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Modeling the Formation of Flare Active Sunspots

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14. ABSTRACT

We have performed numerical simulations of the rise and emergence of untwisted toroidal flux ropes and studied the physics of the emergence process. We developed techniques to understand the development of magnetohydrodynamic instabilities responsible for the appearance of some active regions on the Sun. In addition, we modelled the process of energy injection and release in active regions, and analyzed the consequences of this process for the heating of the solar atmosphere. We combined magnetohydrodynamic and thermodynamic modeling in an entirely new and novel manner to study both the processes of magnetic energy storage and release, as well as the solar plasma's response to the magnetic field dynamics.

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EXECUTIVE SUMMARY

This report presents research conducted by Kalman Knizhnik under his Karle's Fellowship tenure at NRL. Active regions on the Sun are responsible for explosive solar phenomena such as flares and coronal mass ejections, which abruptly change conditions in the Earth's upper atmosphere and magnetosphere. This can have a direct impact on Naval operations by degrading communication and radar systems. The ultimate goal of this work is to understand how these explosive phenomena are initiated in order to facilitate better prediction and advance forecasting capabilities. This work therefore addresses the goals of the Department of Defense Space Science and Technology Strategic Plan.

We have performed numerical simulations of the rise and emergence of untwisted toroidal flux ropes and studied the physics of the emergence process. We developed techniques to understand the development of magnetohydrodynamic instabilities responsible for the appearance of some active regions on the Sun. In addition, we modelled the process of energy injection and release in active regions, and analyzed the consequences of this process for the heating of the solar atmosphere. We combined magnetohydrodynamic and thermodynamic modeling in an entirely new and novel manner to study both the processes of magnetic energy storage and release, as well as the solar plasma's response to the magnetic field dynamics.

This work represents a unique and valuable contribution to solar physics because it makes three major advances. Our work represents the first time 1) a lower limit has been placed on the twist required for magnetic fields to rise coherently from the solar interior to the solar surface, 2) the mechanism of active region formation on the Sun has been rigorously quantified, which leads to better understanding of active region dynamics, and therefore of their explosive behavior, and 3) magnetohydrodynamic and thermodynamic modeling have been combined into one coherent model to understand the plasma's response to explosive magnetic events.

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MODELING THE FORMATION OF FLARE ACTIVE SUNSPOTS

1. THE RISE AND EMERGENCE OF UNTWISTED TOROIDAL FLUX ROPES ON THE SUN

An important unsolved problem in solar physics is the origin of active regions observed on the solar photosphere and in the overlying corona. Since these active regions are the source regions for solar flares and coronal mass ejections, understanding their origin and behavior is of critical importance to predicting solar storms. It is generally agreed that active regions, and the sunspots that comprise them, are formed by the emergence of bundles of magnetic field, known as magnetic flux ropes, formed in the solar interior by a dynamo process. The general picture (Parker, 1955) is that once these flux ropes are formed, they become buoyant and start to rise through the solar interior up to the photosphere, the visible surface of the Sun. As each flux rope crosses the two-dimensional surface, its cross section with the photosphere forms a set of sunspots, and as the flux rope continues to rise into the corona, an active region is formed.

One issue with this model is that it has been thought that buoyantly rising flux ropes require a critical twist in order to maintain their integrity against vortical flows generated by viscous forces during their rise (Moreno-Insertis, 1996, Emonet, 1998). These flows act to fragment the flux ropes and halt their rise. Furthermore, even if the flux ropes make it to the photosphere, it has been shown that flux ropes need to be significantly twisted in order to make it through the photosphere, since the buoyancy drastically decreases just above the photosphere, so a large magnetic pressure gradient force is necessary to overcome the weight of the plasma on the flux rope (Murray, 2006). Whether the twist required to rise coherently through the solar interior is the same twist required to cross through the photosphere is an open question. To date, no study has set a lower limit on the twist required to rise coherently through the solar interior and emerge through the photosphere into the corona.

Several studies have loosely addressed this issue by simulating the rise and emergence of weakly twisted flux ropes. Previous authors (Archontis, 2013) performed a numerical simulation of the rise and emergence of a weakly twisted cylindrically shaped flux rope. They found that it was able to rise coherently and emerge through the photosphere, and that its emergence was governed by the magnetic buoyancy, or undular, instability, in which portions of magnetic field lines which are able to emerge through the photosphere are shaped concave down, which allows the plasma on the field lines to drain, enhancing the buoyancy of the magnetic field lines and enabling further emergence. Evidence of this instability during the emergence process has also been seen by other authors (Murray, 2006, Leake, 2013). However, Toriumi, (2011) performed a simulation with the same twist as that used by Archontis, (2013) and found that their flux rope was not able to emerge. Thus, the question of how twisted a flux rope needs to be in order for it to be able to rise coherently through the solar interior and emerge through the photosphere remains open.

In this report, we describe numerical simulations that we performed of the rise and emergence of a completely untwisted toroidally shaped flux rope. Our simulations demonstrate that twist is not required for the coherent rise and emergence of magnetic flux ropes. Furthermore, our simulations show that the emergence itself does, indeed, occur through the action of an undular instability, and we quantify the properties of this instability (Dacie 2016). We argue that the active region resulting from the emergence of this untwisted flux rope bears qualitative similarity to active regions observed on the Sun.

We performed a numerical simulation in which we placed an untwisted, toroidally shaped flux tube in the solar interior. We made it buoyant and allowed it to rise self consistently to the photosphere. The numerical model solved the equations of MHD in three Cartesian dimensions, and we initialized the

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simulation with a stratified atmosphere, in which the density, pressure and temperature increase with depth below the photosphere.

In Figure 1, we show the rise of the toroidal flux rope through the solar interior. The green field lines are traced from the bottom boundary of our simulation, and are plotted in front of two planes, whose color shading shows vertical momentum, while the black contours show out of plane vorticity. As the flux rope rises, it straightens out its legs and flattens out on top. Vortical motions are generated, but they do not appear to have any impact on the integrity of the flux rope. This can also be seen in the cross sectional cut of magnetic field magnitude shown in Figure 2. The cross section of the flux rope expands as it rises, but it is clearly not becoming fragmented.

