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A Survey of Methods for Estimating Pulse Width and Pulse Repetition Interval

by Kenneth Ranney and Kwok Tom

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A Survey of Methods for Estimating Pulse Width and Pulse Repetition Interval

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Contents

1. Introduction

Researchers at the US Combat Capabilities Development Command Army Research Laboratory recently performed a survey of pulse-identification techniques presented in the open literature. This effort represented the first step in the development of a capability to identify radar pulses without access to a priori information about the transmitting system. The final objective is to monitor a suspected radar's behavior and note changes that might indicate an increased (or decreased) threat level. For example, a change in pulse width (PW) and/or pulse repetition interval (PRI) could signal a change from an acquisition mode to a track mode. If a vehicle under radar surveillance were to recognize this change, it could implement appropriate countermeasures.

Many different approaches have already been proposed, some attempting to characterize pulse modulation and others attempting to characterize only PW and PRI. The majority of the modulation-estimation algorithms focus on classifying communications signals. The ones adapted for radar, however, usually require the preliminary determination of the radar's carrier frequency. Some even require the presence of only the signal and noise samples from a single radar system. In these cases, a preprocessing stage must be included to extract the pulse data input to the modulation estimator.

In what follows, we summarize many representative approaches to radar pulse detection and characterization found in the open literature. We restrict attention in this report to PW and PRI estimation algorithms, including (as part of PRI estimation) the pulse de-interleaving problem. A sister report addresses the identification of intra-pulse modulations present within a given pulse sequence. We begin in Section 2 with techniques for directly estimating PW. Section 3 describes representative approaches for estimating PRI based on autocorrelation and histogram techniques. Section 4 outlines other PRI estimation approaches based on detection statistics (features) and methods associated with them. These include clustering techniques and network-based approaches. A list of references is included at the conclusion of the report.

2. Direct PW Estimation

An accurate pulse detector and PRI estimator constitute critical parts of any pulse characterization algorithm. These pulse detection schemes can operate in the time domain, in the frequency domain, or in both via a spectrogram or other time–frequency transform. They must all, however, first reduce their processed spectrum to a band centered about a particular carrier frequency. If the detector functions agnostically (i.e., without a priori information about the radar system), this carrier frequency must be extracted via preliminary surveys of the electromagnetic (EM) environment. Thus, the input data for all algorithms consist of a stream of pulses that occupy some bandwidth and exhibit either constant or variable PRIs. In the most general case, the PW and intra-pulse modulation may also vary.

Many authors present approaches that operate directly on the time–domain pulse sequence. An appealing and intuitive approach incorporates a moving average filter and attempts to identify the transitions between low and high levels within the filter's output stream.[1–5] While Adam et al.[3] describe the most straightforward implementation of a moving average of measured power, their methodology would not be readily transferable to an operational system. Ahmad et al. [4] employ a global threshold, based on the maximum amplitude of the smoothed (filtered) pulse, to extract pulse information. Their technique could be realized in an operational system. Several authors[1,2,5] describe well-formulated systems exploiting the difference among outputs of spatial filters that have been displaced in time. Their approach is based on edge-detection concepts, and their difference of boxes reduces to a slightly modified application of a Haar filter. Rather than computing a bipolar filter output directly, Fan et al.[5] calculate normalized moving averages within two boxes in an effort to simplify the threshold selection process. Their final edge detection statistics are defined as

$$y(k)_{rising} = \left\lfloor \frac{\sum_{u=k-N+1}^{k} |S(u)+N_\sigma(u)|}{\sum_{v=k-2N}^{k-N-1} |S(v)+N_\sigma(v)|} \right\rfloor \tag{1a}$$

$$y(k)_{falling} = \left\lfloor \frac{\sum_{v=k-2N}^{k-N-1} |S(v)+N_\sigma(v)|}{\sum_{u=k-N+1}^{k} |S(u)+N_\sigma(u)|} \right\rfloor \tag{1b}$$

