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12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

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a. REPORT	b. ABSTRACT	c. THIS PAGE			Samuel Burden
UU	UU	UU	UU		19b. TELEPHONE NUMBER 206-221-3545

RPPR Final Report
as of 05-Aug-2019

Agency Code:

Proposal Number: 68303EGYIP **Agreement Number: W911NF-16-1-0158**

INVESTIGATOR(S):

Name: Samuel Burden
Email: sburden@uw.edu
Phone Number: 2062213545
Principal: Y

Organization: **University of Washington**
Address: Office of Sponsored Programs, Seattle, WA 981959472
Country: USA

DUNS Number: 605799469 EIN: 916001537
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Final Report for Period Beginning 01-May-2016 and Ending 30-Apr-2019
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Begin Performance Period: 01-May-2016 **End Performance Period:** 30-Apr-2019
Report Term: 0-Other
Submitted By: Samuel Burden Email: sburden@uw.edu
Phone: (206) 221-3545
Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 4 **STEM Participants:** 7

Major Goals: We propose to develop generalizable tools for deriving and applying simple models that provide accurate quantitative predictions for dynamic locomotion and self-manipulation behaviors executed by robots and animals. We are motivated by questions targeting the analytical, computational, and experimental foundation of modeling dynamic sensorimotor control.

Q1: What constitutive physical laws yield predictable and stable locomotion mechanics?
Q2: Which model properties enable implementation of scalable computational techniques?
Q3: Can such models provide accurate predictions for robotics and neuromechanics?

We seek a modeling framework that can adapt to new circumstances and environments on-line, and that can facilitate translation of principles between scientific and engineering disciplines.

The preceding objectives will be achieved through six aims that establish a unified (1.) analytical, (2.) computational, and (3.) experimental framework for predictive modeling.

Aim 1.1 (Predictable locomotion mechanics) Characterize impact restitution laws and reduced-order deformable substrates that yield models for multi-legged gaits that satisfy (a) existence and uniqueness and (b) orbital and structural stability properties.

Aim 1.2 (Stable locomotion mechanics) Develop incremental stability theory for nonsmooth dynamics derived in Aim 1.1, and use this theory to explore tradeoffs between morphological and digital feedback control.

Aim 2.1 (Scalable inverse modeling) Derive scalable system identification algorithms that estimate predictive dynamical models for legged locomotion from Aim 1.1 from data.

Aim 2.2 (Scalable behavior synthesis) Derive scalable nonsmooth optimization algorithms to synthesize robot maneuvers that exploit dynamics, e.g. favorable energetic transformations and morphological feedback control.

Aim 3.1 (Perturbation recovery strategies of cockroaches) Identify neuromechanical perturbation recovery strategies in cockroaches by fitting a reduced-order model from Aim 1.1 using the algorithms from Aim 2.1.

Aim 3.2 (Intrinsically stable gaits and maneuvers of robots) Test algorithms for system identification and behavior synthesis from Aims 2.1 and 2.2 by automatically generating gaits and maneuvers for a legged robot.

Accomplishments: Aim 1.1 (Predictable locomotion mechanics)
In models for locomotion mechanics on hard ground (specifically, mechanical systems subject to unilateral constraints), we derived conditions that ensure trajectories exist uniquely for all forward time, and found that inertially decoupling limbs is generally necessary and sufficient to ensure trajectory outcomes vary continuously with respect to initial state and system parameters (i.e. trajectories are orbitally and structurally stable, as desired). Surprisingly, we found that the condition required for continuity is actually sufficient to ensure trajectories vary piecewise-differentiably with respect to initial conditions; a further force-decoupling assumption ensures classical differentiability. This insight lays the foundation to derive scalable algorithms for optimization, learning, and control for a broad class of models for contact-rich locomotion mechanics.

Aim 1.2 (Stable locomotion mechanics)
We extended the classical dynamical systems theory of averaging to apply to locomotion mechanics. By averaging the cumulative effect of forces applied over multiple steps of a periodic behavior, we obtain a smooth approximation for the nonsmooth dynamics of locomotion that certifies the existence and stability of a gait in the presence of perturbations. The stability of the gait arises from intrinsic mechanisms in the model's locomotion mechanics (namely, viscoelasticity in the limbs), effectively leveraging within-step morphological feedback control. This provides the potential to evaluate the tradeoff between within-step feedback implemented digitally versus mechanically. These results were published in the International Journal on Robotics Research.

We extended the classical dynamical systems theory of contraction to apply in hybrid systems that combine continuous-time and discrete-time dynamics. By analyzing the local properties of continuous-time flow and discrete-time impact, we derive conditions under which all trajectories converge toward one another at an exponential rate. Importantly, convergence is established with respect to the intrinsic distance in the hybrid system, so that discontinuities arising from impacts do not disrupt convergence. Conversely, we prove that hybrid systems that are contractive with respect to the intrinsic distance metric satisfy our infinitesimal contraction conditions. This work has been submitted and is under review in the IEEE Transactions on Automatic Control.

Aim 2.1 (Scalable inverse modeling)
Under the conditions derived in our work on Aim 1.1 that ensure differentiability of trajectory outcomes in models of legged locomotion (namely, inertial and force decoupling of limbs), we are justified in applying scalable techniques originally derived for classical smooth nonlinear systems to model dynamics near periodic orbits in the nonsmooth dynamics of locomotion. Specifically, we leverage Floquet normal form theory for oscillators and estimate linear phase-varying models; these oscillators can represent rhythmic locomotion behaviors like periodic gaits. We have applied these techniques to synthetic simulation data as well as experimental data collected from robots and organisms, including cockroaches and humans.

We have begun collecting a rich dataset on a vertical hopper robot testbed wherein a single leg, constrained to move along a vertical rail, intermittently contacts variable-height terrain; see. By recording synchronized sensor data collected onboard (accelerometer, encoder) and offboard (motion capture) at a rate of 1kHz, we can precisely characterize both the continuous dynamics of contact as well as the discrete dynamics of impact. Our preliminary findings suggest that data-driven models are predictive of robot behavior, but simple analytical models for impact are inaccurate and must be revised. To estimate high-precision models from data, we derived and validated a novel state estimation algorithm for hybrid systems that leverages a recent advance in nonsmooth optimization theory termed nonsmooth variable projection; this work has been submitted and is currently under review in Automatica.

