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DESIGN OF MAXIMUM THRUST PLUG NOZZLES WITH VARIABLE INLET GEOMETRY

VOLUME II. COMPUTER PROGRAM MANUAL

Gearold R. Johnson, H. Doyle Thompson, and Joe D. Hoffman

Jet Propulsion Center
Purdue University
Lafayette, Indiana 47907

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The problem of designing two-dimensional or axisymmetric plug nozzles with variable inlet geometry which produce the maximum axial thrust when an engineering constraint is imposed on the plug contour, for example, fixed length, has been formulated and numerically solved. The formulation was written to consider a gas mixture whose composition is either fixed or in chemical equilibrium. The effects of the boundary layer thickness and the wall shear stress are included in the problem formulation. The optimization analysis and the results from selected parametric studies are presented in Volume I of this report. This volume, Volume II, contains a description of the computer program, a discussion of the input parameters, and the results of six sample cases.

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FOREWORD

The present study is part of the program "An Analytical Study of the Exhaust Expansion System (Scramjet Scientific Technology)" being conducted by the Jet Propulsion Center, Purdue University, Lafayette, Indiana, under United States Air Force Contract No. F33615-67-C-1068, Project 3012, Task 301209, BPSN 7(63 301209 6205214). The Air Force program monitor was Capt. Gary J. Jungwirth of the Air Force Aero Propulsion Laboratory (AFAPL/RJT). Volume I of this report presents the formulation, numerical solution procedure and the results of selected parametric studies of the design of maximum thrust plug nozzles with variable inlet geometry. Volume II is the computer program manual.

This report was submitted by the authors on 15 October 1971.

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Gary J. Jungwirth
Captain, USAF
Project Engineer
Ramjet Technology Branch
Ramjet Engine Division
AF Aero Propulsion Laboratory

ABSTRACT

The problem of designing two-dimensional or axisymmetric plug nozzles with variable inlet geometry which produce the maximum axial thrust when an engineering constraint is imposed on the plug contour, for example, fixed length, has been formulated and numerically solved. The formulation was written to consider a gas mixture whose composition is either fixed or in chemical equilibrium. The effects of the boundary layer thickness and the wall shear stress are included in the problem formulation. The optimization analysis and the results from selected parametric studies are presented in Volume I of this report. This volume, Volume II, contains a description of the computer program, a discussion of the input parameters, and the results of six sample cases.

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SECTION I

INTRODUCTION

A computer program has been developed that can design maximum thrust plug nozzles with variable inlet geometry. The optimization analysis which is the basis for the computer program is developed in Volume I of this report. This volume (Volume II) contains a description of the computer program, a discussion of the input data pack and a discussion of six sample cases.

The program is written in FØRTRAN EXTENDED language for the CDC-6500 and CDC-6600 computers. The program can be modified to be compatible with other computing machines. The sample cases presented in this manual were executed on the CDC-6500 computer. Execution times are given for this machine.

SECTION II

PROGRAM ORGANIZATION

1. GENERAL

The program consists of a main program and 22 subroutines which are used to perform the three primary tasks of: 1) data input and initialization, 2) calculation of the flow field in a plug nozzle, and 3) calculation of the Lagrange multipliers and the relaxation of the nozzle geometry. The program is overlayed and the primary tasks are performed in sequence. The overlay arrangement is presented in Fig. 1.

A wide variety of options are available to the user. The possible combinations permit the following:

A. Flow field analysis for a specified plug nozzle geometry which includes the capability of inputing a tabular contour.

B. The design of an optimal plug nozzle for a specified ambient pressure (when the ambient pressure is specified, the cowl lip radius, the orientation of the initial-value line, and the plug contour are determined from the optimization analysis).

C. The inverse of option (B). For this case the user specifies the cowl lip radius, and the ambient pressure, the orientation of the initial-value line and the plug contour are obtained from the optimization analysis.

D. The design of an optimal plug nozzle with a specified initial-value line and cowl lip radius (referred to as the fixed inlet option).

The second and third options are referred to as variable inlet options.

In addition, the nozzle geometry can be either planar (i.e., two-dimensional), or axisymmetric. The possible engineering design constraints that can be imposed upon the optimization analysis are: 1) fixed nozzle length, 2) constant plug surface area, 3) constant plug contour arc length, and 4) an arbitrarily weighted linear combination of 1), 2), and 3).

The program can treat either a calorically and thermally perfect gas, or variable thermodynamic properties and stagnation conditions can be input in tabular form. In addition, the flow field analysis option can treat rotational flows for which the variable thermodynamic properties

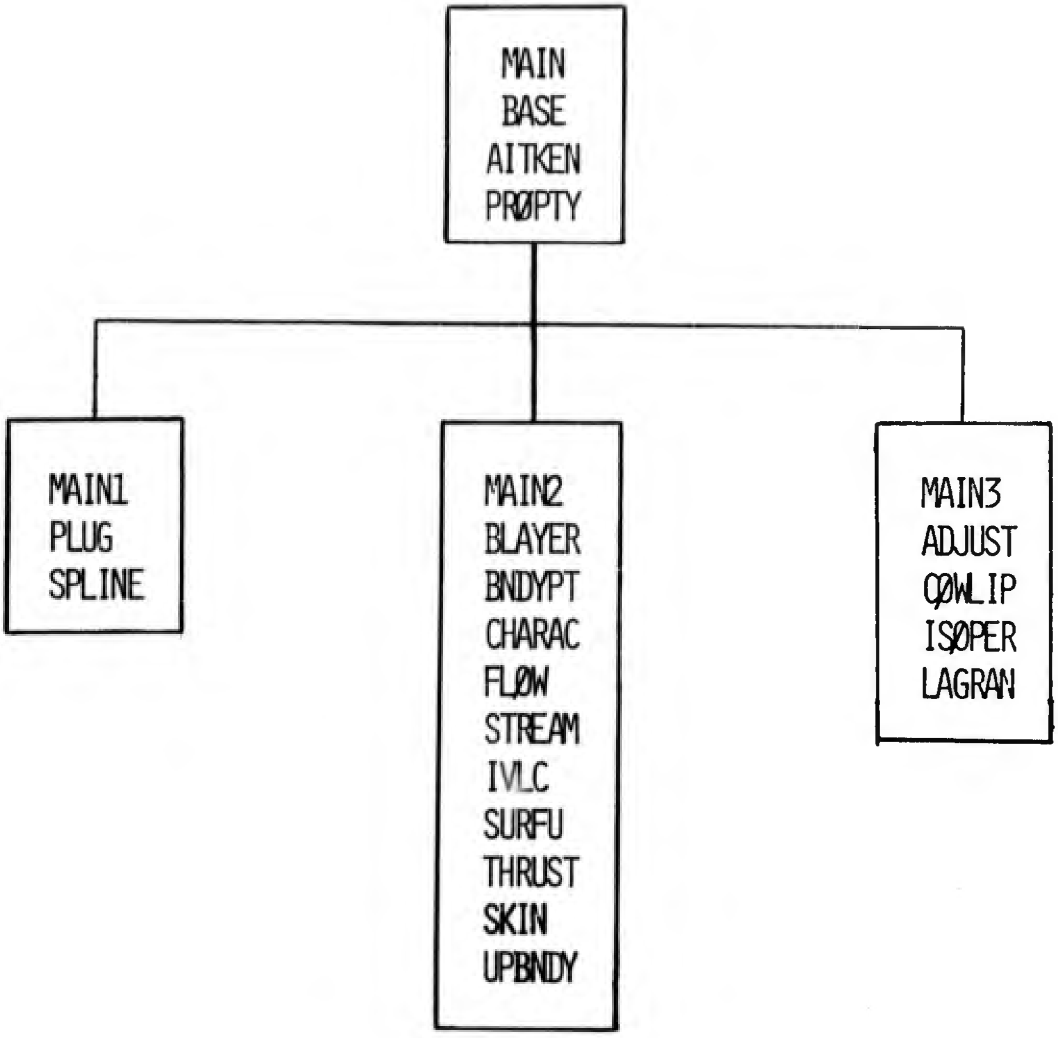

FIGURE 1. PROGRAM OVERLAY STRUCTURE

and stagnation conditions are specified separately on up to eleven separate streamlines. The effects of boundary layer thickness and wall shear stress can also be included.

The possible options should provide the user with a wide range of design options. Program modifications are, of course, also possible. A modular, subroutine approach was used for such items as the base pressure model, the boundary layer model, and the calculation of the initial-value line. These subroutines can be modified without disturbing the overall logic to suit the preference of each individual user.

2. DATA INPUT AND INITIALIZATION

All required data and control parameters are input by means of a data pack. The data are read by program MAIN1. The data are then output to identify the particular case being executed. Once the input data and control parameters have been read in, initialization of the subroutines takes place. This consists of changing units for the program calculations, calculating various program constants and the initialization of program variables. The necessary parameters are transmitted to the other subroutines through labeled common blocks.

