AFRL-AFOSR-VA-TR-2020-0197

Microresonator-Based Optical Frequency Combs - Initiation, Characterization, Control

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07/28/2020
Final Report

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Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB1
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Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
28/07/2020

2. REPORT TYPE
FINAL

3. DATES COVERED *(From - To)*
15/06/2015-31/12/2019

4. TITLE AND SUBTITLE
Microresonator-Based Optical Frequency Combs: Initiation, Characterization, Control

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-15-1-0211

5c. PROGRAM ELEMENT NUMBER

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

6. AUTHOR(S)
Andrew M. Weiner

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Purdue University
401 South Grant Street
West Lafayette, IN

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
875 North Randolph Street, RM 312
Arlington, VA 22203-1954

10. SPONSOR/MONITOR'S ACRONYM(S)
USA, AFRL, AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution A - Approved for Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Optical frequency comb generation via nonlinear wave mixing in high quality factor microresonators offers a route to take frequency combs out of the laboratory, which can potentially revolutionize a diverse set of applications, including optical frequency synthesis and metrology, time-keeping, telecommunications, lidar, and radio-frequency signal processing, among others. Our research investigates the science of frequency combs generated in chip-scale microring resonators fabricated in silicon nitride films on silicon substrates. Our research encompasses frequency combs from both anomalous and normal dispersion microresonators and explores fundamental comb phenomenology, power considerations and applications.

15. SUBJECT TERMS
frequency combs, optical microresonators, nonlinear optics, radio-frequency photonics

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
			UU	7	Andrew M. Weiner
					19b. TELEPHONE NUMBER *(Include area code)* (765)494-5574

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Contract/Grant Title: Microresonator-Based Optical Frequency Combs: Initiation, Characterization, Control

Contract/Grant #: FA9550-15-1-0211

Principal Investigator: Prof. Andrew M. Weiner, Purdue University

Optical frequency comb generation via nonlinear wave mixing in high quality factor microresonators is now a very active research topic. Microresonator-based Kerr frequency comb (microcomb) generation offers a route to take frequency combs out of the laboratory, which can potentially revolutionize a diverse set of applications, including optical frequency synthesis and metrology, time-keeping, telecommunications, lidar, and radio-frequency signal processing, among others. Our research investigates the science of frequency combs generated in chip-scale microring resonators fabricated in silicon nitride films on silicon substrates. Our research on Kerr combs under this grant may be characterized broadly according to three main themes: (1) fundamental comb phenomenology; (2) power considerations; and (3) representative applications that both explore new opportunities and elucidate the research directions. In this report I describe several examples of our work in each of these three categories.

It is important to note that frequency combs can be generated in microresonators with either anomalous or normal dispersion. Anomalous dispersion microresonators have been most widely studied, both because combs are easier to initiate and because highly stable cavity soliton states that consist of one or more ultrashort "bright" pulses circulating within the cavity can be generated. Originally, it was widely believed that initiation of combs in normal dispersion microresonators would not be possible. However, at Purdue we were able to show that comb generation in normal dispersion microresonators could be realized due to various effects not considered in the simplest models [1]; we were also the first to demonstrate that the resulting combs can form novel mode-locked "dark" pulse waveforms [2] with substantially higher comb powers [3] than the ultrashort pulse cavity soliton combs. Broadly speaking, both the bright pulse solutions and dark pulse solutions have different advantages and disadvantages. Our research summarized below involves both bright pulse soliton combs in anomalous dispersion microresonators and dark pulse combs from normal dispersion microresonators.

1. Fundamental comb phenomenology

Ideally a frequency comb provides a large set of highly stable optical frequencies, locked with precisely equal frequency separations and low noise over large bandwidth. However, in order to reach such a state with a microresonator frequency comb (also known as a Kerr comb), one must deal with a number of complex issues related both to initiation of the comb and navigation through chaotic and other operating regimes in the nonlinear phase space. Such effects are interesting both from a fundamental nonlinear physics and nonlinear dynamics perspective and due to their importance in reaching the ideal low noise comb states relevant to most applications. Some of our work related to fundamental comb phenomenology is summarized below.