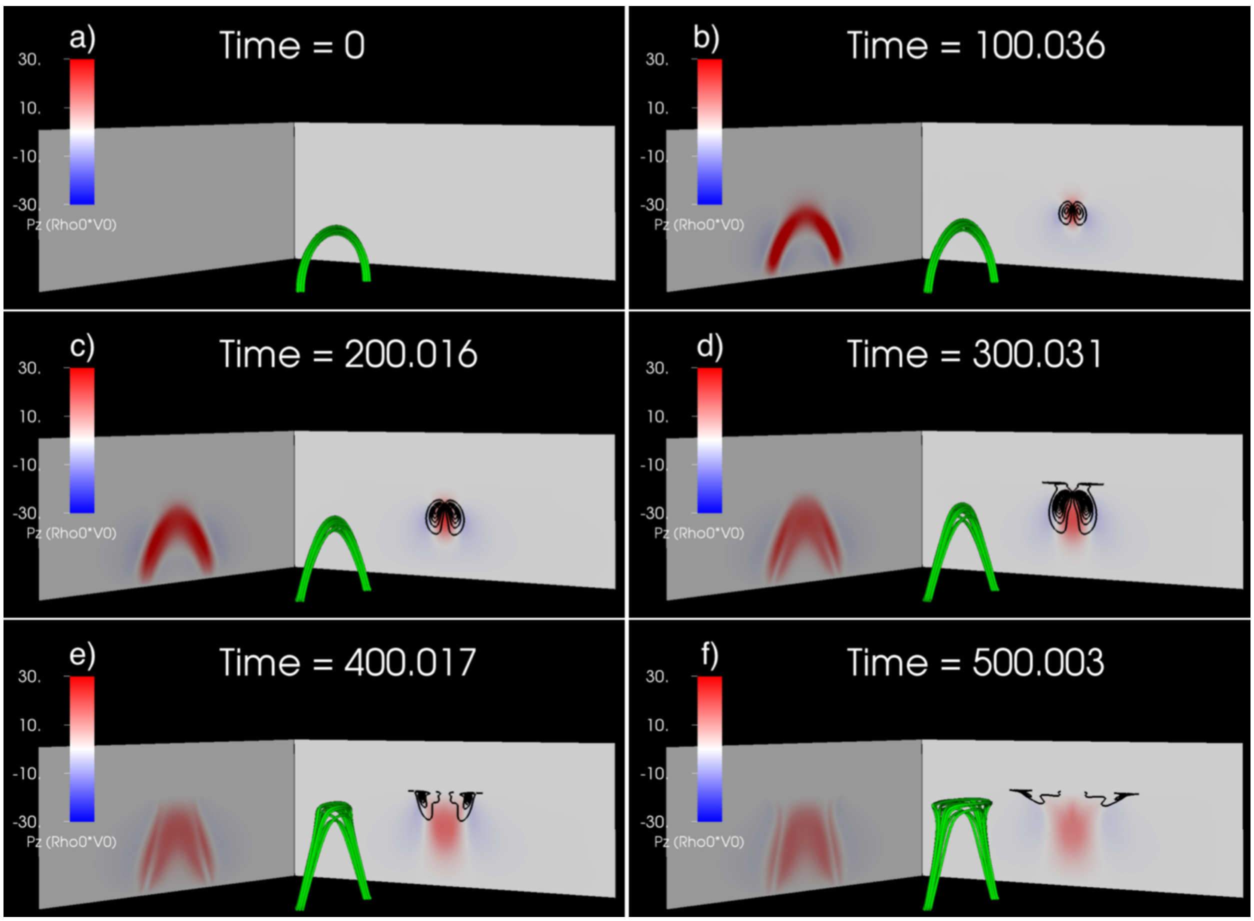

Figure 1 - Field lines in the torus at different times during the simulation, traced from the bottom boundary. Color shading shows vertical momentum, and contours show out of plane vorticity. The two vertical cuts are taken in the x = 0 and y = 0 mid-planes, respectively.

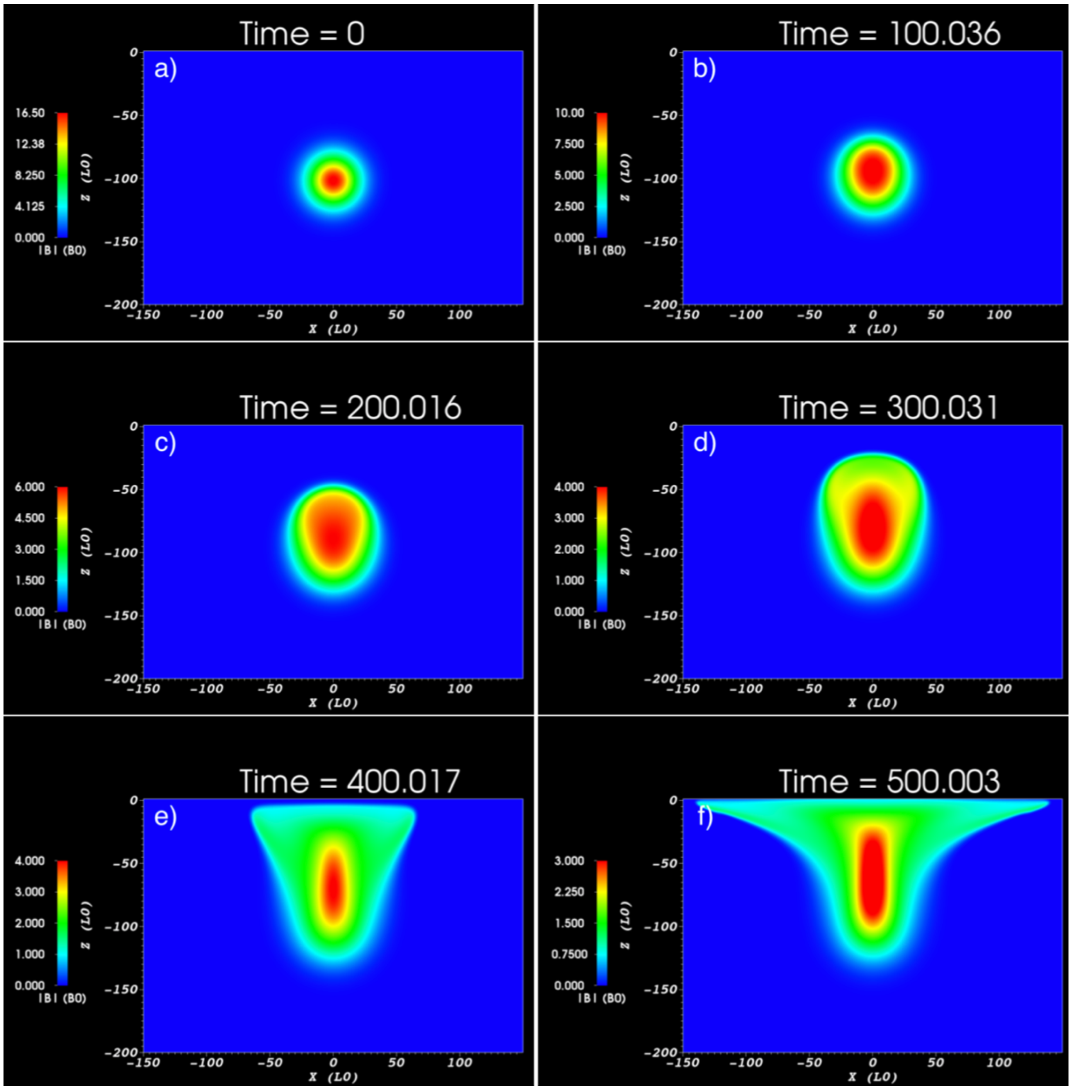

Figure 2 - Contours of magnetic field strength at different times during the simulation in the x-z plane