3. CALCULATION OF THE FLOW FIELD

Upon completion of the data input and initialization the program logic control is transferred to program MAIN2. The flow field initial-value line is either input or calculated in subroutine IVLC. In the present calculation analysis the start line points have a constant Mach number, and the flow angle has a linear variation along this line. Subroutine IVLC then controls the logic for the calculation of the kernel (region Q, see Fig. 2). The left-running Mach line IK then becomes the initial-value line for the calculation of the remainder of the flow field. The cowl lip contour is calculated in SURFU. The initial-value line, the plug contour and the cowl lip contour are the initial and boundary conditions for the method of characteristics solution of the flow field. Right-running Mach lines originating from the initial-value line are followed until the plug contour is intersected. The logic necessary for the flow field calculations is controlled by subroutine FLØW. Subroutine CHARAC determines the solution at an interior mesh point in the flow field. Subroutine BNDYPT determines the solution at the intersection of a right-running Mach line and the plug contour. This process continues until a right-running characteristic has been calculated from each point on the initial-value line. Subroutine UPBNDY is then called to calculate the flow properties at a specified point on the cowl contour. A right-running Mach line is then followed from that point to the plug contour. This process is repeated until the nozzle length specified by the value of FIXEDL is reached. Subroutine FLØW outputs the solution for each right-running Mach line after the entire Mach line has been calculated. Subroutines THRUST, STREAM, BLAYER and SKIN are used to determine the thrust, stream function, boundary layer thickness and wall shear stress, respectively. The necessary parameters are saved for the optimization analysis.

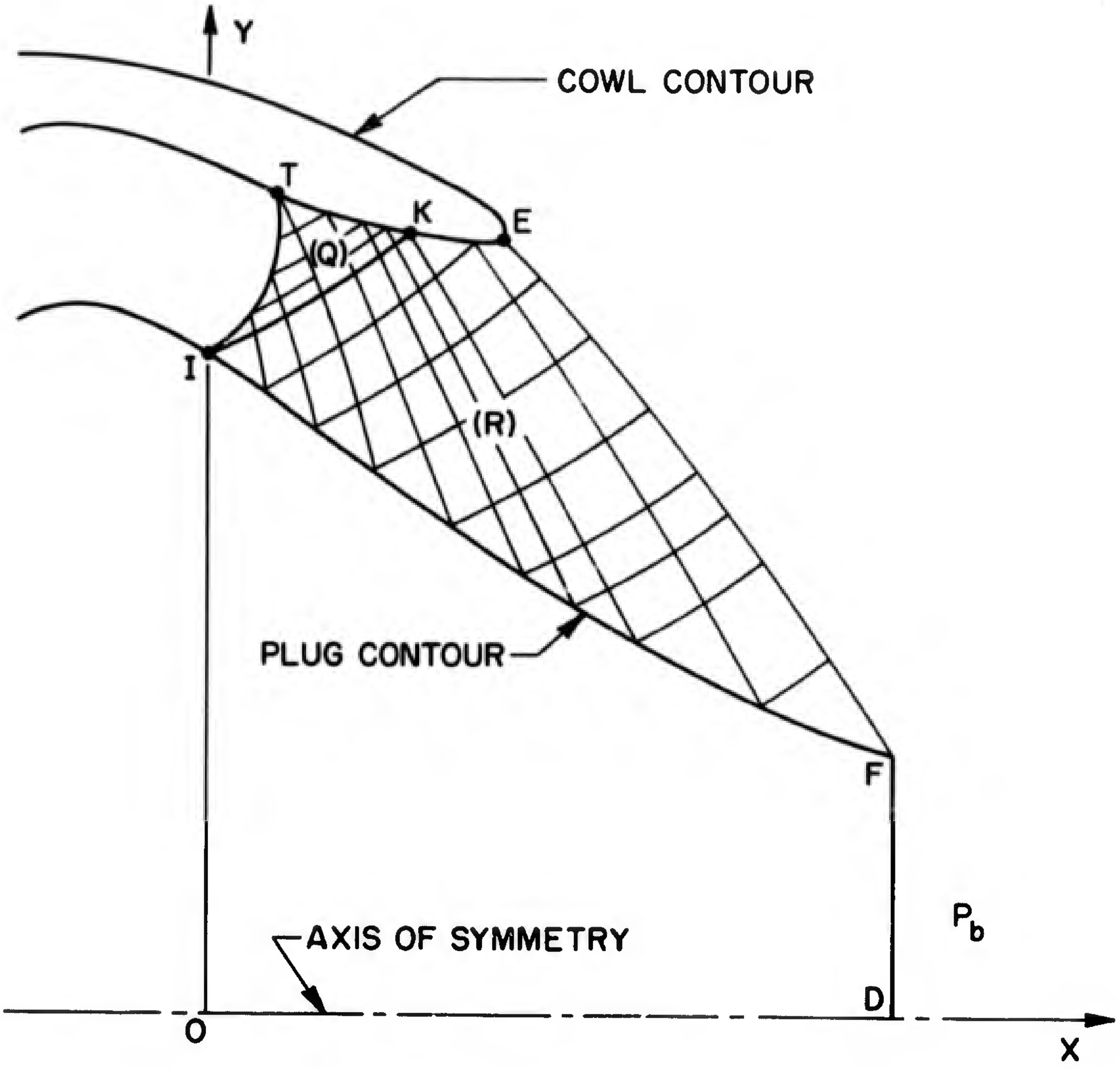

FIGURE 2. PLUG NOZZLE GEOMETRY AND FLOW FIELD

4. CALCULATION OF THE MULTIPLIER FIELD

In the optimization modes the program logic control is transferred to program MAIN3 upon completion of the flow field calculations. Program MAIN3 calls subroutine LAGRAN for the calculation of the Lagrange multipliers. The Lagrange multipliers at point F (the plug base) are calculated from the corner conditions (see Fig. 2). The transversality condition along EF (the exit right-running Mach line) is used to determine the multipliers at each mesh point that was calculated in the flow field calculations. The values of the multipliers along the boundary EF serve as initial conditions for the method of characteristics solution for the multiplier field. The solution proceeds along right-running Mach lines from the plug contour boundary to the cowl lip contour or initial-value line. Subroutine LAGRAN also evaluates the error equation along the plug contour. Once the multipliers have been calculated the program returns control to MAIN3. The error at each wall point is checked to determine if the solution is within the convergence criterion. If it is, the final wall solution is output and control is returned to program MAIN for input data for the next case, if any. If convergence is not satisfied, subroutine ADJUST is used to calculate the next estimate of the wall contour. Subroutine C0WLIP calculates the error at point E and translates the contour up or down as needed in order to drive the error at point E to zero. At this point the program returns control to MAIN and the flow field solution and multiplier solution are repeated for the new wall contour. This iterative process is repeated until the solution criteria are satisfied or the number of iterations exceeds MAXITR.

SECTION III

SUBROUTINE DESCRIPTIONS

A brief description of each program and subroutine in the computer program is given here to supplement the information available in the form of comments within the program.

1. LINKO

a. MAIN. This is the main program. It controls the proper calling sequence of the overlay scheme. No calculations are made in this program. The overlay structure is shown in Fig. 1.

b. BASE. This subroutine calculates the base pressure using the empirical equation

$$p_b = C_B\ p/M^{E_B} \qquad (1)$$

where the constants C_B and E_B are program input parameters, and p and M are the pressure and Mach number on the plug contour at point F (see Fig. 2). Default values of $C_B = 0.846$ and $E_B = 1.3$ are built into the program. Since this subroutine is the only place in the program where the base pressure is calculated, the base pressure model employed by the program can be changed easily by rewriting this subroutine. To facilitate the replacement of the base pressure model an expanded argument list has been used. The input arguments to BASE are the x and y coordinates of point F, x_F and y_F; the flow properties at point F, M_F, V_F, θ_F, p_F, ρ_F; the ambient pressure p_a; stagnation conditions P_o, T_o, ρ_o; the thermodynamic properties γ and gas constant R; gravitational constant g_o; and the constants for the empirical base pressure model C_B and E_B. The output argument is the base pressure p_b in lbf/ft^2.

c. AITKEN. This is an interpolation routine, obtained from Wright-Patterson Air Force Base, Ohio, which uses Aitken's method of interpolation. Its arguments are described on the comments cards in the source deck. A one-dimensional array X of independent variables and a one-dimensional array Y of dependent variables of number N are used by the subroutine to interpolate at the independent variable XB. A polynomial of degree K is used to calculate YB, the interpolated result. The maximum degree of the interpolating polynomial is ten. The degree of the interpolating polynomial is specified by the input variable NDEG1.

d. PR0PTY. This subroutine determines the flow velocity, Mach number, pressure, density, and temperature for a given value of either pressure, Mach number, or velocity. The argument IP specifies the given property. If IP equals 1, pressure is given; if 2, Mach number is specified; and if 3, the velocity is given. If the gas model is specified as an ideal gas, the isentropic relations are used to calculate gas properties. For variable thermodynamic properties but constant stagnation properties throughout the flow the property determination is by interpolation in a single table. If the flow model is rotational, i.e., with tabular properties for specified streamlines, the property determination is by double interpolation with the stream function.

