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THESIS

LOGISTICS IN A CONTESTED ENVIRONMENT: NETWORK ANALYSIS AND FLOW OPTIMIZATION

by

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June 2020

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The present state of the naval logistics system is efficient and effective at supplying forward-deployed naval forces; however, it is optimized for operations in uncontested environments. As the United States enters another era of great-power competition, it must determine the type of logistics system that is able to operate in a region where friendly forces do not have sea control. This study creates a network of logistics hubs in the Pacific theater. Using network science and prominent centrality measures, we examine the structure of a nominal Pacific theater logistics network for an array of logistic assets. In this thesis, we develop a maximum flow algorithm and apply it to our logistics network. Our proposed maximum flow algorithm optimizes vessel configuration and routing to satisfy as much demand as possible. We hope our study provides insights into the capabilities of the current logistics system and potential areas for improving logistics capabilities in contested environments.

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LOGISTICS IN A CONTESTED ENVIRONMENT: NETWORK ANALYSIS AND FLOW OPTIMIZATION

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ABSTRACT

The present state of the naval logistics system is efficient and effective at supplying forward-deployed naval forces; however, it is optimized for operations in uncontested environments. As the United States enters another era of great-power competition, it must determine the type of logistics system that is able to operate in a region where friendly forces do not have sea control. This study creates a network of logistics hubs in the Pacific theater. Using network science and prominent centrality measures, we examine the structure of a nominal Pacific theater logistics network for an array of logistic assets. In this thesis, we develop a maximum flow algorithm and apply it to our logistics network. Our proposed maximum flow algorithm optimizes vessel configuration and routing to satisfy as much demand as possible. We hope our study provides insights into the capabilities of the current logistics system and potential areas for improving logistics capabilities in contested environments.

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List of Acronyms and Abbreviations

ASBM	anti-ship ballistic missile
ASCM	anti-ship cruise missile
CSBA	Center for Strategic and Budgetary Assessments
ESB	Expeditionary Sea Base
ESD	Expeditionary Transfer Dock
JHSV	Joint High Speed Vessel
LCS	Littoral Combat Ship
LHA	Landing Helicopter Assault Ship
LPD	Amphibious Transport Dock
LSD	Dock Landing Ship
MOLA	Marine Operations Logistics Asset
MSC	Military Sealift Command
ORCA	Extra Large Unmanned Undersea Vessel
OSV	Offshore Service Vessel
SEA	Systems Engineering Analysis
T-AK	Maritime Prepositioning Ship
T-AKE	Dry Cargo/Ammunition Ship
T-AKR	Maritime Prepositioning Ship
T-AO	Fleet Replenishment Oiler

T-AOE Fast Combat Support Ship

USN U.S. Navy

UUV Unmanned Underwater Vehicle

Executive Summary

The United States has been the dominant world power since World War II; therefore, it has been able to operate its logistics systems with relative impunity. In the dawning of the age of great power competition, maritime logistics forces are no longer able to guarantee that they will operate in a risk-free environment.

Network science is a prominent field in modern academia. Researchers are able to use analytical techniques to both visualize and examine the structure of networks. Popular measures of importance within networks are the degree, eigenvector, closeness, and betweenness centrality metrics. We present an analysis of the importance of various nodes in a nominal contested Pacific maritime logistics network, based on a multitude of current and potential logistics assets. We use the results of our centrality analysis to draw conclusions about the suitability of the current logistics force structure for operations in contested environments.

Logistics networks are not static, they are organized to enable the flow of resources through the network. In this thesis we develop a maximum flow algorithm, which is a mixed integer linear program that represents a dynamic network flow model. The maximum flow algorithm considers a given network, set of assets, and demand signal. Our algorithm outputs vessels configurations and routing assignments, optimized to satisfy maximum demand across the logistics network.

We believe that the concepts of network science and network flow provide key insights into the strengths and vulnerabilities of the U.S. maritime logistics system in contested environments. Our research can aid in examining the structural characteristics of logistics networks and identifying proper assignment of assets to logistics duties in contested environments.

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CHAPTER 1: Introduction

Logistics form the backbone of any military operation; therefore, threats to their supply chains have been a primary concern for battlefield commanders since the genesis of warfare. In World War II, the United States organized a logistics force that enabled Allied forces to project power and succeed in military operations far from friendly territory [1]. The greatest threats faced by maritime logistics forces in World War II came from hostile submarine and air forces. These platforms were the disruptive military technology of the period, and military strategy had to rapidly adapt to counter the threat. In the Battle of the Atlantic, this was manifested in the adoption of convoy tactics and the introduction of persistent aerial support [2]. The 20$^{\text{th}}$ century saw the rapid development of precision weapons capable of long-distance targeting and strikes. Maritime logistics in the 21$^{\text{st}}$ century face their own forms of disruptive technology: modern submarines, cyber threats, and cruise and ballistics missiles.

The United States has been able to operate using a peacetime maritime logistics architecture essentially since the end of World War II. The current system employed by the Military Sealift Command (MSC) is honed for operations in uncontested environments. In 2019, the Center for Strategic and Budgetary Assessments (CSBA) contended that the current maritime logistics architecture is not capable of supporting U.S. strategy and operational concepts against adversaries. The CSBA report recommends a shift in the underlying concepts, capabilities, and force posturing of the maritime logistics force [1]. By viewing the challenge of maritime logistics through the lens of network science, it is possible to recommend shifts to the logistics force structure for success in a contested environment.

Over the last few decades, network science has come to prominence as a burgeoning academic field. Network scientific methods allow researchers to reduce systems to simple graphical structures, from which information can be easily extracted. The roots of modern graph theory can be traced to the seminal "Seven Bridges of Königsberg" problem [3]. In the city of Königsberg there are four major land masses connected by bridges. In this 18$^{\text{th}}$ century problem, Euler pondered how to cross every bridge exactly once and still reach every region of the city. Euler's solution is notable, as it mapped the physical network of islands and bridges to a mathematical structure known as a graph. We discuss the "Seven Bridges of Königsberg" problem in greater detail in Chapter 2 of this thesis.

Euler's problem is small enough to be solved through a simple exhaustive search. One can attempt every path in a short time and prove that it is in fact impossible to solve the "Seven Bridges of Königsberg" problem. The true utility of network science is mapping large, complex networks to abstract mathematical structures. Network science has been used to analyze a variety of networks, spanning diverse fields including biology, transportation analytics, and sociology.

As a subset of transportation networks, physical logistics networks can be analyzed using a network theoretic approach. Geographic locations can be viewed as nodes in a network, with connections represented by arcs between the nodes. Network analytics and a maximum flow algorithm can be applied to the maritime logistics problem, offering insights and recommendations for improvement.

1.1 Problem Definition

The current U.S. maritime logistics system is not optimized for performance in a contested environment. In highly contested environments, such as the South China Sea, logistics vessels are confronted with a multitude of threats. The main threat categories are hostile submarines, anti-ship cruise missiles (ASCMs), and anti-ship ballistic missiles (ASBMs). The ASCM threat is complicated because ASCMs may be air, surface, or submarine launched. Each threat category relies on cuing and targeting sensor systems. The U.S. Navy (USN) must meet the logistical demand in a contested environment while minimizing losses. In order to determine the most appropriate vessel for a particular supply run, we must be able to first evaluate the expected threat level. This threat level stems from the enemy order of battle and it is dependent on the number and capability of hostile assets. Secondly, we must be able to utilize a logistics asset that is survivable against the threats that it faces. The crux of the issue is determining the optimal asset for operations in the face of an expected threat level.

1.2 Research Questions and Contributions

The purpose of this research is to examine the U.S. maritime logistics network in the Pacific theater. Other organizations, such as the CSBA, have considered this problem at a strategic level. We will develop a network of logistics hubs connected by sea lanes. We will apply prominent network analytic algorithms to inspect the features of the maritime logistics network. We will use the insights from these algorithms to highlight vulnerabilities and strengths within the network. Additionally, we will apply a maximum flow algorithm to our network. The results of the maximum flow algorithm will allow us to discern the most efficient use of different logistics assets. We will also use this analysis to make recommendations concerning the logistics force structure that is best suited to succeed in a contested environment.

We use data that was created in collaboration with the Systems Engineering Analysis (SEA)-29 Capstone group [4]. In Chapter 3, we provide a summary of the data relevant to the development of our network model.

1.3 Thesis Organization

This thesis is organized into five chapters: 1) Introduction, 2) Background, 3) Methodology, 4) Results and Analysis, 5) Conclusions and Future Work. In the following chapter, we provide a review of prior contributions to the field of network science and flow optimization. In Chapter 3, we describe the methods used to examine the network of projected logistics hubs in the Pacific. In Chapter 4, we present the results of our network analysis and maximum-flow algorithm. The final chapter concludes the thesis and recommends future work that will refine our recommendations for providing logistics support in a contested environment.

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CHAPTER 2:
Background

In this chapter, we review the foundations of network science. Additionally, we discuss the mathematical formulation of our maximum-flow algorithm that is used to optimize logistics flow. This chapter is divided into three main sections: 1) Network Science Overview, 2) Topological Characteristics, 3) Maximum-Flow Problems. In the network science overview, we cover rudimentary terms and ideas that are commonplace in academia. The topological characteristics section will review metrics that describe structural aspects of the network, mostly to highlight nodes and communities of importance. The maximum-flow algorithm we develop in the final section describes the data, variables, and constraints that optimize network flow.

2.1 Network Science Overview

Network science is a burgeoning field of modern academia. Network scientists have used the concepts for analyzing social networks and the structure of communications frameworks, such as the Internet. Network science is also a useful tool in the analysis of transportation networks. In the study of transportation systems, nodes, also known as vertices, are used to represent the presence of physical hubs and delivery points within the network. Arcs, also known as edges, are used to connect vertices when a route exists between two physical components of the transportation system. In mathematical terms, a graph G, composed of vertices and edges, is defined as:

Definition 1 (Graph)**.**

> *A* ***graph*** *is an ordered pair of finite disjoint sets* (V,E) *such that* E *is a subset of the set* VxV *of unordered pairs of V. The set* V *is the set of vertices* V *and* E *is the set of edges. If* G *is a graph, then* $V = V(G)$ *is the vertex set of* G*,* $E = E(G)$ *is the edge set. An edgex,y is said to join the vertices* x *and* y *and is denoted by* xy*. Thus* xy *and* yx *mean exactly the same edge; the vertices* x *and* y *are the end vertices of this edges* G*.* [5]

Figure 2.1 depicts the overlay of the physical system and mathematical representation of the "Seven Bridges of Königsberg" problem.

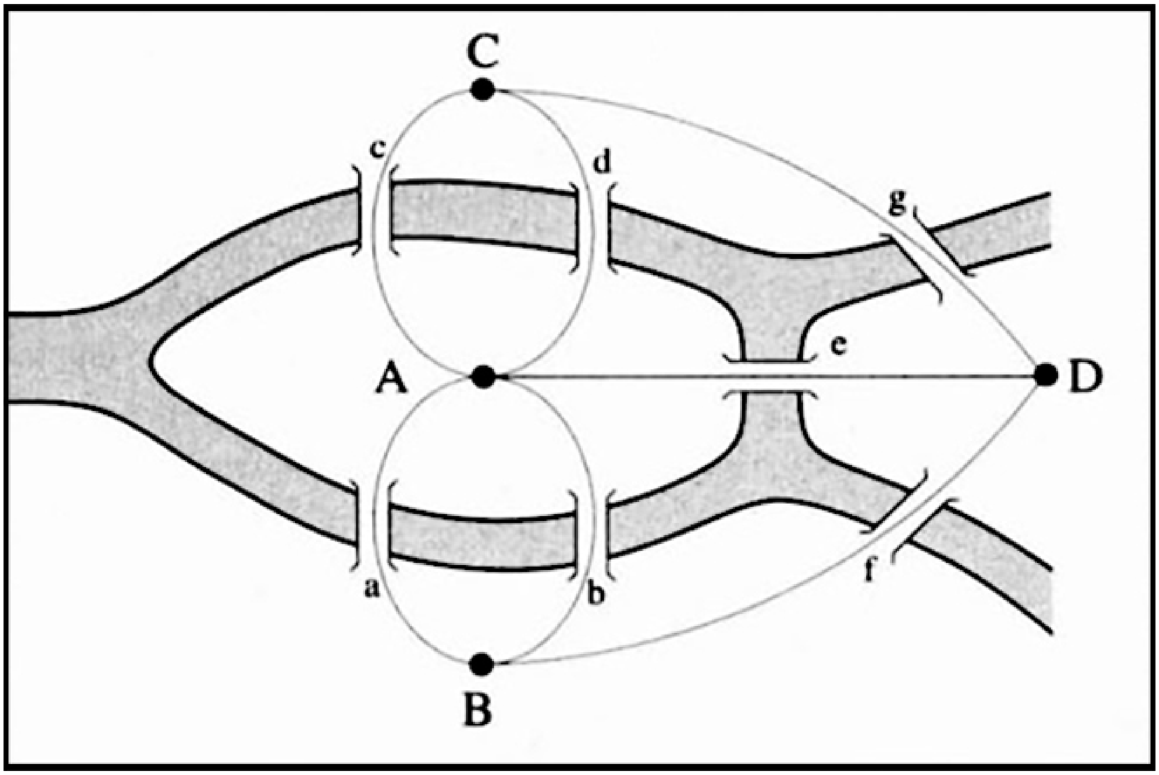

Figure 2.1. Graphical Representation of "Seven Bridges of Königsberg" Problem. Source: [6].

The seminal "Seven Bridges of Königsberg" problem, as a transportation network itself, serves as a relevant proving ground for these foundational concepts. Each of the four vertices describes a portion of Königsberg and every edge corresponds to a bridge connecting two portions of the city.

Many graphs have binary valued edges; a 1 represents the presence of an edge between to vertices while a 0 means an edge is not present. Edges do not have to be binary valued; an edge can possess what is referred to as a weight. An edge weight attaches a level of importance to an edge. For instance, edge weights in a social network might depict the number of shared acquaintances between two individuals. In the context of a transportation network like the Bridges of Königsberg graph, edge weights could represent the length of each bridge, or the time it takes to cross each bridge.

Real-life networks are often complex, and the number of vertices and edges makes meaningful analysis difficult. In order to extract more insight from a graph, we often focus on smaller portions of the total graph. These sub-regions of the graph are characterized by lesser numbers of vertices and edges, and are referred to as subgraphs. A subgraph is defined by Ray as:

Definition 2 (Subgraph)**.**

> *A **subgraph** of a graph G is a graph H such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and the assignment of endpoints to edges in H is the same as in G. We then write $H \subseteq G$.* [7]

A vertex is considered to be adjacent to another vertex if there is an edge connecting the two vertices. Adjacent vertices are called neighbors, leading to our next definition. A neighborhood is defined as:

Definition 3 (Neighborhood)**.**

> ***Neighborhood**, $N_G(a)$, is a set of nodes which are adjacent to node a.* [8]

As West describes, the degree of a vertex, v, is the number of edges containing v as an end vertex [9]. The degree of a vertex is equivalent to the cardinality of its neighborhood. Just as graphs can have weighted edges, a vertex can have a weighted degree. The weighted degree of a vertex, v, is the sum of the weights of the edges that have v as an end vertex.

A network is a construct that encapsulates more data than a graph. Networks are useful because they act as a simple mathematical rendering of a complex system of connections. Newman defines a network as:

Definition 4 (Network)**.**

> *A **network** is a simplified representation that reduces a system to an abstract structure capturing only the basics of connection patterns and little else.* [10]

Networks represent complex systems and can have multiple connections between the same vertices. For instance, a public transportation network may have multiple subway lines that can be used to get from one station to another. In graph theoretic terms, graphs that have multiple edges between the same vertices are known as multigraphs. According to Chartrand, a multigraph is defined as:

Definition 5 (Multigraph)**.**

> *A **Multigraph** M consists of a finite nonempty set V of vertices and set E of edges, where every two vertices of M are joined by a finite number of edges (possibly zero). If two or more edges join the same pair of (distinct) vertices, then those edges are known as parallel edges.* [8]

Multilayer graphs are a subset of multigraphs. Multilayer graphs are composed of multiple layers of connectivity that can be interacting or non-interacting [11]. For our purposes, non-interacting means that the structure of one layer does not affect the structure of other layers in the multigraph.

2.2 Topological Characteristics

Section 2.1 described many of the basic terms that are necessary to understand networks. Next, we turn our attention to methods and metrics for understanding the structure, or topology, of networks.

Definition 6 (Network Topology)**.**

> ***Network Topology** is the physical arrangement and structure of the network.* [10]

Network topology can be quantified by a host of different metrics. We will examine the following metrics in more detail: centralities, modularities, and diameters. The concept of centralities rely on the definitions of paths and walks. A walk is defined by Chartrand as:

Definition 7 (Walk)**.**

> *A $u - v$ **walk** is a sequence of vertices in graph, beginning with vertex u and ending at vertex v such that consecutive vertices in the sequence are adjacent.* [8]

The above definition of a walk refers to a sequence of adjacent vertices, but does not require that no vertex is repeated on the walk. A path is a walk in which each vertex is distinct. Now that we have reviewed definitions for a path and a walk, we can address centralities. In this thesis, we will use four of the most popular measures of centrality: 1) degree centrality; 2) eigenvector centrality; 3) closeness centrality; 4) betweenness centrality.