In Figure 3, we show the vertical magnetic field component on the photosphere at several times during the emergence of our flux rope. Around t = 548, magnetic flux first appears at the photosphere as a pair of triangular shaped opposite polarity regions, enclosed by a circular band of flux. By t = 640, a small perturbation can be seen to have developed between these, resembling a pair of dimples in the photosphere. Around t = 700, the entire region drastically changes, and is overwhelmed by narrow lanes, or dimples, of adjacent positive and negative flux. The primary polarities have disappeared by this time. The narrow lanes have grown significantly, forming many extended dimples on the photosphere. These dimples are characteristic signatures of the undular instability seen by other authors. At t = 760, these dimples have grown in size and are oriented at $\pm 45^{o}$ to the x-y axes. The distance between the two primary polarities is approximately equal to the diameter of the initial toroidal flux rope. In addition, there are bands of flux oriented along the x direction, separating the main polarities. The photospheric magnetic field configuration remains relatively stable over the next several hundred time units, merely separating slightly. One of the features of the photospheric magnetic field seen in Figure 3 which is so striking is the presence of lots of salt-and-pepper features of alternating positive/negative polarity. This is frequently seen in observations, and it is encouraging that our simulations qualitatively matches observational results.

The structure of the active region in the photosphere and overlying corona can be seen in Figure 4, where we have plotted magnetic field lines at several different times during the simulation. In panels a-b, the yellow and red field lines in the middle of the flux rope undulate into and out of the photosphere, which is behavior associated with the undular instability. In panels c-d, a pair of magnetic lobes, evident in the red field lines, form as a result of the growth of the undular instability.

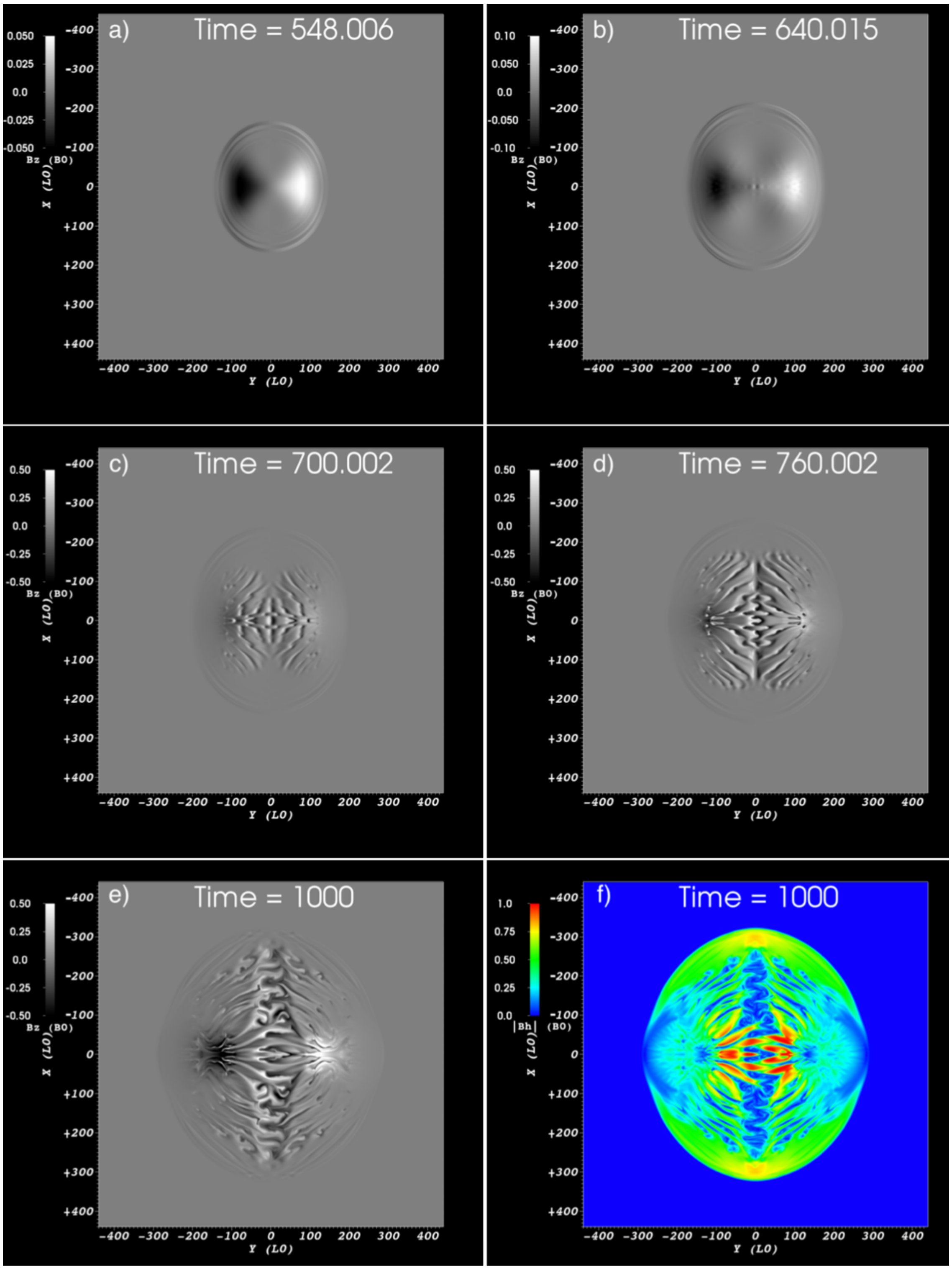

Figure 3 - Vertical component of the magnetic field seen on the photosphere at several different times during the simulation. In the last panel, we show the magnitude of the in-plane component of the magnetic field at the end of the simulation.

