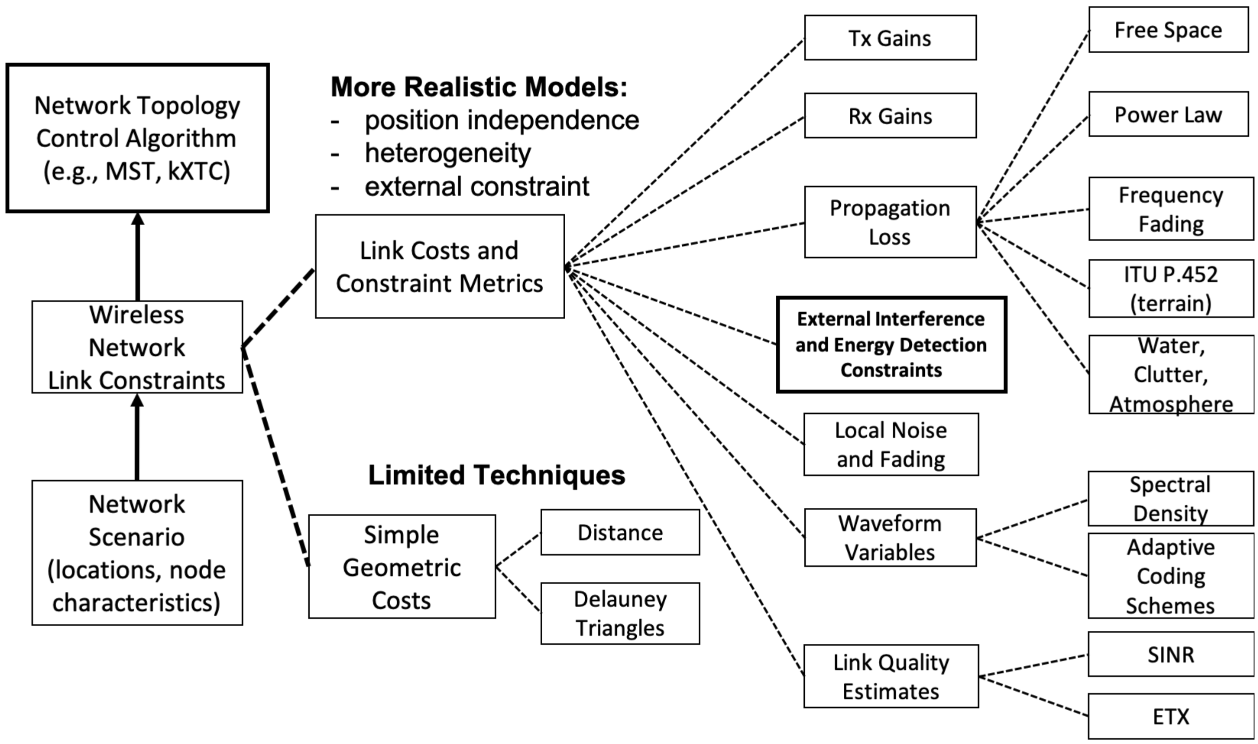


Fig. 2—Overview of Various Transmit Cost Considerations

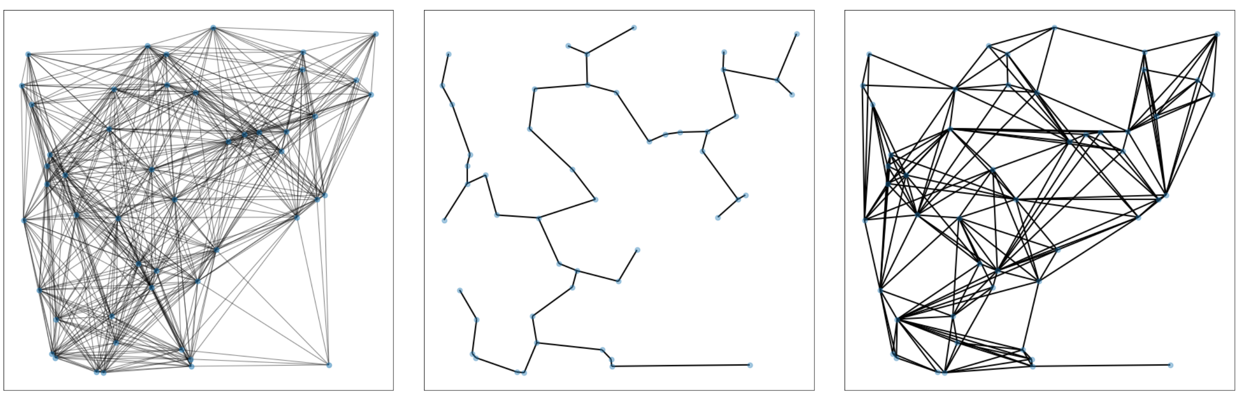
Figure 2 captures a sampling of the methods that may influence the construction of a cost value for establishing edges in a topology control algorithm. Cost can be as simple as the distance between nodes, which captures at a rough level the concept that nodes further from each other require greater power to establish a connection. Although as we earlier discussed, simple distance-based cost models do not capture all the nuances of communication between nodes, and a more accurate cost formulation is required to build effective topology control algorithms for operations in realistic environments. Concepts such as signal-to-interference noise ratio (SINR), accurate radio wave propagation loss, or other network link quality metrics can all be used to inform a more accurate cost value. Prior to presenting more complex cost models, we will present illustrative distance-based cost models to introduce basic topology control concepts, so we can then expand our analyses to more realistic forms of cost modeling.

2. POWER-CONSTRAINED WIRELESS NETWORK TOPOLOGY CONTROL

2.1 Illustrative Models

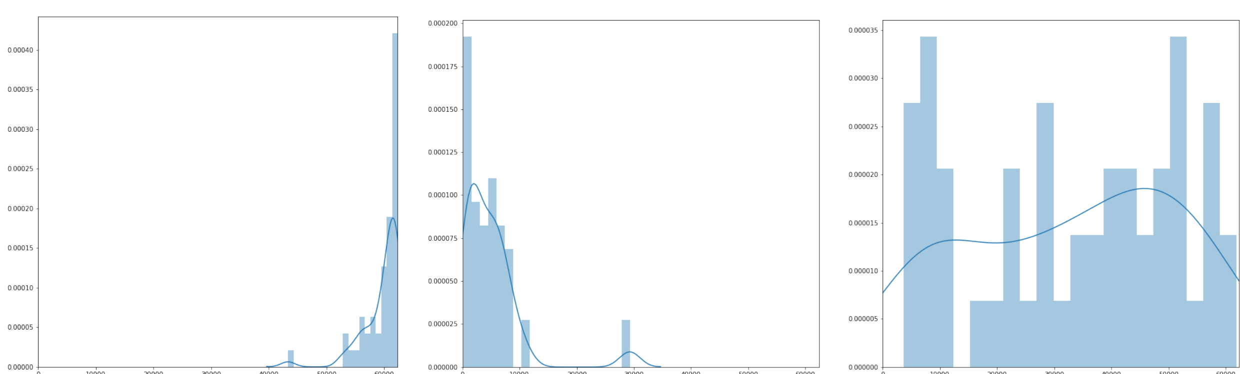
To initially illustrate a basic concept of power-constrained topology control, we present examples using simplified edge cost models. Figure 3 demonstrates a set of methods for basic topology control and Fig. 4 represents the associated maximum transmit power (MTP) profiles. The MTP profile is a graphical means to represent the distribution of maximum transmit powers required by each node to maintain its highest cost network neighbor link in the topology. Since initially propagation loss is calculated using basic free space propagation, the transmit cost on the x axes shown in Fig. 4 represents a distance-based proportional cost metric raised to an exponent.

In Figs 3(a) and 4(a) we model a simple two dimensional random geometric network layer within a 500m $\times$ 500m square region. For this toy scenario, wireless transmitters use a common upper range constraint based upon 250m as their maximum reliable wireless link range and connect to all neighbors they can within this range constraint. Using this same basic illustrative example, in Figs. 3(b) and 4(b) a topology is computed using an MST algorithm (Kruskal's Algorithm [14]) from the graph of all potential candidate edges in the range limited case. The MST topology result demonstrates, as predicted, the sparsity and large diameter of the network along with a significant reduction in the maximum transmit power budget. A more balanced approach to topology control is shown in Figs. 3(c) and 4(c) which begins with the maximum range network and uses a sparsification algorithm [15] to remove certain edges until a stretch factor constraint is satisfied. The stretch factor constraint keeps the network diameter from growing too large as we adjust our network, but this global knowledge approach does not lend itself to distributed operation. The design tradeoff between using simple sparsification versus the MST approach increases the overall MTP profile of the network due to some longer range links requiring greater transmit power to achieve a denser, though smaller diameter, network.



(a) Fixed Range Topology (b) MST Topology (c) Sparsifier Topology

Fig. 3—Basic Topology Control: Graph Examples



(a) Max Transmit Power (b) MST Power (c) Constrained Sparsifier Power

Fig. 4—Basic Topology Control: Proportional MTP Costs

2.2 Overall Design Goals

Our goal is to reduce overall MTP costs while also supporting the addition of resiliency to counteract the fragility prone, extremely sparse solutions, like the MST. This is generally a non-trivial problem in basic graph theory even without the additional technical challenges of supporting heterogeneity and distributed operations [16].

