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14. ABSTRACT
The objective of this ARO)project was to explore the abundant possibilities of sculpting stress waves in mechanical nonlinear lattice structures represented by woodpile mechanical metamaterials and their derivatives. These nonlinear mechanical metamaterials offer an ideal setting for investigating the interplay among various geometrical and system parameters, such as disorder, nonlinearity, impurities, and resonance, in a controllable manner. This project specifically focused on three research themes: (i) Energy transport and localization mechanisms under the

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Report Title

Final Report: Woodpile Mechanical Metamaterials for Sculpting Stress Waves

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The objective of this ARO)project was to explore the abundant possibilities of sculpting stress waves in mechanical nonlinear lattice structures represented by woodpile mechanical metamaterials and their derivatives. These nonlinear mechanical metamaterials offer an ideal setting for investigating the interplay among various geometrical and system parameters, such as disorder, nonlinearity, impurities, and resonance, in a controllable manner. This project specifically focused on three research themes: (i) Energy transport and localization mechanisms under the simultaneous influence of disorder and nonlinearity, (ii) Wave scattering and trapping in locally resonant woodpile systems for energy harvesting; and (iii) Turbulence-like energy cascade phenomena in woodpile system for impact mitigation purposes. All these tasks are closely linked under the theme of stress wave sculpting. The PIs investigated these fundamental wave dynamics by (i) an experimental design, prototyping, and testing by PI Yang; a theoretical understanding by PI Kevrekidis; and finally a numerical framework for simulating such settings, formulated by both teams. The findings from this study suggest that the woodpile or similar mechanical metamaterials allow us to better control the fundamental characteristics (e.g., waveform, speed, and frequency contents) of nonlinear waves, which can lead to creation of novel engineering devices, e.g., impact mitigator and wave filter.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
01/20/2017	1	H. Yasuda, C. Chong, E. G. Charalampidis, P. G. Kevrekidis, J. Yang. Formation of rarefaction waves in origami-based metamaterials, Physical Review E, ( ): . doi:
01/20/2017	4	Gil-Yong Lee, Christopher Chong, Panayotis G. Kevrekidis, Jinkyu Yang. Wave mixing in coupled phononic crystals via a variable stiffness mechanism, Journal of the Mechanics and Physics of Solids, ( ): 501. doi:
01/20/2017	5	Panayotis G. Kevrekidis, Atanas G. Stefanov, Haitao Xu. Traveling Waves for the Mass in Mass Model of Granular Chains, Letters in Mathematical Physics, ( ): 1067. doi:
01/20/2017	6	Lifeng Liu, Guillaume James, Panayotis Kevrekidis, Anna Vainchtein. Breathers in a locally resonant granular chain with precompression, Physica D: Nonlinear Phenomena, ( ): 27. doi:
01/20/2017	7	E. Kim, R. Chaunsali, H. Xu, J. Jaworski, J. Yang, P. G. Kevrekidis, and A. F. Vakakis. Nonlinear low-to-high-frequency energy cascades in diatomic granular crystals, PHYSICAL REVIEW E, ( ): . doi:
01/20/2017	2	Alejandro J. Martínez, Hiromi Yasuda, Eunho Kim, P. G. Kevrekidis, Mason A. Porter, Jinkyu Yang. Scattering of waves by impurities in precompressed granular chains, Physical Review E, ( ): . doi:
01/20/2017	3	C. Chong, E. Kim, E. G. Charalampidis, H. Kim, F. Li, P. G. Kevrekidis, J. Lydon, C. Daraio, J. Yang. Nonlinear vibrational-state excitation and piezoelectric energy conversion in harmonically driven granular chains, Physical Review E, ( ): . doi:

TOTAL: 7

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

- Jinkyu Yang (PI): Faculty Early Career Development (CAREER) Award, National Science Foundation (2016)
- Riley Pratt and Yelisey Makarevich (Undergraduate students): Summer Undergraduate Research Program (SURP) Fellowship, NASA Space Grant Consortium
- Panos Kevrekidis (co-PI) Inaugural T. Brooke Benjamin Award on Nonlinear Waves by The Society for Industrial and Applied Mathematics (2016).
- Panos Kevrekidis (co-PI) Member of the Inaugural Class of Fellows of the Greek Diaspora Fellowship Program (2016)