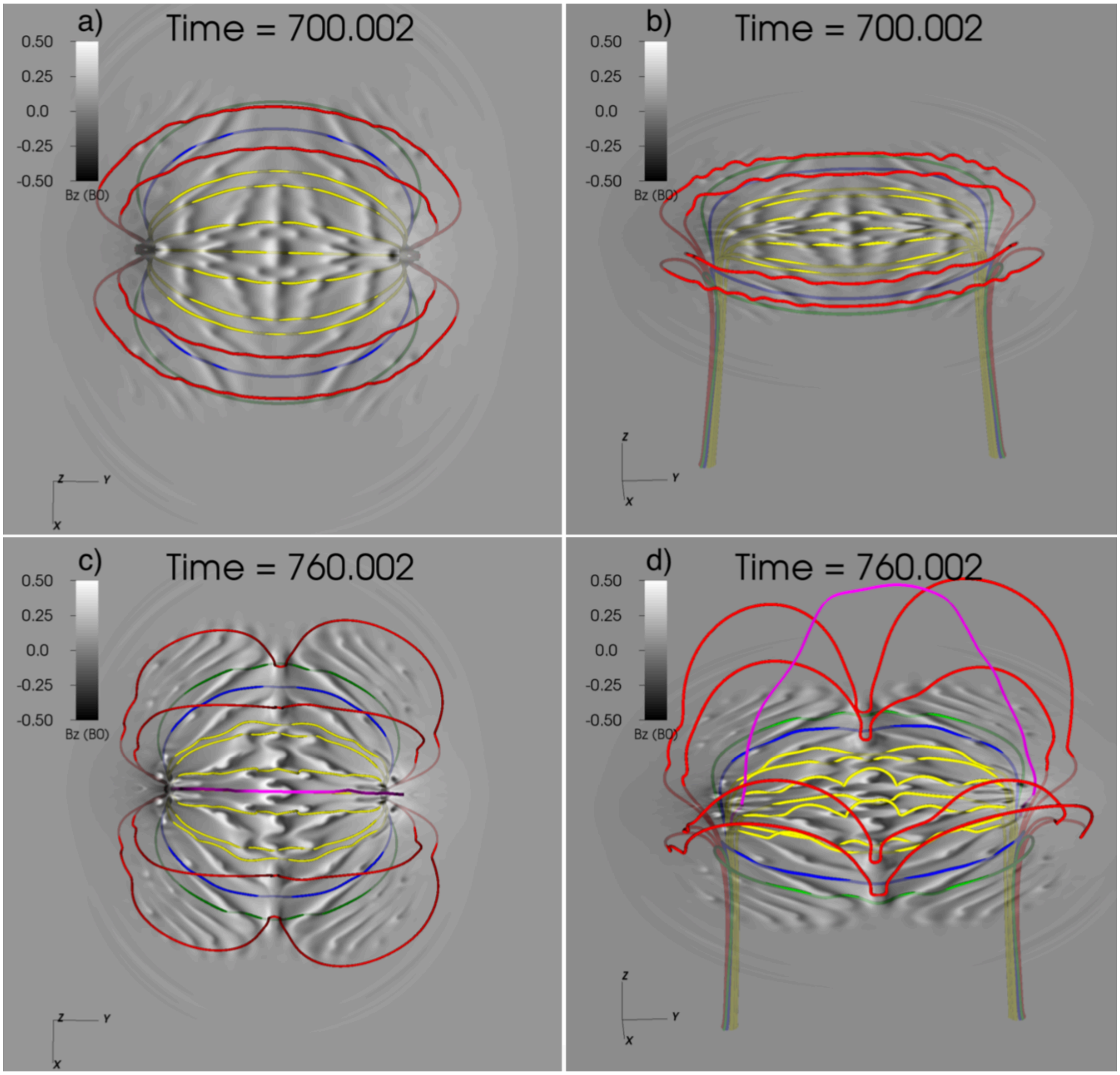

Figure 4 - Field lines, overplotted on maps of the vertical component of the photospheric magnetic field, at various stages of the simulation.

The nature of the undular instability is quantified in Figure 5-Figure 7. In Figure 5 and Figure 6, we take a cut of the vertical component of the magnetic field at the photosphere along two perpendicular cuts. The central peak of each cut was fit to a sinusoid (or cosinusoid), and the wavenumber of the fit is listed in each panel of each figure. The red numbers denote peaks whose amplitudes will be studied in Figure 7. The theoretical wavenumber expected for the undular instability is approximately 0.3 L_0^{-1} (Acheson, 1979, Fan, 2001), which is in excellent agreement with our finding. In Figure 7 we plot the amplitude of each of the three peaks along each cut as a function of time. Each curve is fit to a function of the form $B_z \sim b_0 \exp(\sigma t)$, where σ is the growth rate of the instability, and whose value is shown by the color magnitude in the plots. Theoretically, the instability should have a growth rate of approximately 0.2

t_0^{-1} (Chandrasekhar, 1961, Acheson, 1979, Spruit, 1982, Fan, 2001), also in excellent agreement with our measurement.

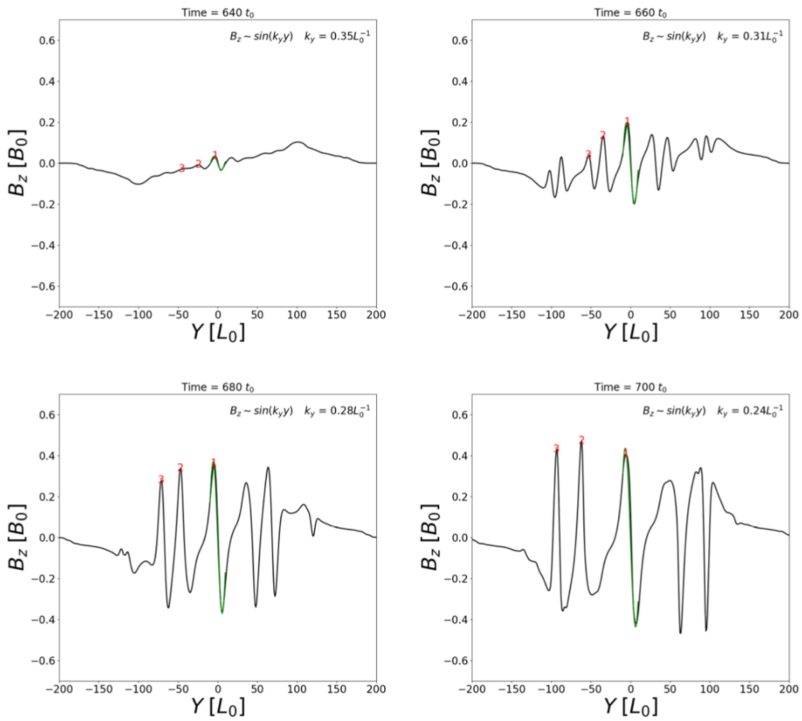

Figure 5 - A cut of the vertical component of the magnetic field taken at x=z=0 at several times during the simulation. The green curve is a portion of a sinusoid fit to the central peak, with fit properties stated in the panel, and the red numbers label the peaks whose amplitude will be studied below in Figure 7.