Distributed forms of graph spanners and minimum cost CDS algorithms are examples of graph-theoretic areas we feel can address a more balanced approach between minimizing cost and improving resiliency. This paper will primarily focus on a distributed approach with generally favorable spanning properties. A spanner of graph G in a geometric sense can be thought of as a subgraph $H \in G$ that has a smaller number of edges with relatively short distances. More formally an α-spanner of G can be defined as a subgraph H of G (where $V_G = V_H$) and the following constraint holds:

$$d_H(s,t) \leq \alpha * d_G(s,t) \forall (s,t) \in V_G \tag{1}$$

$d_G(s,t)$ represents the cost or the distance of the path between nodes s and t in the network G. This can be interpreted as meaning the resulting spanner has network path lengths within a stretch factor α of the original graph's path lengths given the same endpoints. The basic graph spanner concepts were first discussed by L.P. Chew in 1986, although the term "spanner" was not introduced in the original paper [17]. Spanners have been used in computational geometry to solve proximity problems and are also found in fundamental applications to motion planning. For example, they have found use in telecommunication networks to optimize network reliability and mobile network roaming. When considering more heterogeneity in our networks as we have discussed, good spanning algorithms are not necessarily in the physical geometric plane, but rather in a multi-dimensional space represented by costs that may not be directly correlated to geospatial distance.

3. FLEXIBLE COST ADAPTATION AND RESILIENCY

3.1 XTC: Introduction and Theory

In [1], the authors introduced a topology control algorithm called XTC and discussed its use in mobile ad hoc networks. Unlike many other fundamental topology control concepts, it has several desirable advantages: simplicity, weighted graph operation, node position independence, and the potential for distributed operation. At a basic level, XTC works by locally carrying out the removal of potential neighbor communication edges from consideration in the calculation of its resultant network topology. From a general weighted graph G of potential edges, XTC computes a resulting subgraph G_x while maintaining connectivity. In the more basic XTC operation, two hop potential edge information is shared within the network and if there is a two-hop path between nodes u and v requiring less energy or cost than a direct connection (u,v) between the nodes, then the edge (u,v) will be removed from the resultant XTC subgraph G_x.

XTC has been shown to have some theoretical characteristics making it an attractive foundation on which to build topologies with reduced edge costs. The basic XTC algorithm will produce a topology with a girth of 4, which means it will only have cycles of length 4 or greater (any cycle for any node of length 4 or greater) [1]. Girth inversely relates to the edge-connectivity of a graph, so a girth of no less than 4 provides a topology with a greater resilience over other algorithms that end up producing sparser topologies such as a MST [18]. This insight will prove important as we later demonstrate resiliency extensions that progressively produce higher levels of edge connectivity, and thus even lower girth. Further evidence of XTC's attractive topology characteristics are its spanner and bounded degree properties. A perfect spanner will evaluate the ratio between the costs of the shortest path between u and v on a given topology control graph (TC) and the shortest path on the Unit Disk Graph (UDG) model:

$$s(u,v) := \frac{|p_{TC}(u,v)|}{|p(u,v)|}$$

as 1.0. XTC produces spanner values near one for both some general notion of energy (edge weight) and Euclidean distance (hop count), with numerical studies demonstrating spanner value averages of ~ 1.06 and ~ 1.25 respectively [1]. For a general graph edge cost weighting scheme, XTC provides topologies that balance graph weight across a network making it useful for generalized power control or other constraints.

While XTC is heuristic in nature, there are some theoretical foundations of XTC that give us confidence in its behavior. In UDG, it has been shown that XTC exhibits a bounded degree property with each node possessing at most a degree of six [1]. Degree values for the UDG model rise accordingly with the network density, so many networks with a lower density of potential connections will not necessarily reach the degree upper bound of six. In practice, the maximum degree for an XTC topology is usually around four or five, which suggests that when combining XTC with power constraints, the resultant topologies should aid in reducing interference in ad-hoc networks by increasing the spatial diversity across the network [1]. The limited degree properties may also later assist in optimizing topologies for multi-channel RF architectures, including the incorporation of directional antennae constraints.

3.2 Adapting XTC to More Complex Models

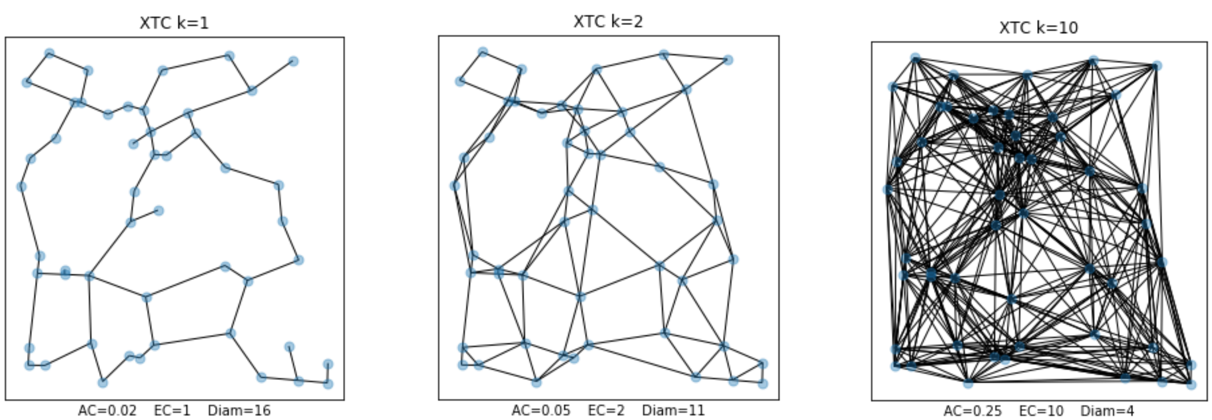
The XTC framework provides flexibility to adapt complex transmission costs (e.g., terrain, fading effects, heterogeneous environments) to the topology construction process. Therefore, we construct more accurate cost models using advanced propagation models and heterogeneous parameters from our AdaptNet analysis library. Once these costs are constructed we formulate a cost matrix representing all potential network communication links. In our present analytic simulation of the XTC algorithm (see Appendix A), 2-hop local potential graph edge information is assumed to be successfully shared between each network node. The present XTC algorithm also requires symmetric link weights, so we apply the MTP required in two directions as the link weight. In the distributed case of XTC, a requirement for correctness is that each node obtains an ordered list of its potential network topology neighbors and associated costs. The potential errors that could occur in the ordering of cost information and proper ordering between neighbor databases has led to the development of additional resiliency concepts. As we shall demonstrate, these resiliency concepts have additional benefits in terms of more adaptive and denser topology construction when desired.

3.3 Resilient Topology Control Extensions: kXTC

The idea for a k resilient variant of XTC was first discussed in [19]. The author's main design purpose sought to protect against k levels of mis-ordering of neighbor link costs during the topology pruning process of XTC in the distributed form of the algorithm. The process is accomplished by having k neighbor nodes agree on the removal of the local link based upon the existence of a lower cost two-hop path. While an important practical feature for protecting against errors or misinformation during distributed operations, this process also forms topologies with resilient k edge connectivity properties, where k equals the minimum number of edges that must be removed to disconnect a graph G. Therefore, the resultant topology graph will have some topological redundancy built-in from this process. We implemented the k resilient extension with an ability to handle generic transmit power costs in our XTC simulation prototype and we will be referring to this algorithm implementation from now on as kXTC, where kXTC is equivalent to the original XTC when $k = 1$.

In the following examples, we demonstrate kXTC using some scenarios and initial free space radio frequency (RF) propagation models. Along with the resultant topology graphs, we provide the MTP distributions. In Figs. 5(a), 5(b), and 5(c), we present the topological results for kXTC when $k \in \{1, 2, 10\}$ respectively. Some invariant graph metrics are also displayed to assess the resultant network resiliency and density. The algebraic connectivity (AC) represents the second smallest eigenvalue of its normalized Laplacian matrix and has known useful properties in assessing graph structure [20]. The relative magnitude of this value reflects how well connected the overall graph is and has many uses related to examining a network's robustness [21, 22]. AC is also known to be a lower bound for edge connectivity (EC), which is the number of edges needed to be removed from a graph in order to render it trivial [23]. The last metric included in the figure is the network diameter representing the maximum eccentricity of the graph, where eccentricity of a node v is the maximum distance from v to all other nodes in G.

We can see that when $k = 1$ the kXTC algorithm is similar to an MST result in terms of sparsity. The next series of experiments increase values of k resilience in the election implementation. As k increases, one can see the algorithm is adding redundant neighborhood edges throughout the network versus the $k = 1$ scenario. AC and EC values increase along with increasing k indicating an increase in topological resilience. If we continue to increase k the network will eventually become fully connected if it is possible for the algorithm to do so within the constraints of the cost objective established. As a more extreme example, we show kXTC results where $k = 10$. A minimum of 10 edges now must be removed before the network will initially disconnect globally.



(a) kXTC: k=1 (b) kXTC: k=2 (c) kXTC: k=10

Fig. 5—kXTC Resiliency Level Examples

Figure 6 plots the resultant topology MTP profiles against each other for cases of $k \in \{1, 2, 3, 4, 10\}$. This illustrates the basic effect of adding increased topological resiliency on the overall MTP distributions. It's clear the MTP distribution requires increased maximum power across the nodes to achieve this resiliency, but this increase is somewhat constrained and is built up in a localized manner by adding as many additional low cost edge connections as possible. However, it is also clear there is less to gain in resilience versus constraining the MTP profile as k becomes large.

