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SOLID LOAD DEFINITION STUDY: THE VIBRATION ENVIRONMENT

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Prepared by
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College of Engineering
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TECHNICAL REPORT AFRPL TR-68-140

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Sponsored by
Air Force Rocket ~~Propulsion Laboratory~~
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Edwards Air Force Base, California
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This report was prepared at the Solid Rocket Structural Integrity Information Center, College of Engineering, University of Utah, Salt Lake City, Utah 84112, under the Technical Direction of M. L. Williams.

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PREFACE

This report contains the findings and recommendations of a specific study undertaken by the *Solid Rocket Structural Integrity Information Center* at the recommendation of the Structural Integrity Committee of the ICRPG Working Group on Mechanical Behavior. The study was conducted under the direction of Professor M. L. Williams. The SRSIIC is supported by the U. S. Air Force Rocket Propulsion Laboratory under Contract No. F04611-67-C-0042 with Captain Scott W. Beckwith as Project Engineer.

The basic motivation for this study, as expressed by the Structural Integrity Committee, is that the ability to reliably assess the structural integrity of a solid propellant rocket motor is as dependent upon an accurate knowlecge of the loads imposed on the motor as it is upon characterization of the materials, accurate prediction of the stresses and strains in the motor, and the availability of a valid failure criteria. Since considerable progress has been made in the latter, it was deemed timely to investigate the load definition problem. The vibration environment was specifically selected for study since it is a load for which there is considerable uncertainty and one for which considerable effort and funds have been expended to assure structural integrity.

The findings and conclusions presented herein are the combined thinking and recommendations of numerous engineers responsible for solid rocket motor and weapon system development. They were condensed from countless technical discussions and personal contacts. Although there were too many individuals to list separately, appreciation is expressed for the assistance of the following organizations:

Atlantic Research Corp.
Hercules, Inc.
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Naval Missile Center at Pt. Mugu
Rocketdyne
Space Technology Laboratory
Thiokol Chemical Corp.

We are also indebted to the following members of the solid rocket community for critically reviewing and commenting on a preliminary draft of the report.

Most of the comments thus obtained have been incorporated into the report. Reviewers were:

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The invaluable assistance of Professors James W. Mar of Massachusetts Institute of Technology and M. L. Williams of the University of Utah is also gratefully acknowledged. During his tenure at the University of Utah as the 1968 Visiting Clyde Professor, Dr. Mar contributed freely of his knowledge and experience, particularly in a description of the dynamic design process. To Dr. Williams we are indebted for that special insight into engineering problems and methodology, and for his overall direction of the study.

F. R. Wagner

F. R. WAGNER
Assistant Professor,
Mechanical Engineering
Principal Investigator
F04611-67-C-0042

This technical report has been reviewed and is approved.

Scott W. Beckwith

SCOTT W. BECKWITH
Capt., USAF
Project Engineer

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INTRODUCTION

During the past decade, intensive research and study has brought about many significant advancements in the ability to predict the structural adequacy of solid propellant rocket motors, and, in particular, the influence and behavior of the viscoelastic propellant itself. A proper evaluation of the structural integrity of the solid rocket grain has required (1) development and standardization of adequate means for measuring the stress-strain behavior, or constitutive law, for the viscoelastic material, (2) development of sophisticated techniques for predicting the stress states in linear viscoelastic media, (3) development of new strain measuring instrumentation for obtaining experimental data in laboratory specimens and prototypes, (4) introduction of the structural test vehicle (STV) concept to verify analytical techniques, and (5) evaluation of failure criteria for viscoelastic media, including the effects of cumulative damage. The accurate demonstration of the structural integrity of any design, however, depends as much upon a precise knowledge of the applied load itself, as it does upon a subsequent evaluation of the effect when imposed on the structure.

This study therefore was initiated to investigate the rationale behind present load determinations and specifications for solid propellant rocket motors. As a point of departure, special emphasis has been focussed upon interactions between the viscoelastic solid propellant grain and its housing when the motor assembly is subjected to a vibration environment. The vibration environment was selected for this study since it is the load which has the greatest uncertainty associated with it, and it is a load for which significant amounts of development funds and time have been expended in order to achieve a satisfactory design. From a purely design point of view and depending on the grain design and mission requirements, the thermal, pressurization, and acceleration may be more important than the vibration environment Temperature specifications and their effect upon motor performance and cost has already been studied,(1)* and the other loadings do not have the uncertainty associated with them that the vibration environment does.

The first section of this report sets the stage for latter sections with a general discussion of the dynamic design problem. It is followed

* Numbers in parentheses refer to references list at the end of the report.

by sections devoted to summarizing the specifications that are presently imposed upon solid rocket designers and to discussions of rational means for establishing vibration requirements. Final sections are devoted to a presentation of techniques for predicting the response of solid propellant rocket motors to vibration environments and criteria for failure under random loading. Although the latter sections are excursions from the basic topic of load definition, they are necessary to illustrate the dependency of the dynamic design problem upon the method of load definition.

DISCUSSION OF THE DYNAMIC DESIGN PROBLEM

The general system of interest is the solid rocket, air or ground launched missile. From the structural standpoint, it is a thin shell of revolution almost completely filled by a viscoelastic, load supporting material approximately two thirds the density of aluminum. Aerodynamic surfaces and a nozzle are attached to the rocket case, and other components of the system include fusing mechanisms, warhead, guidance and control electronics, and ballast, all of which contribute mass to the structural system. We wish to examine those factors which bear on the structural integrity of such a system with particular emphasis upon effects that dynamic loads impose on the rocket grain. The relatively new feature in this situation is that, in contrast to metals, a viscoelastic material is very sensitive to the vibration frequency. Its loss modulus which determines the rate at which energy is dissipated in the body, is a function of applied frequency and temperature. In certain ranges of these latter parameters,(see Figure 1) significant and possibly dangerous, internal heating of the propellant can occur. In this sense, we can anticipate a thermo-mechanical-ballistic coupling which is important to design against. On the other hand ultra-conservatism in the design is inefficient and extremely costly during the development stage.

The Design Process

In order to give a proper perspective to the further discussions, it is well to briefly review the structural design process. The system requirements in terms of such parameters as range, payload, vulnerability, and probability of success will define the environment in which the missile must operate. Table I lists the various operational phases of such a missile as it performs its intended mission. The landing phase of the aircraft is also shown in case the missile is not fired. Table II lists the frequency content of the sources of dynamic loads. The environment as portrayed can be quantized by: (1) limit load factors, (2) spectrum of load factor versus vibrational frequency, (3) alternating load factor versus number of cycles, (4) time histories of the aerodynamic loads, (5) power spectral density of the random loads, and (6) temperature.

The designer, generally in terms of his past experience, decides upon a configuration for his missile; he must next subject his design to the qualification of the mission profile. This is the stage during which structural analysis and component structural testing play the dominant roles and lead to analytical and experimentally substantiated predictions for stresses, deflections, frequencies, and temperatures.

Next the structural designer must determine, by analysis or test, the ultimate capability of the chosen design. This phase is generally a more difficult task than the determination of stresses or deflections under given loads because the ultimate capability of a structure involves a knowledge of failure modes on the one hand or destruction of the test vehicle on the other.

Finally, the designer must determine if the predicted response is within the prescribed factors of safety or, alternatively, if the requisite probabilities of success have been met. The factors of safety are numbers which are subjectively determined. A number representing the probability of success, to many people, can be arrived at more rationally and is more meaningful, but the calculations required to demonstrate that a given structure has a given probability of not-failing are very involved and not sufficiently understood at this time.

The Loads Environment

The air-launched missile will experience all of the loads of the carrier aircraft, loads engendered by the atmosphere during captive flight, loads due to interactions between the aircraft and the missile, and loads during powered free flight. Presently the design criteria for aircraft utilize the envelope concept wherein limit loads for flight maneuvers, gust encounters, landing, etc. are specified. This concept is neatly summarized in the form of a v-n (velocity-load factor) diagram. For airplanes of the 1930 and early 1940 era, a few v-n diagrams for each new design could insure structural integrity. As the speed, altitude, range, and cost of airplanes increased, so did the number of v-n diagrams. The pilot induced maneuvers are essentially discrete and deterministic occurrences but the encounters with atmospheric gusts are more random. The true factor of safety of the airplane structure has also decreased due to the use of elaborate manufacturing measures to trim excess material and reduce weight. Consequently, fatigue as a failure mechanism has become an over-riding consideration. In order to properly assess the fatigue life, the random nature of atmospheric turbulence must be taken into account. This fact has led to the development of power spectral techniques both for the gust and fatigue design.