Aim 2.2 (Scalable behavior synthesis)
We conducted a feasibility study to assess whether state-of-the-art algorithms for trajectory optimization and reinforcement learning are applicable to contact-rich locomotion mechanics. When trajectory outcomes are continuous and piecewise-differentiable, the classical gradient-based methods can be generalized using established techniques from nonsmooth optimization; in particular, the Bouligand derivative that provides the first-order approximation of a piecewise-differentiable function generalizes the classical gradient that provides the first-order approximation of a smooth function, and this generalized derivative can be used to rapidly descent the nonsmooth cost landscape. However, in the general case where trajectory outcomes are discontinuous – whether arising from intrinsic properties of rigid-body mechanics or as side-effect of contact-dependent controllers – we showed that the cost landscape is fractured into many disjoint pieces, precluding application of scalable derivative-based optimization algorithms.

Aim 3.1 (Perturbation recovery strategies of organisms)
We have evaluated a variety of models for perturbation recovery in organisms, including cockroaches and humans. Although data-driven models explain large fractions of the variance in experimental data on small time scales, they provide poor long-term predictions due to inherent model instability. In contrast, reduced-order analytical models predict long-term recovery, but provide poor predictions on small time scales. We bridge these two extremes to estimate oscillator models from data using structure derived from reduced-order analytical models.

Aim 3.2 (Intrinsically stable gaits and maneuvers of robots)
The contractive systems discovered in Aim 1.2 enjoy a strong stability property: every feedforward input yields a unique trajectory that asymptotically attracts all other trajectories. We have conducted simulation studies to characterize contractive models of contact-rich dynamics and synthesized intrinsically-stable behaviors using these simulations. These behaviors have been demonstrated on our one-leg robot testbed.

Training Opportunities: I mentored each of the graduate students listed as Participants one-on-one during weekly or biweekly individual and group meetings. The training and professional development they received included instruction in domain- or application-specific analytical, computational, and/or experimental techniques, as well as providing opportunities to practice and received feedback on written and oral communication skills.

Bora Banjanin attended and presented at the following conferences:
- Yearly Meeting of the Society for Integrative and Comparative Biology (SICB) 2016 in Portland, OR, USA
- Dynamic Walking 2018 meeting in Pensacola, FL, USA
- Dynamic Walking 2019 in Canmore, Alberta, Canada

Andrew Pace attended and presented at the following conferences:
- Dynamic Walking 2016 in Holly, MI, USA
- Hybrid Systems: Computation and Control (HSCC) 2017 in Pittsburgh, PA, USA
- IEEE International Conference on Robotics and Automation (ICRA) 2017 in Singapore
- Dynamic Walking 2017 in Mariehamn, Finland, USA
- Dynamic Walking 2019 in Canmore, Alberta, Canada

Jacob Baldassini earned the MS ECE degree based on work conducted on this project.

Liam Han earned the MS ME degree based on work conducted on this project

Tianqi Li earned the MS ME degree based on work conducted on this project.

Yana Sosnovskaya earned the MS ECE degree based on work conducted on this project.

Results Dissemination: Invited speaker:
- Dynamic Walking Meeting, Canmore, Alberta, Canada, June 2019
- Robotics Seminar at NVIDIA Research, Seattle, WA, USA, April 2019
- Robotics Seminar at California Institute of Technology, Pasadena, CA, USA, Mar 2019
- Neuroscience & Robotics Seminar at Northwestern University, Evanston, IL, USA, Mar 2019
- Dynamic Walking Meeting, Pensacola, FL, USA, June 2018
- Institute for Neuroengineering Seminar at the University of Washington, Seattle, WA, USA, Mar 2017
- Physics of Living Systems at the Georgia Institute of Technology, Atlanta, GA, USA, February 2017
- Dynamics and Controls Seminar, University of California, San Diego, CA, USA, April 2017
- GRASP Lab Seminar, University of Pennsylvania, Philadelphia, PA, USA, April 2017
- IEEE SIMPAR Workshop on Robot Simulation, San Francisco, CA, USA, December 2016
- Microsoft Research, Redmond, WA, USA, October 2016

Developed and delivered presentations on human interaction with autonomy:
- Washington Aerospace Scholars, Seattle, WA, USA, 2016 (approximately 60 in attendance)
- Electrical Engineering prospective undergraduate students, Seattle, WA, USA, 2017 (approximately 60 in attendance)
- UW MathDay, Seattle, WA, USA, 2017 (approximately 1200 in attendance), 2018

Developed and taught a First Course in Robotics to 24 participants in the University of Washington Summer Institute for Mathematics (SIMUW), Seattle, WA, USA, 2016.

RPPR Final Report
as of 05-Aug-2019

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Co-supervised a Postdoctoral Researcher, Dr. Ryan Robinson, with Dr. William Nothwang from Army Research Lab, Adelphi, MD. This collaboration involved weekly meetings with Dr. Robinson and quarterly meetings with Dr. Nothwang, and visits (1 each) by PI Burden to ARL Adelphi and Dr. Nothwang to UW Seattle. PI Burden co-authored 2 conference papers with Dr. Robinson (1 of these was co-authored with Dr. Nothwang).