Definition 8 (Centrality)**.**

> *1) **Degree Centrality**: the degree of a vertex.* [10]
>
> 2) ***Eigenvector Centrality**: a degree centrality score proportionate to the sum of the degree centrality scores of its neighbors*. [12]
>
> 3) ***Closeness Centrality**: measures the mean distance from a vertex to other vertices connected to it*. [13]
>
> 4) ***Betweenness Centrality**: measurement of how well a vertex lies on the shortest paths connecting other vertices*. [13]

While all centrality measures indicate the importance of a node, each centrality metric reveals different insight into the structure of a network.

In the context of a logistics network, each of these centrality measures reveal a different aspect of a vertex's importance. Degree centrality of a vertex, v, provides insight on how many other vertices v can directly interact with. Eigenvector centrality reveals how collocated a vertex, v is with important hubs in the network. Closeness centrality accounts for the weight of the edges connecting a vertex and its neighborhood. In the context of logistics, this directly relates to the supply flow rate. A higher closeness centrality implies that a vertex can rapidly disseminate supplies within its neighborhood. Betweenness centrality signifies whether a vertex is a part of optimal paths within the network, corresponding to more efficient supply delivery.

Average path length is a metric that provides holistic assessment of the general closeness of a graph, G. Small average path length has implications when considering the redundancy and resiliency of a network. Newman defines average path length as:

Definition 9 (Average Path Length)**.**

> *Consider an unweighted graph G with the set of vertices V where |V|=n. Let $d(v_i,v_j)$ be the weighted distance between v_i and v_j for $v_i, v_j \in V$. The* ***average path length*** *l_G is* [14]:
>
> $$l_g = \frac{1}{n(n-1)} \sum_{i \neq j} d(v_i, v_j).$$

Networks also have a property, network diameter, which describes the size of the network. First, we note that a geodesic is the shortest path between two vertices of a graph. Diameter is defined as:

Definition 10 (Diameter)**.**

> ***Diameter*** *of a network is the length of the longest geodesic path in the network.* [10]

The longest geodesic path returns the maximum number of edges needed to connect any two vertices in the network. The practical application of this metric provides an indication of how influence disseminates within a network. In the context of the Internet, the diameter could reveal the number of hops a packet would take while transiting from one IP address to another. Complex networks have structural features that are not apparent from cursory examination. Communities are one of those features. Radicchi defines communities as:

Definition 11 (Community)**.**

> *A* ***community*** *is a subset of vertices within the graph such that connections between vertices within a community are denser than connections with the other communities in the rest of the network.* [15]

Communities identification provides a way to subdivide the total network, and an opportunity to probe the denser portions. Network modularity is a metric that parametrizes the strength of the communities within a network. A very modular network is characterized by segregation into very dense communities. Newman's definition for network modularity is:

Definition 12 (Network Modularity)**.**

> ***Network modularity*** *is the difference between the edges in the network that connect vertices within the community, and the expected value with the same community divisions but with random generated connections between the vertices.* [16]

2.3 Maximum Flow Problems

Maximum flow problems are a subset of operations research problems. In these problems, networks are represented as graphs made of nodes and arcs. In classic maximum flow problems, there is one specified source node, s, and one specified sink node, t. The problem assumes that arc weight, $u_{i,j}$, represents the maximum rate of resources traversing the arc (i, j) [17]. The solution to the maximum flow problem returns the optimal route from the source node to the sink node. Additionally, many classical maximum flow problems return an output of maximum resource flow per unit of time.

For example, flow of traffic within a network of roads and checkpoints can be examined as a maximum flow problem. The arc weights, $u_{i,j}$, could represent automobiles per hour. The solution of the maximum flow problem would give the maximum number of automobiles that travel from checkpoint s to checkpoint t per hour. One way of solving the maximum flow problem is application of a linear program. A linear program composed of sets, parameters, variables, an objective function, and constraints will be able to computationally solve small order maximum flow problems [19]. Figure 2.2 depicts a example of a maximum flow problem with multiples sources and sinks. By creating a series of artificial arcs between physical nodes and a "supersource/supersink", it is possible to expand the concept of the maximum flow problem. The supersource is represented as s' and the supersink is denoted t' in graph (b) of Figure 2.2. This expansion permits a solution that optimizes flow from one set of supply nodes to a disjoint set of sink nodes through a network. The basic framework of maximum flow problems is pivotal to optimally routing supplies within a logistics network.

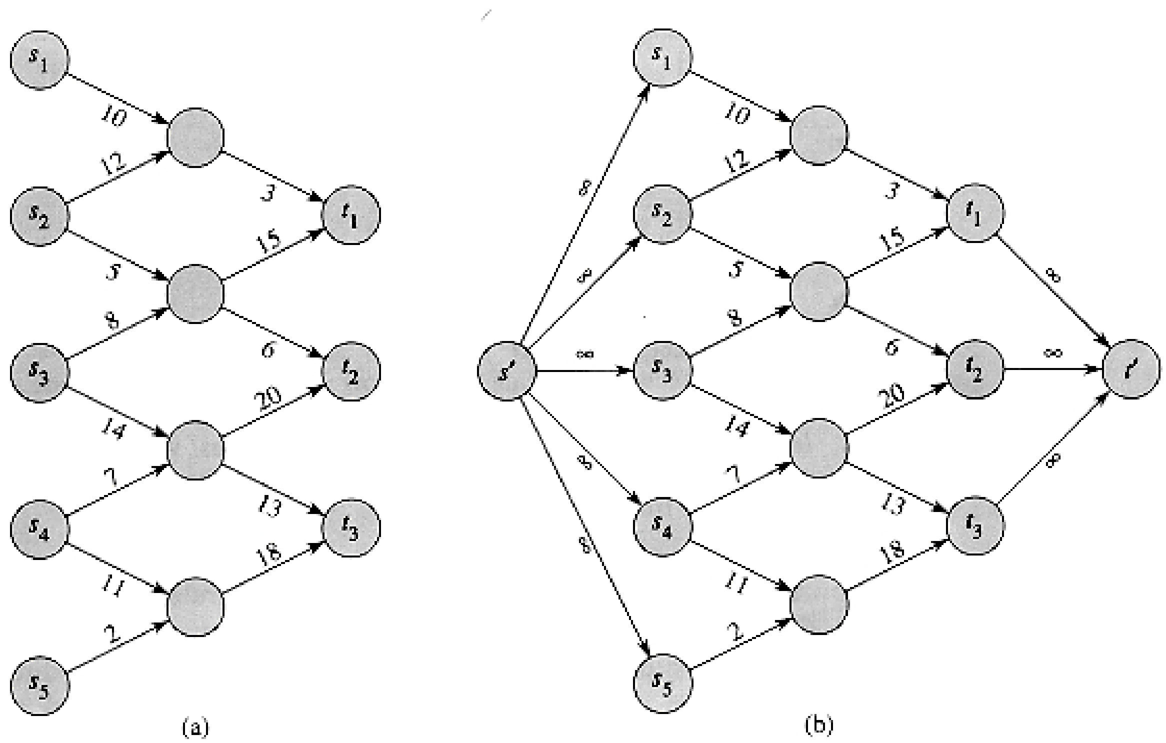

Figure 2.2. Illustration of Maximum Flow Problem with Multiple Sinks and Multiple Sources. Source: [18].

CHAPTER 3:
Data and Methodology

3.1 Data Creation

The data we use in this thesis was created in collaboration with the SEA-29 Capstone group [4]. We will not include the raw data in this thesis, but will highlight some important components of the data set. In order to access the raw data contact Dr. Fotis Papoulias, in the Systems Engineering Department at the Naval Postgraduate School.

Table 3.1. List of Nodes in Pacific Theater Logistics Network

Nodes List	
Singapore	Cebu
Guam	Palau
Darwin	Hawaii
Kwajalein	Zuoying
Bandar Seri Begawan	Yokosuka
Sasebo	Okinawa
Busan	Cam Ranh Bay
Haiphong	Manila
Puerto Princesa	Diego Garcia
Phuket	Pattaya
Perth	

The structural framework of any network is reliant upon the nodes and arcs that comprise it. Table 3.1 presents the logistics hubs that were considered in this network. These nodes were chosen based on their past or current use as a logistics hub by the USN or for their potential to serve as a logistics hub in the future. These nodes were chosen to encompass ports in areas of critical strategic importance. These nodes also allow for flow of supplies into theater via the Pacific or Indian Oceans.

Our network is multilayered, with each layer pertaining to a specific class of logistics vessel. Each layer of our multigraph has the same connections between the nodes, but the edge weights and edge type are dependent on the layer. Consult Appendix A for a table of sample edges from one layer of the network. It is important to note that our network is undirected, so each node can act as both a source and a target.

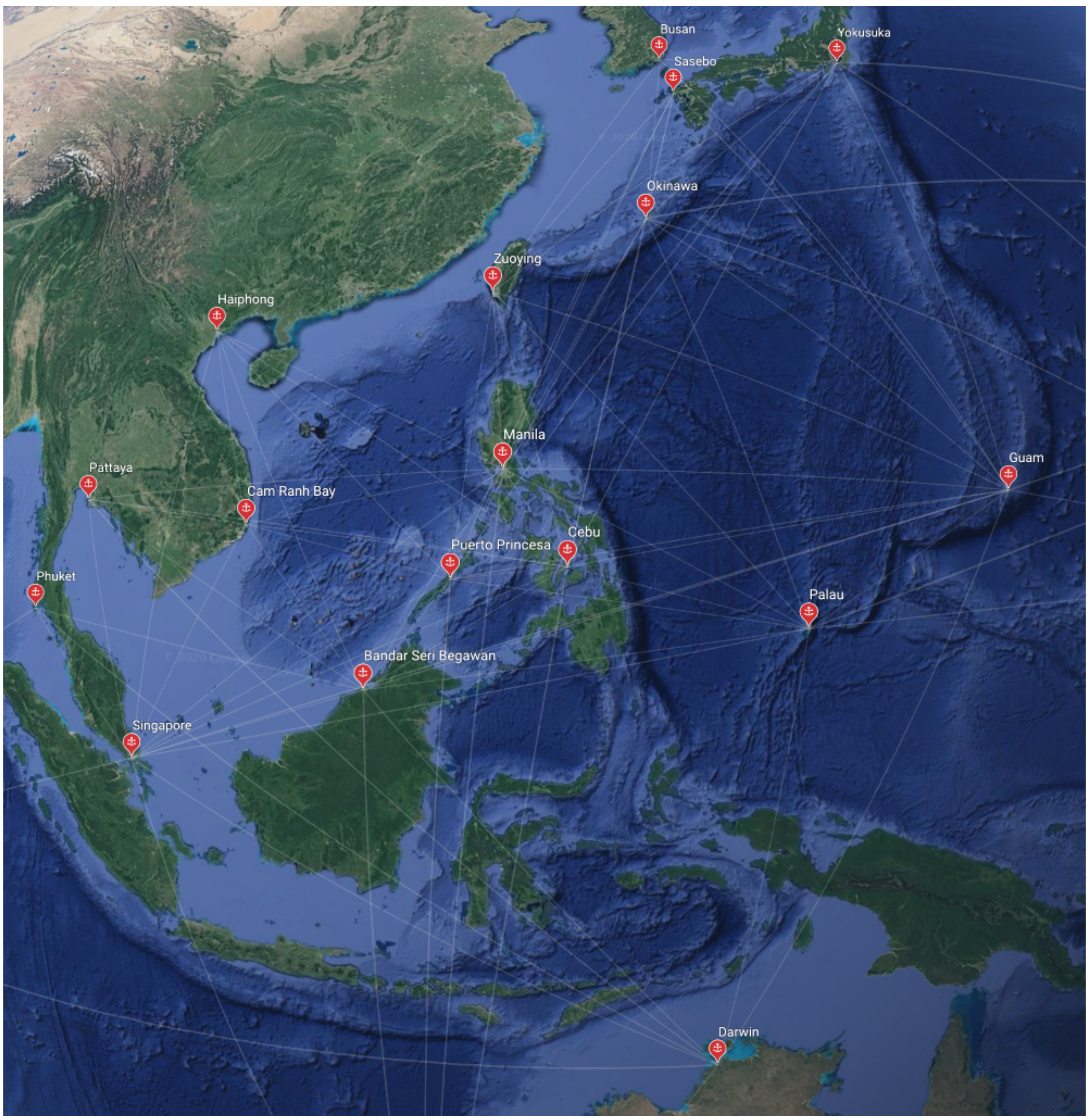

Figure 3.1. Overlay of the Logistics Network on Satellite Imagery

Figure 3.1 depicts the overlay of the graphical representation of the logistics network on satellite imagery. Some nodes are located outside of the bounds of the image displayed in Figure 3.1: Hawaii to the East, Kwajalein to the East, Perth to the South, and Diego Garcia to the West. Note that edges in Figure 3.1 do not correspond to specific routes between hubs, they simply represent sea lanes between nodes in the network. As such, some of the edges in this visualization cross over land.

For the maximum flow portion of this thesis, we require the demand signal at each node in the network. We develop an algorithm that optimizes flow to satisfy demand at sink nodes throughout the network. Table 3.2 presents the daily tonnage requirement of each node that has a demand signal. The daily tonnage demanded is an aggregate of the three subsets of supplies: fuel, ordnance, and stores. For the formulation and breakdown of the demand signal, consult the SEA-29 Capstone Report [4].

Table 3.2. Demand Signal of the Logistics Network

Node	Demand (tons/day)		
	Stores	Fuel	Ordnance
Singapore	102	1810	25
Guam	2067	3813	601
Darwin	468	719	268
Yokosuka	11	1884	48
Sasebo	1117	722	467
Okinawa	813	976	497
Busan	569	1070	356
Manila	923	1692	482

Table 3.3 presents each class of logistics vessel considered in this thesis. Each vessel class is represented as a discrete and non-interacting layer in our multigraph. We will not delve into the characteristics of these vessels; they are are partially described in Appendix C and covered in extensive detail in the SEA-29 Capstone Report [4].

Table 3.3. List of Logistics Vessels

Vessel List	
LCS	Mothership
JHSV	MOLA
T-AOE	ORCA
T-AO	T-AK
T-AKE	T-AKR
LHA	RORO
LSD	LCU
LPD	ESD
OSV	ESB
Sea Train	

Every edge in each layer has an associated threat level. For our purposes, threat level is used to calculate the probability that a vessel survives traversing the edge. It should be noted that some edges lie completely outside of the threat region, so the probability of survival is one. From the vessel characteristics, threat level, and route distance, we create several different edge weighting systems.

3.2 Edge Weighting Systems

We used centrality and modularity calculation programs in NetworkX and Gephi, a network visualization software [20], [21]. Not every centrality or modularity metric treats edge weights the same. Some metrics treat weight as a penalty to connectivity, while some metrics view weight as a positive attribute when evaluating closeness.

The degree centrality of a node is ratio of the cardinality of the node's neighborhood to the total number of nodes in the network. Eigenvector centrality is proportionate to the sum of the degree centrality of a vertex's neighborhood, so it also views high weight as an indication of closeness. Network modularity and community assignments are created using the Louvain algorithm, which observes a positive correlation between edge weight and connectivity [22].

Before we define our edge weighting schemes, we will define the terms used in their creation. $CostofLosses$ is the average monetary value that is lost on a one-way traversal of an edge. This value was created by taking the multiplying a vessel's worth (the sum of the vessel cost and the cost of embarked supplies) and the probability that the vessel is sunk while traversing the edge. $\frac{TonnageDelivered}{day}$ is the average tons daily tonnage delivered by a vessel traversing an arc. The average tonnage delivered is the product of a vessel's probability of surviving a one way traversal and the cargo capacity (in tons) of the vessel. This value is created by dividing the average tonnage delivered by the vessel by the transit time between nodes (in days).

In order to properly determine communities and calculate eigenvector centrality, we propose the following edge weighting system:

$$W_{i,j} = \frac{\frac{TonnageDelivered}{day}}{CostofLosses}. \tag{3.1}$$

Closeness and betweenness centrality values are negatively correlated to edge weight. Closeness centrality emphasizes nodes with the shortest mean distance to its neighbors, so nodes with a low mean distance are considered important within the network. Betweenness centrality values are calculated using the shortest paths determined via Dijkstra's algorithm. Dijkstra's algorithm determines shortest paths based on the assumption that edge weights act as penalties; therefore, edges with lower weight are give preference in determining shortest paths [23].

In order to properly calculate closeness and betweenness centrality, we propose the following edge weighting system, which is the inverse of the previous edge weighting system:

$$W_{i,j} = \frac{CostofLosses}{\frac{TonnageDelivered}{day}}. \tag{3.2}$$

Some edges in each layer are outside of the threat region; therefore, they have no losses. In order to avoid zero-weighted and infinity-weighted edges, we make the cost of losses on these edges equal to one, so that our edge weight is simply equivalent to the daily tonnage delivery rate.

Through the proper application of these edge weighting schemes, we are able to properly determine the most important nodes according to each of the prominent measures of centrality.

3.3 Maximum Flow Formulation

This section presents a model that optimizes vessel selection, configuration, and routing to maximize the amount of demand satisfied at logistics hubs in the network. The sets and parameters used in this model were developed in collaboration with the SEA-29 Capstone group [4]. The model we are presenting is a form of the canonical maximum network flow problem [17]. The model we develop differs from normal network flow models in two key ways: 1) flow between nodes is not continuous; it is conducted via discrete runs conducted by logistics vessels 2) each arc has a level of risk associated with it, prohibiting certain transits. We account for these differences in the formulation of our model. Our model expands on the framework of Christafore and Danielson's theses, specifically their development of linear programs that model discrete vessel transits and different vessel configurations by supply class [19], [24] .