2. LINK1

a. MAIN1. This program controls the first phase of the program. MAIN1 is called only once per case. This program reads the input data, initializes the indices, converts units, calculates program constants and writes out the input data. The logic for the point numbering scheme is developed in this program and designated NP0INT (I). The point numbering scheme employed in the remainder of the program is as follows: point I (see Fig. 2) is numbered 1; the second point on the left-running Mach line IK is point 2; the point where the right-running Mach line originating from point 2 intersects the plug contour is point 3; the third point on IK is labeled point 4, etc. NP0INT (I+1) gives the point number of the wall point on the I-th right-running Mach line. This counter is used to set indices on D0 loops, to store the flow field solution as it is calculated, and to retrieve this information for the optimization analysis. If the program is modified in any manner extreme care must be taken to ensure that the points are numbered correctly.

b. PLUG. If called, this subroutine calculates the first estimate of the plug contour by fitting specified end point conditions to a quadratic equation in x. The initial conditions at point I (x,y,θ) specify one end point. At point F (see Fig. 2), either y or θ can be specified by the value of the input flag IPLUG. IPLUG = 1 corresponds to specifying θ at point F, and IPLUG = 2 corresponds to specifying y at point F.

c. SPLINE. This subroutine fits a cubic interpolation spline to an array of tabular data. A cubic polynomial is fit between each consecutive pair of points such that the slopes of the adjoining cubic polynomial are matched at the data points. The slopes at the two end points of the array of data must be specified. The parameters of the argument list are: KN0T, the number of data points in the table; X(I), the value of the independent variable at the I-th data point; Y(I), the value of the dependent variable at the I-th data point; T(I) the calculated slope at the I-th data point. Subroutine SPLINE is called to evaluate the plug contour slope when a tabulated nozzle contour is input.

3. LINK2

a. MAIN2. This program controls the logic for the second phase of the program which consists of the calculation of (1) the initial-value line, (2) the cowl lip contour and (3) the plug nozzle flow field. In addition, this subroutine writes out the plug nozzle contour for each iteration.

b. BLAYER. This is the subroutine that determines the boundary layer thickness ε^* which is evaluated according to the following expression:

$$\varepsilon^* = \varepsilon_o^* \, (Re_y/Re_{y_o})^n = (\varepsilon^*/Re_y^n)_o \; Re_y^n) \tag{2}$$

The parameters ε_0^* and Re_{y_0} are the values of the boundary layer thickness and Reynolds number evaluated at the reference stagnation condition. The entire coefficient $(\varepsilon^*/Re_y^n)_o$ is input as the parameter DELSI.

In practice, representative values of DELSI are determined by computing values of ε_0^* and Re_{y_0} from a boundary layer analysis of nozzles similar to the one being designed. In this way the user can incorporate the boundary layer model of his choice. In the present model n is internally specified as DELEXP = 1.5.

Variations on the boundary layer model can easily be accomplished by varying the input value of DELSI or by internally varying the value of DELEXP. Further, the entire subroutine can be replaced; however, the replacement subroutine must also evaluate the Reynolds number (REY) since the value calculated in BLAYER is used by subroutine SKIN.

c. BNDYPT. This subroutine finds the point where a right-running Mach line intersects the plug contour (or inviscid core boundary) IF and the flow properties at that point. The location and flow properties are known at points 1 and 2 in Fig. 3. Since the boundary location, slope and the boundary stream function are known, it is only necessary to locate point 3 on this boundary and to calculate one flow property at point 3 (see Fig. 3). Since the program follows right-running Mach lines, only the right-running characteristic and compatibility equations are used. The equations programmed are:

1) for the irrotational flow model

$$dy/dx = \tan(\theta-\alpha) \tag{3}$$

$$(\cot\alpha/V)dV + d\theta - \nu \sin\theta \, dy/(My \sin(\theta-\alpha)) = 0 \tag{4}$$

2) for the rotational flow model

$$dy/dx = \tan(\theta-\alpha) \tag{5}$$

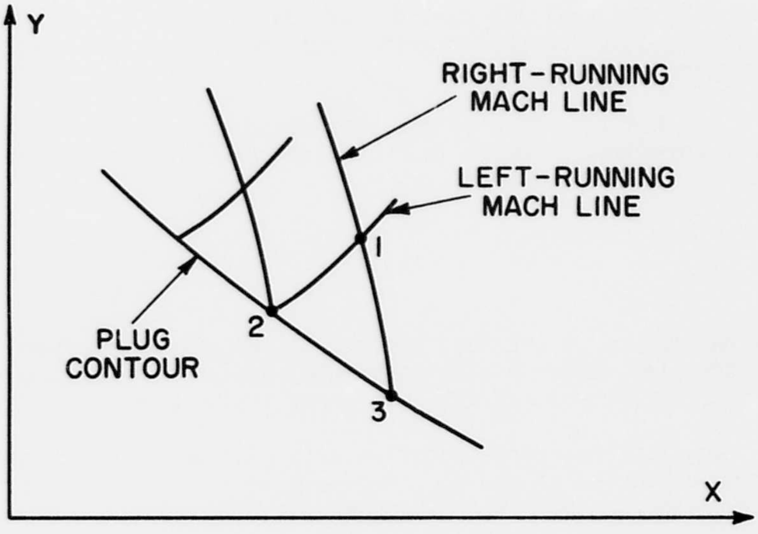

FIGURE 3. PLUG CONTOUR LABELING SCHEME FOR BNDYPT

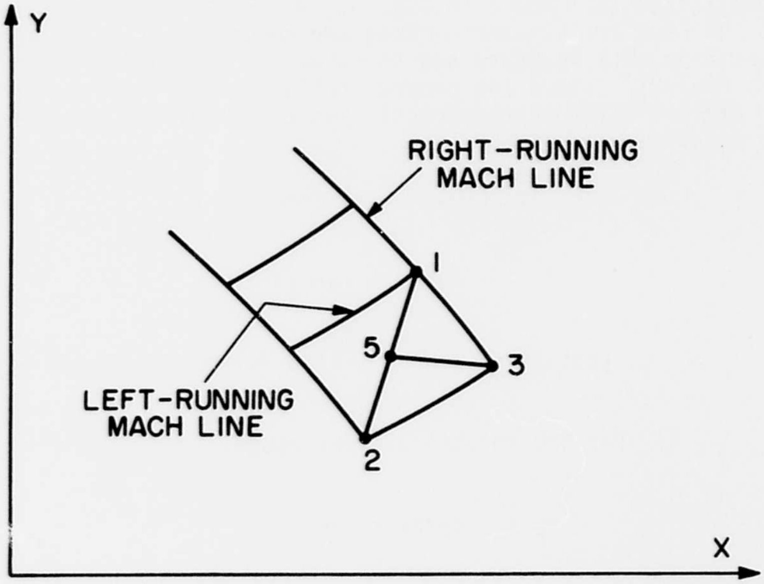

FIGURE 4. MESH POINT LAB   SCHEME FOR CHARAC

$$(\cot\alpha/\rho V^2)dp - d\theta + \nu \sin\theta dy/(My \sin(\theta-\alpha)) = 0 \qquad (6)$$

A modified Euler predictor-corrector technique with averaged coefficients is employed in the present analysis. The flow angle is known at point 3 for the irrotational flow model and for the pressure at point 3 in the rotational flow case. The values of the boundary stream function and p or V (depending on the flow model) are transferred to subroutine PRØPTY to determine the remaining flow variables.

d. CHARAC. This subroutine calculates the interior point flow field solution. The solution process employs the modified Euler predictor-corrector technique for solving the flow field characteristic and compatibility equations. When averages of the coefficients are evaluated, the technique of averaging the entire coefficient of each differential is used. The point labeling scheme for this subroutine is shown in Fig. 4. The solution is sought at point 3, and the value of the stream function at point 5 is obtained by linear interpolation between points 1 and 2. For the irrotational flow model it is not necessary to locate point 5 since the stream function is not needed. This routine uses both the left-running and right-running characteristic and compatibility equations. The equations programmed are as follows:

(1) the irrotational flow model

$$dy/dx = \tan(\theta \pm \alpha) \qquad (7)$$

$$(\cot\alpha/V)dV \mp d\theta - \nu \sin\theta\, dy/(My \sin(\theta \pm \alpha)) = 0 \qquad (8)$$

(2) the rotational flow model

$$dy/dx = \tan(\theta \pm \alpha) \qquad (9)$$

$$(\cot\alpha/V^2)dp \pm d\theta + \nu \sin\theta\, dy/(My \sin(\theta \pm \alpha)) = 0 \qquad (10)$$

$$dy/dx = \tan\theta \qquad (11)$$

where the upper signs are along left-running Mach lines and the lower signs are along right-running Mach lines. The remaining flow variables are obtained by calling subroutine PRØPTY. The stream function and p or V (depending on the flow model) are arguments in the calling statement.

e. FLØW. This subroutine contains the logic for calculating the flow field in Region R of Fig. 2. Calculations begin on and are carried out down the right-running Mach line designated by the subprogram variable I.