PARTICIPANTS:

Participant Type: Graduate Student (research assistant)
Participant: Bora Banjanin
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Andrew Pace
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Joey Sullivan
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Jacob Baldassini
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Yana Sosnovskaya
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:

International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Liam Han
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Tianqi Li
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

ARTICLES:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: The International Journal of Robotics Research
Publication Identifier Type: DOI Publication Identifier: 10.1177/0278364918756498
Volume: 37 Issue: First Page #: 266
Date Submitted: 7/30/19 12:00AM Date Published: 3/1/18 8:00AM
Publication Location:
Article Title: A hybrid dynamical extension of averaging and its application to the analysis of legged gait stability
Authors: Avik De, Samuel A Burden, Daniel E Koditschek
Keywords: legged robots, dynamics, motion control
Abstract: We extend a smooth dynamical systems averaging technique to a class of hybrid systems with a limit cycle that is particularly relevant to the synthesis of stable legged gaits. After introducing a definition of hybrid averageability sufficient to recover the classical result, we illustrate its applicability by analysis of first a one-legged and then a two-legged hopping model. These abstract systems prepare the ground for the analysis of a significantly more complicated two legged model: a new template for quadrupedal running to be analyzed and implemented on a physical robot in a companion paper. We conclude with some rather more speculative remarks concerning the prospects for further extension and generalization of these ideas.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: Hybrid Systems: Computation and Control
Date Received: 11-Aug-2017 Conference Date: 18-Apr-2017 Date Published:
Conference Location: Pittsburgh, Pennsylvania, USA
Paper Title: Piecewise - Differentiable Trajectory Outcomes in Mechanical Systems Subject to Unilateral Constraints
Authors: Andrew Pace, Samuel Burden
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: IEEE International Conference on Robotics and Automation (ICRA)
Date Received: 11-Aug-2017 Conference Date: 29-May-2017 Date Published:
Conference Location: Singapore, Singapore
Paper Title: Decoupled limbs yield differentiable trajectory outcomes through intermittent contact in locomotion and manipulation
Authors: Andrew Pace, Samuel Burden
Acknowledged Federal Support: **Y**

DISSERTATIONS:

Publication Type: Thesis or Dissertation
Institution: University of Washington, Seattle
Date Received: 11-Aug-2017 Completion Date: 6/15/17 11:38PM
Title: An Examination of the Effects of Deformable Foam Contact Surfaces on Robotic Locomotion
Authors: Jacob Baldassini
Acknowledged Federal Support: **Y**

Publication Type: Thesis or Dissertation
Institution: University of Washington, Seattle
Date Received: 11-Aug-2017 Completion Date: 6/15/17 10:11PM
Title: External Measurement System for Robot Dynamics
Authors: Yana Sosnovskaya
Acknowledged Federal Support: **Y**

Publication Type: Thesis or Dissertation
Institution: University of Washington, Seattle
Date Received: 31-Aug-2018 Completion Date: 6/15/18 4:59PM
Title: Experimental Realization of Feedforward Deadbeat Control on a Hybrid Model of Legged Locomotion
Authors: Tianqi Li
Acknowledged Federal Support: **Y**

Publication Type: Thesis or Dissertation
Institution: University of Washington, Seattle
Date Received: 31-Aug-2018 Completion Date: 6/15/18 7:00AM
Title: Automating Perturbation Experiments for a Hopping Robot using a Cable-Driven Impedance Haptic Device
Authors: Liam Han
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 05-Aug-2019

Project Summary - Grant #W911NF-16-1-0158
(Reporting Period: May 01 2016 – April 30 2019)

Predictive models for sensorimotor control of legged locomotion

Dr. Samuel Burden
Electrical Engineering
University of Washington, Seattle, WA, 98119

Objective

We propose to develop generalizable tools for deriving and applying simple models that provide accurate quantitative predictions for dynamic locomotion and self-manipulation behaviors executed by robots and animals. We are motivated by questions targeting the analytical, computational, and experimental foundation of modeling dynamic sensorimotor control.

Q1: What constitutive physical laws yield predictable and stable locomotion mechanics?

Q2: Which model properties enable implementation of scalable computational techniques?

Q3: Can such models provide accurate predictions for robotics and neuromechanics?

We seek a modeling framework that can adapt to new circumstances and environments on-line, and that can facilitate translation of principles between scientific and engineering disciplines.

Approach

The preceding objectives will be achieved through six aims that establish a unified (1.) analytical, (2.) computational, and (3.) experimental framework for predictive modeling.

Aim 1.1 (Predictable locomotion mechanics) Characterize impact restitution laws and reduced-order deformable substrates that yield models for multi-legged gaits that satisfy (a) existence and uniqueness and (b) orbital and structural stability properties.

Aim 1.2 (Stable locomotion mechanics) Develop incremental stability theory for nonsmooth dynamics derived in Aim 1.1, and use this theory to explore tradeoffs between morphological and digital feedback control.

Aim 2.1 (Scalable inverse modeling) Derive scalable system identification algorithms that estimate predictive dynamical models for legged locomotion from Aim 1.1 from data.

Aim 2.2 (Scalable behavior synthesis) Derive scalable nonsmooth optimization algorithms to synthesize robot maneuvers that exploit dynamics, e.g. favorable energetic transformations and morphological feedback control.

Aim 3.1 (Perturbation recovery strategies of [organisms]) Identify neuromechanical perturbation recovery strategies in cockroaches and humans by fitting a reduced-order model from Aim 1.1 using the algorithms from Aim 2.1.

Aim 3.2 (Intrinsically stable gaits and maneuvers of robots) Test algorithms for system identification and behavior synthesis from Aims 2.1 and 2.2 by automatically generating gaits and maneuvers for a legged robot.

Relevance to Army

To move over, around, and through obstacles in the real world, autonomous robots need to employ a repertoire of dynamic and dexterous behaviors. Since the world is ever-changing, these behaviors must be synthesized on-the-fly and adapted to diverse environmental conditions. On rubble-strewn terrain, for instance, a legged robot tasked with reconnaissance may intermittently encounter perturbations to its motion or barriers taller than its hip height. Confronted with such challenges, the robot must be able to autonomously reject the perturbations and synthesize maneuvers to overcome obstacles. At present, these capabilities are out of reach. Our proposed objectives provide a path to overcome these limitations by enabling a robot to automatically estimate a model of its self and its environment and use this model to synthesize dynamically-stable gaits and maneuvers that traverse terrain and obstacles. Since animals outperform their robot counterparts, we seek techniques that enable translation of principles from natural to engineered systems. We are specifically focused on techniques applicable to the *Minitaur* platform developed under the *ARL Robotics CTA*.