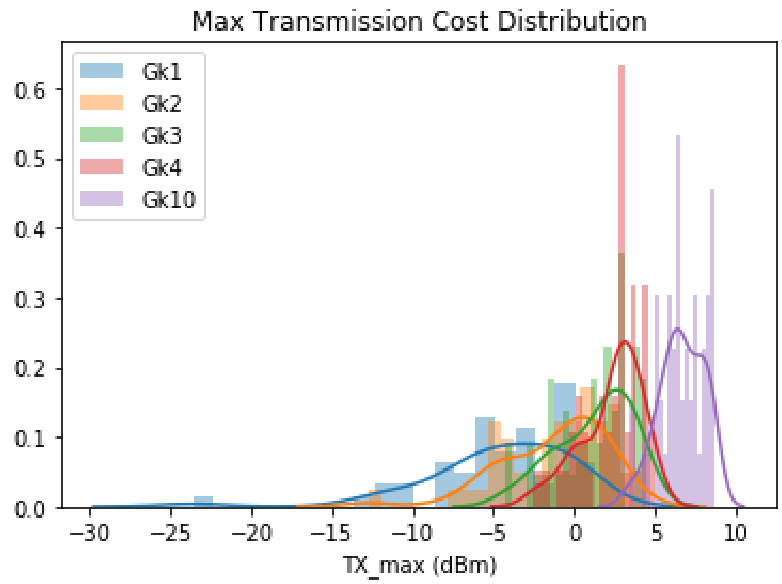


Fig. 6—Maximum Power Profiles vs. k Values

To wrap up this section of simple kXTC examples, we perform a basic simulation of network topology results with increasing k while plotting multiple metrics against increasing k values. In Fig. 7 we use the three metrics described before, but additionally plot the 95th percentile point of the resulting power profile in dBm. This value means that 95 percent of the transmitters achieved a maximum transmit power below this value in order to support the resultant topology. From this we can see that a range of 10dBm is achievable throughout these topology extremes.

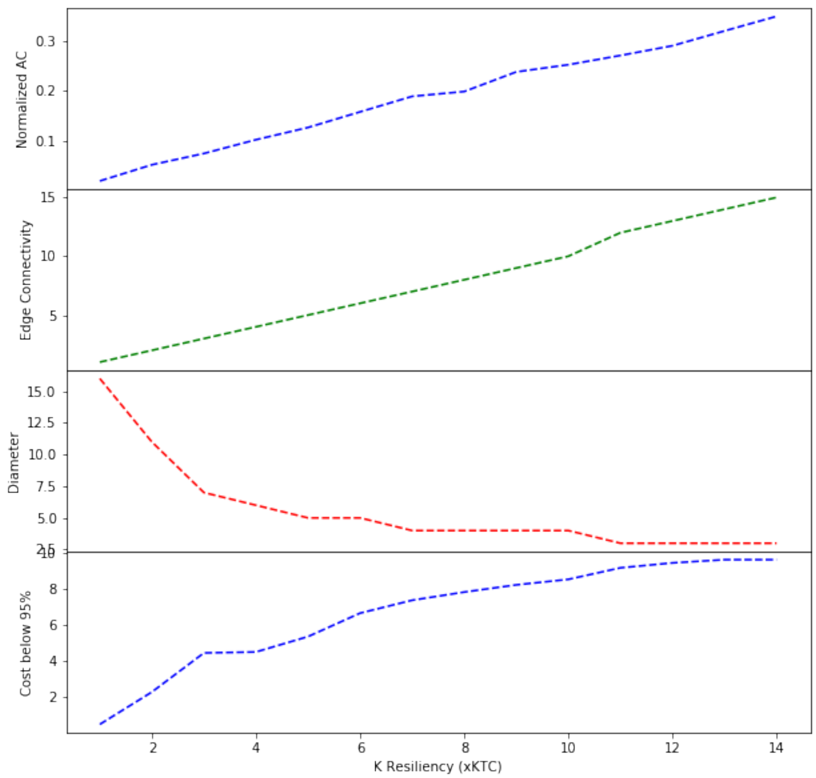


Fig. 7—Metrics vs. *k* resiliency

This set of initial experiments demonstrates that kXTC has the ability to increase resilience and decrease the network diameter in a somewhat controlled manner which appears balanced across all nodes of the network. This balance is achieved largely because XTC operates in a distributed manner and does not treat any particular node preferentially during the topology construction phase.

4. EXPERIMENTATION AND ANALYSIS: COMPLEX RF LOSS ENVIRONMENTS AND RESILIENCE

4.1 Experiment Setup

A primary thrust for examining kXTC is to characterize its average case performance across various node positions and cost models. To that end, we conducted a range of experiments to test resilience, power control capability, and adaptivity of kXTC as we examine more complex loss environments. Referring to the abstract cost model, Fig. 2, our experiments will determine the edge cost as the transmission power required to establish reliable communication as defined in Appendix B.1. Specifically, our experiments examine how merely changing the loss model in Fig. 2 to more accurately reflect real-world RF propagation dramatically changes the resulting topology.

4.2 Analysis Overview

In this section we examine the average performance of kXTC to balance tradeoffs between increasing network resilience and power distributions across a series of randomly generated geometric networks. The first analysis examines basic measures of resilience and MTP distribution characteristics across increasing values of k. The second part of our experiment uses percolation analysis by degrading the resultant network topologies via a series of edge failure simulations. Initially, for simplicity, the cost of transmission calculates loss using the free space loss equation, Equation B2. To demonstrate the adaptivity of our approach and to highlight areas of future work, we also present scenarios derived from more complex terrain and atmosphere affected propagation as modeled by the Python library Pycraf which uses ITU standard 452-16 [24, 25], detailed in Appendix C.

4.2.1 Experiment Description

We synthesize our scenarios with a given number of nodes and corresponding randomly generated positions to study kXTC across different random graphs. Each synthesized scenario uniformly distributes the given number of nodes within a confined area, and we use random geometric (RG) placements to generate topologies. To produce the candidate scenarios, we randomly generate n GPS (latitude, longitude) coordinates uniformly in a circular area around a coordinate point ρ using a random heading η satisfying $-\pi \leq \eta \leq \pi$ and a radius $\mathbf{r}$ from center ρ. The radius is randomly generated such that $\mathbf{r_0} \leq \mathbf{r} \leq \mathbf{R}$, where $\mathbf{r_0}$ is a specified minimum radius and $\mathbf{R}$ is a specified maximum radius, in the following way. First, we define a uniform random variable $u \in [0,1)$ and use this to generate the new random radius $\mathbf{r}$ via:

$$\mathbf{r} = (\sqrt{u} * (\mathbf{R} - \mathbf{r_0}) + \mathbf{r_0}) \tag{2}$$

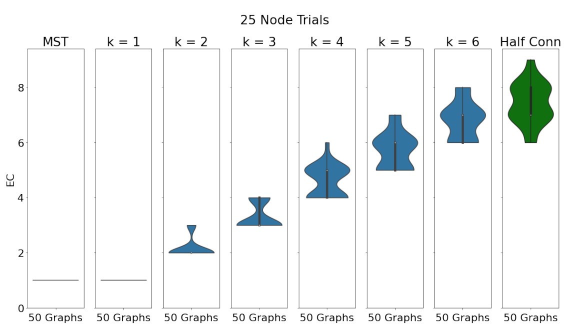
which prevents biasing the random radii toward the center ρ [26]. We chose our ρ value to offer some unique terrain and littoral profile: Stinson Beach, just north of San Francisco, California. This location allows us to simultaneously model propagation effects caused by water and both steep and non-imposing terrain based upon elevation data. We additionally limited the maximum radius $\mathbf{R}$ to 5000 meters (an area of $\sim 78.5\text{km}^2$), and set the default altitude for every antenna to be 2 meters (about the antenna height of a vehicle borne radio), and used node population sizes of $n \in \{25, 50, 100\}$ to generate 50 graphs for each population. As population size increased, the node per area density increased as we held $\mathbf{R}$ constant at 5000 meters. For each of the 50 graphs within a population size, we generated connection topologies using kXTC for $k \in \{1,2,3,4,5,6\}$.

For this experiment, the RF parameters were uniform across all nodes and populations. As stated, the SINR values for all nodes followed the simple model of reliable communication outlined in Appendix B.1 at a value of 10 dB. Each node had a noise figure of 4 dB (a standard default value representative of most receivers [27]), no transmit or receive gain, and transmitted at a bandwidth of 5 GHz. The maximum possible transmission power for each node was 50 dBm, so if a link required more power than this to establish communication, the cost function became undefined for that link and unusable by our topology control algorithms. This set of RG simulations produced 900 graphs to analyze for resilience and power control characteristics for our two different objective cost functions.

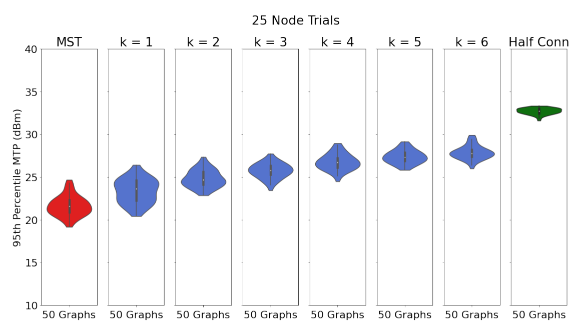
4.2.2 Analysis of kXTC Modalities

For each kXTC topology generated from the original 50 graphs within a population size, we represented the resiliency measure of a graph as its EC. The MTP distribution of each kXTC topology was evaluated at the 95th percentile, thus capturing a power threshold representative of nearly all nodes without being excessively affected by the outlier maximum transmission values. As reference for the performance gains that kXTC provides for both EC and power control, we show additional results for an MST topology as a lower bound on performance measure generated using Kruskal's Algorithm [14]. For an average case network void of any topology control, we provide results for a network that randomly connects half of all potential edges meeting the cost criteria, which we refer to as simply "Half-Conn." Since our experiments produced 50 graphs per node population, we present our relevant statistics as distributions of these graphs. When showing distribution results we use violin plots, which are similar to box plots, except they also show the probability density of the data at different values, smoothed by a kernel density estimator.