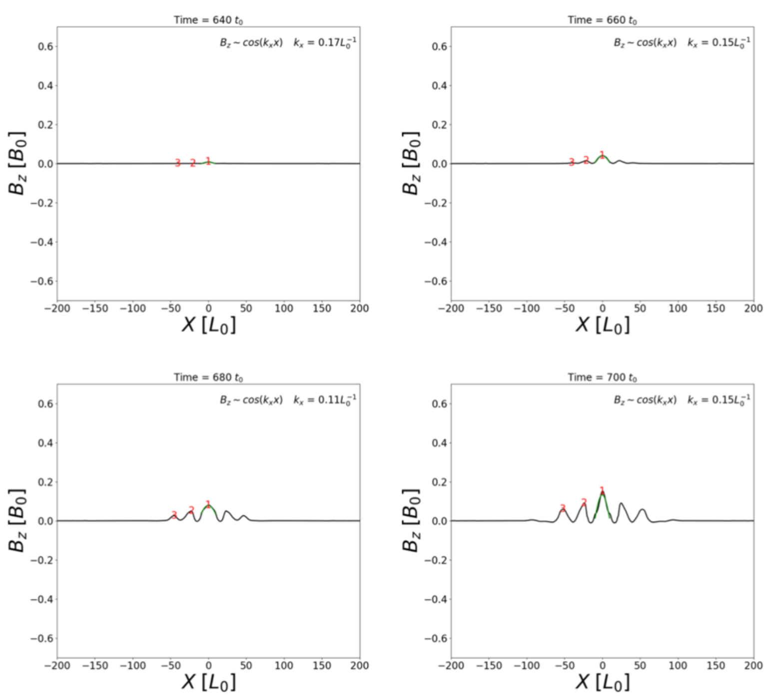

Figure 6 - Same as Figure 5 but at y=z=0.

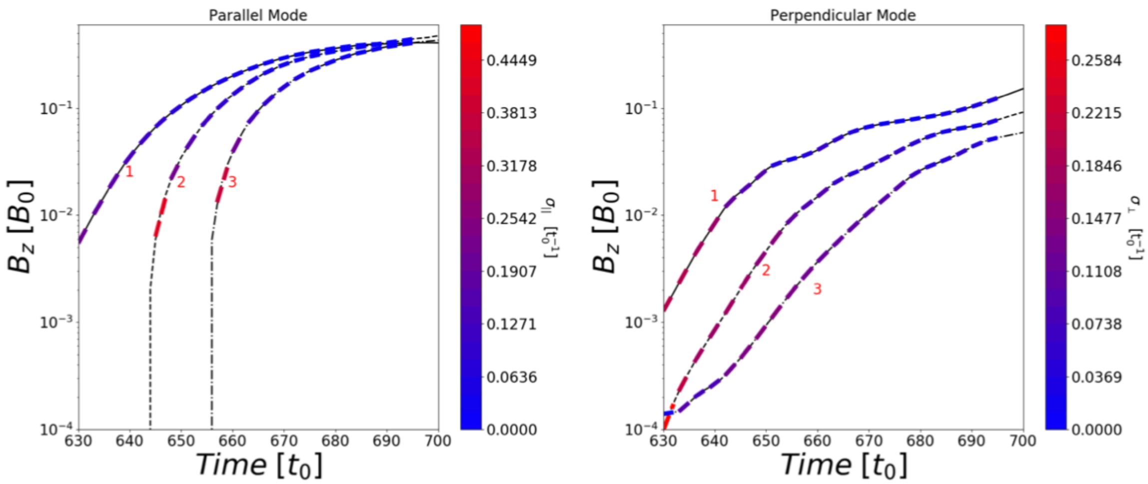

Figure 7 - The amplitude of the three peaks labeled in Figure 5 and Figure 6 as functions of time. The color shading represents the instantaneous growth rate, determined by fitting the data to an exponential function.

In this section of the report, we have summarized our work on the emergence of untwisted toroidal flux ropes. The key finding of our work was that buoyantly rising magnetic flux ropes need not be twisted in order to rise coherently and emerge through the photosphere into the corona. A second crucial finding of our work was that the emergence process proceeds through the growth of the undular instability, which produces salt and pepper features on the photosphere, reminiscent of those seen in solar observations. A major focus of our work was quantifying the fastest growing wavenumbers and growth rates of the instability. This was critical to identifying what instability was occurring, and the fact that our measured values were in excellent agreement with the theoretical predictions for the undular instability gave us confidence that this is the instability that was responsible for the emergence of our flux rope.

2. NANOFLARE DIAGNOSTICS FROM MAGNETOHYDRODYNAMIC HEATING PROFILES

In addition to solar storms, the active regions produced by emerging magnetic flux ropes described in Section 1 are also responsible for heating the solar corona to multi-million degree temperatures. The process by which this occurs is a matter of debate. One of the most widely accepted models of coronal heating is that localized heating events, known as nanoflares, supply enough energy to heat the corona to its observed temperatures. These heating events are thought to be produced by magnetic reconnection, the topological reconfiguration of oppositely oriented magnetic field lines which interact with each other. The process of magnetic reconnection converts magnetic energy into heat, thus producing the hot corona. The magnetic energy stored in the magnetic field, in turn, ultimately comes from the photosphere, where the jostling and braiding of magnetic field lines by photospheric convection injects stress into the magnetic field. It is this stress that is stored and ultimately converted into heating by reconnection.

The details of coronal heating are more complex than this simple picture suggests. The frequency of nanoflares is a critical parameter that needs to be understood in order to explain coronal heating. If nanoflares occur with high frequency, then the coronal plasma has no time to cool, and would be nearly isothermal, in direct contrast with observations, which show a narrow distribution of temperatures peaked around 4 MK (Winebarger, 2011; Warren, 2012) On the other hand, if the frequency of nanoflares is low, then the coronal plasma will cool fully before being reheated, and the observed distribution of temperatures will be too broad to match observations (Bradshaw, 2012, Lopez Fuentes, 2015). Thus, the frequency of nanoflares must be naturally finely tuned in order to match the observed distribution of temperatures.

In this work, we use magnetohydrodynamic (MHD) simulations to study the question of nanoflare frequencies. In particular, we modelled the photospheric jostling and braiding of the coronal magnetic field and did a statistical analysis to determine how much time passes between consecutive reconnection events. In addition, we used the information from the MHD simulation as input to the thermodynamic (TD) EBTEL code (Klimchuk, 2008, Cargill, 2012) to model the response of the coronal plasma to each reconnection event. In this way, we combined information from MHD and TD in a manner that, to our knowledge, has never been done before.

We performed a series of three simulations. The first two were published in Knizhnik, (2020) and the last one has been submitted for publication to ApJ.

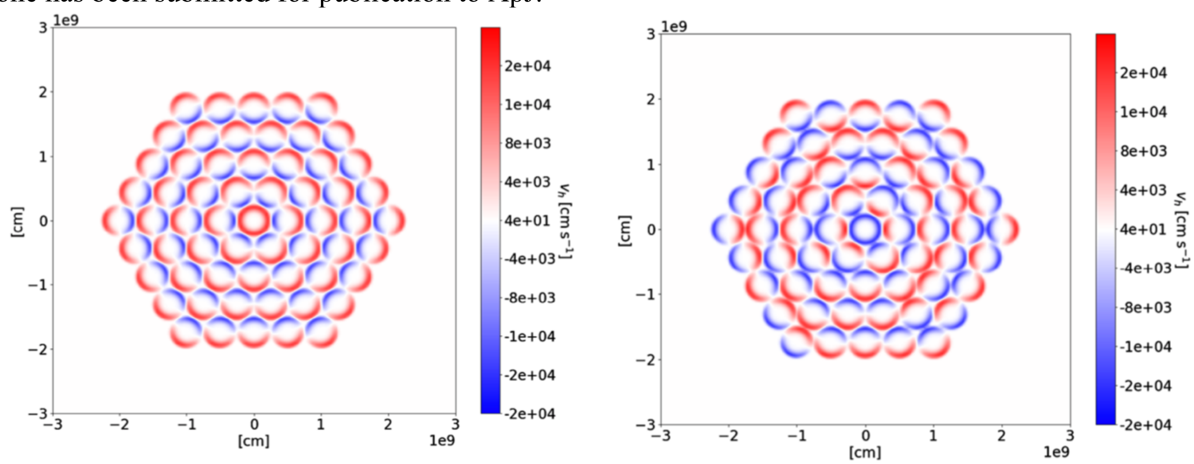

Figure 8 – Azimuthal velocity on the photosphere in the simulations in Knizhnik (2020)