Accomplishments for Reporting Period

Aim 1.1 (Predictable locomotion mechanics)

In models for locomotion mechanics on hard ground (specifically, mechanical systems subject to unilateral constraints), we derived conditions that ensure trajectories exist uniquely for all forward time, and found that inertially decoupling limbs is generally necessary and sufficient to ensure trajectory outcomes vary continuously with respect to initial state and system parameters (i.e. trajectories are orbitally and structurally stable, as desired). Surprisingly, we found that the condition required for continuity is actually sufficient to ensure trajectories vary piecewise-differentiably with respect to initial conditions; a further force-decoupling assumption ensures classical differentiability; see Fig. 3 and [PaBu17a, PaBu17b]. This insight lays the foundation to derive scalable algorithms for optimization, learning, and control for a broad class of models for contact-rich locomotion mechanics.

Aim 1.2 (Stable locomotion mechanics)

We extended the classical dynamical systems theory of *averaging* to apply to locomotion mechanics. By averaging the cumulative effect of forces applied over multiple steps of a periodic behavior, we obtain a smooth approximation for the nonsmooth dynamics of locomotion that certifies the existence and stability of a gait in the presence of perturbations; see Fig. 4. The stability of the gait arises from intrinsic mechanisms in the model's locomotion mechanics (namely, viscoelasticity in the limbs), effectively leveraging within-step morphological feedback control. This provides the potential to evaluate the tradeoff between within-step feedback implemented digitally versus mechanically. These results were published in the *International Journal on Robotics Research* [DeBu+18].

We extended the classical dynamical systems theory of *contraction* to apply in hybrid systems that combine continuous-time and discrete-time dynamics. By analyzing the local properties of continuous-time flow and discrete-time impact, we derive conditions under which all trajectories converge toward one another at an exponential rate. Importantly, convergence is established with respect to the intrinsic distance in the hybrid system, so that discontinuities arising from impacts do not disrupt convergence. Conversely, we prove that hybrid systems that are contractive with respect to the intrinsic distance metric satisfy our infinitesimal contraction conditions. This work has been submitted and is under review in the *IEEE Transactions on Automatic Control* [BuLi+18].

Aim 2.1 (Scalable inverse modeling)

Under the conditions derived in our work on Aim 1.1 that ensure differentiability of trajectory outcomes in models of legged locomotion (namely, inertial and force decoupling of limbs), we are justified in applying scalable techniques originally derived for classical smooth nonlinear systems to model dynamics near periodic orbits in the nonsmooth dynamics of locomotion. Specifically, we leverage Floquet normal form theory for oscillators and estimate linear phase-varying models; these oscillators can represent rhythmic locomotion behaviors like periodic gaits. We have applied these techniques to synthetic simulation data as well as experimental data collected from robots and organisms, including cockroaches [BaBu+16] and humans [BaRo+19].

We have begun collecting a rich dataset on a vertical hopper robot testbed wherein a single leg, constrained to move along a vertical rail, intermittently contacts variable-height terrain; see Fig. 1. By recording synchronized sensor data collected onboard (accelerometer, encoder) and offboard (motion capture) at a rate of 1kHz, we can precisely characterize both the continuous dynamics of contact as well as the discrete dynamics of impact. Our preliminary findings suggest that data-driven models are predictive of robot behavior, but simple analytical models for impact are inaccurate and must be revised. To estimate high-precision models from data, we derived and validated a novel state estimation algorithm for hybrid systems that leverages a recent advance in nonsmooth optimization theory termed *nonsmooth variable projection*; this work has been submitted and is currently under review in *Automatica* [PaZh+19].

Aim 2.2 (Scalable behavior synthesis)

We conducted a feasibility study to assess whether state-of-the-art algorithms for trajectory optimization and reinforcement learning are applicable to contact-rich locomotion mechanics. When trajectory outcomes are continuous and (piecewise-)differentiable as in Fig. 3 and [PaBu17a, PaBu17b], the classical gradient-based methods can be generalized using established techniques from nonsmooth optimization; in particular, the *Bouligand* derivative that provides the first-order approximation of a piecewise-differentiable function generalizes the classical *gradient* that provides the first-order approximation of a smooth function, and this generalized derivative can be used to rapidly descent the nonsmooth cost landscape. However, in the general case where trajectory outcomes are discontinuous – whether arising from intrinsic properties of rigid-body mechanics or as side-effect of contact-dependent controllers – we showed that the cost landscape is fractured into many disjoint pieces as in Fig. 5, precluding application of scalable derivative-based optimization algorithms [BaBu19].

Aim 3.1 (Perturbation recovery strategies of [organisms])

We have evaluated a variety of models for perturbation recovery in organisms, including cockroaches [BaBu+16] and humans [BaRo+19]. Although data-driven models explain large fractions of the variance in experimental data on small time scales, they provide poor long-term predictions due to inherent model instability. In contrast, reduced-order analytical models predict long-term recovery, but provide poor predictions on small time scales. We bridge these two extremes to estimate oscillator models from data using structure derived from reduced-order analytical models [LiSu+19].

Aim 3.2 (Intrinsically stable gaits and maneuvers of robots)

The contractive systems discovered in Aim 1.2 enjoy a strong stability property: every feedforward input yields a unique trajectory that asymptotically attracts all other trajectories. We have conducted simulation studies to characterize contractive models of contact-rich dynamics [BuLi+18, Section V-B] and synthesized intrinsically-stable behaviors using these simulations [BuLi+18]; see Fig. 2. These behaviors have been demonstrated on our one-leg robot testbed, depicted in Fig. 1.

Collaborations and Technology Transfer

- No technology transfers to report.
- Co-supervised a Postdoctoral Researcher, Dr. Robinson, with Dr. Nothwang from Army Research Lab, Adelphi, MD. This collaboration involved weekly meetings with Dr. Robinson and quarterly meetings with Dr. Nothwang, and visits (1 each) by PI Burden to ARL Adelphi and Dr. Nothwang to UW Seattle. PI Burden co-authored 2 conference papers with Dr. Robinson (1 of these was co-authored with Dr. Nothwang).