3.3.1 Model Description

First, we introduce the various sets and indices used in the model. The logistics network consists of disjoint sets of nodes, N, and arcs, A. Denote $i \in N$ as a specific node and $(i, j) \in A$ as a specific arc within the network. There are two subsets of the total set of N nodes: demand nodes, $D \subset N$, and source nodes, $S \subset N$. Denote C as the set of supply classes, in our model: fuel, ordnance, cargo. Each demand node $i \in D$ requires some amount of supply class $c \in C$. We prescribe a finite list of vessels, denoted $v \in V$, available for use. The set v is broken into two subsets, V_1 and V_2, which allow some vessels to have set starting locations and other starting locations to be determined by the algorithm. This list includes various manned and unmanned vessels. Each vessel is a distinct element of the set V. Each vessel can be configured in different ways; configurations determine how the vessel's storage capacity is split amongst the supply classes. Define $g \in G_v$ as the possible configurations of some vessel v. For example, one configuration might fill the vessel's capacity completely with fuel, another configuration might have an even distribution between supply classes. Two artificial accounting nodes are also included in the model. We define a $superSink$ node that calculates all the total demand satisfied. Additionally, a $vesselSink$ node accounts for all vessels completing their voyages.

Next, we introduce the parameters used in the model. The parameter denoting the maximum time allowed by the model is $maxTime$. The $maxTime$ parameter sets how long the logistics assets will have to move supplies from the supply nodes $s \in S$ to the demand nodes $d \in D$. The demand for commodity c at node i is $demand_{i,c}$. The source nodes where the logistics vessels $v \in V_1$ load supplies and start their journey are labeled $start_{n,v}$. Configuration g of vessel v carries at most $vesselCap_{v,g,c}$ of supply class c. The model maximizes logistics deliveries in a contested environment, and must accordingly account for the risk inherent to transiting arcs in the network. Risk is incorporated via $r_{i,j,v}$, the probability of destruction of a vessel v transiting along arc (i, j). The parameter $r_{i,j,v}$ is generated from a threat analysis model that was generated in cooperation with the SEA 29 Capstone Project. For more information covering the details behind the determination of the $r_{i,j,v}$ parameter, consult Chapter III, the Modeling Effort chapter of the SEA 29 report [4]. The parameter P_v defines the minimum acceptable probability of survival for each vessel. A vessel will not be routed on an arc that has a lower probability of survival than P_v. Each arc is representative of a sea lane between nodes, and the length of the route is denoted $distance_{i,j}$. Each vessel has a set speed of advance, given by vel_v. The length of transit is calculated from $distance_{i,j}$ and vel_v for each possible vessel and arc combination, and used to account for timing constraints. There are also times $load_{i,v}$ and $unload_{i,v}$ for a vessel v to load/unload supplies at node i. The model incorporates various penalty parameters, which may be tuned to change the model's priorities. The weight attached to a specific supply class is $weight_c$, which can emphasize delivery of a certain supply class. The penalty for the loss of a specific vessel is $losses_v$. The β_1 parameter penalizes the objective function based on the time of the vessels' arrivals. The β_2 parameter penalizes the objective function for prescribing a greater total number of transfers between vessels. The β_3 parameter penalizes the objective function for sending vessels on high risk routes. The β_4 parameter penalizes the objective function for sending a single vessel on a large number of routes. The β_5 parameter penalizes the objective function based on the time of vessels' departures, ensuring all vessels leave supply nodes as soon as they are able.

Finally, we introduce the decision variables used in the model. The main decision variable determines how to route each vessel through the network. We will use the binary decision variable $Y_{i,j,v}$ to represent whether vessel v traverses arc (i, j). From these $Y_{i,j,v}$ variables we can output the routes of the different vessels. We use another binary decision variable $W_{v,g}$ to represent whether vessel v has configuration g. We represent the amount of flow of a supply class c along an arc (i, j) using vessel v using $X_{i,j,c,v}$. We can determine $X_{i,j,c,v}$ from $Y_{i.j.v}$ and $W_{v,g}$. We are most interested in the final flow of goods, namely how much of each supply class is received by each demand node within the $maxTime$ parameter. The total flow of supply class c along arc (i, j) is denoted by $Z_{i,j,c}$. Using the decision variable $Z_{i,j,c}$, we can determine the amount of demand satisfied. In order to account for the transfer of supplies between vessels, we need to include the variables $A_{i,v}$ and $D_{i,v}$, which account for the arrival and departure times of a vessel $v \in V$ from a node $i \in N$. The variable $U_{i,v,w}$ is binary and represents whether supplies are transferred from a vessel v to a distinct vessel w at node i. We represent the amount of a supply class c transferred from vessel v to w at node i with the variable $S_{i,c,v,w}$. For simplicity, this formulation fixes the configuration of a vessel throughout the duration, even if the vessel takes part in supply transfers. The variable $SL_{i,v}$ determines the starting locations of vessels for $v \in V_2$. The development of the transfer constraints introduces nonlinearities into constraints in the model.As shown by Leandro, we can keep the model linear via the introduction two new variables and a series of constraints [25]. The variable $YD_{i,j,v}$ is the linearization of $Y_{i,n,v}D_{i,v}$. The variable $UA_{i,v,w}$ is the linearization of $U_{i,v,w}A_{i,v}$. The variable $XSL_{n,j,c,v}$ is the linearization of $X_{n,j,c,v}SL_{n,v}$.

Our goal is to deliver the maximum amount of supplies to demand nodes over a set time period. We can tune the length of the time period to observe how much demand can be met if logistics forces are operating on certain timelines. By solving the algorithm repeatedly, over a variety of $maxTime$ values, we can analyze how time constraints affect logistics service. Additionally, we can vary the penalty parameters, β, and observe how flow is routed for different optimization priorities.

3.3.2 Mixed Integer Linear Program

Indices and Sets

$v \in V_1$ logistics vessels with set start nodes

$v \in V_2$ logistics vessels with variable start nodes

$v \in V$ all vessels = $V_1 \cup V_2$

$s \in S$ supply nodes

$d \in D$ demand nodes

$ss \in superSink$ artificial accounting node where all supply flow terminates

$vs \in vesselSink$ artificial accounting node where all vessels terminate

$n \in N$ all nodes = $S \cup D \cup SuperSink \cup VesselSink$

$c \in C$ supply classes

$g \in G_v$ configuration of vessel v

$(i, j) \in A$ arc directed from node i to j

Data [units]

$load_{i,v}$ load time at node i by vessel v [hours]

$unload_{i,v}$ unload time at node i by vessel v [hours]

$distance_{i,j}$ route length of arc (i, j) [nautical miles]

vel_v speed of advance of vessel v [knots]

$demand_{i,c}$ demand for class c at node i [supply units]

$vesselCap_{v,g,c}$ capacity for class c on configuration g of vessel v [supply units]

$weight_c$ weight put on commodity c [fraction]

$losses_v$ weight put on losses for vessel v [millions of dollars]

$r_{i,j,v}$ probability of destruction of vessel v along arc (i, j) [fraction]

P_v acceptable probability of survival for a vessel v [fraction]

$start_{n,v}$ whether vessel $v \in V_1$ starts at node n [binary]

β_1 penalty for late arrivals [fraction]

β_2 penalty for transfers [fraction]

β_3 penalty for traversing risky routes[fraction]

β_4 penalty for number of legs traveled [fraction]

β_5 penalty for late departures [fraction]

M arbitrary very large number

$maxTime$ time limit to complete deliveries [hours]

Decision Variables [units]

$SL_{i,v}$ whether vessel $v \in V_2$ starts at node $n[binary]$

$Y_{i,j,v}$ whether vessel v travels on arc $(i, j)[binary]$

$W_{v,g}$ whether vessel v has configuration $g[binary]$

$X_{i,j,c,v}$ flow of commodity c along arc (i, j) using vessel v [supply units]

$Z_{i,j,c}$ flow of commodity c along arc (i, j) [supply units]

$A_{i,v}$ arrival time of vessel v to node i [hours]

$D_{i,v}$ departure time of vessel v to node i [hours]

$U_{i,v,w}$ whether supplies are moved from vessel v to w at node i [binary]

$S_{i,c,v,w}$ amount of class c moved from vessel v to w at node i [supply units]

$YD_{i,j,v}$ lineariziation of $Y_{i,n,v}D_{i,v}$ [hours]

$UA_{i,v,w}$ lineariziation of $U_{i,v,w}A_{i,v}$ [hours]

$XSL_{i,j,c,v}$ lineariziation of $X_{i,j,c,v}SL_{i,v}$ [supply units]

Objective Function

$$\begin{aligned}
\max \sum_{i \in N} \sum_{c \in C} & weight_c Z_{i,ss,c} \\
& - \beta_1 \sum_{v \in V} A_{vs,v} \\
& - \beta_2 \sum_{i \in N} \sum_{v \in V} \sum_{w \neq v \in V} U_{i,v,w} \\
& - \beta_4 \sum_{(i,j) \in A} \sum_{v \in V} Y_{i,j,v} \\
& - \beta_5 \sum_{i \in N} \sum_{v \in V} D_{i,v} \\
& + \beta_3 \sum_{v \in V} \sum_{(i,j) \in A} \log(1 - r_{i,j,v}) Y_{i,j,v}
\end{aligned}$$

Subject to

$$\sum_{g \in G_v} W_{v,g} = 1 \qquad \forall v \in V \tag{3.3}$$

$$\sum_{j \in N,(n,j) \in A} Y_{n,j,v} = start_{n,v} + \sum_{i \in N,(i,n) \in A} Y_{i,n,v} \quad \forall v \in V_1, n \in N/(superSink \cup vesselSink) \tag{3.4}$$

$$\sum_{j \in N,(n,j) \in A} Y_{n,j,v} = SL_{n,v} + \sum_{i \in N,(i,n) \in A} Y_{i,n,v} \qquad \forall v \in V_2, n \in N/(superSink \cup vesselSink) \tag{3.5}$$

$$\sum_{j \in N,(n,j) \in A} Y_{n,j,v} \leq 1 \qquad \forall v \in V, n \in N/(superSink \cup vesselSink) \tag{3.6}$$

$$\sum_{i \in N} Y_{i,vs,v} = 1 \qquad \forall v \in V, \tag{3.7}$$

$$Z_{i,ss,c} \leq demand_{i,c} \qquad \forall i \in D, c \in C \tag{3.8}$$

$$X_{i,j,c,v} \leq \sum_{g \in G_v} W_{v,g} vesselCap_{v,g,c} \qquad (i,j) \in A, c \in C, v \in V \tag{3.9}$$

$$X_{i,j,c,v} \leq MY_{i,j,v} \qquad (i,j) \in A, c \in C, v \in V, \tag{3.10}$$

$$Z_{i,j,c} = \sum_{v \in V} X_{i,j,c,v} \qquad (i,j) \in A, j \in N/(superSink \cup vesselSink), c \in C \tag{3.11}$$

$$\sum_{j \in N,(n,j) \in A} Z_{njc} = \sum_{i \in N,(i,n) \in A} Z_{inc} \qquad \forall n \in D, c \in C \tag{3.12}$$

$$\sum_{(i,ss) \in A} Z_{i,ss,c} = \sum_{n \in S,(n,j) \in A} \sum_{v \in V1} X_{n,j,c,v} start_{n,v} + \sum_{n \in S,(n,j) \in A} \sum_{w \in V_2} XSL_{n,j,c,w} \qquad \forall c \in C \tag{3.13}$$

$$\sum_{n \in N} SL_{n,v} = 1 \qquad n \in N/(superSink \cup vesselSink) v \in V_2 \tag{3.14}$$

$$XSL_{n,j,c,v} \leq SL_{n,v} \sum_{g \in G_v} vesselCap_{v,g,c} \qquad \forall (n,j) \in A, c \in C, v \in V_2 \tag{3.15}$$

$$XSL_{n,j,c,v} \leq X_{n,j,c,v} \qquad \forall (n,j) \in A, c \in C, v \in V_2 \tag{3.16}$$

$$XSL_{n,j,c,v} \geq X_{n,j,c,v} - (1 - SL_{n,v}) \sum_{g \in G_v} vesselCap_{v,g,c} \qquad \forall (n,j) \in A, c \in C, v \in V_2 \tag{3.17}$$

$$\sum_{(i,ss)\in A}\sum_{v\in V} Y_{i,ss,v} = 0 \tag{3.18}$$

$$\sum_{(n,vs)\in A}\sum_{c\in C} Z_{n,vs,c} = 0 \tag{3.19}$$

$$\sum_{(n,vs)\in A}\sum_{c\in C}\sum_{v\in V} X_{n,vs,c,v} = 0 \tag{3.20}$$

$$\sum_{(i,j)\in A}\sum_{v\in V} losses_v log(1 - r_{i,j,v})Y_{i,j,v} \geq \sum_{v\in V} losses_v log(P_v)$$

$$\forall v \in V, j \in N/(superSink \cup vesselSink) \tag{3.21}$$

$$U_{i,v,v} = 1 \qquad i \in N, v \in V \tag{3.22}$$

$$\sum_{j\in N,(i,j)\in A} X_{i,j,c,v} = \sum_{w\in V} S_{i,c,w,v} \qquad (i, j) \in A, c \in C, v \in V, \tag{3.23}$$

$$S_{i,c,w,v} \leq MU_{i,v,w} \qquad i \in N, c \in C, v \in V, w \in V \tag{3.24}$$

$$\sum_{v\in V} S_{i,c,w,v} \leq \sum_{k\in N,(k,i)\in A} X_{k,i,c,w} + \sum_{j\in N} X_{i,j,c,w} start_{i,w} \qquad i \in N, c \in C, w \in V_1 \tag{3.25}$$

$$\sum_{v\in V} S_{i,c,w,v} \leq \sum_{k\in N,(k,i)\in A} X_{k,i,c,w} + \sum_{j\in N} XSL_{i,j,c,w} \qquad i \in N, c \in C, w \in V_2 \tag{3.26}$$

$$\sum_{v\in V}\sum_{w\in V} S_{i,c,w,v} = \sum_{j\in N,(i,j)\in A} Z_{i,j,c} \qquad i \in N/superSink, c \in C \tag{3.27}$$

$$A_{vs,v} \leq maxTime \qquad \forall v \in V \tag{3.28}$$

$$A_{n,v} = \sum_{i\in N,(i,n)\in A} YD_{i,n,v} + \frac{distance_{i,n}}{vel_v} Y_{i,n,v} \qquad \forall v \in V, n \in N/(superSink) \tag{3.29}$$

$$YD_{i,j,v} \leq maxTime Y_{i,j,v} \qquad \forall (i, j) \in A, v \in V \tag{3.30}$$

$$YD_{i,j,v} \leq D_{i,v} \qquad \forall (i, j) \in A, v \in V \tag{3.31}$$

$$YD_{i,j,v} \geq D_{i,v} - (1 - Y_{i,j,v}) maxTime \qquad \forall (i, j) \in A, v \in V \tag{3.32}$$

$$UA_{n,w,v} \leq maxTime U_{n,w,v} \qquad \forall n \in N, v, w \in V \tag{3.33}$$

$$UA_{n,w,v} \leq A_{n,w} \qquad \forall n \in N, v, w \in V \tag{3.34}$$

$$UA_{n,w,v} \geq A_{n,w} - (1 - U_{n,w,v}) maxTime \qquad \forall n \in N, v, w \in V \tag{3.35}$$

$$D_{n,v} \geq load_{n,v} + (UA_{n,w,v} + unload_{n,w}U_{n,w,v})$$
$$\forall v, w \in V, n \in N/(superSink \cup vesselSink) \tag{3.36}$$

- Objective Function maximizes the amount of demand satisfied. Supply classes are weighted by $weight_c$ in case it is more important to deliver one type vs another. The β penalty terms can be tuned to adjust priorities of the optimization.
- Constraint (3.3): ensures each vessel is of exactly one configuration and remains in that configuration throughout the model.
- Constraint (3.4): flow balance constraint for vessels with fixed starting points. A vessel that enters a demand node must must leave the node. The start variable allows for flow to originate at the starting locations. Ensures each vessel travels a continuous path in the network. Equality allows vessels to end their path at any node by using the $vesselSink$ as the last node. Vessel might be dispatched directly to the $vesselSink$, in which case the vessel is not used in reality.
- Constraint (3.5): same as Constraint (3.4), but for vessels with variable starting locations.
- Constraint (3.6): ensures total outgoing flow per vessel is at most 1. Coupled with Constraints (3.4) and (3.5), it means that if vessels starts at a node it cannot return to the same node, avoiding round-trips.
- Constraint (3.7): ensures each vessel has to have an arc to the $vesselSink$, which means every vessel's journey ends at the $vesselSink$.
- Constraint (3.8): ensures the model delivers no more than demanded.
- Constraint (3.9): ensures the amount of each supply class flowing along an arc in a vessel fits within capacity of the vessel's configuration.
- Constraint (3.10): if there is no vessel on the arc, then there is no flow on the arc. If there is a vessel, then M essentially makes this constraint irrelevant.
- Constraint (3.11): total flow of supply classes on an arc must equal the sum of flow carried on vessels traversing the arc. This constraint does not apply for the final demand going to $superSink$.
- Constraint (3.12): another flow balance constraint. The total flow of supply entering a node must equal the flow exiting. Note that some of the flow exiting the node may be satisfying demand (which occurs when flow travels from the node to $superSink$).
- Constraint (3.13): total demand satisfied by the model must equal the flow originated

from the source nodes for vessels with fixed and variable starting points.