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Haitao Xu	0.54	
Domonic Mei	0.13	
FTE Equivalent:	**0.67**	
Total Number:	**2**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Gil-Yong Lee	0.25
FTE Equivalent:	**0.25**
Total Number:	**1**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Jinkyu Yang	0.18	
Panoyotis Kevrekidis	0.09	
FTE Equivalent:	**0.27**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Riley Pratt	0.25	Aeronautics and Astronautics
Yelisey Makarevich	0.25	Aeronautics and Astronautics
FTE Equivalent:	**0.50**	
Total Number:	**2**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 2.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 2.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 2.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 2.00

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The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Haitao Xu
Total Number: 1

Names of other research staff

NAME | PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Sub Contractors (DD882)

1 a. University of Massachusetts - Amherst

1 b. 70 Butterfield Terrace
Amherst MA 010039242

Sub Contractor Numbers (c): AR0000000009112
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e): UMass' role in the proposal will be to provide theoretical leadership and computationalco
Sub Contract Award Date (f-1): 9/21/15 12:00AM
Sub Contract Est Completion Date(f-2): 9/20/16 12:00AM

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1 a. University of Massachusetts - Amherst

1 b. Research Administration Building
70 Butterfield Terrace
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Sub Contractor Numbers (c): AR0000000009112
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Sub Contract Est Completion Date(f-2): 9/20/16 12:00AM

Inventions (DD882)

Scientific Progress

See Attachment

Technology Transfer

Final Report for W911NF-15-1-0604
Attention: **Dr. David Stepp**

Woodpile Mechanical Metamaterials for Sculpting Stress Waves

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1 Foreword

The objective of this Army Research Office (ARO) project was to explore the abundant possibilities of *sculpting* stress waves in mechanical *nonlinear* lattice structures represented by woodpile mechanical metamaterials and their derivatives. These nonlinear mechanical metamaterials offer an ideal setting for investigating the interplay among various geometrical and system parameters, such as disorder, nonlinearity, impurities, and resonance, in a controllable manner. This project specifically focused on three research themes: (i) Energy transport and localization mechanisms under the simultaneous influence of disorder and nonlinearity, (ii) Wave scattering and trapping in locally resonant woodpile systems for energy harvesting; and (iii) Turbulence-like energy cascade phenomena in woodpile system for impact mitigation purposes. All these tasks are closely linked under the theme of stress wave sculpting. The PIs investigated these fundamental wave dynamics by (i) an experimental design, prototyping, and testing by the PI Yang's team at the University of Washington; a theoretical understanding by PI Kevrekidis's team at the University of Massachusetts Amherst; and finally a numerical framework for simulating such settings, formulated by both teams. The findings from this study suggest that the woodpile or similar mechanical metamaterials allow us to better control the fundamental characteristics (e.g., waveform, speed, and frequency contents) of nonlinear waves, which can lead to creation of novel engineering devices, e.g., impact mitigator, wave filter, and mechanical logic devices.

2 Statement of the Problem Studied

In this one-year project (10/2015 - 9/2016), the PIs focused on investigating novel nonlinear wave dynamics in woodpile mechanical metamaterials and their derivative systems. Particularly, the major efforts were spent on the following problems, which are consistent with the tasks described in the "Scope of the Proposed Research" document submitted prior to the start of the project:

- **Energy transport mechanisms (e.g., super- and sub-diffusive energy transport and localization phenomena) in mechanical metamaterials**

- **Wave scattering and trapping in *mass-with-mass* woodpile mechanical metamaterials for energy harvesting**
- **Low-to-high frequency/wavenumber cascades in woodpile mechanical metamaterials**

The PIs' teams pursued these projects in a simultaneous fashion, but the purpose of these tasks is directed all towards unveiling novel nonlinear wave dynamics in mechanical metamaterials. In the process of this project, the PIs utilized a mutually synergistic approach towards the technical approaches and findings of these tasks, developing a strong collaborative link between the two groups, as was one of the key intentions of the program. The highlights of new findings and accomplishments obtained from this project are as discussed below and as is reflected in the resulting publications and preprints [1, 2, 2, 4–10].

3 Summary of the most important results

3 .1 Experimental/computational verifications of super-/sub-diffusive energy transport and Anderson-like localization in mechanical metamaterials

While energy transport and localization have been an important subject of research (e.g., Anderson localization) over a long time in the field of condensed matter physics, their implications in mechanical metamaterials have not been fully explored, particularly in the setting of combined *nonlinearity* and *disorder*. Through this ARO project, the PIs conducted the first experimental demonstration of versatile energy transport and localization phenomena in carefully designed discrete lattice systems, which allowed simultaneous application of disorder and nonlinearity in a controllable manner. The visualization of propagating waves was enabled by the state-of-the-art non-contact laser Doppler vibrometry.

