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THE EFFECT OF ANISOTROPY AND REORIENTATION OF PRINCIPAL STRESSES ON THE SHEAR STRENGTH OF SATURATED CLAY

by
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and
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University of California · Berkeley

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THE EFFECT OF ANISOTROPY AND REORIENTATION OF PRINCIPAL STRESSES ON THE SHEAR STRENGTH OF SATURATED CLAY

A Report of an Investigation

by

J. M. Duncan

and

H. Bolton Seed

Under

Contract No. DA-22-079-CIVENG-62-47

with

U. S. Army Engineers Waterways Experiment Station

CORPS OF ENGINEERS

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FOREWORD

The work described in this report was performed under Contract DA-22-079-CIVENG-62-47, "Shear Properties of Undisturbed Weak Clays," between the U. S. Army Engineer Waterways Experiment Station and the University of California.

The general objective of the research, which was begun in February, 1962, is to investigate the influence of pore-water pressure on the strength characteristics of undisturbed weak clays. Work on this project is conducted under the supervision of Professor H. Bolton Seed, Professor of Civil Engineering and Clarence K. Chan, Associate Research Engineer. The project is administered by the Office of Research Services of the College of Engineering.

The phase of the investigation described in this report was performed by J. M. Duncan, now Assistant Professor of Civil Engineering, and the report was prepared by J. M. Duncan and H. Bolton Seed. This is the second report on investigations performed under this contract. The previous report, "The Effects of Sampling and Disturbance on the Strength of Soft Clays," Report No. TE-64-1, was completed in February, 1964.

SUMMARY

The objectives of this investigation were to establish the fundamental nature of the anisotropy of saturated clays with respect to undrained strength, and to evaluate the practical significance of anisotropy and reorientation of the principal stresses in short-term stability problems.

The undrained shear strength which can be mobilized in a clay deposit in the field may vary with the orientation of the failure plane for either of two different reasons: Either (1) anisotropy of the clay with respect to its physical properties, or (2) reorientation of the principal stresses during construction, may result in a variation of undrained strength with orientation of the failure plane. These two phenomena must be considered separately in order to establish the fundamental nature of anisotropy and to evaluate their practical significance.

Anisotropy of one-dimensionally consolidated clay with respect to undrained strength may arise as a result of (a) anisotropy with respect to the shear strength parameters in terms of effective stresses, either c' and Φ' or c_e and Φ_e, or (b) anisotropy with respect to the development of the changes in pore-water pressure induced by the changes in total stress which occur during construction. Both of these types of anisotropy are probably fundamentally related to the fact that plate-shaped clay particles tend to become oriented with their flat sides parallel to the major principal plane during one-dimensional consolidation.

The degree of reorientation of the principal stresses during construction, which is related to the orientation of the failure plane, influences the magnitude of the change in pore-water pressure during construction and thus the undrained strength. Since both the orientation of the failure plane and the undrained strength depend on the degree of reorientation of the principal stresses, the undrained strength of a clay would vary with the orientation of the failure plane even if the clay were isotropic with respect to all of its physical properties.

Consolidated-undrained strength tests have shown that both over-consolidated, artificially prepared kaolinite and normally consolidated,

undisturbed San Francisco Bay Mud are anisotropic with respect to undrained strength. For both soils, the anisotropy with respect to undrained strength results principally from anisotropy with respect to the development of pore-water pressure. Both soils were also found to be anisotropic, to a small degree, with respect to the strength parameters in terms of effective stresses.

Two different types of anisotropically consolidated undrained plane strain strength tests were performed on undisturbed Bay Mud in order to simulate the consolidation and subsequent undrained failure at two different points in-situ where the failure plane has two different orientations. From the results of these tests and consideration of the probable variation of the value of the pore pressure parameter $\bar{A}_f$ with orientation of the failure plane in-situ, it has been possible to hypothesize the entire variation of the undrained strength of Bay Mud in-situ with orientation of the failure plane.

Two different types of undrained plane strain strength tests were also performed where "perfect sampling" of anisotropically consolidated Bay Mud was simulated in the laboratory. The results of the tests show that, for practical purposes, "perfect sampling" has a negligible influence on the undrained strength for either of the two orientations of the failure involved in the two types of "perfect sampling" tests.

A comparison has been made of the hypothesized in-situ variation of the undrained strength of Bay Mud with the variation determined by performing unconsolidated undrained triaxial tests on "undisturbed" samples of Bay Mud which were trimmed in different directions; this comparison shows that the effect of sampling on the form of the relationship between the undrained strength and the orientation of the failure plane is small if disturbance of the samples is minimized.

Experimental studies performed by other investigators have shown that other saturated clays are also anisotropic with respect to undrained strength to a significant degree; but these studies indicate that the variation of undrained strength with orientation of the failure plane may be different from the type of variation which was found to be characteristic of both overconsolidated, artificially prepared kaolinite and normally consolidated, undisturbed Bay Mud.

A stability analysis performed using the variation of undrained strength with orientation of the failure plane in-situ which was hypothesized for Bay Mud has shown that the computed minimum factor of safety is less than that computed by means of the ordinary "$\Phi = 0$" method of analysis, but that the position of the critical failure surface is the same. Neglecting to include the effect of disturbance on the undrained strength tends to reduce the factor of safety computed by means of the ordinary "$\Phi = 0$" method of analysis, whereas neglecting the effect of anisotropy and reorientation of principal stresses tends to increase the computed factor of safety. The fact that neglecting to account for the effect of disturbance on the one hand and neglecting to account for anisotropy and reorientation of principal stresses on the other hand have counteracting influences on the computed factor of safety may be in part responsible for the agreement between actual factors of safety and those computed by the ordinary "$\Phi = 0$" method of analysis.

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LIST OF SYMBOLS

ENGLISH LETTERS

A	Activity, ratio of plasticity index to percent of soil by weight smaller than two microns - (dimensionless)
$\bar{A}$	pore pressure parameter, the ratio of the change in pore pressure caused by change in deviator stress to the change in deviator stress - (dimensionless)
$\bar{A}_f$	the value of $\bar{A}$ at failure
AC-U	Anisotropically Consolidated-Undrained, a test where the sample is anisotropically consolidated and then sheared undrained; in the normal AC-U test, the major principal stress acts in the direction of the axis of the sample both during consolidation and at failure
AC-UU	Anisotropically Consolidated-Unconsolidated Undrained, a test in which a sample is anisotropically consolidated, then unloaded to an isotropic state of stress undrained, then sheared undrained; also described as a "perfect sampling" test
B	pore pressure parameter, the ratio of the magnitude of the change in pore pressure caused by a change in all-around pressure to the magnitude of the change in all-around pressure - (dimensionless)
C_c	compression index, change in void ratio on consolidation curve for one log (base 10) cycle change in pressure - (dimensionless)
C_s	change in void ratio on swelling curve for one log (base 10) cycle change in pressure - (dimensionless)
c	as a subscript, indicates consolidation
c	intercept, on the shear stress axis, of a strength envelope which is plotted in terms of total normal stress - (kg/cm^2)
c'	intercept, on the shear stress axis, of a strength envelope which is plotted in terms of effective normal stress - (kg/cm^2)
c_e	effective cohesion as defined by Hvorslev; the intercept, on the shear stress axis, of a strength envelope which is plotted in terms of effective normal stress, when all samples defining the envelope have the same void ratio at failure - (kg/cm^2)
c_1, c_3	cohesion associated with the major and minor principal strength axes in anisotropic strength theories - (kg/cm^2)
c_u	$1/2(\sigma_1 - \sigma_3)_{max}$ in an undrained shear test - (kg/cm^2)

c_v	coefficient of consolidation - (cm^2/sec)
f	as a subscript indicates failure
HPS	Horizontal Plane Strain, a laboratory plane strain compression test simulating consolidation and undrained failure of an element of soil in-situ where the major principal stress is vertical during consolidation and horizontal at failure
IC-U	Isotropically consolidated-undrained, a triaxial test in which the sample is isotropically consolidated and then sheared undrained
k	σ_3'/σ_1' during consolidation - (dimensionless)
k_o	coefficient of earth pressure at rest, that value of σ_3'/σ_1' which will result in consolidation without lateral strain - (dimensionless)
p'	major principal effective stress during consolidation, also used without the prime - (kg/cm^2)
UU	Unconsolidated Undrained, a laboratory compression test performed on a sample which is trimmed and tested without change in water content
u	pore pressure - (kg/cm^2)
V	volume - (cm^3)
VPS	Vertical Plane Strain, a laboratory plane strain compression test simulating consolidation and undrained failure of an element of soil in-situ where the major principal stress is vertical during consolidation and at failure
VPS-UU	Vertical Plane Strain - Unconsolidated Undrained, a test in which a plane strain sample is anisotropically consolidated, then unloaded to an isotropic state of stress undrained, then caused to fail in compression undrained; also described as a "perfect sampling" test

GREEK LETTERS

α	angle between failure plane and horizontal - (degrees)
β	angle between the axis of a triaxial or plane strain sample and the horizontal - (degrees)
Δ	a prescript indicating a change in the quantity appended
δ	angle by which principal stresses are reoriented during shear - (degrees)
ϵ_{ac}	axial strain during consolidation - (dimensionless or percent)
ϵ_{lc}	lateral strain during consolidation - (dimensionless or percent)

ϵ_v	volumetric strain = $\Delta V/V$ - (dimensionless or percent)
θ	an angle - (degrees)
λ	ratio of the slope of a swelling curve to the slope of a consolidation curve - (dimensionless)
ρ_A	ratio of the average shear strength mobilized on a failure plane in the field to the shear strength for vertical plane strain - (dimensionless)
ρ_D	ratio of $1/2(\sigma_1 - \sigma_3)$ for UU triaxial tests to the shear strength for vertical plane strain - (dimensionless)
σ	normal stress - (kg/cm^2) - prime indicates effective normal stress
σ_e'	equivalent consolidation pressure, the consolidation pressure corresponding to a certain water content on the virgin curve - (kg/cm^2)
σ_ℓ	lateral stress - (kg/cm^2)
σ'_{ps}	effective stress after "perfect sampling" - (kg/cm^2)
σ'_t	effective stress after trimming - (kg/cm^2)
σ_1	major principal stress - (kg/cm^2)
$(\sigma_1 - \sigma_3)$	deviator stress - (kg/cm^2)
$(\sigma_1 - \sigma_3)_f$	deviator stress at failure, the maximum deviator stress - (kg/cm^2)
σ'_{1c}	effective major principal stress during consolidation - (kg/cm^2)
σ'_{1f}	effective major principal stress at failure - (kg/cm^2)
σ_2	intermediate principal stress - (kg/cm^2)
σ_3	minor principal stress - (kg/cm^2)
τ	shear stress - (kg/cm^2)
τ_{ff}	shear stress on the failure plane at failure - (kg/cm^2)
ϕ	angle of inclination of a strength envelope which is plotted in terms of total normal stress - (degrees)
ϕ'	angle of inclination of a strength envelope which is plotted in terms of effective normal stress - (degrees)
ϕ_e	effective angle of internal friction as defined by Hvorslev; the angle of inclination of a strength envelope which is plotted in terms of effective normal stress, when all samples have the same void ratio at failure - (degrees)
ϕ_1, ϕ_3	friction angles associated with the major and minor principal strength axes in anisotropic strength theories - (degrees)

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I. ANISOTROPY AND REORIENTATION OF PRINCIPAL STRESSES RELATED TO STABILITY PROBLEMS

When an undrained failure takes place in a deposit of clay in the field, the shear strength mobilized on a failure arc may vary with the orientation of the arc either as a result of anisotropy of the clay or as a result of reorientation of principal stresses. In order to show how these two factors may affect stability, the problem of the stability of a cut in normally consolidated clay is discussed below.

The Possible Types of Anisotropy of Shear Strength

Figure 1a shows a normally consolidated clay deposit as it might exist in the field. The major principal stress, p'* is vertical and is equal to the effective overburden pressure. The horizontal stress is the same in all directions and is equal to $k_o p'$. Both stresses increase linearly with depth.

The clay deposit shown in figure 1a has been consolidated one-dimensionally under an anisotropic system of stresses. Studies discussed in the review of previous work have shown that clay particles tend to become oriented perpendicular to the major principal stress during one-dimensional consolidation. Parallel orientation of clay particles could conceivably cause both the strength and compressibility of the clay to vary with direction. If the strength of a clay varies with the orientation of the failure plane, the clay will be said to exhibit anisotropy with respect to shear strength.

There are two possible types of anisotropy with respect to undrained shear strength:

(1) Anisotropy with respect to the values of the shear strength parameters in terms of effective stresses, either c' and ϕ' or c_e and ϕ_e.

(2) Anisotropy of the soil with respect to the development of pore water pressure; application of the same change in total stress

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*A complete list of symbols used is given immediately preceding Section I.

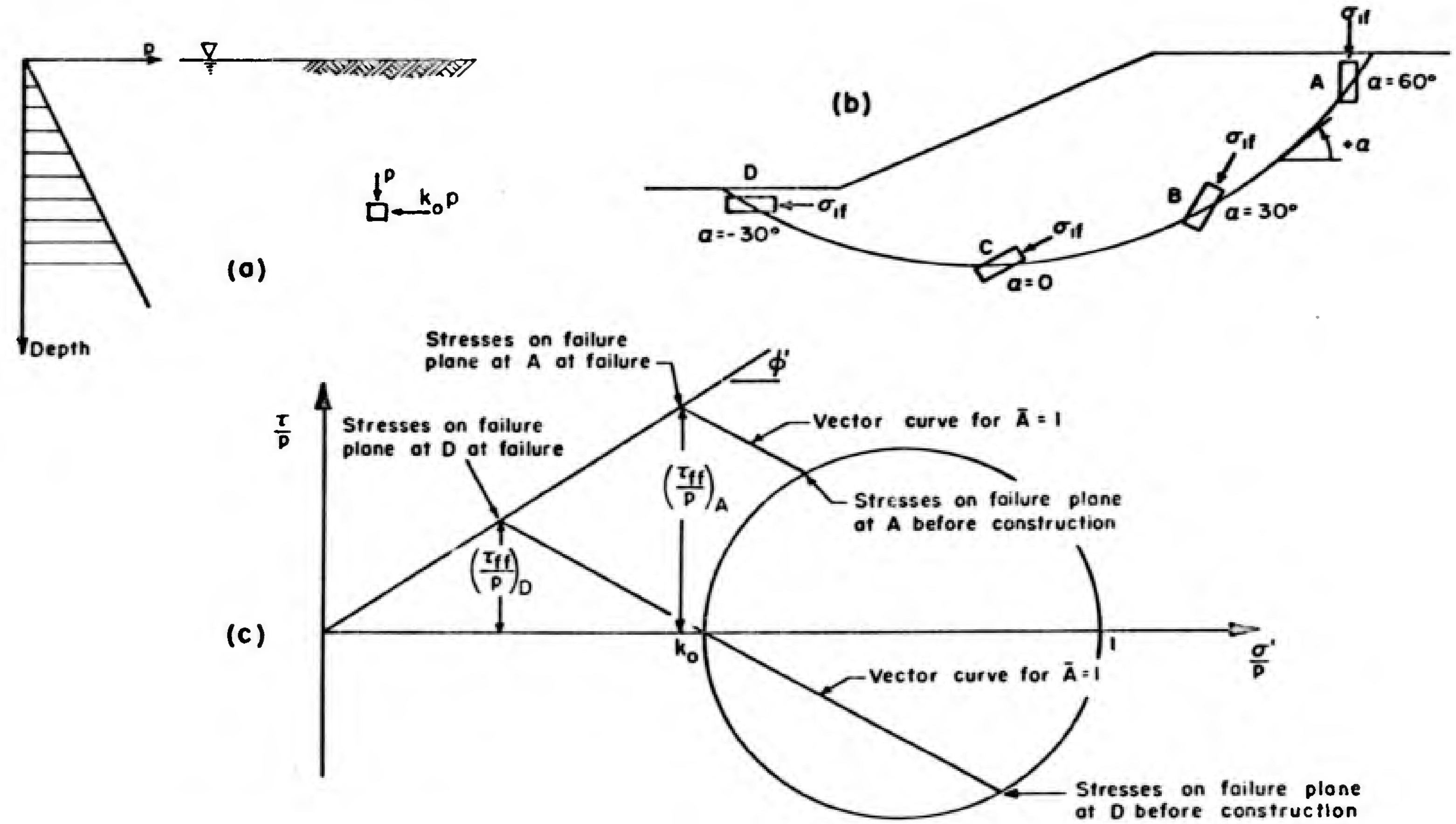

Fig. 1. (a) A NORMALLY CONSOLIDATED CLAY DEPOSIT, (b) A CUT IN THE DEPOSIT WITH THE CUT SLOPE IN AN INCIPIENT STATE OF FAILURE, AND (c) VECTOR CURVES FOR POINTS A AND D IF THE CLAY IS ISOTROPIC WITH RESPECT TO c', ϕ' AND $\bar{A}_f$.

would result in different changes in pore water pressures depending on the orientation of the applied stresses. This may be thought of as anisotropy with respect to the pore pressure parameter $\bar{A}_f$.

Both of these types of anisotropy would result in anisotropy with respect to the shear strength parameters in terms of total stresses or anisotropy with respect to the ratio of the undrained strength to consolidation pressure, c_u/p, since undrained shear strength reflects both the effective stress parameters and the pore pressure at failure.

The Shapes of Sliding Surfaces

The problem of stability analysis involves the determination of the shape of the sliding surface as well as the shear strength of the soil involved. For analyses of the stability of slopes in homogeneous cohesive soils, the failure surface is usually assumed to be an arc of a circle. This assumption is made on the basis of experience; observation of many slope failures has shown that the shape of the failure surfaces could be approximated closely by such an arc. Detailed reports of such observations are given by Collin (1846), and the report of the Swedish Geotechnical Commission (1922) for example.

In figure 1b is shown the same clay deposit as in figure 1a, after construction of a cut. Suppose that, because of the changes in shear stress caused by the construction, the cut slope is in a state of incipient failure; that is, any increase of load at the top of removal of material from the bottom will cause a rotational slide along the failure arc ABCD. The orientation of the failure plane (sliding surface) is different at every point. If the angle between the failure plane and the plane on which the major principal effective stress acted during consolidation (the horizontal plane) is called α, then it can be seen that α is about 60° at point A, 30° at point B, zero at point C and about minus 30° at point D.

Since the failure plane has a different orientation at every point along the sliding surface, it can be concluded that if the clay is anisotropic with respect to shear strength, the ratio of the undrained strength to the consolidation pressure will also vary from point to point along the sliding surface. To perform a proper analysis of the stability of such a slope, it would be necessary to know the relationship between the undrained strength ratio c_u/p and the orientation of the failure plane.

Reorientation of Principal Stresses in Anisotropic Stress Systems

In order to describe completely an anisotropic system of stress it is necessary to specify both the magnitudes of the stresses and the directions in which the stresses act. For instance, in describing the state of stress acting on an element of clay in a deposit like the one shown in figure 1a, it is necessary to specify the directions of the principal stresses as well as their magnitude. The state of stress would be described completely by the statements that the major principal stress is vertical and equal to $p' + u$, and that the intermediate and minor principal stresses are horizontal, and that both are equal to $k_o p' + u$. The orientations of the principal stresses could also be specified by indicating the orientations of the planes on which they act; the major principal stress acts on the horizontal plane, and the intermediate and minor principal stresses act on vertical planes.

The magnitudes of the stresses acting on an element may also be specified by the Mohr's circle of stresses, as shown by the stress circle in figure 2a which is labelled "initial stresses". Since the Mohr's circle indicates only the magnitude of the stresses, but not the direction in which they act, the stress circle must be supplemented by information about the directions in which the principal stresses act in order to describe completely the stress system acting on an element. The orientations of the principal stresses may be specified by indicating the orientations of the stresses themselves, as shown in figure 2c.

An anisotropic system of changes in stress has all of the features of a system of stresses; it has a major principal change in stress and a minor principal change in stress, both of which have given orientations. In order to describe completely a system of changes in stress it is necessary to indicate both the orientation and the magnitude of

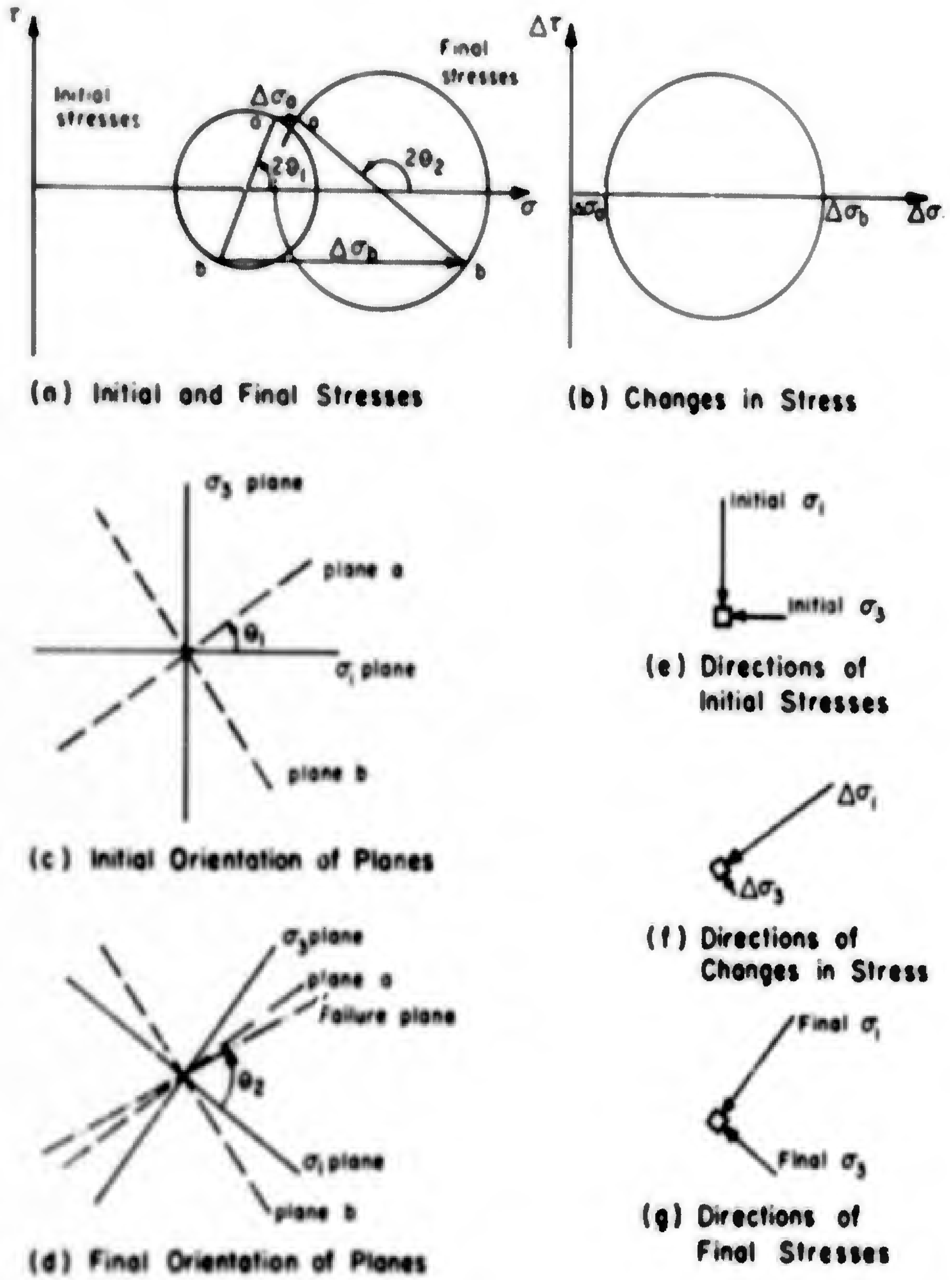

Fig. 2 – AN EXAMPLE OF ROTATION OF PRINCIPAL STRESSES.

the changes in stress. The magnitude of the changes in stress may be described by a Mohr's circle of changes in stress, as shown in figure 2b. The orientations of the principal changes in stress may be designated by indicating the orienations of the principal changes in stress themselves, as shown in figure 2f, or by showing the orientation of the planes to which major and minor principal changes in stress are applied; plane a in figure 2c is the plane to which the minor principal change in stress shown in figure 2f is applied, and plane b in figure 2c is the plane to which the major principal change in stress is applied.

In addition to a major principal change in stress and a minor principal change in stress, a system of changes in stress has a deviator of change in stress, which is the difference between the major principal change in stress, $\Delta\sigma_1$, and the minor principal change in stress, $\Delta\sigma_3$, i.e. the deviator of the change in stress is equal to $(\Delta\sigma_1 - \Delta\sigma_3)$. The magnitude of the deviator of the change in stress is believed to be important because the change in pore pressure induced by a change in the total stresses acting on an element of soil is related to the deviator of the change in stress by the equation

$$\Delta u = B \cdot \Delta\sigma_3 + \bar{A}\,(\Delta\sigma_1 - \Delta\sigma_3), \tag{1-1}$$

which was proposed by Skempton (1954).

When a system of changes in stress such as those shown in figure 2b is applied to an element initially acted upon by a system of stress such as the one shown in figure 2a, the magnitude and orientation of the final principal stresses depend on the magnitude and orientation of the initial stresses and the magnitude and orientation of the changes in stress. If the orientations of the principal changes in stress do not coincide with the orientations of the initial principal stresses, then the final orientations of the principal stresses will be different from the initial orientations of the principal stresses, i.e. the principal stresses will be reoriented. In addition, when the directions of the initial principal stresses do not coincide with the directions of the principal changes in stress, the final major principal stress will not be equal to the algebraic sum of the initial major principal stress plus the major principal change in stress; neither will the final minor principal stress nor the final deviator stress be equal to the algebraic sums of the initial values plus the values of the changes.

When the principal stresses are reoriented during application of a change in stress, the magnitudes and orientations of the final stresses may be determined by considering the initial stresses and the changes in stress on a pair of planes of known orientation. For example, consider the case where the initial stresses on an element are those shown by the stress circle marked "initial stresses" in figure 2a, and the orientations of the initial principal stresses are shown in figure 2e. The magnitudes of the changes in stress are those shown in figure 2b, and the orientations of the principal changes in stress are shown in figure 2f. By considering the initial stresses and changes in stress on a pair of planes of known orientation, the final state of stress may be determined. The initial stresses, and the changes in stress, on planes a and b may be used to determine the final state of stress. The initial stresses on planes a and b (figure 2c) may be determined analytically or graphically by means of the Mohr's circle of stress; the initial stresses are indicated by the points marked "a" and "b" on the Mohr's circle of initial stresses in figure 2a. Since planes a and b are the planes to which the major principal changes in stress are applied, the changes in normal stress are $\Delta\sigma_a = \Delta\sigma_3$ and $\Delta\sigma_b = \Delta\sigma_1$, and the changes in shear stress are zero. The changes in stress on planes a and b are indicated in figure 2a by the arrows labelled "$\Delta\sigma_a$" and "$\Delta\sigma_b$".

The final stresses on planes a and b are indicated by the points marked "a" and "b" on the Mohr's circle of final stresses in figure 2a. Planes a and b have fixed orientations in space and are perpendicular to one another; since they are perpendicular planes, the stresses acting on these planes must be represented by points at opposite ends of a diameter of the Mohr's circle of final stresses. This fact may be used to determine the position of the Mohr's circle of final stresses as shown in figure 2a. Once the Mohr's circle of final stresses has been drawn, the magnitudes of the final major and minor principal stresses may be determined; the final principal stresses are indicated by the points of intersection of the final stress circle with the normal stress axis.

The final orientations of the principal stresses may be determined from the relative orientations of plane a and the final orientation of the major principal plane. Plane a may be used as a reference because it has a fixed orientation, whereas the orientation of the major principal plane changes during application of the changes in stress. The angle between plane a and the initial major principal plane is θ_1, as indicated by the fact that the angle between the radii which intersect the Mohr's circle of initial stresses at the points representing the stresses on plane a and the major principal stress is equal to $2\theta_1$; the relative orientation of plane a and the initial major principal plane is shown in figure 2c. The angle between the radii which intersect the Mohr's circle of final stresses at the points representing the stresses on plane a and the major principal stress is equal to $2\theta_2$, indicating that the angle between plane a and the final major principal plane is equal to θ_2, as shown in figure 2d. The final orientations of the principal stresses, which are perpendicular to the principal planes, is shown in figure 2g.

If the final state of stress shown in figure 2a would cause failure, the failure plane would make an angle of about $45-\Phi'/2$ with the final orientation of the major principal stress. The approximate orientation of this failure plane is shown in figure 2d; a different final orientation of the major principal stress would correspond to a different orientation of the failure plane.

The magnitude of the changes in stress which are required to produce the final state of stress shown in figure 2a, beginning with the initial stresses shown in the same figure, depends on the orientations of the principal changes in stress. Because the magnitude of the required change in stress is related to the direction of application of the principal changes in stress, the deviator of the change in stress will depend on the amount of reorientation of the principal stresses. If a laboratory triaxial sample were acted on by the initial stresses shown in figure 2a, the stresses could be changed easily to the final stresses shown in figure 2a in two different ways. The initial major principal stress is vertical, as shown in figure 2e, indicating that the axial stress acting on the sample is larger than the radial stress. If the axial stress acting on the sample was increased by the difference between the initial and final values of the major principal stress, and the lateral stress acting on the sample was increased by the difference between the initial and final values of the major principal stress, then the stresses acting on the sample would be equal to the final stresses shown in figure 2a. This manner of changing the stresses results in no reorientation of the principal stresses; it also requires a smaller deviator of change in stress than the case discussed previously, where the principal stresses were reoriented.

There is another simple way in which the stresses acting on a laboratory sample could be changed from the initial values shown in figure 2a to the final values shown in the same figure. Initially the major principal stress acts in the axial direction, and the minor principal stress acts in the lateral direction on the sample; if the lateral stress was increased from the initial value of the minor principal stress to the final value of the major principal stress shown in figure 2a, and the axial stress was reduced from the initial value of the major principal stress to the final value of the minor principal stress shown in the same figure, then the stresses acting on the sample would be those indicated by the circle of final stresses. To reduce the axial stress to a value less than the lateral stress (cell pressure) would require a cell piston threaded into the sample cap, to which tensile loads could be applied. This method of changing the stresses acting on the sample would result in a 90° reorientation of the principal stresses, since the major principal stress acts initially in the axial direction and finally in the lateral direction; it also requires a larger deviator of change in stress than either of the two changes described previously.

Changing the stresses from the initial to the final values shown in figure 2a without reorientation of the principal stresses requires a deviator of change in stress which is equal to the difference between the lengths of the diameters of the initial and final Mohr's circles of stress. Changing the stresses from the initial to the final values shown in figure 2a with a 90° reorientation of principal stresses requires a deviator of change in stress which is equal to the sum of the lengths of the diameters of the initial and final Mohr's circles of stress. Changing the stress from the initial to the final values with an amount of reorientation of the principal stresses which is between zero and 90°, such as the change described previously and illustrated fully in figure 2, requires a deviator of the change in stress which is more than the difference between the diameters of the initial and final Mohr's circles of stress, but less than their sum. A change in stress accompanied by some reorientation of the principal stresses may be considered to be the general case for field conditions; although no reorientation between zero and 90° is possible in conventional undrained laboratory triaxial compression tests, the two possibilities of no reorientation and a 90° reorientation are seen to be the extremes of the general case for the field.

Since the deviator of the change in stress is different for each amount of reorientation of principal stresses, it would be expected that the change in pore pressure accompanying the change in total stress would also be different for each amount of reorientation of principal stresses, because the change in pore pressure is related to the deviator of the change in stress by the equation:

$$\Delta u = B \cdot \Delta\sigma_3 + \bar{A}\,(\Delta\sigma_1 - \Delta\sigma_3)$$

The larger the degree of reorientation of principal stresses, the larger is the deviator of the change in stress and the larger will be the increase in pore pressure if $\bar{A}$ is larger than one-half, as it usually is for normally consolidated clays, and if the same value of $\bar{A}$ applies for any degree of reorientation of principal stresses. Thus the pore water pressure at failure in an undrained element of soil depends on the amount of reorientation of principal stresses as well as on the value of $\bar{A}$; reorientation of principal stresses tends to increase the pore pressure at failure and reduce the undrained strength.

In considering changes in stress during which the principal stresses are reoriented, it is important to distinguish between the change in deviator stress, $\Delta(\sigma_1 - \sigma_3)$, and the deviator of the change in stress, $(\Delta\sigma_1 - \Delta\sigma_3)$. The change in deviator stress is simply the difference between the diameters of the final and the initial Mohr's circles of stress, whereas the deviator of the change in stress is equal to the difference between the major and the minor principal changes in stress. The change in deviator stress and the deviator of the change in stress are identical when the principal stresses are not reoriented during the change in stress, as in most laboratory tests. In this case the commonly used expression

$$\Delta u = B \cdot \Delta\sigma_3 + \bar{A} \cdot \Delta(\sigma_1 - \sigma_3)$$

is precisely equivalent to the correct expression

$$\Delta u = B \cdot \Delta\sigma_3 + \bar{A} \cdot (\Delta\sigma_1 - \Delta\sigma_3).$$

When the principal stresses are reoriented, however, as they are in the field, $\Delta(\sigma_1 - \sigma_3)$ is not equal to $(\Delta\sigma_1 - \Delta\sigma_3)$, and the equation proposed by Skempton, relating changes in pore pressure to changes in total stress, should be written in terms of $(\Delta\sigma_1 - \Delta\sigma_3)$, the deviator of the change in stress, because there may be a considerable difference between the change in deviator stress and the deviator of the change in stress.

Reorientation of Principal Stresses Along Sliding Surfaces

If the cut shown in figure 1b is at the point of failure, the direction of the principal stresses can be determined from the fact that the stress circle for each point must be tangent to the Mohr failure envelope in terms of effective stresses. The orientations of the major principal stresses at failure are shown at points A, B, C, and D in figure 1b. During consolidation of the clay deposit, the major principal stress was vertical everywhere, but at failure the major principal stress is essentially vertical only at point A, the top of the sliding surface. At other points such as B and C it can be seen that the major principal stress has rotated through some angle; if the angle by which the principal stresses are reoriented is called δ, then it can be seen that $\delta = 60° - \alpha$. Thus the amount of reorientation of principal stresses is zero at A, 30° at B, 60° at C, and 90° at D.

In order to show what effect this reorientation of principal stresses might have on the strength of the clay, it will be assumed that the clay is isotropic with respect to c', ϕ' and A_f. Suppose that in the laboratory two samples are consolidated anisotropically to the same pressure and then caused to fail, undrained. If one sample simulates failure at point A, the major principal stress will have the same orientation during consolidation and at failure. If the other sample simulates failure at point D, the major principal stress at failure must be perpendicular to the major principal stress during consolidation. Since the effective stress envelopes and the pore pressure parameters $\bar{A}_f$ are known (they are assumed to be the same for both samples), the strengths can be determined graphically as shown in figure 1c. The vector curves for the two samples are shown as parallel straight lines. The slope of a vector curve is a function of the pore pressure parameter $\bar{A}$; the vector curves in figure 1c correspond to $\bar{A}_f = 1$, a typical value for normally consolidated clay.

The ratio of the shear stress on the failure plane at failure to the consolidation pressure, τ_{ff}/p, is equal to the ordinate of the intersection of the vector curve and the effective stress envelope. It can be seen that if both samples had $\bar{A}_f = 1$, the sample which represents point A would have a higher strength ratio than the sample which represents point D.

Points A and D are the only points on the failure arc where the principal stress directions would not rotate during construction of the cut. At point A the major principal stress is vertical before construction, during construction, and at failure. At point D the major principal stress is vertical before construction, and it will remain vertical throughout the initial stages of construction if point D is on a vertical plane of symmetry of the cut at every stage. For simplicity, only this case is considered. During construction the vertical normal stress at point D decreases and the horizontal normal stress increases, but there are still no shear stresses on the horizontal or vertical planes, so the principal stress directions remain exactly the same. At some stage of construction, however, the vertical and the horizontal stresses will become equal. Any further decrease in the vertical stress or increase in the horizontal stress will cause the horizontal stress to become greater than the vertical stress, and thus the major principal stress σ_1, will act in the horizontal direction. The major principal stresses at D will thus be reoriented by 90°, but the principal stress directions will not have changed from their original orientations.

At any other point on the failure arc, such as B or C, the principal stresses rotate gradually during construction. These cases are more difficult to analyze, but the analysis explained in section II shows that if the clay is isotropic with respect to $\bar{A}_f$, c' and ϕ', then, as a result of reorientation of principal stresses, the strength ratio, τ_{ff}/p, at any point on the arc between A and D will be less than the strength ratio at point A, and more than the strength ratio at point D. Points A and D thus represent the extreme values of strength ratio if the assumptions of isotropy of $\bar{A}_f$, c' and ϕ are correct, and the strength ratio τ_{ff}/p varies continuously along the failure arc.

Isotropy with respect to c', ϕ' and $\bar{A}_f$ has been assumed in order to demonstrate that even in this simple case, the strength ratio τ_{ff}/p would vary along the failure arc as a result of the reorientation of principal stresses. In an actual case neither c', ϕ' nor $\bar{A}_f$ is likely to be constant. The strength ratio in such a case would be a function of the variations of the values of c', ϕ' and $\bar{A}_f$ with the orientation of the failure plane. Because the values of c', Φ' and $\bar{A}_f$ as well as the amount of reorientation of principal stresses may be different for different values of α, there are two separate parts to the problem of determining the undrained strength in-situ for any orientation of the failure plane. One part of the problem consists of analyzing the influence of reorientation of principal stresses; Hansen and Gibson (1949) have performed this analysis, and the solution is described in Section II of this report. The other part of the problem of determining the variation of undrained strength in-situ consists of determining how the values of c', Φ' and $\bar{A}_f$ vary with the orientation of the failure plane; this part of the problem must be solved experimentally. The experiments used to determine these variations are described in Section IV of this report.

The Relationship Between the Strength of a Laboratory Sample and the Strength of an Element of Soil in the Field

In many practical cases the stability of a cut like the one shown in figure 1b is analyzed using the strengths measured in unconfined compression or UU triaxial tests. The strengths measured in these tests will be the same if the soil is a normally consolidated saturated clay.

The question of interest here is "Is the strength measured in a UU triaxial test equal to the strength of an element in the field?" Noorany and Seed (1965) and Skempton and Sowa (1963) have shown that, for two different clays, the UU strength was within 7 percent of the strength at point A, if the UU strength had not been reduced by disturbance, swelling, or temperature change. The experimental techniques in the two investigations were the same. The strength of an element at point A was simulated by making an "ordinary" AC-U triaxial test. This is, the triaxial sample was consolidated to stresses p and $k_o p$ where p is the axial stress. This simulated consolidation without lateral strain in the field. When consolidation was complete, further drainage was prevented and the sample was caused to fail by either reducing the lateral stress or increasing the axial stress. Except for the difference between plane strain and radially symmetrical deformation, this normal AC-U sample duplicates both the consolidation and the failure conditions at point A; thus the strength at point A can be represented by the strength of this AC-U sample.

Before a sample is removed from the ground, the stresses acting on it are anisotropic, p and $k_o p$. When it has been sampled, i.e. removed from the ground, the stresses acting on it are isotropic. When the UU strength is measured, the axial stress is increased until the sample fails. Because the sample has been trimmed so that its axis is parallel to the direction in which the major principal stress acted in the ground, the major principal stress acts in the same direction at failure in the laboratory as it did during consolidation in the ground. To simulate the effect of removal and reapplication of stresses during sampling without confusing the results by allowing disturbance, swelling or temperature change, the anisotropically consolidated samples were "sampled" in the laboratory, i.e. when consolidation was complete, further drainage was prevented, and the axial load was first removed and then reapplied. The first step represents the stress relief due to sampling and the second step represents the re-application of load during a UU test on a "perfectly sampled" sample.

By this technique Noorany and Seed found that the UU strength of "perfectly sampled" undisturbed Bay Mud was from 93 to 96 percent of the strength of the sample simulating point A. Skempton and Sowa found that the UU strength of "perfectly sampled" remoulded clay from Dorking, Surrey was from 98 to 99 percent of the strength of the sample simulating point A.

Seed, Noorany and Smith (1964) and Ladd and Lambe (1963) have investigated the effects of "imperfect sampling" on the strength. The effects of an increase in temperature have been discussed by Duncan and Campanella (1965). Both of these factors further reduce the strength, that is, make the UU strength less than the strength at point A. Even if the best available techniques were used for estimating the strength at point A, the strength so determined would only simulate the conditions at one point on the failure arc. The strengths at other points might be different because of the combined effects of anisotropy and reorientation of principal stresses.

Summary of the Problem and Scope of the Investigation

From the preceding discussion it may be seen that the undrained shear strength which can be mobilized on a failure surface in the field may vary with the orientation of the failure plane as a result of anisotropy of the soil, either with respect to the strength parameters in terms of effective stresses or with respect to the development of pore pressure. Furthermore, reorientation of the principal stresses during construction can result in changes in pore pressure which are different for different orientations of the failure plane, and thus the undrained strength in-situ may vary with the orientation of the failure plane as a result of reorientation of principal stresses also. The methods presently used for measuring undrained strength and for analyzing short-term stability problems do not take account of either of these factors.

The investigation described in the following pages was performed in an attempt to answer the following questions which have been raised by the preceding discussion:

(1) Are soils anisotropic with respect to undrained strength?

(2) If so, does this anisotropy result from anisotropy with respect to the strength parameters in terms of effective stresses or from anisotropy with respect to development of pore pressure?

(3) As a result of both anisotropy and reorientation of principal stresses, how would the undrained strength in-situ vary with the orientation of the failure plane?

(4) Is the variation of undrained shear strength with orientation of the failure plane the same in-situ and in the laboratory?

(5) How do anisotropy and reorientation of principal stresses affect the stability of a clay slope?

The first part of the investigation (Section II) consists of a review of the literature concerned with anisotropy and reorientation of principal stresses. The second part (sections III through VI) consists of an experimental study of the effects of anisotropy and reorientation of principal stresses on undrained strength. The last part (Section VII) is an example of the analysis of the short-term stability of a clay slope using anisotropic strength.

II. REVIEW OF PREVIOUS WORK

The objective of this section is to examine the results of previous investigations which are related to the influence of anisotropy and reorientation of principal stresses on undrained strength. The studies made by other investigators consist of experimental and analytical investigations of four different types: (1) experimental investigations of the orientation of clay particles and the influence of this orientation on strength, (2) experimental investigations of anisotropy with respect to undrained strength, (3) analysis of the strengths of materials with anisotropic strength parameters, and (4) analyses of the effect of reorientation of principal stresses on the undrained strength.

Consideration of Particle Orientation

Anisotropy of clays with respect to compressibility and strength is probably fundamentally related to the orientation of plate-shaped clay particles. In view of this inferred causality, the results of the more direct experiments concerning anisotropy may be better understood when considered in the light of the results of experiments concerning the structure of clays and its effect on strength and stress-strain behavior.

By studying specially prepared thin sections under a microscope, Mitchell (1956) was able to conclude that seven undisturbed marine clays and one lacustrine clay had some degree of parallel orientation of particles. With one exception these clays had been consolidated in the field to nearly 3 kg/cm^2. Rosenqvist (1959) studied lightly consolidated Norwegian marine clays using electron microscopy. He found that all of the clays conformed nearly exactly to the model for salt-flocculated clays hypothesized by Lambe (1953); they were characterized by edge-to-face contact and random particle orientation. Martin (1962) compared the peak amplitudes of diffracted X-rays from the 002 and 020 planes of kaolinite clay. He concluded that the clay was approximately "ideally oriented" after one-dimensional consolidation to 197 kg/cm^2, and approximately "ideally random" after isotropic consolidation to 1 kg/cm^2.