Figs. 8 and 9 provide the initial results for resilience and MTP distributions when cost is constrained using free space loss across the 25 and 100 node RG populations. The comparison of these figures reinforces our intuition that resilience increases proportionally as k increases, as seen in the increasing EC distributions in Figs. 8(a) and 9(b). Furthermore, by looking at the MTP distributions for both 25 node (Fig. 8(b)) and 100 node (Fig. 9(b)) graphs, for $k = 1$ kXTC provides near MST (in red) levels of power control. Increasing k causes the distributions to increase overall, but when compared to a more naive "Half-Conn" approach to establishing connectivity, kXTC provides consistent power control (about 10 dB lower than "Half-Conn") while ensuring that the overall resilience of a topology also increases as k does.

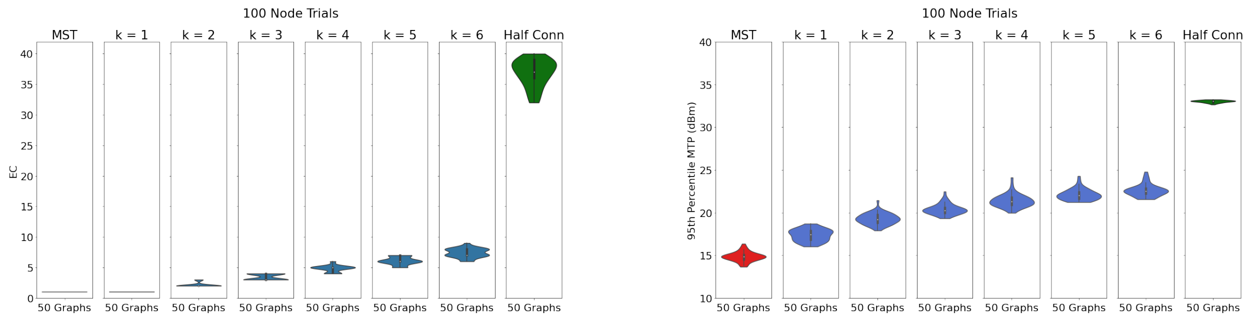


(a) EC Distribution



(b) 95th Percentile MTP Distribution

Fig. 8—Free Space Loss: 25 Nodes

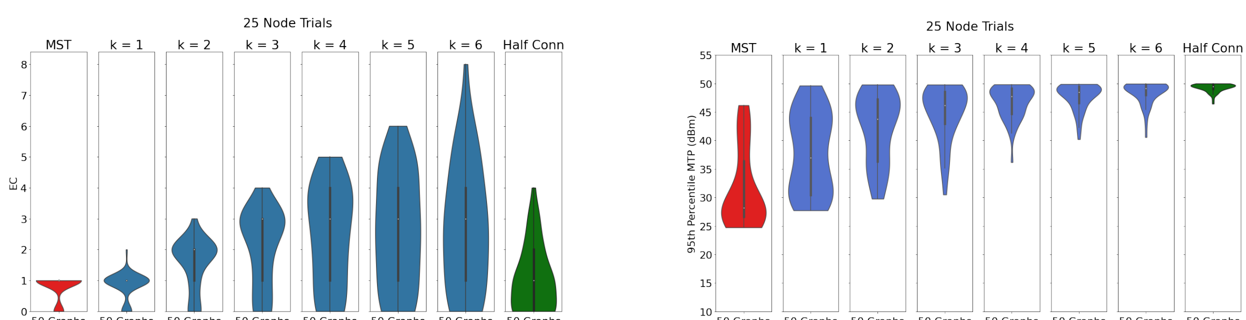


(a) EC Distribution

(b) 95th Percentile MTP Distribution

Fig. 9—Free Space Loss: 100 Nodes

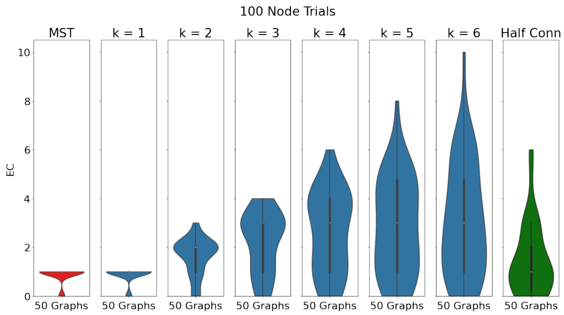
For terrain induced propagation in Figs. 10 and 11, the transmit costs have become increasingly complex and many of the graphs became disconnected by simply introducing terrain affected loss in the cost function. When fragmented due to link cost constraints, kXTC still establishes topologies in the remaining connected components. EC, however, becomes 0 for disconnected graphs and cannot measure these conditions for partial resilience (in Figs 10(a) and 11(a)). For this reason, we develop and use an alternative metric to gain insight into more detailed structural resilience in the next section. Despite the more extreme and non-distance proportional cost when using the complex loss models, XTC still adheres to the *k*-connected graph guarantee when the creation of a connected graph is possible. Even the simple "Half-Conn" topologies fail to consistently establish well connected graphs or provide the high levels of resilience that we saw in the free space loss examples. This striking decrease in resilience by the "Half-Conn" topology speaks to how difficult it is to establish networks when costs shift to be more complex and less proportional to distance. Noticeably in Figs. 10(b) and 11(b), the "Half-Conn" topology pushes the MTP to the default maximum power of 50 dB for both the 25 and 100 node population graphs, while the cost informed topology control algorithms (both MST and kXTC) manage to deliver the better power distributions in light of the resources and connections available. Despite wireless links in general requiring significantly more power to establish when using terrain induced path loss estimates, kXTC achieves its goal of resiliently connecting the network as best as it can while optimizing the MTP distributions.



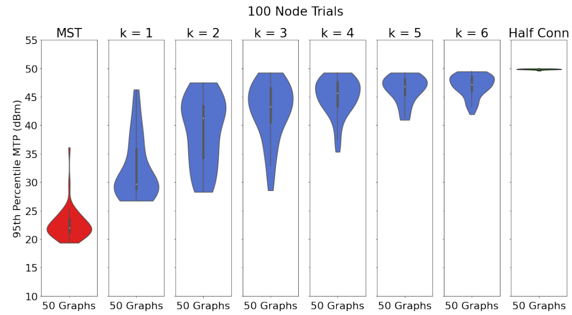
(a) EC Distribution

(b) 95th Percentile MTP Distribution

Fig. 10—Terrain Induced Loss: 25 Nodes



(a) EC Distribution



(b) 95th Percentile MTP Distribution

Fig. 11—Terrain Induced Loss: 100 Nodes

As demonstrated in our terrain-based examples, where many distance and geometric-based algorithms would fail, kXTC manages to produce topologies from the potential connections available with increased edge connectivity and constrained power control, almost as good as an MST for $k = 1$. Given kXTC's inherent ability to adapt to cost while maintaining resilience, we plan further work using it as a foundation on which to develop additional radio or network driven cost adaptations, which we will discuss in later sections. These adaptations include hybrid optimizations of additional directive RF resources and using the algorithm output to inform heterogeneous asset deployment and positioning of nodes in challenging scenarios.

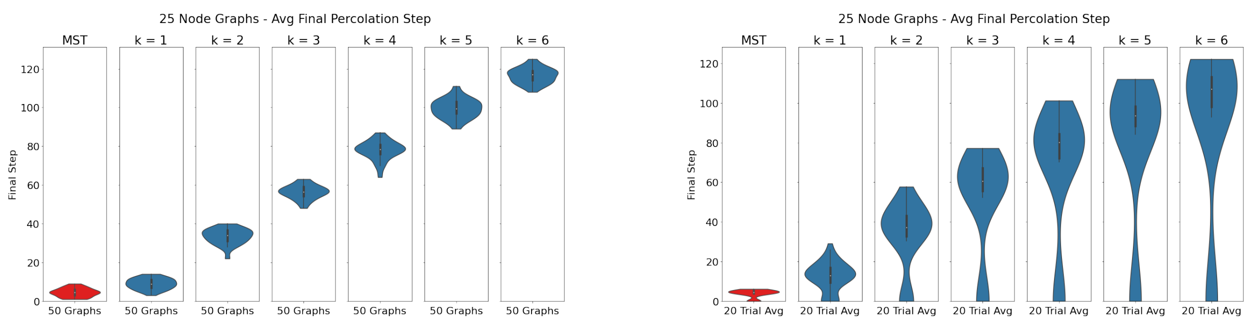
4.2.3 kXTC: Evaluating Structural Resilience to Communication Edge Failure

The resilience of a network's topology consists both of robust measures of its connectivity and its ability to maintain functional operation despite internal failures. Traditional percolation studies focus on the removal of nodes to model the collapse or destruction of a specific network entity to determine how the removal of these nodes affects the ability for other nodes to receive information [28, 29]. This model lends itself to analysis of networks created by static infrastructure or centralized network models [28].

Our focus on edges, or links, shifts the measurement of resilience from basic topological network approaches. Node-centric percolation (most prevalent) defines resilience as ensuring that most of the nodes remain connected even when a fraction of nodes disappear, essentially ensuring the network will continue to survive [29]. The size of the largest remaining graph component, the greatest connected component (GC), is often the metric used in the past for node-centric percolation studies, but some graph performance is abstracted away when only looking at the size of components. Rather than focusing on the number of nodes connected, our edge centric approach looks at resilience through a lens of information flow and communication links. That is, despite the removal of edges, how well can a network support the transmission of information across the remaining links from source to destination. One such measure of interest is referred to as the average Inverse Geodesic Distance (IGD), which is based on the average geodesic distance (GD) that measures the average of the shortest paths between any two nodes. Though the average GD is a strong measure of a network's ability to support flow of information, it becomes undefined for disconnected graphs. To enable measurement of network performance as graphs become disconnected, the average IGD inverts the lengths of the shortest paths between any two nodes and calculates the average of these inverted lengths over all nodes. It remains a finite value approaching zero as the graph becomes more disconnected, while still capturing a network's structural capability to handle information flow with remaining components [30].