The experimental results of wave propagation, in comparison with computational results, are shown in Fig. 1. The top and bottom panels show ordered and disordered systems, respectively. The nonlinearity of the system was controlled by imposing different ratios of dynamic disturbances to static precompression on Hertzian contact-based lattices. From the left to the right panels, the level of nonlinearity decreases gradually. In Fig. 1a, we observe efficient transport of energy in the form of solitary waves (dashed lines show the wave front), which is in agreement with the numerical simulation results in Fig. 1d. As we reduce the level of nonlinearity, we start to observe linear oscillatory waves, but the wave fronts remain straight. If we characterize this energy spreading trend by calculating the second moment of energy ($m_2(t)$) [12, 13], we find that the asymptotic energy spreading in the homogeneous chain is ballistic (i.e., $m_2(t) \sim t^2$ as $t \to \infty$)) in all three cases (Fig. 1a-c).

If the chain is under disorder, we observe noticeable wave scattering in the spatio-tempral maps (Fig. 1g-i). What is interesting in this disordered system is that as we change the level of linearity, the landscape of energy transport also changes accordingly. Under the highly nonlinear regime (Fig. 1g), the energy spreading trend follows super-diffusive wave propagation. However, as we lower the level of nonlinearity, it gradually changes to sub-diffusive energy transfer. Thus, we find that the nonlinearity acts as a dial to control the energy spreading trends in the disordered system, unlike its ordered counterpart.

What is more intriguing in this simultaneously disordered and nonlinear system is the interplay between Anderson-like energy localization and nonlinear effects. In linear systems, it is well known that disorder contributes to the localization of waves. Imagine the propagation of harmonic waves in homogenous vs. disordered systems. It is evident that disordered systems will be susceptible to more wave scattering, thereby causing wave localization. However, if the system starts to be governed by nonlinear dynamics, this trend is completely flipped. In an ordered system like a homogeneous chain, we have a high localization of waves, while in a disordered system, we witness relatively low localization of waves. Solitary waves are good examples of highly localized waves in strongly nonlinear and homogeneous systems. This ARO project verified experimentally this interplay between disorder and nonlinearity for the first time in the setting of mechanical metamaterials.

More detailed results will be available from the PIs' upcoming publication [4]. While this study has demonstrated the experimental/computational patterns of super-/sub-diffusive energy transport and Anderson-like localization in mechanical metamaterials, the findings can be applied to other physical systems under the influence of disorder and nonlinearity, without losing their generality. Also, the effect of the two key parameters – disorder and nonlinearity – can be combined with other system parameters and configurations, such as local resonant and strain hardening/softening effects. These tasks will be tackled in the future studies of the PIs based on the findings from this study.

3 .2 Energy trapping/harvesting via *mass-with-mass* woodpile mechanical metamaterials

Related to the previous studies on energy transport and localization, the quest for long-lived localized energy in metamaterials is of significant interest for the purpose of energy harvesting. For this, the PIs have investigated two effects, which have not been fully explored by previous studies for the purpose of energy harvesting: nonlinearity and local resonance. The study on the nonlinearity effect has been conducted in the setting of granular crystals that embed piezoelectric elements for the conversion of localized nonlinear waves to electrical energy. We found experimentally and computationally that the vibration states formed in these nonlinear systems can allow us to garner energy in broad regions of frequencies, where different vibration states are possible. The electromechanical model that we have introduced for modeling the interaction between the lattices and piezoelectric components proved to be accurate and efficient in this study. Details can be found in the PIs' publication [9].

Building on this nonlinear system, the PIs further implemented the local resonance (i.e., *mass-with-mass*) features into mechanical metamaterials. We considered a single- or double-defect in woodpile elastic lattices, which naturally exhibit local resonances due to the intrinsic bending modes of the woodpile elements [14]. As a result, we could evidently observe long-lived, robust nonlinear wave structures both experimentally and computationally. The PIs are currently working on preparing for a manuscript that includes preliminary results from this study [5]. It is envisioned that this nonlinear, locally resonant system may lead to a drastic improvement of energy harvesting efficiency compared to conventional linear wave-based energy harvesting approaches.