Hvorslev (1960) consolidated a silty clay one-dimensionally from the liquid limit. Thin slices of the clay, when put in water, expanded in the direction in which they had been compressed and fissures developed parallel to the plane on which the major principal stress had acted during consolidation.

Seed and Chan (1959) compacted pairs of samples of silty clay and of kaolinite using kneading compaction. One sample of each pair was compacted dry of optimum and the other wet of optimum. The samples were then allowed to swell under a confining pressure until they were almost completely saturated. The only difference between members of a pair of samples after swelling was the structure induced by compaction. Samples compacted dry of optimum had flocculated particle orientation, whereas those compacted wet had dispersed particle orientation. The flocculated structure is characterized by edge-to-face contact and random particle orientation. Dispersed structure is characterized by face-to-face particle contact within groups or books which are themselves randomly oriented.

IC-U triaxial tests with pore pressure measurements were performed on the samples. The conclusions from the pairs of tests on silty clay and kaolinite were the same. The samples compacted dry of optimum had steeper deviator stress-axial strain curves (a higher modulus) and lower pore pressures at small strains than those compacted wet of optimum. The maximum deviator stress was the same for samples compacted dry or wet, as was the effective principal stress ratio at all strains.

Conclusions

It may be concluded from these studies that there is a tendency for particles to become oriented parallel to the plane on which the major principal stress acts during consolidation. In the case of lightly consolidated marine clays, which are believed to be flocculated when sedimented, the degree to which particles are oriented may increase with increasing consolidation pressure. If a clay possesses anisotropy because particles are oriented parallel to the plane on which the major principal stress acted during consolidation, then the normal to that plane should be an axis of radial symmetry of the anisotropy. In this type of anisotropy, properties associated with any plane should vary with the angle between the plane in question and the plane on which the major principal stress acted during consolidation. The properties might also vary with direction within any plane except the plane on which the major principal stress acted during consolidation.

One-dimensionally consolidated clays may possess either of the two types of anisotropy with respect to shear strength described in Section I, as a result of the tendency for parallel particle arrangement. Since two different types of random particle arrangement give rise to differing stress-strain and pore pressure-strain characteristics, it is possible that an oriented structure might result in anisotropy with respect to these properties. Furthermore, since the shear strength parameters in terms of effective stresses must be fundamentally related to the number and type of interparticle contacts, these parameters might vary with direction if particles are oriented parallel to one another.

Experimental Studies of Anisotropy with Respect to Undrained Strength

The simplest method of investigating variation of strength with orientation of the failure plane is to trim samples with their axes at different angles to the plane on which the major principal stress acted during consolidation, and perform UU triaxial or direct shear tests. Such samples differ from an element of soil in the field in that the anisotropic stress system under which the samples were consolidated has been released during sampling and replaced by an isotropic stress sufficient to maintain constant volume. In the field, the amount of reorientation of the principal stresses, and thus the change in deviator stress before failure, is related to the angle between the failure plane and the horizontal. In the laboratory, all UU samples are under an isotropic state of stress before testing, and the change in deviator stress is equal to the deviator stress at any stage of the test no matter in what direction the sample was trimmed. An element of soil in the field which does not undergo reorientation of principal stresses can be simulated by a normal AC-U triaxial sample, and the corresponding UU triaxial sample can be simulated by a "perfectly sampled" anisotropically consolidated sample (an AC-UU sample). The changes in deviator stress during the strength tests are quite different for the AC-U and the AC-UU samples, as they would be for an element of soil in the ground and the corresponding UU triaxial sample with the same orientation of the failure plane. It has recently been demonstrated that AC-U and AC-UU samples have about the same strength because the two samples have quite different values of the pore pressure parameter $\bar{A}_f$. These AC-U and AC-UU tests involve only one orientation of the failure plane, $\alpha = 60°$: whether or not the strengths of samples simulating field conditions would be nearly the same as the strength of UU samples for all orientations of the failure plane has not been determined.

In addition to having undergone stress release, UU samples may also have been disturbed, have absorbed water or dried out, and undergone a change in temperature. Any of these things can alter the strength of a given sample. Hopefully, but not necessarily, the change in strength due to these factors would be an equal percentage for each sample, independent of the direction in which it is trimmed.

The objective of these comments is to show that because so many factors may influence anisotropy with respect to strength, it should not be assumed that strength is a unique function of orientation of the failure plane and is the same in every type of test. In spite of these complications, however, UU tests can show if soil behaves anisotropically, at the very least. It would be surprising if there were not some similarity between UU strengths and field strengths for all orientations of the failure plane.

UU Triaxial Tests on Samples Trimmed in Different Directions

In describing the manner in which triaxial samples are trimmed, the adjectives "vertical", "inclined (angle)", and "horizontal" refer to the axis of the sample. A vertical sample is one trimmed in the normal way, with its axis perpendicular to the horizontal. An inclined sample is one trimmed so that the angle between its axis and the horizontal is less than 90 degrees and more than zero; the angle between the axis and the horizontal is given in parentheses. A horizontal sample is one trimmed so that its axis is horizontal. "Horizontal" will be used as an exact synonym for the more explicit but longer "plane on which the major principal stress acted during virgin consolidation."

Bishop (1948) has described tests in which samples of London clay from the soft upper few feet were taken in an inclined borehole. The objective was to determine the strength on a horizontal plane, so the axes of the samples were presumably inclined at about 30° to the horizontal. The samples were reconsolidated in an oedometer and tested in unconfined compression. The strength of the inclined samples was 28 percent less than the strength of vertical samples.

Unconsolidated undrained tests were made on horizontal and vertical samples of London clay from another location (Ward, 1957 and Ward, Samuels and Butler, 1959). More than 130 samples were cut from blocks trimmed out of the walls of tunnels at several locations. All of the samples were from greater depths than those tested by Bishop. With the exception of one group of samples described as exceptionally fissured, the horizontal samples were from 15 to 39 percent stronger than the vertical samples. The only inclined (45°) samples tested were one percent weaker than the vertical samples. There was no correlation between the strength of samples from the various locations and the relative strengths of horizontal and vertical samples. Skempton (1961) has shown that the ratio of horizontal to vertical stress in the overconsolidated London clay varies from 2.5 at the top to 1.5 at a depth of 100 feet below the surface. The fact that Bishop found that the strength of inclined samples from the shallow depths were 28 percent less than the strength of vertical samples, whereas Ward found that the strength of inclined samples from greater depths were only one percent less than the strength of vertical samples may be related to the difference in the ratio of horizontal to vertical stresses at the two depths.

Jacobson (1955) cut samples of a Swedish post-glacial marine clay from about 3 meters below the top of a slope. Vertical, inclined (45°) and horizontal samples were taken alternately in a line. Jacobson states that the inclined samples had strengths intermediate between those of the horizontal and vertical samples but his tabulated values do not agree with this statement. He tested 12 vertical, 12 inclined and 10 horizontal samples. The greatest average difference in strength was 14 percent, but because of the discrepancy between statement and tabulated values, it is not known which were the weakest and which the strongest. The mean error of his measurements was about 14 percent, and Jacobson concluded that the clay was isotropic with respect to strength.

Hvorslev (1960) consolidated remoulded Vienna and Little Belt clays one-dimensionally to 5.0 kg per sq cm, and trimmed vertical, inclined (45°) and horizontal samples. The samples had a 2 cm square cross section and were 4 cm high. During trimming the samples dried out somewhat, and when tested had water contents corresponded to about 6 kg/cm^2 consolidation pressure. Because of slight variations in water content and strength of individual samples, the compressive strengths could not be compared directly. Hvorslev could, however, compute the equivalent consolidation pressure, σ'_e, for each sample and thus the ratio $(\sigma_1 - \sigma_3)_f/\sigma'_e$. Strength comparisons were therefore made on the basis of this ratio, which is equal to twice c_u/p.

The relative strengths of the samples tested by Hvorslev are given in Table 1. Hvorslev computed relative strengths using the strength of

Table 1. Relative Strengths of Vertical, Inclined and Horizontal Samples of Vienna and Little Belt Clays (data from Hvorslev, 1960).

Type of Sample	Vertical	Inclined (45°)	Horizontal
Vienna	1.00	0.92	0.87
Little Belt	1.00	1.08	1.20

the inclined samples as unity, because he was interested in differences from the case where the failure plane was horizontal. For comparison with other UU test results, however, it is more convenient to let the strength of the vertical (the normal type) samples be unity.

It is surprising that two clays with very similar stress histories, tested by the same investigator, should be found to exhibit opposite trends with respect to the influence of orientation of the failure plane on strength. One might reasonably expect the trend to be the same for all clays, though with the magnitude of the strength difference varying from one clay to another. There is one feature about Hvorslev's tests which is unfortunate. As mentioned above, the clays dried during trimming; the amount of drying presumably was different for each sample because strengths could not be compared directly, but only by correlation with water content. This means that while the primary purpose of the investigation was to study the influence of anisotropic consolidation on the strength, the strengths were controlled to some extent by isotropic consolidation (drying). Also, drying would probably result in non-uniform water content distribution and non-uniform strength within the samples. Never-the-less, a consistent difference in the amount of drying of the different types of samples would be required to reverse the observed trend of the variation of undrained strength with orientation of the failure plane.

Undrained Direct Shear Tests

Bieber (1958) made undrained direct shear tests on Vicksburg silty clay and kaolinite which had been mixed at high water contents and consolidated one-dimensionally. He measured the undrained strength of two types of samples. In one type of sample the failure plane was horizontal (normal direct shear) and in the other it was vertical. The latter samples were trimmed so that relative motion across the failure plane was vertical.

It was found that the strength which could be mobilized on a vertical failure plane was greater than the strength on a horizontal failure plane for both kaolinite and Vicksburg silty clay. The ratio of these strengths increased with increasing consolidation pressure in the range $0.5 < p < 2$ kg/cm^2 but for all higher pressures the strength ratio was about the same: 1.4 for kaolinite and 1.15 for Vicksburg silty clay.

Field Vane Shear Tests

A very comprehensive investigation of field vane shear strengths was made by Aas (1965) at four sites around Oslofjord, Norway. By using vanes with different diameter-to-height ratios, he was able to determine that the shear stress mobilized on the horizontal vane surface was from 1.1 to 2.0 times as large as the shear stress mobilized on the vertical vane surface. These results reflect several factors besides the anisotropy of the soils at the sites where the tests were performed, and the results are not directly applicable to the problem outlined in Section I.

Summary of Experimental Studies

The results of the undrained strength tests on normally consolidated clays performed by Hvorslev and Bieber, and those on overconsolidated clay performed by Bishop and Ward are summarized in figure 3. If normally consolidated clays and overconsolidated clays are considered separately, the four series of UU triaxial tests shown in the upper part of figure 3 show four different types of behavior*. Assuming that horizontal samples of London clay from shallow depths would be weaker than the vertical samples, the behavior of the clays define the following four different categories.

(1) Normally consolidated clay, vertical samples stronger than horizontal (Vienna clay).

(2) Normally consolidated clay, horizontal samples stronger than vertical (Little Belt clay).

(3) Overconsolidated clay, vertical samples stronger than horizontal (probably shallow London clay).

(4) Overconsolidated clay, horizontal samples stronger than vertical (deep London clay).

Inclined samples of the normally consolidated clays always had intermediate strengths, whereas for the deep samples of London clay the inclined samples were the weakest (by 1 percent).

At this time it does not seem possible to make a consistent interpretation of all these tests. The differences in the case of London clay might be related to the differences in stress history throughout the depth of the overconsolidated deposit. In the case

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*The results of the tests performed by Jacobson are not shown, because it is not known which samples were weakest and which were strongest.

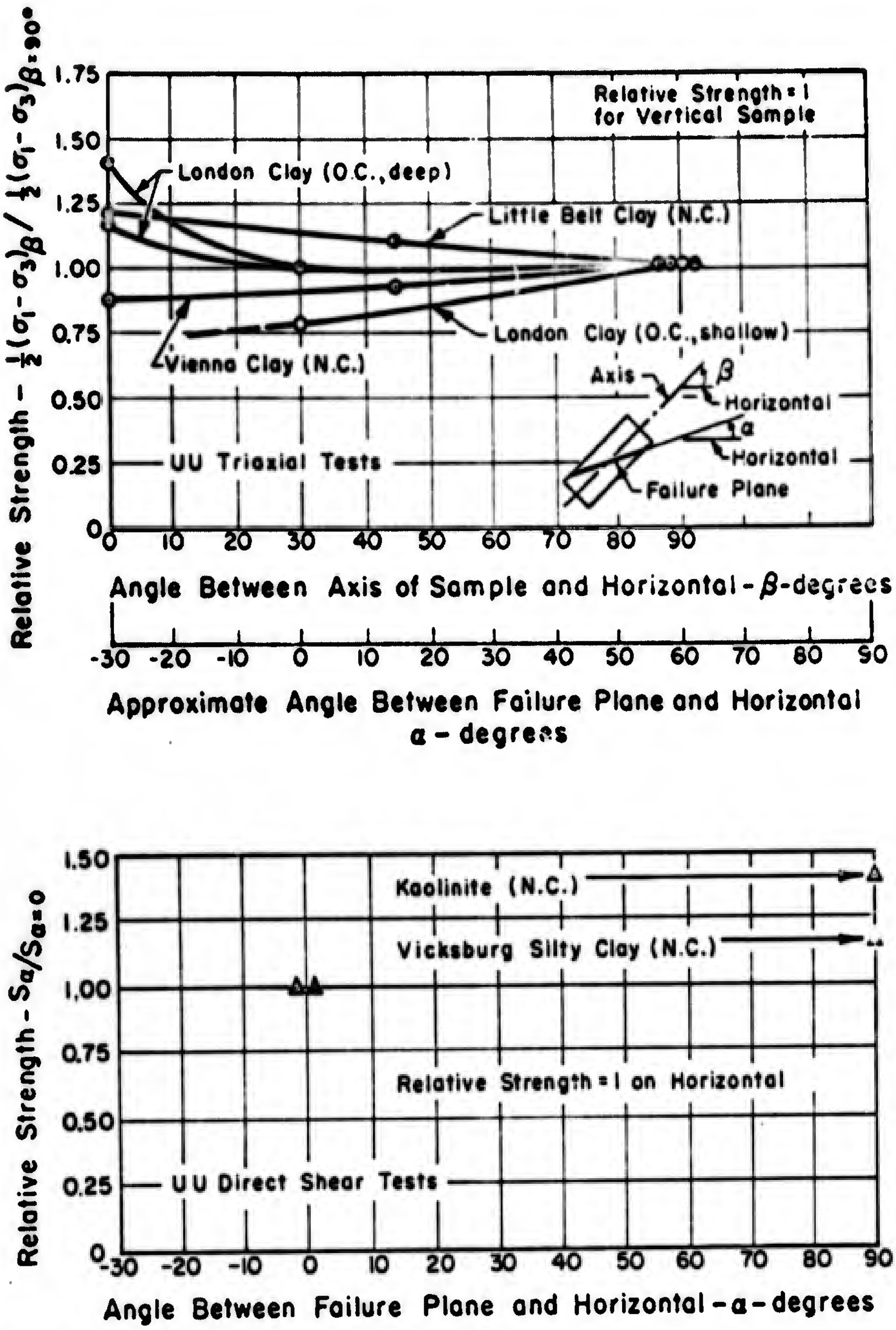

Fig.3 – SUMMARY OF AVAILABLE DATA CONCERNING ANISOTROPY WITH RESPECT TO UNDRAINED STRENGTH.

of the normally consolidated clays, the only explanation other than a fundamental difference in behavior would seem to be some change in behavior caused by allowing the samples to dry during trimming.

The two series of direct shear tests shown in figure 3 both show the same trend. The strength on a vertical failure plane is greater than the strength on a horizontal failure plane, although the ratio is greater for kaolinite than for Vicksburg silty clay.

Conclusions from Previous Experimental Studies

It would appear from the studies previously discussed that soil is anisotropic with respect to undrained strength, but the relationship between strength and orientation of the failure plane measured in laboratory tests is different for different soils. The amount of variation in strength is probably related to the clay content. There is no way to separate differences in pore pressure from differences in strength parameters in terms of effective stresses as they affect the results of UU tests. The influence of both of these factors must be understood in order to be able to interpret strength tests correctly and to solve the problem outlined in Section I.

Analyses of Strength of Materials With Anisotropic Strength Parameters

If one could determine the maximum and minimum values of the shear strength parameters and the way in which the parameters vary between these extremes, it would be possible to predict the orientation of the failure plane and the strength when the material is caused to fail by a deviatoric stress system making any angle with the axis of anisotropy. The studies described in this section are solutions to this general problem for various assumed variations of shear strength parameters and relative orientations of the stress and anisotropy axes.

Theories

Casagrande and Carillo (1944) considered the two cases of a purely cohesive anisotropic material and a purely frictional anisotropic material, characterized by values of c or ϕ which were assumed to vary elliptically between maximum and minimum values. The problems were solved in general for the case where the principal stress axes coincide with the principal strength axes. A trial and error graphical method was described for this case and the more general case where the principal stress axes are inclined to the principal stress axes.

Hank and McCarty (1948) considered the more general problem of a material possessing both cohesion and friction. The only case treated was the one in which the principal stress and the principal strength axes coincide; both c and ϕ were assumed to vary elliptically between the extreme values. Although the analytical procedure is involved, the final result can be achieved directly from Casagrande and Carillo's solution for cohesive materials if the expression $(c_1 + \sigma_1 \tan \phi_1)$ used by Hank and McCarty is substituted for the term c_1 used by Casagrande and Carillo, and the expression $(c_3 + \sigma_3 \tan \phi_3)$ used by Hank and McCarty is substituted for the term c_3 used by Casagrande and Carillo. The terms c_1, ϕ_1 and c_3, ϕ_3 are the cohesion and friction associated with the major and minor principal strength axes.

Jaeger (1960) found solutions for the strength of cohesive materials which exhibit two different types of anisotropy. One of the cases was that of a sample which contains a single plane of weakness making an arbitrary angle with the major principal stress. The other was the case in which the strength of the material varies sinusoidally between extremes associated with perpendicular axes, and the principal stress axes are inclined at an arbitrary angle to the principal strength axes.

Livneh and Shklarsky (1963) investigated the case of a material possessing both cohesion and friction which were assumed to vary as the square of the sine of the angle between the major principal axis of strength and the failure plane. The variation of strength assumed in the analysis was suggested by the behavior of asphalt concrete.

Conclusions

All of the theories predict that the minimum strength may be measured in a compression test, but the maximum strength cannot. This is due to the fact that in a compression test, a failure plane may develop on either side of the major principal stress. If one of these possible planes is weaker than the other because of anisotropy, then failure will occur on the weaker one. On the other hand, the maximum strength could only be measured in a test where failure is forced to occur on a particular plane. However, the maximum strength may be mobilized in the field, where kinematic compatability as well as stress controls the orientation of the rupture surface.

The theories show that in the general case the line which expresses the Mohr strength criterion is not tangent to the envelope of stress circles at failure, and that the line drawn tangent to the circles lies above the Mohr strength line. Hvorslev (1960) noted that the same situation may also arise because of slight inhomogeneities in the sample. The difference in orientation of the true failure plane and the plane defined by $45 + \phi'/2$ leads to appreciable differences in computed values of τ_{ff} and σ'_{ff}, but very small differences between the Mohr strength line and the envelope of stress circles at failure. This means that accurate Mohr strength parameters may be determined from an envelope of circles even though the correct position of the failure plane is not known.

The strength parameters used in the analyses discussed previously could be either the strength parameters in terms of total stresses, or the strength parameters in terms of effective stresses. If this approach was used with the shear strength parameters expressed in terms of total stresses, all of the factors which determine the variation of the shear strength parameters (variation of effective stress strength parameters, variation of pore pressure parameters, and reorientation of principal stresses) would be lumped together. Nothing could be learned concerning the orientation of the failure plane with respect to the axes of applied stress, because the orientation of the failure plane is controlled by the "true" friction angle. If this approach were used with the shear strength parameters expressed in terms of effective stresses it would treat only a minor portion of the whole problem of variation of strength with orientation of the failure plane, the variation of the effective stress strength parameters. Experiments made on San Francisco Bay Mud and kaolinite have shown this variation to be of relatively minor importance when compared to variations in pore pressure at failure. The orientation of the failure plane could be predicted if the variation of the "true" friction angle with α were known.

The principal difficulty with this approach either in terms of total or effective stresses is to determine the maximum and minimum values of the shear strength parameters and the manner of variation between these limits. These theories would be of value if it were found that the shear strength parameters varied between the limiting values in a certain way for all soils of a given type; then when the maxima and minima were determined, the shear strength could be predicted for any intermediate relative orientation of stress and anisotropy axes. However, since the extreme values of the shear strength parameters cannot be measured easily, and since the manner of variation between the extreme values is not known, it is doubtful that theories of the type outlined previously can be used practically at the present time.

Analyses of the Effects of Reorientation of Principal Stresses in Isotropic Soils

Undrained Strength on a Sliding Surface

Hansen and Gibson (1949) derived an expression for the undrained strength which could be mobilized on a failure plane, with any orientation, in terms of the initial stress conditions and the Hvorslev shear strength parameters. This work was done before Skempton had suggested the pore pressure parameters A and B (Skempton, 1954), and the earlier "λ-theory" (Skempton, 1948a), was used to relate changes in pore pressure to changes in total stress. The expression derived by Hansen and Gibson, in terms of the symbols used in this report, is

$$c_u/p = \frac{c_e}{p} \cos \Phi_e + \frac{1}{2}(1 + k_o) \sin \Phi_e - \sin \Phi_e \left(\frac{1 - \lambda}{1 + \lambda}\right)\left[\left(\frac{c_u}{p}\right)^2\right.$$

$$\left. - \frac{c_u}{p} (1 - k_o) \cos 2(45 + \frac{\Phi_e}{2} - \alpha) + \left(\frac{1 - k_o}{2}\right)^2\right]^{1/2} . \qquad (2\text{-}1)$$

In this equation, λ is the ratio of the slope of a swelling curve to the slope of the virgin consolidation curve. It was assumed in the derivation that the values of λ, c_e and Φ_e were the same for all values of α, i.e. for any orientation of the failure plane. The solution gives the strength for any orientation of the failure plane when the initial stress conditions are anisotropic and the soil is isotropic with respect to c_e, ϕ_e and λ.

In the λ-theory proposed by Skempton, the change in pore pressure due to a change in total stresses is expressed by:

$$\Delta u = \Delta\sigma_3 + \frac{1}{1 + 2\lambda} (\Delta\sigma_1 - \Delta\sigma_3) \qquad (2\text{-}2)$$

By comparison with equation 1-1, it can be seen that

$$\bar{A} = \frac{1}{1 + 2\lambda} \qquad (2\text{-}3)$$

Solving for λ in terms of $\bar{A}$, it is found that

$$\lambda = \frac{1 - \bar{A}}{2\bar{A}} \qquad (2\text{-}4)$$

and

$$\frac{1 - \lambda}{1 + \lambda} = \frac{3\bar{A} - 1}{\bar{A} + 1} \qquad (2\text{-}5)$$

Using this relationship between λ and $\bar{A}$, equation 2-1 may be written in terms of the more familiar $\bar{A}$ as follows,

$$\frac{c_u}{p} = \frac{c_e}{p} \cos \Phi_e + \frac{1}{2}(1 + k_o) \sin \Phi_e - \sin \Phi_e \left(\frac{3\bar{A} - 1}{\bar{A} + 1}\right) \left[\left(\frac{c_u}{p}\right)^2 - \frac{c_u}{p}(1 - k_o) \cos 2(45 + \frac{\Phi_e}{2} - \alpha) + \left(\frac{1 - k_o}{2}\right)^2\right]^{1/2} \qquad (2\text{-}6)$$

The corresponding expression for c_u/p in terms of c', Φ' obtained by simply substituting c', Φ' for c_e, Φ_e, in equation 2-6 is:

$$\frac{c_u}{p} = \frac{c'}{p} \cos \Phi' + \frac{1}{2}(1 + k_o) \sin \Phi' - \sin \Phi' \left(\frac{3\bar{A} - 1}{\bar{A} + 1}\right) \left[\left(\frac{c_u}{p}\right)^2 - \frac{c_u}{p}(1 - k_o) \cos 2(45 + \frac{\Phi'}{2} - \alpha) + \left(\frac{1 - k_o}{2}\right)^2\right]^{1/2} \qquad (2\text{-}7)$$

It seems likely that the shear strength parameters c_e and Φ_e are more fundamental than the parameters c' and Φ', i.e., that c_e and Φ_e better express the relationship between the effective normal stress on the failure plane at failure and the shear strength. If this is true, then using c' and Φ' rather than c_e and Φ_e in an analysis of the type performed by Hansen and Gibson would overestimate the dependency of the shear strength on the effective stress on the failure plane at failure, because the parameter Φ' is always larger than the parameter Φ_e. To investigate the magnitude of the discrepancy, the influence of the reorientation of principal stresses on the shear strength of a soft clay has been computed using both equation 2-6, which is written in terms of c_e and Φ_e, and equation 2-7, which is written in terms of c' and Φ'; the results of these computations are shown in figure 4. The upper part of figure 4 shows that the values of the parameters were selected so that the c_e, Φ_e failure criterion and the c', Φ' failure criterion give the same undrained strength

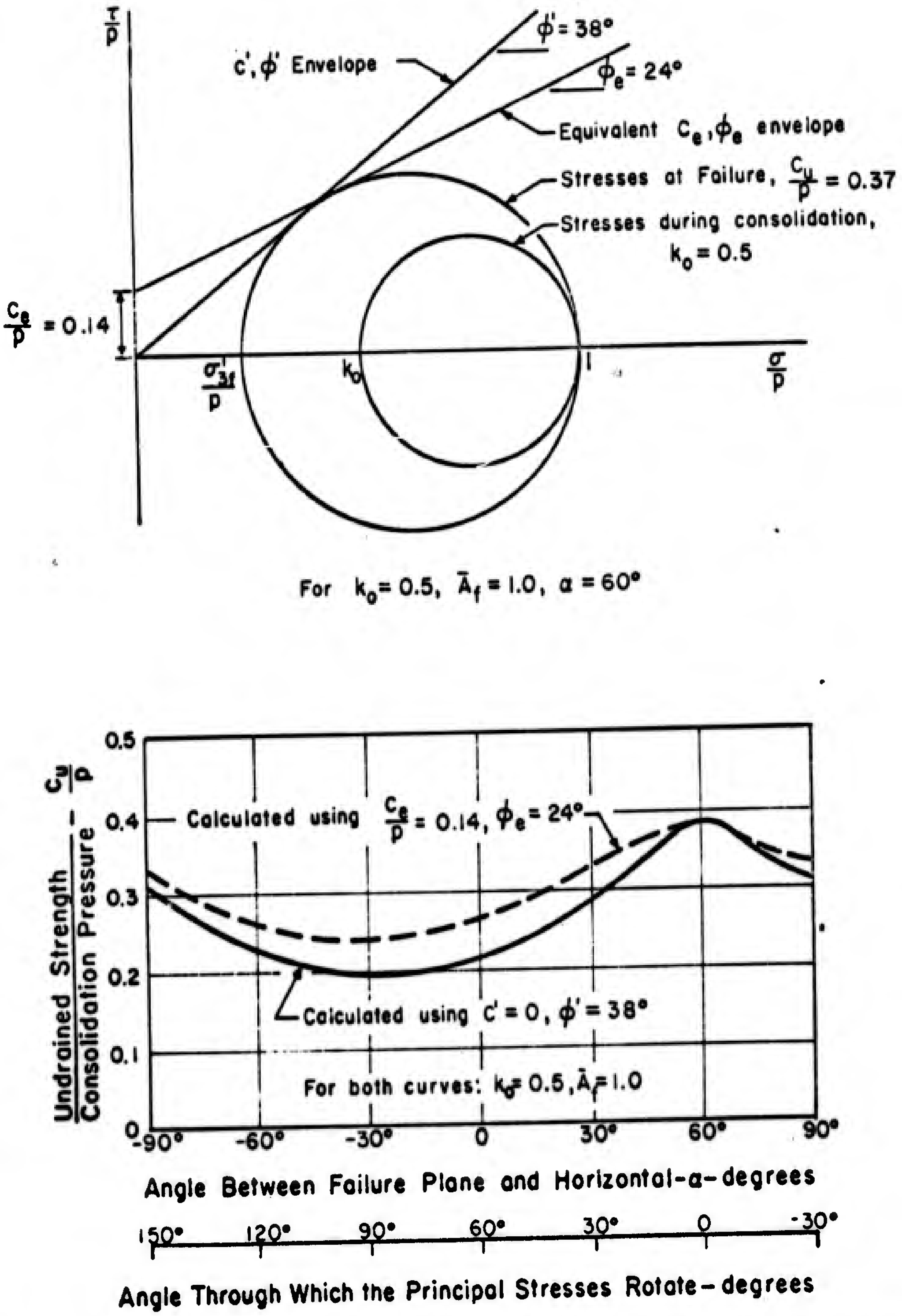

Fig.4- THE EFFECT OF ROTATION OF THE PRINCIPAL STRESSES ON THE UNDRAINED STRENGTH OF A SOIL WHICH IS ISOTROPIC WITH RESPECT TO $\bar{A}_f$ AND c_e, ϕ_e OR c', ϕ'.

ratio c_u/p for $k_o = 0.5$, $\bar{A}_f = 1$ and $\alpha = 60°$; these are typical values for a laboratory AC-U test. As shown in the lower part of figure 4, for any other value of α the c_e, Φ_e failure criterion predicts higher values of the ratio c_u/p than the c', Φ' failure criterion. In figure 4, the degree of reorientation of the principal stresses is related to the orientation of the failure plane; the angle by which the principal stresses are reoriented during undrained shear is approximately equal to $60° - \alpha$.

From the results of the computations, shown in figure 4, it may be seen that if soil were isotropic with respect to the more fundamental shear strength parameters, c_e and Φ_e, and also with respect to $\bar{A}_f$, and if correct values of the ratio c_u/p for any value of α could therefore be calculated using equation 2-6, then equation 2-7 would give incorrect values. As a matter of fact, soils are probably anisotropic with respect to c_e, Φ_e, with respect to c', Φ' and with respect to $\bar{A}_f$*. This means that equations 2-6 or 2-7 will predict correct values of c_u/p only if the proper values of c_e, Φ_e or c', Φ' and $\bar{A}_f$ are used for each value of α. Since c', Φ' can be determined by making tests on only normally consolidated samples, whereas c_e, Φ_e cannot, there is a practical advantage to using equations 2-7 although equation 2-6 expresses the fundamental relationships more clearly.

Equations 2-1, 2-6 and 2-7 give the strength which could be mobilized on a failure plane making an angle α with the horizontal if the principal stresses rotate through an angle $(45 + \phi/2 - \alpha)$. An example of the type of failure to which these equations apply is the case shown in figure 1; it is characterized by the fact that the motion of any particle would describe a path in a vertical plane. Only the horizontal plane has a single strength independent of the direction of motion of the particles in that plane, and that is because the amount of rotation of principal stresses is independent of the path

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*San Francisco Bay Mud has been found to be anisotropic with respect to Φ' and $\bar{A}_f$, whereas c' is equal to zero for two different orientations of the failure plane. The experimental results also indicate that Bay Mud is anisotropic with respect to c_e or Φ_e or both.

of motion. In all other planes the path of motion determines the amount of reorientation of principal stresses, and there are an infinite number of possibilities for each plane.

Bearing Capacity and Earth Pressures

Hansen (1952) extended the earlier analysis (by himself and Gibson) by finding solutions to the problems of bearing capacity and lateral earth pressures for saturated undrained clay. These solutions include the effect of rotation of principal stresses by assuming that shear strength varies sinusoidally with the angle between the failure plane and the horizontal. No further assumptions regarding anisotropy are required.

Pore Pressures in a Thin Silt Layer

Bishop and Henkel (1953) analyzed the stability of a dam built on a foundation which contained a thin layer of clayey silt (L.L. = 28, P.L. = 18). Because the thin layer was the weakest part of the foundation of the dam, it was desired to predict the increase in pore pressure in the layer caused by construction of the dam, and to compute the rate of dissipation of this pore pressure during construction. This procedure allowed the use of higher strengths in the analysis, and made it possible to check the analysis by measuring pore pressures in the field during construction.

To predict the pore pressures which would result from construction of the dam, Bishop and Henkel derived an expression for the pore pressures induced in a thin clay layer when a change in normal stress, $\Delta\sigma$, and a change in shear stress, $\Delta\tau$, are applied to its upper surface. Under this type of loading the principal stresses are reoriented. The relationship between the applied stresses and the induced pore pressure is

$$\Delta u = \Delta p + p\left(\frac{1 - k_o}{2}\right) + (2\bar{A} - 1)\left[p^2\left(\frac{1 - k_o}{2}\right)^2 + \Delta\tau^2\right]^{1/2}$$

Unfortunately, no information is given concerning the agreement between the predicted and measured pore pressures.

Conclusions from Previous Analytical Studies

When the principal stresses rotate during a change in stress, the deviator of the change in stress may be much larger than the difference between the deviators of the initial and final states of stress. If changes in pore pressure are assumed to be proportional to the deviator of the change in stress, then it can be shown that reorientation of principal stresses tends to reduce the strength by increasing pore pressure, as illustrated in figure 4.

In actual fact, the assumption of proportionality between changes in pore pressure and changes in deviator stress is probably not correct in the sense that there is not a unique constant of proportionality which applies to all possible orientations of the failure plane. This is to say that soils are probably anisotropic with respect to $\bar{A}_f$. The fact that $\bar{A}_f$ varies with the type of test and reorientation of the failure plane means that Hansen and Gibson's analysis is not a complete solution to the problem of variation of strength with orientation of the failure plane. Never-the-less, their analysis is a completely sufficient analytical tool, provided only that one can measure or infer the correct soil properties for each value of α.

General Summary of Previous Work

Unconsolidated, undrained triaxial and direct shear tests have shown that five different one-dimensionally consolidated clays were anisotropic with respect to undrained strength, and that the variation of undrained strength with orientation of the failure plane may be different for different clays. Anisotropy with respect to undrained strength probably results from parallel orientation of clay particles during anisotropic consolidation, and the degree of anisotropy probably increases with increasing amounts of clay.

Hansen and Gibson (1949) have made an analysis of the effect of reorientation of principal stresses on the undrained strength of anisotropically consolidated soils. This analysis shows that reorientation of principal stresses tends to reduce the undrained strength by increasing the pore water pressure and reducing the effective stress at failure. In order to use this analysis for solution of practical problems, it is necessary to know the values of c', ϕ' or c_e, ϕ_e and the values of $\bar{A}_f$ which apply to each orientation of the failure plane.

The studies described above have answered the first question which arose from the discussion in section I, i.e.

(1) Are soils anisotropic with respect to undrained strength? Both the unconsolidated, undrained triaxial tests and the unconsolidated undrained direct shear tests described above indicate that soils are anisotropic with respect to undrained strength.

Still unanswered are the questions concerned with the fundamental nature of this anisotropy, and with the practical implications, i.e.

(2) Does anisotropy with respect to undrained strength result from anisotropy with respect to the strength parameters in terms of effective stresses, or from anisotropy with respect to development of pore water pressure?

(3) As a result of both anisotropy and reorientation of principal stresses, how would the undrained strength in-situ vary with the orientation of the failure plane?

(4) Is the variation of undrained shear strength with orientation of the failure plane the same in-situ and in the laboratory?

(5) How do anisotropy and reorientation of principal stresses affect the stability of a clay slope?

The experimental work and analyses which were performed to answer these questions are described in the following sections: An investigation of the cause of anisotropy is described in section III, investigations and analyses of the in-situ variation of undrained strength is made in section V, and an example of a stability analysis using anisotropic strength is described in section VII.

III. EXPERIMENTAL INVESTIGATION OF THE CAUSE OF ANISOTROPY WITH RESPECT TO UNDRAINED STRENGTH

Objective

The experiments described in this section were performed in order to answer one of the questions which remained unanswered after the literature review, i.e.

> Does the anisotropy of soil with respect to undrained strength result from anisotropy with respect to the strength parameters in terms of effective stresses, or from anisotropy with respect to development of pore water pressure?

To answer this question, undrained tests with pore pressure measurement were made on samples of one-dimensionally consolidated kaolinite which were trimmed in different directions. These tests gave the desired information for kaolinite; c' and ϕ' could be determined for each orientation of the failure plane, and the pore pressures or pore pressure parameters could be compared directly for samples trimmed in different directions.

Properties of the Kaolinite

A commercially available kaolinite, ASP-900, was used in all of the strength tests described below. The liquid limit of this kaolinite is 45 percent, the plastic limit is 34.8 percent, and 48 percent of the particles are smaller than 2 microns. (Seed, Woodward and Lundgren, 1964). The coefficient of consolidation c_v, is about 0.01 cm^2/sec; the compression and swelling indices are $C_c = 0.33$, $C_s = 0.06$ respectively.

Preparation of the Samples

Tests were performed on two different batches of overconsolidated kaolinite. The methods of consolidating these batches are described below.

Batch I.

Dry kaolinite was mixed with enough deaired tap water to make the water content equal to 66.2 percent. The mixture was put into three six-inch diameter, eight-inch high molds and consolidated one-dimensionally to 1.5 kg/cm^2. Then the three 6-inch diameter blocks, each about 5 inches high, were stacked up and consolidated further in a large triaxial cell. The three-part "sample" was consolidated anisotropically to $\sigma_1' = 9\ kg/cm^2$, $\sigma_3' = 5.5\ kg/cm^2$, and rebounded to an isotropic stress of 1 kg/cm^2. The high value of c_v for the kaolinite (0.01 cm^2/sec) made this method of consolidation practicable even though the drainage path was quite long. This method of preparation was chosen so that the samples from all blocks would be as nearly the same as possible, and so that the stress conditions at the end of consolidation would be isotropic, making the influence of the stress release nil. After consolidation and rebound, the three blocks were separated, triple-wrapped in evacuated plastic bags and stored in the wet room until needed. Cylindrical samples, 1.4 inches in diameter and 3.5 inches long, were trimmed from the blocks with their axes vertical, inclined at 45°, and horizontal. The direction of the plane on which the major principal stress had acted during consolidation was marked to that the orientation of this plane could be compared with that of the failure plane. Water contents were taken from the sample trimmings. All of the data in table 2 and in figures 5, 7 and 8a were measured in tests performed on samples from Batch I.

Batch II.

Dry kaolinite was mixed with deaired tap water to give a water content equal to 67.6 percent, and this mixture was consolidated to 16 kg/cm^2 and rebounded to 1 kg/cm^2 in a six-inch diameter, eight-inch high mold. Horizontal and vertical samples, 1.4 inches in diameter and 3.5 inches high, were trimmed from the consolidated block. The samples were sealed inside two membranes with a layer of silicone grease between and stored under water until needed for testing. The data obtained from tests on these samples are shown in figure 8b.

UU Triaxial Tests

In the initial phase of this study, UU triaxial tests were performed on vertical and horizontal samples of kaolinite from batches other than Batch I and Batch II described above. These exploratory tests showed that one-dimensionally consolidated kaolinite was anisotropic with respect to strength but the data are not given here. At this earliest stage it was assumed, incorrectly, that the horizontal and vertical samples would have the maximum and minimum strengths and inclined samples an intermediate strength, as in Hvorslev's tests. Batch I was prepared primarily for the purpose of performing consolidated undrained tests with pore pressure measurements, and these tests were made on horizontal and vertical samples. When the consolidated undrained tests had been completed, horizontal, inclined (45°) and vertical samples were trimmed from the remainder of the material from Batch I, and unconsolidated undrained tests were made on the three types of samples. These tests (figure 5) showed that in actual fact the inclined samples were weakest. If this had been known before the CU tests were made, they would have been made on vertical and inclined rather than vertical and horizontal samples.

Variation of Strength with Orientation of Failure Plane

The strengths measured in the UU tests made on samples from Batch I are shown in figure 5. The minimum strength is associated with inclined (45°) samples where the failure plane was nearly coincident with the horizontal. The ratios of strengths are:

$$\frac{(\sigma_1 - \sigma_3)_f \text{ inclined } (45°)}{(\sigma_1 - \sigma_3)_f \text{ vertical}} = 0.75$$

$$\frac{(\sigma_1 - \sigma_3)_f \text{ horizontal}}{(\sigma_1 - \sigma_3)_f \text{ vertical}} = 0.87$$

The latter ratio is the same as that measured in the preliminary tests which are not reported in detail.

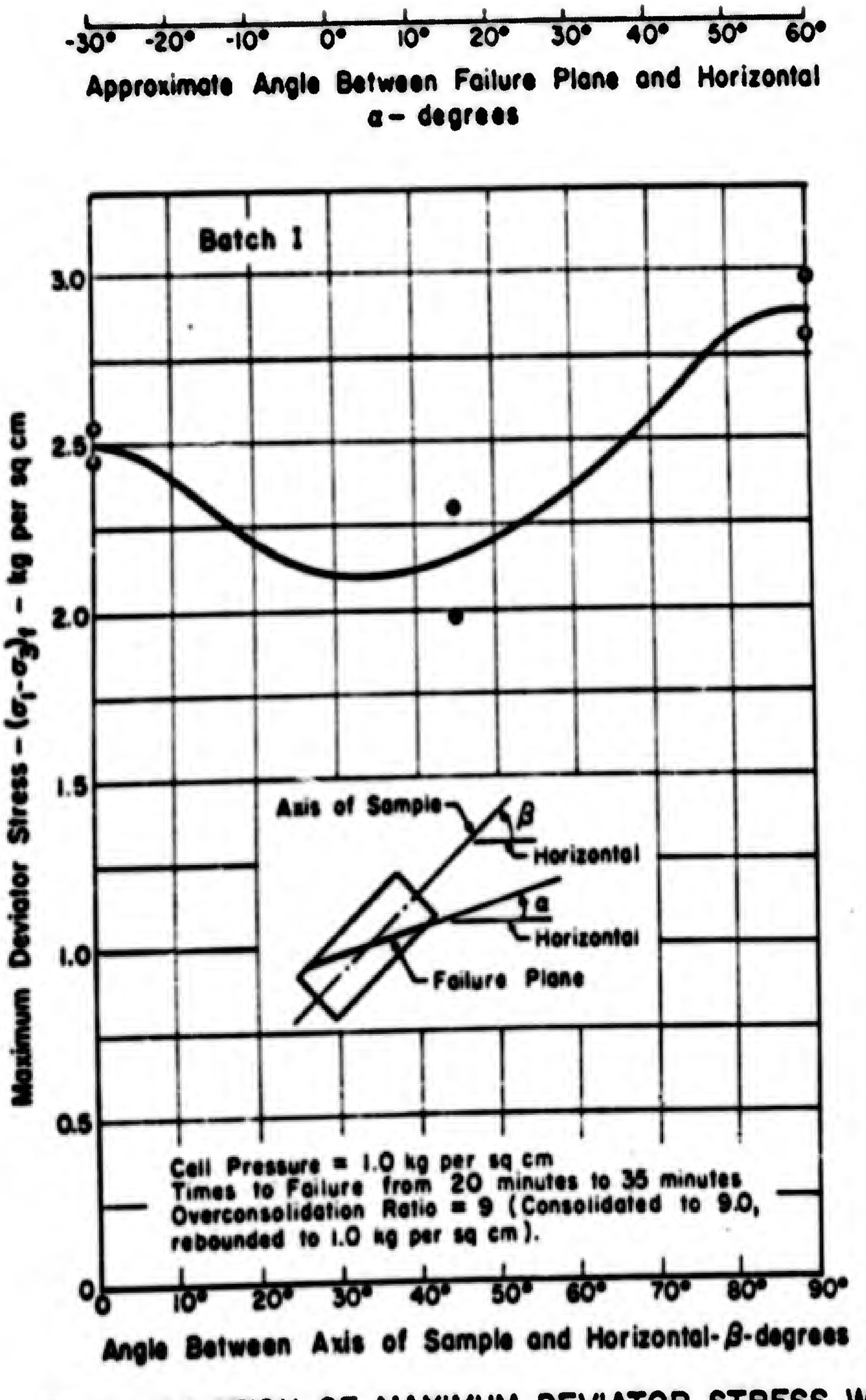


Fig.5 – VARIATION OF MAXIMUM DEVIATOR STRESS WITH THE ORIENTATION OF THE FAILURE PLANE IN UU TRIAXIAL TESTS ON OVERCONSOLIDATED KAOLINITE.