Figure 8 shows the driving profile for the first two simulations. In the first simulation, 61 photospheric vortices rotate clockwise about their respective centers. In the second simulation, 30 rotate counterclockwise and 31 rotate clockwise. The goal of these simulations is to twist the coronal magnetic field, and in doing so, inject energy into the magnetic field, which will be released into plasma heating when it reconnects.

Figure 9 shows the driving profile during two rotation cycles of the third simulation. This time, the entire pattern of vortices rotates about its common center after every rotation cycle of each individual cell. This time, the idea is that, in addition to twisting of the coronal magnetic field, there is also braiding of the magnetic field, adding to the complexity of the evolution.

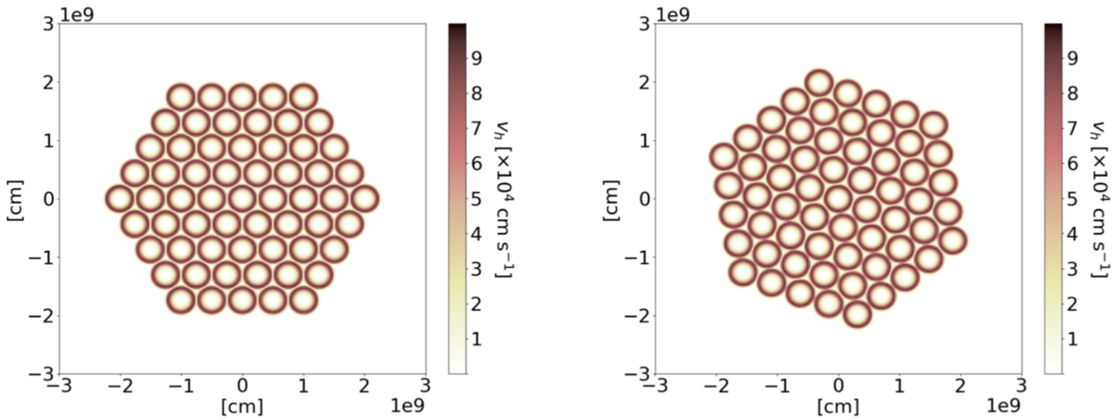

Figure 9 - Azimuthal velocity profile during two rotation cycles of the third simulation

In all three simulations, the photospheric driving is applied at one end of a rectangular box, and the magnetic field at the other end of the box is constrained not to move. For each of the three simulations, we study the frequency of reconnection events by tracing magnetic field lines from the fixed boundary, and looking for locations at the driven boundary where the field lines have moved by a distance greater than the driving velocity times the time increment. Field lines which move by such large distances are identified as having reconnected.

In Figure 10, we show the Poynting flux (the energy injected into the corona) at the photosphere for the two simulations studied in (Knizhnik, 2020). While the energy injected into the corona by the photospheric motions is larger for the simulation with half of the cells rotating in opposite directions, both simulations inject an energy of order 10^7 erg cm^{-2} s^{-1}, which is the canonical energy input constrained by observations that is required to explain the observed heating (Withbroe, 1977). This gives us confidence that our photospheric driver injects enough energy to heat the corona.

In Figure 11, we show the locations of reconnection events at the base of our simulation box at several times during the third simulation that we ran. The white dots represent locations where reconnection has occurred. Knowing where and when these events occur, we calculate their frequency (or, alternatively, time delay) distribution, plotted in Figure 12. This is a key result of our studies, and it shows that the distribution of time delays between reconnection events follows a power law distribution with a slope of approximately -1.3. This is important for coronal heating models that study the response of the coronal plasma to reconnection events. These models often suppose power law slopes of about -2, which is somewhat steeper than we find, but these assumptions are completely ad hoc. The major advance of our model is that it provides a physically motivated basis for this previously ad hoc input to thermodynamic models.

As part of our work, we took representative field lines from our MHD simulation and used them to populate a TD model that studied the plasma’s response to reconnection events. Since our model gives us the time evolution of each field line (whether or not it reconnected at each time step), this information can be used to inform the thermodynamic model.

Figure 13 shows the coronal plasma’s response to reconnection on a field line chosen at random from our simulation. The left panel shows the heating rate, temperature, and density of the plasma on the field line, and the right panel shows the emission measure, which quantifies the amount of plasma at a given temperature. What is evident in this figure is that the heating events take place extremely infrequently. So infrequently, in fact, that the plasma cools fully before the next reconnection event. This is also reflected in the emission measure, which quantifies the amount of plasma at a given temperature. This quantity is integrated over the entire lifetime of the field line, and shows a very shallow slope of around 1.7, showing that the plasma goes through a large range of temperatures during its evolution. In contrast, observations suggest that the slope should be between 2 and 5 (Warren, 2012).

Figure 14 shows the emission measure produced by averaging the emission measure of different combinations of field lines. We try combinations of 25 field lines ("multi-strand"), 350 field lines ("cluster") and all of the field lines in the simulation ("all"). None of these combinations produce an emission measure slope in the range reported in observations.