- Constraint (3.14): vessels in V_2 must start at a physical node.
- Constraint (3.15): the amount of a of a supply class carried on a vessel must be less than its capacity.
- Constraint (3.16): ensures the linearization $XSL_{n,j,c,v}$ is less than $X_{n,j,c,v}$.
- Constraint (3.17): ensures the linearization $XSL_{n,j,c,v}$ is greater than $X_{n,j,c,v}$. If $SL_{n,v}$=1, combining constraints (3.15)-(3.17) yields the linearization $XSL_{n,j,c,v}$=$X_{n,j,c,v}$ as desired.
- Constraint (3.18): ensures vessels do not traverse $superSink$ arcs. $superSink$ arcs are artificial arcs, not physical arcs. They are merely an accounting tool to track the amount of demand satisfied at each demand node. Thus it is not possible for a vessel to traverse this arc.
- Constraint (3.19): ensures there is no flow of supply classes on $vesselSink$ arcs.
- Constraint (3.20): ensures there is no flow of supply classes on vessels on $vesselSink$ arcs.
- Constraint (3.21): is an aggregate risk constraint. It ensures that the probability each vessel, v, survives traversing an arc is at least P_v. Allows us to weigh different assets by $losses_v$ because we are able to accept higher risk for certain assets.
- Constraint (3.22): a vessel always transfers to itself, an accounting constraint.
- Constraint (3.23): total amount of a supply class c on vessel v leaving node i must equal the total transferred to v from other vessels w. Note, a vessel can transfer supplies to itself.
- Constraint (3.24): the total amount of a supply class transferred is only positive if a transfer occurs.
- Constraint (3.25): the total amount transferred from vessel $w \in V_1$ to all other vessels must be less than or equal to the amount of supplies that vessel w arrived with at node i, i.e. a vessel cannot transfer more supplies than it has.
- Constraint (3.26): the total amount transferred from vessel $w \in V_2$ to all other vessels must be less than or equal to the amount of supplies that vessel w arrived with at node i, i.e. a vessel cannot transfer more supplies than it has.
- Constraint (3.27): the total amount transferred at node i across all vessels must equal flow out of the node. Here we exclude the $superSink$ because transfers only correspond to physical flow.

- Constraint (3.28): ensures that all vessels arrive to the $vesselSink$ in the alotted time.
- Constraint (3.29): calculates the arrival time of a vessel v to a node n. The arrival time is calculated by summing the departure time from the previous node and the transit time.
- Constraint (3.30): forces the linearization $YD_{i,j,v}$ to zero if no vessel transits the arc.
- Constraint (3.31): ensures the linearization $YD_{i,j,v}$ is less than $D_{i,v}$.
- Constraint (3.32): ensures the linearization $YD_{i,j,v}$ is greater than $D_{i,v}$. If $Y_{i,j,v}$=1, combining constraints (3.30)-(3.32) yields the linearization $YD_{i,j,v}$=$D_{i,v}$ as desired.
- Constraint (3.33): forces the linearization $YA_{n,w,v}$ to zero if no transfer occurs.
- Constraint (3.34): ensures the linearization $UA_{n,w,v}$ is less than $A_{n,w}$.
- Constraint (3.35): ensures the linearization $UA_{n,w,v}$ is greater than $A_{n,w}$. If $U_{n,w,v}$=1, combining constraints (3.33)-(3.35) yields the linearization $UA_{n,w,v}$=$A_{n,w}$ as desired.
- Constraint (3.36): calculates the departure time of a vessel v from a node n. The departure time is calculated by summing the arrival time at node n and the unloading/loading time. The combination of loading and unloading times gives the total transfer time. The optimization penalizes for late delivery, so a vessel will always leave an intermediary node as soon as it is able.

CHAPTER 4:
Results

In this chapter, we present the results from our two different methods of analysis. The first method applies network science to analyze the structure of the logistics network. The second method applies our maximum flow algorithm to the network, providing feedback on routing assignments and demand satisfaction.

4.1 Network Analysis

Figure 4.1 is a visualization of the uncontested layer of the logistics network. In this visualization, the edges are weighted based solely upon the distance between nodes. Thicker edges correspond to longer routes. Nodes are the same as presented in Table 3.1, and are sized based upon the betweenness centrality of the node. A larger node has a higher betweenness centrality value. Table 4.1 displays the characteristics of the network presented in Figure 4.1. The colors of the nodes and edges represent the communities in the network.

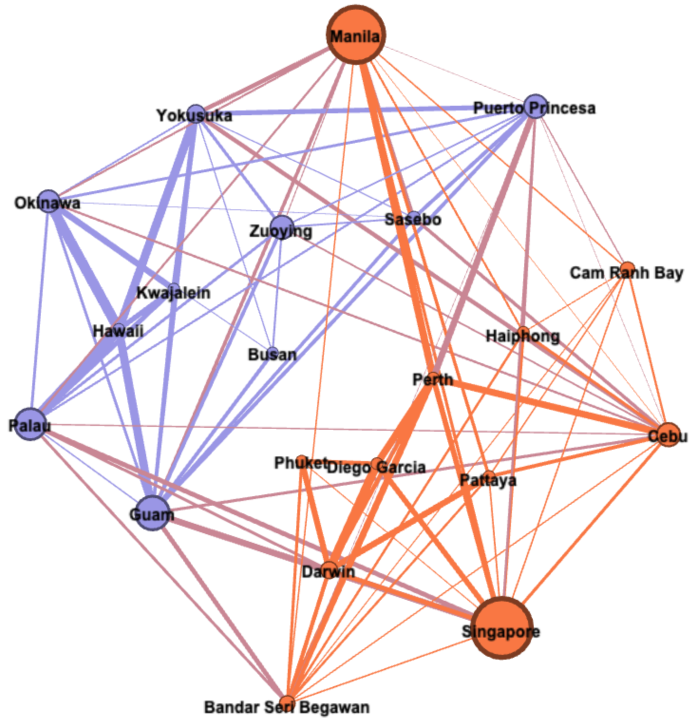

Figure 4.1. Network Visualization of Uncontested Layer of Logistics Network

The network modularity value indicates that communities are present; however, there is a high level of connection across communities. We note that the communities are geographically biased, a South China Sea and an East China Sea/Western Pacific community. After applying community detection algorithms to all layers (accounting for contested environments), we observe the same trend of low modularity.

Each layer in the contested environment was characterized by the presence of three communities. Geographically, these communities represented the South China Sea, East China Sea, and Western Pacific. We did not gain major structural insight from community detection and analysis, so we will not discuss it further.

We note that nodes in the Philippines have the highest degree. These nodes are geographically central; therefore, they connect to nodes in the South China Sea, East China Sea, and the Western Pacific. The least connected nodes in this network are on the fringe of the

Pacific Theater, such as Hawaii and Diego Garcia.

Table 4.1. Logistics Network Characteristics

Characteristic	**Value**
Number of Nodes	21
Number of Edges	91
Average Unweighted Degree	8.66
Network Diameter	3
Average Shortest Path Length	1.69
Network Modularity	0.263
Number of Communities	2

Next we introduce and analyze the average nodal cost of a selection of nodes in three layers.

4.1.1 Average Nodal Cost

We now introduce the concept of an average nodal cost.

Definition 13. *The* **average nodal cost**, $CostofLosses$ *(in dollars per ton delivered), is the average time-independent cost of one-way transit to/from a node.*

The edge weighting scheme, Equation (3.2) in Section 3.2, gives the average cost of losses for daily tonnage delivery; however, we divide by the transit time (in days) to get a time-independent edge weight equivalent to $\frac{CostofLosses}{TonnageDelivered}$. We note that this time-independent edge weight is equivalent for vessels traversing to or from a given node. Since both of these values are vessel dependent, each layer will have different average nodal costs, as we describe next. The average nodal cost is calculated by dividing the aggregate incident edge value (using the edge weights previously described), or weighted degree, by the unweighted degree of the node.

This measure provides a means of comparison between layers without bias regarding a vessel's velocity. Table 4.2 presents the average nodal cost of a selection of ten nodes for the LCS, ORCA, and T-AKR layers of the logistics network.

Table 4.2. Average Nodal Cost

Rank	ORCA		LCS	
1	Okinawa	$136,905	Bandar Seri Begawan	$247,105
2	Yokosuka	$136,444	Zuoying	$244,558
3	Zuoying	$124,882	Manila	$244,463
4	Bandar Seri Begawan	$124,589	Cebu	$242,991
5	Puerto Princesa	$124,500	Okinawa	$241,751
6	Manila	$124,246	Palau	$239,532
7	Guam	$144,712	Puerto Princesa	$228,097
8	Cebu	$123,241	Guam	$208,963
9	Palau	$120,620	Singapore	$208,280
10	Singapore	$108,794	Yokosuka	$193,601
Rank	**T-AKR**			
1	Zuoying	$365,859		
2	Okinawa	$351,879		
3	Bandar Seri Begawan	$351,632		
4	Manila	$341,736		
5	Palau	$339,610		
6	Puerto Princesa	$337,183		
7	Cebu	$333,401		
8	Guam	$294,803		
9	Singapore	$281,368		
10	Yokosuka	$266,169		

Table 4.2 shows that nodes located within high threat regions are generally more expensive, on the order of 20%, than nodes on the fringe of the threat region. This is an indication that is is advantageous to route vessels through nodes with lower risk whenever possible. The next paragraph details the strategic takeaways from Table 4.2 and the overall average nodal cost analysis.

The vessel parameters discussed in this paragraph are detailed in Appendix C. For more information on the vessels considered, consult the SEA-29 Capstone Report [4]. The LCS is an combat-oriented naval vessel with a high cost and a small tonnage capacity. The LCS also has defensive layers, so it is more survivable in contested environments. The ORCA is a very small tonnage capacity, inexpensive vessel with no defenses. The ORCA is submerged, and has the highest survivability values on every route. The T-AKR is a vessel used by the MSC in the current logistics force structure. It is an expensive, large tonnage capacity, undefended asset. The high monetary losses associated with the T-AKR indicate that conventional logistics assets are ill-suited for operations in contested environments. The LCS is cheaper to use per ton, despite its small capacity and high cost. This indicates that the presence of defensive layers on logistics vessels may decrease the average nodal cost. The ORCA has the smallest tonnage capacity, but its low cost and high survivability make it the cheapest asset. This indicates that UUVs are viable assets for use in contested environments.

Next we examine the betweenness centrality of nodes in the same layers of the network.

4.1.2 Betweenness Centrality Analysis

Betweenness centrality is a measure of the proximity of a node to geodesics in the network [13]. In logistics networks, with edge weights given as penalties based on distance or other factors, betweenness centrality is a prominent metric in determining nodal importance. This makes logical sense, since the shortest path between two nodes minimizes the penalty experienced for transiting between those two nodes.

Table 4.3 ranks the top five nodes in the uncontested, LCS, ORCA, and T-AKR layers by betweenness centrality.

Table 4.3. Betweenness Centrality Ranking

Rank	**Uncontested**	**LCS**
1	Singapore	Singapore
2	Manila	Guam
3	Guam	Manila
4	Palau	Okinawa
5	Cebu	Darwin
Rank	**ORCA**	**T-AKR**
1	Palau	Guam
2	Bandar Seri Begawan	Singapore
3	Cebu	Diego Garcia
4	Manila	Palau
5	Phuket	Darwin

The uncontested layer of the logistics network is an indication of the geographic betweenness centrality of nodes in the network. The LCS, ORCA, and T-AKR all represent betweenness centrality in a contested environment.

As a product of its defenses the LCS is able to operate within threat regions with a reasonable chance of survival, but it still prefers to travel through nodes on the fringe of the threat region.

The ORCA has very high survivability within the threat regions, but is slow. In order to maximize the daily tonnage delivery, the ORCA exhibits clear bias towards transiting through a geographically proximate cluster of nodes within the first island chain despite their location in a high threat region.

The T-AKR is undefended, so it has a low probability of survival along edges within the threat region. The low survivability of the T-AKR in the threat region skews the betweenness centrality metric to favors nodes on the fringe, or completely outside the threat region. If assets similar to the T-AKR are used in contested environments, they perform better on short transits, providing increased survivability and a higher daily tonnage delivery rate.

Betweenness centrality is an indicator of the importance of a node to flow throughout the network, so nodes with high betweenness values are crucial to facilitating resource flow. A small subset of nodes have high betweenness centrality values across layers. For example, Singapore and Guam are consistently important across layers that have large tonnage capacity. Their importance means that they are vulnerable to adversary action, closure of these hubs would have a severe negative effect on the performance of the logistics network.

4.1.3 Centrality Comparison

Figure 4.2 compares the degree centrality values of the nodes in the logistics network. While the edge weights may be different, all layers have the same nodes and edges and therefore each layer has the same degree centrality distribution, according to the equation detailed in Chapter 3.

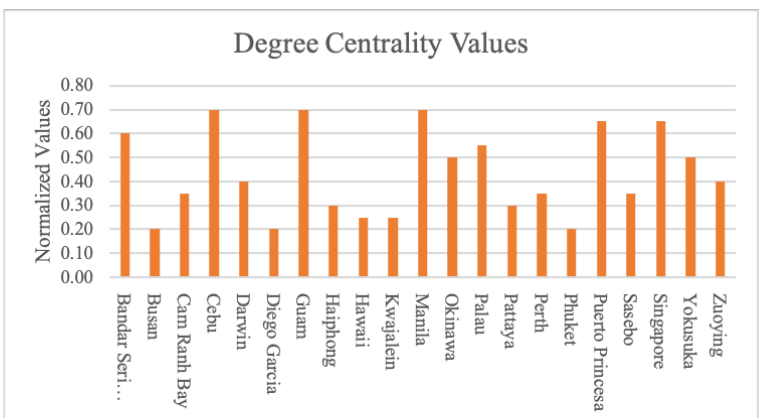

Figure 4.2. Comparison of Degree Centrality Values

The degree centrality metric is important in logistics networks because it indicates how many hubs a node can directly send supplies to. In our graph, the most important nodes by degree centrality are located at positions of geographic significance such as the first island chain cluster, Singapore, and Guam.

For full centrality tables, consult Appendix B, which has the different centrality rankings and values for each layer of the network.

Figures 4.3-4.5 compare the eigenvector, closeness, and betweenness centrality metrics for the LCS, ORCA, and T-AKR layers. In these graphs, eigenvector and betweenness centralities are plotted on the primary vertical axis, since they are normalized within the range [0,1]. The closeness centrality metric is not normalized, so it is plotted on the secondary vertical axis.

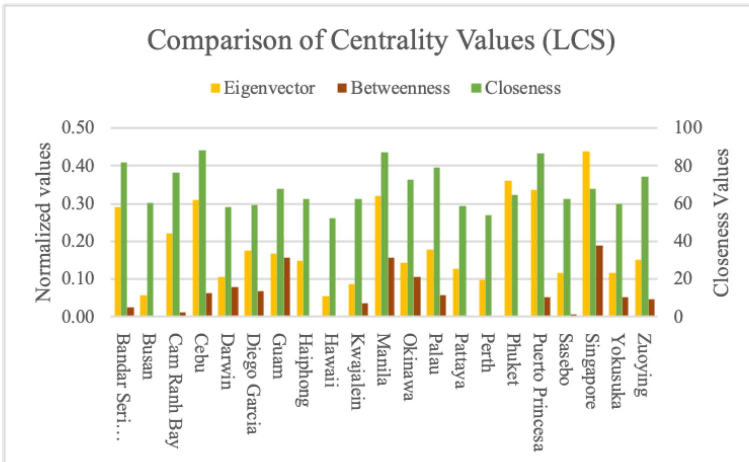

Figure 4.3. Comparison of Centrality Metrics in the LCS Network Layer

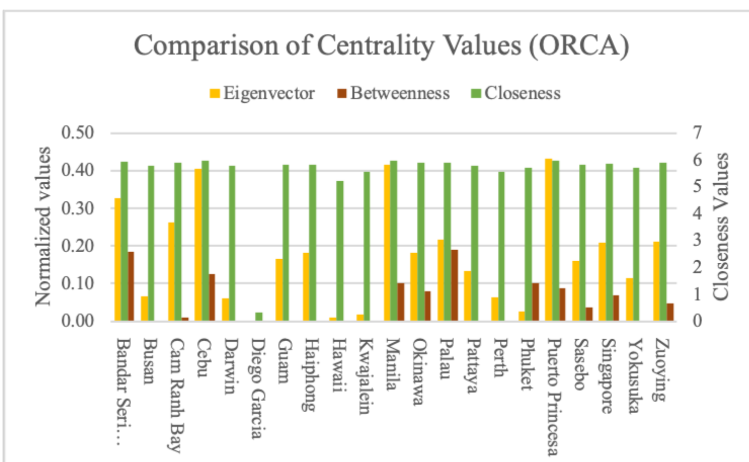

Figure 4.4. Comparison of Centrality Metrics in the ORCA Network Layer

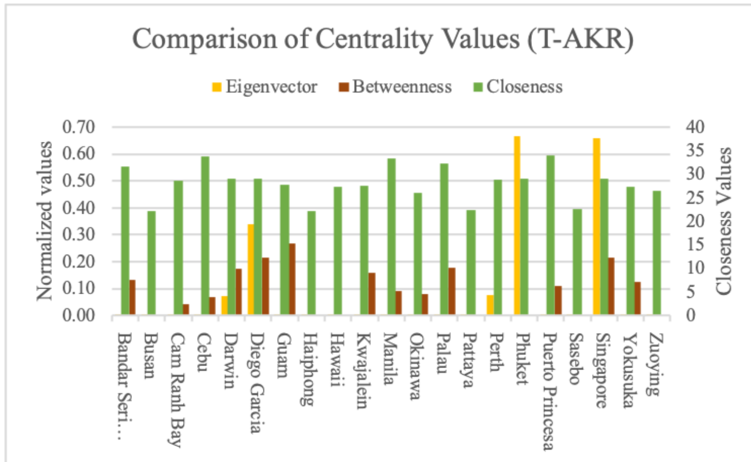

Figure 4.5. Comparison of Centrality Metrics in the T-AKR Network Layer

Every layer has approximately the same top nodes ranked by closeness, geographically they are the first island chain node cluster. The nodes ranked after the top five exhibit variability across layers, while large assets have roughly the same rankings. Similarly, small tonnage vessels have roughly equivalent closeness centrality rankings, due to the edge weighting scheme. The edge weighting scheme values the daily tonnage delivery rate, which is much smaller for low capacity carries traversing long routes. Closeness centrality has utility is differentiating between general classes of vessels, but is less useful in comparing specific layers.