The curve through the data is drawn with horizontal tangents at $\beta = 0$ and $\beta = 90°$ because these are the extremes of sample orientation. A sample with $\beta = -5°$ is identical to a sample with $\beta = +5°$, and a sample with $\beta = 85°$ is identical to a sample with $\beta = 95°$, so the variation of strength with β must be periodic as shown. The variation of strength with α, however, may be periodic in a different way in different types of tests, or in the field. For instance, it is possible that instead of reaching a maximum at $\alpha = 60°$, the strength will be a maximum at $\alpha = 90°$. (If the strength reached a maximum at $\alpha = 90°$, the variation would be approximated by the equation $s_\alpha = s_{max} - (s_{max} - s_{min}) \cos^2 \alpha$, for example). The strength associated with $\alpha = 90°$ could not be measured in a triaxial test if it was the maximum. There are two possible failure planes in any triaxial sample, one inclined at about 30° on one side of the sample axis, the other 30° the other side of the axis. A sample trimmed so that $\beta = 60°$ has possible failure planes with $\alpha = 90°$ and $\alpha = 30°$. If the strength associated with $\alpha = 90°$ is larger than the strength associated with $\alpha = 30°$, then only the latter can be measured. The difference between the strengths which could be mobilized on the two planes ($\alpha = 90°$ and $\alpha = 30°$) would necessarily arise from a difference between the values of the strength parameters in terms of effective stresses associated with the two planes, because the same pore pressures act throughout the sample and the effective stresses on the two planes are the same. (Under the laboratory test conditions, the two planes have the same inclination to the major principal stress and are thus subjected to the same total stresses).

Relative Orientation of the Failure Plane and the Horizontal

The plane called the horizontal is the plane on which the major principal acted during virgin consolidation. In a sample trimmed in any direction the angle between the horizontal and the failure plane is approximately $\beta - 30°$. The orientation of the horizontal plane was known for all samples in Batches I and II, and the relative position of the failure plane and the horizontal was noted when each sample was disassembled.

The following observations are true of every test in which the orientation of the horizontal plane was known:

(1) In both horizontal and inclined samples, the line of intersection of the failure plane and the horizontal was always practically perpendicular to the axis of the sample.

(2) In inclined samples the failure plane was nearly parallel to, rather than nearly perpendicular to the horizontal.

These observations are shown in figure 6. The other possibilities, which were not observed, are that the failure plane intersect the horizontal so that the line of intersection would not be perpendicular to the axis of the sample as shown by the dotted plane in the upper part of figure 6, or that the failure plane intersect the horizontal nearly perpendicularly as shown by the dotted plane in the lower part of figure 6.

The fact that these latter possibilities were not observed must mean that the effective stress envelopes associated with these orientations of the failure plane are higher than those associated with the actual failure planes. The amount of difference is not known, but it must have been more than the usual inhomogeneities associated with any sample because the observations were consistent.

IC-U Triaxial Tests

The reason for using heavily overconsolidated samples in this phase of the study was to be able to reconsolidate a sample isotropically in the triaxial cell without obliterating the anisotropic properties induced by its previous stress history. By using high anisotropic consolidation pressures it was hoped to produce samples which were insensitive to subsequent lower stresses. This procedure appears to have been successful; the samples were anisotropic with respect to strength even when reconsolidated isotropically.

The results of the consolidated undrained tests are summarized in table 2 and figures 7 and 8a. Unfortunately, the degree of saturation of the samples when trimmed was less than 100 percent, and they were still unsaturated when tested. This means that the absolute magnitudes of the measured pore pressures and calculated values of the

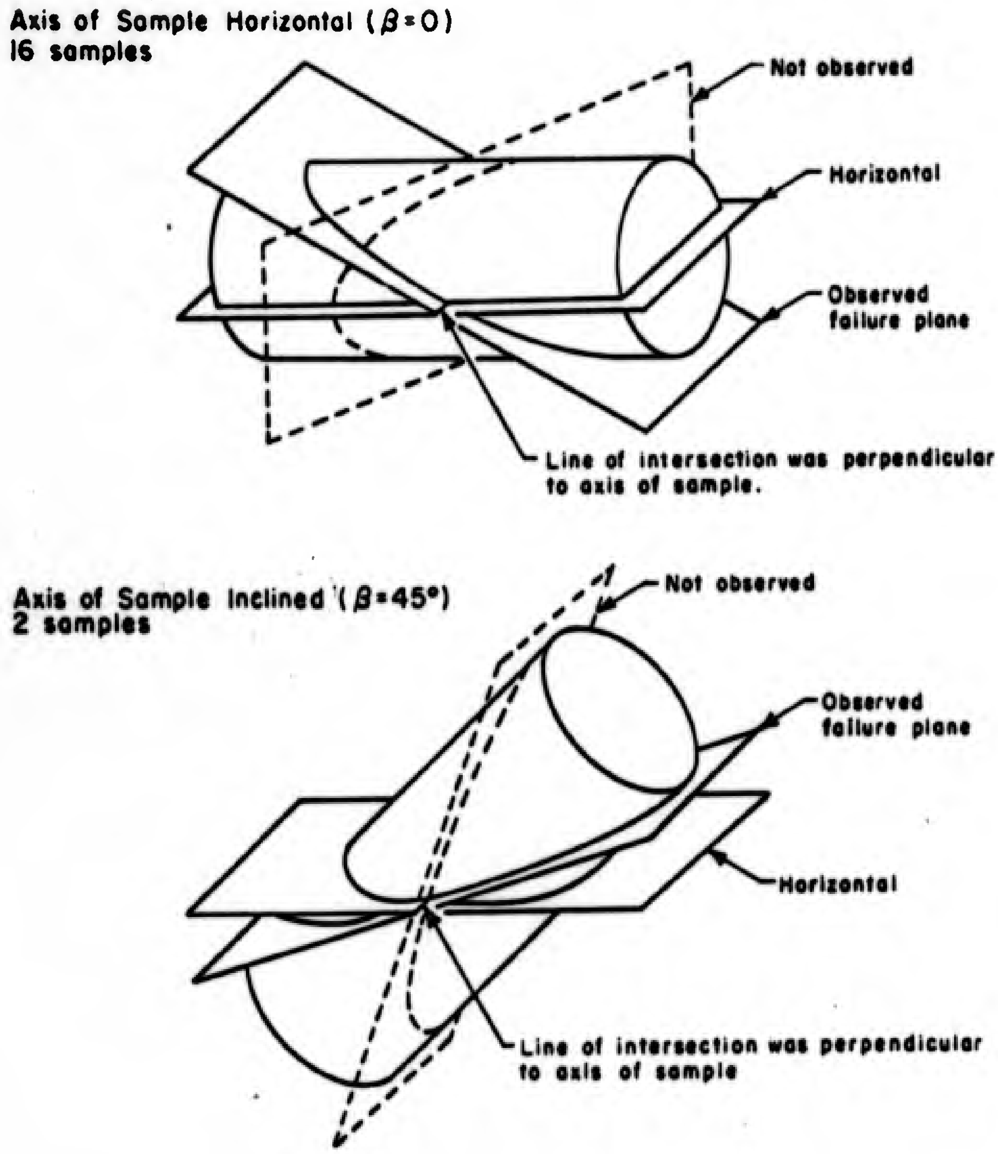


Fig.6 - ORIENTATION OF THE FAILURE PLANE WITH RESPECT TO THE HORIZONTAL IN TRIAXIAL TESTS ON OVERCONSOLIDATED KAOLINITE.

effective stress at failure are probably not correct. It does not preclude valid qualitative conclusions being drawn from comparison of horizontal (CU-H) and vertical (CU-V) samples, however, since there was no prejudice in the degrees of saturation. None of these results has been corrected for the loads carried by filter paper drains and rubber membranes or for piston friction. The corrections would be the same for any pair of samples however, and could not be responsible for the measured differences in behavior of horizontal and vertical samples.

The measured strengths of the samples are shown in the upper part of figure 7. The average strength ratio shown by the lines drawn through the data is

$$\frac{(\sigma_1 - \sigma_3)_{f \text{ horizontal}}}{(\sigma_1 - \sigma_3)_{f \text{ vertical}}} = 0.90$$

The strength ratio increases slightly with increasing consolidation pressure, indicating that the isotropic reconsolidation does tend to obscure the anisotropic properties.

The effective stress strength envelope is shown in the lower part of figure 7. Although the observations of the failure planes indicated that the soil is anisotropic with respect to the effective stress strength parameters, the scatter in the data obscures the difference between the horizontal and vertical samples. The results of drained tests, also shown in the figure, are consistent with the results of the undrained tests.

The pore pressure parameters $\bar{A}_f$ calculated from the pore pressures measured in these tests are shown in figure 8a. For both horizontal and vertical samples $\bar{A}_f$ decreased with increasing overconsolidation ratios, but $\bar{A}_f$ was always higher (or less negative) for horizontal than for vertical samples. Thus the horizontal samples had higher pore pressures at failure and lower strengths than the vertical ones. This is the most distinctive difference in behavior of the two types of samples.

Table 2. Summary of Results of Undrained and Drained Triaxial Tests on Samples of Overconsolidated Kaolinite, Batch I.

Type of Test	Sample	Consol. Pressure (kg/cm^2)	w/c at Failure (percent)	S at Failure (percent)	B after Consol.	$(\sigma_1-\sigma_3)_f$ (kg/cm^2)	$\bar{A}_f$	ϵ_{af} (percent)
Consolidated Undrained	CU-V-1	0.50	34.9	99.9	0.96	2.17	-0.30	9.0
	CU-V-2	0.75	34.9	100	0.98	2.68	-0.27	8.5
	CU-V-3	1.00	35.0	99.3	0.76	2.57	-0.17	10.0
	CU-V-4	1.50	34.3	99.6	0.86	2.70	-0.06	7.0
	CU-V-5	3.00	34.0	98.9	0.68	3.69	+0.21	9.0
	CU-H-1	0.50	34.9	99.8	0.92	1.71	-0.26	8.0
	CU-H-2	0.75	34.9	100	0.98	2.33	-0.14	10.5
	CU-H-3	1.00	34.4	99.7	0.88	2.07	-0.02	8.5
	CU-H-4	1.50	34.0	99.8	0.92	2.45	+0.11	8.0
	CU-H-5	3.00	33.8	98.7	0.64	3.22	+0.34	10.5
Consolidated Drained	CD-V-1	1.00	35.0			2.13		5.5
	CD-V-2	1.00	-	99.7	0.90	2.26		8.5
	CD-H-1	1.00	35.1			1.96		5.8
	CD-H-2	1.00	34.4	99.9	0.97	2.18		7.0

Continued

Table 2. Continued

Type of Test	Sample	Time to Failure (hours)	σ'_{1f} (kg/cm^2)	σ'_{3f} (kg/cm^2)	$\frac{1}{2}(\sigma_1-\sigma_3)_f$ (kg/cm^2)	$\frac{1}{2}(\sigma_1+\sigma_3)_f$ (kg/cm^2)	$\frac{1}{2}(\sigma'_1+\sigma'_3)_f$ (kg/cm^2)	Over-Consolidation Ratio
Consolidated Undrained	CU-V-1	5.7	3.32	1.15	1.08	1.58	2.23	18
	CU-V-2	7.3	4.15	1.47	1.34	2.09	2.81	12
	CU-V-3	7.1	4.00	1.43	1.28	2.28	2.71	9
	CU-V-4	7.1	4.37	1.67	1.35	2.85	3.02	6
	CU-V-5	10.1	5.90	2.21	1.84	4.94	4.05	3
	CU-H-1	4.5	2.66	0.95	0.85	1.35	1.80	18
	CU-H-2	6.3	3.41	1.08	1.16	1.91	2.24	12
	CU-H-3	5.5	3.09	1.02	1.03	2.03	2.05	9
	CU-H-4	7.1	3.67	1.22	1.22	2.72	2.44	6
	CU-H-5	9.9	5.12	1.90	1.61	4.61	3.51	3
Consolidated Drained	CD-V-1	6.0	3.13	1.00	1.06	2.06	2.06	9
	CD-V-2	6.1	3.26	1.00	1.13	2.13	2.13	9
	CD-H-1	5.7	2.96	1.00	0.98	1.98	1.98	9
	CD-H-2	5.9	3.18	1.00	1.09	2.09	2.09	9

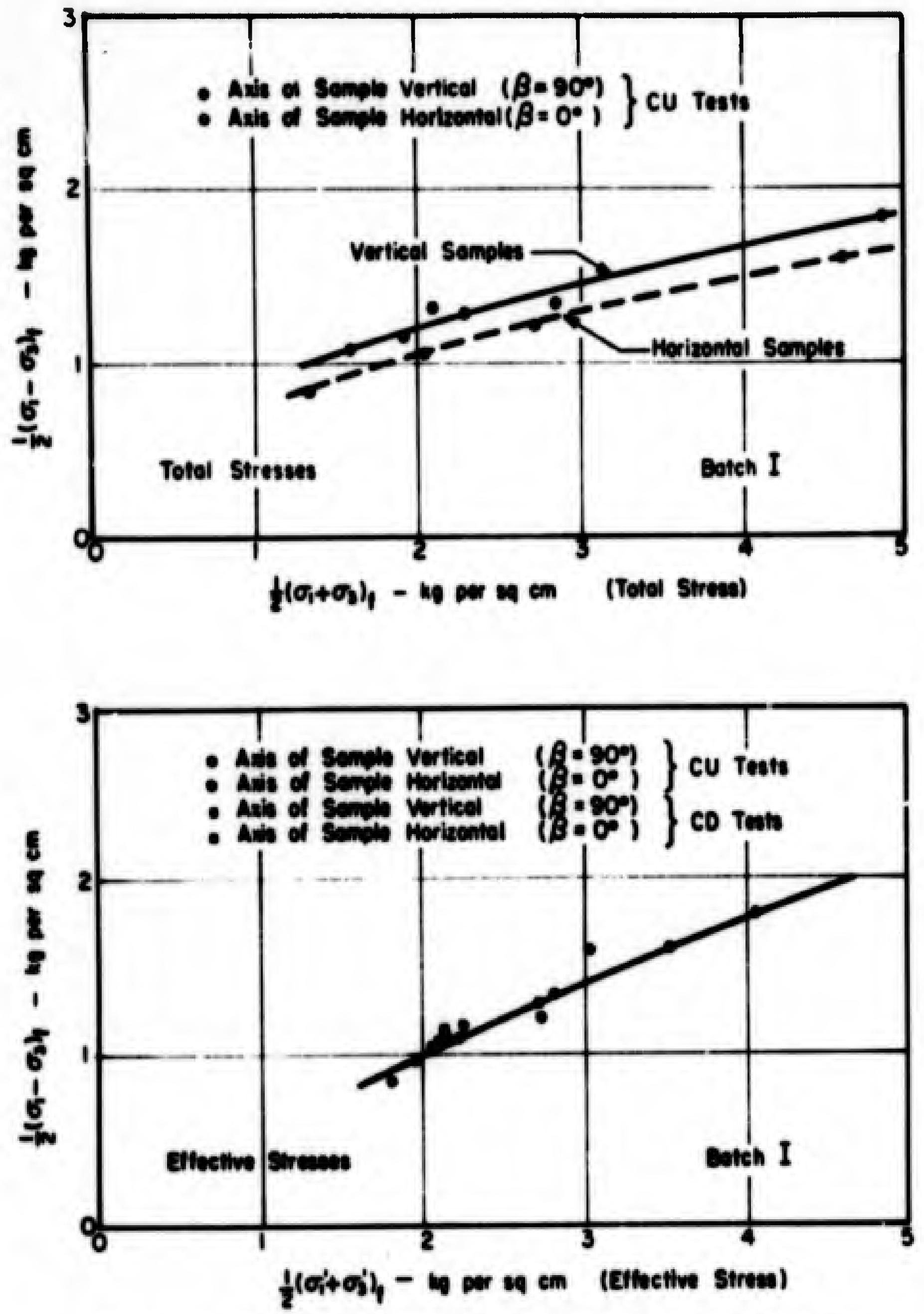


Fig.7 – VARIATION OF MAXIMUM SHEAR STRESS WITH AVERAGE NORMAL TOTAL STRESS AND AVERAGE NORMAL EFFECTIVE STRESS IN CU AND CD TRIAXIAL TESTS ON OVERCONSOLIDATED KAOLINITE.

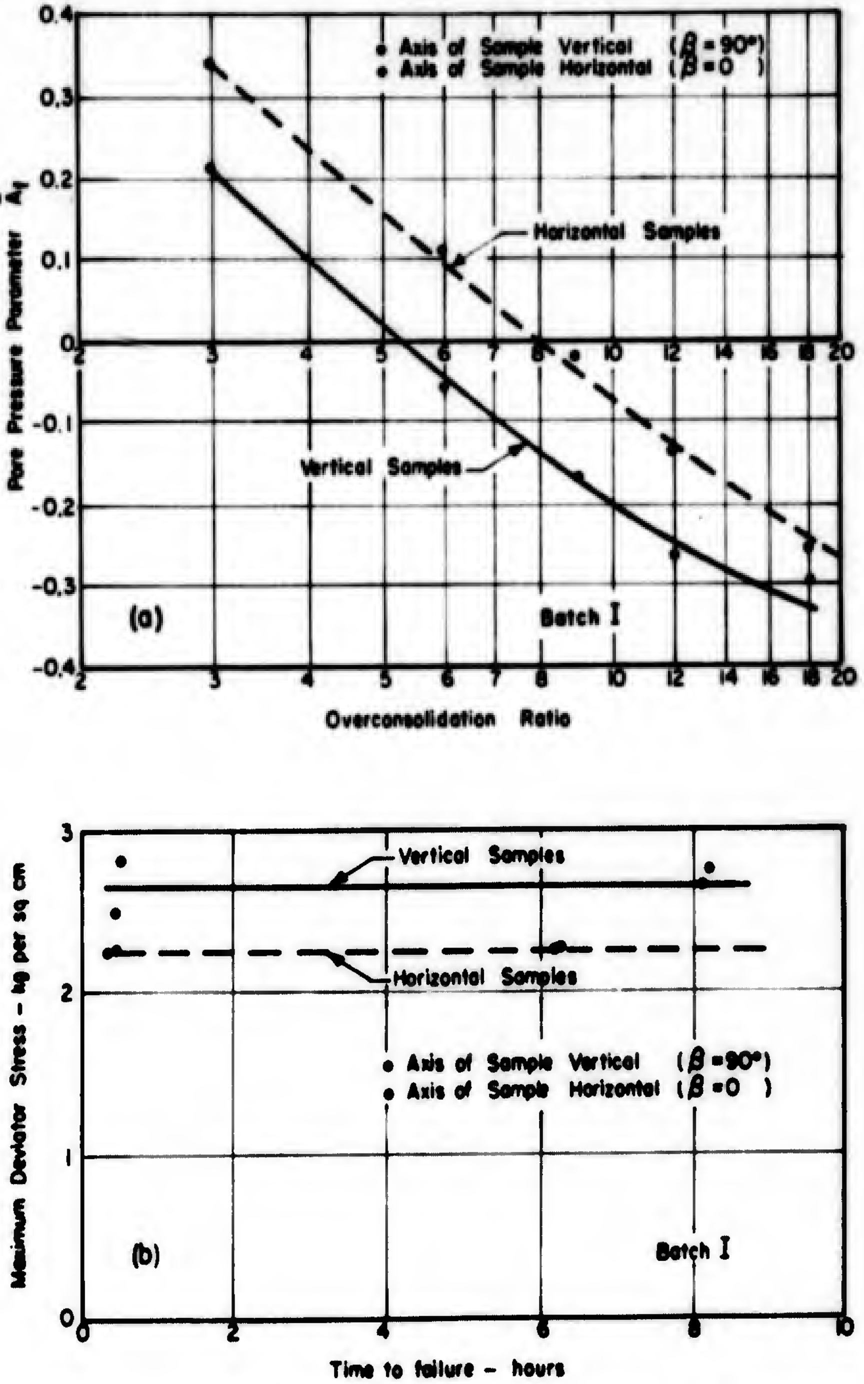


Fig.8-(a) VARIATION OF PORE PRESSURE PARAMETER $\bar{A}_f$ WITH OVERCONSOLIDATION RATIO AND (b) VARIATION OF MAXIMUM DEVIATOR STRESS WITH TIME TO FAILURE IN UNDRAINED TRIAXIAL TESTS ON OVERCONSOLIDATED KAOLINITE.

Because the samples tested in these tests were unsaturated, it was decided to make tests where the times to failure were quite different to see if time to failure had any effect on the strength. This was done because theories available for predicting the time required for a given degree of equalization of pore pressure only apply to fully saturated samples. Batch I1 was prepared for this purpose, and duplicate tests were made on horizontal and vertical samples in which the times to failure were from 20 minutes to 8 hours. The results are shown in figure 8b. The ratio of strengths was practically identical even when the time to failure was varied by a factor of 15, and the conclusion was that pore pressures must have been equal throughout the samples in the IC-U tests and that pore pressure gradients could not have been responsible for the observed differences in strength.

The axial strains at failure were for practical purposes identical for the two types of sample in all types of tests. In the IC-U tests the average strain at failure was 8.7 percent for vertical samples and 9.1 percent for horizontal samples.

IC-D Triaxial Tests

One of the inferences drawn from the results of the IC-U tests was that the drained strength of the two types of sample would be practically the same. In order to check this directly, four drained tests were performed; the data from these tests are summarized in table 3 and shown in figure 7. One vertical sample and one horizontal sample were almost completely saturated, and the others were only partially saturated when tested. The strength of the saturated vertical sample was about 4 percent greater than the strength of the saturated horizontal sample. The relationship between the quantities $1/2\ (\sigma_1 - \sigma_3)_f$ and $1/2\ (\sigma_1' + \sigma_3')_f$ are consistent with those from the IC-U tests, as shown in figure 7.

Conclusions

These tests demonstrate that the kaolinite used is anisotropic with respect to undrained strength after anisotropic consolidation.

So far as it is possible to tell, the variation of strength with the orientation of the failure plane is similar to that found by Bishop for undisturbed London clay from the upper few feet (see figure 3).

Most of the difference in the undrained strength of samples trimmed in different directions is due to the difference in pore pressure which accompanies an increase in deviator stress, i.e. the anisotropy with respect to undrained strength results principally from anisotropy with respect to the pore pressure parameter $\bar{A}_f$.

Observations of the failure planes and drained test results indicate that these samples are also anisotropic, to a small degree, with respect to effective stress strength parameters or drained strength. The number of drained tests was too small, and the scatter in the results of the undrained tests was too large to allow a quantitative estimate of this anisotropy.

IV. THE VARIATION OF UNDRAINED SHEAR STRENGTH WITH ORIENTATION OF THE FAILURE PLANE IN-SITU

Objective

The experiments described in this section were performed to answer the second of the questions which remained unanswered at the end of the literature review; i.e.,

> How would the undrained strength of a soil in the field vary with orientation of the failure plane along a rotational sliding surface as a result of both anisotropy and reorientation of principal stresses.

San Francisco Bay Mud was chosen for use in this study because a large number of uniform, undisturbed samples of the soil could be obtained fairly easily, and because Bay Mud is typical of the soft, sensitive marine clays which are frequently associated with short-term stability problems.

The technique used to answer the question above was to perform two types of anisotropically consolidated, undrained plane strain tests which simulated as closely as possible in the laboratory the conditions of undrained failure at two points on a rotational sliding surface in the field (points A and D in figure 1). From the results of these tests, and consideration of the probable variation of $\bar{A}_f$ with orientation of the failure plane, it is possible to make an estimate of the variation of the in-situ undrained strength with orientation of the failure plane using the equation derived by Hansen and Gibson (equation 2-7).

Description of Bay Mud

San Francisco Bay Mud is the uppermost sedimentary unit in the San Francisco Bay area. It is a marine clay deposited in drowned valleys and on drowned sedimentary flats. Of particular interest here is the top one of the three members into which the Bay Mud unit was divided by Trask and Rolston (1951). This member (called A-1) is the only one probed in borings at the University of California field test site at Hamilton Air Force Base where the undisturbed samples used in this study were obtained. At this site the Bay Mud is a soft to very soft, grey, slightly organic silty marine clay with isolated silt lenses and a few roots and shells. The sensitivity is about 8, and the present electrolyte concentration of the pore water is about 17 gm/litre total dissolved solids taken as equivalent NaCl. This combination of salt concentration and sensitivity agrees with a correlation established by Skempton and Northey (1952) and by Bjerrium (1954), for English and Norwegian marine clays.

Figure 9 is a summary of the properties of Bay Mud determined in field and laboratory tests. The upper few feet of the deposit at this site have been weathered and overconsolidated by desiccation, but before weathering were probably essentially the same as below. Preconsolidation pressures indicate that the effect of desiccation extends nearly to 20 feet below the surface, although the lowest water level observed in bore holes is 12 feet below the surface. Below 18 feet the strength is proportional to the effective overburden pressure, and the ratio c_u/p is about 0.32 as determined from the UU triaxial and field vane shear tests shown in figure 9. (The results of the UU triaxial tests shown in figure 9 have been corrected for the loads carried by piston friction and rubber membranes.)

The average Atterberg limits are LL = 88, PL = 43 when the limit tests are performed on soil which has not been allowed to dry. After air drying the liquid limit is lowered by about 20 percent, and the plastic limit increased by about 6 percent. Mitchell (1961) says that this could be due to either alteration of the organic matter or cation fixation during drying. Most of the scatter in the measured water contents shown in figure 9 is probably due to the fact that samples may contain greater or lesser amounts of organic matter and silt which are inhomogeneously distributed. Krone (1962) found that a recent sediment in the Bay Area contained 2.5 percent organic matter. Sixty percent of the sample he tested was finer than 2 microns and 100 percent was finer than 40 microns. Silt lenses are usually not continuous over more than a few square inches but they may greatly affect the horizontal permeability of a small laboratory sample. As determined in oedometer tests, $C_c = 1.1$, $C_s = 0.028$, and $c_v = 2 \times 10^{-4}$ cm^2/sec.

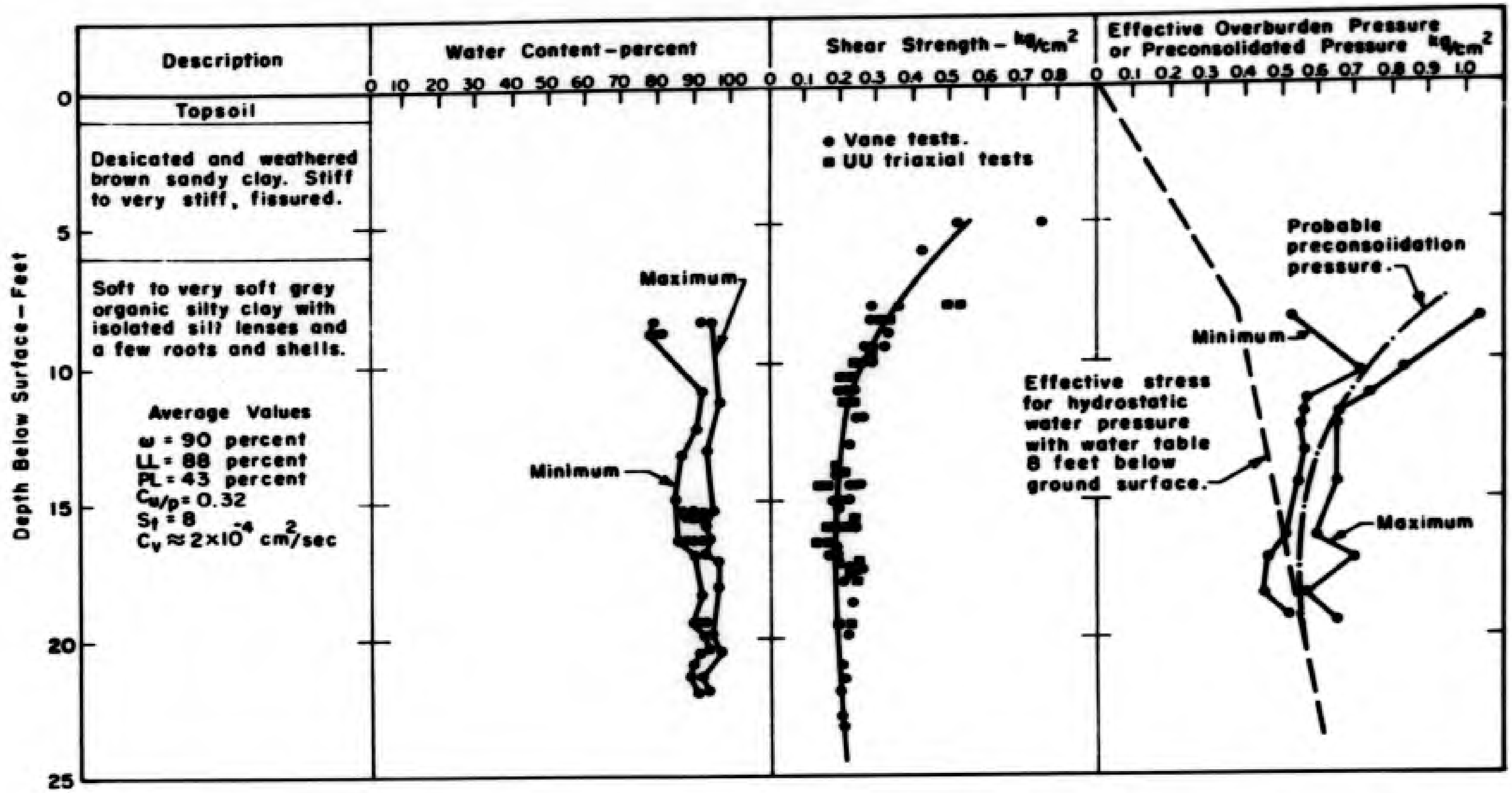


Fig.9— SUMMARY OF THE PROPERTIES OF SAN FRANCISCO BAY MUD (SAMPLES FROM HAMILTON AIR FORCE BASE, MARIN COUNTY).

Mitchell (1961) used X-ray diffraction and electron photomicrographic techniques to analyze the composition of recently deposited muds. He found that Bay Mud contains montmorillonite, kaolin, vermiculite, and mica and/or hydrous mica. Tube-shaped particles are evident in the electron photomicrographs, indicating that the kaolin mineral may be halloysite. The activity, A, as defined by Skempton (1953) is in the range $0.8 \leq A \leq 1.0$.

Obtaining Undisturbed Samples of Bay Mud

All of the undisturbed samples of Bay Mud tested in the laboratory were secured by hand sampling with a 5-inch diameter fixed-piston sampler. The bore holes are made with hand augers, to a maximum depth of about 25 feet. Samples for research investigations are taken from 15 to 25 feet below the surface. Sample tubes are machined brass, with an area ratio of 10 percent (Hvorslev, 1949) and are either 12 inches or 16 inches long. The piston is made of lucite and has an "O"-ring seal. The piston is left in place to seal the top of the sample after it is secured, and a lucite, "O"-ring sealed cap seals the bottom. Samples are transported to the laboratory and stored in a 68°F wet-room in the sample tubes. As Bay Mud is needed for testing, it is extruded from the tube, and afterward the tube is resealed.

Occasional soft spots are noticed, and these samples are rejected. Usually the samples are firm and appear to be of the highest quality. A one half inch thick ring of soil around the edge of the tube is not used for strength or consolidation tests, but may be used for water content determinations.

Corrections to Experimental Data

Need for Corrections

Before discussing the methods of test utilized or the results obtained it is desirable to comment briefly on the need for applying appropriate corrections to strength test data. In any type of triaxial compression test it is necessary to surround the sample by a rubber membrane. Furthermore, in all except vacuum triaxial tests, it is necessary to set up the sample in a pressure cell and apply load to it by means of a piston passing through the head of the cell. When making tests on soils of low permeability, it is common practice to place filter paper drains along the sides of the sample to expidite drainage. In plane strain tests, it is also necessary to use end plates to insure that the sample will deform in plane strain. The loads carried by filter paper drains, rubber membranes and cell piston friction can amount to a significant part of the total axial load apparently carried by an undrained plane strain or triaxial sample. In the case of plane strain samples there is also friction between the sample and the end plates. Neglecting to correct for these factors results in an over-estimate of the axial load carried by a triaxial or plane strain sample at the end of consolidation and at failure.

To determine the variation of undrained strength in-situ with orientation of the failure plane, two different types of plane strain tests were performed on undisturbed Bay Mud. One of these types of test simulated the conditions of consolidation and subsequent undrained failure at point A in figure 1, and the other simulated the conditions at point D.

Both of these types of sample were consolidated approximately one-dimensionally, but the strains during consolidation, and the cross-sectional areas at the end of consolidation, were different for the two types of samples, as shown in figure 10. Those samples representing point A were compressed axially during consolidation, and as a result, the cross-sectional areas did not change significantly. Those samples representing point D, on the other hand, were compressed laterally during consolidation and the cross-sectional areas did change.

Because of the differences in strains during consolidation and cross-sectional areas at the end of consolidation, the necessary corrections to the axial stress for loads carried by filter paper drains, rubber membranes, cell piston friction and end plate friction are different for the two different types of test. This means that valid comparison of the results of the two types of test could not be made until the results had been corrected. The results of the tests which are of primary interest are the stresses at the end of consolidation and at

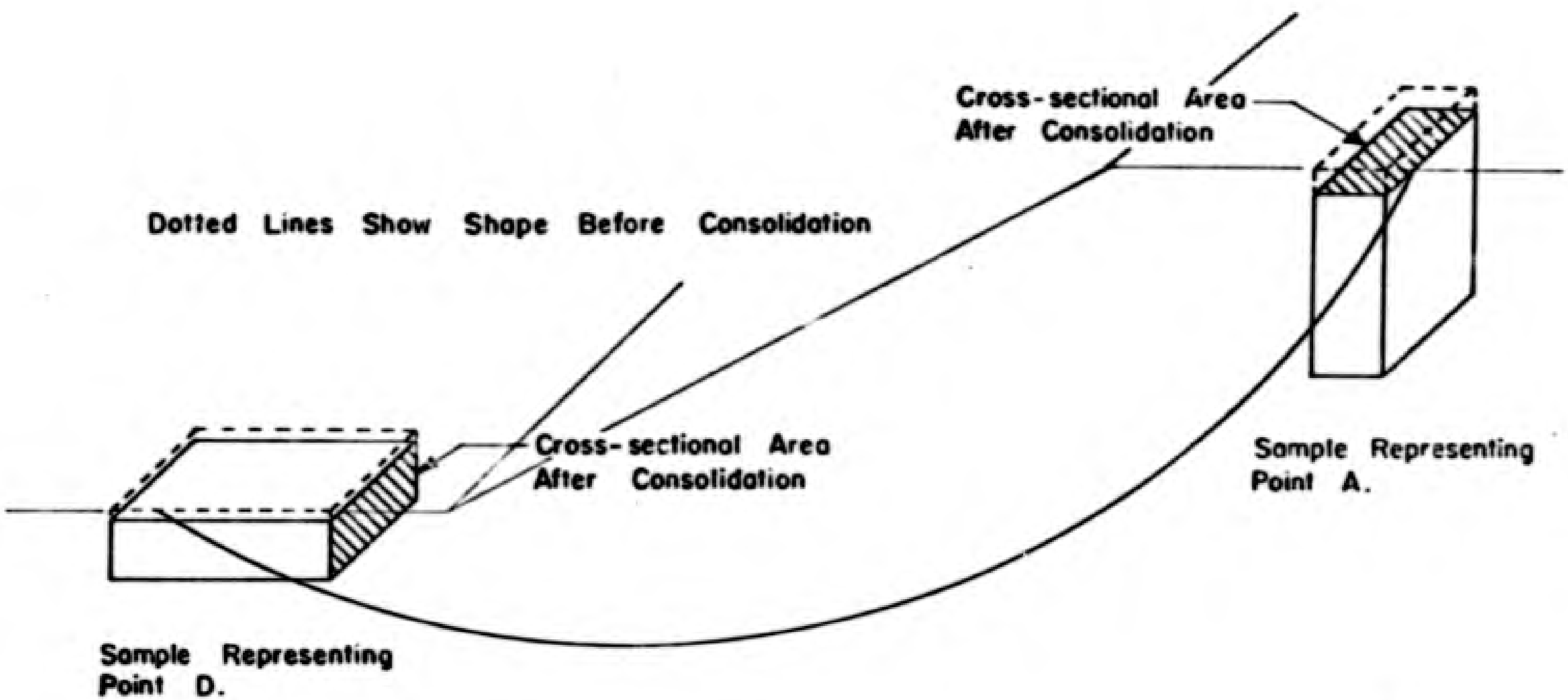


Fig.10- STRAINS DURING CONSOLIDATION FOR TWO TYPES OF PLANE STRAIN SAMPLE.

failure, the differences in these stresses, and parameters computed using these stresses. In order to be able to make comparisons of these quantities, the corrections which are described in detail by Duncan and Seed (1965) were applied at the end of consolidation and at failure. The entire variations of deviator stress, pore pressure parameter $\bar{A}$ and major principal stress ratio with strain were not corrected however, because the corrections are tedious to compute and because it is believed that the information required from the variations themselves, as opposed to the end points, can be derived as well from the uncorrected as from the corrected variations. For instance, it is possible to see if deviator stress drops off or stays constant after failure without making corrections. If, for some reason, more detailed information were required concerning the variation of any quantity with strain, then each point defining that variation could be corrected.

The necessity for correcting the results of these two types of plane strain test before comparing them illustrates a general principal: If different corrections apply to a pair of results to be compared, then comparison should not be made until the results have been corrected for the effects of extraneous influences.

Furthermore, if the objective of a series of tests is to measure the strength, the strength parameters, the pore pressure parameter $\bar{A}$ or the effective major principal stress ratio for use in practical problems, then the results must be corrected or the answers will be wrong. All of these quantities are influenced by the axial load in the sample, and failure to correct the axial load may lead to significant errors in the measured quantities.

On the other hand, if the objective of an investigation is simply to compare the results of two tests, and if the same corrections would apply to both of a pair of results to be compared, then comparison can be made without first correcting the results. An example is the investigation described in Section III; the conclusions drawn with regard to the relative strengths of vertical and inclined samples must all be qualitatively correct, because the same corrections would apply to any pair of samples. Quantitatively, however, the results must be somewhat in error because the absolute magnitudes of the strengths are wrong.

Magnitude of the Corrections

The necessary corrections to the axial stress at failure in the ordinary (IC-U) triaxial tests and in the two types of plane strain test described in this chapter are listed in table 3. The values in the table are the sums of the necessary corrections for the loads carried by filter paper drains, rubber membranes, cell piston friction and end plate friction, expressed as a percentage of the uncorrected axial stress at failure. The necessary corrections, expressed as a percentage of the uncorrected stress, decrease with increasing consolidation pressure and strength; since the consolidation pressures given in table 3 are the lowest and highest used in the tests on Bay Mud, the corresponding corrections are the maximum and minimum values for the tests in this investigation.

The values of the corrections shown in table 3 are quite significant, especially at low consolidation pressures; use of uncorrected strengths or strength parameters from any of these three types of tests conducted at low consolidation pressures would be unconservative by 20 percent to 30 percent. Since the corrections for the two types of plane strain test are not the same, comparison of uncorrected data could lead to erroneous conclusions, as could comparison of uncorrected data from IC-U triaxial tests with data from plane strain tests simulating point D in figure 1.

Table 3. Corrections Applied to the Axial Stress at Failure in Undrained Tests on Undisturbed Bay Mud.

Type of Test	Required Correction to Axial Stress (in percent of applied deviator stress)	
	$\sigma'_{1c} = 0.8$ kg/cm^2 (Plane Strain) $\sigma'_{1c} = 1.0$ kg/cm^2 (Triaxial)	$\sigma'_{1c} = 4.0$ kg/cm^2 (Plane Strain) $\sigma'_{1c} = 4.0$ kg/cm^2 (Triaxial)
AC-U Plane Strain Simulating Point A	21.8	9.6
AC-U Plane Strain Simulating Point D	28.1	11.5
IC-U Triaxial	21.9	8.2

The necessary corrections to the axial stress in UU triaxial tests are smaller than those shown in table 3 for IC-U tests conducted at low pressures, because no filter paper drains are used on the samples. The loads carried by piston friction and rubber membranes amount to about 0.4 kg at six percent axial strain, or about 10 percent of the axial load at failure of a 1.4 inch diameter UU triaxial sample of undisturbed Bay Mud. The UU triaxial test data shown in figure 9 have been corrected by subtracting 0.02 kg/cm^2 from the measured strengths.

The influence of the corrections of the test results was found to be qualitatively the same in all the types of test, as shown in table 3, and not much different in magnitude for any of them. The corrected results differ from the uncorrected results in the following ways:

(1) Corrected consolidation pressures are lower than the uncorrected ones.

(2) Corrected strengths are lower than the uncorrected ones.

(3) Corrected values of the pore pressure parameter $\bar{A}_f$ are higher than the uncorrected ones.

(4) The corrected effective stress failure envelope lies below the uncorrected one. The corrected value of c' is less, and the corrected value of ϕ' is more than the corresponding uncorrected value.

(5) The corrected effective principal stress ratio at failure is less than the uncorrected one.

(6) The corrected stress-strain curves are less steep than the corrected ones. The difference in slope is larger if the total required correction increases significantly during the undrained test than if the required correction is practically constant.

The influences of the corrections on the strengths, the effective stress failure envelope, the pore pressure parameter $\bar{A}_f$ and the effective principal stress ratio at failure determined in IC-U triaxial tests are shown in figure 11. The effects of the corrections on the positions of plane strain test vector curves are shown in figure 21.