During captive flight, the missile,when it is carried as an external store is subjected to aerodynamic forces. The missile goes through the same turbulence environment which designs the carrier aircraft. Considerably less research has been accomplished in the field of unsteady aerodynamics on slender bodies. There are also interaction effects which give rise to additional aerodynamic forces. At supersonic speeds, there are additional loads due to the impingement of starting shocks and standing shocks.

During powered free flight, the missile will experience the atmospheric turbulence and its maneuvering loads. These latter may be significantly higher than those induced by the carrier aircraft.

At launching, whether from the ground or from captive flight, ignition and launching loads will arise. They may be either vibration or shock in nature and in extreme cases may induce stress waves into the grain. While viscoelastic materials characteristically attenuate wave propagation rather strongly, some launching loads and many countermeasure shock loads can reach significant importance. In the ensuing discussion, however, vibration will generally be distinguished from shock in the sense of the time of load application and intensity. Finally, wind shear loadings during the vertical acceleration of a rocket should be mentioned, particularly if the turn-over maneuvering loads occur at altitudes where sudden changes in horizontal wind velocity with altitude are significant.

Determination of the anticipated loading for the various environments enumerated above is not an easy matter. In many cases they have been bracketted statistically. As such there is an inevitable indeterminacy. The entire objective of load definition is to establish reasonable limits to impose upon the designer. Too often however, arbitrary limits become sanctified by time, even after new considerations have entered the design process. In the present case, the loading specifications have generally not previously considered the peculiarities of viscoelastic media and, understandably enough, do not reflect them. After reviewing the pertinent loading specifications, as presently defined, we shall illustrate some of the characteristics of viscoelastic materials which may in turn lead to suggestions for an improved load definition.

SPECIFICATION OF VIBRATION ENVIRONMENTS

Present Specification Practice

The vibration environment specifications to which solid propellant rocket motors have been designed and qualified can be divided into three different categories. These methods are:

1. sinusoidal oscillation at each of the resonant frequencies of the motor.(2-8)
2. sinusoidal oscillation while continuously varying the frequency over a specific range.(4,7-13)
3. wide-band random excitation.(4,10-13)

The most common vibration specification is the first type, namely sustained, sinusoidal oscillation at each of the resonant frequencies of the motor.(2-8) The qualification test required by this type of specification is conducted in two phases, namely a resonance search and a resonance dwell or endurance test. A typical specification for the resonance search phase requires that sinusoidal "excitation...shall be applied to each of the rocket motors [in the test sample] throughout the frequency range of 2 to 500 cps and critical resonances noted. Vibration measuring pickups shall be used at a sufficient number of locations on the rocket motor and components to detect and define critical resonances and associated vibration mode shapes. Vibration measures shall be supplemented by visual observations with the aid of a stroboscope to assure proper definition of the vibration mode or by audible response." The oscillation is applied along the longitudinal axis of the motor as well as in one or more lateral directions. The vibration amplitude is kept lower than that which the motor is expected to experience during its endurance tests.

After the resonance search, the motor is subjected to sustained, sinusoidal oscillation at each of the resonance frequencies found in the resonance survey. The first specifications of this type required 15 hours of vibration at an input amplitude of 0.100 inch double amplitude* or 5 g's peak acceleration at each resonance frequency of the motor.[2] It is further required that the rocket motor will be subjected to a minimum of 60 hours of vibration. A later standard[4] reduced the duration to 30 minutes at each resonance and a total test period of at least two hours.

The second type of specification requires that the motor be subjected to sinusoidal vibration but in this case the frequency of the excitation is continuously varied.[4] Typical frequency ranges are 2 to 500 cps and 5 to 2000 cps. A complete scan from the low frequency to the high and back to the low requires from 15 to 20 minutes, and is continued for 2 hours for each direction of load application.[4]

The random type of specification requires that the motor be subjected to an excitation which has components over a broad band of frequencies (50 to 2000 cps).[4] The exact nature of a random vibration specification is given by a power spectral density curve which describes how the energy of the excitation is spread over the frequency range. Integration of the power spectral density (PSD) curve leads to a root-mean-square (rms) value for the excitation level which, although not conveying the full significance of the PSD curve, does give a convenient number for comparisons. Specified values of random vibration begin at 5.3 g (rms).[4]

From the design standpoint however, the important question is not "What are the values of the vibration environment that are presently specified?", but rather, "Are the current vibration specifications realistic?". In other words do the specifications presently used reflect the vibration levels that an actual solid rocket motor might reasonably expect to encounter during its lifetime?

Table III contains a comparison of some measured vibration levels with the latest military standard, MIL-STD-810B. MIL-STD-810B is a general specification which contains a range of different test environments of increasing severity. In this manner the magnitude of a test requirement can be modified to specific applications. A simple comparison of the

*The double amplitude is the total displacement measured from one extreme position to the other.

magnitudes of the specified and measured vibration levels, however, shows that for every case except the captive flight of a missile contained in the bay of a tactical aircraft with the doors open, the low end of the specifications are equivalent or more severe than the maximum condition measured. For instance, the transportation and handling vibration for shipment by common carrier specifies an applied vibration of from 1.3 to 5 g's which compares reasonably with the 2.2 g's maximum measured on a railroad car. On the other hand, the 2.2 g maximum noted was a reasonably rare occurrence. Over 93 percent of the time the measured acceleration never exceeded 0.75 g's.

In the captive flight of an air-launched rocket carried in a bay with the bay doors open, a measured acceleration of 11.8 g's (rms) was noted whereas the maximum value in the specification is 10 g's (peak). It is possible since most of the energy in the measured power-spectral-density curve is contained in one or two peaks that instrumentation errors might have caused a larger reading than actually occurred. In general, it appears that the *magnitude* of specified environments are consistent with those measured under operational conditions although the duration may be short. There is a record of a missile which has been carried on over 100 missions,[14] which represents from about 100 to 250 hours of captive flight and the resulting vibration. Not all of this vibration, however, will have been at the maximum amplitude and certainly not at the resonant frequency of the motor.

Before passing judgement on whether or not the specifications are reasonable or not it is necessary to review the specifications in light of the failure modes of the body. The comparison of peak accelerations is adequate to ascertain the equivalence of two different vibration environments provided the body subjected to the vibration fails due to a fatigue fracture which depends only upon the amplitude of the stress and the number of stress reversals. Failure in a solid rocket motor grain subjected to a vibration environment, however, may be aggravated in two different ways. First, the normal structural failure associated with the initiation and growth of a fatigue crack can expose additional burning surface area which may lead to a runaway pressure rise on ignition, and subsequent explosion.

Second, solid propellant materials exhibit viscoelastic behavior such that their fatigue properties also depend upon frequency, not simply the stress level and number of cycles.[29,30] Thus a simple amplitude comparison technique between test sample and prototype is not always adequate. Furthermore, since most naturally occurring vibrations are random phenomena and not simple sinusoidal oscillations, it will be necessary to examine the fatigue properties of propellants when subjected to random oscillations including effects of variations in power spectral characteristics. In this connection it is important to note that the usual analysis approach considers the propellant material to be linearly viscoelastic, which may or may not be appropriate, particularly when considered in conjunction with the coupled thermo-mechanical response. In addition, the internal heating characteristic which can be rather dominant in viscoelastic media[31,32] has an important scale difference between laboratory specimen and prototype due to different scaled rates of conduction and surface radiation. These matters make it difficult to rationally establish the equivalence of two vibration tests or to establish constant-frequency or sweep-frequency, sinusoidal vibration tests which will verify the resistance of the grain to fatigue in its temperature sensitive environment. It is therefore impossible to quarantee the adequacy of substantial sections of the present specifications for viscoelastic solid propellant rocket motors.