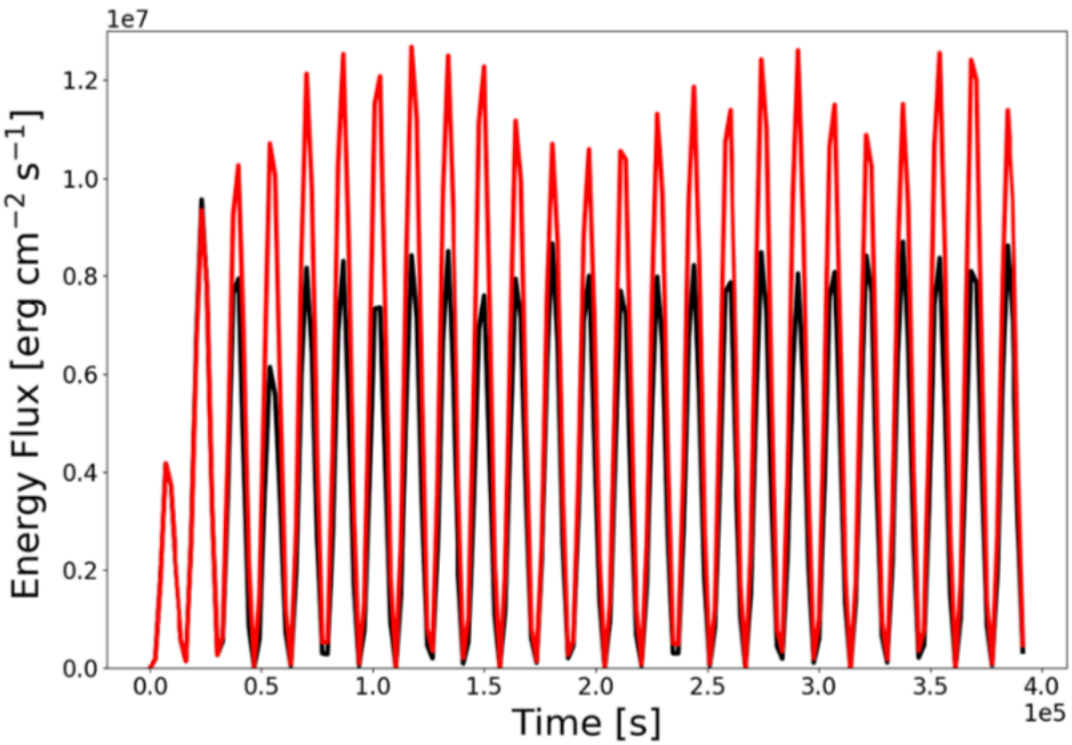

Figure 10 - Poynting Flux injected into the corona by our simulations with all cells rotating clockwise (black) and with half of the cells rotating clockwise and half counterclockwise (red).

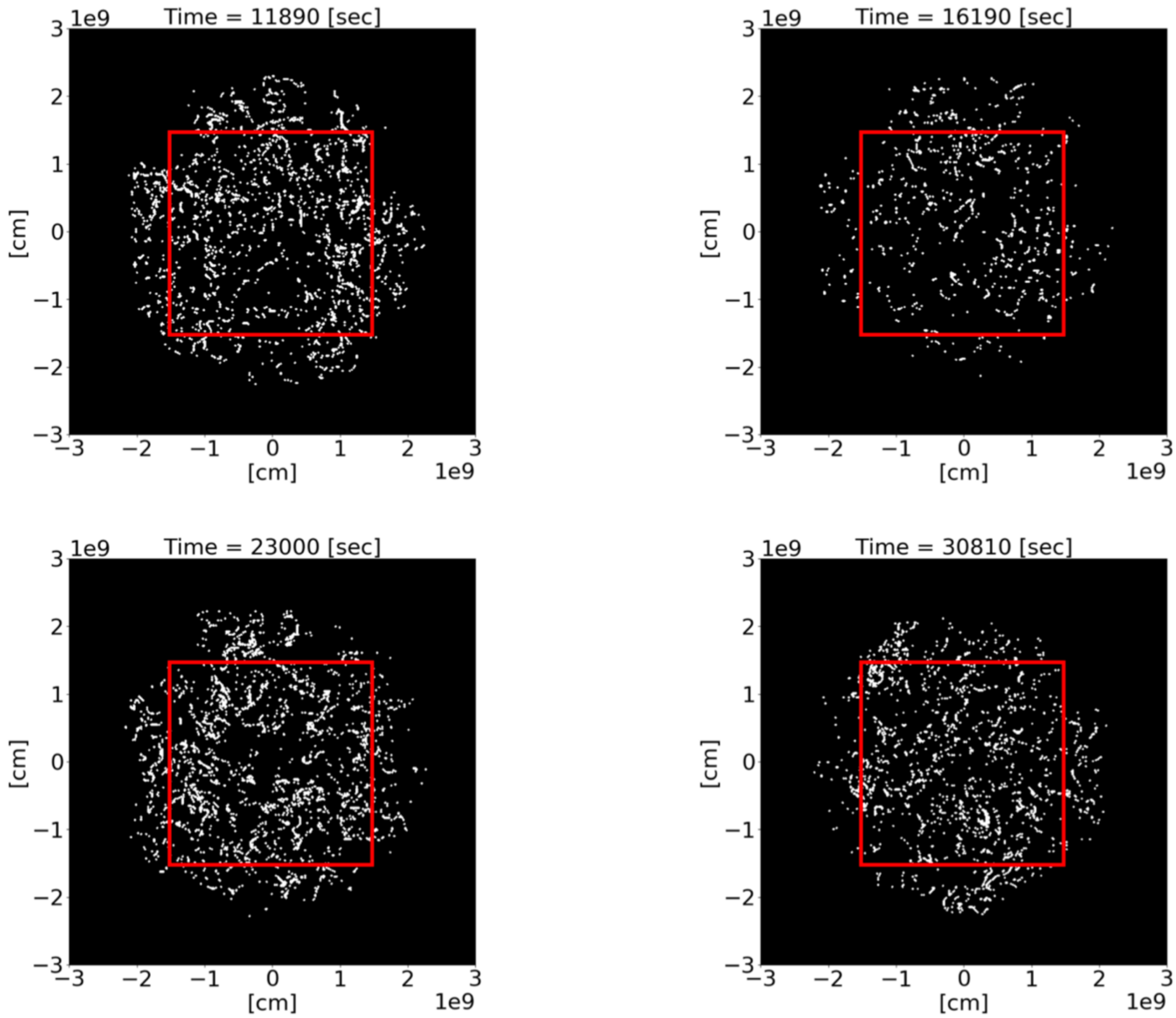

Figure 11 - Map of reconnected field lines at several times during our simulation. The maps show white if reconnection has occurred, and black if there has been no reconnection.

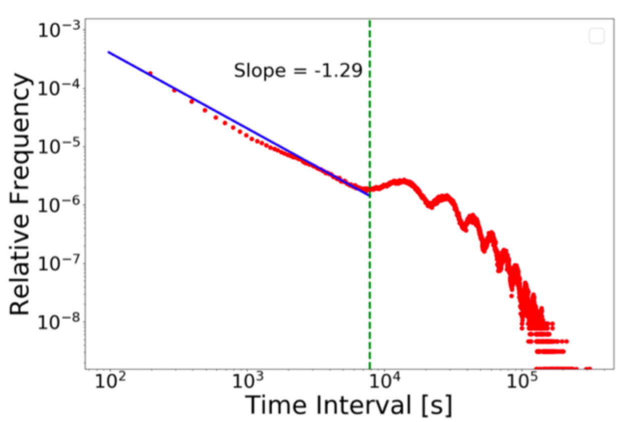

Figure 12 - Distribution of time delays between reconnection events for our simulation, along with a power law slope of best fit.