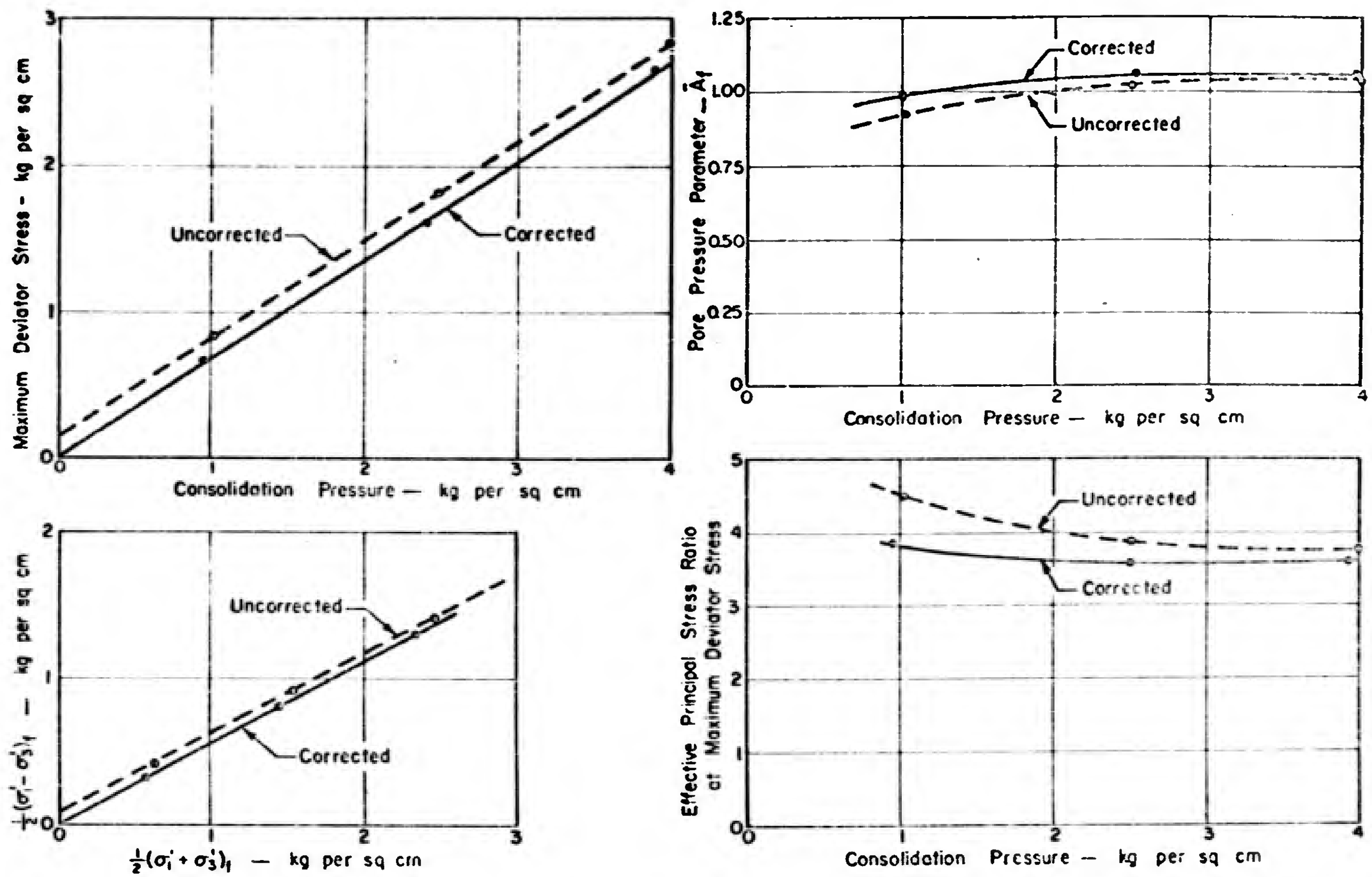


Fig.11– EFFECTS OF CORRECTIONS FOR FILTER PAPER DRAINS, RUBBER MEMBRANES, AND CELL PISTON FRICTION ON THE UNDRAINED STRENGTH, EFFECTIVE STRESS ENVELOPE, $\bar{A}_f$, AND (σ_1'/σ_3') IN IC-U TRIAXIAL TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

Description of the Plane Strain Tests

Tests Simulating Conditions at Point A

These tests are the normal type of AC-U plane strain tests. The samples are consolidated approximately one-dimensionally with the major principal stress acting in the direction of the axis of the sample (see figure 10), and are caused to fail by either increasing the axial stress or decreasing the lateral stress when consolidation is complete, while preventing further drainage. The samples are forced to deform in plane strain by end plates which are held a fixed distance apart so that they prevent the samples from increasing in length. (Appendix A contains detailed descriptions of the apparatus and techniques for performing plane strain tests.)

Since the major principal stress acts in the direction of the axis of the sample both during consolidation and at failure, there is no reorientation of principal stresses. These plane strain tests simulate the conditions at point A in figure 1, where the axis of the element is vertical, and for this reason they are called VPS (vertical plane strain) tests.

Tests Simulating Conditions at Point D

These tests were devised especially for this study. The samples are consolidated approximately one-dimensionally with the major principal stress perpendicular to the axis of the sample (see figure 10). During both consolidation and shear the samples are constrained to deform in plane strain by end plates, as are the samples simulating point A.

The samples are caused to fail by first reducing the lateral stress (the major principal stress during consolidation) until it becomes equal to the axial stress (the minor principal stress during consolidation) after consolidation is complete and while preventing further drainage. Then the axial stress is increased, at constant lateral pressure, until the sample fails.

In an undrained compression test performed on a saturated sample, both the position of the vector curve and the strength of the sample are the same whether the lateral stress is changed during the undrained test or not. The change in pore pressure which results from a change in deviator stress and a change in lateral stress is given by the expression

$$\Delta u = B \cdot \Delta\sigma_{\ell} + \bar{A}(\Delta\sigma_a - \Delta\sigma_{\ell}) \tag{4-1}$$

where $\Delta\sigma_{\ell}$ = change in total lateral stress - (kg/cm^2)

and $(\Delta\sigma_a - \Delta\sigma_{\ell})$ = deviator of the change in stress - (kg/cm^2)

If the samples will fail in compression then the deviator of the change in stress, defined as $(\Delta\sigma_a - \Delta\sigma_{\ell})$, rather than as $(\Delta\sigma_1 - \Delta\sigma_3)$, must be greater than zero, but the change in lateral stress may be either greater than or less than zero. Since the pore pressure parameter B is equal to one for a saturated sample, equation (4-1) may be rewritten as

$$\Delta u = \Delta\sigma_{\ell} + \bar{A}(\Delta\sigma_a - \Delta\sigma_{\ell}) \tag{4-2}$$

Since the change in total lateral stress is equal to $\Delta\sigma_{\ell}$, then the change in effective lateral stress must be

$$\Delta\sigma'_{\ell} = \Delta\sigma_{\ell} - \Delta u = \Delta\sigma_{\ell} - \Delta\sigma_{\ell} - \bar{A}(\Delta\sigma_a - \Delta\sigma_{\ell}) = -\bar{A}(\Delta\sigma_a - \Delta\sigma_{\ell}), \tag{4-3}$$

which shows that the change in effective lateral stress is independent of the change in total lateral stress. Since the effective lateral stress is unaffected by changes in total lateral stress, then the effective stress on the failure plane (and any other plane also) must be independent of changes in total lateral stress. Because the effective stress on the failure plane is independent of changes in total lateral stress, then the position of the vector curves and the strength measured in an undrained compression test on a saturated sample must be the same whether the lateral stress is changed during the test or not. As a matter of practical convenience, in the tests simulating conditions at point D, the first stage of the undrained test was conducted by decreasing the lateral stress and the second stage was conducted by increasing the axial stress. Since the pore pressure parameter B was equal to one for all samples, the results of the tests would have been the same if the tests had been conducted by increasing the axial stress while maintaining the lateral stress constant.

It should be noted that equation (4-2) shows that the definition of the pore pressure parameter $\bar{A}$ is

$$\bar{A} = \frac{\Delta u - \Delta_{\ell}}{(\Delta_{a} - \Delta_{\ell})} \tag{4-4}$$

when the lateral stress is changed during an undrained test on a saturated sample. This is the definition used to compute the values of $\bar{A}$ for all tests in this investigation.

In the tests simulating conditions at point D, the major principal stress acts in the lateral direction during consolidation, and in the axial direction at failure; the principal stresses are reoriented by 90° during the test. These plane strain tests simulate the conditions at point D in figure 1, where the axis of the element is horizontal, and for this reason they are called HPS (horizontal plane strain) tests.

Since the purpose of both the VPS and HPS tests is to simulate the consolidation and undrained failure of two elements of soil in the ground, they are described below as "in-situ" tests in order to distinguish them from plane strain tests where the objective was to simulate the effects of "perfect sampling". The apparatus and techniques for the plane strain tests are further described in Appendix A.

Results of "In-situ" tests

The results of six VPS and 7 HPS tests at different consolidation pressures, corrected as discussed previously, are summarized in table 4. The test numbers are not completely sequential because some of the tests could not be completed satisfactorily; tests performed while the techniques were being developed were repeated after the techniques had been established, and other tests could not be completed as a result of leaks or laboratory power failure which occurred during testing.

The uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$ and effective principal stress ratio with strain are shown in figures 12 through 16. (The consolidation pressures tabulated in these figures have been corrected). Note that the deviator stress has been defined as $(\sigma_a - \sigma_\ell)$ rather than $(\sigma_1 - \sigma_3)$, and therefore the deviator stress is negative at the end of consolidation and during the first part of the shearing phase of the HPS tests. The results are plotted to show comparisons between VPS and HPS tests with approximately equal major principal stresses during consolidation.

Comparison of the strengths of any pair of samples shown in figures 12 through 16, or of the corrected results in figure 17, shows that the ratio c_u/p is higher for VPS tests (point A in figure 1) than for HPS tests (point D in figure 1); the average value of the ratio c_u/p is 0.37 for VPS tests and 0.28 for HPS tests.

The average value of the ratio of the minor to the major principal stress, $((\sigma'_3/\sigma'_1)_c = k)$, during consolidation in HPS tests was 0.42, whereas the average value of this ratio was 0.50 in VPS tests. These ratios were not the same for the two types of test because the loads carried by filter paper drains and by rubber membranes were different for the two types of sample, and because neither type of sample was consolidated exactly one-dimensionally. Axial strain of the HPS samples was prevented approximately by maintaining the top cross-bar on the loading yoke in a constant position as indicated by a dial gage on top of the cross-bar as shown in figure 18. A load cell used to measure the force required to prevent movement was positioned between the bottom of the cross-bar and the cell piston. The small amount of elastic deformation required to activate the load cell required the same amount of upward movement of the cell piston and the top of the sample, so that these HPS samples did strain axially a small amount during consolidation. Thus the measured value of k must be somewhat less than k_o. In the case of the VPS samples, lateral strain during consolidation was prevented by forcing the side plates against the samples to maintain a constant thickness. (The construction and function of these plates are described in Appendix A.) When consolidation was essentially complete, the side plates were removed from contact with the samples, and at this

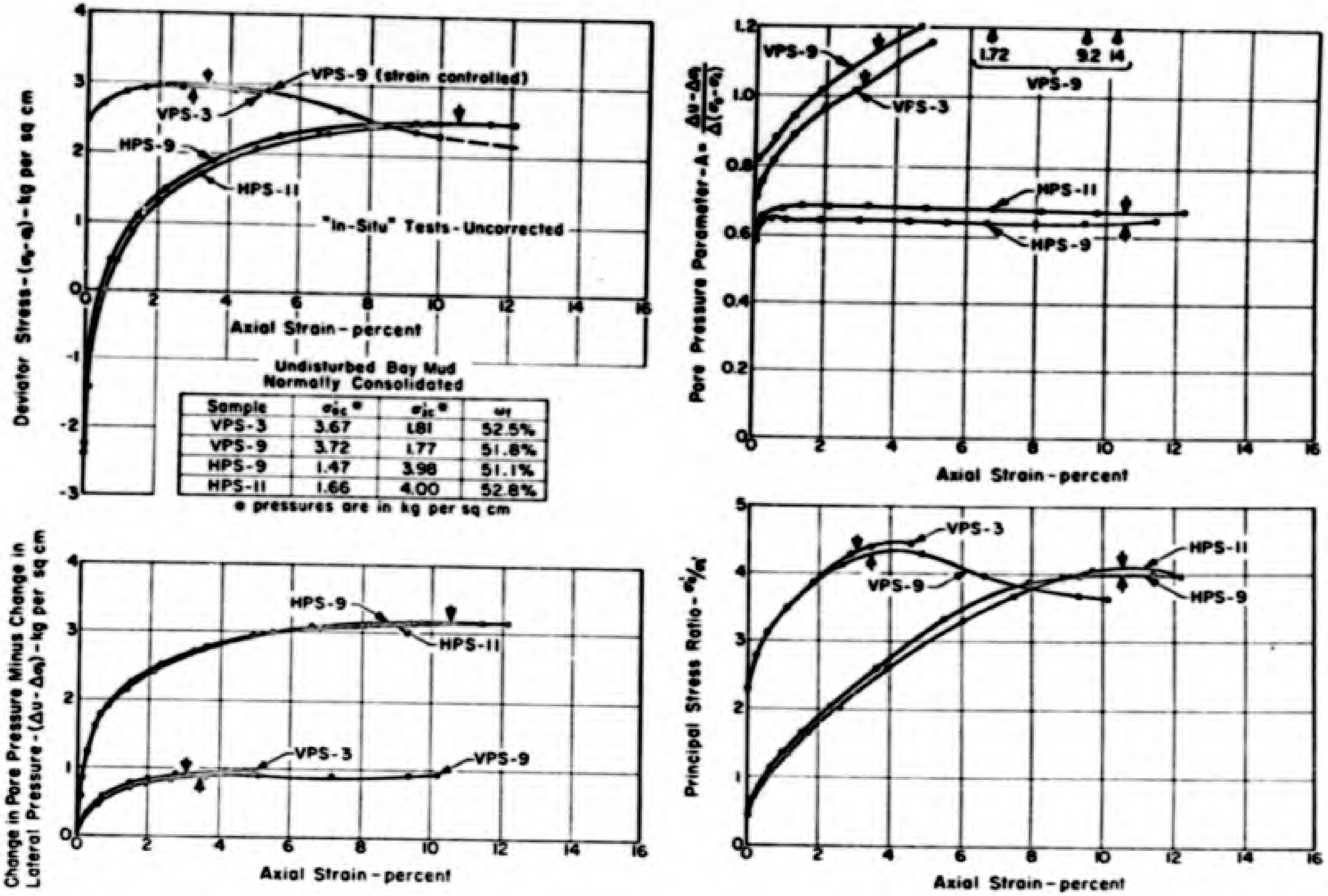
Table 4. Summary of Results of "In-Situ" Plane Strain Tests on Undisturbed San Francisco Bay Mud

Sample	Depth (feet)	Initial w/c (percent)	σ'_{ac} (kg/cm^2)	$\sigma'_{\ell c}$ (kg/cm^2)	$\left(\frac{\sigma'_3}{\sigma'_1}\right)_c$	Consol. w/c (percent)	$\left(\frac{\Delta V}{V}\right)_c$	$(\sigma_a-\sigma_\ell)_f$ (kg/cm^2)	$\Delta(\sigma_a-\sigma_\ell)_f$ (kg/cm^2)
VPS-3	18.5	87.5	3.67	1.81	0.49	52.9	0.256	2.70	0.84
VPS-4	18.5	85.3	0.64	0.35	0.55	77.4	0.042	0.51	0.22
VPS-5	18.5	87.3	1.39	0.70	0.50	64.3	0.169	1.03	0.34
VPS-6	19.0	89.5	2.14	1.07	0.50	59.1	0.207	1.58	0.51
VPS-8	19.0	87.7	2.90	1.44	0.50	53.7	0.255	2.12	0.66
VPS-9	21.0	91.0	3.72	1.77	0.48	51.8	0.283	2.76	0.81
HPS-4	17.0	93.1	0.69	1.61	0.43	65.2	0.191	0.81	1.73
HPS-5	17.0	85.8	1.20	3.15	0.38	52.1	0.243	1.65	3.60
HPS-6	16.5	90.1	0.34	0.81	0.42	74.8	0.093	0.41	0.88
HPS-7	16.5	89.3	0.99	2.40	0.41	57.5	0.226	1.28	2.69
HPS-9	15.5	88.8	1.47	3.98	0.37	51.1	0.277	2.23	4.74
HPS-11	19.0	92.3	1.66	4.00	0.42	52.8	0.282	2.24	4.68
HPS-12	20.5	85.5	1.52	3.18	0.48	52.5	0.238	1.77	3.43

(Continued)

Table 4. Continued

Sample	Depth (feet)	$(\Delta u-\Delta\sigma_{\ell})_f$ (kg/cm^2)	$\bar{A}_f$	$\left(\frac{\sigma'_1}{\sigma'_3}\right)_f$	ϵ_{af} (percent)	Time to Failure (hours)	σ'_{1f} (kg/cm^2)	σ'_{3f} (kg/cm^2)	$\frac{(\sigma_1-\sigma_3)_f}{2}$ (kg/cm^2)	$\frac{(\sigma'_1+\sigma'_3)_f}{2}$ (kg/cm^2)
VPS-3	18.5	0.90	1.07	3.97	3.1	5.25	3.61	0.91	1.35	2.26
VPS-4	18.5	0.20	0.91	4.40	4.0	5.75	0.66	0.15	0.26	0.41
VPS-5	18.5	0.39	1.15	4.33	3.2	6.0	1.34	0.31	0.52	0.83
VPS-6	19.0	0.58	1.14	4.22	4.2	7.0	2.07	0.49	0.79	1.28
VPS-8	19.0	0.78	1.18	4.27	3.8	6.75	2.77	0.65	1.06	1.71
VPS-9	21.0	0.87	1.07	4.07	3.5	4.5	3.66	0.90	1.38	2.28
HPS-4	17.0	1.30	0.75	3.61	11.1	4.50	1.12	0.31	0.41	0.72
HPS-5	17.0	2.52	0.70	3.62	9.6	4.50	2.28	0.63	0.83	1.46
HPS-6	16.5	0.61	0.69	3.05	9.0	5.0	0.61	0.20	0.21	0.41
HPS-7	16.5	1.94	0.72	3.91	10.0	5.25	1.72	0.44	0.64	1.08
HPS-9	15.5	3.17	0.67	3.76	10.6	5.25	3.04	0.81	1.12	1.93
HPS-11	19.0	3.20	0.68	3.80	10.6	4.50	3.04	0.80	1.12	1.92
HPS-12	20.5	2.24	0.65	3.68	9.8	5.25	2.43	0.66	0.89	1.55



Sample	σ'_{1c}*	σ'_{3c}*	w_f
VPS-3	3.67	1.81	52.5%
VPS-9	3.72	1.77	51.8%
HPS-9	1.47	3.98	51.1%
HPS-11	1.66	4.00	52.8%

Fig.12— VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS.

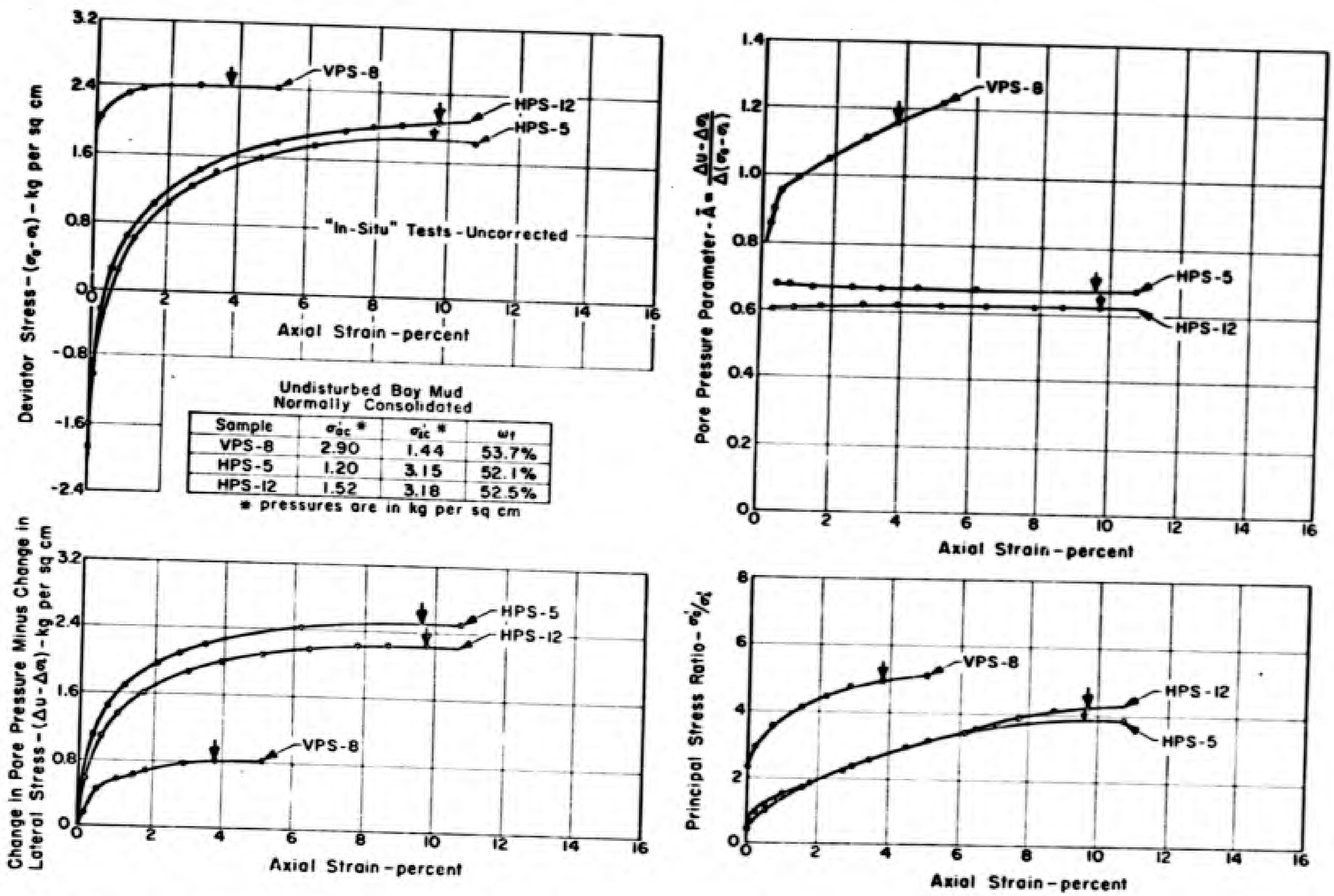


Fig.13 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS.

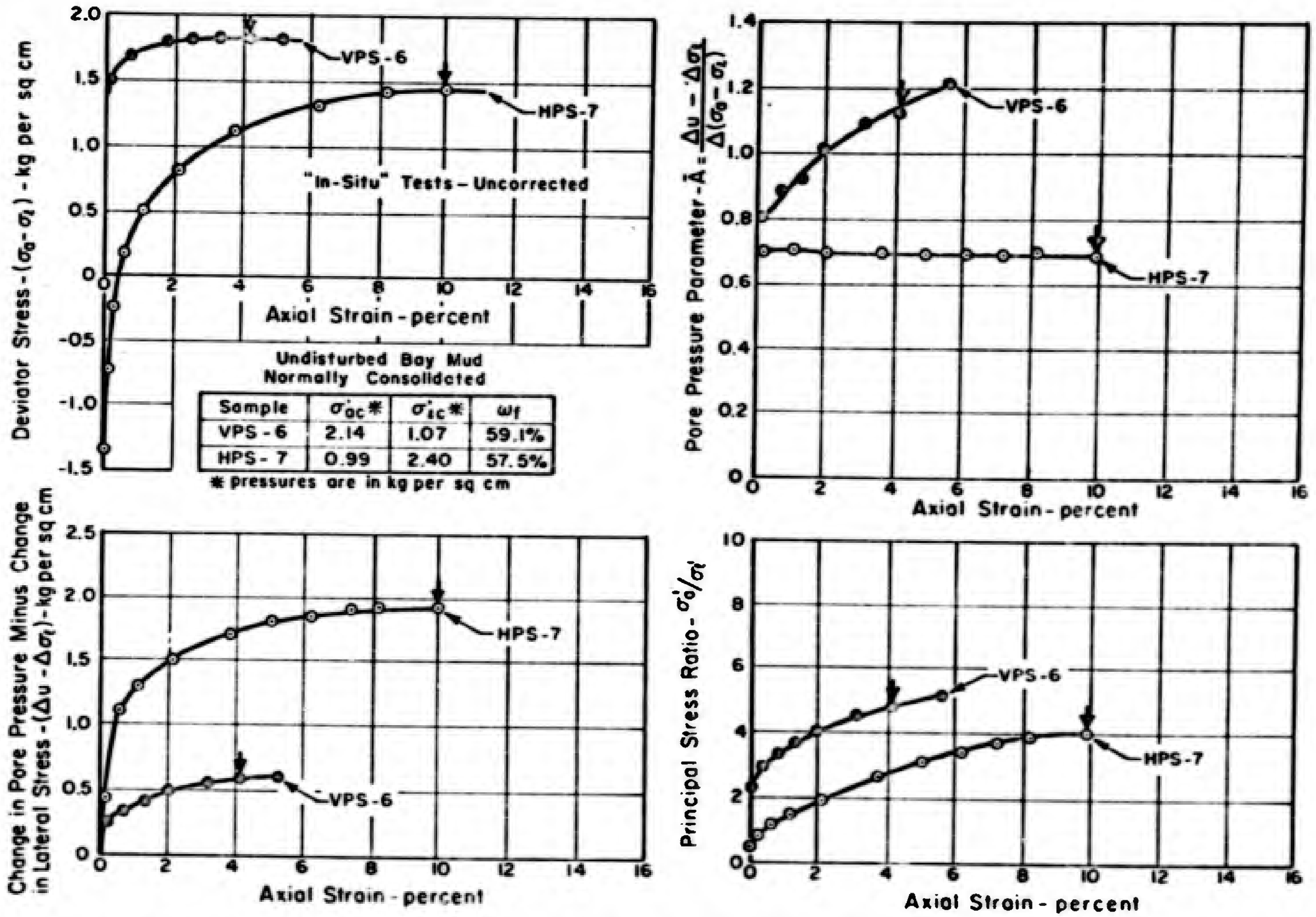


Fig.14–VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS.

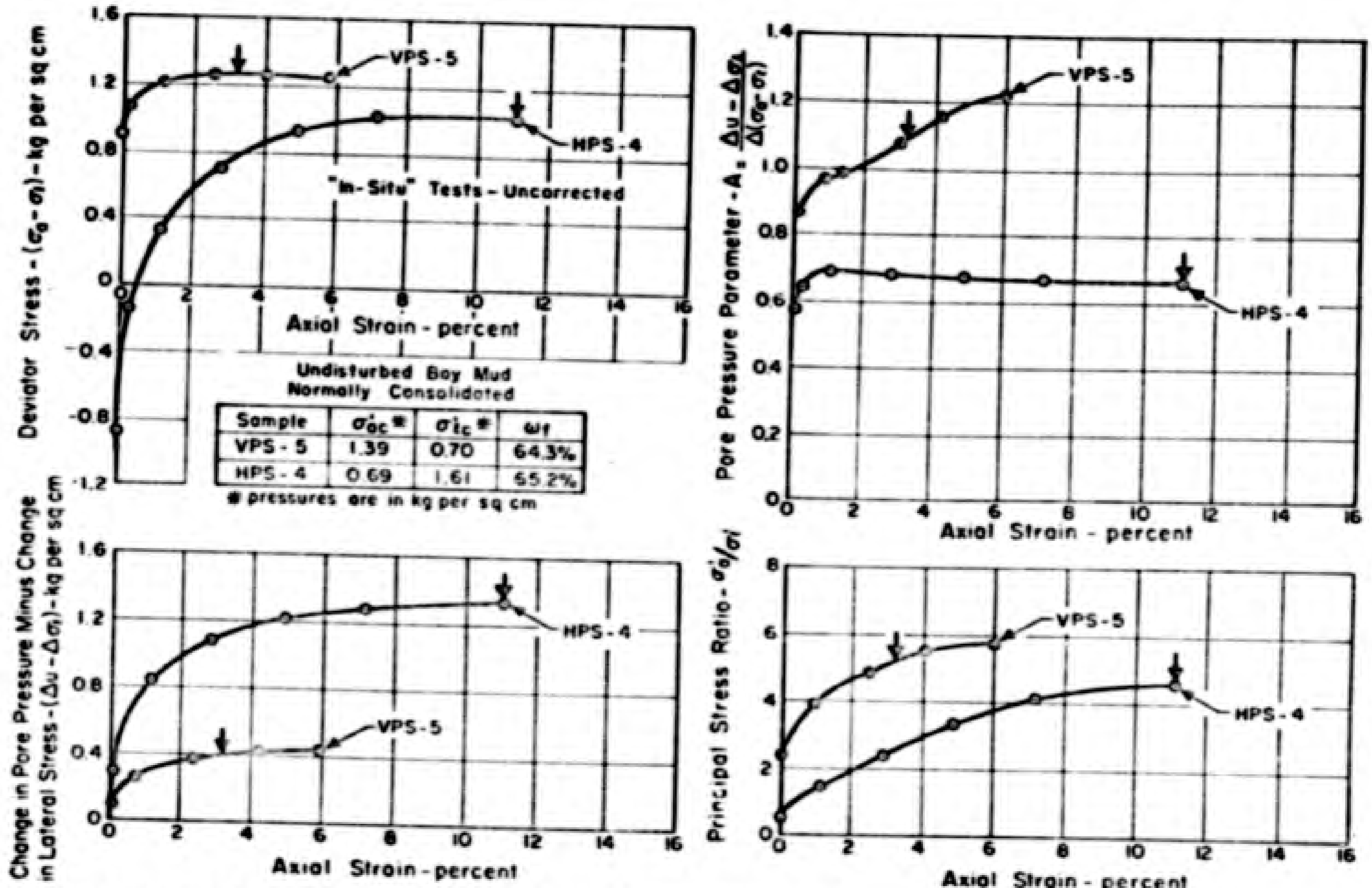


Fig.15 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS.

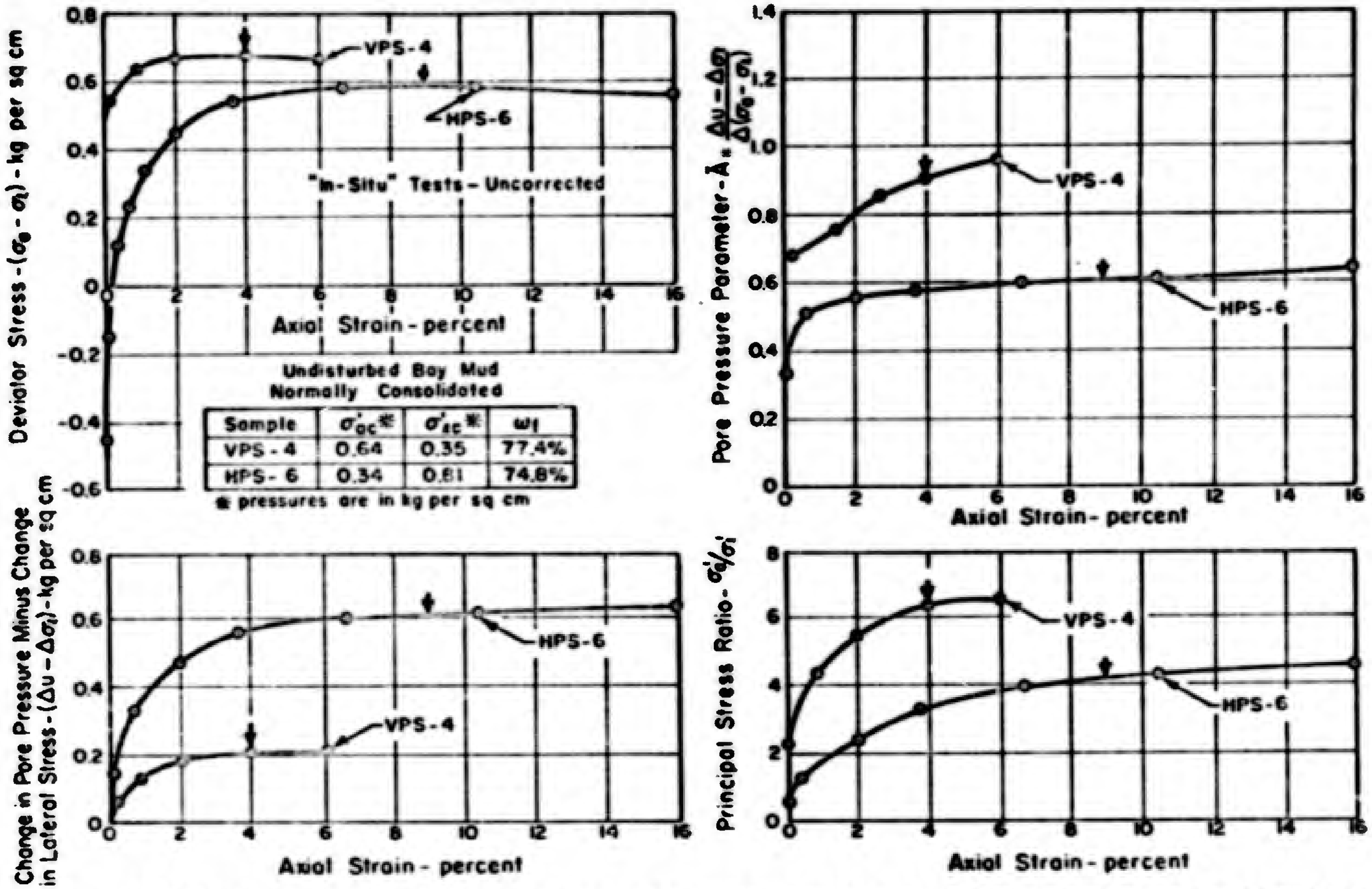


Fig.16 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS.

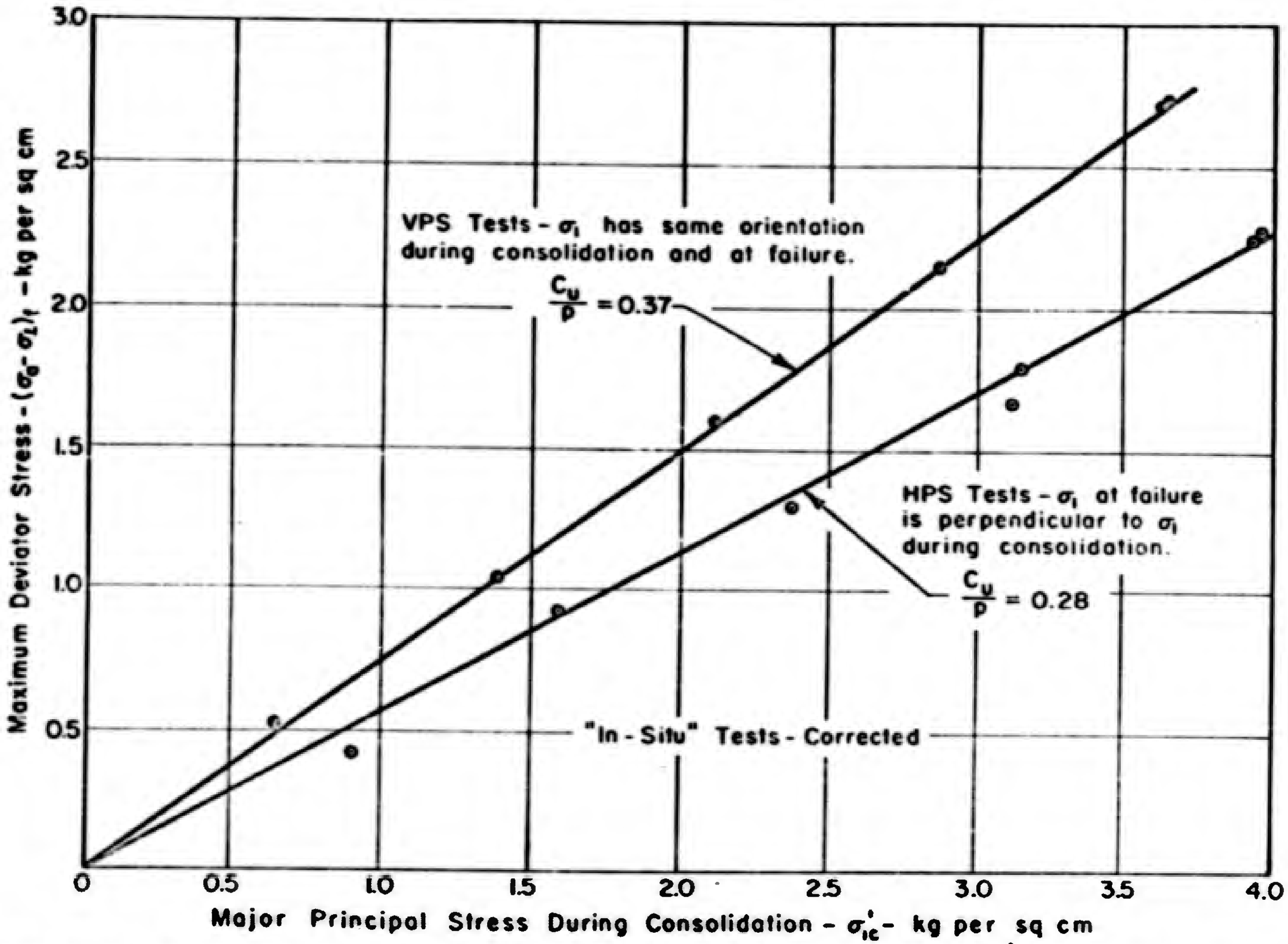


Fig.17 - VARIATION OF UNDRAINED STRENGTH WITH CONSOLIDATION PRESSURE IN HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS ON SAN FRANCISCO BAY MUD.

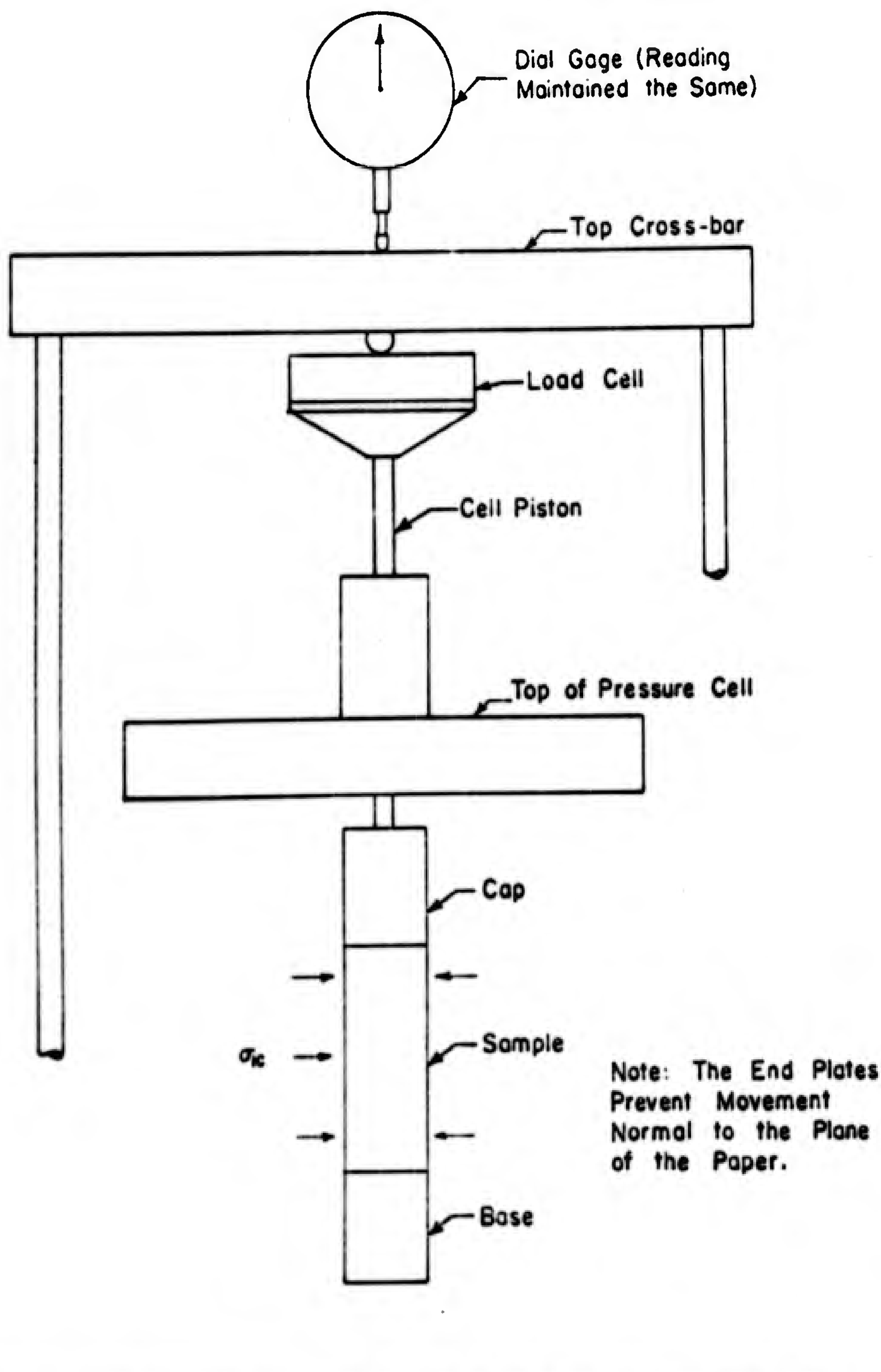


Fig.18- SKETCH OF AN HPS SAMPLE DURING CONSOLIDATION

time the samples always deformed somewhat because the load carried by friction between the side plates and the samples was transferred to the samples.

In order to estimate the effect on the measured strengths of the difference in the average values of k, the strengths which would have been measured if the stress ratio had been k_o in both types of test instead of 0.42 in HPS and 0.50 in VPS tests were estimated using the equation derived by Hansen and Gibson (equation 2-7). For the reasons discussed above, k_o must be slightly more than 0.42, perhaps 0.45. Using equation 2-7 it was found that the strength of VPS samples would be the same for k = 0.45 as for k = 0.50, and that the strength ratio c_u/p for HPS samples would be about 3.5 percent higher if k were 0.45 than if k were 0.42. If corrected to the same value of k, in this way, the strength ratios for the two types of test would be:

for VPS samples, $c_u/p = 0.37$

for HPS samples, $c_u/p = 0.29$.

Thus if an HPS and a VPS sample were consolidated to exactly the same anisotropic consolidation pressure with the value of k equal to 0.45, the ratio of the strengths of the two samples would be 0.29/0.37 or 0.78, i.e., the strengths of the HPS sample would be only about 78 percent of the strength of the VPS sample. Since the VPS tests simulate conditions at point A in figure 1, and HPS tests simulate conditions at point D, the conclusion may be drawn that the ratio c_u/p at point D would be only about 78 percent as large as the ratio c_u/p at point A.