As an amplification to the foregoing remarks, however, it is possible to compare the constant and sweep frequency sinusoidal vibration tests. The rate at which heat is generated in a linear viscoelastic material subjected to a uniaxial, sinusoidally varying strain of amplitude ε_0 is given by[33]

$$\dot{q} = \pi f \, E''(f) \, \varepsilon_0^2 \qquad (1)$$

where f is the frequency of the oscillating strain and E"(f) is the loss modulus of the material. This internal heating raises the temperature of the body and creates thermal gradients. Therefore, since the moduli of viscoelastic materials are temperature dependent the loss modulus will change as the body heats up. The loss modulus may either increase or decrease depending upon the slope of the E"(f) variation at the particular frequency being imposed (see insert in Figure 1). As an approximation, consider the isothermal response of a typical rocket motor subjected to sinusoidal vibration (see Figure 1). It was found that the peak heating rate occurred at the resonant frequency, i.e. the frequency which produces the highest strain amplitude (570 cps), and that this heating rate is over ten times higher than the average heating rate. In terms of the vibration tests, the heating rate that a solid rocket motor could experience during a resonance dwell vibration test may be as much as ten times greater than it would experience during a sweep frequency type of test. Since in a random vibration all frequencies occur in an unpredictable fashion it is to be expected that random vibration would be more like a sweep frequency than a resonance dwell test. It is therefore concluded that due to the internal heating, a resonance dwell test conducted at similar amplitudes to random vibration is more severe than the random vibration.

The Ideal Vibration Specification

In the preceding discussion, the vibration environment was specified in terms of an acceleration level and its spectral characteristics. This tacitly assumes that the presence of the motor or its configuration did not affect the vibration environment. In addition there apparently has been little or no attempt to tailor the vibration specifications to specific motor applications.* In studying the response of a simple, two-degree-of-freedom, linear system consisting of one single degree-of-freedom system placed upon another single-degree-of-freedom-system to random vibration, Curtis and Boykin[34] found that the existence of the second single-degree-of-freedom system has a significant effect upon the response of the lower system. By analogy then, one might expect that under some conditions the presence of the solid rocket motor itself can affect the response of the system and hence the environment. In addition, the existence of the second system always decreased the response of the first. In other words the second system acts as a vibration absorber and reduces the amplitude of the environment to which it is subjected. Most significant effects occur whenever the mass of the second system approaches the mass of the first

* MIL-STD-810B has the capability of modifying the vibration amplitudes through the selection of specific curves within the specification but there is little flexibility over the shape of the spectrum.

($M_2 > 0.1\ M_1$), or whenever the resonant frequency of the second system is less than twice the lower system.

The above absorption effect has also been noted in solid propellant rocket motor vibration tests. The observed effect is that at certain frequencies known as "antiresonances" the test equipment is incapable of providing sufficient power to maintain the input vibration at the specified acceleration level. Under these conditions the force required to drive the motor at the required acceleration level will usually exceed the maximum loads which can be applied to the carrier vehicle. This has caused several motor contractors to insist that motor vibration environments be specified in terms of applied force instead of input acceleration. Force specifications are certainly more meaningful but require that the vibration environment be predicted for each rocket-carrier system.

To further illustrate the fact that the design of the rocket-carrier system can have a significant effect upon the vibration environment, examine the power spectral density curves given in Figures 2 and 3. Figure 2 illustrates how the power spectral density of the vibration changes with speed of the vehicle, and Figure 3 shows the variation in the spectra with direction of measurement. Note that in all cases a peak or spike in the spectrum occurs between 80 and 100 cps. This peak is attributed to a resonance in the suspension of the carrier. This of course would not be a characteristic of tracked vehicles in general but is a peculiarity existing in this specific design.

To conclude this discussion, it is noted that the ideal vibration specification for a solid propellant rocket motor will reflect (1) both the dynamic characteristics of the motor and the system in which it is incorporated, and (2) the mission profile.

PREDICTION OF VIBRATION ENVIRONMENTS

In order to incorporate vibration loads into the design, it is necessary to analytically determine the vibration environment for different motor and system designs as well as various mission requirements. Reference 35 contains a summary of known random vibration prediction techniques with emphasis upon predicting spacecraft environments. The techniques presented fall into four main categories, namely:

1. Analytic predictions for continuous bodies and structures.
2. Analytic predictions for approximate, lumped-parameter systems.
3. Semi-theoretical extrapolations of experimental data.
4. Analogs and model studies.

Ideally one would like to predict the design requirements for the solid rocket motor by (1) determining those excitation forces for the system which are independent of the system design and then (2) calculating the response of the total rocket-vehicle system to these forces. Prediction methods 1, 2 and 4 fall into these categories and differ only in the manner in which the system is modeled and in the way in which the calculations are carried out.

As a simple illustration, consider a linear spring-mass-damping mechanical system. Its governing differential equation says that certain characteristics of the system (mass, spring constants, damping coefficients) and time derivatives when multiplied by the response, equal the input. Stated alternately there are three major quantities: the input, the response, and the system representation or "transfer function." Within certain approximations, essentially linearity, a knowledge of any two of these will permit the calculation of the third.

Now associate the same concept with an airplane or missile, and consider again the same basic quantities. The input are those atmospheric gusts, rough surfaces, etc. which the system may encounter and which are independent of the system. The response is the loads environment and is predictable from a knowledge of the input and the transfer function for the system. The first step in the process is to identify and measure those fundamental phenomena which are the exciting forces. To illustrate the measurement procedure, consider atmospheric gusts. In this case it is possible to use an airplane as a device to measure the energy content in the atmosphere as manifested by the atmospheric gust intensity. First the airplane must be calibrated. For this purpose a known input (step elevator pulse) is imposed onto the airplane and the resulting vertical acceleration (response) is measured. Now since two quantities (input and response) are known, it is possible to compute the third, namely, the transfer function which in this example characterizes the physical parameters of the aircraft. Thus, with the transfer function known, the calibration of the measuring instrument is complete and it can be used to measure desired unknown quantities, namely the gust intensity. All that is required is to measure the responses for a specified set of conditions. With those measurements as one known quantity plus the second known quantity, namely, the transfer function of the airplane, the third quantity -- the desired input -- is deduced.

If it could then be established that the characteristics of the atmospheric input are similar under widely varying conditions, or that the variations could be suitably measured such as at various stations and altitudes throughout the world, the input would be (statistically) determined once and for all.

From the design standpoint, this means that for any specified or known new structure, plus the known atmospheric gust energy input, the response of the new structure could be calculated. Eventually a more difficult problem could be attacked; design an optimum transfer function (airplane or missile) for a specified (known) response and input.

Characterization of these fundamental input quantities is underway and includes such items as atmospheric turbulence,[36-39] wind profile,[40] road roughness,[41] and runway roughness.[42-43] From this data it is thus possible to predict the response of various systems.[39,44-53]

This basic approach was employed in predicting the sinusoidal transportation and handling environment for the third stage motor of the MINUTEMAN missile.[19] In this study the motor-transporter system was modeled using two different two-dimensional, lumped parameter systems. The motor was treated as a rigid body in both models. This assumption was later shown to be satisfactory since the forcing frequencies encountered in the transportation and handling environment were much lower than the first natural frequency of the motor. Determination of the parameters for the model was accomplished by experimentally measuring the response of the system while driving the transporter over a spaced board test track. The loading function for the analysis was assumed to be sinusoidal, and the determination of the input force was taken from a special track designed to give sinusoidal excitation. An amazingly good correlation with road test measurements was achieved. The maximum acceleration was predicted to be 0.84 g(peak) whereas the maximum measured was 0.80 g(peak).

No reference to analytical prediction of the response of a solid rocket-carrier system to a random vibration input has been uncovered. In the absence of sufficient data to apply such an analysis, Curtis(54) and others(24,35,55-57) have proposed semi-theoretical methods for extrapolating experimental data from one application to a similar one. The method of Curtis(54) is particularly useful since it is relatively simple to use and is directly applicable to rockets carried in aircraft. Reasonable success was reported for this method in predicting the environment for the F-111/Phoenix system.(22) In the Curtis method the random vibration environment is assumed to be a broadband random vibration spectra with several narrowband peaks superimposed. The broadband portion of the environment is assumed to be proportional to the free stream dynamic pressure of the aircraft and is derived from statistical analyses of a large amount of experimental data. The narrow band spectraum peaks represent center frequencies which depend upon the natural frequencies of the rocket-aircraft system. Similar techniques have been used for the flight portion of ground launched rockets.(24)

DYNAMIC RESPONSE OF SOLID PROPELLANT ROCKET MOTORS

Several papers have been written outlining the status of the dynamic response problem for solid propellant rocket motors.(58,59) The essential requirement is a satisfactory, general technique for predicting the stress-strain response of the grain due to the oscillatory loads. The purpose of this discussion is to summarize those portions of particular interest in the solid rocket motor vibration problem. It is convenient to divide the discussion into three parts: (1) predictions of the response to sinusoidal vibration under isothermal or known steady state thermal gradients, (2) effect of mechanically induced internal heating (thermo-mechanical coupling), and (3) the treatment by random analysis.