Resulting Journal Submissions (under review)

[PaZh+19] A. M. Pace, J. Zhang, S. A. Burden, A. Aravkin. "Offline state estimation for hybrid systems via nonsmooth variable projection." Submitted to *Automatica*, 2019. https://arxiv.org/abs/1905.09169

[BaBu19] B. S. Banjanin and S. A. Burden. "Nonsmooth optimal value and policy functions for mechanical systems subject to unilateral constraints". Submitted to *IEEE Control Systems Letters,* 2019. https://arxiv.org/abs/1710.06745

[BuLi+18] S. A. Burden, T. Libby, and S. D. Coogan. "On contraction analysis for hybrid systems." Submitted to *IEEE Transactions on Automatic Control*, 2018. https://arxiv.org/abs/1804.04122

Resulting Journal Publications

[DeBu+18] A. De, S. A. Burden, and D. E. Koditschek. "A Hybrid Dynamical Extension of Averaging and Its Application to the Analysis of Legged Gait Stability." *The International Journal of Robotics Research* 37 (2-3): 266–86, 2018. http://dx.doi.org/10.1177/0278364918756498

Resulting Conference Publications and Presentations (peer-reviewed)

[PaBu17a] A. M. Pace and S. A. Burden. *Piecewise-Differentiable Trajectory Outcomes in Mechanical Systems Subject to Unilateral Constraints*. International Conference on Hybrid Systems: Computation and Control (HSCC), pp. 243-252, 2017. http://dx.doi.org/10.1145/3049797.3049807

[PaBu17b] A. M. Pace and S. A. Burden. *Decoupled limbs yield differentiable trajectory outcomes through intermittent contact in locomotion and manipulation*. IEEE International Conference on Robotics and Automation (ICRA), pp. 2261-2266, 2017. http://dx.doi.org/10.1109/ICRA.2017.7989259

Resulting Conference Abstracts and Presentations (not reviewed)

[LiSu+19] T. Libby, J. Sullivan, and S. A. Burden. *Design of Legged Robots Using Haptic Mixed Reality*. Dynamic Walking Meeting, 2019.

[PaZh+19] A. M. Pace, J. Zhang, S. A. Burden, and A. Aravkin. *Estimating State and Contact Mode for Legged Locomotion without Dedicated Contact Sensors*. Dynamic Walking Meeting, 2019.

[BaRo+19] B. S. Banjanin, M. C. Rosenberg, M. Yamagami, K. M. Steele, and S. A. Burden. *Subject-specific models for predicting human locomotor response to ankle foot orthoses*. Dynamic Walking Meeting, 2019.

[BuLi+18] S. A. Burden, T. Libby, and S. D. Coogan. *Why (or, more specifically, when) do feedforward inputs yield stable behaviors?* Dynamic Walking Meeting, 2018.

[PaBu17] A. M. Pace and S. A. Burden. *How (de)coupled are Minitaur limbs?* Dynamic Walking Meeting, 2017.

[PaBu16] A. M. Pace and S. A. Burden. *Assessing stability and controllability of multi-legged gaits*. Dynamic Walking Meeting, 2016.

[BaBu+16] B. Banjanin, S. A. Burden, T. Y. Moore, S. Revzen, and R. J. Full. *Estimating predictive dynamical models of legged locomotion from data*. Yearly meeting of the Society for Integrative and Comparative Biology (SICB), 2016.

Graduate Students Involved During Reporting Period

Current:

- Bora Banjanin (PhD ECE Su 2019)
- Andrew Pace (PhD ECE Fall 2019)
- Joey Sullivan (PhD ECE 2023)

Graduated:

- Liam Han (MS ME 2018)
- Tianqi Li (MS ME 2018)
- Jacob Baldassini (MS EE 2017)
- Yana Sosnovskaya (MS EE 2017)

Awards, Honors and Appointments

- None to report.