The difference in strength between HPS and VPS samples occurs as the combined effect of the difference in reorientation of principal stresses and the difference in the values of the pore pressure parameter $\bar{A}_f$ in the two tests. The principal stresses are reoriented in HPS tests, but not in VPS tests and, as a result, the deviator of the change in stress during the undrained portion of an HPS test is much larger than the deviator of the change in stress in the undrained portion of a VPS test. On the other hand, the pore pressure parameter $\bar{A}_f$ is smaller for HPS tests than for VPS tests; the average values of $\bar{A}_f$, shown in the upper part of figure 19, are $\bar{A}_f = 0.70$ for HPS tests and $\bar{A}_f = 1.12$ for

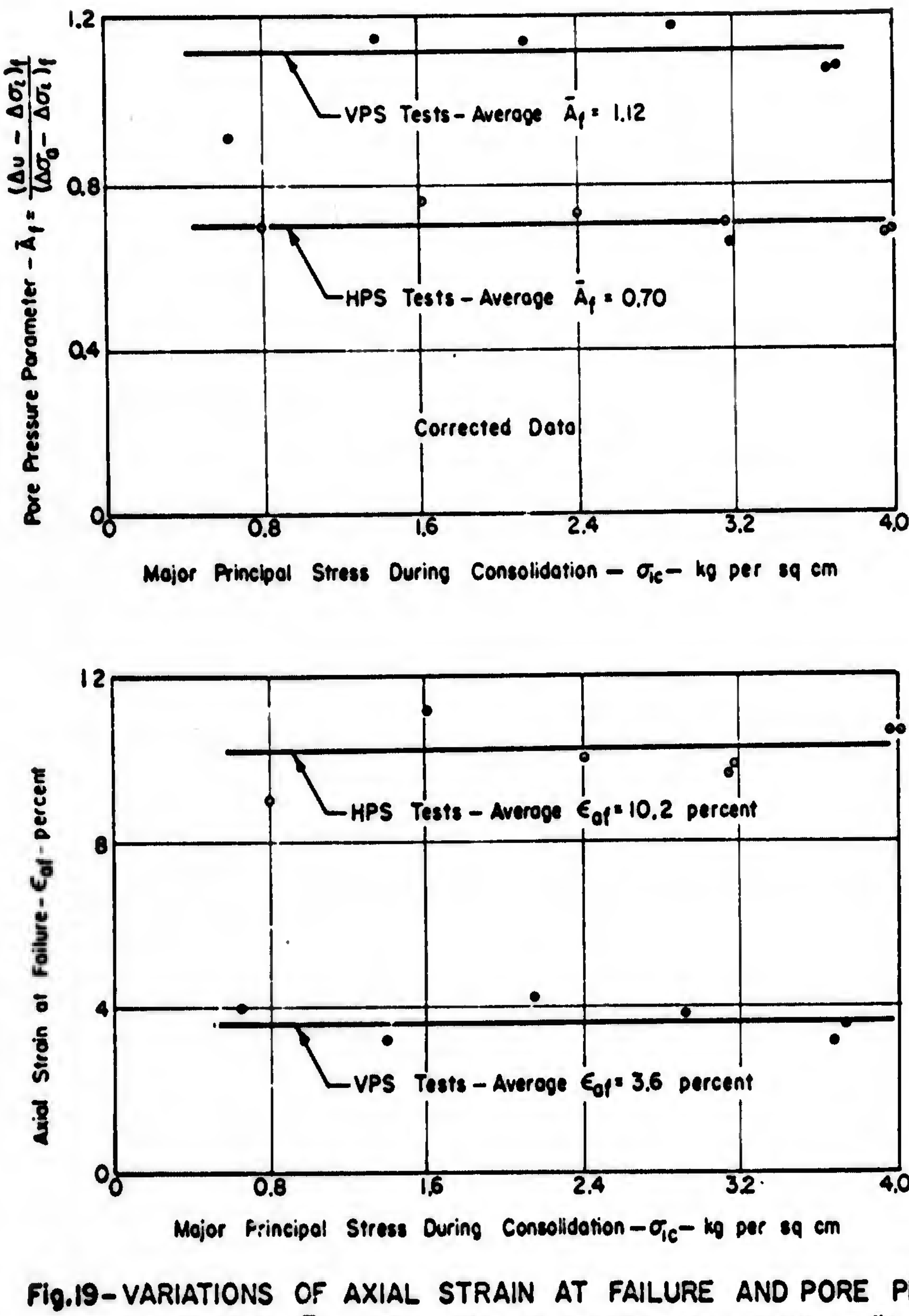


Fig.19 - VARIATIONS OF AXIAL STRAIN AT FAILURE AND PORE PRESSURE PARAMETER $\bar{A}_f$ WITH CONSOLIDATION PRESSURE IN "IN-SITU" PLANE STRAIN TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

VPS tests. Thus as a result of reorientation of principal stresses, the strength of samples representing point D in figure 1 (HPS samples) tends to be smaller than the strength of samples representing point A (VPS samples), whereas, as a result of anisotropy with respect to $\bar{A}_f$, the strength of samples representing point D tends to be larger than the strength of samples representing point A. Since the former effect exceeds the latter, the strength of a sample representing point D is smaller than the strength of a sample representing point A. These statements can be summarized as follows:

Samples Representing	$\bar{A}_f$	Deviator of the change in stress	Undrained Strength
A	larger	smaller	larger
D	smaller	larger	smaller

The average strain at failure in VPS tests was only about one third of the average strain at failure in HPS tests, as shown in the lower part of figure 19. The fact that the strains at failure are significantly different in samples representing points A and D may mean that the peak strength cannot be mobilized at both the upper and lower ends of a failure arc in the field simultaneously. The problem of predicting the relative magnitudes of the strains at the top and bottom of a failure arc is a difficult one, and beyond the scope of this investigation. If, however, the strains at point A and point D were equal, the element of soil at point A would fail before the strength at point D was fully mobilized. If the stress on the sample representing point A does not decrease after failure, then the strains at point A and point D could increase until the peak strength was mobilized at both A and D. If, however, the stress on the sample representing point A decreases after failure, then additional strain beyond that required to mobilized the peak strength at point A would result in a decrease in stress at A and an increase in stress at D. Most of the plane strain tests were performed using controlled stress loading, so the behavior after peak stress could not be determined. In order to find out whether or not the stress on samples representing point A decreases after failure, an additional test, VPS-9, was performed using controlled-strain loading in the later stages of the undrained test. The stress-strain curve for this test is shown in figure 12, where it can be seen that the stress acting on sample VPS-9 decreased to even less than that acting on the HPS samples at strains larger than 8 percent. Since the stress on a sample representing point A does decrease after failure, it may not be possible to mobilize the peak strengths at both point A and point D simultaneously. Thus, anisotropy with respect to strain at failure may contribute to the occurrence of progressive failure in the field by preventing simultaneous development of peak strength all along a rupture surface.

If failure is defined as that stage in the test at which the deviator stress reaches its maximum value, as it is throughout this investigation, then the Bay Mud is anisotropic with respect to c', Φ'. The method used to determine the strength parameters c', Φ' is shown in the upper part of figure 20, and the effective principal stress ratios at maximum deviator stress are shown in the lower part of the same figure. The angle between the failure plane and the horizontal in VPS tests is approximately 60°, and for this orientation of the failure plane the data in figure 20 show that $c' = 0$, $\phi' = 38°$. The angle between the failure plane and the horizontal in HPS tests is approximately -30° and the data in figure 20 show that for this orientation of the failure plane $c' = 0$, $\phi' = 35°$. Since Bay Mud is anisotropic with respect to ϕ' then it must also be anisotropic with respect to one or both of the Hvorslev strength parameters c_e, ϕ_e.

Figure 21 shows the uncorrected vector curves for the "in-situ" plane strain tests. The corrected positions of the end points are indicated by arrows at the ends of the curves. All of the effects of the corrections on the measured data which were discussed above (except the effect of the slope of stress-strain curves) are illustrated by the relative positions of the corrected and uncorrected end points; the uncorrected positions indicate higher consolidation pressures, higher strengths, higher values of c', lower values of $\bar{A}_f$, and higher effective principal stress ratios at failure than the corrected ones.

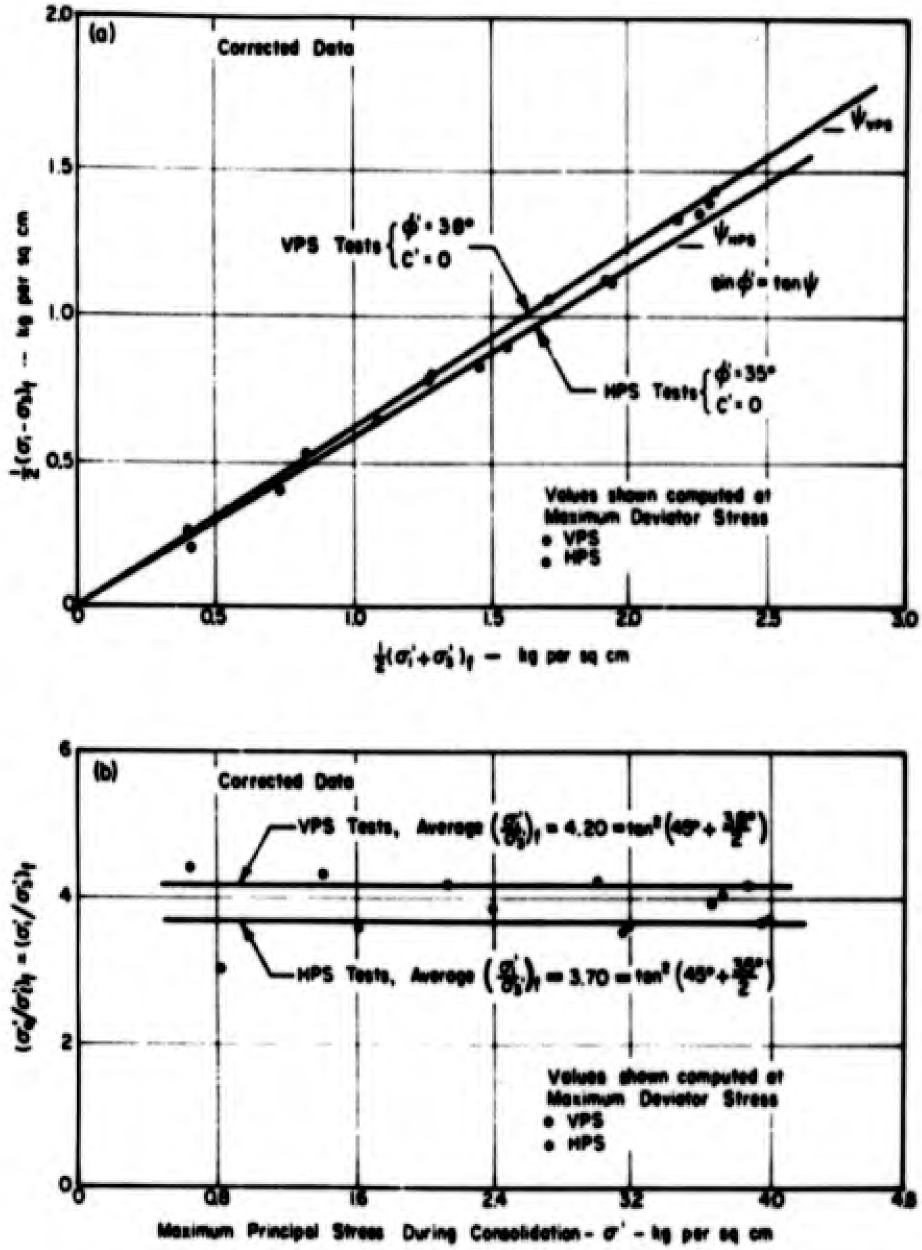


Fig.20 – (a) DETERMINATION OF EFFECTIVE STRESS STRENGTH PARAMETERS, AND (b) VARIATION OF EFFECTIVE PRINCIPAL STRESS RATIO AT FAILURE WITH MAJOR PRINCIPAL STRESS DURING CONSOLIDATION, IN PLANE STRAIN TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

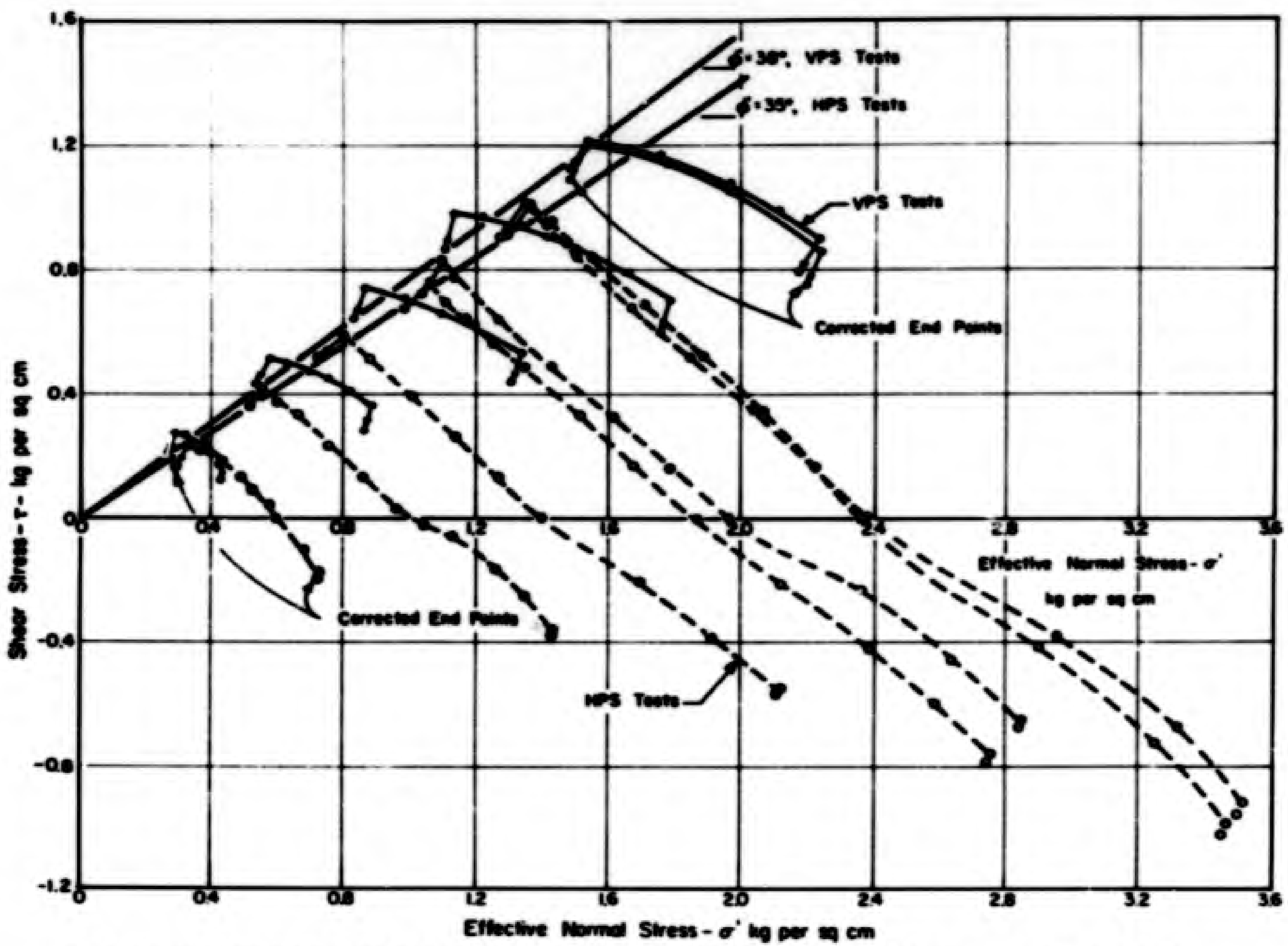


Fig. 21– UNCORRECTED VECTOR CURVES AND CORRECTED END POINTS FOR "IN-SITU" PLANE STRAIN TESTS.

Corrected vector curves for all of these "in-situ" plane strain tests are shown in figure 22. These curves were corrected by calculating the end points from corrected data and sketching in the corrected curve with a shape similar to the uncorrected ones. This method was adopted because the corrections are tedious to calculate and because the vector curves were not used to derive quantitative information. If for some reason precisely corrected vector curves were required, then each point defining the curve could be corrected separately. All of the differences in behavior between the HPS and VPS samples except the difference in strains at failure are illustrated by these vector curves. The differences in strength are shown by differences in shear stress at the intersections of the vector curves and the strength envelopes, the differences in $\bar{A}_f$ are responsible for the differences in slope of the vector curves, and the fact that the left-hand end points of the vector curves define two different envelopes demonstrates the anisotropy with respect to the effective stress strength parameters.

Conclusion

The VPS and HPS tests described above duplicate as closely as possible the consolidation and undrained failure of two different elements of normally consolidated clay in the ground. They represent the most direct measurements of in-situ strengths which it is possible to make in the laboratory for the two different orientations of the failure plane which they involve ($\alpha = 60°$ and $\alpha = -30°$). There is no reason to believe that the relative values of the real in-situ strengths would be significantly different from those measured in these tests.

The variation of undrained strength between $\alpha = 60°$ and $\alpha = -30°$ cannot be determined by direct experiment using equipment available at the present time (1965). However, the tests described above have shown that the two most important factors governing this variation of strength are:

(1) Reorientation of principal stresses during undrained shear.

(2) The variation of the pore pressure parameter $\bar{A}_f$ with the orientation of the failure plane.

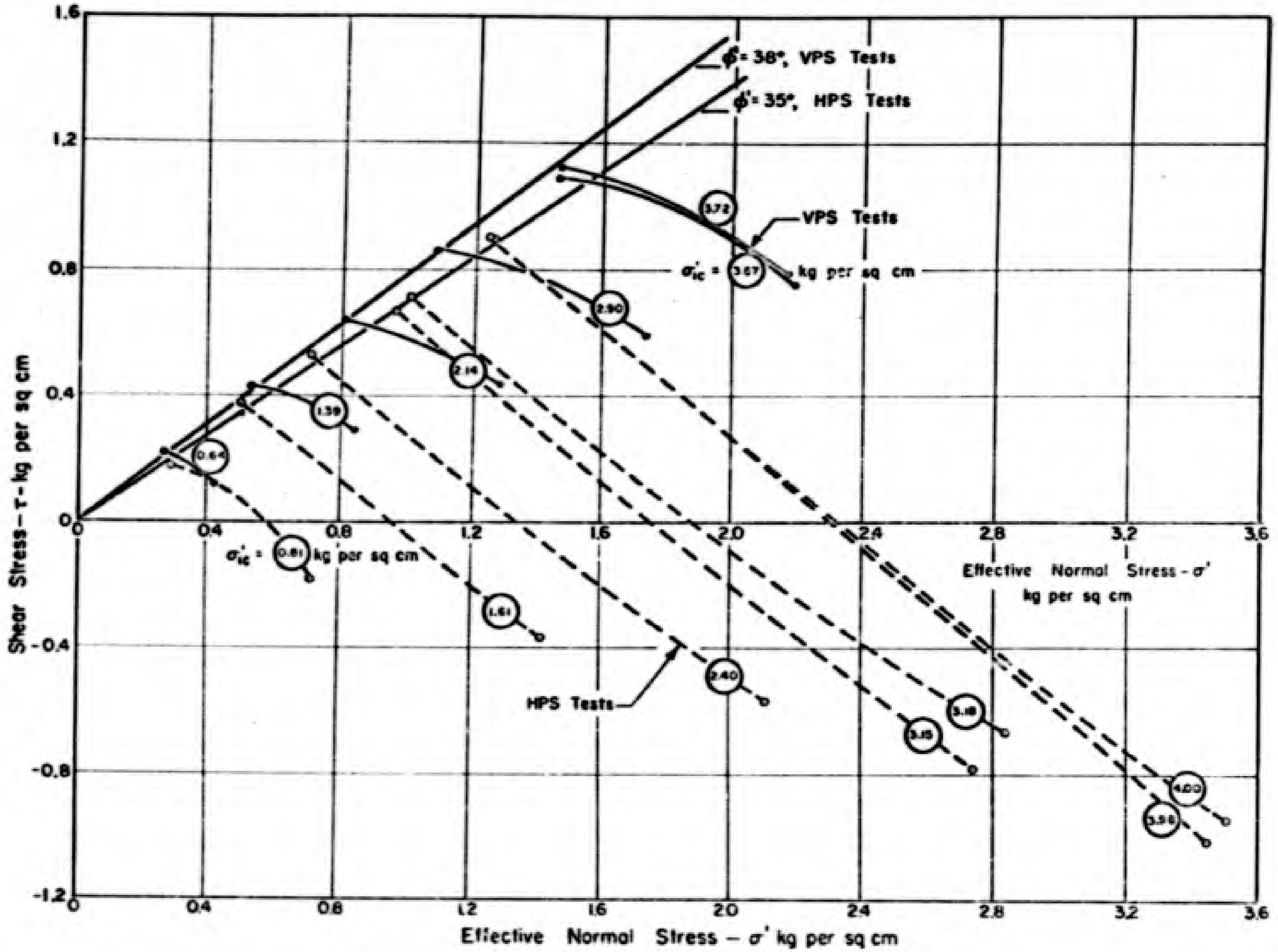


Fig.22 - CORRECTED VECTOR CURVES FOR HORIZONTAL AND VERTICAL "IN-SITU" PLANE STRAIN TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

The effect of reorientation of the principal stresses during undrained shear is expressed by the equation derived by Hansen and Gibson (equation 2-7). This equation can be used to find the entire variation of undrained strength if the variations of c', Φ' and $\bar{A}_f$ are known. The variation of $\bar{A}_f$ is more important than the variation of c', Φ' because the difference in the two values of $\bar{A}_f$ for $\alpha = 60°$ and $\alpha = -30°$ (shown in figure 19) is significant, whereas the difference in the two values of Φ' for $\alpha = 60°$ and $\alpha = -30°$ (shown in figure 20) does not appear to be significant. Since the value of c' is zero for both values of α, it seems probable that c' = 0 for all values of α.

The Variation of $\bar{A}_f$ with α

The situation with respect to $\bar{A}_f$ is similar to the situation with respect to undrained strength: Two values, for $\alpha = 60°$ and $\alpha = -30°$, are known from experiments and the other values must be inferred. It is easier, however, to get an insight into the variation of $\bar{A}_f$ in the field than undrained strength in the field. $\bar{A}_f$ is a function of the compressibility of the clay in the particular direction in which the clay is compressed, among other things. Since the anisotropy of one-dimensionally consolidated clay is probably fundamentally related to the parallel orientation of plate-shaped clay particles, and in the ground these particles will tend to be horizontal, it follows that the vertical direction will be an axis of symmetry of the anisotropy, and the horizontal plane will be a plane of symmetry of the anisotropy. Therefore, it seems logical to expect that one extreme value of $\bar{A}_f$ would obtain when the major principal stress at failure is vertical, and the other when the major principal stress at failure is horizontal; and that the variation of $\bar{A}_f$ with a α would be symmetrical about its extreme values. When the major principal stress at failure is vertical, the orientation of the failure plane corresponds to $\alpha = 60°$ and when the major principal stress at failure is horizontal, the orientation of the failure plane corresponds to $\alpha = -30°$. The VPS and HPS tests have shown that $\bar{A}_f = 1.12$ when $\alpha = 60°$ and $\bar{A}_f = 0.70$ when $\alpha = -30°$; these are probably the extreme values of $\bar{A}_f$.

In addition to the considerations outlined previously, it seems likely that the variation of $\bar{A}_f$ with α will be represented by a smooth curve. Considering that the maximum value of $\bar{A}_f$ would probably be associated with $\alpha = 60°$, and that the minimum value of $\bar{A}_f$ would probably be associated with $\alpha = -30°$; that the variation of $\bar{A}_f$ with α should be symmetrical about the extreme values; and that the variation should be smooth and contain no abrupt variations, it seems reasonable to believe that the variation will be closely approximated by the curve shown in the upper part of figure 23. The variation shown in the upper part of figure 23 can be expressed as

$$\bar{A}_{f\alpha} = \bar{A}_{f(\alpha=-30°)} + (\bar{A}_{f(\alpha=60°)} - \bar{A}_{f(\alpha=-30°)}) \sin^2 (\alpha+30°)$$

or

$$\bar{A}_{f\alpha} = 0.70 + 0.42 \sin^2 (\alpha+30°).$$

<u>The Variation of Undrained Strength with α</u>

The considerations outlined previously may be used to develop a picture of the complete variation of the undrained strength of Bay Mud with the orientation of the failure plane in-situ. Briefly restated, these considerations are:

(1) The strengths for $\alpha = 60°$ and $\alpha = -30°$ should be those measured in VPS and HPS tests, respectively.

(2) The variation of $\bar{A}_f$ with α should be represented by a smooth, symmetrical curve with the maximum and minimum values of $\bar{A}_f$ at $\alpha = 60°$ and $\alpha = -30°$, respectively. Such a variation is shown in the upper part of figure 23.

The shear strength ratio c_u/p for any value of α can be found by substituting the proper values of α, $\bar{A}_f$, k_o, Φ' and c' into equation 2-7. In order to make the solution of equation 2-7 more convenient, it was programmed for a computer and solved for several values of α and $\bar{A}_f$, using $k_o = 0.45$, $\Phi' = 35°$ and $c' = 0$. Even though these values of k_o and Φ' do not correspond exactly to the results of the VPS tests, the calculated value of c_u/p is 0.367 for that case, compared to the measured

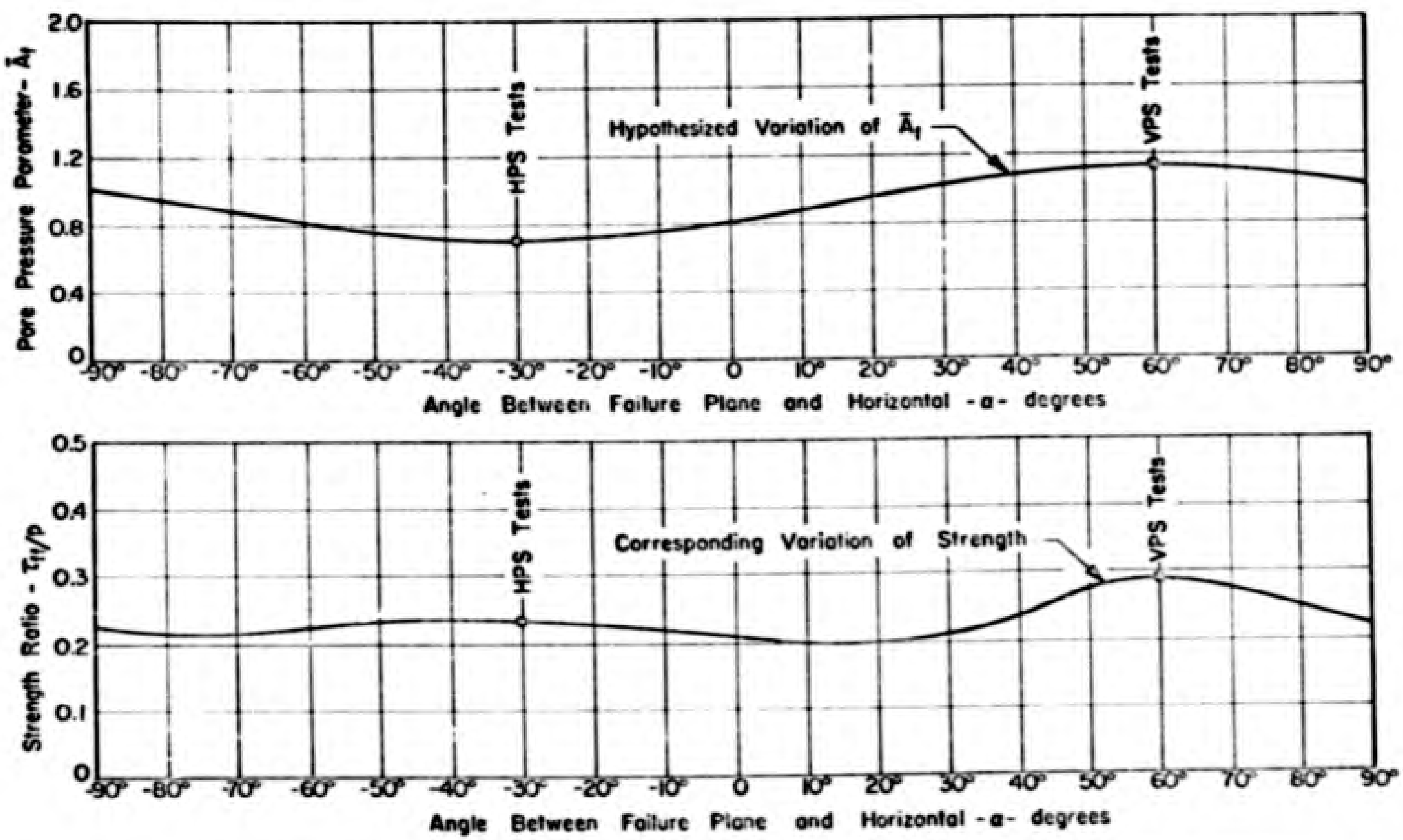


Fig.23 - HYPOTHESIZED VARIATION OF $\bar{A}_f$ AND CORRESPONDING VARIATION OF STRENGTH WITH ORIENTATION OF THE FAILURE PLANE IN SAN FRANCISCO BAY MUD.

value, 0.37. The results of the computations are shown in figure 24; the strength ratio shown in this figure is τ_{ff}/p which is equal to $(c_u/p) \cos \Phi'$. It is interesting to note that figure 24 shows that the sensitivity of the undrained strength to a change in the value of $\bar{A}_f$ is quite different for the different values of α. For instance, a change in $\bar{A}_f$ from 0.8 to 1.6 corresponds to only about a 12 percent decrease in strength when $\alpha = 60°$, but the same change in $\bar{A}_f$ corresponds to an 85 percent decrease in strength when $\alpha = -30°$. This difference results from the fact that the principal stresses are reoriented in the latter case, and the deviator of the change in stress is quite different in the two cases.

Figure 24 may be used to obtain the variation of τ_{ff}/p with α, if the variation of $\bar{A}_f$ with α is known, by finding the values of τ_{ff}/p for the six values of α shown in the figure, and plotting the results. This has been done using the hypothesized variation of $\bar{A}_f$ with α shown in the upper part of figure 23; the corresponding variation of τ_{ff}/p with α is shown in the lower part of the figure.

The variation of undrained strength with α shown in figure 23 is based on the two independent considerations enumerated above, and it appears to constitute the best estimate of the in-situ variation of undrained strength with α in Bay Mud which it is possible to make on the basis of available evidence.

Since the experimental investigations of anisotropy with respect to UU laboratory strength made by other investigators on other soils seem to define four different types of behavior, it should not be inferred that the variation of undrained strength in-situ with orientation of the failure plane in Bay Mud would necessarily hold for all soils. Furthermore, the variation of strength shown in figure 23 was developed only for the strength mobilized by the type of motion associated with a rotational slide or a strip bearing capacity failure, i.e., plane strain deformation in which the path of motion of any particle lies in a vertical plane. It would not be applicable for other types of deformation or other paths of motion of particles. For instance, it would not apply to a field vane shear test.

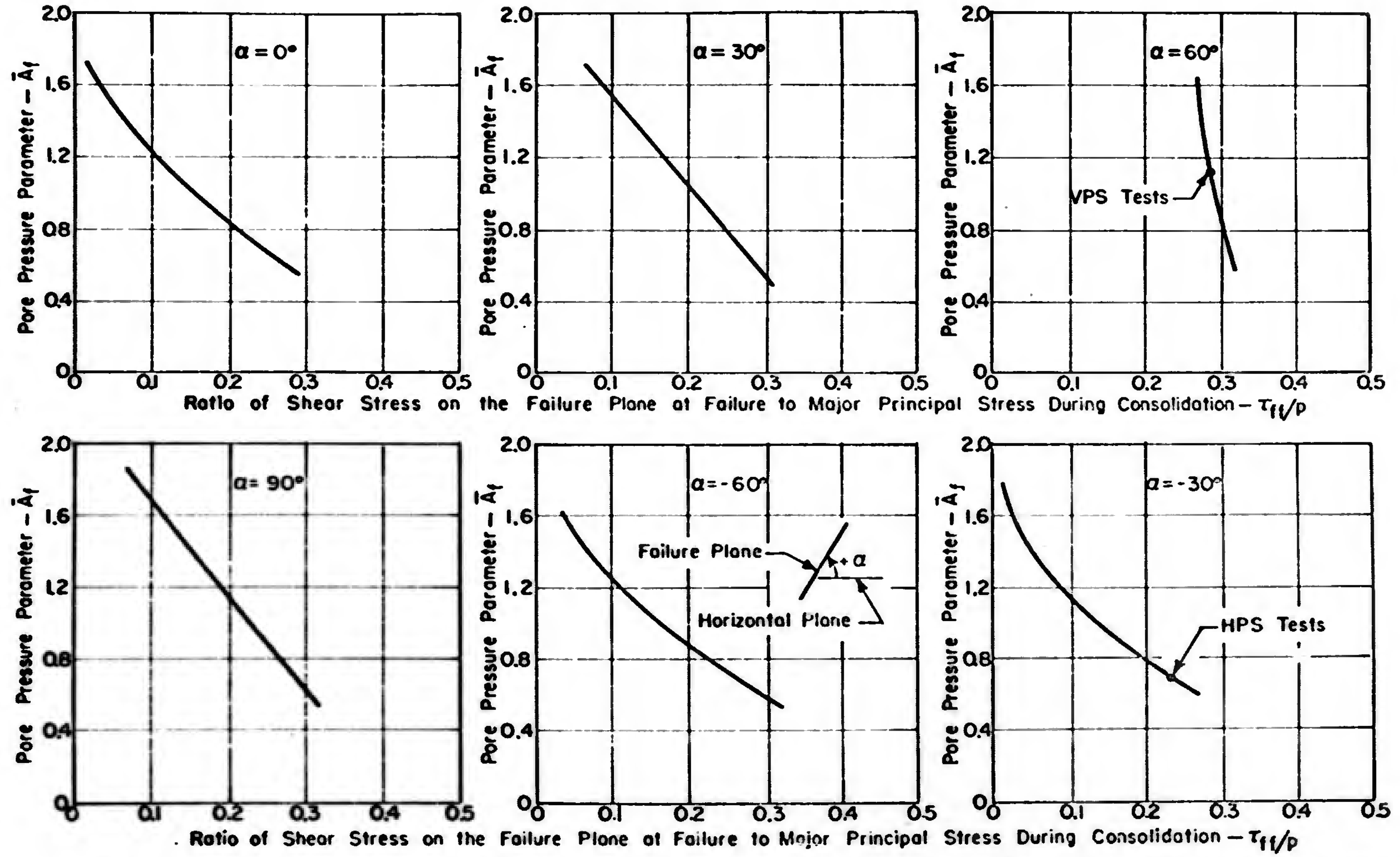


Fig.24—VARIATION OF STRENGTH RATIO τ_{ff}/p WITH THE PORE PRESSURE PARAMETER $\bar{A}_f$ FOR SIX POSSIBLE ORIENTATIONS OF THE FAILURE PLANE, FOR $c'=0$, $\phi'=35°$ AND $k_o=0.45$.

V. THE EFFECT OF SAMPLING ON THE RELATIONSHIP BETWEEN UNDRAINED STRENGTH AND REORIENTATION OF THE FAILURE PLANE

One of the questions which was not answered by the literature review was concerned with the effect of sampling on the relationship between undrained strength and orientation of the failure plane, i.e.

How would the variation of strength with orientation of the failure plane determined using UU laboratory tests compare to the variation of undrained strength with orientation of the failure plane in-situ?

The operation of sampling consists of two parts: One is the release of the anisotropic system of stresses under which the clay was consolidated; this part of the sampling operation is called "perfect sampling". The second part of real, or imperfect sampling, is the combination of disturbance, absorption of water or drying out, and temperature change. Since no change in water content is associated with "perfect sampling", and since the negative pore water pressure is the same in all directions, the anisotropic system of stresses is replaced by an isotropic stress just sufficient to prevent change in volume. The effect of "perfect sampling" on undrained strength has been investigated by Noorany and Seed (1965), Ladd and Lambe (1963) and by Skempton and Sowa (1964). Disturbance, absorption of water and temperature increase will reduce the magnitude of the isotropic stress acting on the sample after "perfect sampling", whereas drying out or reduction in temperature will increase the isotropic stress. Reduction in the isotropic stress results in a reduction in strength, and conversely, an increase in the isotropic stress results in an increase in strength. The effects of disturbance have been investigated by Seed, Noorany and Smith (1964) and by Ladd and Lambe (1963) who showed that the reduction in strength which results from a decrease in isotropic stress is about the same whether the decrease in isotropic stress is due to disturbance or to swelling. The effects of an increase in temperature on the isotropic stress and on the strength have been discussed by Duncan and Campanella (1965).

The effects of disturbance, of absorption of water or drying out and of temperature change in the strength of samples would probably be about the same for any orientation of the failure plane. The effects of stress release or "perfect sampling" however, might be different for each orientation of the failure plane. In order to determine the effect of "perfect sampling" on stress release, "perfect sampling" plane strain tests were performed on undisturbed Bay Mud using the same apparatus as used for the HPS and VPS tests described in the last section.

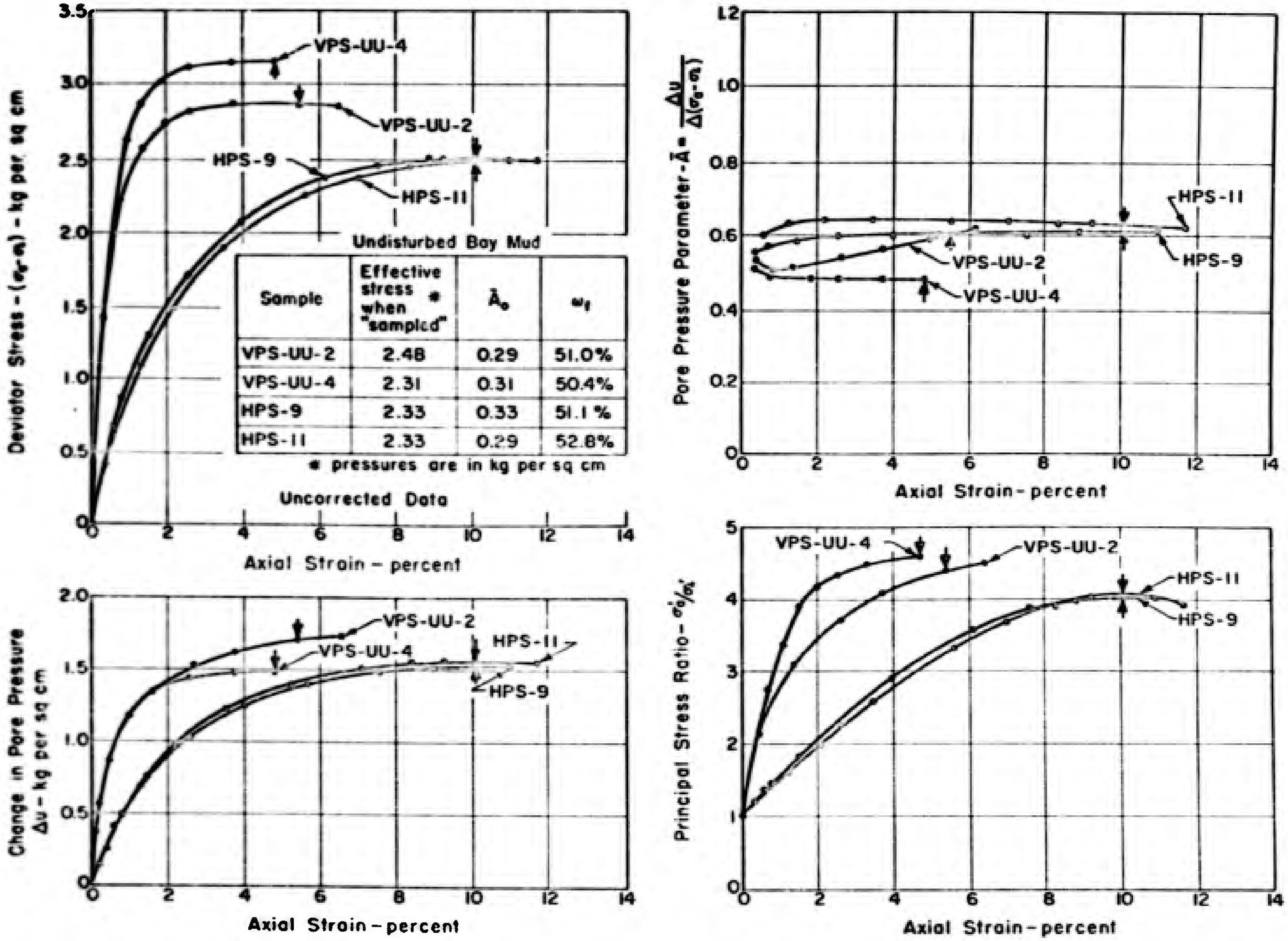
"Perfect Sampling" Tests

The "in-situ" tests described in Section IV were devised to simulate conditions at points A and D, the upper and lower ends of the failure arc shown in figure 1. These tests provide a means of measuring the ratio c_u/p for the two orientations of the failure plane which they involve. Similar plane strain tests were performed where the samples were anisotropically consolidated just as for the "in-situ" tests. Before increasing the axial load to cause undrained failure, however, the major principal stress was reduced to the value of the minor principal stress: this phase of the tests was the "perfect sampling" phase.

As explained in Section IV, in the first part of an undrained test on an HPS "in-situ" sample, the lateral stress (which was the major principal stress during consolidation) was reduced until the lateral and axial stresses were equal, and at this stage of the test the sample was under an isotropic state of stress. Thus the only difference between an HPS sample at this stage of the test and an HPS sample which has been "perfectly sampled" by reducing both the axial and the lateral stress to atmospheric pressure is the absolute value of the pore water pressure. Unless the pore water in the "perfectly sampled" sample cavitates, the absolute magnitude of the pore water pressure is inconsequential; since the samples are saturated and the pore pressure parameter B is equal to one, any change in total isotropic stress results in an equal change in pore water pressure, and no change in effective stress. If the pore water in the "perfectly sampled" sample does not cavitate, then the effective stress acting on the HPS sample when the lateral stress has been reduced to the same value as the axial stress will be the same as the effective stress on the HPS sample after "perfect sampling". Thus the first part of the HPS test is similar to "perfect sampling", and the second stage of the test (increasing the axial stress until the sample fails) can be considered as either the second stage of "in-situ" test simulating point D in figure 1, or an undrained test on a "perfectly sampled" plane strain sample with $\beta = 0$. Therefore the strengths measured using "in-situ" and "perfect sampling" tests on samples simulating point D are identical because the two tests are in fact the same.

The results of the HPS tests performed on samples consolidated with the values of the major principal stresses during consolidation equal to 0.8, 2.4 and 4.0 kg/cm^2 have been reinterpreted as "perfect sampling" tests. The uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$ and effective principal stress ratio, beginning with the time when the axial and lateral stresses were equal, are shown in figures 25, 26 and 27. The corrected results of the tests are summarized in table 5.

A series of three "perfect sampling" tests on vertical plane strain samples (tests VPS-UU-1 through -3) was conducted using three different values of major principal stress during consolidation. These "perfect sampling" tests were performed by reducing the axial stresses until they were equal to the lateral stresses (after consolidation was complete and while preventing further drainage) and then increasing the axial stress until the samples failed. The uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$ and effective principal stress ratio with axial strain are shown in figures 25, 26 and 27; these figures have been plotted to show comparisons of VPS-UU and HPS "perfect sampling" tests conducted at approximately equal values of the major principal stress during consolidation. The average value of the ratio c_u/p determined in the VPS-UU "perfect sampling" tests was 0.364, about 98 percent of the value determined for the vertical plane strain "in-situ" samples. The two percent reduction in strength resulting from "perfect sampling" is about the same as the reductions determined by other investigators



Sample	Effective stress * when "sampled"	$\bar{A}_o$	w_f
VPS-UU-2	2.48	0.29	51.0%
VPS-UU-4	2.31	0.31	50.4%
HPS-9	2.33	0.33	51.1 %
HPS-11	2.33	0.29	52.8%

Fig.25 - VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO FOR HORIZONTAL AND VERTICAL "PERFECTLY SAMPLED" PLANE STRAIN SAMPLES.

