AFRL-AFOSR-VA-TR-2020-0009

Intelligent Distributed Systems

A Stephen Morse
YALE UNIV NEW HAVEN CT

11/15/2019
Final Report

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Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTA2
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
12-11-2019	Final	01-08-2016 --31-07-2019

4. TITLE AND SUBTITLE
Intelligent Distributed Systems

5a. CONTRACT NUMBER
FA9550-16-1-0290

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
A Stephen Morse

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Yale University

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF OFFICE OF SCIENTIFIC RESEARCH
875 NORTH RANDOLPH STREET, RM 3112
ARLINGTON VA 22203-1954

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Distibution A - Approved for public release

13. SUPPLEMENTARY NOTES

14. ABSTRACT

We have invented a distributed algorithm for finding a common fixed point of a family of suitably defined nonlinear functions mapping a finite dimensional space into itself.
Necessary and sufficient conditions for convergence have been derived and several generalizations have been developed. The utility of these findings have been demonstrated by noting their applicability to distributed optimization.

We have developed algorithms for enabling a networked family of autonomous agents to estimate the state of an unforced linear system. All of the algorithms exhibit exponential convergence at pre-assigned rates and some are able to function correctly in the face of changing communications graphs.

15. SUBJECT TERMS
Distributed Control

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			A Stephen Morse
			UU		**19b. TELEPHONE NUMBER** *(Include area code)* 203-641-5320

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The aim of this project has been to develop new tools for analyzing distributed dynamical networks and for controlling them. More specifically, our work has focused mainly on three interrelated-objectives. First, we have sought to develop algorithms for the distributed computation of solutions to systems of equations of all types across a network of mobile autonomous agents. Second we have focused on crafting a variety of algorithms for estimating the state of a linear system whose sensed outputs are distributed across a network. Third we have used graph rigidity theory and nonlinear system theory to develop techniques for autonomously maintaining the correct relative positions of mobile autonomous agents in a large agent network.

We have made important advances in the area of distributed computation. In particular we have invented a distributed algorithm for finding a common fixed point of a family of m suitably defined nonlinear maps M_i from $\mathbb{R}^n$ to $\mathbb{R}^n$ [1, 2]. S common fixed point is computed simultaneously by m agents assuming each agent i knows only its own private map M_i, the current estimates of a common fixed point generated by its neighbors, and nothing more. Each agent recursively updates its estimate by utilizing the current estimates generated by each of its neighbors. Neighbor relations are characterized by a time-dependent directed graph $\mathbb{N}(t)$ whose vertices correspond to agents and whose arcs depict neighbor relations. We have shown that for any family of maps M_i which are "paracontractions" from $\mathbb{R}^n$ to $\mathbb{R}^n$ with a common fixed point and any sequence of "repeatedly jointly strongly connected graphs" $\mathbb{N}(t)$, $t = 1, 2, \ldots$, the algorithm causes all m agents' estimates to converge to a common fixed point of the M_i [1]. These results have been generalized to asynchronous systems in [3] and necessary and sufficient conditions for convergence to a common fixed point have been given in [4]. The effects of limited data transmissions have been examined in [5] and extension of the algorithm to a broader class of maps called "strongly quasi-nonexpansive" has been carried out in [6]

There are many meaningful examples of paracontractions. For example, the orthogonal projection $x \longmapsto \arg\min_{y \in \mathcal{C}} \|x - y\|_2$ on a given closed convex set $\mathcal{C}$ is a paracontraction with respect to the two-norm. So is the gradient descent map $x \longmapsto x - \alpha \nabla f(x)$ where $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a convex and differentiable function, ∇f is Lipschitz continuous with parameter $\lambda > 0$, and α is a constant satisfying $0 < \alpha < \frac{2}{\lambda}$. The proximal map $x \longmapsto \arg\min_{\{} y \in \mathcal{C}\} f(y) + \frac{1}{2}\|x - y\|_2$. associated with a closed proper convex function $f : \mathbb{R}^n \to (-\infty, \infty]$ is another example of a paracontraction. These types of paracontractions typically arise in optimization algorithms.