Eigenvector centrality, in logistic terms, ranks nodes based on their proximity to highly connected hubs in the network. In eigenvector centrality, the rankings are almost identical across all classes of vessel. Upon closer inspection, the values have different distributions. Carriers with lower $Cost of Losses$ exhibit a flatter distribution, while vessels with higher $Cost of Losses$ have a small subset of nodes with significantly higher eigenvector centrality values, and many nodes with near-zero values. Eigenvector centrality is ill-suited for comparing layers of the logistics network.

Betweenness centrality appears to be the most discerning centrality metric. It takes the total network structure into account when determining a node's centrality, rather than just a node's neighborhood. Betweenness centrality also appears to be the most apt for distiguishing between layers of the network, valuing low-threat nodes preferentially for expensive and vulnerable assets. As expected, betweenness centrality is the most applicable to examining nodal importance in logistics networks.

Centrality analysis supports the hypothesis that conventional logistics assets are unsuitable for operations in contested environments. We find that assets with integrated defensive capabilities are able to operate in the threat region with higher effectiveness. We also find that expendable assets are favored for operations in contested environments, pointing to the utility of smaller vessels and unmanned assets as logistic platforms.

4.2 Maximum Flow Analysis

In this section we review the inputs and results of the maximum flow algorithm presented in Chapter 3. For detailed inputs and results data from the maximum flow algorithm, consult Appendix C. The inputs in Appendix C were created in cooperation with the SEA-29 Capstone group [4].

The $losses_v$ parameter is the combined worth of the vessel and its supply load in millions of dollars. We prescribe acceptable risk levels, P_v to each vessel based upon their cost, cargo capacity, and military utility. We consider four possible configurations: A, B, C, and D. Configuration D evenly partitions $vesselCap_v$, with one third of the cargo space allocated for each supply class. Configurations A, B, and C each favor a different supply class; stores-heavy, fuel-heavy, and ordnance-heavy respectively. Each of these configurations allocate 50% of $vesselCap_v$ to the preferred supply class and 25% of $vesselCap_v$ to each other supply class. Constraint 3.21 in our mixed linear program is not constructed to handle arcs with no risk. In order to prevent an undefined value, from the value of the logarithm evaluated at zero, we set a near-zero probability of destruction for risk free arcs. For simplicity's sake, the values of $load_v$ and $unload_v$ are set to five, and equal for every vessel. The $maxTime$ parameter for each scenario is set to 600 hours. The demand signal for each scenario is calculated by multiplying $maxTime$ and $demand_{n,c}$, the average hourly demand of each node for a given supply class. This gives the total demand based on the prescribed delivery timescale. All supply classes are assumed to have equal importance, $weight_c$. We set two source nodes in the network: Hawaii and Diego Garcia. Setting these two nodes as the only sources represents pushing all supplies from out of theater hubs. Having one source in the Pacific Ocean and one source in the Indian Ocean allows us to compare the efficiency of routing supplies into theater along Easterly or Westerly routes. Due to the limitations of the mixed integer linear program, not every vessel can have a variable starting location. In order to have a non-empty V_1, we dictate that TAKR1 started at Hawaii. All of the aforementioned assumptions hold for every test scenario.

We use Pyomo, an optimization modeling language based in Python, to create our algorithm [26], [27]. We use the CPLEX solver to solve our Mixed Integer Linear Program. The full theater problem, considering more than 150 total vessels, is too large for the solver to handle. We scope the vessel set to two vessels per class, leading to 30 total vessels. By scoping the problem, we allow the solver to arrive at a reasonably optimal solution while still providing insight on vessel routing and configuration assignment. All four scenarios arrive at a solution within a factor of two of optimal.

Most integer programming solvers use a sequence of steps to bring the solution successively closer to optimality. One step improves the best current solution for the objective function. The other step solves for an upper bound on the objective function, and successively lowers this upper bound. Our solutions are within a factor of two of optimal, meaning that the current solution for the objective function is over half of the upper bound of the objective function. In the case of Scenario 4, the solver's best solution for the objective value function is 73716.5489, while the upper bound is 144327.5500. We guarantee that our solution is within a factor of two of optimal, but it may be better than that. The actual optimal value for the objective function could lie anywhere between the current solution and the upper bound of the objective function.

Next, we present the results of four different scenarios. Due to the limited set of vessels, and without scoping the demand signal, the total demand is unable to be satisfied. We also note that due to an issue with the linear program, many vessels are not loaded to full capacity, contributing to the lack of demand satisfaction. All scenarios saw the same amount of supplies delivered: 73932.0 tons aggregated across all demand nodes.

4.2.1 Scenario 1: Full Demand Signal Uncontested

Table 4.4 presents the demand signal at each node in the network, where all nodes not listed have zero demand. Table 4.4 also shows the amount of demand satisfied at each node within the allotted time period.

Table 4.4. Demand Satisfaction of Maximum Flow Scenario 1

Node	Demand (tons)			Demand Satisfied (tons)		
	Stores	Fuel	Ordnance	Stores	Fuel	Ordnance
Singapore	2551	9049	629	999	0	0
Guam	51679	19064	15023	15435	14936	14785
Darwin	11696	17971	6706	5695	5695	6395
Yokosuka	279	9417	1191	0	0	0
Sasebo	27913	18057	11665	0	0	0
Okinawa	20313	24407	12424	2498	2498	2997
Busan	14237	26747	8910	0	0	0
Manila	23069	42300	12053	0	999	999

Of the vessel set, 27% start at Diego Garcia and 66% start at Hawaii. Due to the slow transit speed of the ORCA, it is unable to make any transits within the given time constraints, so 7% of the vessels traverse no arcs. All commodities are prioritized evenly; however, stores-heavy is the most common vessel configuration. Routing assignments ensure that despite supply class preferential configurations each node receives approximately equivalent amounts of each supply class. Since all routes in this scenario have negligible risk, routing is determined based on the quickest paths from source to demand nodes. Vessels that resupply Southern demand nodes originate from Diego Garcia, while vessels resupplying north of the Philippines start from Hawaii. It is important to note that this model flows supplies from both sides of the theater, showing the value of the Indian Ocean path in logistics supply routing.

The objective function is designed to promote timely deliveries. Since the model is unable to satisfy the entire demand signal, it delivers to demand nodes that are the shortest distance from source nodes. Demand nodes that are farther from source nodes have little or no demand satisfied.

4.2.2 Scenario 2: Full Demand Signal Contested

Table 4.5 presents the demand signal at each node in the network, where all nodes not listed have zero demand. Table 4.5 also shows the amount of demand satisfied at each node within the allotted time period.

Table 4.5. Demand Satisfaction of Maximum Flow Scenario 2

Node	Demand (tons)			Demand Satisfied (tons)		
	Stores	Fuel	Ordnance	Stores	Fuel	Ordnance
Singapore	2551	9049	629	1849	2497	629
Guam	51679	19064	15023	14836	13986	14137
Darwin	11696	17971	6706	1998	1998	1998
Yokosuka	279	9417	1191	0	0	0
Sasebo	27913	18057	11665	0	0	0
Okinawa	20313	24407	12424	5495	5494	5495
Busan	14237	26747	8910	0	0	0
Manila	23069	42300	12053	0	2018	999

Of the vessel set, 20% start at Diego Garcia and 73% start at Hawaii. Due to the slow transit speed of the ORCA, it is unable to make any transits within the given time constraints, so 7% of the vessels traverse no arcs. All commodities are prioritized evenly; however, stores-heavy is the most common vessel configuration. Routing assignments ensure that despite supply class preferential configurations each node receives approximately equivalent amounts of each supply class. Assets with high $losses_v$ and $r_{i,j,v}$ parameters are routed along low risk routes. The algorithm tasks more survivable and cheaper vessels to supply runs inside the threat region. The assignment of the T-AKR class to the risk-free Hawaii-Guam route and LCS/JHSV vessels to routes within the threat region support this conclusion. Since the vessel set is unable to meet all demand, vessels are primarily used to meet demand at nodes in low threat areas that lie closer to source nodes. Similar to Scenario 1, supplies flow into the theater from both sides. In this scenario, Hawaii is the heavily favored source node, while Diego Garcia is still as the starting point for resupplying Southerly demand nodes.

4.2.3 Scenario 3: Partial Demand Signal Uncontested

Table 4.6 presents the demand signal at each node in the network, where all nodes not listed have zero demand. Table 4.6 also shows the amount of demand satisfied at each node within the allotted time period.

Table 4.6. Demand Satisfaction of Maximum Flow Scenario 3

Node	Demand (tons)			Demand Satisfied (tons)		
	Stores	Fuel	Ordnance	Stores	Fuel	Ordnance
Sasebo	27913	18057	11665	3996	3996	3996
Okinawa	20313	24407	12424	11988	12988	11988
Busan	14237	26747	8910	0	0	0
Manila	23069	42300	12053	8642	8342	7993

In order to force vessels to resupply nodes in high threat regions, we remove the demand at the Singapore, Guam, Yokosuka, and Darwin nodes for this scenario.

Of the vessel set, 33% start at Diego Garcia and 60% start at Hawaii. Due to the slow transit speed of the ORCA, it is unable to make any transits within the given time constraints, so 7% of the vessels traverse no arcs. All commodities are prioritized evenly; however, stores-heavy is the most common vessel configuration. Routing assignments ensure that despite supply class preferential configurations each node receives approximately equivalent amounts of each supply class. Since all routes in this scenario have negligible risk, routing is determined based on the quickest paths from source to demand nodes. Vessels that resupplied Southern demand nodes originated from Diego Garcia, while vessels resupplying north of the Philippines started from Hawaii. It is important to note that this model flows supplies from both sides of the theater, showing the importance of the Indian Ocean route in logistics supply routing.

As in Scenario 1, the model is unable to satisfy the entire demand signal, it delivers to demand nodes that are the shortest distance from source nodes. Nodes that are farther from source nodes have less demand satisfied.

4.2.4 Scenario 4: Partial Demand Signal Contested

Table 4.7 presents the demand signal at each node in the network, where all nodes not listed have zero demand. Table 4.7 also shows the amount of demand satisfied at each node within the allotted time period.

Table 4.7. Demand Satisfaction of Maximum Flow Scenario 4

Node	Demand (tons)			Demand Satisfied (tons)		
	Stores	Fuel	Ordnance	Stores	Fuel	Ordnance
Sasebo	27913	18057	11665	4846	5694	4846
Okinawa	20313	24407	12424	6993	6993	6993
Busan	14237	26747	8910	4695	3847	3847
Manila	23069	42300	12053	8441	8292	8442

In order to force vessels to resupply nodes in high threat regions, we remove the demand at the Singapore, Guam, Yokosuka, and Darwin nodes for this scenario.

Of the vessel set, 33% start at Diego Garcia and 60% start from Hawaii. Due to the slow transit speed of the ORCA, it is unable to make any transits within the given time constraints, so 7% of the vessels traverse no arcs. All commodities are prioritized evenly; however, stores-heavy is the most common vessel configuration. Routing assignments ensure that despite supply class preferential configurations each node receives approximately equivalent amounts of each supply class. Assets with high $losses_v$ and $r_{i,j,v}$ parameters are routed along low risk routes. The algorithm tasks more survivable and cheaper vessels to supply runs inside the threat region. In support of this claim, ESB2 and T-AKR1 transport supplies to Yokosuka which are subsequently shuttled to demand nodes along higher threat routes by more attritable assets. As in previous scenarios, supplies flow into the theater from both directions. In this scenario, Hawaii is the favored source node, while Diego Garcia is still the starting point for resupplying Southerly demand nodes. The bias towards the Hawaii source node is less extreme that in Scenario 2, which is because the model flows more supplies to southern demand nodes in this scenario. In this scenario, demand is satisfied more equally across all demand nodes. It is interesting to note that the introduction of risk causes some vessels to route to more distant nodes, because it is advantageous to satisfy demand at those nodes.

The outputs of the maximum flow algorithm support the hypothesis that the current MSC force structure is unsuitable for operations in contested environments. We observe that vessels are routed differently in contested and uncontested environments, given the same demand signal. Vessels have higher probability of survival in the threat area if they are either less detectable or have integrated defensive capabilities. The vessels that are assigned to transport supplies to high threat nodes are characterized by their expendability or higher probability of survival. In order to maintain logistic support in contested environments, consideration should be given to the development and acquisition of assets that are expendable or more survivable in the face of hostile threats.

CHAPTER 5: Conclusion and Future Work

In this thesis, we examine the maritime logistics network in the Pacific theater. In this chapter we summarize our findings and suggest potential areas for future research.

5.1 Conclusion

In the first portion of this thesis, we summarize the data that we created in collaboration with the SEA-29 Capstone Group [4]. Using that data, we use network theoretic principles to examine the structure of the maritime logistics network in the Pacific theater. Our primary measure of importance, betweeness centrality, relies on shortest paths that maximize daily tonnage delivery while minimizing losses. We observe that vessels that are currently in the MSC inventory prefer to traverse nodes in uncontested or low-threat areas. Vessels with defensive capabilities prefer low-threat regions, but are more capable of traversing edges in contested environments. Inexpensive vessels are also more capable of operations in high threat environments than current logistics assets, due to their expendability. Additionally, we present a comparison of centralities across the layers of our network. A small subset of nodes are highly central across layers, indicating points of vulnerability in the network.

Logistics systems are not static, in reality supplies must flow through the network. In the case of our maritime logistics network, discrete vessels carry supplies between nodes. We present an algorithm that maximizes the demand satisfied at nodes throughout the network. We compare scenarios that maximize flow in uncontested and contested environments. We observe that expensive conventional logistics assets are routed to preferentially deliver to nodes in low-threat environments. Defended or inexpensive vessels are routed to higher threat nodes. The results of our maximum flow algorithm concur with the conclusions developed through network analysis. Current MSC logistics vessels are not optimal for operations in contested environments.

Consideration should be given to using expendable or more survivable assets to provide logistic support in contested environments.

5.2 Future Work

5.2.1 Further Application of Methodology

Our nominal logistics network only features major hubs in the Pacific theater. The incorporation of more nodes into the network would affect the structure of the network, providing insight on the effectiveness of small expeditionary logistics hubs. The methodology used in this thesis can also be applied to the logistics problem in other theaters, notably the Atlantic and Mediterranean regions. The military logistics system relies on airborne and land-based transportation of supplies. Incorporating connections via overland and air routes would provide insight into the overall structure of the U.S. logistics network. Additionally, incorporating air and ground assets into the maximum flow algorithm would provide the capability to compare the efficiency of seaborne, airborne, and land-based logistics connectors. Convoys are a common naval strategy, the methods presented in this thesis can be used to examine their worth in modern contested environments. Further trials will be able to analyze the effect of changing the demand signal on the solution generated by the maximum flow algorithm.

5.2.2 Distance Centrality Analysis

In 2018, Roginski created a new centrality metric: distance centrality. Distance centrality results in a determination of which vertices whose removal have the greatest (and least) change in a matrix's distribution of distances [28]. Used in conjunction with the weighting systems used in our logistics network, the distance centrality measure could indicate the effect of removing a node on the structure of the logistics network.

5.2.3 Maximum Flow Algorithm Improvements

The maximum flow algorithm presented in this thesis has potential for improvement. Changes to the linear program could allow for round-trips between nodes, enabling expendable assets to serve as shuttles for supplies within high threat regions. The $beta$ parameters can be tuned, realigning the priorities of the objective function. Adjusting the demand signal, either by simple scaling or different nodal distribution, would have an impact on routing. Changes to the constraints will be able to fix the partial loading issue. In practicality, not all nodes are equal. The demand signal at forward operating locations is prioritized over demand at nodes removed from combat. Modifications to this maximum flow algorithm would allow for demand at certain nodes to be preferentially satisfied.

The concept of heuristics can be applied to the optimization presented in this thesis. The development of a vessel assignment heuristic would allow for faster solutions and the ability to solve larger problems.