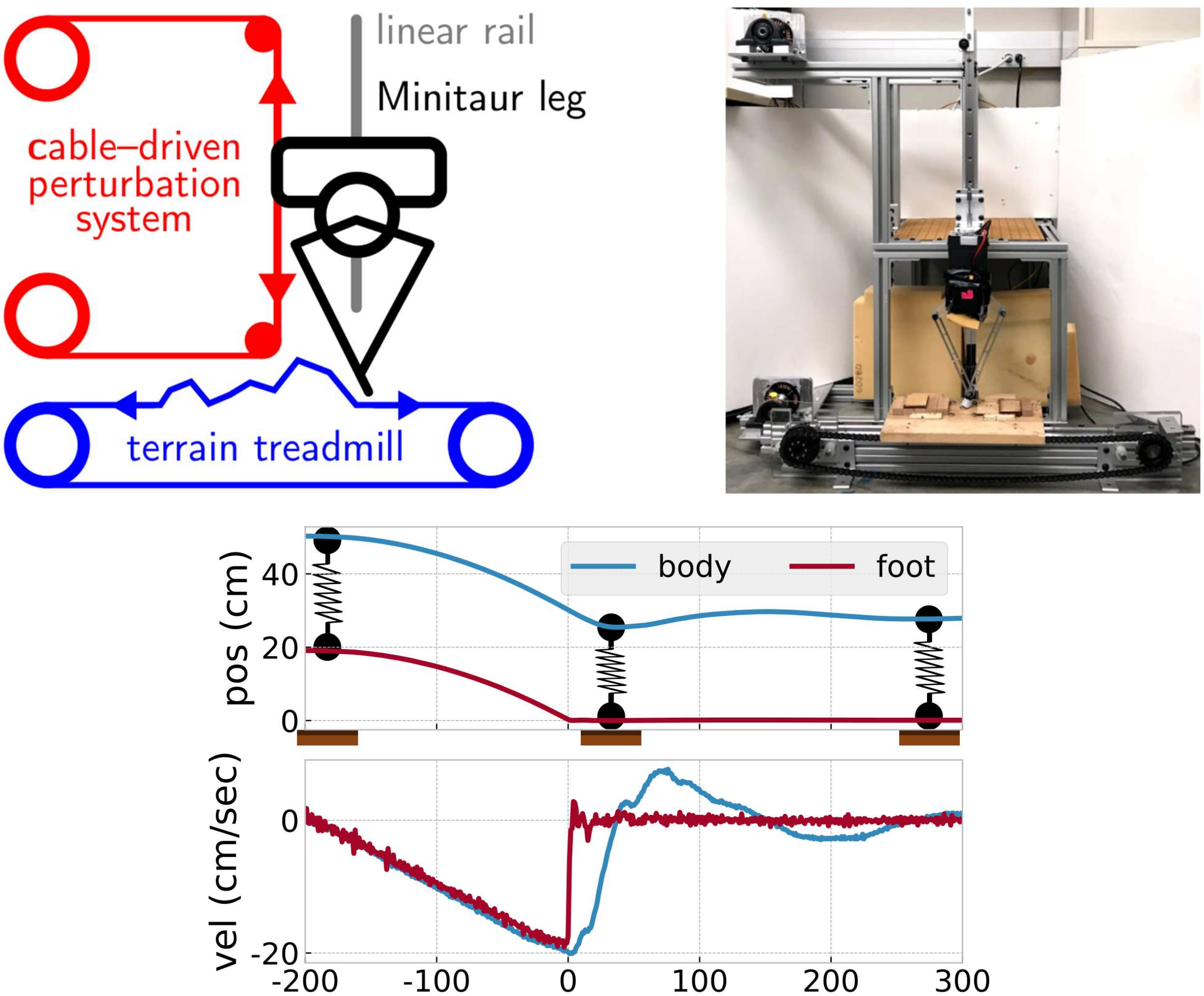


Fig. 1: Schematic, photograph, and impact data from haptic robot testbed constructed in our lab. (*top left*) A single robot leg (from Ghost Robotics' Minitaur) is constrained to move vertically in a gravitational field using a linear rail. A treadmill underfoot presents the robot with variable terrain. A Cable-driven perturbation system enables application of force profiles ranging +/- 2x robot weight at 80% maximum testbed speed. Onboard motor encoders and accelerometers record data at 1kHz; a high-speed motion capture camera records data at 1kHz. (*top right*) Photograph of testbed prototype. (*bottom*) Sample data collected on testbed showing position and velocity traces in 500msec window around impact.

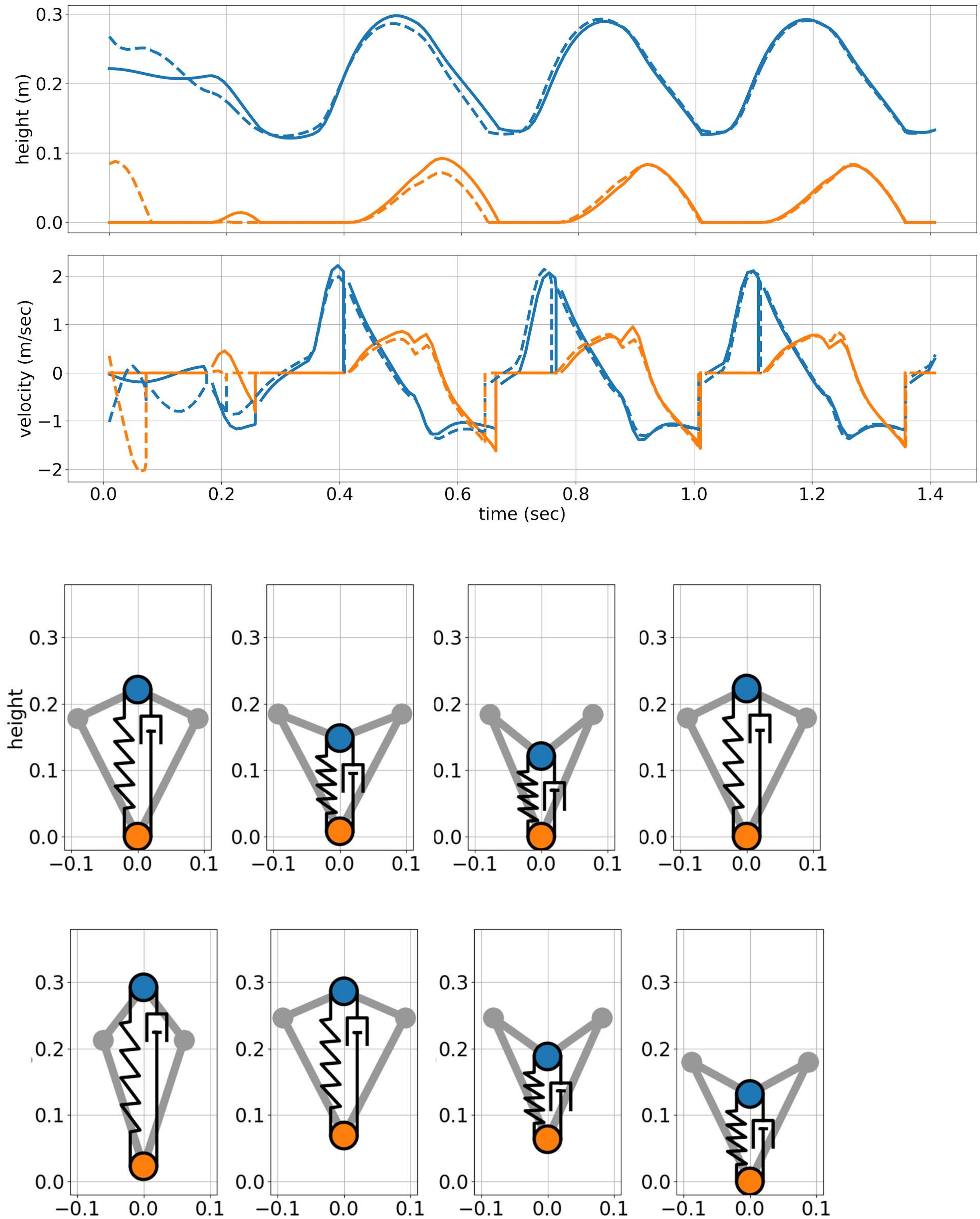


Fig. 2: Contractive model for *vertical hopper* (Minitaur leg constrained to vertical motion) [BuLi+18]. (*top*) Height and velocity traces for two initial conditions from different contact states rapidly converge. (*bottom*) Sequence of simulation snapshots illustrating hopping of Minitaur leg model.