Our original motivation for studying the problem of computing a common fixed point of a family of maps grew out of our efforts to improve on our earlier work in [7]. The work in [7] is concerned with the development of a distributed algorithm for finding a solution to the linear equation $Ax = b$ assuming the equation has at least one solution and agent i knows a pair of the matrices $(A_i^{n_i \times n}, b_i^{n_i \times 1})$ where $A = \text{block column}\{A_1, A_2, \ldots, A_m\}$ and $b = \text{block column}\{b_1, b_2, \ldots, b_m\}$. While the algorithm in [7] solves the problem, one of its limitations is that it requires each agent to initially compute a solution to its own private equation $A_i x_i = b_i$, since for large A_i such a computation can be daunting. To circumvent this problem we successfully developed a new algorithm with the same convergence properties as the one in [7], which consists of m local update rules of the form $x_i(t+1) = A_i' G_i(A_i x_i(t) - b_i), \;\; i \in \{1, 2, \ldots, m\}$ where G_i is a suitably defined positive definite gain matrix [8]. The affine linear maps $x \longmapsto x - A_i' G_i(A_i x - b_i)$ are paracontractions with respect to the two norm and any common fixed point they have is a solution to $Ax = b$. Over the course of this project we've pursued other questions related to the algorithms in [7] and [8]. For example, necessary and sufficient conditions for exponential convergence are derived in [9] and an asynchronous version of the algorithm in [7] is studied in [10]. A continuous-time gradient-descent version of the algorithm in [8] is developed in [11] and a more efficient version of the algorithm in [7] is proposed in [12]

In the area of estimation, we have developed a number of algorithms for enabling a networked family of $m > 1$ agents to estimate the state of the multi-channel, time-invariant, linear system

$$\dot{x} = Ax \qquad y_i = C_i x, \;\; i \in \{1, 2, \dots, m\} \tag{1}$$

in a distributed manner, assuming that for each i, agent i can sense y_i and, in addition, receive the current state-estimates of each of its neighbors . Neighbor relations are characterized by a directed graph $\mathbb{N}$, which depending on the problem of interest, may be fixed or varying with time. For the case when $\mathbb{N}$ is fixed and strongly connected, we have developed a linear time-invariant distributed observer whose state-estimation errors are guaranteed to converge to 0 exponentially fast at any preassigned rate [13, 14]. More recently we have crafted simplified distributed observers for both discrete and continuous time which are also capable of estimating state exponentially fast at any given preassigned rates [15, 16]. For the case when $\mathbb{N}$ changes with time, we have devised a hybrid observer with an exponential convergence rate which can function either synchronously or asynchronously [17, 18]. An alternative hybrid observer based the observer architecture developed in [15] is currently under development.

A major problem with existing distributed observer techniques is that none can deal with the situation when instead of (1) the goal is to estimate the state of the multi-channel system with inputs; i.e.,

$$\dot{x} = Ax + \sum_{i=1}^{m} B_i u_i \qquad y_i = C_i x, \;\; i \in \{1, 2, \dots, m\} \tag{2}$$

We have made a major breakthrough along these lines by figuring out how to modify the observer developed in [14] *to handle this situation provided each input u_i is a state feedback law of the form* $u_i = F_i x$ [19]. This has resulted in what we believe is the first distributed feedback control system capable on stabilizing *any* jointly controllable, jointly observable multi-channel linear system of the form (2) with time-invariant distributed feedback control provided the associated communication graph is strongly connected. This finding contrasts sharply with well-known classical result [20] for the decentralized control of (2) which states that no matter what linear time-invariant decentralized controls are applied, the spectrum of the resulting closed loop system will contain a uniquely determined set of eigenvalues called the system's "fixed spectrum" [21, 22]. This means that if the fixed spectrum of such a system contains an unstable eigenvalue, then stabilization with decentralized control is impossible whereas the introduction of communication between agents enables stabilization with distributed control.

In the area of formation control we have extended to encompass three dimensional formations, our earlier discovery in [23] of robustness issues with rigidity based formation control of two dimensional systems [24]. We have also extended the results in [23] to formations with more realistic double integrator dynamic agent models [25]. Finally we have developed a technique for maintaining a rigidity based undirected formation when there are measurement biases [26].

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