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APPENDIX A:
Sample Edge Table: LCS Layer of the Network

Type	Weight	Source Name	Target Name
LCS	252.631579	Hawaii	Kwajalein
LCS	162.367865	Hawaii	Guam
LCS	28.8142826	Hawaii	Palau
LCS	158.912208	Hawaii	Yokosuka
LCS	27.6079811	Hawaii	Okinawa
LCS	83.8696436	Yokosuka	Guam
LCS	242.708804	Yokosuka	Kwajalein
LCS	183.77038	Yokosuka	Sasebo
LCS	182.961978	Yokosuka	Busan
LCS	145.939603	Yokosuka	Okinawa
LCS	64.4956005	Yokosuka	Cebu
LCS	64.2852586	Yokosuka	Manila
LCS	265.447947	Okinawa	Sasebo
LCS	214.015477	Okinawa	Busan
LCS	93.4637616	Okinawa	Guam
LCS	43.1546509	Okinawa	Kwajalein
LCS	390.130624	Guam	Kwajalein
LCS	167.424508	Guam	Palau
LCS	60.891319	Guam	Darwin
LCS	75.6784634	Guam	Sasebo
LCS	69.3447147	Guam	Busan
LCS	76.5934884	Guam	Manila
LCS	93.3296345	Guam	Cebu
LCS	58.5690376	Guam	Bandar Seri Begawan
LCS	44.2288778	Guam	Singapore
LCS	92.1981267	Palau	Darwin
LCS	95.5788661	Palau	Okinawa

Type	**Weight**	**Source Name**	**Target Name**
LCS	61.5721691	Palau	Kwajalein
LCS	116.473241	Palau	Manila
LCS	175.428334	Palau	Cebu
LCS	90.9357201	Palau	Bandar Seri Begawan
LCS	58.5193238	Palau	Singapore
LCS	117.416917	Cebu	Okinawa
LCS	78.2741906	Cebu	Sasebo
LCS	296.890134	Cebu	Manila
LCS	113.054701	Cebu	Cam Ranh Bay
LCS	90.1023466	Cebu	Haiphong
LCS	175.963472	Cebu	Bandar Seri Begawan
LCS	81.1431437	Cebu	Singapore
LCS	119.314798	Manila	Okinawa
LCS	86.1787874	Manila	Sasebo
LCS	157.078037	Manila	Cam Ranh Bay
LCS	120.902967	Manila	Haiphong
LCS	163.675118	Manila	Bandar Seri Begawan
LCS	84.9904619	Manila	Singapore
LCS	190.667451	Haiphong	Cam Ranh Bay
LCS	97.3013554	Haiphong	Bandar Seri Begawan
LCS	87.8299192	Haiphong	Singapore
LCS	207.736356	Cam Ranh Bay	Bandar Seri Begawan
LCS	151.842519	Cam Ranh Bay	Singapore
LCS	151.910985	Bandar Seri Begawan	Singapore
LCS	63.6544933	Bandar Seri Begawan	Darwin
LCS	62.5894074	Singapore	Darwin
LCS	246.153846	Singapore	Diego Garcia
LCS	154.571593	Darwin	Diego Garcia
LCS	295.709571	Phuket	Diego Garcia
LCS	995.555556	Phuket	Singapore
LCS	85.3631191	Phuket	Bandar Seri Begawan
LCS	49.4688874	Phuket	Darwin

Type	Weight	Source Name	Target Name
LCS	150.231203	Pattaya	Singapore
LCS	111.811617	Pattaya	Bandar Seri Begawan
LCS	145.376519	Pattaya	Cam Ranh Bay
LCS	79.7363207	Pattaya	Manila
LCS	72.5881156	Pattaya	Cebu
LCS	46.4690639	Pattaya	Darwin
LCS	128.060495	Zuoying	Busan
LCS	131.506502	Zuoying	Sasebo
LCS	85.2086819	Zuoying	Yokosuka
LCS	73.5419911	Zuoying	Guam
LCS	91.3227165	Zuoying	Palau
LCS	211.369876	Zuoying	Manila
LCS	138.862685	Zuoying	Cebu
LCS	139.937249	Zuoying	Puerto Princesa
LCS	300.503074	Perth	Darwin
LCS	188.697789	Perth	Diego Garcia
LCS	52.0540555	Perth	Singapore
LCS	42.0135953	Perth	Cebu
LCS	42.5463537	Perth	Puerto Princesa
LCS	38.9055008	Perth	Manila
LCS	41.4462999	Perth	Bandar Seri Begawan
LCS	70.453574	Puerto Princesa	Guam
LCS	115.28331	Puerto Princesa	Palau
LCS	96.280586	Puerto Princesa	Okinawa
LCS	55.5658837	Puerto Princesa	Yokosuka
LCS	70.5122329	Puerto Princesa	Sasebo
LCS	341.051164	Puerto Princesa	Manila
LCS	323.494307	Puerto Princesa	Cebu
LCS	168.400559	Puerto Princesa	Cam Ranh Bay
LCS	106.892929	Puerto Princesa	Haiphong
LCS	300.262263	Puerto Princesa	Bandar Seri Begawan
LCS	104.721977	Puerto Princesa	Singapore

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APPENDIX B:
Centrality Rankings

Uncontested				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Puerto Princesa	0.426
2	Guam	0.700	Manila	0.409
3	Manila	0.700	Cebu	0.395
4	Singapore	0.650	Bandar Seri Begawan	0.325
5	Puerto Princesa	0.650	Cam Ranh Bay	0.259
6	Bandar Seri Begawan	0.600	Singapore	0.215
7	Palau	0.550	Palau	0.215
8	Yokusuka	0.500	Zuoying	0.212
9	Okinawa	0.500	Okinawa	0.193
10	Darwin	0.400	Haiphong	0.179
11	Zuoying	0.400	Guam	0.177
12	Sasebo	0.350	Sasebo	0.166
13	Cam Ranh Bay	0.350	Yokusuka	0.140
14	Perth	0.350	Pattaya	0.134
15	Haiphong	0.300	Busan	0.080
16	Pattaya	0.300	Perth	0.062
17	Hawaii	0.250	Darwin	0.060
18	Kwajalein	0.250	Phuket	0.059
19	Busan	0.200	Kwajalein	0.033
20	Diego Garcia	0.200	Hawaii	0.018
21	Phuket	0.200	Diego Garcia	0.015

Uncontested				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Cebu	334.038	Singapore	0.192
2	Manila	332.951	Manila	0.179
3	Puerto Princesa	332.622	Guam	0.089
4	Bandar Seri Begawan	312.986	Palau	0.079
5	Cam Ranh Bay	292.571	Cebu	0.047
6	Zuoying	292.230	Zuoying	0.047
7	Palau	291.059	Puerto Princesa	0.047
8	Okinawa	282.234	Okinawa	0.042
9	Singapore	264.333	Yokusuka	0.026
10	Guam	255.168	Darwin	0.021
11	Haiphong	247.793	Bandar Seri Begawan	0.016
12	Sasebo	247.280	Cam Ranh Bay	0.011
13	Busan	236.313	Sasebo	0.005
14	Pattaya	234.202	Hawaii	0.000
15	Yokusuka	223.958	Kwajalein	0.000
16	Phuket	214.911	Busan	0.000
17	Darwin	212.402	Haiphong	0.000
18	Kwajalein	166.159	Diego Garcia	0.000
19	Perth	157.483	Phuket	0.000
20	Diego Garcia	135.720	Pattaya	0.000
21	Hawaii	108.684	Perth	0.000

LCS				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Singapore	0.439
2	Guam	0.700	Phuket	0.362
3	Manila	0.700	Puerto Princesa	0.336
4	Singapore	0.650	Manila	0.319
5	Puerto Princesa	0.650	Cebu	0.309
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.290
7	Palau	0.550	Cam Ranh Bay	0.220
8	Yokusuka	0.500	Palau	0.177
9	Okinawa	0.500	Diego Garcia	0.174
10	Darwin	0.400	Guam	0.167
11	Zuoying	0.400	Zuoying	0.153
12	Sasebo	0.350	Haiphong	0.147
13	Cam Ranh Bay	0.350	Okinawa	0.143
14	Perth	0.350	Pattaya	0.128
15	Haiphong	0.300	Yokusuka	0.118
16	Pattaya	0.300	Sasebo	0.117
17	Hawaii	0.250	Darwin	0.106
18	Kwajalein	0.250	Perth	0.097
19	Busan	0.200	Kwajalein	0.087
20	Diego Garcia	0.200	Busan	0.058
21	Phuket	0.200	Hawaii	0.054

LCS				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Cebu	87.999	Singapore	0.189
2	Manila	87.154	Guam	0.158
3	Puerto Princesa	86.475	Manila	0.158
4	Bandar Seri Begawan	81.682	Okinawa	0.105
5	Palau	79.249	Darwin	0.079
6	Cam Ranh Bay	76.385	Diego Garcia	0.068
7	Zuoying	74.251	Cebu	0.063
8	Okinawa	72.442	Palau	0.058
9	Singapore	67.961	Yokusuka	0.053
10	Guam	67.923	Puerto Princesa	0.053
11	Phuket	64.868	Zuoying	0.047
12	Sasebo	62.681	Kwajalein	0.037
13	Kwajalein	62.419	Bandar Seri Begawan	0.026
14	Haiphong	62.230	Cam Ranh Bay	0.011
15	Busan	60.231	Sasebo	0.005
16	Yokusuka	59.839	Hawaii	0.000
17	Diego Garcia	59.331	Busan	0.000
18	Pattaya	58.540	Haiphong	0.000
19	Darwin	58.269	Phuket	0.000
20	Perth	53.865	Pattaya	0.000
21	Hawaii	52.408	Perth	0.000

JHSV				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.656
2	Guam	0.700	Singapore	0.653
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Darwin	0.097
5	Puerto Princesa	0.650	Perth	0.094
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.060
7	Palau	0.550	Pattaya	0.039
8	Yokusuka	0.500	Puerto Princesa	0.037
9	Okinawa	0.500	Manila	0.030
10	Darwin	0.400	Cam Ranh Bay	0.029
11	Zuoying	0.400	Cebu	0.029
12	Sasebo	0.350	Palau	0.018
13	Cam Ranh Bay	0.350	Guam	0.013
14	Perth	0.350	Haiphong	0.008
15	Haiphong	0.300	Zuoying	0.007
16	Pattaya	0.300	Kwajalein	0.007
17	Hawaii	0.250	Okinawa	0.004
18	Kwajalein	0.250	Yokusuka	0.004
19	Busan	0.200	Hawaii	0.004
20	Diego Garcia	0.200	Sasebo	0.002
21	Phuket	0.200	Busan	0.001

JHSV				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	36.902	Palau	0.432
2	Palau	36.828	Guam	0.384
3	Cebu	36.594	Puerto Princesa	0.279
4	Manila	35.314	Bandar Seri Begawan	0.263
5	Bandar Seri Begawan	35.274	Kwajalein	0.258
6	Guam	33.143	Cebu	0.226
7	Kwajalein	31.824	Yokusuka	0.221
8	Singapore	30.163	Singapore	0.205
9	Phuket	30.056	Diego Garcia	0.205
10	Diego Garcia	30.049	Darwin	0.189
11	Darwin	29.570	Cam Ranh Bay	0.100
12	Yokusuka	29.254	Manila	0.058
13	Perth	28.803	Hawaii	0.000
14	Hawaii	28.767	Okinawa	0.000
15	Zuoying	26.488	Sasebo	0.000
16	Okinawa	25.946	Busan	0.000
17	Cam Ranh Bay	24.067	Haiphong	0.000
18	Busan	21.816	Phuket	0.000
19	Sasebo	21.523	Pattaya	0.000
20	Pattaya	21.181	Zuoying	0.000
21	Haiphong	16.066	Perth	0.000

T-AOE				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.666
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.074
5	Puerto Princesa	0.650	Darwin	0.066
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.000
7	Palau	0.550	Pattaya	0.000
8	Yokusuka	0.500	Cam Ranh Bay	0.000
9	Okinawa	0.500	Puerto Princesa	0.000
10	Darwin	0.400	Palau	0.000
11	Zuoying	0.400	Guam	0.000
12	Sasebo	0.350	Manila	0.000
13	Cam Ranh Bay	0.350	Haiphong	0.000
14	Perth	0.350	Kwajalein	0.000
15	Haiphong	0.300	Cebu	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

T-AOE				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	14.531	Palau	0.411
2	Cebu	14.456	Guam	0.374
3	Palau	13.978	Puerto Princesa	0.284
4	Manila	13.860	Cebu	0.274
5	Bandar Seri Begawan	13.244	Kwajalein	0.247
6	Guam	12.324	Bandar Seri Begawan	0.247
7	Kwajalein	12.319	Singapore	0.226
8	Yokusuka	12.307	Diego Garcia	0.226
9	Hawaii	12.304	Yokusuka	0.216
10	Singapore	11.618	Darwin	0.184
11	Phuket	11.618	Manila	0.053
12	Diego Garcia	11.618	Cam Ranh Bay	0.037
13	Darwin	11.613	Pattaya	0.026
14	Perth	11.611	Hawaii	0.000
15	Zuoying	10.150	Okinawa	0.000
16	Okinawa	9.031	Sasebo	0.000
17	Sasebo	8.567	Busan	0.000
18	Busan	8.524	Haiphong	0.000
19	Cam Ranh Bay	8.162	Phuket	0.000
20	Pattaya	8.132	Zuoying	0.000
21	Haiphong	8.123	Perth	0.000

T-AO				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.665
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.076
5	Puerto Princesa	0.650	Darwin	0.069
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.003
7	Palau	0.550	Pattaya	0.002
8	Yokusuka	0.500	Cam Ranh Bay	0.002
9	Okinawa	0.500	Puerto Princesa	0.001
10	Darwin	0.400	Manila	0.001
11	Zuoying	0.400	Haiphong	0.001
12	Sasebo	0.350	Cebu	0.001
13	Cam Ranh Bay	0.350	Palau	0.001
14	Perth	0.350	Guam	0.001
15	Haiphong	0.300	Kwajalein	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

T-AO				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	35.441	Guam	0.268
2	Cebu	35.252	Singapore	0.216
3	Manila	34.834	Diego Garcia	0.216
4	Bandar Seri Begawan	33.466	Darwin	0.174
5	Palau	33.285	Kwajalein	0.158
6	Singapore	30.785	Manila	0.153
7	Phuket	30.779	Bandar Seri Begawan	0.132
8	Diego Garcia	30.779	Yokusuka	0.126
9	Darwin	30.716	Puerto Princesa	0.111
10	Perth	30.692	Okinawa	0.068
11	Cam Ranh Bay	30.475	Cam Ranh Bay	0.068
12	Guam	29.396	Cebu	0.042
13	Kwajalein	29.352	Hawaii	0.000
14	Yokusuka	29.248	Palau	0.000
15	Hawaii	29.206	Sasebo	0.000
16	Zuoying	27.774	Busan	0.000
17	Okinawa	26.901	Haiphong	0.000
18	Sasebo	24.125	Phuket	0.000
19	Pattaya	23.523	Pattaya	0.000
20	Haiphong	23.429	Zuoying	0.000
21	Busan	23.390	Perth	0.000

T-AKE				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.665
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.076
5	Puerto Princesa	0.650	Darwin	0.070
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.005
7	Palau	0.550	Pattaya	0.003
8	Yokusuka	0.500	Cam Ranh Bay	0.002
9	Okinawa	0.500	Puerto Princesa	0.001
10	Darwin	0.400	Manila	0.001
11	Zuoying	0.400	Palau	0.001
12	Sasebo	0.350	Haiphong	0.001
13	Cam Ranh Bay	0.350	Guam	0.001
14	Perth	0.350	Cebu	0.001
15	Haiphong	0.300	Kwajalein	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

T-AKE				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	33.789	Palau	0.326
2	Cebu	33.376	Guam	0.311
3	Palau	32.325	Singapore	0.216
4	Manila	32.302	Diego Garcia	0.216
5	Bandar Seri Begawan	31.162	Puerto Princesa	0.195
6	Singapore	28.995	Kwajalein	0.184
7	Phuket	28.985	Darwin	0.174
8	Diego Garcia	28.985	Cebu	0.163
9	Darwin	28.883	Yokusuka	0.142
10	Perth	28.844	Bandar Seri Begawan	0.132
11	Guam	27.547	Manila	0.100
12	Kwajalein	27.457	Okinawa	0.063
13	Yokusuka	27.261	Cam Ranh Bay	0.047
14	Hawaii	27.221	Hawaii	0.000
15	Cam Ranh Bay	25.390	Sasebo	0.000
16	Okinawa	25.044	Busan	0.000
17	Zuoying	23.957	Haiphong	0.000
18	Pattaya	22.326	Phuket	0.000
19	Haiphong	20.425	Pattaya	0.000
20	Busan	20.311	Zuoying	0.000
21	Sasebo	20.310	Perth	0.000

LHA				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.665
2	Guam	0.700	Singapore	0.657
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.077
5	Puerto Princesa	0.650	Darwin	0.071
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.011
7	Palau	0.550	Pattaya	0.007
8	Yokusuka	0.500	Cam Ranh Bay	0.007
9	Okinawa	0.500	Puerto Princesa	0.005
10	Darwin	0.400	Manila	0.004
11	Zuoying	0.400	Cebu	0.004
12	Sasebo	0.350	Haiphong	0.004
13	Cam Ranh Bay	0.350	Palau	0.003
14	Perth	0.350	Guam	0.003
15	Haiphong	0.300	Kwajalein	0.001
16	Pattaya	0.300	Hawaii	0.001
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