I = 2 corresponds to point I in Fig. 2, and I = NPTS corresponds to the exit right-running Mach line. The solution is terminated when the nozzle length is equal to the program variable FIXEDL, where FIXEDL is the x-distance between points I and F. When a right-running Mach line is calculated such that FIXEDL is exceeded, a linear interpolation is used to locate a new point on the cowl lip contour such that the right-running Mach line that originates from this point gives the proper nozzle length.

When 0PTI0N is specified as 5, the fixed inlet geometry model, subroutine FL0W performs one additional operation. This operation consists of calculating the left-running Mach line that originates from point D (see Fig. 5). An estimate is made of the slope at point D and is input in the data pack when 0PTI0N = 5. Point D is defined to be a point of tangency for the plug contour downstream of D and the specified circular arc upstream of point D. Only the flow region downstream of the left-running Mach line DS enters the optimization analysis. This treatment allows the initial-value line IK to remain fixed. The optimization process then drives the error to zero by adjusting the contour between points D and F. In this process the slope at point D is allowed to vary so that the error is zero. When the slope changes the program shifts the point D along the circular arc to maintain the tangency condition. For each iteration a new left-running Mach line DS is calculated and stored in the flow field storage array over the left-running Mach line just upstream of DS. Since this region is outside the optimization region these storage locations are not needed during the optimization process. After completion of the adjustment of the contour and the relocation of point D the flow field is recalculated for the new wall starting from the initial-value line IK.

The subroutine also writes out the flow field solution after the calculation of an entire right-running Mach line is completed. The input flags IWRITE and JWRITE control the output. The value input for IWRITE controls the printing of every IWRITE-th right-running Mach line. The value input for JWRITE controls the printing of every JWRITE-th point on the right-running Mach line. The first and last point of each IWRITE-th Mach line are always printed. Also, the first and last Mach lines are always printed. The nozzle performance parameters and boundary layer parameters are also output by FL0W.

f. <u>STREAM</u>. This subroutine integrates the mass flow across each right-running Mach line and determines the stream function for each interior point in Region R. The equation used to calculate the stream function between two points on a right-running Mach line is:

$$\psi = \{2\pi^{\nu} \int_{x_1}^{x_2} y^{\nu}\, \rho(u\dot{y} - v)dx\}/\dot{m} \qquad (12)$$

where $\dot{m}$ is the total mass flow rate through the nozzle. The above integration is performed by the trapezoidal method.

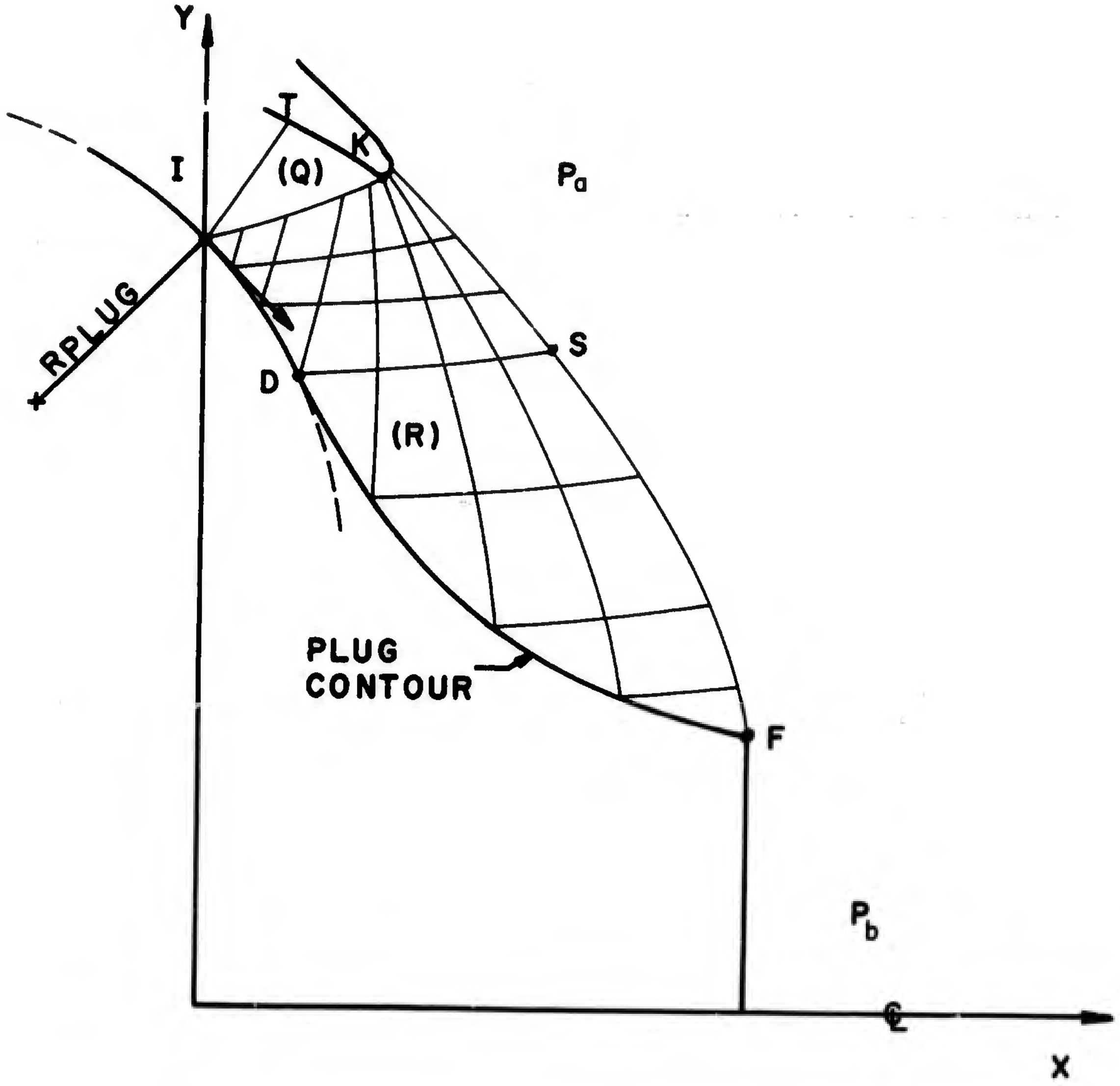

FIGURE 5. GEOMETRY FOR FIXED INLET OPTION

g. IVLC. When initial conditions are not specified subroutine IVLC is called to calculate a start line IT (see Fig. 2) from the specified conditions at point I. The start line has a uniform Mach number, input as M, and is a straight line normal to the flow direction at point I. The flow angle between points I and T has a linear variation specified by DTHETA. This parameter is internal to IVLC and is nominally +6 degrees. The number of points on the initial-value line IK is controlled by the input parameter MPTSSL. Actually, MPTSSL specifies the number of points on line IT which are used to calculate the left characteristic IK and, therefore, the number of points on IK may not quite agree with MPTSSL.