where N is the size of each box, k is the test index, and $S(u)$ and $N_\sigma(u)$ represent, respectively, the signal and noise components of the measured pulse data. Note that, since the summation runs from $k - 2N$ to k, the algorithm will detect a rising or falling edge after a lag of N samples. It should be noted this technique can be applied either in the time domain or in the frequency domain (e.g., in conjunction with a short-time Fourier transform [STFT] to locate the edges of the signal band). Liu et al.[2] outline just such an approach that combines the STFT and edge detection to extend PW estimation accuracy to lower signal-to-noise ratios (SNRs). Here, an additional frequency-domain constant false-alarm rate algorithm must be applied to extract the pulse's spectral content.[2,5] We note this modified approach could, potentially, enable detection of multiple, simultaneous pulses if they are sufficiently separated in frequency.

3. Autocorrelation and Histogram-Based PRI Estimators

Other popular time-domain algorithms for pulse characterization leverage various auto-correlation techniques.[6–10] These approaches have been known for some time,[6] and they often form part of a larger PRI and/or pulse estimation strategy. For example, Schmidt[6] considers the un-normalized autocorrelation of a thresholded pulse sequence (i.e., comprising ones and zeros), and transforms it into the PRI domain. This representation assigns a single value summarizing the number of pulses observed with each particular pulse spacing. Hence, both multiple PRIs and harmonics of the finer-spaced PRIs would both be evident. Nelson[7] compiles several correlation functions designed to track radar video sync pulses. Among these, the cross-power spectrum estimator seems to be the most promising—in particular for a single, non-jittered PRI. Chan et al.[8,9] perform an initial auto-convolution of the pulse envelope to get a coarse estimate of the pulse location. They next perform convolutions of the entire pulse with the left and right halves of the pulse to get a finer estimate of PW and time of arrival (TOA). While they could extend the technique to obtain an estimate of the PRI, they do not discuss this potential application. The approach, although interesting, requires estimates of additional parameters and may not be practical to implement. Nishiguchi and Kobayashi[10] use the autocorrelation function as a baseline and define a complex PRI transform,

$$D(\tau) = \sum_{n=1}^{N-1} \sum_{m=0}^{n-1} \delta(\tau - t_n + t_m) \exp(2\pi i t_n / t_n - t_m)$$

which for a single pulse train with PRI p becomes:

$$D(\tau) = (N-1)\delta(\tau - p)\exp(i2\pi\eta) + \sum_{l=2}^{N-1} \delta(\tau - lp) \frac{\sin(N\pi/l)}{\sin(\pi/l)} \exp\left(\frac{\pi i(N-1+2\eta)}{l}\right), \qquad (2)$$

versus the expression for the autocorrelation function:

$$\boldsymbol{C(\tau) = (N-1)\delta(\tau - p) + \sum_{l=2}^{N-1} (N-l)\delta(\tau - lp)} \qquad (3)$$

Here, $t_n = (n + \eta)p,\ n = 0, 1, 2, \dots N-1$ are the pulse arrival times, and η is a constant used to define the phase of a pulse train. This phase is defined by $\theta = 2\pi\eta \bmod (2\pi)$, which for the single pulse train with PRI $p = t_n - t_{n-1}$ becomes

$$\theta = (2\pi t_n / (t_n - t_{n-1})) \bmod (2\pi)\ , \qquad (3a)$$

providing a link back to the value of η. Plots of $D(\tau)$ and $C(\tau)$ are provided in the paper for a sequence of pulses with three different PRIs, and they indicate the effectiveness of the approach. The authors next note that severe problems arise when the PRIs experience only a small amount of jitter. They proceed to present a modification to address the problem and evaluate it using simulated data. Results are promising, but are not compared with results from other algorithms also designed to address the jitter problem. These alternative approaches attack the problem in stages, rather than attempting to eliminate errors as part of an initial data transformation.