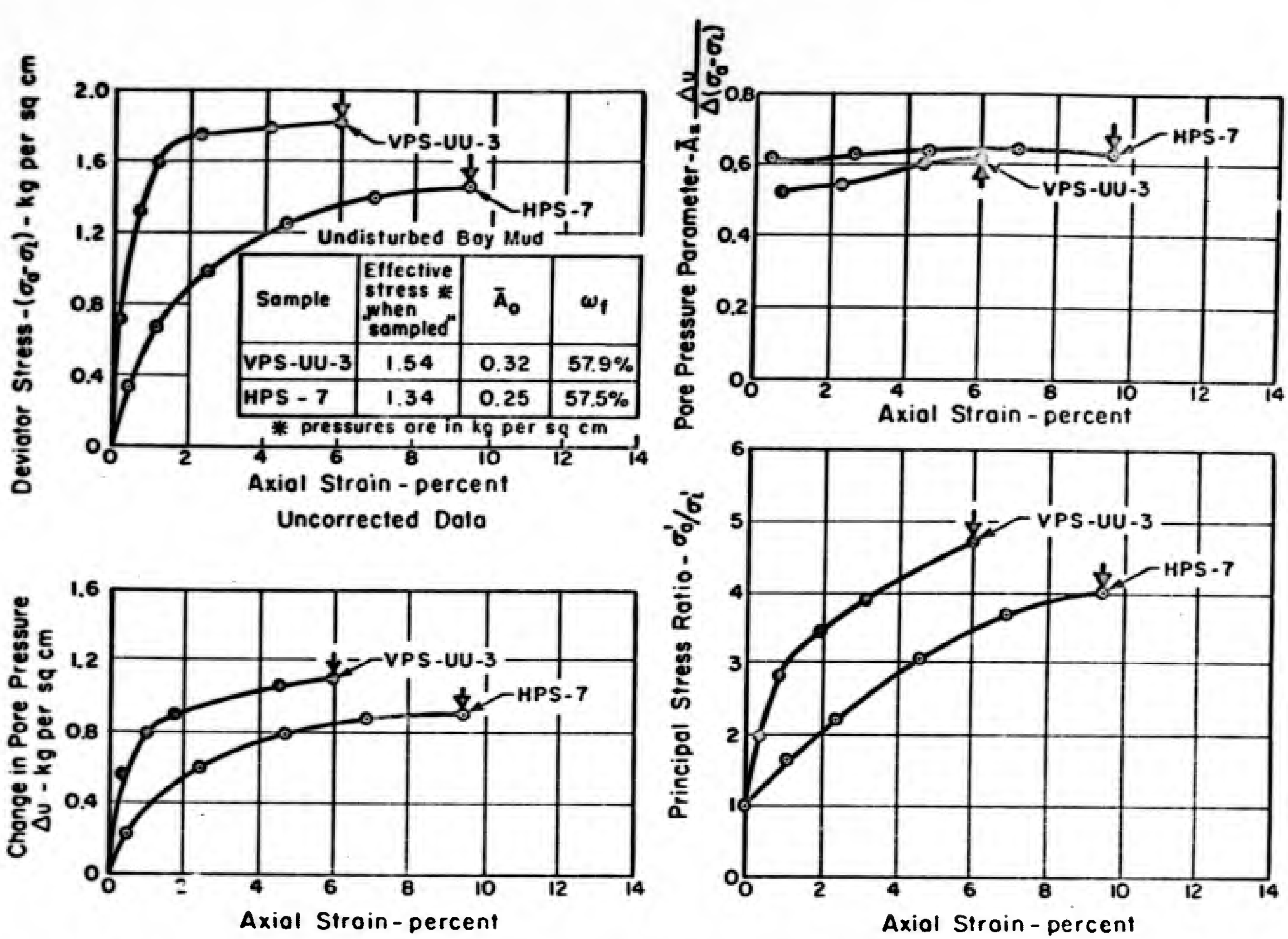


Fig.26 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "PERFECTLY SAMPLED" PLANE STRAIN SAMPLES.

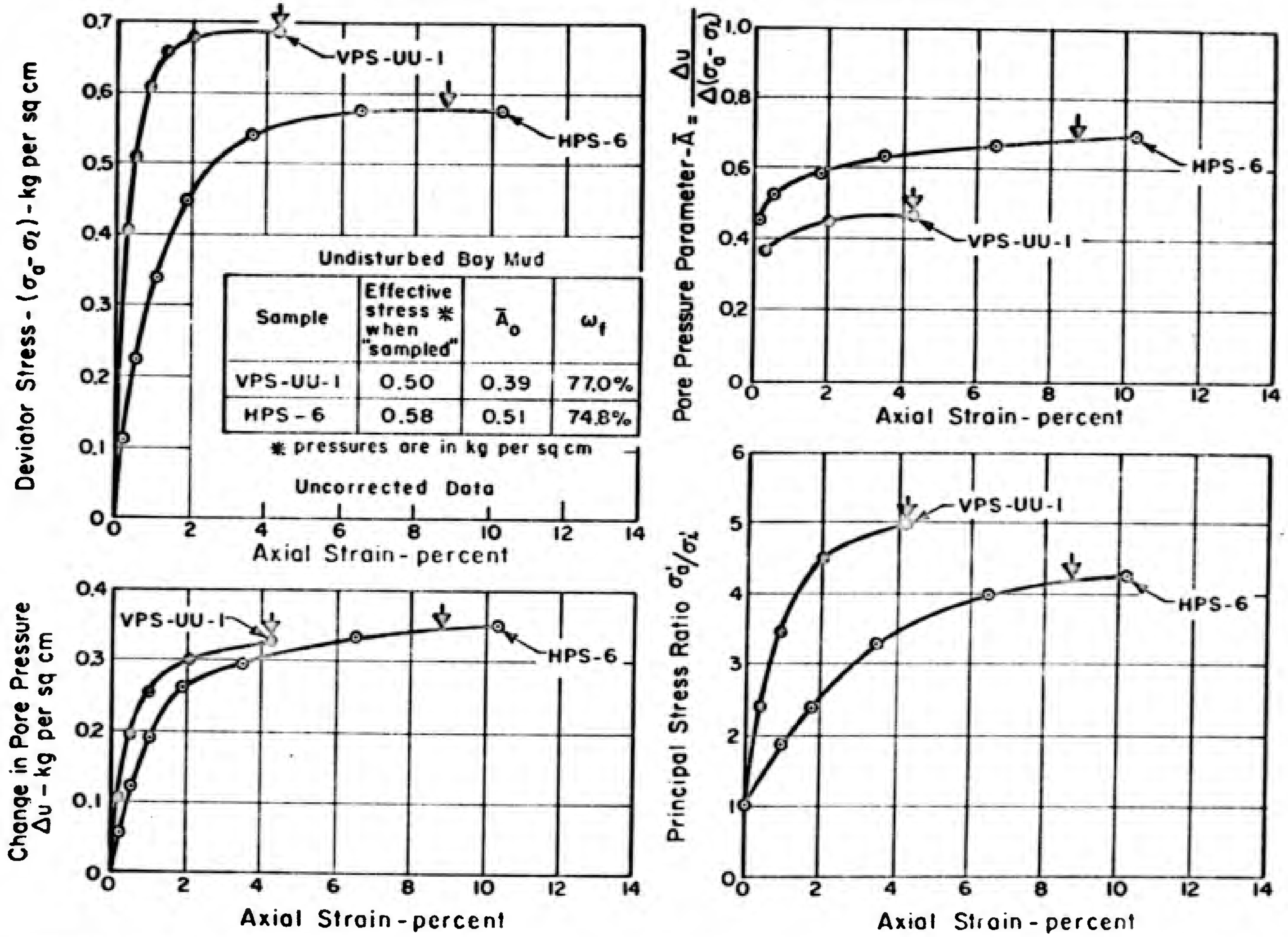


Fig.27–VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT A, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR HORIZONTAL AND VERTICAL "PERFECTLY SAMPLED" PLANE STRAIN SAMPLES.

Table 5. Summary of Results of "Perfect Sampling" Plane Strain Tests on Undisturbed San Francisco Bay Mud

Sample	Depth (feet)	Initial w/c (percent)	σ'_{ac} (kg/cm^2)	$\sigma'_{\ell c}$ (kg/cm^2)	$\left(\frac{\sigma'_3}{\sigma'_1}\right)_c$	Consol. w/c (percent)	$\left(\frac{\Delta V}{V}\right)_c$	Effective Stress When "Sampled" (kg/cm^2)	$\bar{A}_o$
VPS-UU-1	21	88.1	0.67	0.39	0.58	77.0	0.073	0.50	0.39
VPS-UU-2	21	88.1	3.69	1.98	0.54	51.0	0.277	2.48	0.29
VPS-UU-3	21	88.1	2.30	1.19	0.52	57.9	0.221	1.54	0.32
VPS-UU-4	21	90.1	3.87	1.61	0.42	50.4	0.287	2.31	0.31
HPS-6	16.5	90.1	0.34	0.81	0.42	74.8	0.093	0.58	0.51
HPS-7	16.5	89.3	0.99	2.40	0.41	57.5	0.226	1.34	0.25
HPS-9	15.5	88.8	1.47	3.98	0.37	51.1	0.277	2.31	0.33
HPS-11	19.0	92.3	1.66	4.00	0.42	52.8	0.282	2.33	0.29

Table 5. Continued

Sample	Depth (feet)	$(\sigma_a-\sigma_\ell)_f$ (kg/cm^2)	$(\Delta u-\Delta\sigma_\ell)_f$ (kg/cm^2)	$\bar{A}_f$	$\frac{\sigma_1'}{\sigma_3'}\ f$	ϵ_{af} (%)	Time to Failure (hours)	σ_{1f}' (kg/cm^2)	σ_{3f}' (kg/cm^2)	$\frac{(\sigma_1-\sigma_3)_f}{2}$ (kg/cm^2)	$\frac{(\sigma_1'+\sigma_3')_f}{2}$ (kg/cm^2)
VPS-UU-1	21	0.52	0.32	0.62	3.74	4.3	4	0.71	0.19	0.26	0.45
VPS-UU-2	21	2.67	1.73	0.65	4.18	5.5	4.25	3.51	0.84	1.33	2.17
VPS-UU-3	21	1.57	1.06	0.67	4.21	6.0	5	2.06	0.49	0.78	1.27
VPS-UU-4	21	2.84	1.42	0.50	4.19	4.8	4	3.73	0.89	1.42	2.31
HPS-6	16.5	0.41	0.40	0.98	3.05	8.8	3	0.61	0.20	0.21	0.41
HPS-7	16.5	1.28	0.90	0.70	3.91	9.5	3.5	1.72	0.44	0.64	1.08
HPS-9	15.5	2.23	1.51	0.68	3.76	10.1	3.5	3.04	0.81	1.12	1.93
HPS-11	19.0	2.24	1.55	0.69	3.80	10.1	3.75	3.04	0.80	1.12	1.92

using triaxial tests; Noorany found the reduction was four to seven percent for Bay Mud, and Skempton and Sowa found the reduction was one to two percent for Weald clay.

Corrected vector curves for both the VPS-UU and the HPS tests are shown in figure 28 (these vector curves were corrected by computing the end points from corrected data and then sketching the corrected curves with shapes similar to the uncorrected ones). Comparing the vector curves for samples VPS-UU-1 through -3 and the corresponding HPS samples, it can be seen that the HPS samples have both lower strengths and lower effective stresses when sampled than the corresponding VPS samples*. The values of $\bar{A}_f$ listed in table 5 and the slopes of the vector curves in figure 28 are nearly the same for samples VPS-UU-1 through -3 and the corresponding HPS samples. Thus the differences in strength between these three VPS-UU samples and the corresponding HPS samples appear to be due to differences in effective stress after sampling, and not to any difference in behavior of the two types of sample; the average value of the ratio of the undrained strength to the effective stress after sampling, c_u/σ'_{ps}, is 0.52 and for the three VPS-UU samples and 0.48 for the HPS samples**. The average values of the ratios $\sigma'_{ps}/\sigma'_{1c}$ for the three VPS-UU samples is 0.70 and for the HPS samples** is 0.57. There are two features of these results which appear to be anomalous:

(1) The value of the ratio $\sigma'_{ps}/\sigma'_{1c}$ appears to depend on the direction in which the major principal stress was acting during consolidation; the value of the ratio for tests where the major principal stress acted in the lateral direction is different from the value for tests where it acted in the vertical direction. Since "perfect sampling" merely amounts to reducing the major principal stress until it is equal to

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*Test HPS-6 is not believed to be representative of the behavior of Bay Mud when "sampled".

**The value of this ratio for sample HPS-6 was not included in the average.

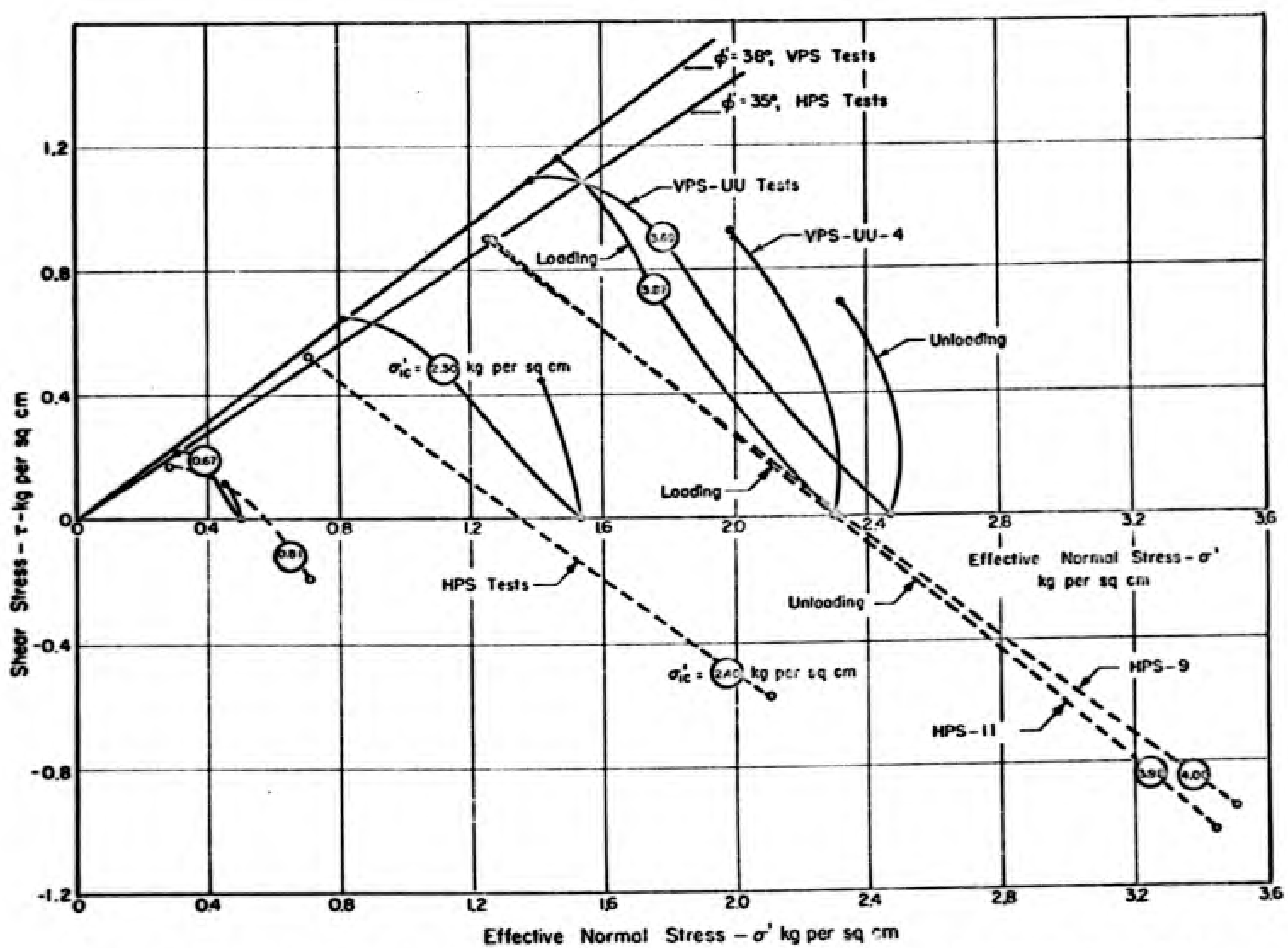

Fig. 28 – CORRECTED VECTOR CURVES FOR HORIZONTAL AND VERTICAL "PERFECT SAMPLING" PLANE STRAIN TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

the minor principal stress in either type of test, the direction in which the major principal stress acted before it was reduced should not influence the value of the ratio $\sigma'_{ps}/\sigma'_{1c}$.

(2) The value of the ratio c_u/σ'_{ps} is almost the same for the two types of test. Since UU triaxial samples which are trimmed horizontally on the one hand and vertically on the other hand presumably are acted upon by the same effective stress, σ'_t, after trimming, then horizontal and vertical UU triaxial samples must have different values of the ratio c_u/σ'_t. Since the value of the ratio c_u/σ'_t is different for horizontal and vertical triaxial samples, it would be expected that c_u/σ'_{ps} would be different for horizontal and vertical "perfectly sampled" plane strain samples.

The explanation of both of these apparent anomalies is that the value of the ratio of the minor to the major principal stress during consolidation, $(\sigma_3'/\sigma_1')_c = k$, was different in HPS tests from that in VPS-UU tests. Noorany and Seed (1965) have shown that

$$\frac{\sigma'_{ps}}{\sigma'_{1c}} = k + \bar{A}_o(1 - k) \qquad (5\text{-}13)$$

where $\bar{A}_o$* is the value of $\bar{A}$ which applies during sampling. The value of $\bar{A}_o$ in all of these "perfect sampling" tests (excepting test HPS-6) was about 0.3. Thus the difference in the value $\sigma'_{ps}/\sigma'_{1c}$ is seen to be due to the fact that the values of k used in the first three VPS-UU tests were significantly different from those used in the HPS tests. (The difference in the values of k results from the fact that the loads carried by filter paper drains, rubber membranes, and piston friction at the end of consolidation were different for the two types of samples, and because neither type of sample was consolidated

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*Noorany and Seed defined $\bar{A}_o$ as the ratio $(\Delta u - \sigma_{3c})/(\sigma_{1c} - \sigma_{3c})$ where Δu is the change in pore pressure due to reducing the stresses σ_{1c} and σ_{3c} to zero.

exactly one-dimensionally. The effect of the different values of k on the results of horizontal and vertical "in-situ" tests has been discussed previously.

In order to determine if the value of the ratio c_u/σ'_{ps} for VPS-UU tests would be different from that measured in HPS tests if the same value of k was used in both types of test, an additional test, VPS-UU-4, was performed. The uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$, and effective principal stress ratio with axial strain for test VPS-UU-4 are shown in figure 25, and the corrected data are summarized in tables 5. The corrected value of the ratio c_u/p for test VPS-UU-4 is 0.367, which is only one percent higher than the average value of this ratio determined in the first three VPS-UU tests.

The values of the ratios $\sigma'_{ps}/\sigma'_{1c}$ and c_u/σ'_{ps} determined in test VPS-UU-4, however, were quite different from those determined in the first three VPS-UU tests. The value of the ratio $\sigma'_{ps}/\sigma'_{1c}$ was 0.59 in test VPS-UU-4 as compared to 0.70 for the first three VPS-UU tests, and the value of the ratio c_u/σ'_{ps} was 0.61 as compared to 0.52 for the first three VPS-UU tests. The value of $\bar{A}_f$ was 0.50 in test VPS-UU-4 as opposed to 0.65 in the first three VPS-UU tests; the fact that $\bar{A}_f$ decreases as the values of k and σ'_{ps}/σ_{1c} decrease seems to be responsible for the fact that c_u/p does not change significantly as k and $\sigma'_{ps}/\sigma'_{1c}$ change.

Because of the differences between the values of k used in the HPS and the first three VPS-UU "perfect sampling" tests, the results are not directly comparable. Test VPS-UU-4, however, was conducted with almost the same value of k as used in the HPS tests, so the results of test VPS-UU-4 are directly comparable with the results of tests HPS-9 and HPS-11, the horizontal plane strain "perfect sampling" tests conducted at approximately the same consolidation pressures. Comparing the vector curves for these tests shown in figure 28, it can be seen that the effective stresses after sampling are about the same for horizontal and vertical "perfectly sampled" samples, but, because the value of $\bar{A}_f$ is less for vertical than for horizontal samples, the vertical samples are stronger.

Whether horizontal or vertical samples are tested, the strengths measured in "perfect sampling" and "in-situ" tests are, for practical purposes, the same. It is interesting to note that the difference between the strength of horizontal and of vertical samples found in "in-situ" tests is the combined effect of anisotropy and reorientation of principal stresses, as explained in Section IV, whereas the same difference in strength was found in "perfect sampling" tests where no reorientation of principal stresses occurs. Thus, whereas the difference between the strengths of horizontal and vertical "in-situ" samples is due to anisotropy and reorientation of principal stresses, the same difference in strengths, when measured in "perfect sampling" tests, is due only to anisotropy.

From the results of these "perfect sampling" tests, and the "in-situ" tests described in Section IV, it seems reasonable to conclude that:

(1) The value of the ratio c_u/p determined using "perfect sampling" plane strain tests is, for practical purposes, the same as that determined using "in-situ" plane strain tests performed on either horizontal or vertical samples.

(2) Although the values of $\sigma'_{ps}/\sigma'_{1c}$ and c_u/σ'_{ps} determined from "perfect sampling" tests are sensitive to the value of the ratio of the minor to the major principal stress during consolidation, k, the value of the ratio c_u/p is not sensitive to the value of k.

Since the value of the ratio c_u/p determined in "perfect sampling" tests is practically the same as that determined using "in-situ" tests for both orientations of the failure plane, it seems logical that the "perfect sampling" and "in-situ" strengths would be similar for any orientation of the failure plane. This inference cannot be checked experimentally, because it is not possible to perform either "perfect sampling" or "in-situ" tests for values of α other than 60° and -30°. It is possible, however, to perform UU tests on undisturbed (imperfectly sampled) samples trimmed in different directions, and to compare the relationship between undrained strength and orientation of the failure plane so determined with the relationship shown in figure 23, which is believed to be the in-situ relationship.

The best type of UU tests for this comparison would be UU plane strain tests, but since the samples will have been imperfectly sampled and the strengths somewhat altered in any case, UU triaxial tests, which are more convenient to perform, will probably serve just as well.

UU Triaxial Tests

To determine the relationship between laboratory undrained strength and orientation of the failure plane, a series of 27 UU triaxial tests was performed using undisturbed samples of Bay Mud trimmed with their axes in different directions. The variations of strength and axial strain at failure with orientation of the failure plane are shown in figure 29. Piston friction was automatically eliminated by the test procedure, and the data have been corrected for the loads carried by rubber membranes.

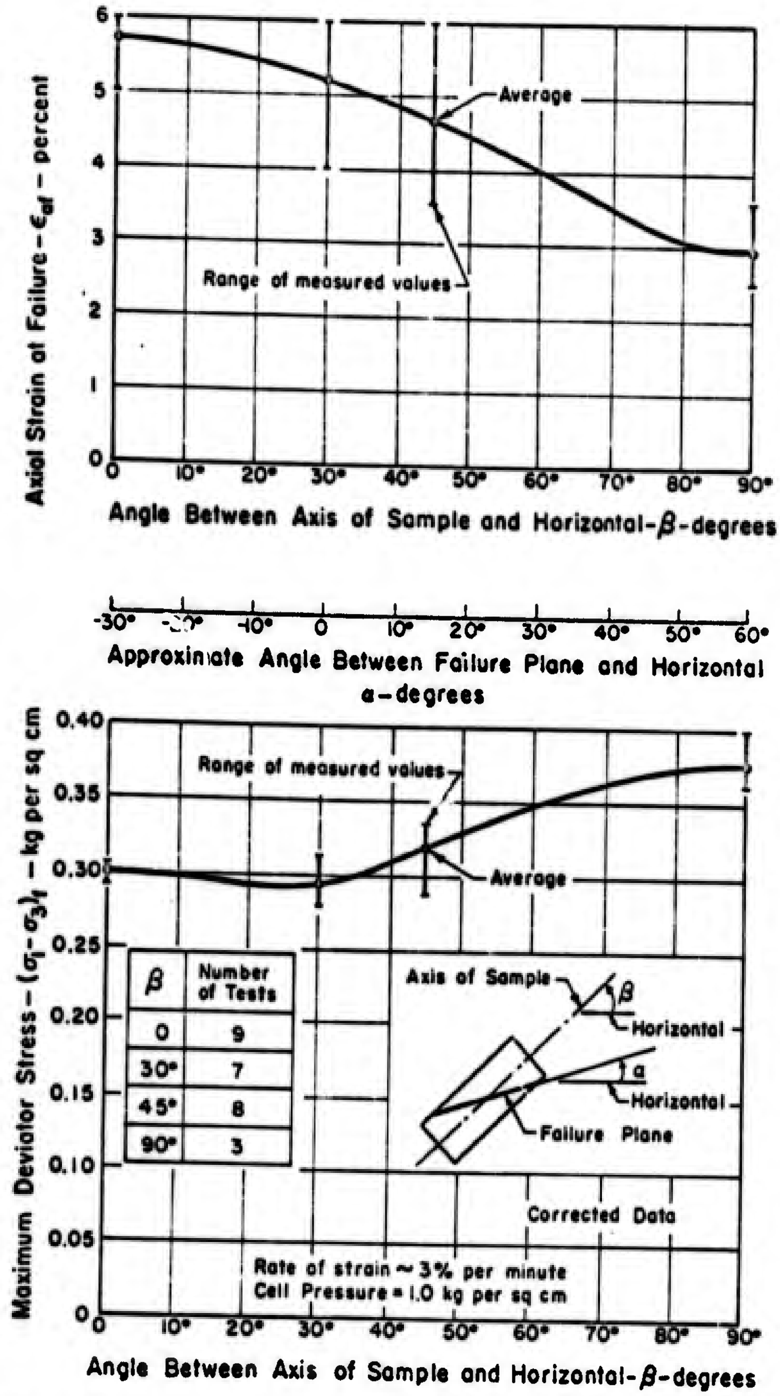
Figure 29 shows that for undisturbed, normally consolidated Bay Mud the strength varies with the orientation of the failure plane in approximately the same way as for the overconsolidated kaolinite (figure 6). The minimum strength is associated with inclined (30°) samples, where the failure plane approximately coincides with the horizontal, and the maximum strength is associated with the vertical samples. The average strength ratios are

$$\frac{(\sigma_1 - \sigma_3)_f \text{ inclined } (30°)}{(\sigma_1 - \sigma_3)_f \text{ vertical}} = 0.79$$

and

$$\frac{(\sigma_1 - \sigma_3)_f \text{ horizontal}}{(\sigma_1 - \sigma_3)_f \text{ vertical}} = 0.81$$

The average strain at failure for the vertical samples is only about half of that for the horizontal ones. The average strain at failure in vertical plane strain tests was about one third of that for horizontal samples in "in-situ" tests, and was about half of that for horizontal samples in "perfect sampling" tests. Thus it appears that



β	Number of Tests
0	9
30°	7
45°	8
90°	3

Fig.29-VARIATION OF MAXIMUM DEVIATOR STRESS AND STRAIN AT FAILURE WITH ORIENTATION OF THE FAILURE PLANE IN UU TRIAXIAL TESTS ON UNDISTURBED BAY MUD.

the strains at failure are more nearly alike after sampling, and that the ratio of the strains at failure for horizontal and vertical samples is about the same in plane strain and triaxial tests.

Conclusions

The measured values of undrained strength shown in figure 29 have been interpreted in terms of the ratio c_u/p and replotted in figure 30. The effective overburden pressures were taken from figure 9. Also shown in figure 30 is the hypothesized in-situ variation of strength which was shown in figure 23 in terms of τ_{ff}/p.

Two separate hypotheses led to the conclusion that the variations of c_u/p determined using UU triaxial tests would be similar to the dashed curve shown in figure 30:

(1) That sampling would not significantly alter the variations of c_u/p with orientation of the failure plane, and therefore the variation of c_u/p determined using UU triaxial tests would be similar to the in-situ variation.

(2) That the in-situ variation of $\bar{A}_f$ with orientation of the failure plane would be as shown in the upper part of figure 23, and therefore the variation of c_u/p in-situ would be as shown by the dashed curve in figure 30.

The fact that the two variations of c_u/p with orientation of the failure plane shown in figure 30 are similar means that both of the hypotheses are probably at least approximately correct. To explain the small differences between the values of c_u/p determined using UU triaxial tests and the hypothesized in-situ variation does not seem to be possible on the basis of available evidence. However, it is possible that either the variation of $\bar{A}_f$ with α would be some smooth curve other than the one shown in figure 23, or that sampling, per se, does affect the strength for some orientations of the failure plane. The other possibilities are that either disturbance or strain effects (triaxial as opposed to plane strain) influence the relationship between undrained strength and orientation of the failure plane.

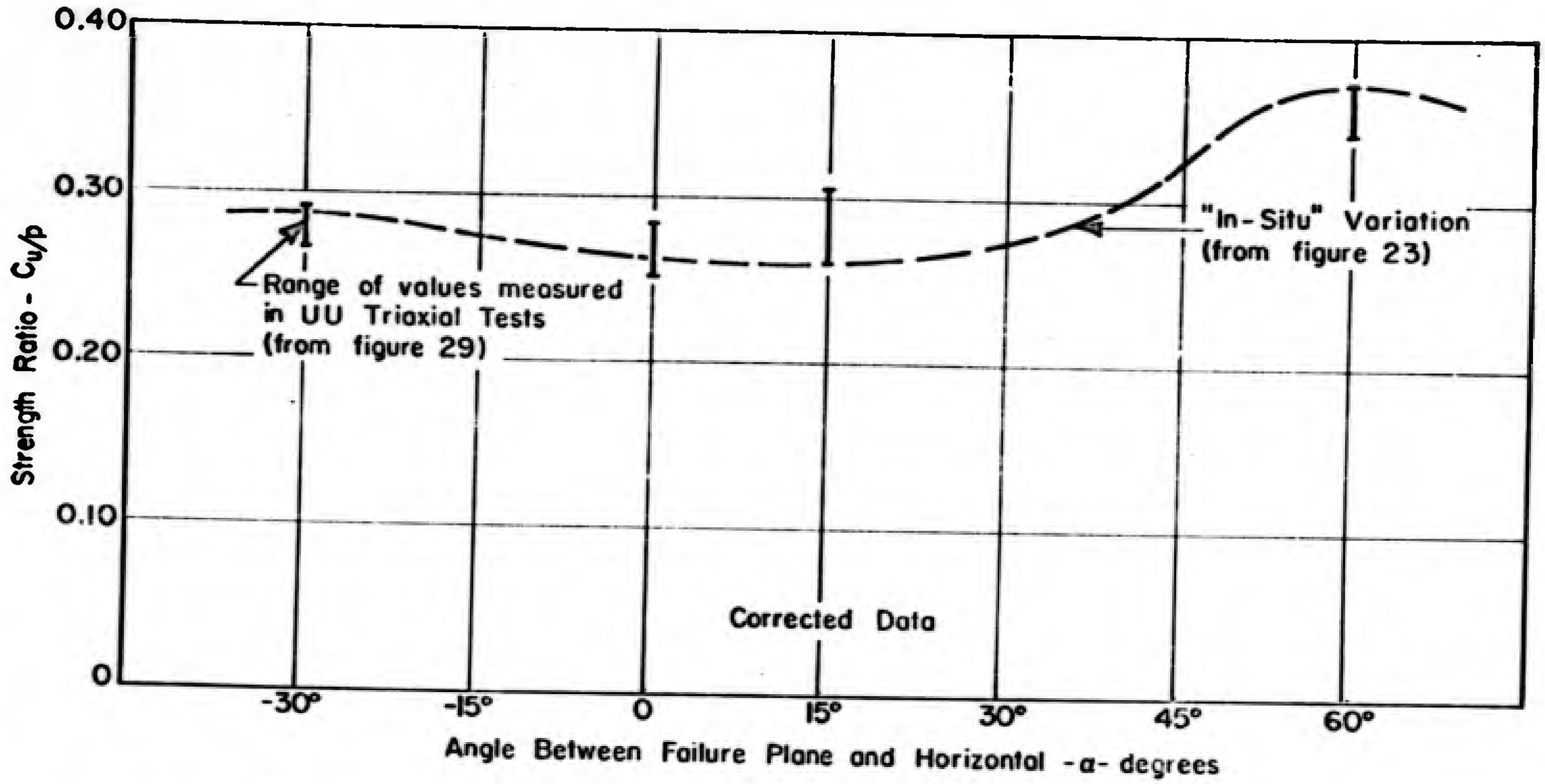


Fig. 30 - COMPARISON OF THE VARIATIONS OF C_u/p WITH ORIENTATION OF THE FAILURE PLANE AS DETERMINED USING "IN-SITU" TESTS AND UU TRIAXIAL TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

VI. THE USE OF THE TRIAXIAL TEST
FOR MEASURING IN-SITU STRENGTHS

The possibility of using two types of AC-U triaxial tests for simulating in-situ conditions, rather than the two types of plane strain test described in section IV, was considered in the early stages of this study. (The methods which might be used for simulating conditions at the upper and lower ends of a failure arc in the field are discussed in Appendix A.) Although it was finally decided to conduct the main study using plane strain tests, one triaxial test of each type was made in order to be able to compare the times required for performing the tests, and the results of the tests, with those for plane strain tests.

In addition, a number of IC-U triaxial tests were performed in order to make a comparison of strengths and effective stress strength parameters determined in these tests with those determined in plane strain tests.

AC-U Tests

Two AC-U tests (tests AC-U-1 and AC-U-2) were performed on undisturbed San Francisco Bay Mud. The samples were consolidated in standard University of California Triaxial cells (Seed, Mitchell, and Chan, 1960) by increasing the cell pressure and the axial load in increments of 20 percent of the load already acting on the sample. The new load increments were applied after 1 to 4 days consolidation under the previous load, and the samples consolidated for two days under the final loads. Altogether, consolidation required 2 weeks. Volume changes were recorded by noting the volume of pore fluid expelled, and the computed volumetric and axial strains were used to correct the area when computing load increments. Back pressures of 1.0 kg/cm^2 were used for both samples, and the pore pressure parameter B was checked and found to be equal to one at the end of consolidation. Electrical pressure transducers were used for measuring pore pressures during the undrained compression tests, which were performed using controlled stress loading. The water contents of the upper and lower parts of the samples were measured separately after failure.

Sample AC-U-1 is analogous to an element of soil at point A in figure 1, where the axis of the element is vertical and the major principal stress acts in the axial direction both during consolidation and at failure. Sample AC-U-1 was consolidated with the major principal stress acting in the axial direction; during consolidation, the ratio of the lateral stress to the axial stress was controlled so that the sample was compressed in the axial direction, approximately without lateral strain. When consolidation was complete, and while preventing further drainage, the axial stress was increased so that the sample failed in compression. Thus, except for the fact that sample AC-U-1 deformed radially symmetrically during shear, whereas the element of soil at point A deforms in plane strain, the two are alike.

Sample AC-U-2 is roughly analogous to an element of soil at point D in figure 1, where the axis of the element is horizontal and the major principal stress acts in the lateral direction during consolidation and in the axial direction at failure. Sample AC-U-2 was consolidated with the major principal stress acting in the lateral direction by applying a tensile load to the cell piston which was threaded into the cap; during consolidation the ratio of the lateral stress to the axial stress was controlled so that the sample was compressed laterally, approximately without axial strain. When consolidation was complete, and while preventing further drainage, the axial stress was increased so that the sample failed in compression. The element of soil at point D is consolidated one-dimensionally, by compression in the lateral direction, whereas sample AC-U-2 was consolidated with radially symmetrical deformation. Sample AC-U-2 also deformed radially symmetrically during shear, whereas the element of soil at point D deforms in plane strain.

The reorientation of the major principal stress during shear is the same in sample AC-U-2 as in the element of soil at point D; in both the sample and the element the major principal stress acts in the lateral direction during consolidation and in the axial direction at failure.

The corrected results of tests AC-U-1 and -2 are summarized in Table 6, and the uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$, and effective principal stress ratio are shown in figure 31. The sample representing an element of soil at point A was stronger than the sample representing an element of soil at point D. The ratios of one half of the maximum deviator stress to the major principal stress during consolidation were found to be:

for the triaxial sample representing point A, $c_u/p = 0.35$,

for the triaxial sample representing point C, $c_u/p = 0.29$

The values of the ratio c_u/p determined using plane strain tests, in which a more exact analogy exists between the laboratory samples and the elements of soil in the field, were $c_u/p = 0.37$ at point A, and $c_u/p = 0.29$ at point D. Thus the difference in the values of the ratio c_u/p for points A and D determined using triaxial tests is slightly less than the difference determined using plane strain tests.

The difference in the values of the ratio c_u/p in the two types of triaxial tests is due to the combined affect of anisotropy with respect to $\bar{A}_f$, and the difference in reorientation of principal stresses between the two samples. The pore pressure parameter $\bar{A}_f$ is larger for the sample representing point A than for the one representing point D, but the deviator of the change in stress is so much smaller in the sample representing point A than in the sample representing point D, that the sample representing point A is the stronger of the two. These statements can be summarized as follows:

Sample Representing	$\bar{A}_f$	Deviator of the Change in Stress	Undrained Strength
A	Larger	Smaller	Larger
D	Smaller	Larger	Smaller

Qualitatively, the difference in behavior of the triaxial samples representing point A and point D is the same as the difference in behavior of the plane strain samples representing points A and D. Quantitatively, the difference in undrained strength found using triaxial tests is somewhat less than that found using plane strain tests.

Table 6. Summary of Results of Consolidated Undrained Triaxial Tests on Undisturbed San Francisco Bay Mud

Sample	Depth (feet)	Initial w/c (percent)	σ'_{ac} (kg/cm^2)	σ'_{rc} (kg/cm^2)	$\frac{\sigma'_a}{\sigma'_r}$	Consol. w/c (percent)	$\left(\frac{\Delta V}{V}\right)_c$	ϵ_{ac} (%)	$\Delta(\sigma_a-\sigma_r)_f$ (kg/cm^2)	Δu_f (kg/cm^2)
AC-U-1	15.5'	92.2	2.91	1.58	1.84	55.0	0.262	0.236	0.71	0.79
AC-U-2	15.5'	-	2.07	3.18	0.65	55.4	0.262	0.012	2.97	2.44
IC-U-1	19	87.7	0.87	1.00	0.87	67.7	0.147	0.041	0.79	0.77
IC-U-2	19	87.5	2.33	2.48	0.94	55.3	0.248	0.097	1.77	1.86
IC-U-3	19	85.6	3.81	3.98	0.96	49.0	0.297	0.123	2.81	2.96

Table 6. Continued

Sample	Depth (feet)	$\bar{A}_f$	$(\sigma_1-\sigma_3)_f$ (kg/cm^2)	$\frac{\sigma_1'}{\sigma_3'}_f$	ϵ_{af} (%)	Time to Failure (hours)	σ_{1f}' (kg/cm^2)	σ_{3f}' (kg/cm^2)	$\frac{(\sigma_1-\sigma_3)_f}{2}$ (kg/cm^2)	$\frac{(\sigma_1'+\sigma_3')_f}{2}$ (kg/cm^2)
AC-U-1	15.5'	1.11	2.04	3.58	6.2	4.2	2.83	0.79	1.02	1.81
AC-U-2	15.5'	0.82	1.86	3.62	12.9	3.5	2.57	0.71	0.93	1.64
IC-U-1	19	0.98	0.66	3.87	11.3	5	0.89	0.23	0.33	0.56
IC-U-2	19	1.05	1.62	3.58	11.0	5	2.25	0.63	0.81	1.44
IC-U-3	19	1.05	2.64	3.56	12.6	5	3.65	1.01	1.32	2.33

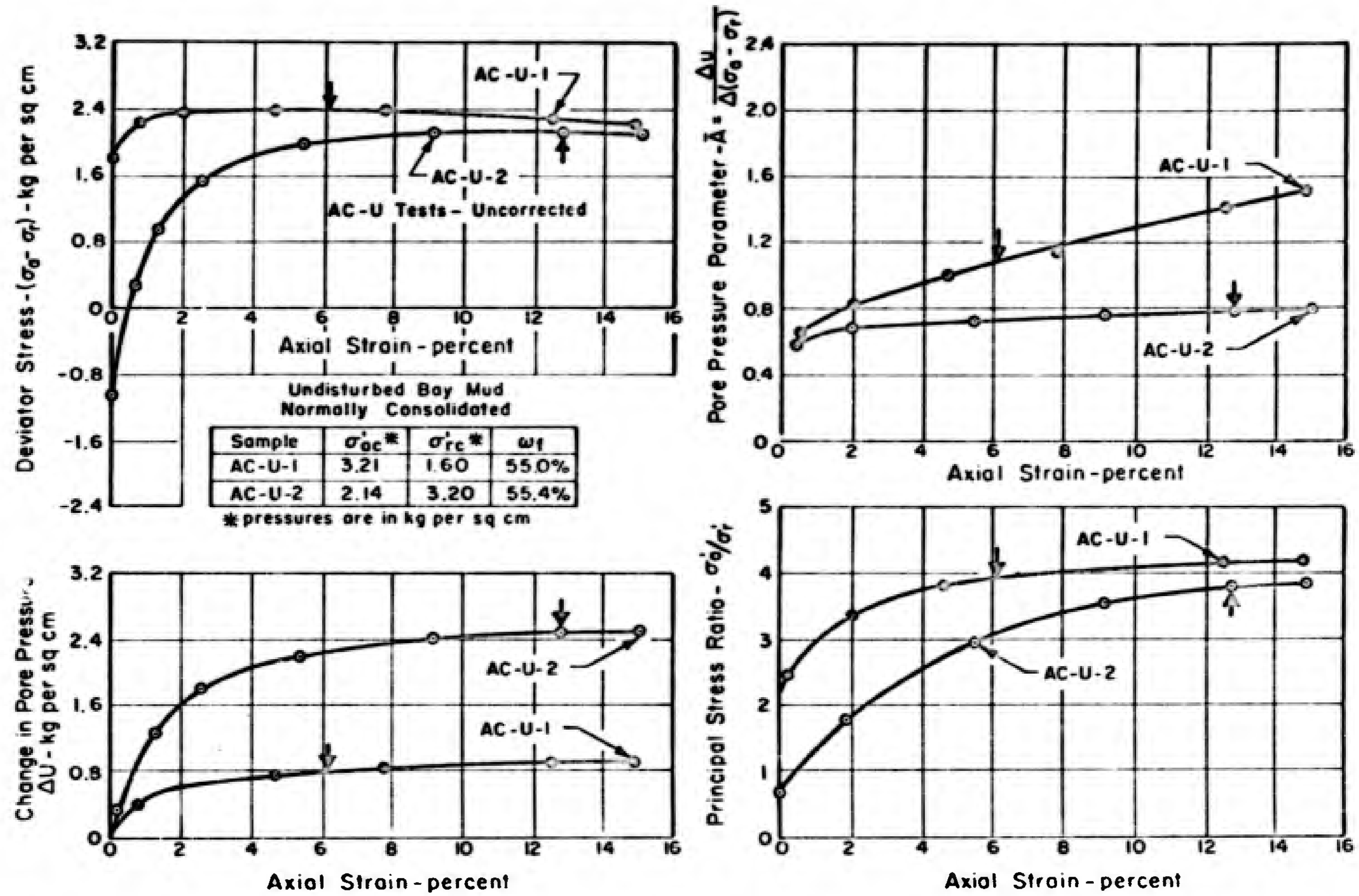


Fig.31 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR AC-U TRIAXIAL TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

Besides providing a less exact analogy to field conditions, and somewhat different results than the plane strain tests, the triaxial tests were found to be less convenient to perform. A larger number of load increments, and consequently a longer consolidation period was required for the triaxial tests than for the plane strain tests. It was necessary to use relatively small load increments in the triaxial tests in order to prevent the samples from failing when a new load increment was applied. On the other hand, large load increments may be used during consolidation in the plane strain tests; the vertical plane strain samples are prevented from failing during consolidation by the presence of the side plates and end plates which prevent lateral strain and change in length of the samples; the horizontal plane strain samples are prevented from failing during consolidation by the presence of the end plates, and because axial strain of the samples was prevented by preventing movement of the cap. Since the plane strain tests provide both a more accurate and a more convenient means of representing in-situ conditions, the detailed study of the influence of anisotropy and reorientation of principal stresses was conducted using plane strain tests.