Response to Isothermal Sinusoidal Loading

The first analysis approach assumes that extended sinusoidal vibration does not generate internal heating of the rocket grain. This isothermal assumption considerably simplifies the analysis, and while physically suspect for other than very low or very high frequencies, it provides a point of departure and permits analytically tractable solutions. Closed form solutions may be found if the rocket motor is modeled as a hollow, right circular cylinder bonded inside a thin elastic case. The response of propellant grain projections which extend inward from the circular walls of propellant in internal-burning star and related configurations are sometimes treated independently, usually as cantilever beams or plates.[60] This basic thick walled cylindrical configuration is then subjected to sinusoidal oscillations in one of three modes -- lateral, axial or torsion. It will be noted that the lateral excitation can give rise to both lobar (breathing) and transverse (bending) modes of vibration.

Before discussing the particular solutions, some related ones should be mentioned. Many of these latter solutions consider the grain to be viscoelastic. References 61 through 64 present solutions for two transient vibrations which result from suddenly applied loads. The first three papers[61-63] deal with radial vibrations induced by a suddenly applied body force in the axial direction, with obvious application to highly accelerated rockets such as Hibex or Sprint. References 65 and 66 present solutions for forced vibration created by sinusoidally varying internal pressures. If uniformly applied along the axis of the motor, this type of loading produces pure radial motion and has been used in studying the possible coupling of mechanical behavior with combustion phenomena during unstable burning.

Using the classical techniques, solutions for forced vibration problems are obtained in two steps. The first step is the solution of the free vibration in which one assumes the body to be deformed from its equilibrium shape, released, and then permitted to oscillate without the action of external forces. This part of the problem leads to an eigen equation which yields the natural frequencies and mode shapes for the body. In a standard solution technique, the stress or strain solutions for known inputs are expressed in terms of these eigen or characteristic functions. Table IV contains a summary of the analytical solutions pertinent to the forced vibration of solid propellant rocket motor grains. Note that with the one exception of torsional vibrations, the grain or core is assumed to be elastic and in some cases incompressible. It should also be noted that the first entry in Table IV is for a solid core. This configuration is a model for a case bonded, end-burning grain. Furthermore, several of the solutions incorporate a rigid case assumption. Only in the case of flexural or torsional vibrations have finite length considerations been introduced and then only because they can be handled in a trivial fashion.

With the lack of readily available analytical solutions for forced vibration problems, there has been considerable impetus for the development of numerical techniques to solve these problems on high speed computers. The lumped parameter technique(82) has been used for some time in the study of aircraft, missile, and spacecraft structures. By adapting the techniques employed for static loadings of continuous bodies already so popular in the solid propellant industry, it has been possible to achieve programs for studying axisymmetric, plane strain and plane stress dynamic problems.(83-86) Two different methods for accounting for the viscoelastic nature of the propellant are used. The first simply employs viscous damping whereas the second utilizes the elastic-viscoelastic correspondence principle to transform an elastic solution into the steady-state viscoelastic solution. It will be noted that arbitrary temperature profiles can be handled in these computer solutions. Recently a more thorough treatment of the core-case interface has been developed.(87) Although this formulation can, in principle, treat dynamic problems, no results have yet been completed on the computer.

Although the combination of the analytical and numerical techniques referred to above permit one to handle a variety of forced vibration problems, there do not appear to be complete treatments for viscoelastic cores or consideration of end effects.

Thermo-Mechanical Coupling in Viscoelastic Vibration Problems

The fact that heat is generated in a cyclic loaded viscoelastic body couples the mechanical behavior of the system to the thermal. In turn, the mechanical properties of viscoelastic materials are strongly temperature dependent; hence the coupling between the thermal and mechanical behaviors works in both directions. Because of this coupling, closed form solutions are possible for only the simplest configurations which include:

1. long rod or slab insulated on the sides and sinusoidally loaded on one end.[88]
2. thin-walled tube subjected to torsional oscillations.[89]
3. sinusoidal shear loadings of slabs and cylinders.[90*,91]
4. cantilevered plate under the action of steady state cyclic loading.[92]
5. time dependent internal pressurization of hollow spheres and cylinders.[93]

An important result of the above studies is that a steady (mechanical) state is achieved after two or three cycles, but the temperature continually changes and by large amounts, depending of course on the heat diffusivity and radiation conditions, long after mechanical equilibrium has been attained. Another conclusion is that the numerical solutions for the sinusoidal loading of viscoelastic bodies can handle arbitrary temperature profiles, thus leading to an ad hoc method for the solution of coupled thermo-mechanical problems. Basically the ad hoc procedure consists of (1) calculating the initial steady state mechanical response using the initial temperature distribution, (2) determining the instantaneous distribution of internal heating which can be combined with a two dimensional transient heat conduction program to calculate a new temperature distribution after a short time Δt, and then (3) using the new temperature distribution start to reiterate the steps.

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* Reference 90 also treats random loadings.

Treatment by Random Analysis

Random analysis arises as an outgrowth of normal Fourier or harmonic analysis.[94] Instead of calculating the exact response to an arbitrary input however, attention is focussed upon the mean square response to the mean square input for loading situations which over reasonable time intervals are sensibly stochastic or random.

Definition of Terms. Having previously described in somewhat general terms the concept of transfer functions, it is important next to become more precise and introduce those definitions and quantities which are needed to reduce the generalities to specific calculation of random phenomena.

1. *Stochastic Process* - a term from the theory of probability. A stochastic process is one which varies in an arbitrary fashion. Unlike deterministic processes the magnitude of a stochastic process is not uniquely defined, i.e., one may not precisely state what the magnitude of the loading function is at a particular time but if the process is stationary it is possible to establish the probability that the amplitude of load lies within a certain small amplitude range at any time. This is in fact one method for describing a random process. A *stationary* stochastic process is one where integrals of the variable over long intervals of time give an average behavior independent of time. From the standpoint of an oscillograph record this means that for any reasonably long interval of tape, the average value of a random trace is the same as over any other reasonably long interval.
2. *Autocorrelation Function* - a definition which relates the value of the random variable at one time instant with its value at a neighboring time and hence measures the interdependence of the function at a given time with what happened before and after. Specifically, the autocorrelation function, $\psi(\tau)$, is defined

$$\psi(\tau) = \int_0^\infty u(t)\, u(t+\tau)\, dt \tag{2}$$

where u(t) is a random variable.

3. *Power Spectral Density Function* - the Fourier transform of the correlation function which depends on the frequency and is essentially a measure of the energy content of the random variable at various frequencies. If $f(\omega)$ is the power spectral density function

$$f(\omega) = \frac{2}{\pi} \int_0^\infty \psi(\tau) \cos \omega\tau \, d\tau \tag{3}$$

Because this is a Fourier transform, it is known that reciprocal relation exists such that

$$\psi(\tau) = \int_0^\infty f(\omega) \cos \omega\tau \, d\omega \tag{4}$$

This therefore gives an alternating method of finding the correlation function, providing the power spectrum $f(\omega)$ is known.

4. *Spectrum (amplitude) Intensity* - a measure of the total energy given up by the random variable, it is the area under the power spectrum curve. It can be shown[94] to be equal to the mean square value of the random variable.

$$\overline{u^2} = \int_0^\infty f(\omega)\, d\omega \tag{5}$$

Note also that

$$\overline{u^2} = \psi(0) = \int_0^\infty u(t)\, u(t+0)\, dt \tag{6}$$

<u>A Particular Example Using a Periodic Forcing Function</u> . Consider a simple mechanical system, that is one for which the differential equation can be written, for example:

$$y'' + \beta y' + \omega_0^{\,2}\, y = F(t) \tag{7}$$

where

$$F(t) = A\, e^{i\omega t}$$

In order to obtain an insight into the relations between the mean square amplitude, the frequency of the forcing and the response function, examine this case where we know or can compute the exact answers.