After an algorithm has grouped pulses into candidate PRIs, it often performs additional processing to eliminate errors due to aforementioned timing jitter and other effects. Histogram-based techniques represent powerful tools for addressing this problem, and many authors have incorporated them into their PRI-estimation paradigms.[11–16] This is particularly true for algorithms relying exclusively on TOA to perform pulse de-interleaving. And, the authors of several works[11, 13–16] also begin with a thresholded pulse sequence, consisting of only 1s and 0s. They next calculate histograms of the sequential differences between TOAs, observing that this output corresponds to an autocorrelation output. For a single (perfect) pulse stream with PRI = p, the histogram and autocorrelation function will have peaks at kp where k is a positive integer. Similarly, if two pulse streams with PRI = p are interleaved with offset = q, the histogram will have large peaks at kp, and smaller peaks at kq and $k(p - q)$. To increase computational efficiency, Mardia[11] introduces a procedure for calculating the cumulative difference histogram, as follows:

1) Calculate the histogram of the difference between adjacent TOAs. That is calculate $t_{i+n} - t_i$ for t_i , t_{i+n} elements of the TOA sequence, and $n = 1$.
2) Identify differences with histogram values above a threshold.
3) Tag this as a PRI value and eliminate those samples from future consideration.
4) Repeat Steps 1–3 for $n \geq 2$ until all samples are exhausted or a stopping criterion has been satisfied.

This author also incorporates a weighted two-pass difference to deemphasize widely spaced, short bursts of contiguous pulses. That is, given a common PRI, the weighting scheme would favor a longer burst of contiguous pulses followed by a longer contiguous non-transmission interval over multiple, short bursts of contiguous pulses interspersed with shorter “dead” intervals. Milojevic and Popovic[13] introduce methods for determining optimum threshold values and guarding against the selection of PRI harmonics instead of the true PRI. By considering a Poisson distribution, they obtain an exponential form for the threshold as a function of histogram bin. Xi et al.[14] modify the approaches to the sequential search and threshold adjustment presented in earlier work and present results obtained using simulated data. Their simulation included three radar systems transmitting different PRIs—two of the PRIs constant and one of them jittered by 5% to 10%. Liu and Zhang[15] describe the sequential difference approach and its shortcomings in some detail. They then propose a clustering scheme to raise histogram values that are artificially low due to jitter. They also incorporate the PRI transform presented by Nishiguchi and Kobayashi.[10] Finally, Ge et al.[16] introduce the Multi-Level time-difference of arrival (TDOA) histogram. They describe both the algorithm and the threshold selection method in detail, providing both pseudocode and flowchart depictions of the algorithm flow.

Many of the above approaches attempt to exploit specific characteristics of the pulse stream to estimate other characteristics, in particular PW and/or PRI. These observations can then be used to associate a set of pulses with a potential threat radar. Some approaches, however, exploit multiple statistics (available from the receiver system) to calculate a set of features.[17–23] These features then serve as input to a PRI estimator or, in some cases, an emitter classifier. In some cases, the data stream itself serves as the input and the decision network extracts the features necessary to perform PRI estimation.

4. Feature-Based PRI Estimators and Clustering Techniques

Song et al.[17] note that a critical piece of the PRI estimation problem resides in correct identification of PRI modulation. They consider TOA estimates and define a difference of PRIs (DPRI) to be

$$d(i) = (t_{i+2} - t_{i+1}) - (t_{i+1} - t_i),$$

where t_i represents the time of arrival of pulse i. Note that some sort of preliminary processing is required to determine these values.

The DPRI sequence is then transformed into a symbol sequence, $s_d(i)$, according to,

$$s_d(i) = \begin{cases} 0, d_{min} \le d(i) < -\varepsilon \\ 1, \quad -\varepsilon \le d(i) < +\varepsilon \\ 2, +\varepsilon \le d(i) < d_{max} \end{cases}$$

This symbol sequence then constitutes the input to multiple feature calculators, each determining whether the PRI belongs to a specific modulation class. The first of these calculates the Shannon entropy of the symbol sequence, and this feature detects if jitter is present. The second calculator determines the sample kurtosis and uses it to segregate out “wobulated” modulation. The final feature uses normalized second-order sample moment to distinguish between constant PRI and PRIs that monotonically increase or decrease across a specified time interval.