IVLC calls CHARAC to calculate the flow region Q shown in Fig. 2. The start line IT is extended beyond point T to calculate the kernel region Q. Point K is located by matching the mass flow rate across IK to that across IT. A numerical Newton-Raphson technique is used to locate point K. The thrust across IK is calculated by IVLC. When variable thermodynamic properties are input the line IT is calculated iteratively. A Newton-Raphson technique is used. In addition, IVLC prints out the left-running Mach line IK which is then the initial-value line for determining the remainder of the plug nozzle flow field.

h. SURFU. This subroutine calculates the cowl lip contour KE shown in Fig. 2. The present subroutine generates a circular arc with points normally located every DELTU degrees apart. DELTU is an input parameter. At point K, the intersection of the initial-value line and the cowl lip contour, the flow field may be only slightly supersonic with strong gradients. For this reason DELTU is modified slightly. The first downstream point of point K on the cowl lip contour is located at DELTU/5 degrees. The next point is at two times this increment, etc., for five points. This spaces five points where the normal distribution of every DELTU would only yield three. Further, in the region approaching the flow angle of zero degrees DELTU is halved for a finer point spacing. This closer spacing is useful for acquiring a length closer to the desired fixed length constraint. Other contours could be treated by rewriting SURFU. This subroutine also writes out the cowl lip contour beyond point K for each iteration.

i. THRUST. This subroutine calculates the thrust function f and its partial derivatives $f_{\dot{\eta}}$ and f_p. The thrust function is given by the following expression:

$$f = [p(\dot{\eta} - \dot{\varepsilon}) + \tau](\eta - \varepsilon)^{\nu} \tag{13}$$

The thrust function is evaluated at each wall point where a right-running Mach line intersects the plug contour. The thrust function is integrated by the trapezoidal method to obtain the thrust contribution on the surface. The thrust across the line IK was evaluated in IVLC. This value is stored in CØMMØN. The derivatives of f are obtained in the subroutine by analytically differentiating the above equation. This subroutine also calculates the thrust due to the pressure force acting on the cowl lip contour KE and the thrust contributions due to the base pressure and the ambient pressure. All of these thrust contributions are summed to yield the total axial thrust for print out purposes.

j. SKIN. This subroutine calculates the skin friction coefficient C_f and the wall shear stress τ, where C_f and τ are determined by the following expressions:

$$\tau = C_f \rho V^2/2 \tag{14}$$

$$C_f = C_{fi}(T/T_r)^{0.6}\, Re_y^{-0.25} \tag{15}$$

where

$$\mu = \mu_o(T/T_r)^{0.6} \tag{16}$$

$$Re_y = \rho V y/\mu \tag{17}$$

C_{fi}, T_r, and μ_o are input parameters.

The meanings of the variables appearing in the argument list are : X, x-coordinate of the wall, inches; Y, y-coordinate of the wall, inches; M, Mach number; V, velocity, ft/sec; THETA, wall slope, radians; P, pressure, lbf/ft^2; RHO, density, lbm/ft^3; and TEMP, temperature, °R. Since this subroutine is the only place the shear stress is evaluated, the shear stress model employed by the program can be changed by simply rewriting this subroutine.

k. UPBNDY. This subroutine calculates the flow field solution on the cowl lip contour. The modified Euler predictor-corrector technique is used to solve the characteristic and compatibility equations, where the average value of the entire coefficient of each differential is employed. Fig. 6 shows the point labeling scheme employed for the wall solution. The wall point is assumed to be fixed in this scheme; therefore, the location and properties of point 4 must also be determined as part of the solution. The location of point 4 is obtained from the intersection of the left-running Mach line between points 3 and 4 and the line connecting points 1 and 2. The properties at point 4 are obtained by linear interpolation of the properties between points 1 and 2. The left-running characteristics and compatibility equations as programmed are as follows:

(1) the irrotational flow model

$$dy/dx = \tan(\theta + \alpha) \tag{18}$$

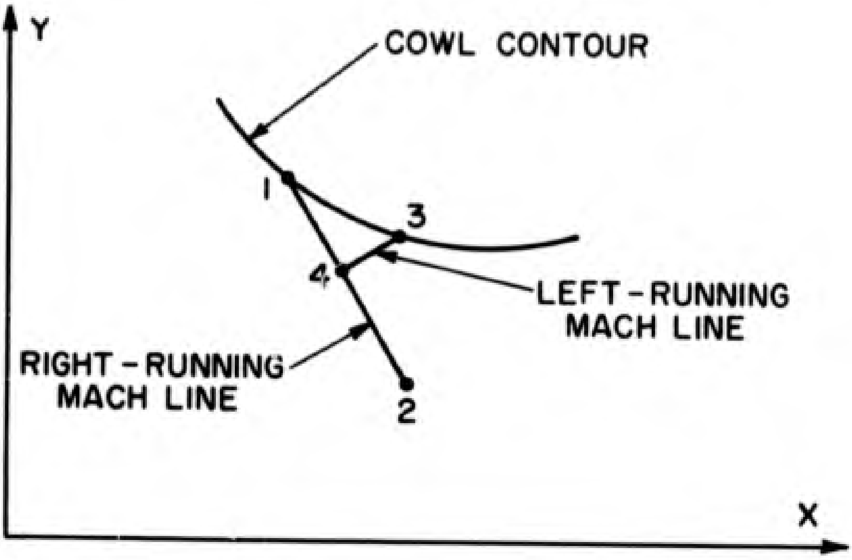

FIGURE 6. COWL LIP CONTOUR POINT LABELING SCHEME FOR UPBNDY

$$(\cot \alpha/V)dV + d\theta - \nu \sin \theta dx/(My \cos (\theta + \alpha)) = 0 \qquad (19)$$

(2) the rotational flow model

$$dy/dx = \tan (\theta + \alpha) \qquad (20)$$

$$(\cot \alpha/\rho V^2)dp - d\theta + \nu \sin \theta dx/(My \cos (\theta + \alpha)) = 0 \qquad (21)$$

The remaining flow parameters are determined by subroutine PRØPTY.

4. LINK3

a. MAIN3. This program controls the logic for the third phase of the program, the optimization analysis. It calls for (1) the calculation of the Lagrange multiplier field, (2) the evaluation of the convergence criteria and, (3) the adjustment of the nozzle geometry. In addition, MAIN3 prints out the final solution, or if the solution has not converged in MAXITR iterations, it punches a new set of input data based upon the final iteration.

b. ADJUST. This subroutine calculates the new plug contour slope from Eq. (35) of Ref. 1 based upon the value of the Lagrange multipliers at each wall point. The slope is then integrated by the trapezoidal rule to obtain the new contour.

c. CØWLIP. This subroutine evaluates the error at point E, Eq. (48) in Ref. 1, and adjusts the cowl lip radius accordingly (ØPTIØN = 3) or calculates the ambient pressure (ØPTIØN = 2). For ØPTIØN = 3, the new boundary obtained from subroutine ADJUST is translated in the y direction to be compatible with the new cowl lip radius.

d. ISØPER. This subroutine evaluates the isoperimetric constraint g and its derivatives, using Eq. (D-6) in Ref. 1, for use in determining the Lagrange multiplier λ_1 along the plug contour. Three design constraints are built into this subroutine. These are: (1) fixed length (specified by setting AK1 = 1.0); (2) fixed surface area (specified by setting AK2 = 1.0); or (3) fixed arc length along IF (specified by setting AK3 = 1.0). Additional design constraints could be included by modifying this subroutine.

e. LAGRAN. This subroutine controls the logic for the calculation of the Lagrange multipliers. The Lagrange multipliers along the exit right-running Mach line and the error along the plug boundary are evaluated in this subroutine. The Lagrange multipliers throughout the flow region are calculated by LAGRAN using Eqs. (K-23 through K-26), (K-31) and (K-32) of Ref. 1. In addition, the Lagrange multiplier values are printed for each point previously printed by subroutine FLØW, and the error is printed at wall points. The write flags IWRITE and JWRITE control this output.

SECTION IV

INPUT

The input data cards are listed below in the same order that they appear in the data card deck. The names of the program input variables appear in upper case letters on the left. The numbers beneath each name denote the card columns in which the value of the variable must appear. All integers must be right adjusted.

CARD 1	FØRMAT (12A6) This card contains job identification information.
HEADER (1-72)	72 alpha-numeric characters identifying the run. The contents of HEADER are printed at the top of each output page. This card may be left blank, but it cannot be omitted.
CARD 2	FØRMAT (6A6) This card contains additional job identification information.
DATE (1-36)	36 alpha-numeric characters for supplemental identifying information. The contents of DATE are printed at the top of each output page. This card may be left blank, but it cannot be omitted.
CARD 3	FØRMAT (4I5) Program options are contained on this card.
MØDEL (5)	Flow model specification. 1 Irrotational flow 2 Rotational flow (only used with ØPTIØN = 1)
NU (10)	Nozzle geometry specification. 0 Two-dimensional, $\nu = 0$ 1 Axisymmetric flow, $\nu = 1$
ØPTIØN (15)	Design option specification. 1 Analysis mode; the flow field for a specified nozzle is calculated. 2 Design mode; the cowl lip radius is specified, and the injection angle, the ambient pressure, and the plug contour are obtained from the optimization analysis.