The solution of equation (7) is:

$$y(t) = B\, e^{i(\omega t+\phi)} \tag{8}$$

where

$$B^2 = \frac{A^2}{(\omega_0^{\,2}-\omega^2)^2 + \beta^2\omega^2} \tag{9}$$

$$\phi = \tan^{-1}\left[\frac{-\beta\omega}{\omega_0^{\,2} - \omega^2}\right] \tag{10}$$

and the mean square average of y(t) can be computed as

$$\overline{y^2} = (B^2/2) \tag{11}$$

and similarly

$$\overline{F^2} = (A^2/2) \tag{12}$$

Therefore using (9), the relation between the mean square input and response is

$$\overline{y^2} = (B^2/2) = \frac{\overline{F^2}}{(\omega_0^{\,2}-\omega^2)^2+\beta^2\omega^2} \equiv \frac{\overline{F^2}}{|z(\omega)|^2} \tag{13}$$

where the impedance,* $z(\omega)$, is defined by

$$|z(\omega)|^2 = (\omega_0^{\,2} - \omega^2)^2 + \beta^2\omega^2 \tag{14}$$

<u>The Case for a Random Input</u>. When the input is random, we can ask the same question regarding the relation between the mean square values. First, however, the frequency properties of the random forcing function must be obtained by using the power spectrum, $f(\omega)$, of $F(t)$ because if it were known then

$$\overline{F^2} = \int_0^\infty f(\omega)\, d\omega \tag{15}$$

The power spectral density function of the input $f(\omega)$ may be determined experimentally in some cases, such as wind tunnel turbulence, by using a band pass filter or mathematically by forming the correlation function $\psi(\tau)$ of $F(t)$

$$\psi(\tau) = \int_0^\infty F(t)\, F(t+\tau)\, dt \tag{16}$$

and taking its Fourier transform

---

* The reciprocal of the impedance is called the admittance.

$$f(\omega) = \frac{2}{\pi} \int_0^\infty \psi(\tau) \cos \omega\tau \, d\tau \tag{17}$$

Knowing $f(\omega)$, it can be proven by generalized harmonic analysis that the power spectrum of the response, say $g(\omega)$, is related to the power spectrum of the forcing function $f(\omega)$ by

$$g(\omega) = \frac{f(\omega)}{\text{impedance of the system}} = \frac{f(\omega)}{|z(\omega)|^2} \tag{18}$$

It is then a simple matter to compute the mean square value of the response from

$$\overline{y^2} = \int_0^\infty g(\omega) d\omega = \int_0^\infty \frac{f(\omega)}{|z(\omega)|^2} d\omega \tag{19}$$

<u>The More General Case</u>. In the previous cases the impedance was known because the characteristics of the spring mass balance system could be expressed mathematically, i.e.

$$|z(\omega)|^2 = (\omega_0{}^2 - \omega^2)^2 + \beta^2\omega^2 \tag{20}$$

but in many cases this is not easy to do. For many dynamic problems, the impedance is quite complicated and is best measured experimentally. For example, from the linearized equations of motion, the unknown differential equation can be represented by Lw, where w for example may be the transverse displacement, which can be equated to the forcing function. Thus,

$$Lw = F(t) \tag{21}$$

By taking the Laplace Transform to find the frequency representation

$$L(\omega)\, w^* = F^*(\omega) \tag{22}$$

where the star indicates the transform. Thus, the response in the frequency field is

$$w^* = \frac{1}{L(\omega)} F^*(\omega) \tag{23}$$

As discussed earlier the quantity $[1/L(\omega)]$ which describes the physical characteristics of the system is called the transfer function, and in the case of the simple system is immediately seen to be associated with the impedance function. For example in that case $L(\omega) = |z(\omega)|^2$.

Now by measuring the response $w^{*}(\omega_0)$ to a series of inputs with fixed frequency $F(\omega_0)$, the only unknown is $L(\omega_0)$, which can be calculated for several (as many as desired) frequencies and the result* plotted as $L(\omega)$ vs. ω. With the impedance, which was the quantity required in (18), being related to the transfer function, it is therefore seen that the impedance and thus the power spectrum and mean square value of the response can be experimentally determined by dividing known inputs into measured responses as initially described qualitatively in the first section.

Application to Solid Rocket Motors. Assume that the stress or strain response of the motor to sinusoidal vibration is known as a function of frequency and amplitude. This may be defined by the transfer function $L(\omega)$ or the impedance $z(\omega)$. We now desire to determine response of the motor to a random vibration.

The oscillatory input being a stochastic process is described in terms of a power spectral density (PSD) curve which reflects how the energy or power of the vibration is distributed with frequency. The ordinate of the PSD curve is the power per unit band width. In the case of vibrations, power is proportional to the peak acceleration squared. Since accelerations are normally expressed in g's, the units for the ordinate of a vibration PSD curve are g^2/cps. The overall intensity of the vibration is given by the rms value of the acceleration. In accordance with equation 5 the rms value of the acceleration is simply the square root of the integral of the PSD curve taken overall possible values of frequency, i.e.

$$a_{rms} = \sqrt{\int_0^{\infty} f(\omega)\, d\omega} \tag{24}$$

When a body or structure is excited by random loads its response will also be random. In the case bodies of linear materials, the simplest approach for determining the power spectral density curve for the response is equation 18,[34,95,96] i.e.

* As a matter of test technique it frequently proves more convenient to use step inputs or pulses rather than fixed sinusoidal forcing functions because the duration of the test is shortened considerably as well as there being other advantages in data reduction.

$$g(\omega) = \frac{f(\omega)}{|z(\omega)|^2} = \frac{f(\omega)}{L(\omega)} \tag{25}$$

where $g(\omega)$ is the PSD of the response, $L(\omega)$ is the transfer function describing the frequency response of the body, $z(\omega)$ is the impedance, and $f(\omega)$ is the PSD of the input.

Reference 97 presents an alternate solution for the response of elastic bodies to random vibration. In this method the classical procedure of summing over natural modes of free vibrations is replaced by an integration over a certain range of frequencies or wave numbers.

The preceding methods apply whenever there is no spatial variation in the random vibration field. A method for computing the correction which is required to take into account the spacewise variation in the input field is presented in reference 98.

FAILURE IN A VIBRATION ENVIRONMENT

The final step in verifying the structural integrity of a body is the comparison of the stresses and strains induced in the body by the anticipated service loads with an appropriate failure criteria. It will be noted that although the fatigue properties of metals show little or no dependency upon frequency they still exhibit strange behavior when subjected to random loads. It is therefore very difficult to anticipate the fatigue characteristics of viscoelastic materials in general and solid propellants in particular.

The approaches to the fatigue failure of metals under random loading fall into three general categories. These categories are:

1. Empirical approach
2. Prediction of random behavior from constant amplitude fatigue data
3. The "dominant crack" or fracture mechanics approach

In the first approach random S-N curves are plotted using the rms stress level and the number of zero crossings with a positive slope as the coordinates.[99,100] Tests are conducted using the same spectral distribution of stresses but varying the level. Lifetimes established are reported to be less than those predicted using the Palmgren-Miner linear cumulative damage law.

Most methods for predicting the fatigue behavior of materials under random loadings from their constant amplitude fatigue data is based on the Palmgren-Miner law.[101-104] This law states that the amount of damage $x(n)$ done to a material after n cycles of fatigue is equal to the sum of the number of cycles at each stress level divided by the demonstrated number of cycles at the same stress level without failure. The strength S remaining after n cycles of fatigue is thus

$$S = S_u[1 - x(n)]$$

where S_u is the ultimate strength of the material for static loadings. The probability of failure on the next cycle, i.e. the $(n + 1)$th cycle is simply the probability of getting a stress greater than the remaining strength.

A modification of Palmgren-Miner law is the double linear damage law proposed by Manson and associates.[105] In this method the life of a material is divided into two phases, namely a crack initiation phase and a crack propagation phase. The essential feature is that the damage due to the crack initiation phase must first be assessed; then the cumulative damage in fatigue is evaluated using crack propagation data.