3 .3 Low-to-high frequency/wavenumber cascades in woodpile mechanical metamaterials

Energy cascades constitute a fundamental principle in turbulent flow, in which fluidic kinetic energy is dissipated through low-to-high frequency and wavenumber conversion. In the PIs' and other scholars' previous work, the feasibility of turbulence-like wave attenuation was validated through

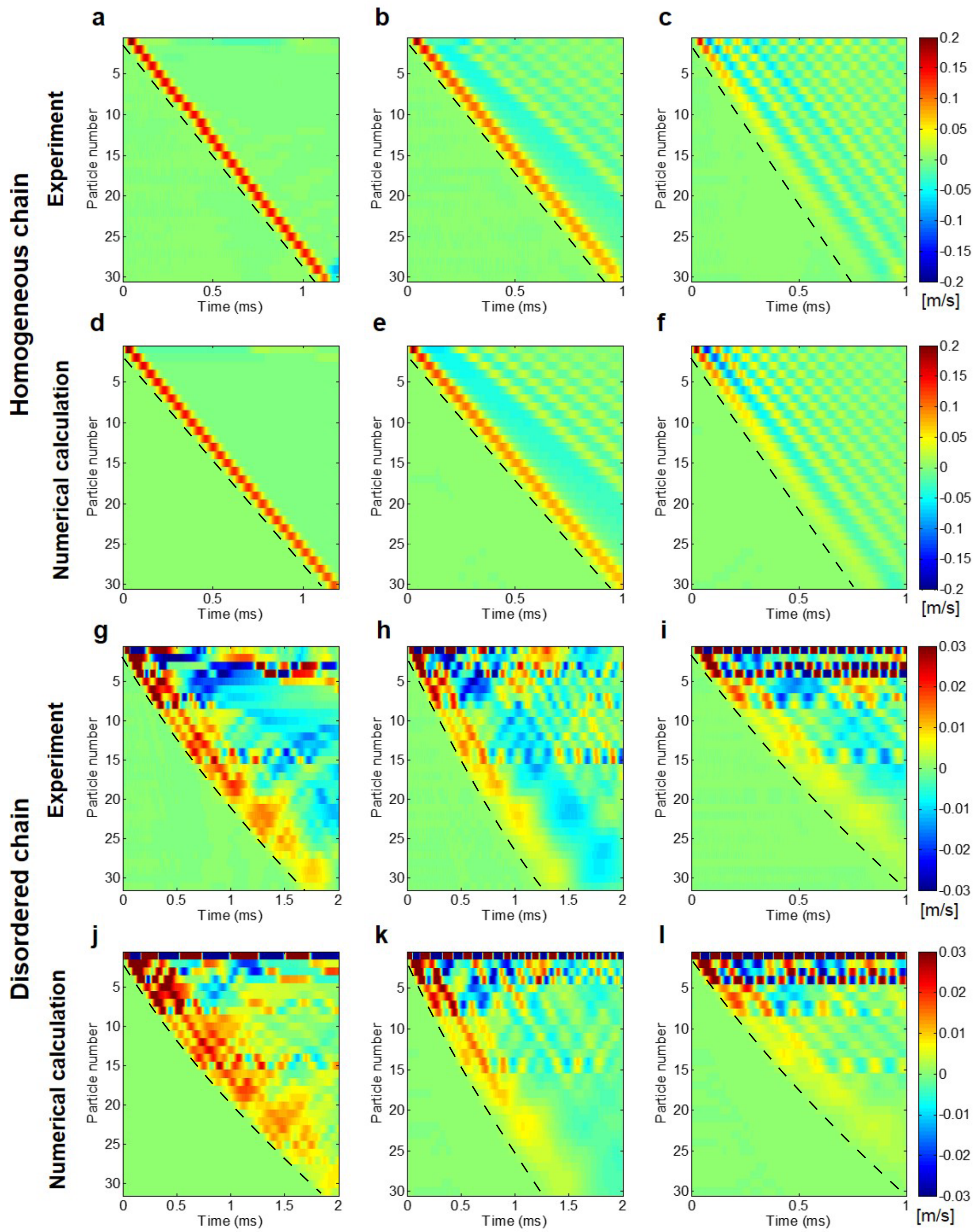

Figure 1: *Wave propagation in homogeneous and disordered chains. (a-f) Spatio-temporal maps of particles' velocities in a homogeneous chain consisting of aluminum beads at the level of high nonlinearity, weak nonlinearity, and linearity. The experiment data (a-c) are compared with numerical calculations (d-f). (g-l) Spatio-temporal maps of particles' velocities in a disordered chain at various levels of nonlinearity. The experiment data (g-i) are compared with numerical calculations (j-l). Dashed lines is displayed to visually track the edge of the velocity distributions.*