The most common form of stable Kerr combs studied to date involves soliton formation in anomalous dispersion microresonators; such Kerr solitons balance cavity dispersion and nonlinearity to realize coherent, ultrashort pulse, low-noise comb operation. The microresonator comb system also presents a laboratory for investigating soliton dynamics, e.g., instabilities that can form from cavity solitons. In [4] we present experimental and numerical observations of a Fermi-Pasta-Ulam recurrence induced by breather solitons in an anomalous dispersion, high-Q SiN microresonator. **Breathers are localized waves in nonlinear systems that undergo a periodic variation in time or space.** In this mode of operation, the comb remains in a short pulse soliton state; however, unlike stable solitons the soliton parameters (e.g., bandwidth, peak intensity, and pulse duration) oscillate – or breathe – quasiperiodically on a time scale slow compared to the microresonator round trip time. We were among the first to show that breather solitons can be excited by increasing the pump power at a relatively small pump detuning in anomalous dispersion microresonators. Out of phase power evolution is observed for groups of comb lines around the center of the spectrum compared to groups of lines in the spectral wings. Numerical simulations based on the generalized Lugiato-Lefever equation are in good agreement with the experimental results. We next investigated breather instabilities that we observe in connection with mode-locked dark pulses in normal dispersion microcavities [5]. Such dark pulse breathers have seen much less experimental study across all subfields in physics compared to soliton-type breathers. We show that with relatively high pump power, dark pulse Kerr combs can enter a breathing regime, in which the time-domain waveform remains a dark pulse but experiences a periodic modulation on a time scale much slower than the microresonator round trip time. In the highly pumped regime, a transition to a chaotic breathing state where the waveform remains dark-pulse-like is also observed; for the first time to our knowledge, such a transition is reversible by

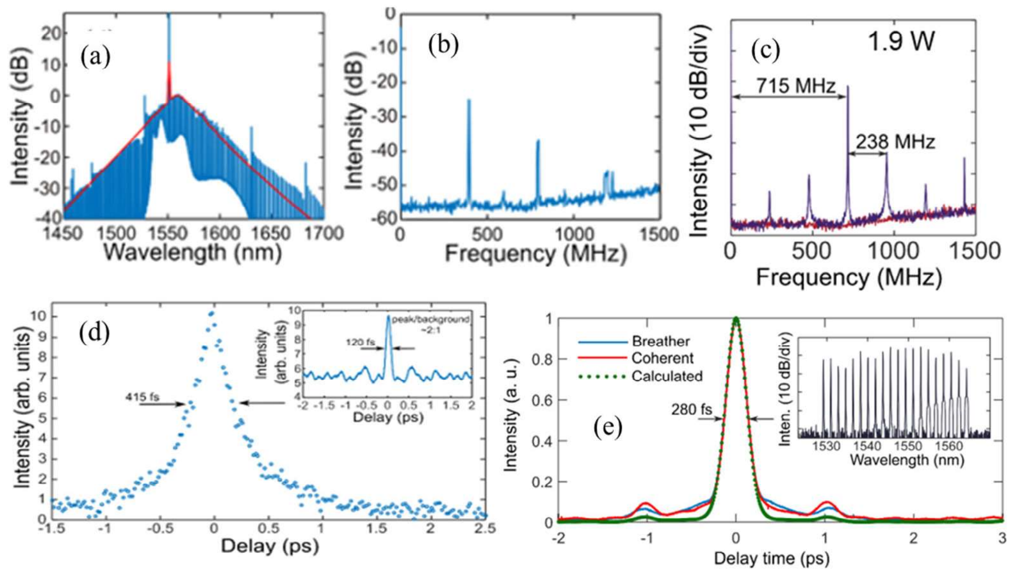

Fig. 1. Representative data for bright and dark pulse breather combs. (a) Optical spectrum of a bright soliton breather from an anomalous dispersion microresonator. (b) RF intensity spectrum for the bright breather, showing peaks at the fundamental breather frequency (~400 MHz) as well as harmonics. The sharp RF lines are indicative of quasiperiodic modulation; the relatively high frequency, compared to the ~100 MHz optical resonance linewidth, differentiates breather action from noisy modulational instability combs. (c) RF intensity spectrum for a dark breather in a normal dispersion microresonator. The sharp line at 715 MHz corresponds to breather action; the weaker peaks spaced by 238 MHz correspond to an unusual period tripling route to chaos. (d,e) Autocorrelation data for (d) bright and (e) dark breathers. The traces provide evidence that in both bright and dark breather cases, the waveform remains similar to that of the stable bright soliton or modelocked dark pulse, respectively. The inset to (d) shows the autocorrelation when the bright soliton and breather transition into chaos; the poor contrast ratio indicates that the waveform loses it clear pulse-like character in the chaotic regime.

reducing the pump power. Examples of data on bright as well as dark breather combs are shown in Fig. 1. These results show that optical microresonators can be exploited as a powerful platform for investigating rich nonlinear dynamics.