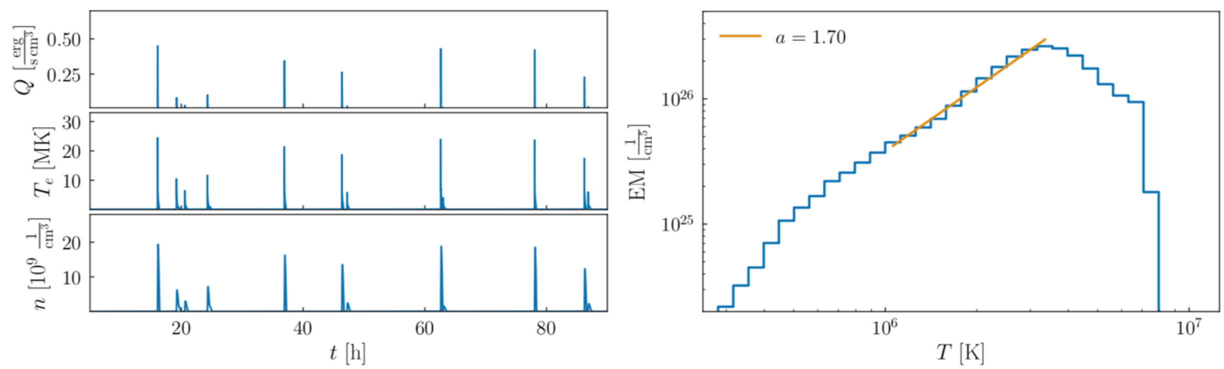

Figure 13 - Left: Heating, temperature, and density profile as determined by EBTEL along a field line chosen at random from our MHD model. Right: Emission measure for this field line.

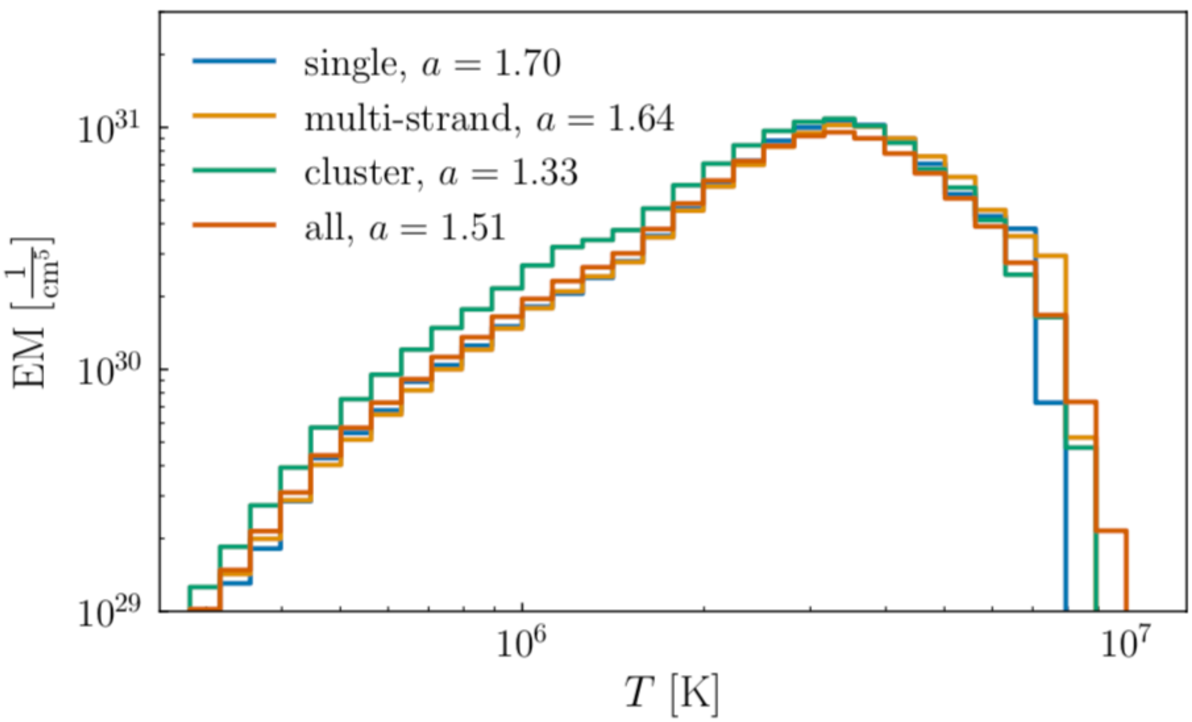

Figure 14 - Emission measure distribution for a single field line, a set of 25 field lines, a set of 350 field lines, and 40000 field lines considered together.

In this work, we combined MHD and TD modeling of magnetic reconnection in an active region to study the heating of the solar corona. We used different types of photospheric driving to twist and braid the coronal magnetic field and induce magnetic reconnection which heated the coronal plasma. We then looked at the temporal behavior of individual field lines and modelled the thermodynamic response of the coronal plasma to individual reconnection events on those field lines.

We found that the photospheric driving that we used was unable to produce magnetic reconnection that was frequent enough to repeatedly heat the coronal plasma prior to its complete cooling. This is an important result because it demonstrates that there is something fundamentally missing from coronal heating models that rely on this type of photospheric driving.

More fundamentally, however, our study represents a tremendous advance in the modelling of coronal heating. To our knowledge, it is the first study that directly used magnetohydrodynamic inputs into thermodynamic simulations. Previously, this was extremely challenging to do because of the vastly different spatial and temporal scales involved in the two regimes. Our study is, mostly, a proof of concept study. Namely, we have shown that it is possible to use magnetohydrodynamic to inform thermodynamic modelling of plasma response to magnetic reconnection occurring in the solar corona. Although we have not been able to generate a plasma response which matches the observed behavior, this presents us with exciting opportunities for future research.

3. CONCLUSIONS

The research conducted during the period covered by Kalman Knizhnik's Karle's Fellowship tenure at NRL saw major advances in the understanding of the formation and behavior of active regions on the Sun. The research modeled the formation of active regions via the rise and emergence of buoyantly rising flux ropes, and quantified the mechanism by which they emerge and evolve. In addition, the research studied the response of the coronal plasma to magnetic reconnection occurring in the coronal magnetic field. Active regions are responsible for explosive solar phenomena which can have a direct impact on the Earth's magnetosphere, impacting communications systems, including HF and VHF radio systems and radar. This work made major advances in understanding how these explosive phenomena are initiated to facilitate better forecasting capabilities. This work therefore addresses the goals of the Department of Defense Space Science and Technology Strategic Plan. Future work will examine different initial configurations of magnetic flux ropes in order to understand how the dynamics observed on the solar surface depend on the properties of rising, emerging flux ropes.

4. ACKNOWLEDGEMENTS

This work is the subject of one published paper that appears in Solar Physics (Knizhnik, 2020) and two that have been submitted for publication to the Astrophysical Journal. The co-authors include Mark G. Linton (Code 7683), Jeffrey W. Reep and William T. Barnes (Code 7680), James E. Leake and Vadim W. Uritsky of NASA Goddard Space Flight Center, and Sally Dacie of Max Planck Institute of Meteorology in Hamburg, Germany.

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