In the third category, the "dominant crack" or Griffith criteria states that failure occurs when the largest or dominant crack in the body propagates to a size large enough to make the applied stress critical. The critical stress is given by a formula of the type

$$S_{crit} = \frac{k}{\sqrt{\ell}}$$

where k is a constant dependent upon the geometry of the crack and material properties, and, for viscoelastic materials, the frequency. The characteristic size of the dominant crack is ℓ. Most work in this area is devoted to establishing how the characteristic crack length varies with time and loading.[30,106-109]

CONCLUSIONS AND RECOMMENDATIONS

Modification of the Design Specifications

The basic conclusion of this study is that due to the viscoelastic nature of solid propellants, the sinusoidal, resonance dwell vibration specifications to which solid propellant rocket motors are commonly qualified are unrealistic and more severe environments than the motors may expect to encounter in service. Even if an unrealistic load requirement results in an overly conservative design, such practice is not necessarily undesirable unless it also results in either reduced performance, increased cost, or both. For the case of solid rocket motors and vibration specifications the proof that performance is significantly reduced or development costs significantly increased is at least an order of magnitude more difficult than the demonstration that the specifications are unrealistic. However, it is the consensus of the numerous solid rocket engineers interviewed as a part of this study that the necessity for satisfying the present vibration requirements has resulted in significantly increased development costs. The difficulties documented in reference 31 also attest to this situation.

It is further recommended that the vibration specifications for solid propellant rocket motors be based upon predictions which account for (1) the dynamic characteristics of the motor, (2) the dynamic characteristics of the rocket carrier, and (3) the mission profile of the system. A variety of prediction techniques are available and the fundamental physical phemomena responsible for vibration loads, i.e., atmospheric turbulence, road and runway roughness, etc., are being characterized and catalogued. It should be noted that the atmospheric gust data which is being gathered uses an aircraft flying horizontally. This data is directly applicable to predicting the vibration environment for a rocket during captive flight but its application to a vertically rising, free flight rocket is suspect. An investigation of potential important altitude variations in the gust spectra, as they may affect the vibrations of rockets, is therefore suggested.

Improvement of Analysis Technique

In addition to improper vibration load definitions, deficiencies have been noted in the available techniques which may be used to predict the effect of vibration upon solid propellant rocket motors. Even those analytical solutions which are available, i.e., composite, concentric cylinder under isothermal (zero internal heating) conditions, do not have the capability for handling viscoelastic or finite length cores. It in fact seems highly unlikely that closed-form, analytic solutions can be found which will have the required coupling of the dynamic loading with the heat conduction and internal heating problem, and include temperature dependent properties. An ad hoc calculation procedure is therefore recommended which utilizes a finite element solution for the steady state dynamic problem and which is stepwise linked to a numerical routine for transient heat conduction. This technique will yield an approximate solution to the thermo-mechanically coupled problem. It is justified by the empirical observation that mechanical transients in dynamically loaded viscoelastic bodies are much shorter than the thermal transients.

Although the prediction of the system response to random excitation is straightforward in the case of isothermal, linear systems, the problem of cyclic loaded viscoelastic bodies is more complicated since the internal heating is a nonlinear phenomena. This makes the application of standard random response techniques to the prediction of solid rocket motor response to random vibration questionable. In the absence of better techniques, however, the aforementioned procedures are recommended.

Utilization of a Structural Test Vehicle (STV)

Further study of the correlation between flight measurements and analysis is highly desirable. It is therefore further recommended that instrumented structural test vehicles be used to measure both the mechanical and thermal states within the simulated grain, and that this vehicle be subjected to programmed sinusoidal and random vibration in addition to the calibration tests. Tests should be conducted at a sufficiently low vibration level that a full spectrum of tests may be conducted before the test vehicle is permanently damaged, but eventually extended to include a reasonably wide range of input intensities and spectral distribution. The sinusoidal tests should be conducted at a variety of frequencies in the range of 2 to 5,000 cps. Tests should also be made at different input amplitudes to determine if the mechanical response is linear. The thermal data should include total temperature rise and the temperature rise rate, and to the extent safety permits, continued to failure.

Administrative Coordination

Inasmuch as several agencies are involved in setting the design requirements and specifications, it is highly recommended that an ad hoc inter-agency task force be assembled to explore the possibility of achieving joint agreement as to proposed modifications. One of the most important conclusions of our study has been the conviction that a proper resolution of the load definition problem for solid rocket motors in conjunction with performance requirements and economic matters should lead to substantial improvement in the overall system design.

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TABLE I

DYNAMIC LOADS SOURCES FOR EXTERNALLY CARRIED AIR-LAUNCHED SOLID PROPELLANT ROCKETS

PHASE	APPROXIMATE DURATION	VIBRATION SOURCES
Transportation and Warmup	30 secs. to 5 min.	Airplane engine, and jet exhaust
Taxi	1 min. to 5 min.	Runway roughness, engine, and jet exhaust
Run-up	30 secs. to 5 min.	Engine, jet exhaust, ground winds
Take-off	30 secs. to 60 secs.	Engine, jet exhaust, ground winds
Climb	3 min. to 20 min.	Engine, jet exhaust, atmospheric turbulence, boundary layer turbulence
Cruise of Aircraft	20 min. to 1-1/2 hr.	Engine, jet exhaust, atmospheric turbulence, boundary layer turbulence, flight maneuvers of aircraft, buffet, blast waves from enemy countermeasures, shock impingement
Air Launch	1 sec. to 5 sec.	Starting impulse of rocket engine, noise from rocket engine, shock impingement
Powered Free Flight of Rocket	1 min. to 3 min.	Rocket engine, atmospheric turbulence, boundary layer turbulence, flight maneuvers, buffet, blast and shock from enemy countermeasures
Descent of Aircraft	5 min. to 10 min.	Atmospheric turbulence, boundary layer turbulence, flight maneuvers, buffet due to flaps and gear-down configuration
Landing	1 sec. to 2 min.	Touchdown impact, deceleration due to brakes and engine reversal, runway roughness

TABLE II

VIBRATION SOURCES

	Freq. range, cps	Dominant range, cps
Turbojet	50 - 10,000	100 - 1,000
Atmospheric Turbulence (Gusts)	0 - 20	0 - 10
Wind Shear	0 - 10	0 - 5
Boundary Layer Turbulence	100 - 10,000	500 - 5,000
Runway Roughness	5 - 30	.5 - 5
Buffeting and Oscillating Shocks	5 - 500	5 - 50