With respect to soliton formation in anomalous dispersion microresonators, one interesting and important point is that the mode-locked state is usually reached after passing through a chaotic operating regime, with the result that the number of solitons generated is probabilistic and is usually greater than one. Practically such multi-soliton combs have a structured spectrum, whereas the smooth spectrum of single soliton is favored for applications. In [6] we introduced, experimentally and numerically, a new, passive mechanism for single temporal soliton formation arising from spatial mode-interaction in microresonators. Deterministic single soliton generation is observed for microresonators with strong mode-interaction in experiments and simulations. Further simulations explain our results based on a nonlinear loss mechanism associated with mode-interaction induced Cherenkov radiation. Our results give important insights into soliton – Cherenkov radiation interaction in cavities.

In another paper [7] we investigated an effect which we term thermo-optical (TO) chaos. Although microresonator frequency combs fundamentally rely on the approximately instantaneous Kerr nonlinearity, slow thermo-optic nonlinearities are also prevalent in most microresonators. Thermal shifts of the microresonator have a prominent influence on comb generation and complicate the attainment of desirable, low noise soliton states. We performed numerical simulations including the thermal dynamics that show that the generated solitons can either survive or annihilate when the pump laser is scanned from blue to red and then stopped at a fixed wavelength; the outcome is stochastic and is strongly related to the number of solitons generated. The random fluctuations of the cavity resonance occurring under TO chaos are also found to trigger delayed spontaneous soliton generation after the laser scan ends, which could enable soliton excitation with slow laser tuning speed. We have observed these interesting effects experimentally in a silicon-nitride microresonator.

Turning our attention now to normal dispersion microresonators, in earlier work we showed that comb generation could be initiated through interactions between different transverse modes in a few-moded waveguide [1]. During this grant period we demonstrated that the interaction between fundamental and second-harmonic waves can provide an entirely new way of phase matching for four-wave mixing in optical microresonators, enabling the generation of optical frequency combs in the normal dispersion regime under conditions where comb creation is ordinarily prohibited [8]. Simultaneous Kerr comb formation and second-harmonic generation with on-chip microresonators can greatly facilitate comb self-referencing for optical clocks and frequency metrology. Moreover, the presence of both second- and third-order nonlinearities results in complex cavity dynamics that is of high scientific interest but is still far from being well-understood. Our study was stimulated by unexpected experimental observations and then explained through theory and simulations. We derived new coupled time-domain mean-field equations and obtained simulation results showing good qualitative agreement with our data. Our findings provide a novel way of overcoming the dispersion limit for simultaneous Kerr comb formation and second-harmonic generation, which might prove to be especially important in the near-visible to visible range where several atomic transitions commonly used for the stabilization of optical clocks are located and where the large normal material dispersion is likely to dominate.

Finally, in recent work performed collaboratively with Chalmers University of Technology (Sweden), we extended our study of the physical dynamics of localized dark pulse structures in normal dispersion combs [9]. In particular, we report the discovery of reversible switching between coherent dark-pulse Kerr combs, whereby distinct states can be accessed deterministically. Furthermore, we reveal that the formation of dark-pulse Kerr combs is associated with the appearance of a new resonance, a feature that has never been observed for dark pulses (such a resonance is now well known for combs in anomalous dispersion microresoantors and is ascribed to soliton behavior [10]). These results contribute to understanding the nonlinear physics in few-mode microresonators and provide insight into the generation of microcombs with high conversion efficiency.

2. Power considerations

One of the fundamental differences between single soliton Kerr combs in anomalous dispersion microresonators and dark pulse Kerr combs in normal dispersion microresonators has to do with the power transferred from the pump light into the comb lines. We discovered that normal dispersion dark pulse combs offer much higher powers in individual combs lines and much more efficient power transfer from the pump into the comb and helped to focus the community's attention on this difference. Such power considerations are very important for applications such as coherent lightwave communications and radio-frequency photonics, among others, where optical signal-to-noise plays a key role.