Liu and Cui[18] create a feature vector comprising TOA, carrier frequency, PW, and direction of arrival (DOA). They then cluster these features using an adaptive approach that does not require prespecification of the number of clusters. Each of the output clusters then represents a radar with the indicated pulse parameters.

Guo et al.,[19] like Song et al.,[17] also use higher-order statistics as well as Shannon entropy. They also assume that noisy radar signals can be modeled as fractals, exhibiting fractal geometric patterns. Hence, they include fractal dimension as part of the feature vector. This vector is then used as input to a K-nearest-neighbors clustering algorithm, and the output clusters indicate emitters with designated characteristics. Wilkinson and Watson[20] employ a 2-D clustering technique based on both DOA and carrier frequency. A table of candidate emitters is maintained, and the distance between table entries and the new data samples is maintained.

Several authors[21–23] also describe clustering techniques of different types. Scherreik and Rigling[21] outline a Bayesian approach that requires no prespecification of the number of clusters. It adjusts the number of clusters dynamically based on the Chinese Restaurant Process and its critical parameter α. Here, the probability that a new data sample is assigned to specific cluster is defined by

$$p(z_n = k|\mathcal{Z}_{\cancel{n}}) \triangleq \begin{cases} \dfrac{{}_kN_{\cancel{n}}}{\alpha + N - 1}, k = 1,2, \dots, K \\ \dfrac{\alpha}{\alpha + N - 1}, k > K \end{cases}$$

where, $\mathcal{Z}_{\cancel{n}}$ is the set of clusters with at least one element without counting element n, and ${}_kN_{\cancel{n}}$ is the count of cluster elements in each cluster, without counting element n. To improve adaptability and speed of execution, the authors define a "minibatch" to be a subset of the entire data stream that is buffered and processed by the clustering routine. Monte Carlo results are presented to illustrate 1) a decrease in the execution time relative to an adaptive approach and 2) an increase in performance (with a slight increase in run time) relative to the standard K-means approach. Liu et al.[22] use the minimum description length to group predefined feature vectors into clusters. Here, some knowledge of pulse location is necessary, because feature vectors are created by sampling the complex data at regular intervals, T. If pulses are present, then these samples are assumed to come from them. Ata'a and Abdullah [23] also define features based on outputs from a preliminary detection algorithm to segregate pulses into sets of signals transmitted by different radars. The first stage of their detection procedure leverages a well-documented clustering technique, the Fuzzy ART network, to perform a de-interleaving operation. The feature vector for this stage comprises quantities extracted from the pulse description words (PDWs) of candidate pulses, namely the DOA and the signal RF information. After pulses have been de-interleaved (clustered), the resulting cluster elements are further processed to obtain PRI estimates.

Tang et al.[24] perform initial preprocessing in an attempt to identify missing pulses, and they are the only ones within this survey to address the problem in this way. Their heuristic, however, may introduce additional problems under more general operating conditions. They define three physics-based, yet intuitively pleasing, features based on differences between estimated PRIs. That is, the authors denote the detected PRI sequence by

$$F(n) = \begin{cases} \mathrm{PRI}_1, n = 1, 2, \cdots, N_1 \\ \mathrm{PRI}_2, n = N_1 + 1, \cdots, N_1 + N_2 \\ \vdots \\ \mathrm{PRI}_m, n = N_{m-1} + 1, \cdots, N_{m-1} + N_m \end{cases},$$

where PRI_1, $\mathrm{PRI}_2 \cdots \mathrm{PRI}_m$ represent the m values in a PRI sequence, while N_1, $N_2 \cdots N_m$ represent the number of pulses. They then define their features using the PRI difference sequence $D(n) = F(n+1) - F(n), n = 1, 2, \cdots, N-1$.

These features are defined as

1) $f_1 = \frac{\mathrm{Num}(D(n) \cdot D(n+1) < 0)}{N-1}, n = 1, 2, \cdots, N-2,$ where $\mathrm{Num}(\cdot)$ indicates the number of n that satisfy the condition in the parentheses.