	3 Design mode; the ambient pressure is specified, and the injection angle, the cowl lip radius, and the plug contour are obtained from the optimization analysis. 5 Design mode; the cowl lip radius, the injection angle, and the ambient pressure are specified, and the plug contour is obtained from the optimization analysis. This is the "fixed inlet" mode.
MAXITR (16-20)	Maximum number of iterations allowed in the optimization process, typically 10. May be 0 or blank when ØPTIØN = 1.
<u>CARD 4</u>	FØRMAT (13I5) Control and size indices are contained on this cards.
MPTSSL (4-5)	Number of data points on the start line IK. Typically 11. Maximum 20.
NPTSLS (8-10)	Number of points on the plug contour to be input in tabular form. Maximum = 100. NPTSLS can be left blank if IPLUG > 0.
NDEG1 (14-15)	Degree of interpolating polynomial for all interpolation of tabular input. $1 \le$ NDEG1 ≤ 10. Typically 1.
NSTART (16-20)	Not used. Leave blank.
IWRITE (24-25)	Flow field output control. Print every IWRITE-th right-running Mach line. If IWRITE > 60, print only the first and last Mach line.
JWRITE (29-30)	Flow field output control. Print every JWRITE-th point on each right-running Mach line output. If JWRITE $\ge$ 60, print only the first and last points on each Mach line.
KWRITE (35)	Cowl lip contour output control. 0 Cowl lip contour is not printed. 1 Cowl lip contour is printed.
LWRITE (40)	Multiplier output control. 0 No multipliers are printed. 1 Multipliers are printed subject to the flags IWRITE and JWRITE. May be left blank when ØPTIØN = 1.
LPUNCH (45)	Punched output control. 0 No punched output of final iteration. 1 Punch out plug contour (format of CARDS 15) and the start line data (format of CARD 6) obtained during the last iteration. These cards can then be used to restart the case.

IPLUG (50)	Controls first wall estimate. 0 Read wall contour (be sure to specify the number of points input in tabular form, NPTSLS (see CARD 4)). 1 Calculate parabolic wall contour, with θ_F specified (see CARD 12). 2 Calculate parabolic wall contour, with y_F specified (see CARD 12).
IPBASE (55)	Base pressure model. 0 Use default values of C_B and E_B ($C_B = 0.846$, $E_B = 1.3$). 1 Input new values of C_B and E_B (see CARD 11).
NSL (59-60)	Number of streamlines on which thermodynamic data are to be specified. If NSL = 0, constant property, homentropic flow is specified and CARD 5 must be included in the data pack (maximum = 11).
NTABL (64-65)	Number of data points in each streamline thermodynamic data set (maximum = 30). When NSL > 1, the same number of points must be included in each table (see CARDS 13 and 14). May be left blank or set equal to 0 when NSL = 0.

<u>CARD 5</u>

FØRMAT (4F10.0)

This card contains the gas stagnation properties and thermodynamic properties. This card is not included in the data pack when NSL > 0.

PO (1-10)	Stagnation pressure P_o, psia.
TO (11-20)	Stagnation temperature T_o, °R.
R (21-30)	Mixture gas constant R, ft-lbf/lbm - °R.
G (31-40)	Specific heat ratio γ.

<u>CARD 6</u>

FØRMAT (5F10.0)

This card contains information for constructing the start line and cowl lip contour (see Fig. 2). These data are required for all design option specifications (ØPTIØN = 2,3, or 5).

M (1-10)	Mach number at point I. M must be greater than 1.0.
THETA (11-20)	Flow angle at point I, degrees. Typically -45.0 to -65.0 degrees. Flow angles are measured relative to the nozzle centerline and are negative for flow toward the nozzle axis.

YPT1 (21-30)	Radius of point I, inches. The x coordinate of point I is assumed to be 0.0.
ZETA (31-40)	Radius of curvature of circular arc used for the cowl lip contour, inches.
DELTU (41-50)	Normal angular increment for locating points along the cowl lip contour, degrees. Typically 2.5°.
CARD 7	FØRMAT (7F10.0) This card contains design information.
PA (1-10)	Ambient pressure, psia. May be left blank when OPTION = 2. If PA is specified when ØPTIØN = 2, the specified value is used in the thrust equation.
FLØWRT (11-20)	Mass flow rate, lbm/sec.
FIXEDL (21-30)	Nozzle length from point I to F, inches.
CFI (31-40)	Constant for the skin friction coefficient expression C_{f_i} (see Eq.(15)). This is not the incompressible flow skin friction coefficient.
DELS1 (41-50)	Coefficient for the boundary layer thickness expression $(\epsilon^*/Re_y^{1.5})_0$, inches (see Eq. (2)).
MUO (51-60)	Viscosity evaluated at the stagnation temperature T_r, lbm/ft-sec (see Eq. (15)).
TR (61-70)	The reference temperature used to calculate the temperature ratio for the viscosity and the skin friction coefficient. May be left blank when CFI = MUO = 0.0. The boundary layer effects are neglected when the values of CFI and DELS1 or MUO are set equal to zero or left blank.
CARD 8	FØRMAT (5F10.0) This card contains convergence criteria and the geometric constraint specifications.
ERS (1-10)	Convergence criterion for the optimal solution. Normally = 0.002. May be left blank when ØPTIØN = 1.
EPS (11-20)	Convergence criterion for the flow field calculations. Normally = 0.001 to 0.0001.
AK1 (21-30)	Constant length constraint weighting factor.
AK2 (31-40)	Constant arc length constraint weighting factor.

AK3 (41-50)	Constant arc length constraint weighting factor.

One particular constraint is specified when its weighting factor has a value of 1.0 and the others are 0.0. A weighted combination is specified when all have values. Only the relative magnitudes are significant. All constraint factors may be left blank when ØPTIØN = 1.

<u>CARD 9</u> FØRMAT (F10.0)

This card is required only when ØPTIØN = 2.

YPT4 (1-10)	Cowl lip radius for fixed cowl lip option, inches.

<u>CARD 10</u> FØRMAT (2F10.0)

This card is required only when ØPTIØN = 5.

RPLUG (1-10)	Downstream radius of curvature of the plug surface, inches.
TPLUG (11-20)	First estimate for the flow angle at the point where the contour first departs from the circular arc, degrees.

<u>CARD 11</u> FØRMAT (2F10.0)

This card is required only if IPBASE = 1 (see CARD 4).

CB (1-10)	Input value for the numerator constant in the base pressure model presented in Eq. (1).
EB (11-20)	Input value for the exponent in the base pressure model as stated in Eq. (1).

<u>CARD 12</u> FØRMAT (2F10.0)

This card is required only if IPLUG > 0.

THETAF (1-10)	The value of the flow angle at the nozzle exit, degrees. Required only if IPLUG = 1 (see CARD 4).
YF (11-20)	The value of the y-coordinate at the nozzle exit, inches. Required only if IPLUG = 2 (see CARD 4).

The data which are input through CARDS 13 and 14 specify the thermodynamic model of the fluid being considered. Three models are available. The first is a thermally and calorically perfect gas. This model is chosen by specifying NSL = 0 on CARD 3. If this model is selected, CARD 5 must also be included in the data deck, and CARDS 13 and 14 are omitted. The second model is a gas with frozen or equilibrium chemical composition in which the pressure, density, temperature and speed of sound are functions of Mach number specified by a one-dimensional tabular input. Actually the velocity is used in the present scheme, but this is directly analogous to the speed of sound when the Mach number is specified. For this model the flow must be homentropic and isoenergetic. The value of NSL on CARD 3 must be NSL = 1. No value is assigned to the stream function on CARD 13, but a blank card must be inserted. The number of values specified in the input table is NTABL on CARD 3. Therefore, this model is specified by NSL = 1, CARD 13 blank and tabular values on CARD 14.

The third thermodynamic model is non-homentropic and nonisoenergetic flows of a real gas having either frozen or equilibrium chemical composition. This model can be used only for the analysis option, i.e., for ØPTIØN = 1. These data are input for a specified value of the stream function STAB (I)($0.0 \leq$ STAB(I) ≤ 1.0). This thermodynamic model is treated as a two-dimensional tabular input. The number of streamlines on which tabular data are specified is set by the value of NSL on CARD 3. Again, NTABL gives the number of CARDS 14 for each streamline. The value of NTABL must be the same for each streamline. Values are obtained from the tabular data by a double interpolation on the stream function and one other property, either the pressure, Mach number or velocity. The interpolation is third-order and is performed by subroutine AITKEN.

<u>CARDS 13</u> FØRMAT (F10.0)

This card is required only when variable property flow is considered, i.e., NSL (see CARD 4) is not zero.

STAB(I) (1-10) I-th value of the stream function corresponding to the set of isentropic thermodynamic property data immediately following. I = 1 corresponds to the plug nozzle boundary, and STAB (1) = 0.0; I = NSL corresponds to the cowl contour, STAB(NSL) = 1.0 These two boundaries must be so specified. These data form NSL sets of one CARD 13 followed by NTABL CARDS 14, one set for each streamline. The remaining I - 2 values must be given as $0.0 <$ STAB(I) < 1.0.

<u>CARDS 14</u> FORMAT (5E15.0)(more than one card)

A set of NTABL (see CARD 4) cards. Each card contains corresponding values of the Mach number, velocity, density, temperature, and pressure, respectively. The cards are arranged in increasing or decreasing values of Mach number and the range of data should bracket the values to be encountered on the I-th streamline. These cards are only included in the data pack when NSL (see CARD 4) is not zero (variable property flow).