theoretical, computational, and experimental approaches [6, 11]. In this ARO project, we took a step further, to propose a wave guiding medium, which can controllably transmit or attenuate propagating waves by using this turbulence-like effect. Specifically, we designed and fabricated a novel device based on intermixing effects of dispersion and nonlinearity in the platform of woodpile mechanical metamaterial. This is highly tunable and can offer two extremes of elastic wave propagation - nearly complete transmission and strong attenuation under impulse excitation. The device is initially uncompressed one-dimensional chain that consists of identical cylinders interacting as per nonlinear Hertz's contact law (Fig. 2). This allows *in-situ* control over the stiffness distribution by setting up appropriate contact-angles of cylinders.

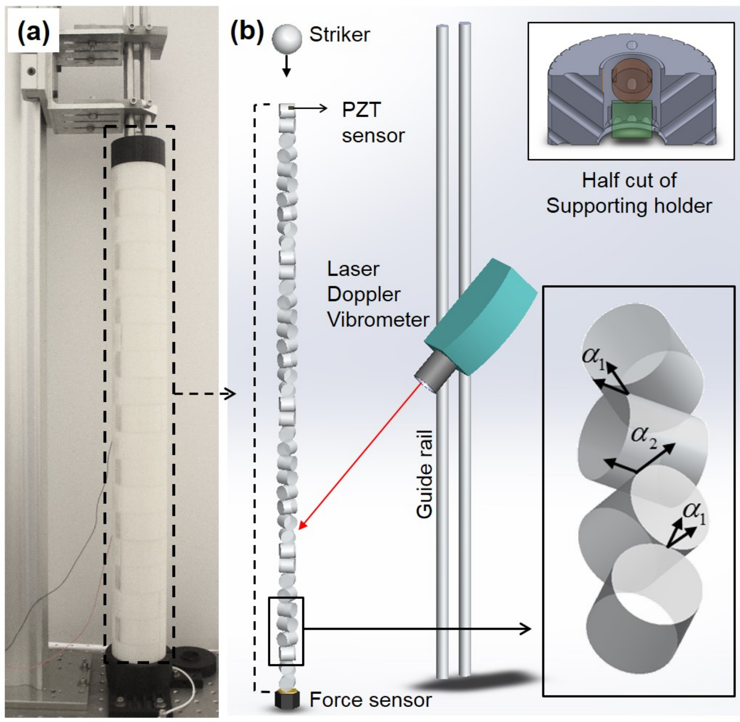

Figure 2: *Experimental setup of tunable wave guide that generates turbulence-like stress waves. (a) The cylinder chain is aligned inside the cylindrical supporting holders. (b) Schematic diagram of the test setup. The upper inset shows a half-cut of cylindrical supporting holder including two orthogonal cylinders and the lower inset represents dimer chain with two alternating contact angles.*

Using this novel experimental setup, we analytically, numerically and experimentally showed that by choosing the appropriate set of contact-angles, the impact excitation can either be localized and transmitted with minimum attenuation, or it can be highly dispersed leading to strong attenuation. Figure 3 shows these two extreme cases based on numerical simulations. The top panel shows a turbulence-like wave propagation (Figure 3a) with low-to-high frequency (Figure 3b) and large-to-small scale energy transfer. This is in sharp contrast to near perfect transmission of waves in the form of solitary waves (Figure 3d), in which we do not observe energy cascades in either frequency or wavenumber domains (Figure 3e-f). We found excellent match of experimental results with the numerics. This study well highlights the key characteristics of the versatile wave propagation mechanisms in the proposed system, that facilitate strong attenuation of incident impulse. These include low frequency to high frequency (LF-HF) scattering, and turbulence-like cascading in an ordered system. The PIs thus envision that these adaptive, cylinder-based phononic crystals, in conjunction with conventional impact mitigation mechanisms, could be used to design highly tunable and efficient impact manipulation devices. The PIs also believe that this turbulence-like mechanism is of critical importance, since it will enable attenuation of impact via this form of energy redistribution in woodpile mechanical metamaterials, without relying on material damping or plasticity effects.