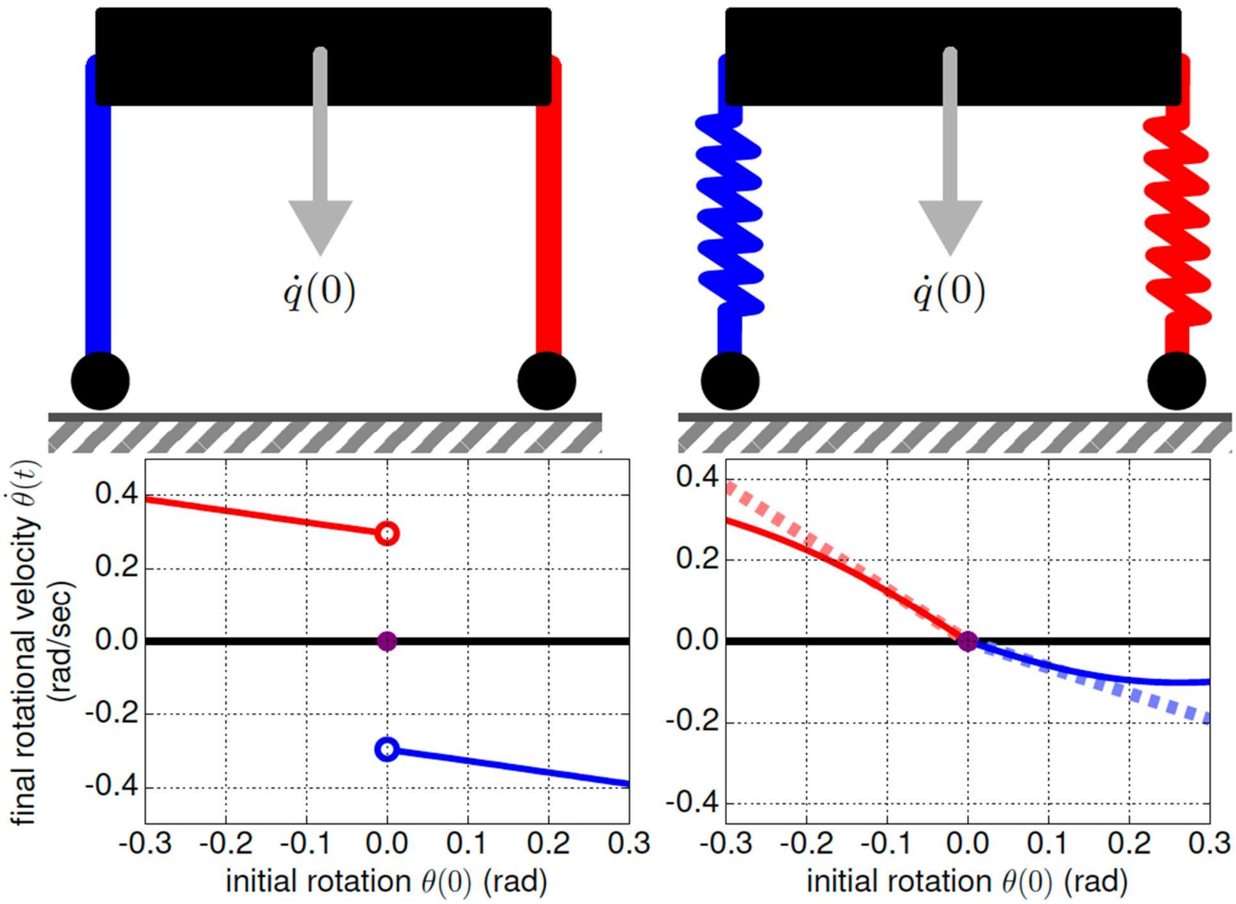


Fig. 3: Trajectory outcomes in mechanical systems subject to unilateral constraints.
(*left*) In general, trajectory outcomes depend discontinuously on initial conditions. In the pictured model for stiff–leg trotting, discontinuities arise when two legs touch down: if the legs impact simultaneously (corresponding to initial rotation $\theta(0) = 0$), then the post–impact rotational velocity is zero; if the rear leg impacts before the front leg ($\theta(0) > 0$) or vice–versa ($\theta(0) < 0$), then the post–impact rotational velocities are bounded away from zero.
(*right*) When limbs are inertially decoupled, trajectory outcomes depend continuously on initial conditions. In the pictured model for soft–leg trotting, trajectory outcomes (solid lines) admit a first–order approximation (dashed lines) that is continuous and piecewise–linear.