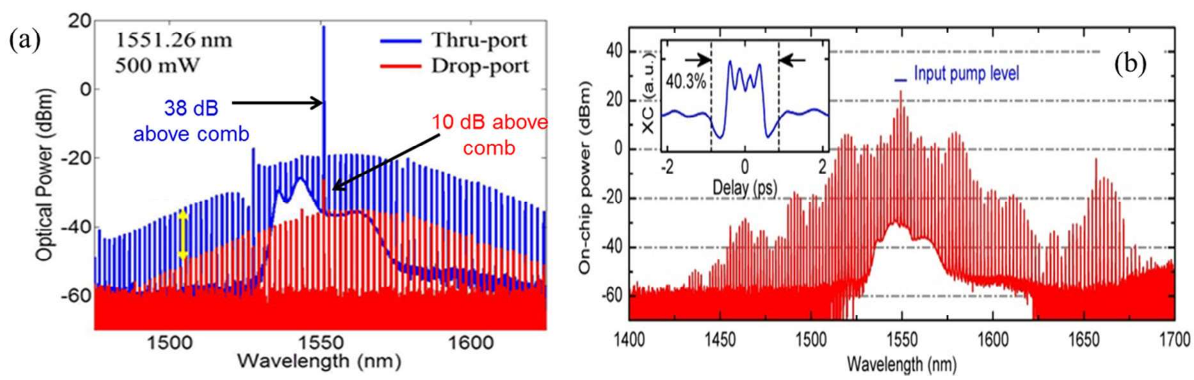

Fig. 2. Power comparison of combs from normal and anomalous dispersion microresonators, showing sharply different behavior. (a) Bright cavity soliton from anomalous dispersion microresonator. The pump exiting the through-port is 38 dB above the neighboring comb lines, the integrated comb output is ~1 mW, roughly 0.2% of the on-chip pump power. In contrast, the pump line at the weakly coupled drop port is only 10 dB above the neighboring comb lines; while the integrated comb power is very weak, it nevertheless but represents 75% of the total power that is actually coupled into the microresonator! (b) Mode-locked dark pulse from normal dispersion microresonator. The pump exiting the through-port is only a few dB above the neighboring comb lines. The integrated comb output has 209 mW average power, approximately 32% of the on-chip pump power.

In our work on cavity solitons in anomalous dispersion microresonators [11], we studied the intracavity waveform of an on-chip microcavity soliton in a microresonator configured with a drop port. Whereas combs measured at the through port are accompanied by a very strong pump line which accounts for >99% of the output power, our experiments reveal that inside the microcavity, most of the power is in the soliton. Time-domain measurements performed at the drop port provide information that directly reflects the intracavity field. Data confirm a train of bright, close to bandwidth-limited pulses, accompanied by a weak continuous wave (CW) background with a small phase shift relative to the comb. Although most of the power inside the cavity is in the soliton (hence in the comb), most of the power in the pump light passes through the input waveguide without every coupling into the cavity and hence does not efficiently transfer into the comb. Thus, cavity soliton based microcombs that are now widely observed in anomalous dispersion cavities have generally had low conversion efficiency of order one percent and below.

In sharp contrast, we observed >30% conversion efficiency (~200 mW on-chip comb power excluding the pump) in the fiber telecommunication band with broadband mode-locked dark-pulse comb in normal dispersion cavities [3]. These results appear to be unique to normal dispersion cavities. In our paper we present a general analysis on the efficiency which is applicable to any phase-locked microcomb state. The effective coupling condition for the pump as well as the duty cycle of localized time-domain structures play a key role in determining the conversion efficiency. Our observation of high efficiency comb states is highly relevant for applications such as optical communications which require high power per comb line. For this reason other groups have begun to explore normal dispersion combs within the last year, reporting power conversion efficiencies that now exceed 40% [12, 13].

Finally, we have collaborated on a more general comparison of power efficiency in soliton combs vs. dark pulse combs [14]. Our study is relevant to applications such as optical communications and several RF signal processing applications where equal power is desired for each comb line. This means that both the raw conversion efficiency of pump power into the comb as well as the power loss incurred in equalizing the shape of the comb power spectrum play a role. The power advantage of dark pulse combs compared to soliton combs after equalization is reduced but still significant. Our study shows that dark pulse combs and soliton combs obey the same power vs. number of comb lines scaling; and equalized dark pulse combs appear to enjoy an approximately 2 dB advantage over equalized soliton combs.