PTAB(J,I,1) (1-15) J-th value of the Mach number.

PTAB(J,I,2) (16-30) J-th value of the velocity, ft/sec.

PTAB(J,I,3) (31-45) J-th value of the density, $\mathrm{lbm/ft}^3$.

PTAB(J,I,4) (46-60) J-th value of the temperature, °R.

PTAB(J,I,5) (61-75)	J-th value of the pressure, psia.

These tables must be input so that the parameters monotonically increase or decrease. A sufficient range of Mach number should be included so that no problems are encountered by having to extrapolate outside the range of the variables. I designates the I-th streamline, of which there are NSL. There are NTABL of these cards for each streamline.

<u>CARDS 15</u> FØRMAT (2F10.0)

These cards contain the first estimate of the plug nozzle contour and are required only when IPLUG = 0. If IPLUG > 0, these cards are not included in the data pack. When the analysis option is used (ØPTIØN = 1), these cards specify the tabular nozzle contour to be analyzed.

XSL(I) (1-10)	I-th value of the x-coordinate, inches.
RSL(I) (11-20)	I-th value of the y-coordinate, inches. I = 2 corresponds to the first card input, and specifies the first wall point in the table. I = NSTSLS corresponds to the final wall point of this table and is usually point F (see Fig. 2). There are NPTSLS - 1 of CARDS 15. For example, for a deck of 24 wall points (24 CARDS 15) NPTSLS must be input as 25 in order to correctly read the data. The first wall point, I = 1, is point I of Fig. 2, and was input on CARD 6. A maximum of 100 wall points are allowed.

SECTION V

SAMPLE CASES

To illustrate the various options that are available the input and selected output from six sample cases are presented in this section. A summary of the options considered in the six sample cases is presented in Fig. 7. Further details are discussed below.

1 SAMPLE CASE I

Figure 8 is a listing of the data pack for Sample Case I. In this case, it is desired to determine the maximum thrust axisymmetric (NU = 1) plug nozzle for a fixed length constraint (AK1 = 1.0, AK2 = AK3 = 0.0) with variable inlet geometry (0PTI0N = 3). The desired length (FIXEDL) is 12.5 inches. The mass flow rate (FL0WRT) is 148.077 lbm/sec. The ambient pressure (PA) is 14.7 psia. No boundary layer effects are to be considered, so MU0 is 0.0.

The thermodynamic properties are input as a single table (NSL = 1), and the table consists of 11 points (NTABL = 11). The irrotational flow model is specified (M0DEL = 1). An inlet Mach number of 1.03 was chosen. For the first approximation the flow angle and y-coordinate of point I were specified as -58.0 degrees and 10.0 inches, respectively. Eleven points are specified on the initial-value line (MPTSSL = 11). The radius of curvature of the cowl lip contour (ZETA) is 0.05 inches. Points along the cowl lip contour are specified in two degree increments (DELTU = 2.0).

A parabolic, first approximation to the nozzle contour is used with the wall slope at point F (IPLUG = 1) specified equal to -13.0 degrees (THETAF = -13.0).

The default values are used for the base pressure model (IPBASE = 0).

In order to print only the first and last point on each right-running Mach line, IWRITE = 1 and JWRITE = 100. The punched output is suppressed by setting LPUNCH = 0. The cowl lip contour is printed (KWRITE = 1) and the Lagrange multipliers are also output (LWRITE = 1). The convergence criteria for the optimization and flow field solution are specified as ERS = 0.0075 and EPS = 0.0001, respectively.

Selected portions of the program output for Sample Case I are presented in Fig. 9. The first page contains a program abstract and selected values of input data. The second page is the tabular thermodynamic tables that are input; page three is the computed first guess for the optimum plug contour; page four is the computed point location and properties on the left characteristic (IK in Fig. 2) used as the initial value line; page five is a tabulation of x, y and θ values on the circular arc expansion (KE in Fig. 2); page six is the beginning of the flow field data for the initial contour; page eleven is the flow field data on the last characteristics for the initial contour (notice that only the end points are printed for each characteristic in accordance with the JWRITE = 100 specification); page twelve is the beginning of the Lagrange multiplier data; page sixteen is the wall contour for the first iteration; and page 102 is the final optimum contour.

This case requires 199 seconds of computational time on Purdue's CDC 6500 machine.

SAMPLE CASE NUMBER	MØDEL	NU	ØPTIØN	IWRITE	JWRITE	NSL	BOUNDARY LAYER	INITIAL PLUG CONTOUR	CONSTRAINT
I	1	1	3	1	100	1	NO	CALC.	LENGTH AK1 = 1.0 AK2=AK3=0.0
II	2	1	1	1	5	3	NO	TAB.	N.A. AK1 = AK2 = AK3 = 0.0
III	1	0	2	1	100	0	NO	CALC.	LENGTH AK1 = 1.0 AK2=AK3=0.0
IV	1	1	5	1	100	0	NO	CALC.	LENGTH AK1 = 1.0 AK2=AK3=0.0
V	1	1	3	1	100	0	YES	CALC.	LENGTH AK1 = 1.0 AK2=AK3=0.0
VI	1	1	3	1	100	0	NO	CALC.	SURFACE AREA AK2 = 1.0 AK1=AK3=0.0

FIGURE 7. SUMMARY OF INPUT OPTIONS FOR SAMPLE CASES

```
    SAMPLE CASE I
    VARIABLE COWL LIP RADIUS
    1    1    3   20
    9    0    1    0    1  100    1    1    0    1    0    1   11
1.03        -58.0      10.0       0.05       1.5
14.7        148.077    12.5       0.0        0.0        0.0
0.0075      0.0001     1.0        0.0        0.0
-13.0
0.0
   1.0000000E+00  3.4533237E+03  1.3349049E-01  5.3811659E+03  2.7935229E+02
   1.2000000E+00  4.0530429E+03  1.1006453E-01  5.1475635E+03  2.2033051E+02
   1.4000000E+00  4.6117294E+03  8.8547512E-02  4.8963604E+03  1.6860687E+02
   1.6000000E+00  5.1281469E+03  6.9781267E-02  4.6353523E+03  1.2579029E+02
   1.8000000E+00  5.6024144E+03  5.4072426E-02  4.3712662E+03  9.1919709E+01
   2.0000000E+00  6.0357090E+03  4.1344295E-02  4.1095890E+03  6.6075356E+01
   2.2000000E+00  6.4299692E+03  3.1292548E-02  3.8545548E+03  4.6907328E+01
   2.4000000E+00  6.7876306E+03  2.3511131E-02  3.6092397E+03  3.3000064E+01
   2.6000000E+00  7.1114062E+03  1.7578051E-02  3.3757173E+03  2.3076095E+01
   2.8000000E+00  7.4041152E+03  1.3104837E-02  3.1552377E+03  1.6080118E+01
   3.0000000E+00  7.6685591E+03  9.7590786E-03  2.9484029E+03  1.1189771E+01
```

FIGURE 8. DATA DECK FOR SAMPLE CASE I

```
SAMPLE CASE I                                                         VARIABLE COWL LIP RADIUS              [illegible]

                 PLUG NOZZLE OPTIMIZATION FOR MAXIMUM THRUST INCLUDING VARIABLE INLET GEOMETRY


ABSTRACT

     THIS PROGRAM WAS WRITTEN BY GEAROLD R. JOHNSON AT THE PURDUE UNIVERSITY, JET PROPULSION CENTER,
     AS PART OF THE REQUIREMENTS OF THE UNITED STATES AIR FORCE CONTRACT NUMBER F33615-67-C-1068 WHICH WAS
     SPONSORED BY THE AERO PROPULSION LABORATORY, WRIGHT-PATTERSON AIR FORCE BASE, OHIO.  PROFESSORS H. DOYLE
     THOMPSON AND JOE D. HOFFMAN WERE THE PRINCIPLE INVESTIGATORS FOR PURDUE UNIVERSITY.


     THE GAS DYNAMIC MODEL IS BASED ON THE ASSUMPTIONS THAT THE FLOW IS A CONTINUUM, STEADY, ISENTROPIC,
     ROTATIONAL, AND EITHER FROZEN OR EQUILIBRIUM CHEMICAL COMPOSITION.  THE GAS THERMODYNAMIC MODEL
     MAY BE EITHER CALORICALLY AND THERMALLY PERFECT OR INPUT IN TABULAR FORM.

     THE TYPE OF NOZZLE (TWO-DIMENSIONAL OR AXISYMMETRIC), THE NOZZLE MASS FLOW RATE, THE GAS PROPERTIES,
     AN INITIAL ESTIMATE OF THE PLUG CONTOUR AND AN INITIAL ESTIMATE OF THE INLET GEOMETRY MUST BE PROVIDED.
     THE FLOW FIELD AND DESIGN EQUATIONS ARE SOLVED BY A NUMERICAL METHOD OF CHARACTERISTICS.  THE PLUG
     CONTOUR AND THE INLET GEOMETRY ARE SYSTEMATICALLY ADJUSTED UNTIL THE CONVERGENCE CRITERIA ARE
     SATISFIED.

     THIS PROGRAM IS CAPABLE OF EXECUTING ONE OF THE FOLLOWING FOUR OPTIONS...

          (1)  ANALYSIS ONLY, SPECIFIED BY SETTING OPTION = 1

          (2)  OPTIMIZATION WITH FIXED COWL LIP RADIUS, SPECIFIED BY SETTING OPTION = 2

          (3)  OPTIMIZATION WITH VARIABLE COWL LIP RADIUS, SPECIFIED BY SETTING OPTION = 3

          (4)  OPTIMIZATION WITH FIXED INLET GEOMETRY, SPECIFIED BY SETTING OPTION = 4
```