TABLE III

COMPARISON OF SOME MEASURED VIBRATION LEVELS WITH VIBRATION SPECIFICATIONS

MODE OF OPERATION	MEASURED VIBRATION LEVELS	SPECIFIED ENVIRONMENT[4]
I. Transportation and Handling		
A. Shipment by Common Carrier		Part 1. Sinusoidal resonance search: 2 to 500 cps Part 2. Sinusoidal resonance dwell: 1.3 to 5.0 g (peak) for 30 min. at each resonance (2 to 500 cps) Part 3. Sinusoidal under sweep frequency: 1.3 to 5.0 g (peak) for 45 min. in each axis (sweep rate = 15 min. for each 5-500-5 cps cycle)
1. Truck		
a. paved road	0.35g(peak)(max at 4 cps)[15]	
b. rough roads (20-25 mph)	1.7 g(peak)(max at 10 cps)[15] 2.1 g(peak)[17]	
c. cross-country (1-10 mph)	5.0 g(peak)[17]	
2. Tractor-Trailer	3.7 g(peak)(max in 240-350 cps region) (occurred less than 1% of time)[16]	
3. Railroad		
a. over the road (50-70 mph)	0.8 g(peak)(max at 1000 cps)[15] 2.2 g(peak)(max value noted 93% of vibration was less than 0.75g)[17] 2.0 g(peak)(predominant frequencies in 2.5 to 7.5 and 50 to 62 cps regions)[27]	
b. switching shack (transient)	35 g(peak) with 8 mph impact[18]	
4. Aircraft, propeller-driven with reciprocating or turbo engine	5.0 g(rms)(max at 400 cps)[15] 5.0 g(peak)(1 to 250 cps)[27]	
5. Aircraft, jet engine	2.8 g(rms)(max at 800 cps)[15]	
6. Helicopter	7.0 g(rms)(max at 400 cps)[15] 1.0 g(peak)(3.5 to 35 cps)[27]	
7. Ships		
a. calm seas	0.08g(peak)(max's at 1.5 & 15 cps)[15]	
b. rough seas	0.8 g(peak)(max's at 0.1 & 15 cps)[15]	
c. emergency maneuvers	3.0 g(peak)(max's at 2.5, 12 & 45 cps)[15]	
B. Shipment by Special Transporters		
1. Minuteman Stage III/Boeing Transporter[19]	0.8 g(peak)(10 times more peaks noted in the 0.25 to 0.5 g range than in the 0.5 to 0.75 range)	
2. Nike I (storage-to-launcher)[20]	1.0 g(peak)--longitudinal 1.5 g(peak)--lateral 3.0 g(peak)--vertical	
3. Titan III Segment/tractor Trailer Transporter[21]	1.3 g(peak)--fore-aft 1.0 g(peak)--lateral and vertical	
II. Air Launched Rocket		
A. Captive Flight (tactical aircraft)		Part 1. Sinusoidal resonance search: 5 to 50 cps or 5 to 2000 cps Part 2. Sinusoidal resonance dwell: 5 or 10 g (peak) for 30 min. at each resonance Part 3. Sinusoidal sweep frequency: 5 or 10 g (peak) for 2 hrs. in each axis (sweep rate = 20 min. per 5-2000-5 cps cycle)
1. Carried in bay (doors open) (g = 1480 psf)[23]	4.3 g(rms)--longitudinal 9.3 g(rms)--lateral 11.8 g(rms)--vertical (spectrum covers 20 to 2000 cps with a maximum in the 500 to 1000 cps region)	
2. Externally Carried	4.7 g(rms) for Mach 1.7 at 35,000 ft. 6.3 to 9.0 g(rms) for 600 knots at 5,000 ft. altitude[23] 3.0 g(peak)(7 to 180 cps)[27]	
B. Powered, Free-Flight	0.86 to 2.8 g(rms)[23]	Part 1. Sinusoidal sweep frequency: 5 to 20 g (peak) for 2 hrs. in each axis (sweep rate = 20 min. for each 5-2000-5 cps cycle) Part 2. Random: 0.04 g^2/cps (7.4 g-rms) to 0.3 g^2/cps (20.7 g-rms) for 30 min. in each axis (100 to 1000 cps with 6 db/octave roll off each end to 50 and 2000 cps respectively)
III. Ground Launched Rockets		
A. Launched from Stationary Site[24]	4.5 g(rms)(max PSD noted was 0.007 g^2/cps at 1200 cps)	Part 1. Sinusoidal with sweep frequency: 5 to 50 g (peak) for 30 min. in each axis (sweep rate = 20 min. for each 5-2000-5 cps cycle) Part 2. Random: 0.02 g^2/cps (5.30 g-rms) to 1.5 g^2/cps (46.3 g-rms) for 30 min. in each axis (100 to 1000 cps with 6 db/octave roll off each end to 50 and 2000 cps respectively)
B. Launched from Mobile Launcher		
1. Captive Transportation, Tracked Vehicle (30-35 mph)	1.65g(peak) at 76 cps--longitudinal[25] 1.04g(peak) at 80 cps--lateral[25] 1.95g(peak) at 80 cps--vertical[25] 2.75g(rms)(PSD spectrum peaks at 100 cps)[25]	
IV. Ship Launched Rockets		
A. Captive Transportation (Tactical Ships)		
1. destroyer	5.0 g(peak)(17 to 170 cps)[27]	
2. PT boat	6.0 g(peak)(10 to 140 cps)[27]	
3. submarine	2.0 g(peak)(15 to 160 cps)[27]	
B. Powered, Free Flight	8.3 g(rms)(max PSD noted was 0.037 g^2/cps in 700 to 1400 cps region)[28]	

TABLE IV

SUMMARY OF SOLUTIONS FOR THE SINUSOIDAL FORCED VIBRATION OF SOLID PROPELLANT ROCKET MOTORS

MODE OF VIBRATION	CONDITIONS OF SOLUTION		SOLUTIONS*	
	Core	Case	Free Vibration Problem, i.e., Natural Frequencies and Mode Shapes	Free Vibration Problem, i.e., Stress or Strain Amplitude vs Frequencies
Lateral-Lobar	Solid, Elastic	Rigid	67	68
	Hollow, Elastic	Rigid	69	70**
	Hollow, Elastic	Flexible, Elastic	71**, 72, 73, 74, 77	77
Lateral-Flexural	Hollow, Elastic	Flexible, Elastic	75, 76, 77	75, 76, 77
Axial-Shear	Hollow, Elastic	Rigid	69, 74, 78, 79**, 80	74
Torsional	Hollow, Elastic	Flexible, Elastic	80, 81***	81***
	Hollow, Viscoelastic	Flexible, Elastic	81***	81***

* Entries refer to references listed at the end of the report
** Core assumed to be incompressible
*** Includes effect of finite length

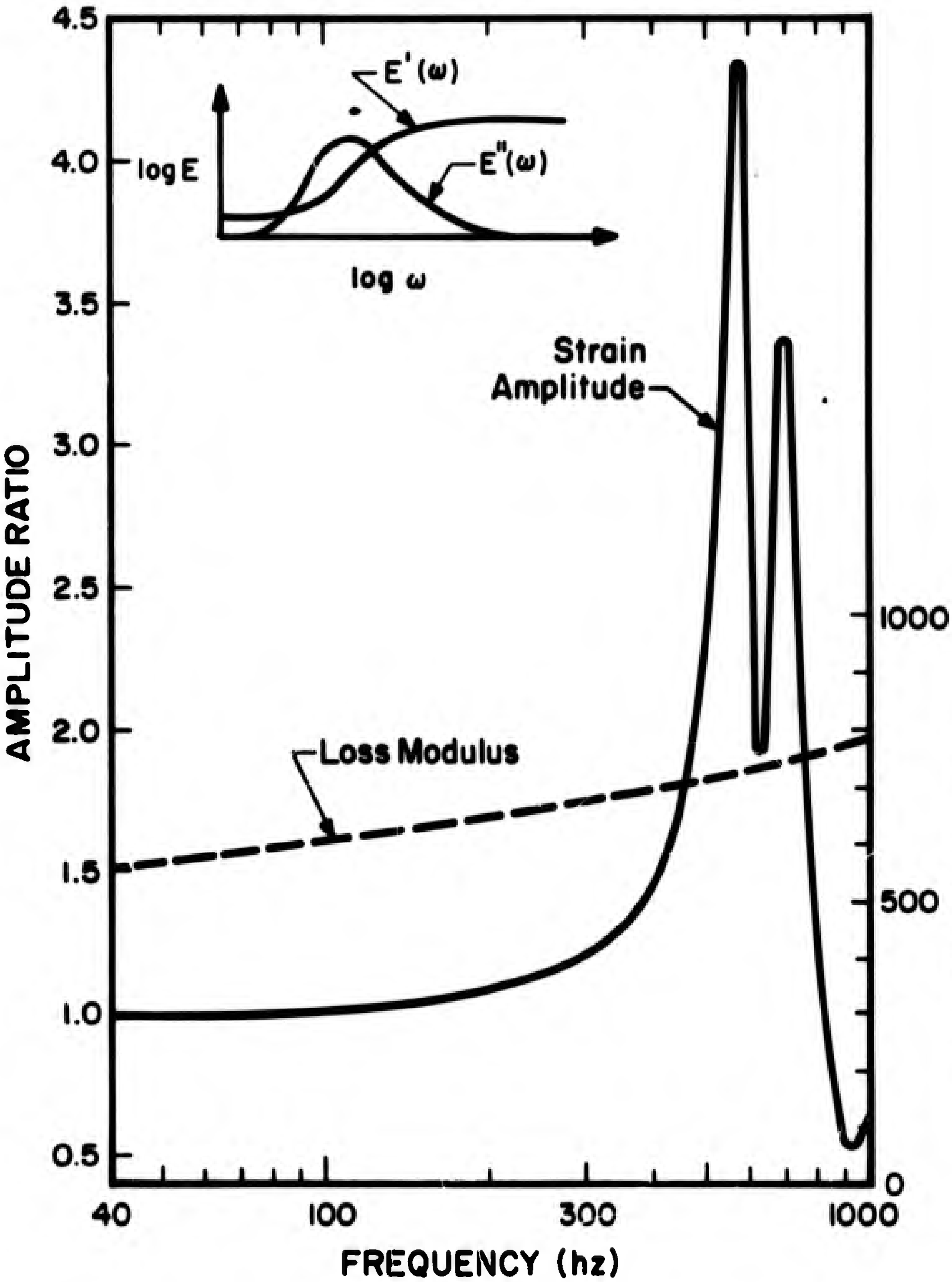

FIGURE 1. Typical Rocket Motor Response to Sinusoidal Vibration (Data from Reference 84). The insert shows broad band response of the real (E') and imaginary (E") parts of the complex (logarithmic) modulus as a function of (logarithmic) frequency.