IC-U Tests

Three isotropically consolidated-undrained triaxial tests were performed using samples of undisturbed San Francisco Bay Mud which were trimmed vertically. The three samples were consolidated in standard triaxial cells using uncorrected values of the effective consolidation pressure equal to 1.0, 2.5 and 4.0 kg/cm^2. Axial and volumetric strains were measured during consolidation, which continued for three days. Back pressures of either 1.0 or 1.5 kg/cm^2 were used to saturate the samples; at the end of consolidation the value of the pore pressure parameter B was checked and found to be equal to one. An electrical pressure transducer was used to measure both the cell water pressure and the pore water pressure. The undrained compression tests were performed by increasing the axial load on the samples at fifteen-minute intervals so that the samples failed in five hours; the water contents of the tops of the samples, the zones containing the failure planes and the bottoms of the samples were measured separately after failure.

The uncorrected variations of deviator stress, pore water pressure, pore pressure parameter $\bar{A}$, and effective principal stress ratio with axial strain are shown in figure 32. The corrected consolidation pressures tabulated in figure 32 are the corrected values of the radial stress at the end of consolidation. Because enough axial strain (more than two percent) occurred during consolidation to fully mobilize the strength of the filter paper drains, the filter papers carried some of the axial load, and as a result, the axial stresses in the samples at the end of consolidation were smaller than the radial stresses. The only significant effect of this inequality of axial and radial stresses at the end of consolidation is on the computed value of $\bar{A}_f$. If it is assumed that the consolidation pressure was isotropic and was equal to the average of the corrected axial and radial stresses, then values of $\bar{A}_f$ equal to 1.17, 1.15 and 1.12 are computed for consolidation pressures of 1.0, 2.5 and 4.0 kg/cm^2, as opposed to the values $\bar{A}_f$ = 0.98, 1.05 and 1.05 which are computed using the corrected (unequal) axial and radial stresses shown in table 6. The latter values of axial and radial stress are probably more nearly correct, and have been listed in table 6. However, there is some uncertainty connected with the application of all corrections, because it is difficult to determine the exact values which apply at any stage of a test. The problem of determining the correct consolidation pressure during "isotropic" consolidation is an example of this uncertainty.

The corrected data and the method used to determine the effective stress strength parameters are shown in figure 33. Just as for the plane strain tests described in Section IV, the shear strength intercept c' was found to be equal to zero. Since the value of the effective principal stress ratio at maximum deviator stress, $(\sigma_1'/\sigma_3')_f = 3.6$, was the same in both the IC-U and the AC-U triaxial tests, it seems likely that the same effective stress envelope (characterized by $c' = 0$, $\Phi' = 34.5°$) would apply to both types of triaxial test.

The value of Φ' determined using the vertical plane strain tests discussed in Section IV was 38°, or about 10 percent higher than the

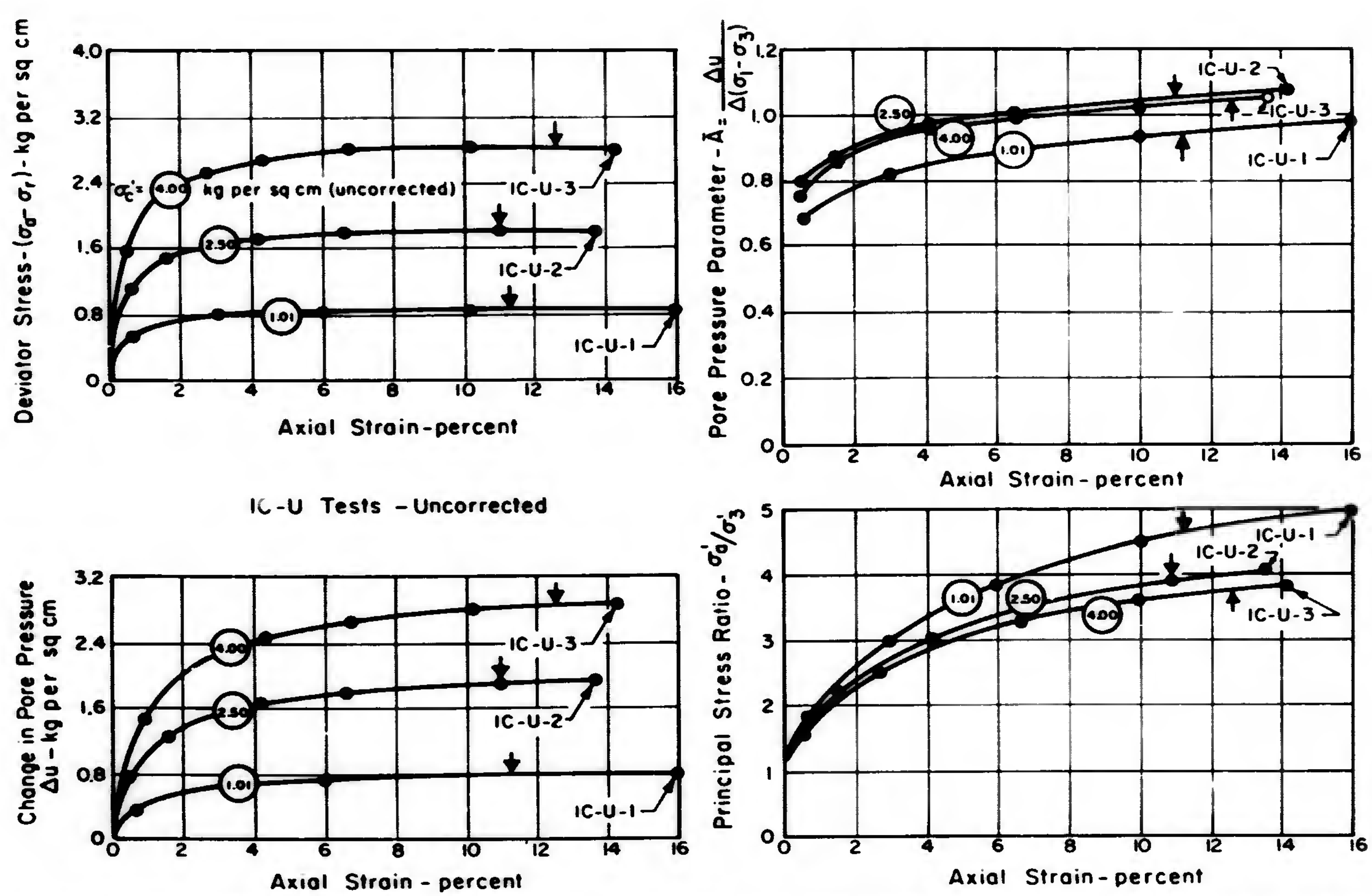


Fig.32 – VARIATIONS OF DEVIATOR STRESS, PORE WATER PRESSURE, PORE PRESSURE COEFFICIENT $\bar{A}$, AND EFFECTIVE PRINCIPAL STRESS RATIO WITH AXIAL STRAIN FOR IC-U TRIAXIAL TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

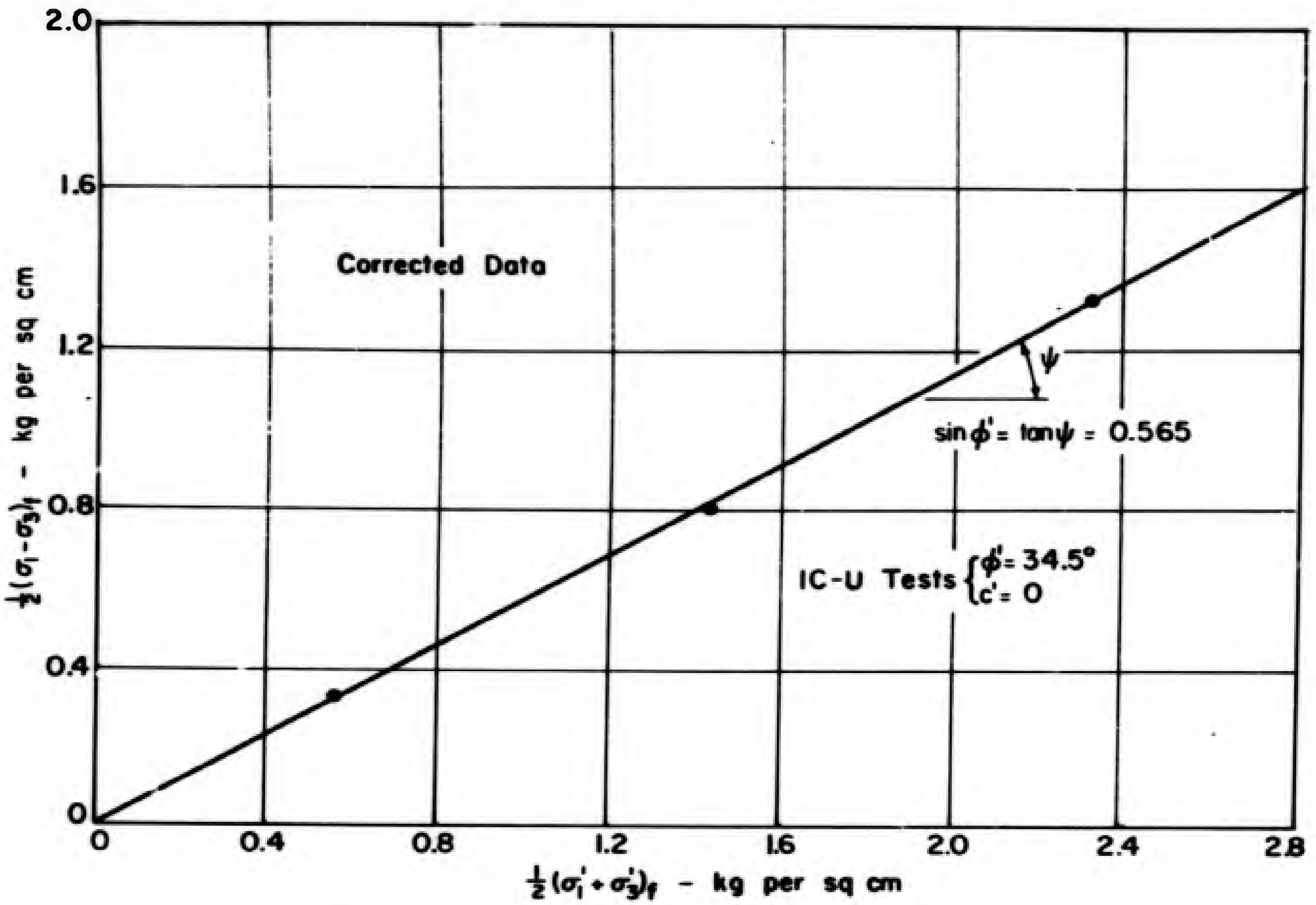


Fig. 33- DETERMINATION OF IC-U TRIAXIAL EFFECTIVE STRESS STRENGTH PARAMETERS FOR UNDISTURBED SAN FRANCISCO BAY MUD.

values determined using triaxial tests. Bishop (1961) reported that the values of Φ' for compacted clay and for a dense sand determined using plane strain tests were about 10 percent higher than those determined using triaxial tests, and Wade (1963) found that the value of Φ' for a saturated, remoulded clay determined using plane strain tests was about five percent higher than that determined using triaxial tests. Both Bishop and Wade compared the results of vertical plane strain tests ($\alpha = 60°$) with the results of triaxial tests. It is interesting to note that the difference in the two values of Φ' determined using two types of plane strain tests involving two different orientations of the failure plane ($\Phi' = 38$ for $\alpha = 60°$ and $\Phi' = 35°$ for $\alpha = 30°$) is nearly as large as the difference in the values of Φ' determined using vertical plane strain tests on the one hand and triaxial tests on the other hand. It seems logical to believe that the maximum and minimum values of Φ' would be associated with $\alpha = 90°$ and $\alpha = 0$, respectively, so that the minimum value of Φ' for plane strain conditions would be somewhat less than 35° and the maximum values of Φ' would be somewhat more than 38°.

Bishop (1961) suggested that since the values of Φ' determined using vertical plane strain were significantly larger than those determined using triaxial tests, the use of values of Φ' determined using triaxial tests would be conservative. It seems likely however, that the variation of the value of Φ' with orientation of the failure plane may compensate for the difference in values of Φ' determined using vertical plane strain tests on the one hand and triaxial tests on the other.

The corrected variations of undrained strength, pore pressure parameter $\bar{A}_f$ and effective principal stress ratio at maximum deviator stress for the IC-U tests are shown in figure 34, along with the axial strains at failure, which require no correction. The ratio of one half of the deviator stress at failure to the major principal stress during consolidation was found to be 0.33, slightly less than the value determined for the AC-U triaxial test representing an element of soil at point A, and more than the AC-U test representing

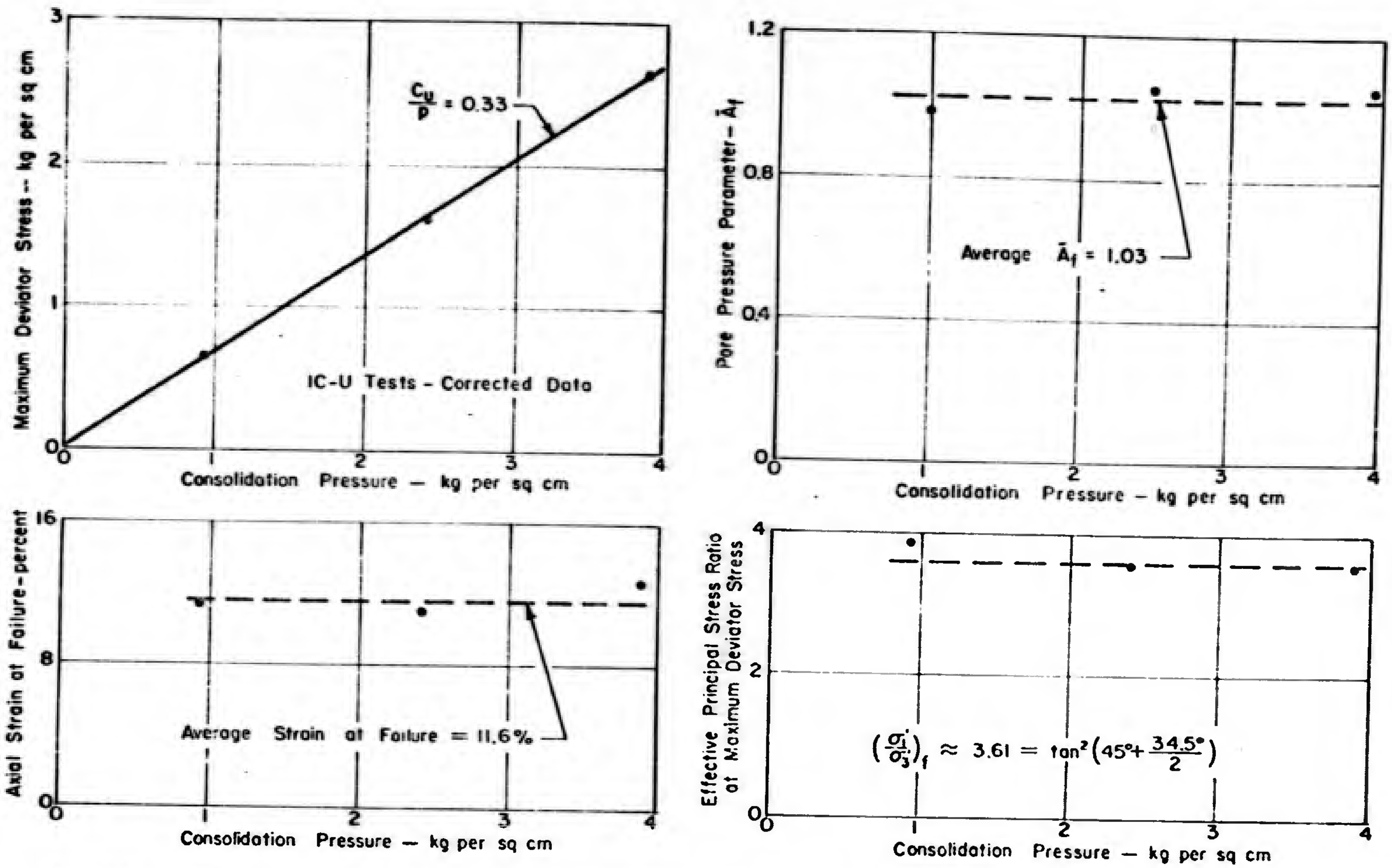


Fig. 34 — VARIATIONS OF MAXIMUM DEVIATOR STRESS, STRAIN AT FAILURE, PORE PRESSURE COEFFICIENT $\bar{A}_f$, AND EFFECTIVE PRINCIPAL STRESS RATIO AT FAILURE WITH CONSOLIDATION PRESSURE FOR IC-U TRIAXIAL TESTS ON UNDISTURBED SAN FRANCISCO BAY MUD.

an element of soil at point D. Other investigators (Bishop and Henkel, 1953) have found that the value of c_u/p determined using IC-U tests was much higher than that determined using AC-U tests; for undisturbed Bay Mud, however, the AC-U triaxial tests simulating point A and the IC-U triaxial tests give values of c_u/p which are for practical purposes the same. The average value of axial strain at failure in the IC-U tests was about the same as that for the AC-U test simulating point D, and the average value of the pore pressure parameter $\bar{A}_f$ for the IC-U test was slightly less than that for the AC-U test simulating point A. With respect to the measured values of strength, strain at failure and pore pressure parameter $\bar{A}_f$, the results of the IC-U triaxial tests are intermediate between those of the two types of AC-U triaxial test. Insofar as it is possible to ascertain, identical effective stress strength parameters apply to both the AC-U and the IC-U triaxial tests.

Conclusions

The difference in the values of the ratio c_u/p determined using the two types of AC-U triaxial test described in this section are qualitatively the same as the difference determined using the two types of plane strain test described in Section IV, but the difference in the values of c_u/p is greater for plane strain than for triaxial tests. Besides representing a less exact analogy to field conditions than the plane strain tests, the AC-U triaxial tests were found to require more time to perform.

The value of Φ' for San Francisco Bay Mud determined using the normal type of plane strain test (vertical plane strain, $\alpha = 60°$) is about 10 percent more than the value determined using IC-U triaxial tests. However, since the values of Φ' for plane strain conditions where the orientation of the failure plane is in the range $-30° \leq \alpha < 60°$ are probably less than the value for $\alpha = 60°$, the value of Φ' determined using triaxial tests may be near an average value for plane strain conditions considering the range of orientations of the failure plane in the field.

The value of the ratio c_u/p determined using IC-U triaxial tests is intermediate between the two values determined using the two types of AC-U triaxial tests described in this chapter, as are the strains at failure and the average value of the pore pressure parameter $\bar{A}_f$.

VII. AN EXAMPLE OF A STABILITY ANALYSIS USING ANISOTROPIC SHEAR STRENGTH

The purpose of making the stability analysis described below was to be able to compare the results of a "$\phi = 0$" analysis made using anisotropic strength with the ordinary "$\phi = 0$" analysis which assumes that clay is isotropic with respect to strength.

The Ordinary "$\phi = 0$" Method of Analysis

The ordinary "$\phi = 0$" analysis is made using the strength measured in unconfined compression or UU triaxial tests. The method is only applied to problems of short-term stability in saturated, intact clays, and the shear strength measured in the laboratory is presumed to be the same as the shear strength of the soil in the field before drainage can occur. For instance, if this method were used to analyze the stability of a slope in Bay Mud at the University of California field test site, the variation of strength with depth shown in figure 9 would be used in the analysis. Application of the method involves the implicit assumption that the clay is isotropic with respect to strength. The "strength" used in the analysis is $1/2\ (\sigma_1 - \sigma_3)_f$, rather than τ_{ff}, the shear stress on the failure plane at failure. If the method were completely rational, τ_{ff} would be used, because the shear stress acting on the rupture surface in situ is τ_{ff}.

The method of computation is shown in figure 35. The resisting moment, R.M., is the sum of individual resisting forces multiplied by the radius of the circular arc, and the overturning moment, O.M. is equal to the weight of the soil above the arc multiplied by the distance from the line of action of the weight force to the center of the arc. The factor of safety is the ratio R.M./O.M.. Several circles are tried until a minimum factor of safety is found.

In cases where actual failures have occurred, the factor of safety at the time of failure must by definition have been equal to one. Analyses of failures, and comparisons of the computed factors of safety with unity have provided the validation of the method for

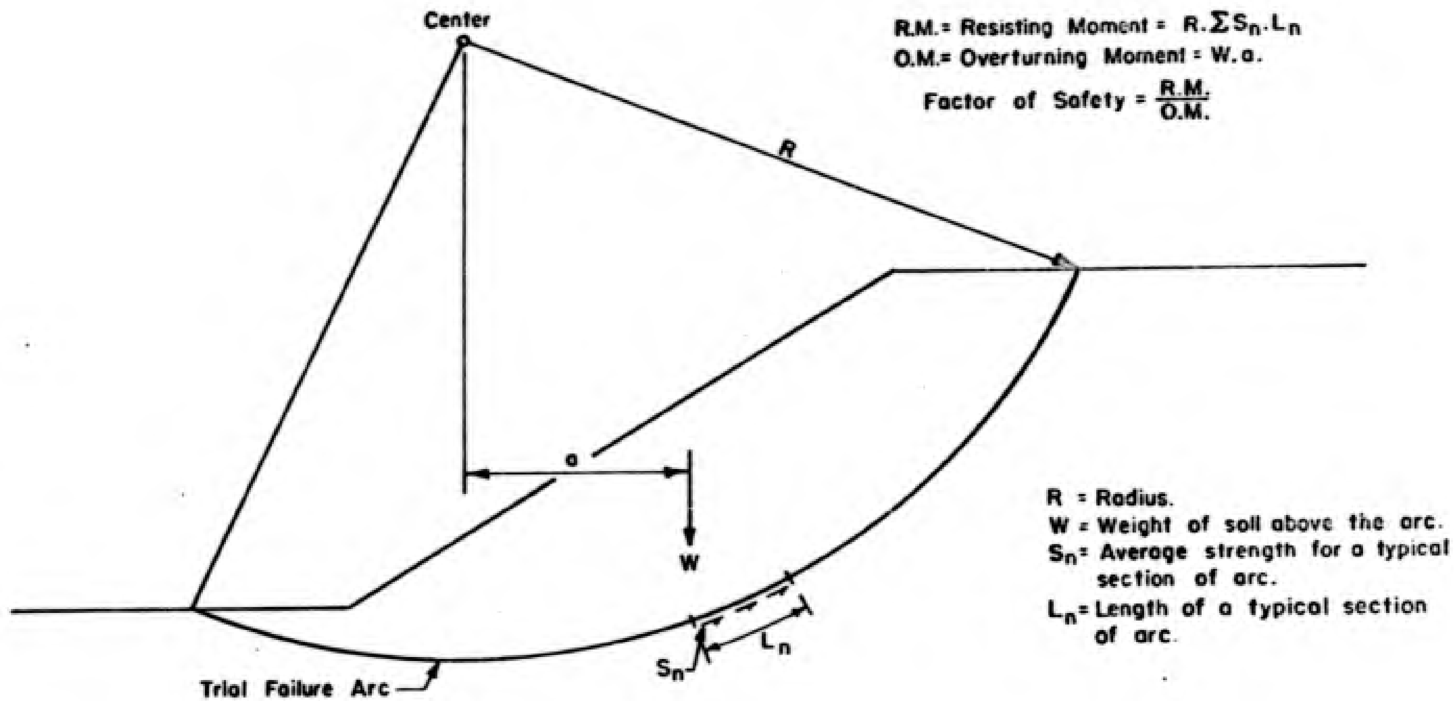


Fig. 35 - ILLUSTRATION OF THE "ϕ=0" METHOD OF STABILITY ANALYSIS.

design purposes (Skempton, 1948b). There are numerous examples of such analyses, many of which have recently been summarized by Bishop and Bjerrum (1960). Computed factors of safety have usually ranged from 0.90 to 1.10 for the section along the length of the failure which gives the minimum factor of safety, and somewhat higher for other sections through the failure. The critical circle (the one giving a minimum factor of safety) lies behind the actual one, and the computed factor of safety for the actual failure surface is usually in the range 1.3 to 1.5.

The "$\phi = 0$" method seems to have been adequately validated for design purposes, and will undoubtedly continue to be used because of its simplicity. The validation, however, is completely empirical, and consists solely of the fact that computed factors of safety are close to unity. Since the "strength" used in the analysis is $1/2\ (\sigma_1 - \sigma_3)_f$ rather than τ_{ff}, the ideally correct factor of safety where a failure has occurred should be $1/\cos\ \phi_e$ rather than unity. Cos ϕ_e is usually about 0.9, so factors of safety computed on the basis of τ_{ff} would be about 90 percent of those computed on the basis of $1/2\ (\sigma_1 - \sigma_3)_f$. However, all laboratory samples are probably disturbed to some degree during sampling, transporting and trimming, and this disturbance probably results in some loss of strength. The loss of strength due to disturbance lowers the computed factor of safety, whereas using the quantity $1/2\ (\sigma_1 - \sigma_3)_f$ in the analysis, rather than the shear strength, τ_{ff}, results in a higher computed factor of safety. Thus the use of the quantity $1/2\ (\sigma_1 - \sigma_3)_f$ in the analysis compensates in some measure for the reduction in strength of laboratory samples due to disturbance.

The basic reason for the discrepancy between the positions of the critical circle and the actual one has not been explained. Skempton (1945) showed that as ϕ increases, the critical circle moves closer to the surface of the slope; for $\phi = 0$ the critical surface is behind the actual one, and for $\phi = \phi'$ it is in front of the actual one. By inference it would appear that the two circles would coincide approximately if $\phi = \phi_e$ and $c = c_e$ were used in the analysis, but this has not been shown. So far as is known, no one has investigated the mechanism by which a change in the value of ϕ used in the analysis causes a change in position of the critical circle.

Since the ordinary "ϕ = 0" analysis is made using 1/2 $(\sigma_1 - \sigma_3)_f$ as the strength, and since there is a significant discrepancy between the position of the critical and the actual failure circles, even the fact that the computed minimum factors of safety are close to one appears to be somewhat anomalous.

The Analysis Using Anisotropic Strength

The most direct application of the results of the investigation of anisotropy and reorientation of principal stresses would be an analysis of a slope failure in San Francisco Bay Mud. Since such a failure was not immediately available, a failure described in the literature was chosen for analysis. The failure analyzed was that of the Congress Street open cut in Chicago (Ireland, 1954); the variation of strength in-situ with orientation of the failure plane was assumed to be shown in figure 36c. This is the same variation of strength with α as shown in figure 23, but plotted in a non-dimensional form. Although the actual variation of strength with α for the Chicago clay may be different, it seems logical that a valid assessment of the influence of the variation of strength shown in figure 36c on the results of the "ϕ = 0" analysis can be made, even if this variation is strictly hypothetical for the case analyzed. The advantages of choosing an actual failure for analysis are that the factor of safety and the position of the actual failure surface are known, and can be compared directly with the results of the ordinary "ϕ = 0" analysis and the analysis using anisotropic strength.

The failure of the Congress Street open cut occurred in 1952, immediately after construction. At the time of the failure, the cut slope had the shape shown in figure 36. The soil in the slope is described as gritty blue clay, has L.L. = 33, P.L. = 18, and is of glacial origin. The clay has apparently been preconsolidated by desiccation to about -10 feet, Chicago City Datum. The borings from which the undrained strength was determined were made eight years before the failure, and samples were obtained with 2-inch diameter

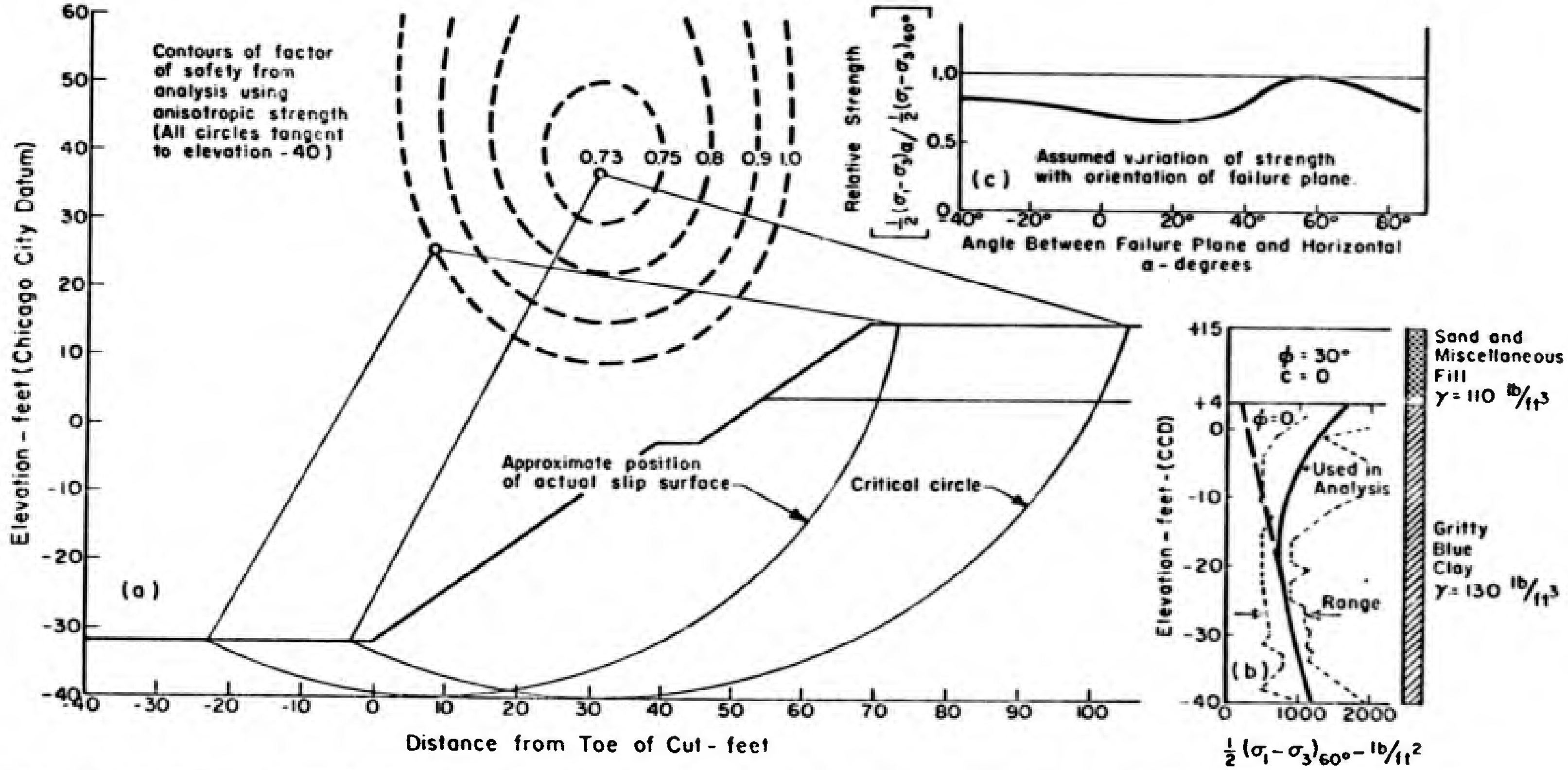


Fig.36–(a) RESULTS OF ANALYSIS OF THE STABILITY OF THE CONGRESS STREET OPEN CUT IN CHICAGO USING ANISOTROPIC STRENGTH, (b) VARIATION OF STRENGTH WITH DEPTH, AND (c) ASSUMED VARIATION OF STRENGTH WITH ORIENTATION OF FAILURE PLANE. SOIL PROPERTIES AND DIMENSIONS FROM IRELAND (1954).

Shelby Tubes. These samples were known to be somewhat disturbed; in order to correct for the effects of disturbance, the strengths of the Shelby tube samples were multiplied by 1.35, according to a correlation between the strengths of samples of different types established by Peck (1943). The maximum and minimum values of corrected strength (strength of Shelby tube samples multiplied by 1.35) are shown by the dashed lines in figure 36b. Ireland divided the clay into three separate layers, each with a constant strength over its thickness, but the continuous variation shown by the solid line has been used in the analysis described here.

This particular failure has been chosen for analysis because it is similar to the problem outlined in Section I and illustrated in figure 1. The only complicating factor is the eleven-foot-thick layer of sand overlying the clay, but the sand contributes so little to the resisting moment that the stability depends almost completely on the shear strength of the clay.

To compute the resisting moment, the trial failure arc was divided into 10-foot lengths, and the depth to the center of each length of arc was measured. Then $1/2\ (\sigma_1 - \sigma_3)_{60°}$ was taken from figure 36b and the relative strength from figure 36c; $1/2\ (\sigma_1 - \sigma_3)_\alpha$ was calculated by multiplying $1/2\ (\sigma_1 - \sigma_3)_{60°}$ by the relative strength. The value $1/2\ (\sigma_1 - \sigma_3)_\alpha$ was multiplied by the 10-foot length of arc to find the resisting forces, which were summed and multiplied by the radius of the arc to find the resisting moment.

To compute the overturning moment, the weights and positions of the centroids of the sand mass and the clay mass were determined separately, and the sum of the overturning moments of these two pieces was computed.

The factors of safety shown in figure 36a were computed assuming the shear strength to be τ_{ff} rather than $1/2\ (\sigma_1 - \sigma_3)_f$. Factors of safety based on τ_{ff} were computed by multiplying the factors of safety based on $1/2\ (\sigma_1 - \sigma_3)_f$ by $\cos \phi_e$, which was estimated to be about 0.9 from a correlation between P.I. and ϕ_e established by Gibson (1953).

Altogether, eight trial failure surfaces were used, and four factors of safety were computed for each one. These four factors of safety were computed using:

(1) Shear strength = τ_{ff} and varying with α as shown in figure 36c. The contours for this case are shown in figure 36a.

(2) Shear strength = $1/2\ (\sigma_1 - \sigma_3)$ and varying with α as shown in figure 36c.

(3) Shear strength = τ_{ff} and constant for all values of α, i.e., the clay was assumed to be isotropic with respect to strength.

(4) Shear strength = $1/2\ (\sigma_1 - \sigma_3)$ and constant for all values of α. This is the ordinary "$\phi = 0$" analysis.

Results

The factors of safety computed on the basis of these four assumptions are given in table 7 for the actual failure surface and the critical circle. The minimum factor of safety by the "$\phi = 0$" method of analysis (critical circle, method 4) is 1.03. Ireland (1954) interpreted the strength data slightly differently and found the minimum factor of safety by the "$\phi = 0$" method to be 1.08. Using either τ_{ff} rather than $1/2\ (\sigma_1 - \sigma_3)_f$ or anisotropic rather than isotropic strength results in a lower minimum factor of safety. The factors of safety for the actual failure surface are about 35 percent more than those for the critical circle for any of the methods of computation. The fact that method number 1 predicts a factor of safety close to unity for the actual failure surface does not appear to be significant, because any circle with its center on the same contour and tangent to elevation -40 feet CCD would have the same factor of safety.

The position of the critical circle was the same for all four methods of analysis and was the same one found by Ireland. The contours of factor of safety shown in figure 36a, for method number 1, have the same shape as the contours for any of the other three methods, the only difference being the magnitudes of the factors of safety.

Table 7. Factors of Safety for Stability Analyses of the Congress Street Open Cut in Chicago

Method of Computation		* No.	Factor of Safety for Critical Circle	Factor of Safety for Annual Circle
Anisotropic Strength	τ_{ff}	1	0.73	0.99
	$1/2\ (\sigma_1 - \sigma_3)$	2	0.81	1.10
Isotropic Strength	τ_{ff}	3	0.93	1.25
	$1/2\ (\sigma_1 - \sigma_3)$	4	1.03	1.39

*refers to numbered descriptions of analyses given previously.

The factors of safety using anisotropic strength are very nearly equal to the factors of safety using isotropic strength multiplied by the average relative strength between $\alpha = -30°$ and $\alpha = 60°$. The average relative strength in this range for the variation shown in figure 36c is 0.77, and the ratio of the factors of safety by method 1 to those by method 3, or of those by method 2 to those by method 4 is 0.79.

Conclusions

If the strength variation is the same as that found for Bay Mud, including anisotropy of strength in a "$\phi = 0$" analysis of stability appears to have no significant influence on the position of the critical circle. The minimum factor of safety with or without anisotropy is significantly less than the factor of safety computed for the actual failure surface, and the critical circle lies behind the actual one.

Application of the ordinary "$\phi = 0$" method of analysis assumes that the clay is isotropic with respect to undrained strength and that this strength can be measured by making UU triaxial tests on imperfectly sampled samples. These two assumptions are both incorrect, but the first tends to increase the computed factor of safety and the second tends to decrease it. The fact that these two assumptions compensate one another may be partly responsible for the agreement between actual factors of safety and those computed by the ordinary "$\phi = 0$" method of analysis. Ladd and Bailey (1964) have expressed a similar view in a discussion of Skempton and Sowa's (1964) conclusion that the "$\phi = 0$" analysis gives the correct answer because the strengths of "perfectly sampled" and "in-situ" samples are nearly the same. The bases for their statement were measurements of the strength of imperfectly sampled samples (Ladd and Lambe, 1963), and triaxial extension tests where the principal stresses rotated during undrained shear.

In order to illustrate the compensating effect of using the strength of disturbed samples and assuming that the undrained strength is the same for any orientation of the failure plane in-situ, it is convenient to define two ratios, a disturbance ratio, ρ_D, and an anisotropy ration, ρ_A. The disturbance ratio is defined by the equation

$$\rho_D = \frac{1/2\ (\sigma_1 - \sigma_3)_f \text{ for UU triaxial tests}}{\text{shear strength for vertical plane strain}} .$$

Besides disturbance, the value of ρ_D reflects the effect of strain conditions (triaxial as opposed to plane strain) on the undrained strength, and also the fact that the shear strength differs from the quantity $1/2\ (\sigma_1 - \sigma_3)_f$ by a factor of $\cos \Phi_e$. It may be possible that the disturbance ratio could be greater than unity if triaxial samples were tested which had undergone practically no strength loss due to disturbance, but in general this ratio will be less than one. For example, consider the results of the UU triaxial tests shown in figure 9; these tests were performed on samples of the highest quality by careful personnel, and the measured strengths are probably as nearly unaffected by disturbance as possible using standard techniques. The value of the ratio c_u/p for the data shown in figure 9 is approximately 0.32. The value of the ratio c_u/p determined using the plane strain tests described in Section IV is 0.37, and using $\Phi_e = 22°$ for San Francisco Bay Mud (Seed, Noorany and Smith 1964), the ratio τ_{ff}/p is found to be 0.34. Thus the disturbance ratio for the data shown in figure 9 is given by

$$\rho_D = \frac{c_u/p \text{ for UU triaxial tests}}{\tau_{ff}/p \text{ for vertical plane strain}} = \frac{0.32}{0.34} = 0.94.$$

The anisotropy ratio is defined by the equation

$$\rho_A = \frac{\text{average shear strength in the field}}{\text{shear strength in vertical plane strain}}$$

The value of ρ_A reflects the effect of the variation of undrained strength in-situ with orientation of the failure plane which results from anisotropy and reorientation of principal stresses. The stability analysis described in this chapter has shown that if the variation of undrained strength with orientation of the failure plane in-situ is as shown in figure 23, then the average shear strength around a circular failure arc would be about 80 percent of the shear strength in vertical plane strain, i.e.

$$\rho_A = 0.8$$

for the variation of undrained strength shown in figure 23.

The disturbance ratio, ρ_D, is a measure of the underestimate of strength which results from the assumption that the quantity 1/2 $(\sigma_1 - \sigma_3)$ determined using UU triaxial tests on vertical samples is the same as the shear strength in-situ, for the same orientation of the failure plane under plane strain conditions. The anisotropy ratio, ρ_A, is a measure of the overestimate of strength which results from the assumption that the shear strength in-situ is the same for any orientation of the failure plane. From the definitions of ρ_D and ρ_A it can be seen that

$$\frac{\rho_D}{\rho_A} = \frac{1/2\ (\sigma_1 - \sigma_3)_f \text{ for UU triaxial tests}}{\text{average shear strength in the field}} ,$$

and for the data shown in figure 9 for UU triaxial tests on Bay Mud, and the variation of shear strength in-situ shown in figure 23,

$$\frac{\rho_D}{\rho_A} = \frac{0.94}{0.80} = 1.17$$

The factor of safety computed using the "$\phi = 0$" method of analysis is proportional to the shear strength used in the analysis. Presumably, if a failure was analyzed, the factor of safety would be equal to one if the shear strength used in the analysis was equal to the average shear strength in the field. Thus the factor of safety computed using the "$\phi = 0$" method of analysis would be equal to ρ_D/ρ_A.

If the samples of Bay Mud had undergone more disturbance during sampling, transporting and trimming, ρ_D/ρ_A might be very nearly equal to one. The factor of safety computed using the strengths of the more disturbed samples would also be nearly equal to one. It seems likely that such compensating errors in testing and analysis may be responsible for the success of the ordinary "$\phi = 0$" method of analysis.

VIII. CONCLUSIONS

From the results of the investigation previously described, the following conclusions may be drawn:

1. The anisotropically consolidated clays tested in this investigation and in previous investigations were found to be anisotropic with respect to undrained strength. Unconsolidated undrained triaxial and direct shear tests performed on samples trimmed in different directions have shown that the undrained strength varies with the orientation of the failure plane. For both of the clays investigated in this study, artificially prepared overconsolidated kaolinite and undisturbed San Francisco Bay Mud, the relationship between the undrained strength and the orientation of the failure plane was qualitatively the same: Samples trimmed in the normal manner (with the axis of the sample in the direction in which the major principal stress acted during consolidation) were the strongest, and samples trimmed so that the failure plane approximately coincided with the plane on which the major principal stress acted during consolidation were the weakest. Studies made by other investigators have shown that the variation of undrained strength with orientation of the failure plane for some clays is different from the variation which is characteristic of both overconsolidated kaolinite and undisturbed Bay Mud.