We conduct our percolation simulations by iteratively removing at each percolation step i a randomly chosen edge $(u,v) \in E_i$, where E_i is the set of edges remaining in the graph at step i. For each edge removal step we measure a network's average IGD until the size (in nodes) of the GC is 50% of the network size, either 25, 50, or 100 for each population.

Intuitively, as the value k increases for the kXTC algorithm (i.e., connectivity increases), a percolation simulation should both take longer to reach 50% sized GC and the network IGD value will approach zero more slowly. Figure 12(a) shows the number of edge removal percolation steps to reach 50% sized GC for a 25 node RG network using an MST-generated topology and kXTC for $k \in \{1,2,3,4,5,6\}$ across 50 graphs using free space loss as the cost criteria. This figure demonstrates that although the MST topology provides a more optimal solution for power reduction, it provides minimal resilience against failure and quickly becomes a severely degraded structure after only half a dozen or so link failures. Figures 12(a) and 12(b) further demonstrate the existence of a proportional relationship between increasing values of k for kXTC and a greater number of steps required to sufficiently degrade the network. This does not necessarily guarantee functional operations of a network in terms of latency, bandwidth, or link quality, but it does demonstrate fundamental structural network resiliency improvements.



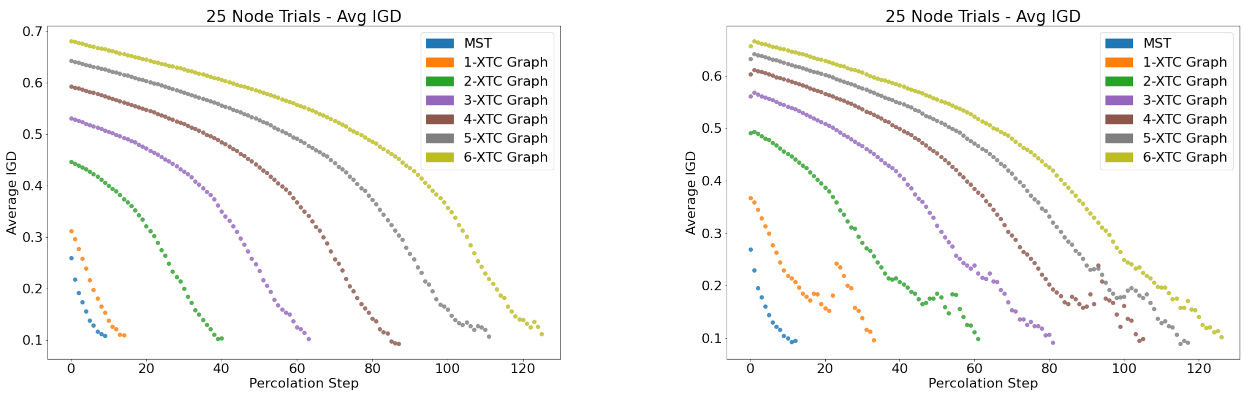
(a) Free Space Loss

(b) Terrain Induced Loss

Fig. 12—Distribution of Edges Removed - RG

For the free space loss driven cost models, the generated kXTC topologies are both more likely to be connected and the edges are distributed near uniformly throughout the topology. It can be assumed that given this uniformity in distribution of edges, that one random edge removal percolation simulation is fairly representative of the $|E|!$ combinations of possible edge removal simulations. The same cannot be said for the case of the terrain influenced kXTC topologies generated using complex loss models to determine transmission power cost. For these cases, due to the non-distance correlated cost, connections and disconnections may be localized and form clusters of either disconnected or lightly connected nodes. Due to the topological structure imposed by terrain, random edge removal will behave differently across percolation simulations depending on the starting edge randomly selected, so some simulations may quickly reach a 50% GC or some may take longer. To account for this inconsistent behavior, the distributions in Fig. 12(b) capture the average number of steps to reach 50% GC for each of the 50 graphs over 20 random edge removal percolation simulations. Even when using more complex cost functions, kXTC delivers comparable resilience to that of the free space loss results, thus demonstrating its flexible and functional ability to adapt to more severe and complex costs.

There exists a similar proportional relationship between k values and the average IGD across a topology's lifetime as seen in figure 13, which shows the average IGD for each edge removal step across 50 nodes for increasing k values using both free space and complex terrain induced cost functions. As k increases, the average IGD approaches zero more slowly and maintains a consistently higher average IGD for a longer percolation period. Similar trends existed for both the 50 node and 100 node populations. With higher k, nodes have a larger set of edges from which to construct shortest paths to other nodes, and the number of possible shortest paths remains higher through longer periods of degradation.



(a) Free Space Loss (b) Terrain Induced Loss

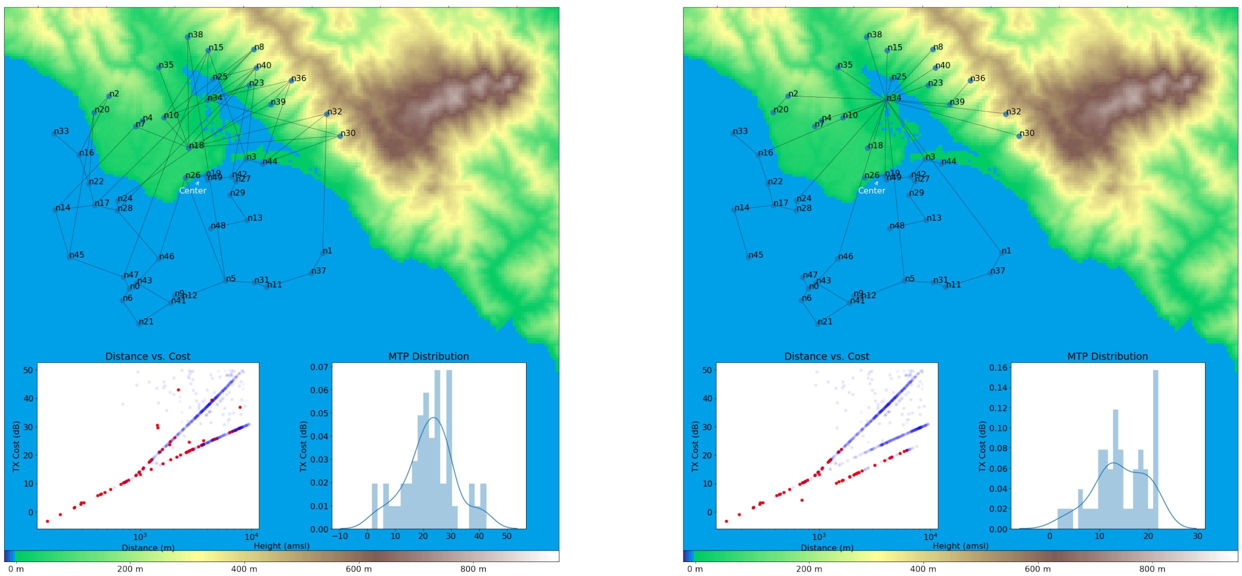
Fig. 13—Average IGD Per k Value - Random Geometric

Of particular interest, despite the more complex cost in terrain, the IGD values are similar to those of the free space loss examples. The steps (Fig. 12) to reach a GC of 50% are also similar for k values in the terrain graphs as those in the free space loss graphs. The small differences could exist for a variety of reasons. In accounting for terrain, 150 of the 900 random geometric graphs started the percolation simulation already disconnected. The IGD value would start lower and the GC would be closer to being 50% of the original graph size when beginning as a disconnected graph. So, while for some terrain graphs this may be the case, the majority of the terrain affected topologies still receive kXTC's resilience benefits and maintain functional ability to promote information flow with similar capability as those kXTC topologies that use free space loss in their cost equations.

4.3 Topology Control for Complex Costs

Terrain effects complicate the MTP problem, primarily, by frequently penalizing communication links with additional exponential loss characteristics. In accounting for terrain, the 95th percentile MTP distribution for $k = 1$ occupies a higher and larger range of powers than for all k values of the free space loss results. In general, our results reinforce that accounting for realistic RF propagation, simply by including terrain loss models, complicates the determination of the kXTC topology. This uncoupling of cost and distance would complicate topology control for many WSN type algorithms that may assume distance or geometric relationships, but kXTC maintains its topology control abilities despite complex costs.

To provide greater topology control assurances in the presence of complex cost, we briefly demonstrate an illustrative advantage of heterogeneous networking that can reduce the MTP profile resulting from kXTC significantly. In a terrain-modeled scenario with more severe and complex cost, we could raise an airborne network asset at a reasonable altitude above the terrain and surface nodes. Airborne assets have a propagation advantage in this case and can also be more easily equipped with antennas that can provide directivity properties and gains for surface nodes. Figure 14 shows the contrast in kXTC topology, cost to distance ratios, and the MTP distribution created by including one network node at an altitude of 1000 meters and giving it 6 dB gain in transmission and reception to surface nodes within an area. AdaptNet provides new costs generated from this heterogeneous scenario to kXTC and the results demonstrate a dramatic reduction in network diameter (for $k = 1$) and an MTP distribution shift to a much lower network power profile.