LHA				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	88.721	Guam	0.268
2	Cebu	86.862	Singapore	0.216
3	Manila	85.923	Diego Garcia	0.216
4	Palau	84.391	Palau	0.200
5	Bandar Seri Begawan	83.408	Darwin	0.174
6	Singapore	75.880	Kwajalein	0.158
7	Phuket	75.826	Bandar Seri Begawan	0.132
8	Diego Garcia	75.822	Yokusuka	0.126
9	Darwin	75.242	Manila	0.111
10	Cam Ranh Bay	75.177	Puerto Princesa	0.111
11	Perth	75.020	Okinawa	0.074
12	Guam	71.778	Cam Ranh Bay	0.042
13	Kwajalein	71.379	Cebu	0.026
14	Yokusuka	70.448	Hawaii	0.000
15	Hawaii	70.077	Sasebo	0.000
16	Zuoying	69.598	Busan	0.000
17	Okinawa	68.493	Haiphong	0.000
18	Sasebo	60.015	Phuket	0.000
19	Pattaya	58.581	Pattaya	0.000
20	Busan	58.185	Zuoying	0.000
21	Haiphong	57.874	Perth	0.000

LSD				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.662
2	Guam	0.700	Singapore	0.656
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.085
5	Puerto Princesa	0.650	Darwin	0.082
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.034
7	Palau	0.550	Cam Ranh Bay	0.023
8	Yokusuka	0.500	Pattaya	0.021
9	Okinawa	0.500	Puerto Princesa	0.018
10	Darwin	0.400	Manila	0.016
11	Zuoying	0.400	Cebu	0.015
12	Sasebo	0.350	Haiphong	0.012
13	Cam Ranh Bay	0.350	Palau	0.010
14	Perth	0.350	Guam	0.010
15	Haiphong	0.300	Kwajalein	0.005
16	Pattaya	0.300	Hawaii	0.003
17	Hawaii	0.250	Yokusuka	0.002
18	Kwajalein	0.250	Zuoying	0.002
19	Busan	0.200	Okinawa	0.001
20	Diego Garcia	0.200	Sasebo	0.001
21	Phuket	0.200	Busan	0.000

LSD				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	88.046	Guam	0.237
2	Manila	87.953	Singapore	0.205
3	Cebu	87.311	Diego Garcia	0.163
4	Palau	82.639	Manila	0.142
5	Bandar Seri Begawan	82.329	Palau	0.132
6	Cam Ranh Bay	77.584	Kwajalein	0.126
7	Zuoying	73.283	Cebu	0.121
8	Singapore	73.047	Darwin	0.116
9	Phuket	72.702	Yokusuka	0.095
10	Diego Garcia	71.913	Okinawa	0.063
11	Darwin	70.095	Bandar Seri Begawan	0.053
12	Guam	70.050	Cam Ranh Bay	0.042
13	Okinawa	69.615	Puerto Princesa	0.026
14	Perth	69.561	Sasebo	0.005
15	Kwajalein	69.024	Hawaii	0.000
16	Yokusuka	66.700	Busan	0.000
17	Hawaii	65.801	Haiphong	0.000
18	Sasebo	61.504	Phuket	0.000
19	Haiphong	61.342	Pattaya	0.000
20	Pattaya	59.492	Zuoying	0.000
21	Busan	58.354	Perth	0.000

LPD				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.661
2	Guam	0.700	Singapore	0.656
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.085
5	Puerto Princesa	0.650	Darwin	0.083
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.037
7	Palau	0.550	Cam Ranh Bay	0.024
8	Yokusuka	0.500	Pattaya	0.022
9	Okinawa	0.500	Puerto Princesa	0.020
10	Darwin	0.400	Manila	0.017
11	Zuoying	0.400	Cebu	0.015
12	Sasebo	0.350	Haiphong	0.013
13	Cam Ranh Bay	0.350	Palau	0.011
14	Perth	0.350	Guam	0.010
15	Haiphong	0.300	Kwajalein	0.005
16	Pattaya	0.300	Hawaii	0.003
17	Hawaii	0.250	Yokusuka	0.002
18	Kwajalein	0.250	Zuoying	0.002
19	Busan	0.200	Okinawa	0.002
20	Diego Garcia	0.200	Sasebo	0.001
21	Phuket	0.200	Busan	0.000

LPD				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	89.851	Guam	0.242
2	Cebu	88.219	Singapore	0.200
3	Manila	87.860	Diego Garcia	0.163
4	Bandar Seri Begawan	83.803	Bandar Seri Begawan	0.153
5	Palau	83.660	Puerto Princesa	0.147
6	Cam Ranh Bay	78.307	Kwajalein	0.126
7	Singapore	73.391	Darwin	0.121
8	Phuket	73.022	Yokusuka	0.095
9	Diego Garcia	72.475	Manila	0.084
10	Zuoying	71.956	Okinawa	0.079
11	Guam	71.842	Cebu	0.037
12	Darwin	70.851	Cam Ranh Bay	0.037
13	Kwajalein	70.668	Palau	0.026
14	Perth	70.225	Sasebo	0.005
15	Okinawa	69.779	Hawaii	0.000
16	Yokusuka	68.091	Busan	0.000
17	Hawaii	67.098	Haiphong	0.000
18	Sasebo	61.773	Phuket	0.000
19	Haiphong	61.442	Pattaya	0.000
20	Pattaya	59.615	Zuoying	0.000
21	Busan	59.133	Perth	0.000

OSV				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.651
2	Guam	0.700	Singapore	0.651
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Darwin	0.119
5	Puerto Princesa	0.650	Perth	0.109
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.076
7	Palau	0.550	Cam Ranh Bay	0.034
8	Yokusuka	0.500	Puerto Princesa	0.032
9	Okinawa	0.500	Pattaya	0.030
10	Darwin	0.400	Manila	0.027
11	Zuoying	0.400	Cebu	0.026
12	Sasebo	0.350	Haiphong	0.020
13	Cam Ranh Bay	0.350	Palau	0.018
14	Perth	0.350	Guam	0.016
15	Haiphong	0.300	Kwajalein	0.008
16	Pattaya	0.300	Hawaii	0.005
17	Hawaii	0.250	Zuoying	0.004
18	Kwajalein	0.250	Yokusuka	0.004
19	Busan	0.200	Okinawa	0.004
20	Diego Garcia	0.200	Sasebo	0.003
21	Phuket	0.200	Busan	0.001

OSV				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	34.922	Guam	0.226
2	Cebu	34.534	Singapore	0.221
3	Manila	34.167	Diego Garcia	0.179
4	Bandar Seri Begawan	32.740	Bandar Seri Begawan	0.174
5	Palau	29.666	Puerto Princesa	0.116
6	Singapore	29.383	Kwajalein	0.111
7	Phuket	29.176	Manila	0.105
8	Diego Garcia	28.707	Darwin	0.105
9	Zuoying	28.522	Yokusuka	0.079
10	Cam Ranh Bay	28.429	Zuoying	0.053
11	Darwin	27.127	Cebu	0.037
12	Perth	27.034	Okinawa	0.011
13	Okinawa	26.702	Cam Ranh Bay	0.011
14	Guam	26.085	Sasebo	0.005
15	Kwajalein	25.532	Hawaii	0.000
16	Yokusuka	24.365	Palau	0.000
17	Hawaii	23.922	Busan	0.000
18	Sasebo	23.106	Haiphong	0.000
19	Haiphong	22.614	Phuket	0.000
20	Busan	22.271	Pattaya	0.000
21	Pattaya	21.429	Perth	0.000

SEATRAIN				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Singapore	0.633
2	Guam	0.700	Phuket	0.607
3	Manila	0.700	Diego Garcia	0.316
4	Singapore	0.650	Bandar Seri Begawan	0.174
5	Puerto Princesa	0.650	Darwin	0.164
6	Bandar Seri Begawan	0.600	Perth	0.136
7	Palau	0.550	Puerto Princesa	0.108
8	Yokusuka	0.500	Manila	0.095
9	Okinawa	0.500	Cam Ranh Bay	0.094
10	Darwin	0.400	Cebu	0.092
11	Zuoying	0.400	Pattaya	0.070
12	Sasebo	0.350	Haiphong	0.057
13	Cam Ranh Bay	0.350	Palau	0.057
14	Perth	0.350	Guam	0.053
15	Haiphong	0.300	Zuoying	0.029
16	Pattaya	0.300	Kwajalein	0.027
17	Hawaii	0.250	Okinawa	0.025
18	Kwajalein	0.250	Yokusuka	0.023
19	Busan	0.200	Sasebo	0.020
20	Diego Garcia	0.200	Hawaii	0.017
21	Phuket	0.200	Busan	0.008

SEATRAIN				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	38.039	Singapore	0.237
2	Cebu	37.256	Bandar Seri Begawan	0.216
3	Manila	37.051	Guam	0.179
4	Bandar Seri Begawan	35.432	Manila	0.116
5	Palau	32.335	Diego Garcia	0.116
6	Singapore	31.526	Puerto Princesa	0.111
7	Zuoying	31.382	Cebu	0.084
8	Cam Ranh Bay	31.204	Okinawa	0.074
9	Phuket	30.518	Yokusuka	0.068
10	Okinawa	29.634	Kwajalein	0.058
11	Diego Garcia	29.253	Darwin	0.058
12	Guam	28.250	Cam Ranh Bay	0.011
13	Kwajalein	26.990	Palau	0.005
14	Darwin	26.927	Sasebo	0.005
15	Perth	26.372	Hawaii	0.000
16	Sasebo	25.747	Busan	0.000
17	Yokusuka	25.233	Haiphong	0.000
18	Haiphong	25.094	Phuket	0.000
19	Busan	24.413	Pattaya	0.000
20	Hawaii	24.027	Zuoying	0.000
21	Pattaya	23.450	Perth	0.000

MOTHERSHIP				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.665
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.075
5	Puerto Princesa	0.650	Darwin	0.068
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.004
7	Palau	0.550	Pattaya	0.002
8	Yokusuka	0.500	Cam Ranh Bay	0.001
9	Okinawa	0.500	Puerto Princesa	0.001
10	Darwin	0.400	Manila	0.001
11	Zuoying	0.400	Haiphong	0.001
12	Sasebo	0.350	Cebu	0.001
13	Cam Ranh Bay	0.350	Palau	0.001
14	Perth	0.350	Guam	0.001
15	Haiphong	0.300	Kwajalein	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

MOTHERSHIP				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	37.845	Singapore	0.263
2	Cebu	37.559	Guam	0.216
3	Manila	37.189	Diego Garcia	0.216
4	Bandar Seri Begawan	35.509	Bandar Seri Begawan	0.184
5	Singapore	33.497	Puerto Princesa	0.163
6	Phuket	33.481	Palau	0.147
7	Diego Garcia	33.445	Okinawa	0.132
8	Darwin	33.307	Darwin	0.132
9	Perth	33.306	Manila	0.111
10	Palau	32.980	Kwajalein	0.105
11	Cam Ranh Bay	29.418	Yokusuka	0.074
12	Zuoying	29.160	Cam Ranh Bay	0.047
13	Okinawa	28.630	Cebu	0.026
14	Guam	28.223	Hawaii	0.000
15	Kwajalein	28.187	Sasebo	0.000
16	Yokusuka	28.101	Busan	0.000
17	Hawaii	28.066	Haiphong	0.000
18	Sasebo	24.128	Phuket	0.000
19	Pattaya	23.227	Pattaya	0.000
20	Busan	23.006	Zuoying	0.000
21	Haiphong	22.736	Perth	0.000

MOLA				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Puerto Princesa	0.422
2	Guam	0.700	Manila	0.400
3	Manila	0.700	Cebu	0.388
4	Singapore	0.650	Bandar Seri Begawan	0.347
5	Puerto Princesa	0.650	Cam Ranh Bay	0.259
6	Bandar Seri Begawan	0.600	Singapore	0.255
7	Palau	0.550	Palau	0.209
8	Yokusuka	0.500	Zuoying	0.197
9	Okinawa	0.500	Okinawa	0.186
10	Darwin	0.400	Haiphong	0.179
11	Zuoying	0.400	Guam	0.172
12	Sasebo	0.350	Sasebo	0.156
13	Cam Ranh Bay	0.350	Pattaya	0.135
14	Perth	0.350	Yokusuka	0.131
15	Haiphong	0.300	Darwin	0.079
16	Pattaya	0.300	Busan	0.075
17	Hawaii	0.250	Perth	0.075
18	Kwajalein	0.250	Phuket	0.046
19	Busan	0.200	Kwajalein	0.023
20	Diego Garcia	0.200	Hawaii	0.012
21	Phuket	0.200	Diego Garcia	0.006

MOLA				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	14.244	Bandar Seri Begawan	0.242
2	Cebu	13.992	Okinawa	0.184
3	Manila	13.909	Puerto Princesa	0.184
4	Bandar Seri Begawan	13.650	Palau	0.168
5	Singapore	12.361	Singapore	0.163
6	Palau	12.330	Manila	0.105
7	Cam Ranh Bay	12.267	Phuket	0.074
8	Zuoying	12.010	Darwin	0.042
9	Okinawa	11.806	Cam Ranh Bay	0.042
10	Darwin	10.516	Cebu	0.037
11	Haiphong	10.429	Zuoying	0.021
12	Guam	10.319	Guam	0.011
13	Sasebo	10.218	Sasebo	0.005
14	Pattaya	9.822	Hawaii	0.000
15	Busan	9.733	Kwajalein	0.000
16	Yokusuka	9.046	Yokusuka	0.000
17	Phuket	8.947	Busan	0.000
18	Perth	8.476	Haiphong	0.000
19	Kwajalein	6.752	Diego Garcia	0.000
20	Hawaii	4.507	Pattaya	0.000
21	Diego Garcia	3.380	Perth	0.000

T-AK				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.664
2	Guam	0.700	Singapore	0.657
3	Manila	0.700	Diego Garcia	0.339
4	Singapore	0.650	Perth	0.080
5	Puerto Princesa	0.650	Darwin	0.075
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.008
7	Palau	0.550	Pattaya	0.005
8	Yokusuka	0.500	Cam Ranh Bay	0.005
9	Okinawa	0.500	Puerto Princesa	0.004
10	Darwin	0.400	Manila	0.003
11	Zuoying	0.400	Haiphong	0.002
12	Sasebo	0.350	Cebu	0.002
13	Cam Ranh Bay	0.350	Palau	0.002
14	Perth	0.350	Guam	0.002
15	Haiphong	0.300	Kwajalein	0.001
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

T-AK				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Cebu	34.788	Guam	0.268
2	Puerto Princesa	34.541	Singapore	0.216
3	Manila	34.074	Diego Garcia	0.216
4	Palau	33.984	Palau	0.179
5	Bandar Seri Begawan	32.431	Darwin	0.174
6	Singapore	30.170	Kwajalein	0.158
7	Phuket	30.154	Bandar Seri Begawan	0.132
8	Diego Garcia	30.153	Yokusuka	0.126
9	Darwin	29.984	Puerto Princesa	0.105
10	Perth	29.920	Cebu	0.095
11	Cam Ranh Bay	29.593	Okinawa	0.068
12	Guam	28.674	Manila	0.068
13	Kwajalein	28.557	Cam Ranh Bay	0.042
14	Yokusuka	28.283	Hawaii	0.000
15	Hawaii	28.173	Sasebo	0.000
16	Zuoying	27.596	Busan	0.000
17	Okinawa	26.599	Haiphong	0.000
18	Sasebo	23.801	Phuket	0.000
19	Pattaya	23.575	Pattaya	0.000
20	Haiphong	23.321	Zuoying	0.000
21	Busan	22.703	Perth	0.000

T-AKR				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.665
2	Guam	0.700	Singapore	0.657
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.078
5	Puerto Princesa	0.650	Darwin	0.072
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.006
7	Palau	0.550	Pattaya	0.004
8	Yokusuka	0.500	Cam Ranh Bay	0.003
9	Okinawa	0.500	Puerto Princesa	0.002
10	Darwin	0.400	Manila	0.002
11	Zuoying	0.400	Haiphong	0.002
12	Sasebo	0.350	Cebu	0.002
13	Cam Ranh Bay	0.350	Palau	0.001
14	Perth	0.350	Guam	0.001
15	Haiphong	0.300	Kwajalein	0.001
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

T-AKR				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	33.938	Guam	0.268
2	Cebu	33.777	Singapore	0.216
3	Manila	33.272	Diego Garcia	0.216
4	Palau	32.368	Palau	0.179
5	Bandar Seri Begawan	31.624	Darwin	0.174
6	Singapore	29.103	Kwajalein	0.158
7	Phuket	29.092	Bandar Seri Begawan	0.132
8	Diego Garcia	29.091	Yokusuka	0.126
9	Darwin	28.976	Puerto Princesa	0.111
10	Perth	28.931	Manila	0.089
11	Cam Ranh Bay	28.617	Okinawa	0.079
12	Guam	27.652	Cebu	0.068
13	Kwajalein	27.572	Cam Ranh Bay	0.042
14	Yokusuka	27.383	Hawaii	0.000
15	Hawaii	27.307	Sasebo	0.000
16	Zuoying	26.541	Busan	0.000
17	Okinawa	25.998	Haiphong	0.000
18	Sasebo	22.628	Phuket	0.000
19	Pattaya	22.449	Pattaya	0.000
20	Haiphong	22.076	Zuoying	0.000
21	Busan	22.072	Perth	0.000

RORO				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.664
2	Guam	0.700	Singapore	0.657
3	Manila	0.700	Diego Garcia	0.339
4	Singapore	0.650	Perth	0.080
5	Puerto Princesa	0.650	Darwin	0.076
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.009
7	Palau	0.550	Pattaya	0.006
8	Yokusuka	0.500	Cam Ranh Bay	0.005
9	Okinawa	0.500	Puerto Princesa	0.004
10	Darwin	0.400	Manila	0.003
11	Zuoying	0.400	Cebu	0.003
12	Sasebo	0.350	Haiphong	0.003
13	Cam Ranh Bay	0.350	Palau	0.002
14	Perth	0.350	Guam	0.002
15	Haiphong	0.300	Kwajalein	0.001
16	Pattaya	0.300	Hawaii	0.001
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