2) $f_2 = \sum_{n=0}^{N-1} s^2(n)/(N-1)$, where

$$s(n) = \begin{cases} -1, D(n) < -\varepsilon \\ \quad 0, |D(n)| \leq \varepsilon \\ +1, \ D(n) > \varepsilon \end{cases}$$

3) $f_3 = \sum_{k=1}^{N-1} \frac{Sp(k)}{t(k)}$, where

$Sp(k) = \sum_{n=1}^{k} \frac{s(n)}{N-1}, k = 1,2 \ldots, N-1$ and

$$t(k) = \begin{cases} k, k \geq 0 \\ 0, k < 0 \end{cases}$$

The authors select a decision tree to decide which PRI is present in the input data stream. Their test data include various PRI configurations, but they do not include any examination of the effects of SNR.

Cain et al.[25] and Li et al.[26] both use neural networks (NNs) to address slightly different problems. Cain et al.[25] first define a 3-D feature vector based on the natural logarithms of the PW, the PRI and the (RF) carrier frequency of each detected pulse in a sequence. These features are first normalized to the interval [0,1]; they then serve as inputs to a convolutional NN (CNN). Since this method uses supervised learning, it requires both full knowledge of the emitter types to be encountered and a large amount of training data. The approach of Li et al.,[26] also based on the CNN, is subject to the same limitations as the approach of Cain et al.[25] Here, however, the authors only attempt to identify certain PRIs by inputting the raw data stream. That is, the authors do not attempt to extract specific features to use as input.

While searching for various PW and PRI estimation strategies, we also uncovered some techniques that did not fit nicely into the broad categories described previously. One of these approaches considers a relatively short Fast Fourier Transform and directly analyzes the frequency domain representation of a pulse chain.[27] The authors essentially perform a STFT and evaluate the spectrum at each location. When multiple pulses are present within the STFT window, then estimates of both PW and PRI can be obtained based on the spectral content of a rectangular pulse train. A preliminary two-filter input stage attempts to improve the system response, based on an assumed range of pulse parameters. A constant PRI was assumed throughout the duration of the STFT (five PRIs). Another approach modelled the received signal as the sum of independent contributions (signals) from multiple sources.[28] This enabled the author to analyze the interpulse covariance matrix, and apply information theoretic concepts to estimate the number of emitters without having to first estimate and associate PRIs.

5. Summary and Conclusion

We have provided a brief overview of various techniques available for estimating PW and PRI. We began with methods for directly estimating PW through the implementation of specialized finite impulse response filters. Following that, we presented autocorrelation-based algorithms that could provide a PW estimate as a byproduct of estimating PRIs. Some authors observed how methods based on histograms of TOA differences (i.e., potential PRI values) were similar to certain autocorrelation formulations, and they described various adaptations of the histogram paradigm. Some authors formulated specialized transforms to highlight the presence of PRIs and perform a degree of de-interleaving.

Some authors exploited TOA differences and other properties of the pulse sequence to create vectors of detection features. These features, when properly normalized, could then be input to clustering algorithms to associate pulses assumed to originate from the same radar system. Other feature-based approaches included NN implementations—one CNN operating on a predefined feature set, while another operated on the raw input data.

We have provided citations for all of the algorithms described in this report. These references also include additional citations for the interested reader to expand horizons even further. This is by no means a comprehensive list; our intention has been to provide an introduction that could guide the initial steps into a rich area of current research.

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List of Symbols, Abbreviations, and Acronyms

2-D	2-dimensional
3-D	3-dimensional
CNN	convolutional neural network
DOA	direction of arrival
DPRI	difference of pulse repetition intervals
EM	electromagnetic
NN	neural network
PDW	pulse description word
PRI	pulse repetition interval
PW	pulse width
RF	radio frequency
SNR	signal-to-noise ratio
STFT	short-time Fourier transform
TDOA	time-difference of arrival
TOA	time of arrival

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