(a) All nodes - 2m Altitude

(b) Node 34 - 1000m Altitude, 6 dB tx/rx gain

Fig. 14—kXTC Topology

In the more generic case, optimizing how to use heterogeneous network assets, node placements, and RF directivity is non-trivial and we plan to focus upcoming work in this area.

5. MITIGATING NETWORK-BASED TRANSMISSION POWER INCIDENT AT AN EXTERNAL NODE

In this section of the paper we introduce a new scenario model and examine topology control adaptation to costs constrained by a receiver external to the communicating network. To allow for this externally-driven cost, we adapt our previous cost constraints by including an estimate of the induced received power (IRP) at a designated location(s) when a network node transmits at the value needed to sustain a potential reliable network neighbor. These cost estimates now become the potential edge costs in the kXTC algorithm and a received power threshold value can also be established that we wish not to exceed in order to maintain low observability or to reduce the transmission node's interference at a specific location(s). With these new cost constraints, the network adapts its topology as before, but instead it also favors links that orient IRP away from external receivers. The topology computation may result in a fragmented network dependent upon the received energy thresholds established, but as in our previous airborne node scenario that can also be informative in pointing out where additional measures such as directivity can be used to further reduce IRP at the external receiver(s).

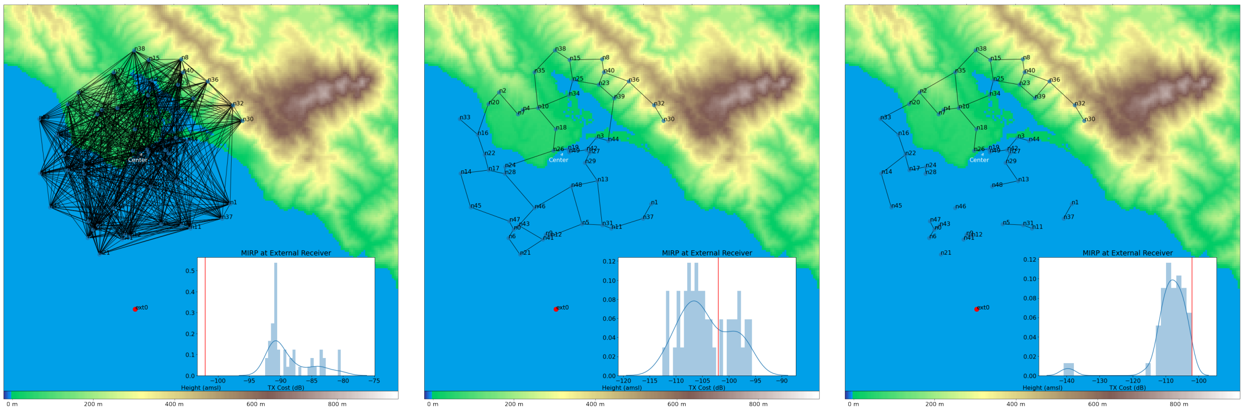
5.1 Reducing Incident Power at an External Location

Ultimately, in the presence of some external receiver, *R*, the network adaptation objective is now to minimize the cost function relative to *R* (whereas, our objective earlier was to minimize the maximum node transmit power needed to connect the network and a set of neighboring communication links). In cases where the network may not possess perfect knowledge of an external receiver, one could establish a probabilistic weighting of the cost function towards a set of areas or sensitivity criteria, which would lead to a more complex formulation of the cost function. We highlight below how adjusting kXTC's cost function to account for the characteristics of a single external receiver reduces the network's maximum incident received power (MIRP) profile, (see Appendix B.2 for details on how we adapt the cost function relative to the external receiver). In this set of experiments we compare the resulting MIRP profiles for an IRP cost optimized kXTC ($k = 1$) to both a regular kXTC ($k = 1$) and our simple "Half-Conn" topology example.

5.1.1 Examples of External Incident Power Reduction

Our primary example scenario shows a coastal wireless network (again, chosen somewhat arbitrarily as Stinson Beach, California). We have a network of 50 nodes, all at an altitude of 2 meters, upon which we will perform adaptive topology control. Additionally, the external receiver (marked in red) resides outside of the network at an altitude of 20 meters and is about 10 times more sensitive than the network nodes with a *noise figure* of 2 db, a *SNR* of 10 db, scanning at a bandwidth of 1 GHz, and a thermal noise floor of -174 dB. There is no receive gain since we are modeling the external receiver as an omnidirectional antenna. This produces a receiver threshold of -102.0 dB, compared to an internal network node sensitivity modeled at about -93 dB. Our first results model cost as transmission power using free space loss for propagating transmissions.

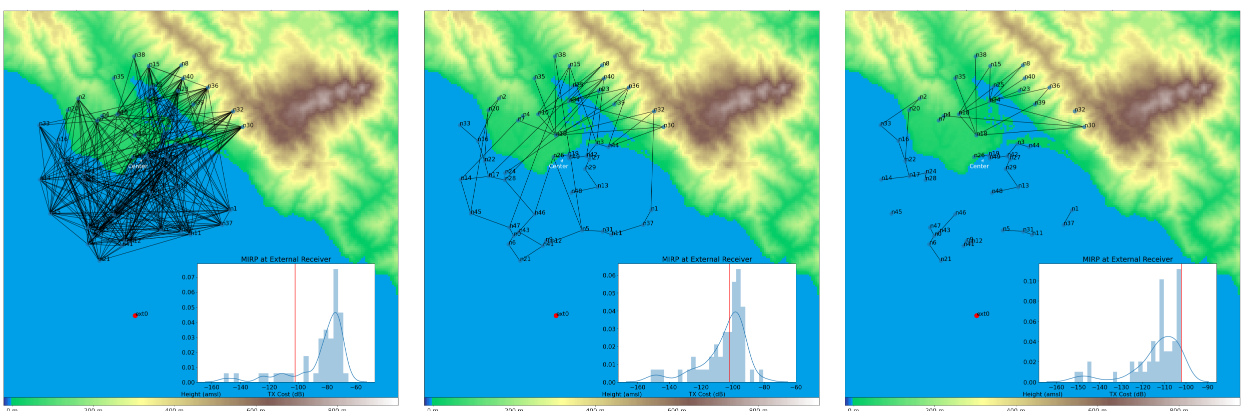
Figure 15 shows the MIRP profile at the external receiver from each network's MTP profiles for the different topology methods. The initial Fig. 15(a), shows that for the simple "Half Conn" topology method, the external receiver is able to sense all network node transmissions (the established external receiver's sensitivity constraint is the vertical red line). This makes sense given that the "Half Conn" is not implementing any real power control and each transmitter covers a reasonable distance to reach its highest cost network neighbor. Figure 15(b) presents the kXTC results that use only internal network power cost constraints. In this case, we see that the generated MIRP violates the established receiver's threshold for several of the kXTC topology links. The MIRP results change quite significantly and promisingly in Fig. 15(c), when the cost function adapts to reduce the IRP costs. Noticeably, this external node constrained cost function prevents kXTC from establishing a connected topology, but all network connections that are established transmit MIRP values beneath the external node's receive sensitivity.



(a) Half Connected (b) kXTC - No External Constraints (c) kXTC - External Constraints

Fig. 15—External Node - Free Space Loss

In Fig. 16, we see that kXTC maintains its ability to adapt the MIRP distributions to the presence of the external receiver, despite the more complex terrain induced cost function. Similar connected components result in Fig. 16(c) as we saw in the free space loss example that transmit below the external receiver's sensitivity and provide a foundation to add in connectivity solutions beyond topology control.



(a) Half Connected (b) kXTC - No External Constraints (c) kXTC - External Constraints

Fig. 16—External Node - Terrain Loss

In order to produce more comprehensive solutions to power control and network management, we plan to perform upcoming research examining hybrid and heterogeneous extensions of work presented here, including analyzing the use of RF directivity when possible and examining gains achievable by additional asset scheduling, such as the use of multiple channels, frequency bands, and scheduling algorithms.

6. CONCLUSION

In this paper we presented an implementation and a series of wireless network simulations for kXTC, a resiliency-enhanced wireless network topology control approach. In our analysis, we targeted the use of kXTC to construct topologies reducing the overall resultant network MTP profiles in the absence of external constraints. We also demonstrated the kXTC adaptation in reducing MIRP profiles in the presence of an external constraint. We presented past work and discussed general advantageous characteristics of the kXTC algorithm for ad hoc network topology control including: graph spanner properties, low complexity, generic edge cost support, flexible resiliency, node position independence, and distributed operation potential.

We initially simulated the topologies and MTP distributions that kXTC obtains for various values of k resilience in a free space propagation environment. In these experiments we used several invariant graph metrics to examine kXTC characteristics as k increases, such as eccentricity and algebraic connectivity. We show that kXTC, for $k = 1$, results are close to the optimal MST results, but as k increases, more deterministic resiliency is added to the graph structure with moderate sacrifices in increased MTP profiles. In conclusion, kXTC with moderate k values appears to present a good tradeoff between topology cost minimization and resiliency features.

We next described a series of experiments, including a large set of randomly generated wireless networks. We presented results and analysis for both free space and terrain-atmospheric propagation models to examine resiliency properties. For comparison we included analysis results for MST (low cost but high fragility) and networks with half of all potential edges connected ("Half-Conn"). For our initial free space experiments, it was clear that kXTC provides moderate power increases over MST, but also adds resiliency features while remaining far below the "Half-Conn" results in terms of required MTP profiles. In the more complex terrain propagation experiments, the distribution of edge connectivity and power profiles had high variance as expected, but moderate values of k showed gains over "Half-Conn" resiliency results. To further examine topological resiliency in these more constrained scenarios, we introduced the concept of edge-based percolation to gain some insight into resiliency when perhaps only partially resilient network solutions are generated due to scenario limitations. The percolation results demonstrated consistent resiliency improvements over MST as values of k increased for terrain modeled cost. These results give us confidence in the partial resilient topologies being formed by kXTC when conditions prevent ideal k connectivity goals from being met.