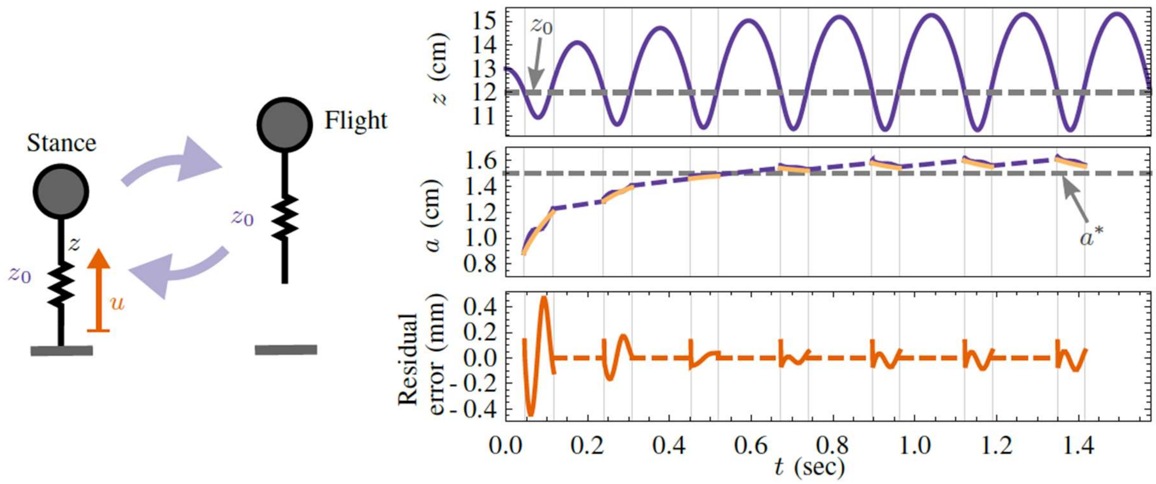
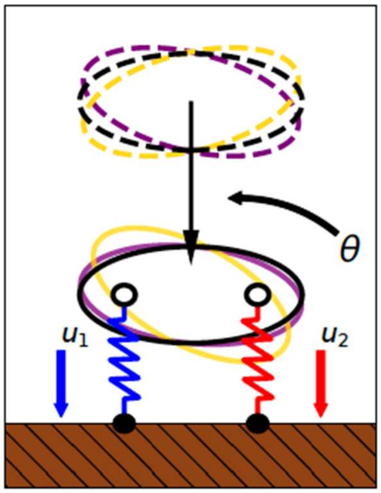
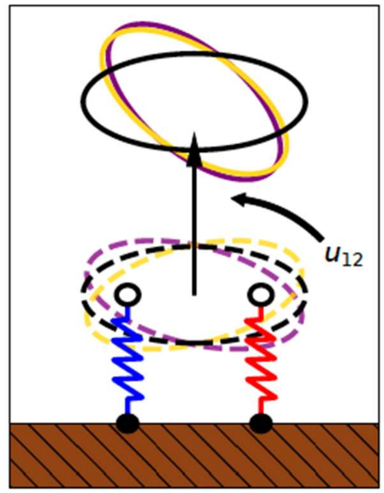


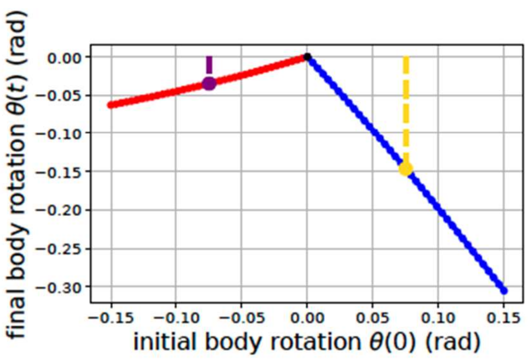
Fig. 4: Averaging the effect of within-step morphological feedback over many steps [DeBu+18].
(*left*) In this model for vertical hopping, a spring-mass moves vertically in a gravitational field. When the limb extends beyond length z_0, the model transitions from *stance* to *flight*. In *stance*, the limb exerts viscoelastic and actuator forces on the mass; in *flight*, the only force is gravity.
(*right*) The cumulative effect of stance forces can be averaged over many steps to obtain a simple proportional-derivative feedback law controlling hopping height. This smooth "averaged" system (in gold) provides a good approximation for the behavior of the original hybrid system (in blue): the trajectories differ by less than 5% of hopping height (top and bottom subfigures), and actuator inputs agree to even greater precision (middle subfigure).



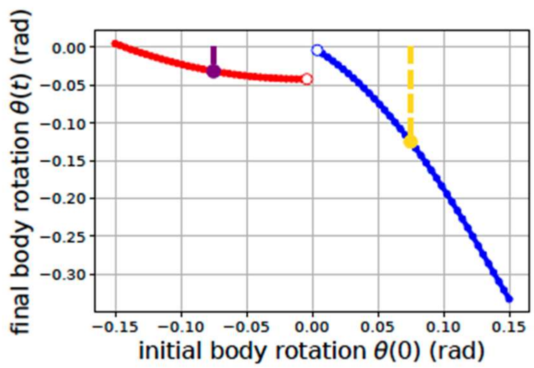
(a) *touchdown* maneuver illustration



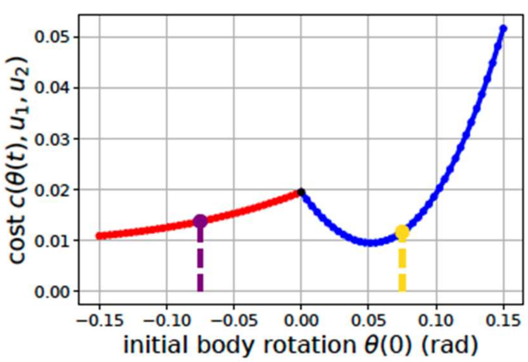
(b) *liftoff* maneuver illustration



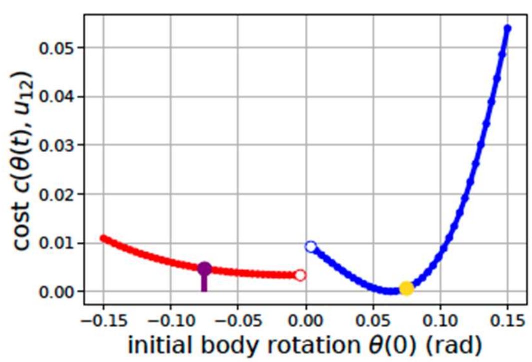
(c) touchdown trajectory outcomes



(d) liftoff trajectory outcomes



(e) touchdown value



(f) liftoff value

Fig. 5: Piecewise-differentiable and discontinuous trajectories in a saggital-plane biped [BaBu18]. (a,b) Illustration of two maneuvers—touchdown and liftoff—performed under non-optimal policies that exert different forces depending on which feet are in contact with the ground. In the touchdown maneuver, feet are initially off the ground and trajectories terminate when the body height reaches nadir; in the liftoff maneuver, feet are initially on the ground and trajectories terminate when the body height reaches apex. (c,d) Trajectory outcomes (final body angle (t)) as a function of initial body angle (0). (e,f) Performance of trajectories as measured by a cost function that penalizes final rotation error and input effort. Dashed colored vertical lines on (c–f) indicate corresponding colored outcomes on (a,b).