RORO				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Cebu	33.549	Guam	0.268
2	Puerto Princesa	33.416	Palau	0.221
3	Manila	33.057	Singapore	0.216
4	Palau	32.547	Diego Garcia	0.216
5	Bandar Seri Begawan	31.564	Darwin	0.174
6	Singapore	29.079	Kwajalein	0.158
7	Phuket	29.061	Bandar Seri Begawan	0.132
8	Diego Garcia	29.060	Yokusuka	0.126
9	Darwin	28.877	Puerto Princesa	0.105
10	Perth	28.806	Manila	0.068
11	Cam Ranh Bay	28.542	Okinawa	0.053
12	Guam	27.775	Cebu	0.053
13	Kwajalein	27.647	Cam Ranh Bay	0.047
14	Yokusuka	27.346	Hawaii	0.000
15	Hawaii	27.225	Sasebo	0.000
16	Zuoying	26.583	Busan	0.000
17	Okinawa	25.940	Haiphong	0.000
18	Pattaya	22.799	Phuket	0.000
19	Sasebo	22.719	Pattaya	0.000
20	Haiphong	22.363	Zuoying	0.000
21	Busan	22.211	Perth	0.000

LCU				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Puerto Princesa	0.425
2	Guam	0.700	Manila	0.403
3	Manila	0.700	Cebu	0.392
4	Singapore	0.650	Bandar Seri Begawan	0.330
5	Puerto Princesa	0.650	Cam Ranh Bay	0.261
6	Bandar Seri Begawan	0.600	Singapore	0.231
7	Palau	0.550	Palau	0.212
8	Yokusuka	0.500	Zuoying	0.206
9	Okinawa	0.500	Okinawa	0.189
10	Darwin	0.400	Haiphong	0.179
11	Zuoying	0.400	Guam	0.174
12	Sasebo	0.350	Sasebo	0.160
13	Cam Ranh Bay	0.350	Pattaya	0.133
14	Perth	0.350	Yokusuka	0.131
15	Haiphong	0.300	Darwin	0.083
16	Pattaya	0.300	Perth	0.078
17	Hawaii	0.250	Phuket	0.078
18	Kwajalein	0.250	Busan	0.075
19	Busan	0.200	Kwajalein	0.038
20	Diego Garcia	0.200	Diego Garcia	0.022
21	Phuket	0.200	Hawaii	0.021

LCU				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Manila	15.769	Singapore	0.226
2	Puerto Princesa	15.663	Manila	0.179
3	Cebu	15.503	Guam	0.147
4	Bandar Seri Begawan	14.807	Okinawa	0.111
5	Cam Ranh Bay	14.186	Puerto Princesa	0.089
6	Zuoying	13.548	Darwin	0.084
7	Palau	13.369	Cebu	0.058
8	Okinawa	12.929	Palau	0.032
9	Singapore	12.788	Yokusuka	0.032
10	Haiphong	11.822	Phuket	0.026
11	Guam	11.788	Cam Ranh Bay	0.011
12	Sasebo	11.409	Sasebo	0.005
13	Darwin	11.124	Hawaii	0.000
14	Pattaya	11.022	Kwajalein	0.000
15	Phuket	10.913	Busan	0.000
16	Busan	10.811	Bandar Seri Begawan	0.000
17	Yokusuka	10.096	Haiphong	0.000
18	Perth	9.020	Diego Garcia	0.000
19	Kwajalein	8.213	Pattaya	0.000
20	Diego Garcia	6.906	Zuoying	0.000
21	Hawaii	5.597	Perth	0.000

ESB				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.666
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.074
5	Puerto Princesa	0.650	Darwin	0.066
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.000
7	Palau	0.550	Pattaya	0.000
8	Yokusuka	0.500	Cam Ranh Bay	0.000
9	Okinawa	0.500	Puerto Princesa	0.000
10	Darwin	0.400	Manila	0.000
11	Zuoying	0.400	Haiphong	0.000
12	Sasebo	0.350	Cebu	0.000
13	Cam Ranh Bay	0.350	Palau	0.000
14	Perth	0.350	Guam	0.000
15	Haiphong	0.300	Kwajalein	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

ESB				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	32.909	Guam	0.268
2	Cebu	32.712	Singapore	0.216
3	Manila	32.142	Diego Garcia	0.216
4	Palau	31.389	Palau	0.179
5	Bandar Seri Begawan	30.585	Darwin	0.174
6	Singapore	28.593	Puerto Princesa	0.163
7	Phuket	28.593	Kwajalein	0.158
8	Diego Garcia	28.593	Bandar Seri Begawan	0.132
9	Darwin	28.585	Yokusuka	0.126
10	Perth	28.582	Manila	0.105
11	Cam Ranh Bay	27.229	Cebu	0.095
12	Guam	26.747	Okinawa	0.079
13	Kwajalein	26.742	Cam Ranh Bay	0.047
14	Yokusuka	26.730	Hawaii	0.000
15	Hawaii	26.725	Sasebo	0.000
16	Okinawa	25.066	Busan	0.000
17	Zuoying	25.030	Haiphong	0.000
18	Pattaya	21.807	Phuket	0.000
19	Sasebo	21.554	Pattaya	0.000
20	Busan	21.061	Zuoying	0.000
21	Haiphong	21.020	Perth	0.000

ESD				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Phuket	0.666
2	Guam	0.700	Singapore	0.658
3	Manila	0.700	Diego Garcia	0.338
4	Singapore	0.650	Perth	0.074
5	Puerto Princesa	0.650	Darwin	0.066
6	Bandar Seri Begawan	0.600	Bandar Seri Begawan	0.000
7	Palau	0.550	Pattaya	0.000
8	Yokusuka	0.500	Cam Ranh Bay	0.000
9	Okinawa	0.500	Puerto Princesa	0.000
10	Darwin	0.400	Manila	0.000
11	Zuoying	0.400	Haiphong	0.000
12	Sasebo	0.350	Cebu	0.000
13	Cam Ranh Bay	0.350	Palau	0.000
14	Perth	0.350	Guam	0.000
15	Haiphong	0.300	Kwajalein	0.000
16	Pattaya	0.300	Hawaii	0.000
17	Hawaii	0.250	Yokusuka	0.000
18	Kwajalein	0.250	Zuoying	0.000
19	Busan	0.200	Okinawa	0.000
20	Diego Garcia	0.200	Sasebo	0.000
21	Phuket	0.200	Busan	0.000

ESD				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Puerto Princesa	32.909	Guam	0.268
2	Cebu	32.712	Singapore	0.216
3	Manila	32.142	Diego Garcia	0.216
4	Palau	31.389	Palau	0.179
5	Bandar Seri Begawan	30.585	Darwin	0.174
6	Singapore	28.593	Puerto Princesa	0.163
7	Phuket	28.593	Kwajalein	0.158
8	Diego Garcia	28.593	Bandar Seri Begawan	0.132
9	Darwin	28.585	Yokusuka	0.126
10	Perth	28.582	Manila	0.105
11	Cam Ranh Bay	27.229	Cebu	0.095
12	Guam	26.747	Okinawa	0.079
13	Kwajalein	26.742	Cam Ranh Bay	0.047
14	Yokusuka	26.730	Hawaii	0.000
15	Hawaii	26.725	Sasebo	0.000
16	Okinawa	25.066	Busan	0.000
17	Zuoying	25.030	Haiphong	0.000
18	Pattaya	21.807	Phuket	0.000
19	Sasebo	21.554	Pattaya	0.000
20	Busan	21.061	Zuoying	0.000
21	Haiphong	21.020	Perth	0.000

ORCA				
Rank	**Degree**	**Value**	**Eigenvector**	**Value**
1	Cebu	0.700	Puerto Princesa	0.433
2	Guam	0.700	Manila	0.417
3	Manila	0.700	Cebu	0.404
4	Singapore	0.650	Bandar Seri Begawan	0.328
5	Puerto Princesa	0.650	Cam Ranh Bay	0.263
6	Bandar Seri Begawan	0.600	Palau	0.216
7	Palau	0.550	Zuoying	0.212
8	Yokusuka	0.500	Singapore	0.209
9	Okinawa	0.500	Haiphong	0.183
10	Darwin	0.400	Okinawa	0.181
11	Zuoying	0.400	Guam	0.166
12	Sasebo	0.350	Sasebo	0.161
13	Cam Ranh Bay	0.350	Pattaya	0.134
14	Perth	0.350	Yokusuka	0.115
15	Haiphong	0.300	Busan	0.067
16	Pattaya	0.300	Perth	0.063
17	Hawaii	0.250	Darwin	0.062
18	Kwajalein	0.250	Phuket	0.026
19	Busan	0.200	Kwajalein	0.017
20	Diego Garcia	0.200	Hawaii	0.009
21	Phuket	0.200	Diego Garcia	0.000

ORCA				
Rank	**Closeness**	**Value**	**Betweenness**	**Value**
1	Cebu	5.965	Palau	0.189
2	Puerto Princesa	5.960	Bandar Seri Begawan	0.184
3	Manila	5.959	Cebu	0.126
4	Bandar Seri Begawan	5.942	Manila	0.100
5	Cam Ranh Bay	5.908	Phuket	0.100
6	Palau	5.904	Puerto Princesa	0.089
7	Zuoying	5.901	Okinawa	0.079
8	Okinawa	5.883	Singapore	0.068
9	Singapore	5.850	Zuoying	0.047
10	Haiphong	5.833	Sasebo	0.037
11	Sasebo	5.819	Cam Ranh Bay	0.011
12	Guam	5.808	Hawaii	0.000
13	Pattaya	5.795	Kwajalein	0.000
14	Busan	5.787	Guam	0.000
15	Darwin	5.782	Yokusuka	0.000
16	Yokusuka	5.722	Busan	0.000
17	Phuket	5.714	Darwin	0.000
18	Kwajalein	5.568	Haiphong	0.000
19	Perth	5.551	Diego Garcia	0.000
20	Hawaii	5.241	Pattaya	0.000
21	Diego Garcia	0.315	Perth	0.000

APPENDIX C:
Maximum Flow Inputs and Results

Vessel	**Defenses**	P_v	$losses_v$	$vesselCap_v$	vel_v
LCS	Y	0.6	527.89364	1400	16
JHSV	N	0.01	229.27156	600	35
T-AOE	N	0.7	686.783907	29291	25
T-AO	N	0.7	670.969279	26838	15
T-AKE	N	0.7	621.822545	11075	20
LHA	Y	0.6	1741.8178	16460	16
LSD	Y	0.6	394.376744	6440	15
LPD	Y	0.6	1716.95255	5691	16
OSV	N	0.5	14.5052	2000	13
ORCA	N	0.01	11.0120208	8	3
T-AK	N	0.7	2345.60955	9578	15
T-AKR	N	0.7	2363.5838	13000	15
RORO	N	0.6	493.000711	8158	15
ESD	N	0.9	1528.11146	193830	15
ESB	N	0.9	1528.11146	193830	15

Uncontested Routing with Full Demand Signal		
Vessel	G_v	**Routing**
LCS1	C	Diego Garcia - Darwin
LCS2	C	Diego Garcia - Darwin
JHSV1	B	Hawaii - Guam
JHSV2	D	Hawaii - Guam
T-AOE1	A	Hawaii - Okinawa
T-AOE2	A	Diego Garcia - Darwin
T-AO1	A	Hawaii - Guam
T-AO2	A	Hawaii - Guam
T-AKE1	A	Diego Garcia - Darwin
T-AKE2	A	Hawaii - Okinawa
LHA1	A	Hawaii - Guam
LHA2	A	Diego Garcia - Darwin
LSD1	A	Hawaii - Guam
LSD2	A	Hawaii - Guam
LPD1	A	Diego Garcia - Singapore - Manila
LPD2	A	Hawaii - Guam
OSV1	A	Hawaii - Guam
OSV2	C	Hawaii - Okinawa
ORCA1	-	-
ORCA2	-	-
T-AK1	A	Hawaii - Guam
T-AK2	A	Hawaii - Guam
T-AKR1	A	Hawaii - Guam
T-AKR2	A	Hawaii - Guam
RORO1	A	Hawaii - Guam
RORO2	A	Hawaii - Guam
ESD1	C	Hawaii - Guam
ESD2	C	Diego Garcia - Darwin
ESB1	A	Diego Garcia - Darwin
ESB2	D	Hawaii - Guam

Contested Routing with Full Demand Signal		
Vessel	G_v	**Routing**
LCS1	A	Hawaii - Guam
LCS2	A	Diego Garcia - Singapore - Manila
JHSV1	C	Diego Garcia - Singapore - Manila
JHSV2	C	Hawaii - Guam
T-AOE1	A	Hawaii - Guam
T-AOE2	A	Diego Garcia - Singapore - Manila
T-AO1	A	Hawaii - Guam
T-AO2	A	Hawaii - Guam
T-AKE1	A	Hawaii - Guam
T-AKE2	A	Diego Garcia - Phuket - Singapore - Manila
LHA1	A	Hawaii - Guam
LHA2	A	Hawaii - Guam
LSD1	A	Hawaii - Okinawa
LSD2	A	Hawaii - Okinawa
LPD1	A	Hawaii - Okinawa
LPD2	A	Hawaii - Okinawa
OSV1	A	Hawaii - Guam
OSV2	C	Hawaii - Okinawa
ORCA1	-	-
ORCA2	-	-
T-AK1	A	Hawaii - Guam
T-AK2	A	Hawaii - Guam
T-AKR1	A	Hawaii - Guam
T-AKR2	A	Hawaii - Guam
RORO1	A	Diego Garcia - Darwin
RORO2	A	Hawaii - Guam
ESD1	C	Diego Garcia - Darwin
ESD2	C	Hawaii - Guam
ESB1	A	Hawaii - Okinawa
ESB2	D	Hawaii - Guam

Uncontested Routing with Partial Demand Signal		
Vessel	G_v	**Routing**
LCS1	B	Diego Garcia - Singapore - Manila
LCS2	A	Hawaii - Okinawa
JHSV1	A	Diego Garcia - Singapore - Manila
JHSV2	D	Hawaii - Okinawa
T-AOE1	A	Hawaii - Yokosuka - Sasebo
T-AOE2	A	Hawaii - Yokosuka - Sasebo
T-AO1	A	Diego Garcia - Singapore - Manila
T-AO2	A	Hawaii - Okinawa
T-AKE1	A	Hawaii - Yokosuka - Sasebo
T-AKE2	A	Hawaii - Yokosuka - Sasebo
LHA1	A	Diego Garcia - Perth - Manila
LHA2	A	Diego Garcia - Singapore - Manila
LSD1	A	Hawaii - Okinawa
LSD2	A	Hawaii - Okinawa
LPD1	A	Hawaii - Okinawa
LPD2	A	Diego Garcia - Singapore - Manila
OSV1	A	Hawaii - Okinawa - Sasebo
OSV2	A	Diego Garcia - Phuket - Singapore - Manila
ORCA1	-	-
ORCA2	-	-
T-AK1	A	Hawaii - Okinawa
T-AK2	A	Hawaii - Okinawa
T-AKR1	A	Hawaii - Okinawa
T-AKR2	A	Hawaii - Okinawa
RORO1	A	Diego Garcia - Singapore - Manila
RORO2	A	Diego Garcia - Singapore - Manila
ESD1	C	Hawaii - Okinawa
ESD2	C	Hawaii - Okinawa
ESB1	A	Hawaii - Okinawa
ESB2	D	Diego Garcia - Singapore - Manila

Contested Routing with Partial Demand Signal		
Vessel	G_v	**Routing**
LCS1	A	Hawaii - Yokosuka - Busan
LCS2	B	Hawaii - Yokosuka - Sasebo
JHSV1	C	Diego Garcia - Singapore - Manila
JHSV2	A	Diego Garcia - Phuket - Singapore - Manila
T-AOE1	A	Diego Garcia - Singapore - Manila - Guam - Okinawa
T-AOE2	A	Hawaii - Guam - Okinawa
T-AO1	A	Hawaii - Yokosuka - Sasebo
T-AO2	A	Hawaii - Kwajalein - Yokosuka - Sasebo
T-AKE1	A	Hawaii - Okinawa - Yokosuka - Sasebo
T-AKE2	A	Diego Garcia - Singapore - Manila
LHA1	A	Hawaii - Okinawa
LHA2	A	Hawaii - Okinawa
LSD1	A	Hawaii - Yokosuka - Busan
LSD2	A	Hawaii - Yokosuka - Busan
LPD1	A	Hawaii - Yokosuka - Sasebo
LPD2	A	Diego Garcia - Singapore - Manila
OSV1	A	Hawaii - Yokosuka - Busan
OSV2	B	Hawaii - Yokosuka - Sasebo
ORCA1	-	-
ORCA2	-	-
T-AK1	A	Diego Garcia - Singapore - Manila
T-AK2	A	Hawaii - Guam - Okinawa
T-AKR1	A	Hawaii - Yokosuka
T-AKR2	A	Diego Garcia - Singapore - Manila
RORO1	A	Hawaii - Kwajalein - Okinawa - Busan
RORO2	A	Diego Garcia - Singapore - Manila - Cebu - Sasebo - Yokosuka - Busan
ESD1	C	Diego Garcia - Singapore - Bandar Seri Begawan - Manila
ESD2	C	Diego Garcia - Phuket - Singapore - Cam Ranh Bay - Manila
ESB1	A	Hawaii - Okinawa
ESB2	D	Hawaii - Yokosuka

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