Next, we presented kXTC in a challenging terrain-induced loss case, but we added a heterogeneous node to the calculation with both increased altitude and additional antenna gains. This heterogeneous "advantaged node" scenario dramatically changes the topology and the maximum transmission cost distribution with some of the nodes using 20dB less transmit power within the connected network than in the non-advantaged scenario.

In our final experiments and analysis, we used an external receive location, outside of the communication network, as the primary network topology formation cost constraint. We presented examples of kXTC using these external cost models and demonstrated that the network organizes itself in terms of these costs, but will also disconnect itself based upon stricter constraints as needed.

This initial report forms the basis for some of our future research in network topology control where we will address several challenging areas raised by these experiments. First, we plan more detailed research in optimizing and adapting network topologies towards external-based constraints and examining temporal adaptation issues. Second, we plan to further explore fundamental mechanisms to utilize heterogeneous network and RF directivity assets to construct and maintain dynamic topologies with low transmission profiles while maintaining effective networks supporting data throughput needs. Significant related network science technical challenges will have to be addressed relating to network wide optimization with heterogeneous assets and asymmetric capabilities. Significant work is also needed in determining how to best model cost and optimization in the presence of temporal network dynamics. Given our flexible cost framework we can examine not only realistic propagation loss models, but introduce temporally changing cost functions, such as dynamic link quality or network mobility models. Future applied research could also begin to examine the relation to distributed control and state sharing of cost functions within adaptive networks to further mature the foundational approaches demonstrated here.

REFERENCES

1. R. Wattenhofer and A. Zollinger, "XTC: A Practical Topology Control Algorithm for Ad-hoc Networks," Proceedings of the 18th International Parallel and Distributed Processing Symposium, 2004. Proceedings. (IEEE), 2004, p. 216.

2. Y. Du, J. Gong, W. Zhangmin, and N. Xu, "A Distributed Energy-Balanced Topology Control Algorithm Based on a Noncooperative Game for Wireless Sensor Networks," *Sensors* **18**, 4454 (12 2018), doi:10.3390/s18124454.

3. G. Hackmann, O. Chipara, and C. Lu, "Robust Topology Control for Indoor Wireless Sensor Networks," Proceedings of the Proceedings of the 6th ACM conference on Embedded network sensor systems, 2008, pp. 57–70.

4. P. Santi, "Topology Control in Wireless Ad-hoc and Sensor Networks," *ACM computing surveys (CSUR)* **37**(2), 164–194 (2005).

5. J. P. Macker, D. Claypool, and N. Hughes, "Improving Routing Path Stability in Mobile Ad Hoc Networks That Use a CDS Control Plane," Proceedings of the 2014 IEEE Military Communications Conference, 2014, pp. 1099–1104.

6. J. P. Macker, J. Dean, and W. Chao, "Simplified Multicast Forwarding in Mobile Ad-hoc Networks," Proceedings of the IEEE MILCOM 2004. Military Communications Conference, 2004., volume 2, 2004, pp. 744–750 Vol. 2.

7. N. Li, J. C. Hou, and L. Sha, "Design and Analysis of an MST-based Topology Control Algorithm," *IEEE Transactions on Wireless Communications* **4**(3), 1195–1206 (2005).

8. D. Wagner and R. Wattenhofer, *Algorithms for Sensor and aAd-hoc Networks: Advanced Lectures* (Springer-Verlag, 2007).

9. R. Wattenhofer, L. Li, P. Bahl, and Y. M. Wang, "Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad-hoc Networks," Proceedings of the Proceedings IEEE INFOCOM 2001. Conference on Computer Communications. Twentieth Annual Joint Conference of the IEEE Computer and Communications Society (Cat. No. 01Ch37213), volume 3 (IEEE), 2001, pp. 1388–1397.

10. A. C. C. Yao, "On Constructing Minimum Spanning Trees in k-Dimensional Spaces and Related Problems," *SIAM Journal on Computing* **11**(4), 721–736 (1982).

11. S. Narayanaswamy, V. Kawadia, R. S. Sreenivas, and P. R. Kumar, "Power Control in Ad-Hoc Networks: Theory, Architecture, Algorithm and Implementation of the COMPOW Protocol, 2002.

12. M. Li, Z. Li, and A. V. Vasilakos, "A Survey on Topology Control in Wireless Sensor Networks: Taxonomy, Comparative Study, and Open Issues," *Proceedings of the IEEE* **101**(12), 2538–2557 (2013).

13. D. Li, B. Wang, and X. Jia, "Topology Control for Throughput Optimization in Wireless Mesh Networks, 12 2008, pp. 161–168. doi:10.1109/MSN.2008.13.

14. J. B. Kruskal, "On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem," *Proceedings of the American Mathematical Society* **7**(1), 48–50 (1956), ISSN 00029939, 10886826. URL `http://www.jstor.org/stable/2033241`.

15. S. Baswana and S. Sen, "A Simple and Linear Time Randomized Algorithm for Computing Sparse Spanners in Weighted Graphs," *Random Structures & Algorithms* **30**(4), 532–563 (2007).

16. E. Lloyd, R. Liu, and S. Ravi, "Approximating the Minimum Number of Maximum Power Users in Ad Hoc Networks," *Mobile Networks and Applications* **11**, 129–142 (01 2006), doi:10.1007/s11036-006-4467-7.

17. P. Chew, "There is a Planar Graph Almost as Good as the Complete Graph," Proceedings of the Proceedings of the Second Annual Symposium on Computational Geometry, SCG '86, New York, NY, USA (Association for Computing Machinery), 1986, pp. 169–177. ISBN 0897911946, doi:10.1145/10515.10534. URL `https://doi.org/10.1145/10515.10534`.

18. J. Cho, Y. Chen, and Y. Ding, "On the (Co)Girth of a Connected Matroid," *Discrete Applied Mathematics* **155**, 2456–2470 (11 2007), doi:10.1016/j.dam.2007.06.015.

19. S. Ghosh, K. Lillis, S. Pandit, and S. Pemmaraju, "Robust Topology Control Protocols," Proceedings of the International Conference On Principles Of Distributed Systems (Springer), 2004, pp. 94–109.

20. M. Fiedler, "Algebraic Connectivity of Graphs," Proceedings of the Czechoslovak Mathematical Journal, 1973.

21. N. Abreu, "Old and New Results on Algebraic Connectivity of Graphs," *Linear Algebra and its Applications* **423**, 53–73 (05 2007), doi:10.1016/j.laa.2006.08.017.

22. J. Feddema, R. Byrne, and C. Abdallah, "Algebraic Connectivity and Graph Robustness (01 2005), doi:10.2172/973665.

23. K. Shekhawat, "Connectivity Algorithm," *CoRR* **abs/1507.06677** (2015). URL `http://arxiv.org/abs/1507.06677`.

24. B. Winkel and A. Jessner, "Spectrum Management and Compatibility Studies with Python," *Advances in Radio Science* **16**, 177–194 (Sept. 2018), doi:10.5194/ars-16-177-2018.

25. "Prediction Procedure for the Evaluation of Interference Between Stations on the Surface of the Earth at Frequencies Above About 0.1 GHz, Recommendation P.452-16," International Telecommunication Union, 2015.

26. E. W. Weisstein, "Disk Point Picking," August 2020. URL `https://mathworld.wolfram.com/DiskPointPicking.html`.

27. J. Freeman, "Principles of Noise (Wiley and Sons.), 1958.

28. D. S. Callaway, M. E. J. Newman, S. H. Strogatz, and D. J. Watts, "Network Robustness and Fragility: Percolation on Random Graphs," *Phys. Rev. Lett.* **85**, 5468–5471 (Dec 2000), doi:10.1103/PhysRevLett.85.5468. URL `https://link.aps.org/doi/10.1103/PhysRevLett.85.5468`.

29. R. Albert and A. L. Barabási, "Statistical Mechanics of Complex Networks," *Reviews of Modern Physics* **74**(1), 47–97 (Jan 2002), ISSN 1539-0756, doi:10.1103/revmodphys.74.47. URL `http://dx.doi.org/10.1103/RevModPhys.74.47`.

30. P. Holme, B. J. Kim, C. N. Yoon, and S. K. Han, "Attack Vulnerability of Complex Networks," *Phys. Rev. E* **65**, 056109 (May 2002), doi:10.1103/PhysRevE.65.056109. URL `https://link.aps.org/doi/10.1103/PhysRevE.65.056109`.

Appendix A

K-XTC ALGORITHM PSEUDOCODE

Algorithm 1 XTC algorithm with k-resiliency

Require: Network neighbors v for each node i with an associated cost function for each neighbor edge $E_{i,v}$

```
kmin ← resiliencythreshold
kxtcNodes ← Nodes
kxtcEdges ← emptySet
for i ← Nodes do
    construct ≻i: ordered list of i neighbors
    exchange ≻i with neighbors
end for
for i ← Nodes do
    inSet_i ← emptySet
    outSet_i ← emptySet
    for v in ≻i do
        k ← 0
        if (inSet_i ∪ outSet_i) isEmpty then
            inSet ← inSet + v
            continue
        else
            inSet ← inSet + v
            for cov in (inSet_i ∪ outSet_i) do
                if (cov in ≻v) and (cov < i in ≻v then
                    k ← k + 1
                    if k >= kmin then
                        inSet ← inSet − v
                        outSet ← inSet + v
                        break
                    end if
                end if
            end for
        end if
    end for
    xktcEdges_i ← inSet
end for
for i ← Nodes do
    kxtcEdges ← kxtcEdges + kxtcEdges_i
end for
```

Appendix B

RADIO PROPAGATION MODEL

B.1 Robust Transmission Power as Edge Cost

Given the parallel between the theoretical notion of graph edge weight and transmission cost, we developed a software framework to model graph edge constraints in more detail. Figure 2 highlights the components within our framework to calculate transmission power that ultimately drives the determination of the cost used in our topology control experiments.