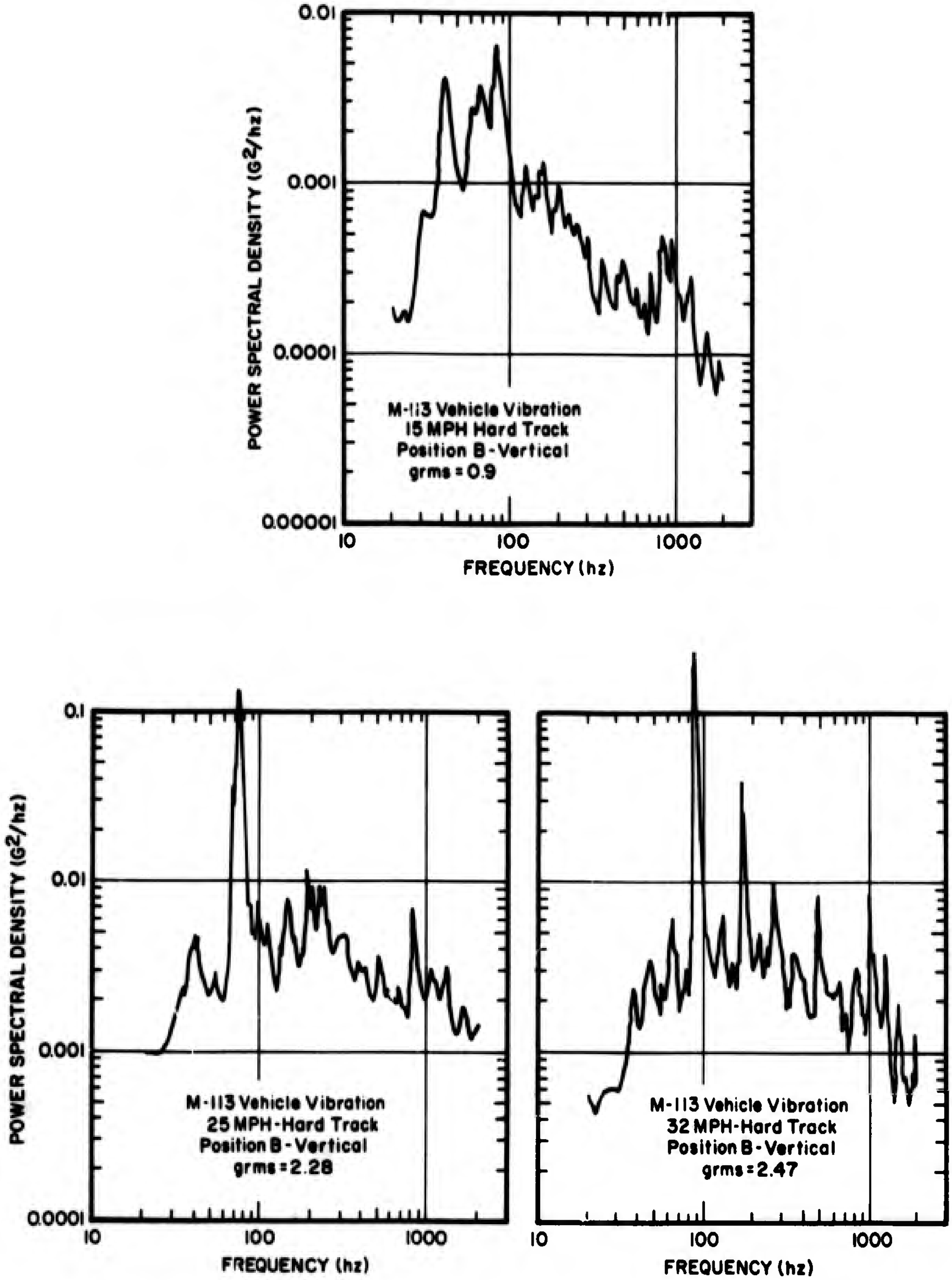

FIGURE 2. Random Vibration Environments for a Solid Rocket Carried in a Tracked Vehicle at Various Speeds (Reference 27).

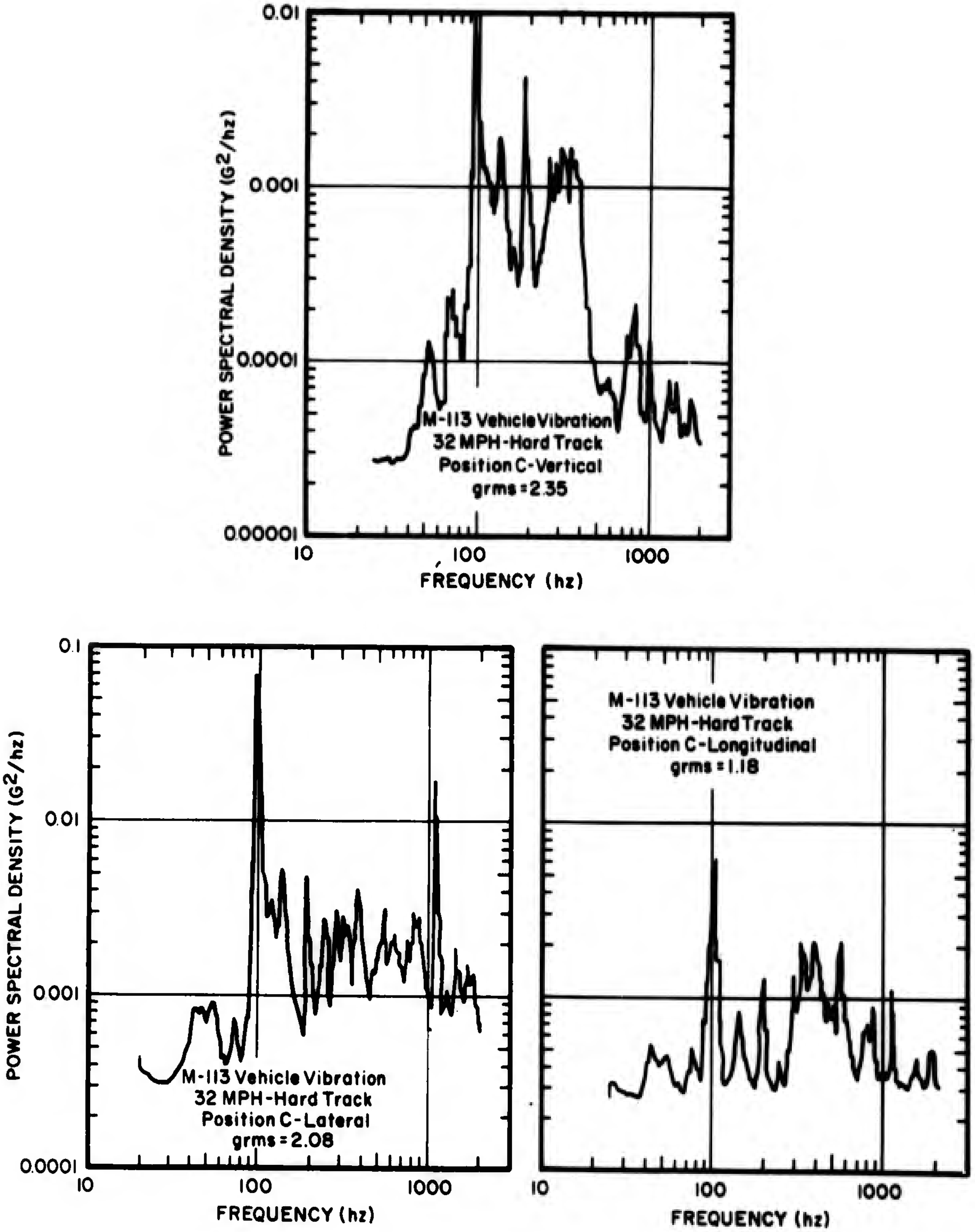

FIGURE 3. Variation of the Vibration Environment with Direction for a Solid Rocket Carried in a Tracked Vehicle (Reference 27).

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13. ABSTRACT

Directed primarily to those responsible for the specification of loads for solid rocket motors, this report attempts to determine if present load specifications and specification practices are realistic. It concludes that present practices and particularly the resonance dwell requirement are not realistic since they (1) fail to account for the viscoelastic nature of the solid propellant grain, and (2) do not adequately handle the interaction between the rocket, its carrier aircraft, vehicle, etc. A summary of established techniques for predicting vibration environments is given and overall problem of design for dynamic loads is discussed.

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