2. For both overconsolidated kaolinite and undisturbed San Francisco Bay Mud, the major part of the difference between the undrained strengths of samples trimmed in different directions is due to anisotropy of the clay with respect to development of pore water pressures during the undrained tests: The change in pore pressure induced during the undrained loading is the same throughout a particular sample, and the same change in pore pressure applies to any plane through the sample, but the ratio of the change in pore pressure to the change in axial stress is different for samples trimmed in different directions. In effect, the clays tested are anisotropic with respect to the pore pressure parameter $\overline{A}_f$. Both overconsolidated kaolinite and undisturbed San Francisco Bay Mud are anisotropic with respect to the strength parameters in terms of effective stresses, but this type of anisotropy has a relatively small effect on the undrained strength.

Parallel orientation of clay particles is probably fundamentally responsible for the anisotropy of clay soil, both with respect to development of pore water pressure and with respect to the strength parameters in terms of effective stresses. Studies of particle orientation made by other investigators have shown that plate-shaped clay particles tend to become oriented with their flat surfaces parallel to the plane on which the major principal stress acts during consolidation. Other investigators have also found that anisotropy with respect to undrained strength is more pronounced in pure clays than in silty clays, and that the degree to which kaolinite is anisotropic with respect to undrained strength increases with increasing consolidation pressure when consolidated one-dimensionally from the liquid limit.

3. The relationship between undrained strength and orientation of the failure plane in-situ is affected by both anisotropy and reorientation of principal stresses. Two different types of anisotropically consolidated-undrained plane strain compression tests, which were devised to simulate the consolidation and subsequent failure of two different elements of soil in-situ, have shown that the ratios of the undrained strength to the major principal stress during consolidation would be significantly different at the upper and lower ends of a failure arc in Bay Mud. From consideration of the way in which the value of the pore pressure parameter $\bar{A}_f$ would be expected to vary with orientation of the failure plane in-situ, it has been possible to hypothesize the relationship between undrained strength and orientation of the failure plane in-situ for Bay Mud.

4. The relationship between undrained strength and orientation of the failure plane determined by means of unconsolidated, undrained triaxial tests on undisturbed Bay Mud is similar to the hypothesized relationship for in-situ conditions.

5. Plane strain tests in which the changes in stress during sampling were simulated in the laboratory have shown that the undrained strength measured in tests simulating in-situ conditions is practically the same as the undrained strength measured in tests simulating "perfect sampling" followed by an unconsolidated undrained test; the two strengths

were either identical, or practically so, for tests simulating conditions at both the lower and upper ends of a failure arc in the field.

6. Failure to account for the influence of disturbance on the strength measured in consolidated undrained tests on the one hand, and failure to account for the influence of anisotropy and reorientation of principal stresses on the in-situ strength on the other hand, have counteracting effects on the factor of safety computed by means of the "$\phi = 0$" analysis. It is possible that these two compensating factors, both of which are neglected in the ordinary "$\phi = 0$" method of analysis, may in part explain the fact that computed factors of safety are nearly equal to unity when failures in normally consolidated clays are analyzed.

7. Undisturbed San Francisco Bay Mud was found to be anisotropic with respect to strain at failure as well as undrained strength, i.e. the maximum deviator stress was reached at different amounts of axial strain depending on the direction in which the sample was trimmed. The strain at failure of a plane strain sample simulating an element of soil at the upper end of a failure arc in the field was only about one-third that of a plane strain sample simulating an element of soil at the lower end of a failure arc in the field. Similarly, the strain at failure in unconsolidated undrained tests performed on samples trimmed in the normal manner was only about one-half of that for samples trimmed with their axes perpendicular to the normal direction. The strains at failure were about the same for samples of overconsolidated kaolinite trimmed in any direction.

8. The value of the strength parameter, ϕ', measured in the normal type of anisotropically consolidated-undrained plane strain compression tests on undisturbed Bay Mud was approximately ten percent more than the value of ϕ' measured in isotropically consolidated-undrained triaxial tests on undisturbed Bay Mud. However, in plane strain tests where the failure plane had a different orientation with respect to the plane on which the major principal stress acted during consolidation, the measured value of ϕ' was only about 1.5 percent more than the value of ϕ' measured in triaxial tests. Since the value of ϕ' measured in

plane strain tests depends on the orientation of the failure plane, it is possible that for some orientations of the failure plane in the field, the value of ϕ' which can be mobilized would be slightly less than the value measured in isotropically consolidated-undrained triaxial tests on vertical samples.

9. In consolidated-undrained tests on soft clays, failure to correct the axial load in the sample for the loads carried by filter paper drains, rubber membranes, and cell piston friction will lead to an overestimate of the maximum deviator stress in either triaxial or plane strain tests; the overestimate of maximum deviator stress may amount to more than 25 percent of the uncorrected value. In plane strain tests it is also necessary to correct for friction between the membrane surrounding the sample and the end plates used to make the sample deform in plane strain, but this correction is not of major importance. The necessary corrections to axial stress decrease as a percentage of the measured strength as the consolidation pressure and the strength increase; the necessary corrections to the axial stress were about ten percent of the maximum deviator stress in consolidated undrained tests on Bay Mud where the consolidation pressure was equal to four kilograms per square centimeter.

10. Attempts to duplicate the in-situ conditions of two elements of soil at the upper and lower ends of a failure arc in the field using two types of anisotropically consolidated-undrained triaxial compression tests have shown that the results are qualitatively the same as the results of plane strain tests where a more exact analogy exists between the laboratory samples and the elements of soil in-situ, but quantitatively the results of the triaxial tests were somewhat different from the results of the plane strain tests. In addition to the fact that the results of the triaxial tests are somewhat less directly applicable to in-situ conditions, the triaxial tests required more time to perform than the plane strain tests.

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APPENDIX A

APPARATUS AND TECHNIQUE FOR PLANE STRAIN TESTING

Purpose of the Tests

The purpose of the plane strain tests was to measure the strengths which could be mobilized at the upper and lower ends of a sliding surface in the field. The tests were made undrained in order to simulate undrained failure in the field. As explained in Section I, the principal stresses do not change direction between the end of consolidation and failure at the upper end of the sliding surface (point A in figure 1). At the lower end of the sliding surface (point D in figure 1) the major principal stress at failure is perpendicular to the direction which it had at the end of consolidation.

The possible methods of conducting a test where the major and minor principal stress directions are interchanged between the end of consolidation and failure are summarized in table 8. Schemes 1 and 3 were rejected in order to avoid the difficulties of interpretation associated with extension tests; these difficulties have been discussed by Roscoe, Schofield and Thurairajah (1963) and by Sowa (1963). Scheme 2 has the disadvantages that the strains during consolidation are not one-dimensional, and that the stress condition at the end of consolidation ($\sigma_1 = \sigma_2 > \sigma_3$) is never found in normally consolidated clays in the field. A single test of this type was performed and was found to give results different from the results of tests performed according to scheme 4. After considering these factors it was decided to adopt scheme 4 (called HPS or horizontal plane strain) to duplicate the conditions at the lower end of the failure arc. The same apparatus designed for these HPS tests was used to simulate conditions at the upper end of the failure arc (this is called VPS or vertical plane strain); in the VPS tests the major principal stress acts in the axial direction both during consolidation and at failure.

Table 8. Methods of Causing a 90° Reorientation of The Major Principal Stress During an Undrained Test

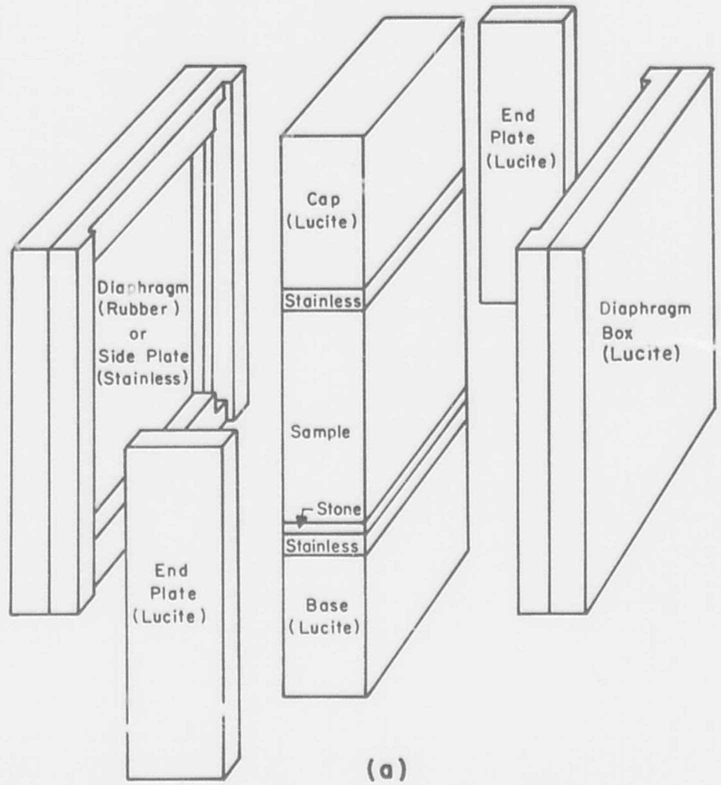
Type of Device	Scheme No.	Direction of Major Principal Stress	
		During Consolidation	At Failure
Triaxial	1	Axial	Radial
	2	Radial	Axial
Plane Strain	3	Axial	Lateral
	4	Lateral	Axial

Apparatus

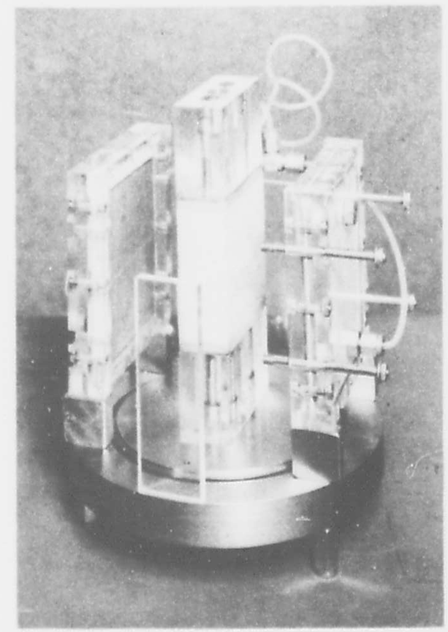
"Frictionless" End Plates

The essential feature of the plane strain device is the pair of plates used to force the sample to undergo plane strain deformation during consolidation and testing. Reducing frictional drag between these plates and the ends of the sample is the principal problem in the design of a satisfactory device. (The ends of the sample are those narrow vertical faces such as the one where the word "sample" is written in figure 37; the other two faces are called the sides and the horizontal surfaces are called the top and bottom). Fortunately, at the time that this investigation was begun, the problem of reducing friction between the end plates and the sample had already been solved. A device had been used at Imperial College by Wood (1958), Cornforth (1964) and Wade (1963) which employed polished end plates and a layer of silicone grease between sample and end plates to reduce the friction. A simple device was designed and used at the University of California (Smith, 1963) which had polished lucite end plates held by tie rods and was small enough to fit inside a triaxial pressure cell. Friction between the end plates and the membrane on the ends of the sample was essentially eliminated by coating the membrane with silicone grease before assembling the end plates.

The end plates of the plane strain device used in this investigation (shown in figures 37 and 38) were made of polished lucite and were



(a)



(b)

Fig.37 – (a) PERSPECTIVE DRAWING OF THE BASIC COMPONENTS OF THE PLANE STRAIN APPARATUS, AND (b) PHOTOGRAPH OF THE COMPONENTS WITH THE BASE OF THE PRESSURE CELL.

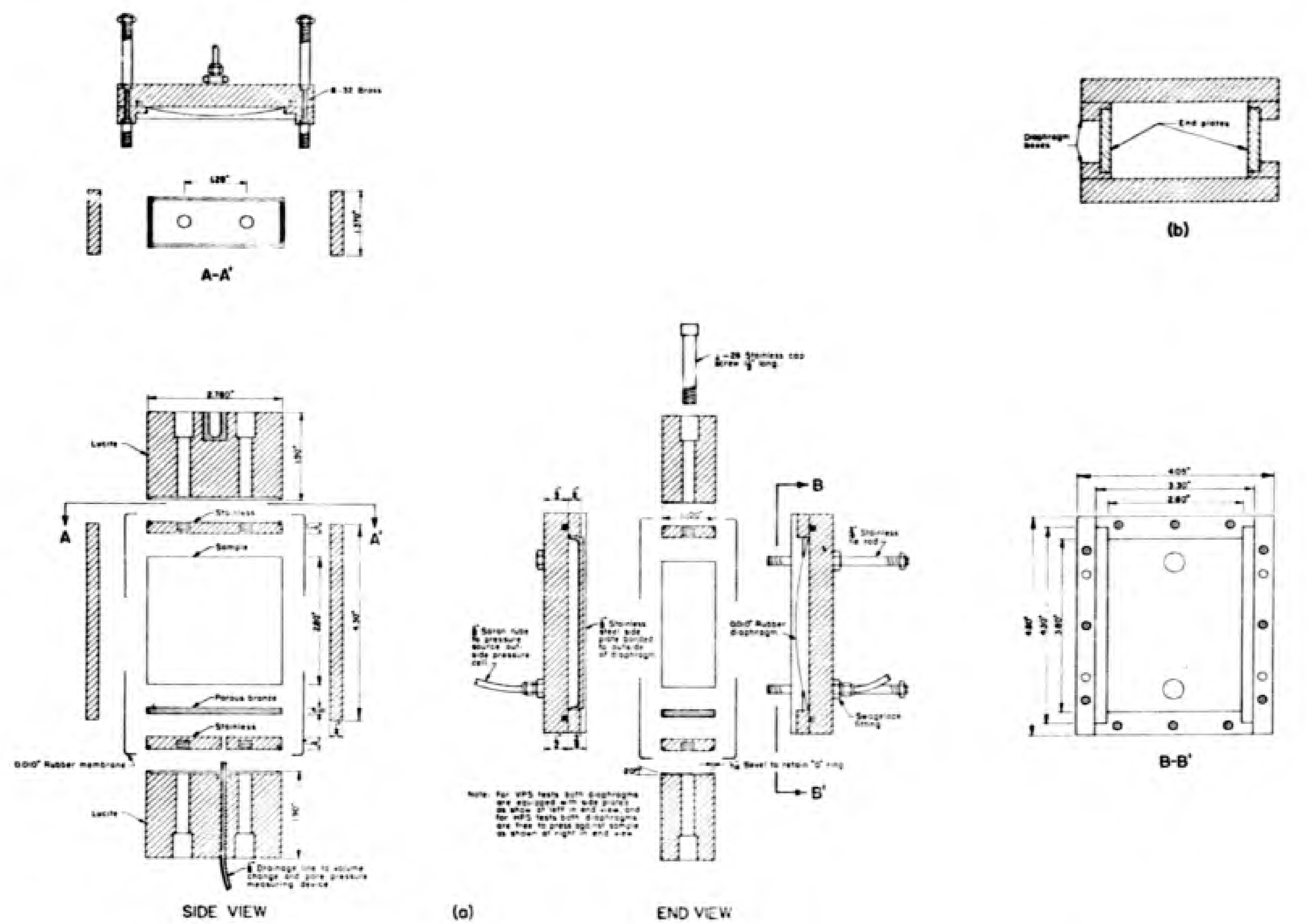


Fig.38-(a) EXPLODED VIEWS OF THE PLANE STRAIN APPARATUS, AND (b) SIMPLIFIED CROSS-SECTION OF THE DIAPHRAGM BOXES AND END PLATES ASSEMBLED.

held in place by the diaphragm boxes which are described below. In order to reduce friction between the end plates and the ends of the sample, the membrane was coated with silicone grease before the apparatus was assembled. Experimental and analytical studies have shown that the use of silicone grease is an effective means of reducing frictional drag between the sample and the end plates; the frictional resistance to relative movement between the end plates and the sample in these plane strain tests is typically about one percent of the axial load in the sample when the sample fails.

Diaphragms and Side Plates

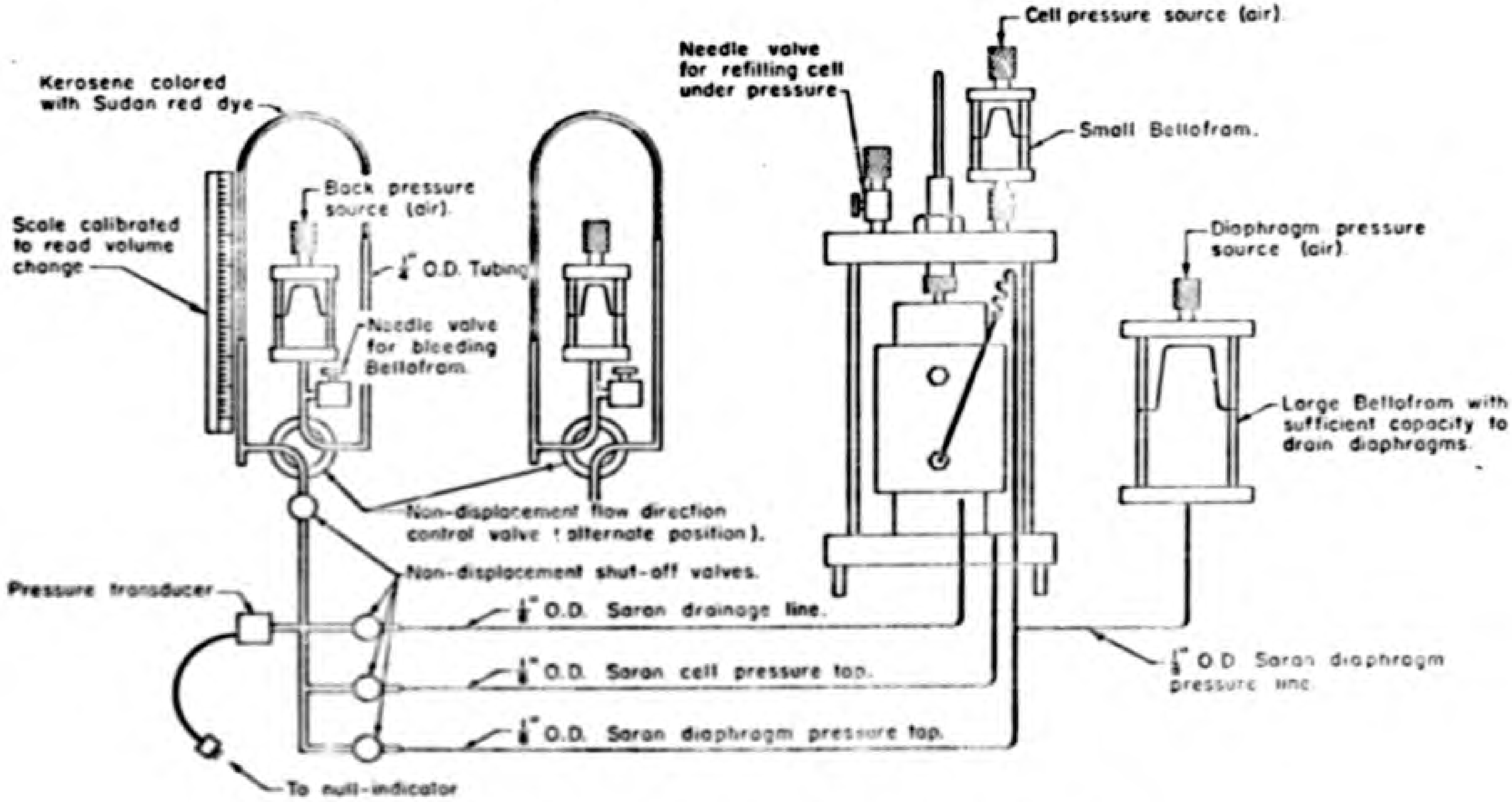
Scheme 4 in table 8 requires that the major principal stress act in the lateral direction (on the sides of the sample) during consolidation. Merely making the cell pressure higher than the axial stress will result in radial, rather than one-dimensional, consolidation of the sample no matter what shape of sample is used. This is because the cell pressure would be greater than the stress required to prevent strain normal to the ends. Some technique for applying a pressure to the sides of the sample which is higher than the cell pressure was required. It was decided to use 0.010-inch thick rubber diaphragms pressing against the sides of the sample to apply this pressure. These diaphragms and the diaphragm boxes on which they were mounted are shown in figures 37 and 38. In order to prevent the diaphragms from squeezing out through the opening between the diaphragm boxes and the cap or base, the clearance between the diaphragm boxes and the cap (or base) was made 0.010" on each side. Pressures in these diaphragms were made as much as 2.4 kg/cm^2 higher than the cell pressure. There was no tendency for the diaphragms to squeeze in between the sample and end plate or out of the opening between the diaphragm boxes and the cap or base. Ten tests each lasting a week or so were made using the same rubber diaphragms, but they never leaked or broke.

An attempt was made to consolidate the first VPS samples using the diaphragms to prevent lateral deformation of the sample, by maintaining a constant volume of water in the diaphragm boxes. This technique was unsuccessful. When the axial load was applied, the sample deformed laterally. Use of smaller load increments to prevent the deformation would have considerably lengthened the required consolidation period, which would have the disadvantage of making consolidation times for HPS and VPS samples quite different, as well as increasing the total time required for testing.

To solve this problem it was decided to bond one-eighth inch thick stainless steel plates (called side plates) to the outside of the rubber diaphragm for use in the VPS tests. These side plates are shown in figure 38(a). The bond was made using water-proof double-backed pressure-sensitive tape. At the beginning of the consolidation increment, these plates were pushed against the sample so that they maintained the cross-sectional area of the sample the same as that of the cap and base. The pressure used to hold the plates in this position was about the minimum which would prevent bulging of the sample. When consolidation was essentially complete (after 200 to 300 minutes), the diaphragm pressure was reduced to less than the cell pressure, and the plates came away from the sample, leaving it standing free. This change was usually accompanied by additional consolidation because the load which had been carried by friction between samples and side plates was now transferred to the sample. Some deformation also occurred, but the sample was strong enough so that the amount was relatively unimportant. At the beginning of the next consolidation load increment the diaphragm pressure was increased so that the side plates moved back against the sample, and the whole process was repeated.

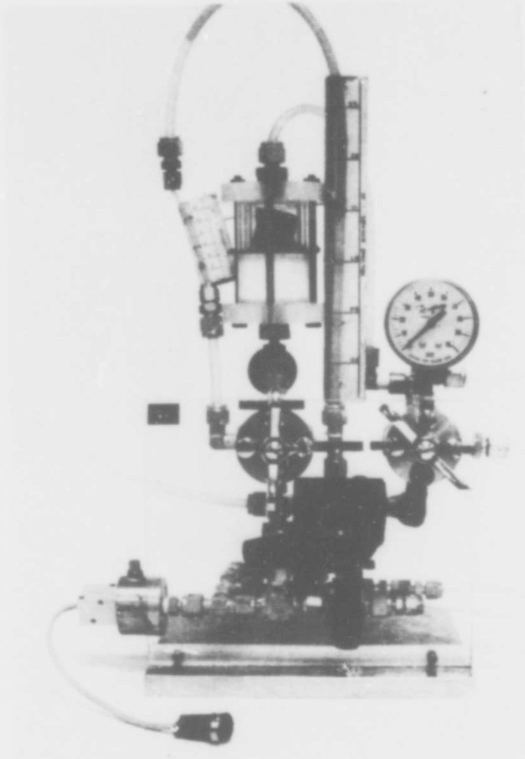
Volume Change and Pressure Measurements

Volume changes during consolidation were measured in order to be able to calculate the cross-sectional area of the samples at the end of consolidation. A one-eighth inch outside diameter saran tubing drainage line from the base of the sample led to a volume change and pressure measuring device which had a calibrated tube for measuring volume change and an unbonded strain gage transducer for measuring pressures. The devide used for measuring volume changes and pressures is shown schematically in figure 39, and photographs of the device are shown in figure 40.

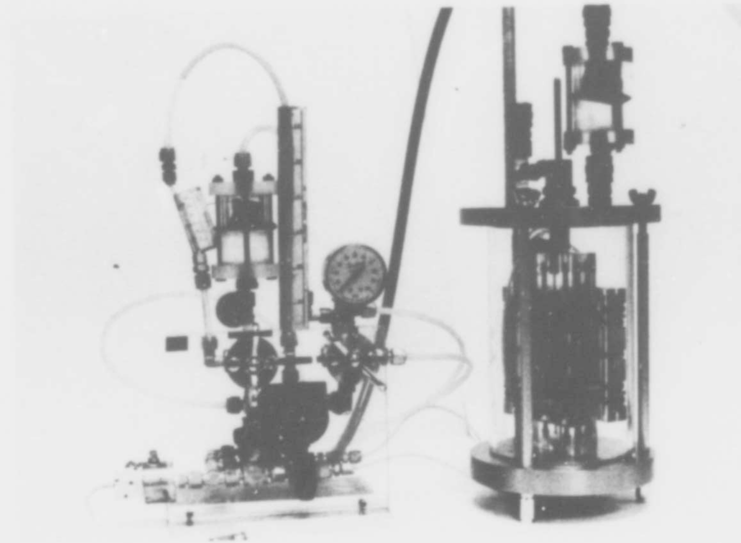


Note: Direction of movement of kerosene-water interface may be reversed by rotating the plug of the flow direction control valve. Position pictured at left will result in upward movement as the sample drains, and position pictured at right will result in downward movement.

Fig.39 – SCHEMATIC DIAGRAM OF VOLUME CHANGE AND PRESSURE MEASURING DEVICE.



(a)



(b)

Fig.40 - PHOTOGRAPHS OF (a) THE VOLUME CHANGE AND PRESSURE MEASURING DEVICE, AND (b) THE PLANE STRAIN APPARATUS ASSEMBLED WITH THE VOLUME CHANGE AND PRESSURE MEASURING DEVICE.

Volume changes were measured by noting the movement of the colored kerosene-water interface in the calibrated tubing. Volumes were recorded to 1/100 cubic centimeter. A back pressure was used to achieve saturation; this pressure was applied by regulating the air pressure acting on bellofram seal which was used to prevent diffusion of air through the drainage line. Similar seals were used at the air-water interfaces where the cell pressures and diaphragm pressures were applied.

All pressures were measured on the water-side of the bellofram seals using the pressure transducer. Tubing from the sample base, the pressure cell and the two interconnected diaphragm boxes led to the volume change and pressures measuring device as shown in figure 40b; by manipulating the non-displacement valves, any of the pressures (back pressure, cell pressure or diaphragm pressure) could be read on the transducer. By means of a simple selector valve for switching the pressures acting on the back pressure bellofram, atmospheric pressure could also be read on the transducer. (This air pressure selector valve is not shown in figure 39). Because the calibration constant of the transducer was about 750 micro-inches per kilogram per square centimeter, and the error in reading should not be more than 3 or 4 micro-inches, the pressures could be measured to within about 0.005 kg/cm^2. An additional advantage of this system is that the effects of electrical zero shift and long-cycle regulator fluctuation are automatically eliminated, because the effective stress is simply the difference in two readings on the same sensitive instrument.

Loading Apparatus

NPS Samples. During consolidation the NPS samples were "loaded" by the diaphragms. The height of the sample was maintained constant during consolidation by means of a small hand-operated screw jack pushing down against the bottom cross-bar of the loading yoke as shown in figure 41 and the length of the sample was maintained constant by the end plates. When consolidation was complete and the drainage valve was closed, the undrained loading part of the test was performed by reducing the diaphragm pressure until it became slightly less than the

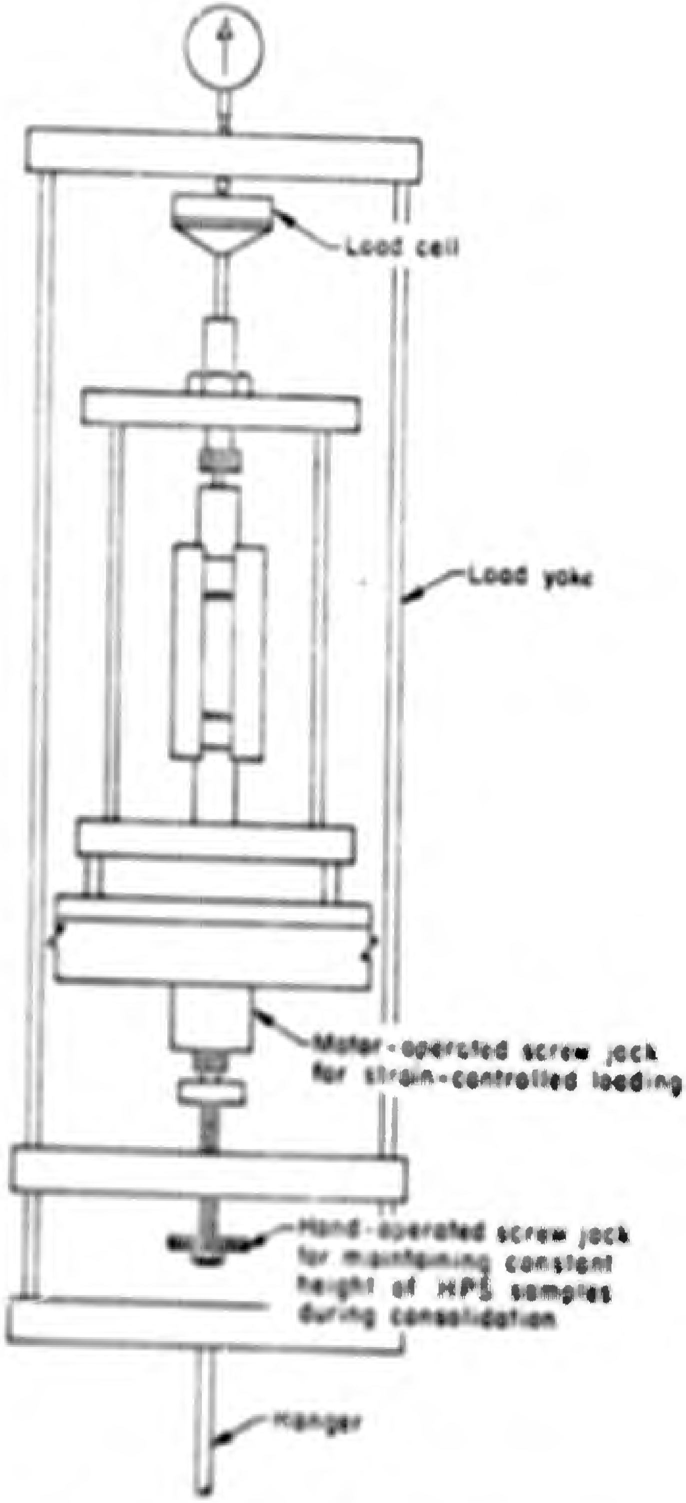


Fig. 41– SCHEMATIC DIAGRAM OF THE LOADING APPARATUS.

cell pressure so that the diaphragms pulled away from the sample. At this stage the sample was acted on by an isotropic stress. Then loads were added to the hanger on the loading yoke so that the sample failed in compression by controlled stress.

<u>VPS Samples</u>. During consolidation the VPS samples were loaded by dead weights which were applied to the hanger shown in figure 41. At the end of consolidation the drainage valve was closed and additional weights placed on the hanger so that the sample failed in compression by controlled stress. One special test was made in which stress control (dead weight) was used up to the peak stress and strain control beyond that point in order to determine the rate of decrease of deviator stress past the peak point. This was accomplished by transferring from dead weight loading to screw jack loading just before peak stress was reached. The screw jack, shown schematically in figure 41, had a variable speed D.C. motor which could be adjusted to the same rate of strain the sample had when transfer was made.

Technique

Preparation of Samples

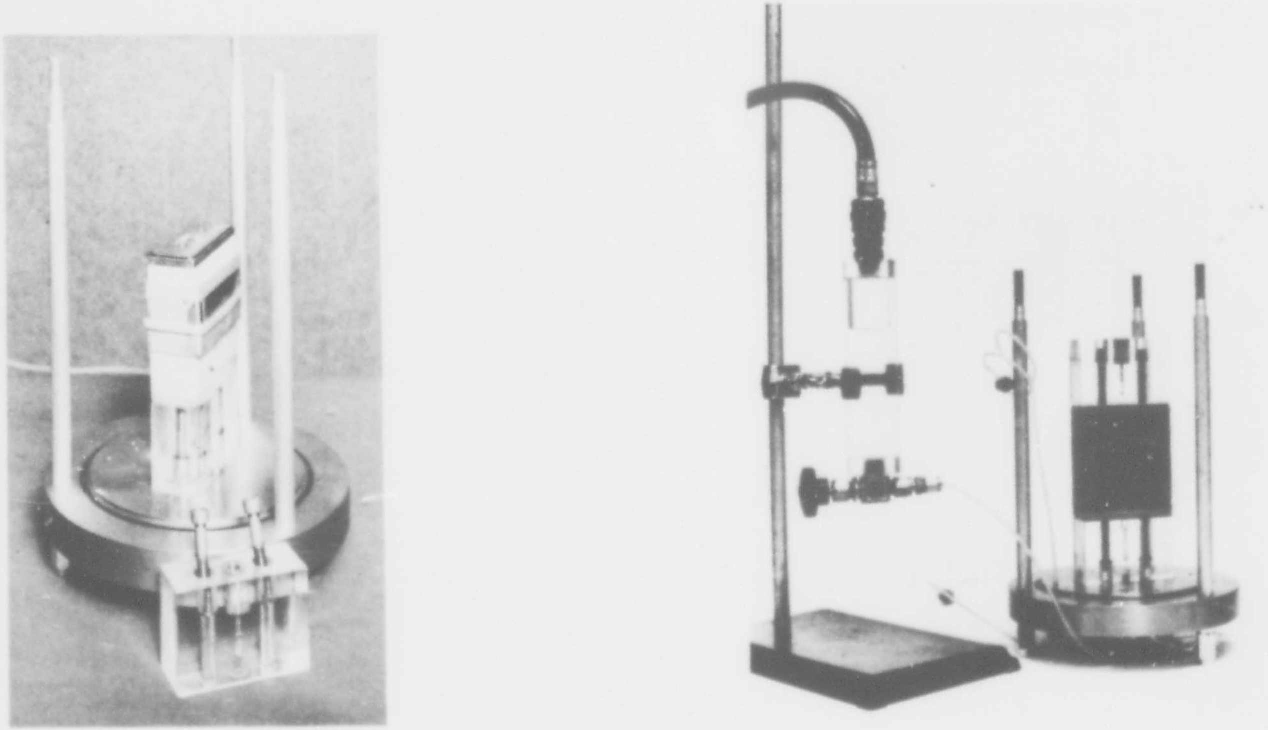
Samples were extruded from the 5-inch diameter thin-walled fixed-piston sample tubes in which they had been stored, and cut to 2.80 by 2.80 by 1.10 inches using a specially constructed lucite mitre box. VPS samples were trimmed with their axes vertical, and HPS samples with their axes horizontal. Three VPS samples could be trimmed from the extruded piece; each of the extra two samples was placed inside three evacuated plastic bags and stored in a plastic container in the wet room until needed. Only one HPS sample was trimmed at a time. Water contents were taken from the larger pieces trimmed from the samples.

Assembly of Apparatus and Sample

A 2.0 inch diameter membrane was cut to a length of 3.8 inches and rolled up on a short section of 1.3 by 3.0 inch rectangular tubing. One end of the membrane was clamped between the stainless steel and lucite parts of the base. (See figure 37 for the parts of the base and figure 42a for a photograph of the membrane being put on the sample). The base was put on the bottom of the pressure cell with the drainage line connected to a one-inch outside diameter, four-inch long lucite reservoir fitted with a needle valve and a vacuum line. The porous stone was boiled and the filter paper was soaked in deaired water; both were blotted dry and assembled on the sample which was then set on the base inside the rectangular tubing. The membrane, which was smaller in perimeter than the sample (6.3 inches as opposed to 7.8 inches), was put on the sample by alternately rolling it up and sliding it off the lower end of the rectangular tubing. By this technique the membrane could be got into place without deforming the sample. The 1/4 inch thick stainless steel portion of the cap was set on top of the sample when the membrane was nearly completely in place as shown in figure 42a. Then water was poured inside the membrane to flush out air bubbles. The rectangular tubing was then pulled free of the membrane, which laid in across the top of the stainless steel part of the cap. The lucite part of the cap was set in position and bolted to the stainless steel part, clamping and sealing the membrane in between. As soon as the seal was made the needle valve was opened and all excess water pulled out the drainage line into the reservoir by applying a 0.3 atmosphere vacuum. This stage of the procedure is shown in figure 42b. The sample was allowed to remain under this vacuum while the rest of the equipment was assembled.

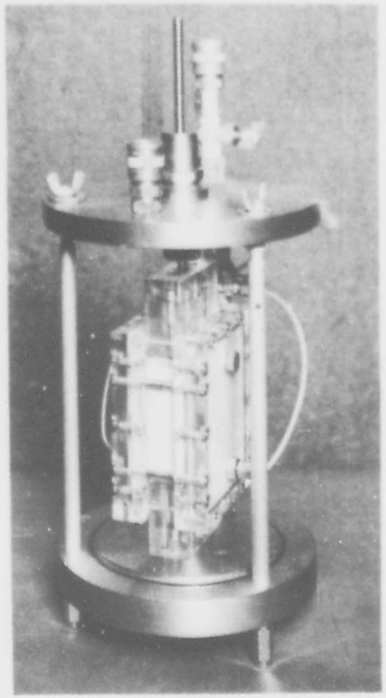
The membrane was sprayed with a friction-reducing spray (Fluoro-Glide, manufactured by Chem-Plast, Inc.) and coated liberally with silicone grease. Then the diaphragm boxes and end plates were assembled around the sample and the tie rods were tightened. A pressure of 0.1 kg/cm^2 was applied to the diaphragms to make them lie in against the sample, the lower portion of the cap, and the upper portion of the base. The rest of the cell was assembled and filled with deaired water. Figure 43 shows a photograph of the cell completely assembled, except for the lucite cylinder.

The cell was placed in position in the loading as shown in figure 43b and the pressure lines and dial gage were put in place. A cell pressure

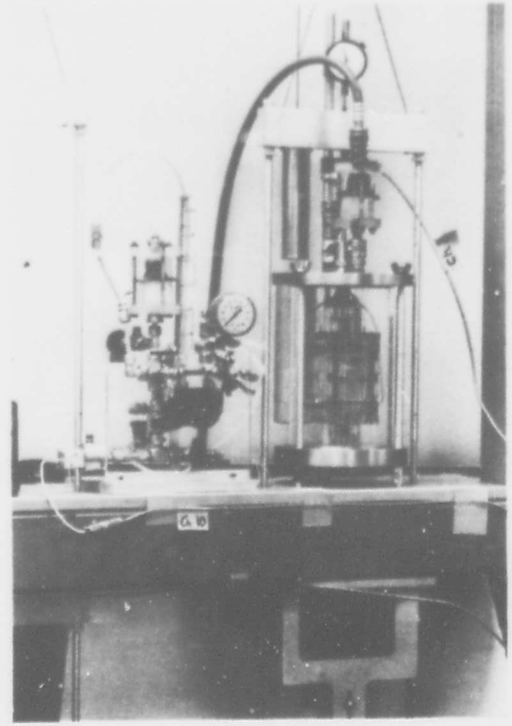


(a) (b)

Fig.42 – PHOTOGRAPHS SHOWING (a) THE TECHNIQUE FOR PUTTING THE MEMBRANE ON THE PLANE STRAIN SAMPLE, AND (b) THE SAMPLE ASSEMBLED WITH THE MEMBRANE IN PLACE AND THE RESERVOIR USED FOR WITHDRAWING EXCESS WATER.



(a)



(b)

Fig.43 – PHOTOGRAPHS SHOWING (a) THE PLANE STRAIN APPARATUS ASSEMBLED IN THE CELL EXCEPT FOR THE LUCITE BARREL, AND (b) THE APPARATUS ASSEMBLED IN THE LOADING FRAME.

of 0.4 kg/cm^2 was applied, and the drainage line was removed from the reservoir. With water dripping from both ends, the drainage line was connected to the volume change and pressure measuring device. Then all pressures (including the back pressure) were adjusted to the values desired for the first consolidation increment, and the hanger load was put in place.

Testing

HPS Samples. As mentioned previously, the height of the HPS samples was maintained constant during consolidation by adjusting a small screw-jack which pushed down against the bottom crossbar of the loading yoke as shown in figure 41. The end plates prevented change in length of the specimen. The load required to prevent a change in height could be measured on a load cell on top of the pressure cell piston as shown in figures 18 and 41.

During consolidation a record was kept of volume change with time. The major principal stress was applied to the sample by the diaphragms, and the cell pressure was adjusted so that it was equal to the sum of (a) k_o multiplied by the major principal effective stress and (b) the back pressure.

The samples were allowed to consolidate under each load increment for one day before the next was applied, and for two days under the final load increment. A load increment ratio of one was used, the schedule of loading being 0.4, 0.8 or 0.4, 0.8, 1.6 or 0.6, 1.2, 2.4 or 0.4, 0.8, 1.6, 3.2 or 0.5, 1.0, 2.0, 4.0 kg/cm^2. After the sample had consolidated for one day under the first load increment, the value of the pore pressure parameter B was checked by increasing both the cell pressure and the diaphragm pressure by about 0.53 kg/cm^2 (so that the change in reading on the null-indicator which was connected to the pressure transducer was 400 micro-inches); the ratio of the increase in pore pressure induced by the change in all-around pressure to the change in the value of the all-around pressure is, by definition, equal to the pore pressure parameter B. Determination that the value of B is equal to unity is indicative of the fact that a sample is completely saturated. No difficulty was found in achieving complete saturation with a back pressure of 1.5 kg/cm^2.

After the second day of consolidation under the last load increment, drainage was stopped and the undrained shear test was performed. The first phase consisted of reducing the diaphragm pressure to the cell pressure in one hour, using four increments, and measuring axial deformation, pore pressure changes and the change in axial load in the sample. The second phase consisted of increasing the hanger load in appropriate increments at 15 minute intervals so that failure occurred in about 5 hours and measuring axial deformation and pore pressure changes. During both phases of the test, the end plates prevented change in length of the sample. Calculations show that the degree of equalization of non-uniform pore pressures within the samples at failure was between 97 and 99 percent for all samples (Bishop and Henkel, 1962). The strain and pore pressure measurements were made after the new load had been in place for 14 minutes.

After failure the cell pressure was checked and the apparatus disassembled as quickly as possible. The sample was first sketched and then cut into 3 pieces (bottom, failure plane, and top) and the water content of each part was measured separately. The measurements indicate that the samples drew water out of the stone during disassembly.

VPS Samples. The width of the VPS samples was maintained constant during consolidation by keeping sufficient pressure behind the side plates to prevent the samples from bulging, and the length was maintained constant by the end plates. After 200 to 300 minutes consolidation, the side plates were pulled away from the samples by reducing the diaphragm pressure. The load increment ratio, loading schedules, and consolidation times were exactly the same as for the HPS samples. The value of the pore pressure parameter B was checked after the first day of consolidation, and a record of changes in volume and height with time was kept.

After the second day of consolidation under the final load increment, drainage was stopped and the undrained shear test performed. This was accomplished by increasing the hanger load in appropriate increments just as for the HPS tests and measuring axial deformation and pore pressure changes. The end plates constrained the sample to deform in plane strain. Failure occurred in 4 1/2 to 6 3/4 hours; this corresponds to degrees of equalization of non-uniform pore pressures at failure of 97 to 99 percent.

Disassembly and water content measurements were the same as for HPS samples.