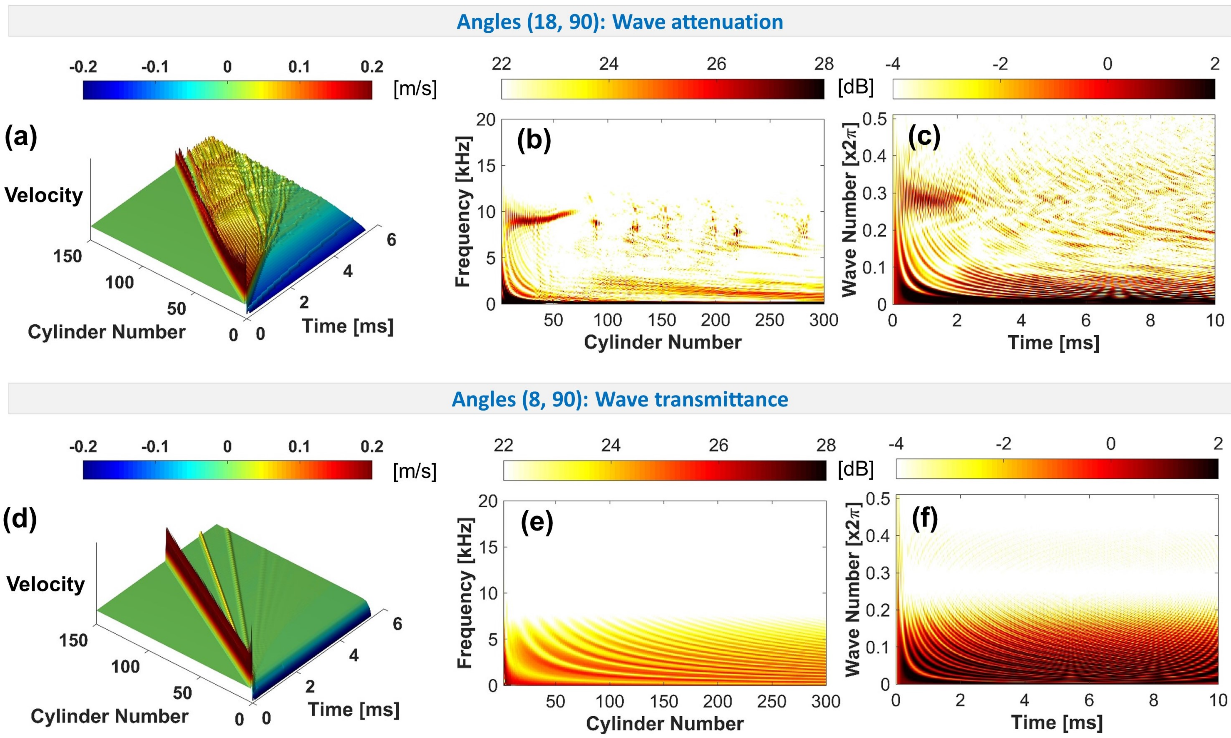

Figure 3: *Energy re-distribution in frequency and wavenumber domains. (a) Spatio-temporal map of cylinder velocity at strong dispersion case. (b-c) Corresponding Fourier transformations in time and space domains. (d-f) The same at wave localization case.*

Besides the outcome mentioned above, the PIs also achieved other scientific findings, such as unveiling a novel wave scattering mechanism in discrete nonlinear media, analogous to the Ramsauer-Townsend (RT) resonance from quantum physics [8], and designing and analyzing multi-channel phononic structures via variable stiffness mechanisms for tunable wave mixing effects [10]. The findings from this one-year project have already been published [6–10] or will be reported in peer-reviewed journals [1–5]. In addition, our teams were involved in high-profile research on the theme of granular crystals at the stage of anticipation of this grant; relevant examples include, but are not limited to, the first identification of a "nanopteron", a nonlocal solitary wave with a resonant tail in [14], as well as the invited submission of a highly visible popular article on the theme of granular crystals [15]. Similarly visible activity has been continued even upon the completion of the present grant through, e.g., the submission of one more manuscript [1], the preparation of another four [2–5], as well as the completion of an extensive invited review summarizing the current state-of-the-art in the field in [16]. The latter manuscript is at the final stage of its consideration by the Journal of Physics: Condensed Matter and we expect that it is likely to become a crucial reference for both beginners and experienced researchers in the field. This array of developments was crucially enabled by the close collaboration of the two teams, fundamentally catalyzed by the existence of this grant, which resulted in cross-pollination of their expertise in experimental mechanics and applied mathematics. It should be also noted that the two teams had frequent tele-conference meetings and a full-day workshop on November 4-5, Friday/Saturday, 2016 at the University of Washington, to discuss the progress of the project. It is the plan of the PIs that they continue exploring this rich area of nonlinear wave dynamics in mechanical metamaterials based on the fruitful intellectual product from this ARO project.

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