3. Selected applications

We have also explored the utility of microresonator Kerr combs for potential applications, with an emphasis on radio-frequency (RF) photonics and lightwave communications. As shown schematically in Fig. 3 [15], such applications generally involve following the comb generator with a modulator. In RF photonics, the modulator replicates copies of the electrical data onto each of the comb lines. These multiple copies can be used for a number of purposes, including tapped delay line and other photonically implemented RF filters, feeding antenna arrays for true-time delay beamforming, channelization for sensing of the RF spectrum, analog links and others. In lightwave communications a bank of modulators can be used to impart different data

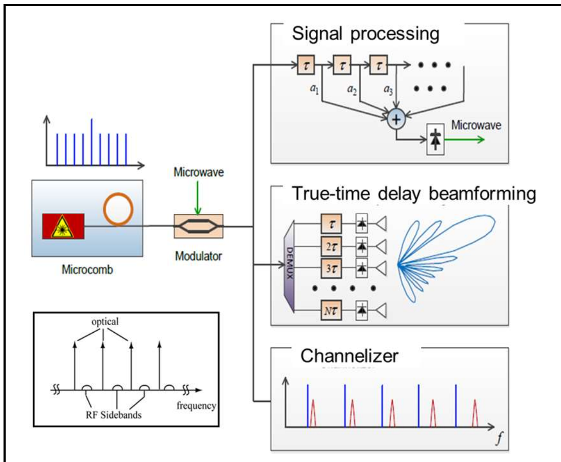

Fig. 3. Schematic illustration of Kerr comb applications in radio-frequency photonics.

signals onto different comb lines, each of which can serve as an independent carrier in wavelength-division multiplexed communication. All of these applications benefit from high power per comb line.

In order to benefit from high power per comb line, we took advantage of high power efficiency combs corresponding to the mode-locked dark pulse operation. In one paper in the RF photonics area, we investigated long haul analog optical links, in which stimulated Brillouin scattering imposes detrimental effects by limiting the optical input launch power [16]. By distributing the optical power among many spectral lines from a Kerr comb, much higher launch power is possible. We also compared the RF link metrics using a soliton or a dark pulse as the sampling frequency comb source. In another study we again used high power efficiency combs corresponding to the mode-locked dark pulse operation, in this case to demonstrate a photonic true time delay beamforming network applicable to broadband phased array antennas [17]. The experimentally demonstrated true time delay beamforming network can support a phased array with 21 elements, over a microwave frequency range of 8-20 GHz, with beam scanning over plus or minus 60 degrees, and is scalable to higher frequencies and a larger number of antenna elements. Furthermore, our work introduces a novel method to achieve both positive and negative weighting of the antenna elements, which enables arbitrary microwave beam pattern control.

We also published two papers, in collaboration with Chalmers University, in which the Kerr comb serves as a multi-wavelength source for wavelength division multiplexed (WDM) optical communications. Experiments using soliton Kerr combs in anomalous dispersion microresonators have already impressively shown the potential of microresonator combs for replacing a multitude of WDM lasers with a single laser-pumped device [18, 19]. However, our group was able to conduct experiments in important regimes not previously investigated. For example, previous demonstrations focused on short-distance few-span links reaching an impressive throughput at the expense of transmission distance. In a first paper we reported the first long-haul coherent communication demonstration using a microresonator-based comb source [20]. We modulated polarization multiplexed (PM) quadrature phase-shift keying-data onto the comb lines allowing transmission over more than 6300 km in a single-mode fiber; to the best of our knowledge, these results represent the longest fiber transmission ever achieved using an integrated comb source. In another experiment conducted over a shorter distance link, we report transmission using a 64-quadrature amplitude modulation format encoded onto the frequency lines of a microresonator Kerr comb [21]. This is the highest order modulation format, equivalent to 6 bits per modulation symbol, demonstrated for Kerr comb sources. An interesting point is that the latter experiments exploit mode-locked dark pulse combs from a normal-dispersion microcavity. Such dark-pulse combs are particularly compelling for advanced coherent communications since they display unusually high power conversion efficiency. The high conversion efficiency of the comb enables transmitted optical signal-to-noise ratios above 33 dB while maintaining a laser pump power level compatible with state-of-the-art hybrid silicon lasers.

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