In general, determining transmission power required to establish communication from a source node to a receiving node comprises the following node specific characteristics:

- Transmit Gain (G_t): Any gain the transmit antenna may possess in dBi (decibels-isotropic). This enables an antenna to be directional in its transmissions by focusing power towards some heading rather than emitting omni-directionally.
- Receiver Gain (G_r): Any gain the receiving antenna may possess in dBi (decibels-isotropic). Similar to transmit gain, receiver gain directs powers towards a direction, which enables a receiver to detect more signal in a specific direction.
- Propagation Loss (L_P): The loss experienced by the transmitting signal as it propagates from source to receiver in decibels. As the signal propagates, it will experience a reduction in power.
- Receiver Threshold (R_t): A signal power threshold below which the receiver (due to hardware constraints and other link quality factors) cannot detect a signal.
- Maximum Transmit Power ($P_{T_{max}}$): The maximum power at which a node can transmit. Any connection requiring power greater than a node's capability cannot be established.

With these factors in mind, calculating required transmit power from node i to node j relies on the following equation, where all values are in some form of decibel (gains are in dBi, which is decibels from an isotropic antenna):

$$P_{T_{i,j}} = R_{t_j} + L_{P_{i,j}} - G_{t_i} - G_{r_j} \tag{B1}$$

There are additional environment conditions to consider when modeling transmit and receiver gain. Future planned work will develop and integrate more accurate directivity abstractions on a per link basis. Modeling propagation loss and receiver threshold can also be more complex, depending on the underlying assumptions being made when accounting for loss and radio hardware/link quality effects.

An oft used simple approach to model propagation loss as previously mentioned, models loss as directly dependent on distance. Varying an exponential value of distance can then provide a measure of loss severity.

In order to provide a more accurate free space propagation estimate, the loss in dB of a signal from node i to node j for units of distance (D) in kilometers and frequency (F) in Gigahertz is as follows:

$$L_{FSL_{i,j}} = 92.45 + 20 * \log_{10}(D_{i,j}) + 20 * \log_{10}(F_i) \tag{B2}$$

This equation informs the loss values with more information than just distance, but still assumes a near perfect, unrealistic propagating environment. To account for more complex terrain and atmospheric effects in our simulations, we have chosen to use loss modeling capabilities from the spectrum management library Pycraf [24]. Pycraf provides radio wave propagation modeling, based upon ITU standard 452-16, that more accurately models loss over terrain and through varying atmospheric conditions [25].

Determining the threshold of a receiver proves complicated to model as well, since the sensitivity depends on many factors that vary with operational posture. Some factors stem from internal technology implementations (e.g., signal modulation, packet coding, etc.), while others arise from the receiver hardware itself. Of these myriad factors, the most approachable to model are the link quality (LQ), a receiver's *noise figure* (N_f), and the bandwidth (BW) at which the receiver is operating.

The quality of a link determines the ability of a receiver to distinguish signal from noise. Often this link quality is measured using a signal-to-interference noise ratio, or SINR. The SINR depends on signal modulation, coding schemes, and bandwidth used. A receiver's N_f is determined by a combination of power supply, input voltage, and circuit design, but generally receivers have a noise figure between 2 and 10 dB [27]. We have chosen a value of 4 dB as the default to represent a fairly sensitive receiver. Additionally, a receiver's sensitivity is informed by the universal thermal noise floor: -174 dBm. Mathematically, this relationship takes the form of the following equation:

$$R_t = -174 + N_f + 10 * \log_{10}(BW) + <LQ> \tag{B3}$$

Since bandwidth is scenario dependent and easily determined, we will choose that value at run time such that it is relevant to our examples (or determined by some type of spectral power density). The link quality metric provides a flexible means to include some measure of link quality in modeling propagation loss. This can vary from SINR values representing thresholds for different levels of reliability or a measure of expected transmission count (ETX) derived from a probability of reception (PoR) value for a given wireless network link architecture. For example, in our scenarios, rather than implement SINR values for a specific waveform, such as 802.11 WiFi, we modeled an initial notion of SINR using a simple tiered approach:

- SINR = 10 dB: Represents a link with a POR 1.0 (i.e., link receives all packets successfully)
- 7.0 dB $\leq$ SINR $<$ 10 dB: Represents a link with POR 0.90 (i.e., link loses 10 percent of packets)
- 4.0 dB $\leq$ SINR $<$ 7.0 dB: Represents a link with POR 0.80 (i.e, link loses 20 percent of packets)
- SINR $<$ 4.0 dB: Below this SINR, the link ceases to function properly and will functionally be non-existent

While an admittedly coarse approximation of SINR, this representation can be used to define communication levels, with a SINR of 10 dB constituting "reliable" communication and SINR values below 7.0 dB providing differing tiers of unreliability. An actual SINR threshold depends both on context of operation and waveform implemented in the receiver, so while this tiered approach gives an initial model, it can change to reflect a more realistic scenario.

The framework we've created enables the use of a variety of link quality metrics that can influence the modeling of propagation loss when transmitting from a source node to a receiving node. Once cost is defined or estimated for every node and its two-hop neighbors, XTC can use those costs to generate a topology exhibiting spanner like graph properties with efficient connections.

B.2 Modeling External Node Cost

In order to establish an energy limit representing undesired interference or detection we need to establish the external receiver's threshold, or sensitivity. There are several factors impacting a receiver's sensitivity, the greatest of which can be seen by referring back to Fig. 2, namely, G_r, N_f, BW, hardware sensitivity, and SINR. The lower SINR, BW, hardware sensitivity, or N_f values are, the more sensitive a receiver will be. Once we define the external receiver sensitivity, we can use it to determine which links internal to our topology incite enough power at the external node to violate its threshold. A process similar to our previous objective function is followed, where we generate a matrix of costs required to establish reliable communication between every node internal to the network:

$$P_{T_{i,j}} = R_{t_j} + L_{P_{i,j}} - G_{t_i} - G_{r_j} \tag{B4}$$

Once we have this matrix of transmit costs, we can then determine the incident power at the external receiver caused by a transmitting network node i and populate a new cost matrix of incident powers at external receiver for every potential node i to node j network connection via the following equation:

$$P_{I_{i,etx}} = P_{T_{i,j}} - L_{P_{i,ext}} + G_{t_i} + G_{r_{ext}} \tag{B5}$$

Where the power incident at the external receiver *ext* from node i is residual power from the transmitting signal used by node i to communicate with node j having experienced propagation loss across the distance from node i to ext. If this incident power, $P_{I_{i,etx}}$, exceeds the external receiver's sensitivity threshold, then the external receiver will be able to detect the power from that internal node's transmission. Any transmission violating the external receiver's sensitivity is marked as unusable by kXTC in the incident cost matrix. We then use this new external receiver cost matrix within kXTC to generate a topology.

Appendix C

ADAPTNET: SOFTWARE LIBRARY AND TOOLS FOR TOPOLOGY CONTROL ANALYSIS

The *AdaptNet* software library is python-based set of analytic approaches and toolkits developed by authors to simulate and analyze distributed, heterogeneous network topology control problems.

The key features include: geospatial position support for wireless network scenarios, various complex propagation models (e.g., ITU-based terrain models), management and calculation of path loss and cost matrices , heterogenous node characteristic (e.g., gains, thresholds), network graph modeling, graph theoretic analysis tools, modeling of external locations for interference analysis, ground-to-space link analysis.

Main Software Dependencies: pycraf, astropy, networkx, numpy

Through the testing of our proposed topology control algorithms and the generation of data contained within this report a workflow utilizing the *AdaptNet* library emerged rather naturally. The fundamental starting point for a topology control algorithm is a set of nodes and their respective locations. Within *AdaptNet*, there exist several methods to generate random sets of nodes and locations according to different random distributions, namely a set of random geometric node coordinates or random gaussian clusters. These methods will all produce a networkx graph object to on which to continue the topology control operations.

Given a set of nodes and their locations, the user now needs to define a cost of some sort for establishing connections between the nodes. *AdaptNet* enables the user to make use of several built in cost functions or easily develop their own. Our general workflow involves determining what type of transmission propagation loss to use, either free space loss or pycraf modeled terrain loss. Free space loss is determined using a simple mathematical relationship between distance and frequency. Modeling terrain effects on loss invokes pycraf to model how terrain impacts the propagating signal. Once a user defines a loss method, *AdaptNet* builds a $n \times n$ matrix ($n =$ number of nodes) of loss values. Using this loss matrix, a $n \times n$ cost matrix can be generated. The default *AdaptNet* mode generates the cost matrix using a simplified version of the link budget equation for establishing reliable communication. Of course, all of this can be adjusted by the user to define their own loss and cost approaches. More basic approaches also exist internally that define cost using simple distance metrics.

After defining the cost matrix within *AdaptNet*, a user is ready generate a topology using either a built in topology control algorithm (e.g., Minimum Spanning Tree, kXTC, or a Gabriel Graph) or their own defined topology control approach. Whichever method the user chooses, the result will be a topology using the given node locations and cost matrix. *AdaptNet* contains several methods for manipulating and analyzing the networkx graph object, including generating graph theoretic analyses.