. . .

```
     FOR THIS CASE OPTION (3) HAS BEEN SPECIFIED IN THE INPUT

     THE AXISYMMETRIC PLUG NOZZLE IS DESIGNED FOR THE FOLLOWING CONDITIONS

          MASS FLOW RATE =    148.077 LBM/SEC

          PLUG LENGTH =   12.500 INCHES

          AMBIENT PRESSURE = 14.70 PSIA
```

FIGURE 9. SELECTED OUTPUT FROM SAMPLE CASE I

```
SAMPLE CASE I
                                                              VARIABLE COWL LIP RADIUS                      PAG.  2

EQUILIBRIUM OR FROZEN FLOW WITH VARIABLE PROPERTIES     NO. STREAMLINES =  1     NO. OF POINTS PER LIN. = 11
                               STREAMLINE NO.  1     STREAMFUNCTION =  [illegible]
            MACH          VELOCITY        DENSITY       TEMPERATURE      PRESSURE

       1.000000E+00    3.453324E+03    1.334965E-01    5.331156E+03    2.743023E+02
       1.200000E+00    4.053043E+03    1.106645E-01    5.147553E+03    2.203350E+02
       1.400000E+00    4.611729E+03    8.854751E-02    4.895360E+03    1.6[illegible]9E+02
       1.600000E+00    5.128147E+03    6.975127E-02    4.635352E+03    1.257303E+02
       1.800000E+00    5.602414E+03    5.407243E-02    4.371206E+03    9.131971E+01
       2.000000E+00    6.035709E+03    4.134436E-02    4.[illegible]E+03    [illegible]E+01
       2.200000E+00    6.429969E+03    3.129255E-02    3.854555E+03    4.[illegible]733E+01
       2.400000E+00    6.787631E+03    2.351113E-02    3.[illegible]E+03    3.3[illegible]E+01
       2.600000E+00    7.111466E+03    1.757865E-02    3.375717E+03    2.3[illegible]E+01
       2.800000E+00    7.404115E+03    1.316484E-02    3.155230E+03    1.[illegible]12E+01
       3.000000E+00    7.668559E+03    9.759079E-03    2.9[illegible]3E+03    1.110977E+01
```

NOT REPRODUCIBLE

FIGURE 9. (Continued)

```
SAMPLE CASE I                                                              VARIABLE COWL LIP RADIUS            PAGE   3
                                            FIRST GUESS FOR THE OPTIMUM PLUG CONTOUR
       X             Y           THETA                                    X             Y           THETA
   (INCHES)      (INCHES)     (DEGREES)                               (INCHES)      (INCHES)     (DEGREES)

    0.00000      10.00000     -59.00000                                4.37983       6.72929     -21.67985
     .50000       9.19963     -59.00000                                4.63221       6.63046     -21.09941
     .56549       9.10101     -54.98821                                4.89205       6.53164     -20.55102
     .63845       9.00219     -52.18283                                5.15938       6.43282     -20.03452
     .71899       8.90337     -49.57549                                5.43410       6.33400     -19.53413
     .80691       8.80455     -47.15563                                5.71646       6.23518     -19.06006
     .90220       8.70572     -44.91148                                6.00620       6.13636     -18.60918
    1.00546       8.60690     -42.83076                                6.30343       6.03753     -18.17771
    1.11540       8.50808     -40.93115                                6.61513       5.93871     -17.76516
    1.23322       8.40926     -39.11071                                6.92331       5.83989     -17.37035
    1.35851       8.31044     -37.44804                                7.23996       5.74107     -16.99220
    1.49126       8.21162     -35.93246                                7.56735       5.64225     -16.62971
    1.63152       8.11279     -34.46408                                7.90169       5.54342     -16.28195
    1.77924       8.01397     -33.12771                                8.24377       5.44460     -15.94807
    1.93443       7.91515     -31.87302                                8.59332       5.34578     -15.62728
    2.09710       7.81633     -30.70436                                8.95035       5.24696     -15.31903
    2.26724       7.71751     -29.61381                                9.31483       5.14814     -15.02264
    2.44486       7.61868     -28.59609                                9.68593       5.04931     -14.73629
    2.62996       7.51986     -27.62447                               10.06529       4.95049     -14.46198
    2.82253       7.42104     -26.72382                               10.45322       4.85167     -14.19956
    3.02257       7.32222     -25.87048                               10.84762       4.75285     -13.93951
    3.23010       7.22340     -25.06312                               11.24936       4.65403     -13.69277
    3.44509       7.12457     -24.31307                               11.65886       4.55521     -13.45366
    3.66756       7.02575     -23.59462                               12.07569       4.45638     -13.22312
    3.89751       6.92693     -22.92281                               12.50030       4.35756     -13.00000
    4.13493       6.82811     -22.28275
```

FIGURE 9. (Continued)

```
SAMPLE CASE I                                                          VARIABLE COWL LIP RADIUS          PAGE  -
                                          DATA FOR THE INITIAL VALUE LINE
                                  MACH                            FLOW
       X              Y          NUMBER        VELOCITY          ANGLE        PRESSURE       DENSITY      TEMPERATURE
   (INCHES)       (INCHES)                     (FT/SEC)        (DEGREES)       (PSIA)       (LBM/FT3)       (DEG-R)

    0.00000       10.00000       1.03000      3545.84691       -58.00000      270.49712       .12959      5346.74224
     .07464       10.02244       1.04924      3601.48960       -57.33354      264.53315       .12773      5320.41319
     .15662       10.04313       1.06402      3649.49412       -56.74334      259.42729       .12566      5307.55965
     .24427       10.06273       1.07851      3693.32654       -56.18331      255.43474       .12415      5291.13023
     .33718       10.08143       1.09216      3734.49472       -55.64604      251.41515       .12254      5275.33254
     .43497       10.09957       1.10525      3773.77767       -55.71058      247.54142       .12104      5260.40235
     .53731       10.11713       1.11793      3811.56521       -54.80259      243.81374       .11952      5245.23943
     .64299       10.13444       1.13033      3848.47997       -53.29137      240.20133       .11807      5230.71231
     .75479       10.15147       1.14243      3884.45613       -52.84365      238.09148       .11666      5215.39174
     .79811       10.15673       1.14615      3895.43815       -52.32037      235.60825       .11623      5211.39973
```

NOT REPRODUCIBLE

FIGURE 9. (Continued)

```
SAMPLE CASE I
                                                                  VARIABLE COWL LIP RADIUS                         PAG.   5
                                    CONTOUR OF THE CIRCULAR ARC THAT FORMS THE COWL LIP
        X            Y           THETA                                       X            Y            THETA
     (INCHES)     (INCHES)     (DEGREES)                                  (INCHES)     (INCHES)      (DEGREES)

      .79011     10.15673     -52.32007                                    .82357     10.13757      -7.02007
      .79027     10.15653     -52.02007                                    .82497     10.13753      -5.52007
      .79059     10.15612     -51.42007                                    .82552     10.13747      -4.77007
      .79108     10.15551     -50.52007                                    .82617     10.13742      -4.02007
      .79193     10.15451     -49.02007                                    .82683     10.13738      -3.27007
      .79284     10.15353     -47.52007                                    .82748     10.13735      -2.52007
      .79373     10.15258     -46.02007                                    .82813     10.13732      -1.77007
      .79462     10.15165     -44.52007                                    .82879     10.13731      -1.02007
      .79556     10.15074     -43.02007                                    .82944     10.13730       -.27007
      .79653     10.14986     -41.52007                                    .83010     10.13730        .47993
      .79752     10.14901     -40.02007                                    .83075     10.13731       1.22993
      .79854     10.14816     -38.52007                                    .83140     10.13732       1.97993
      .79957     10.14738     -37.02007                                    .83206     10.13735       2.72993
      .80063     10.14664     -35.52007                                    .83271     10.13739       3.47993
      .80170     10.14596     -34.02007                                    .83337     10.13743       4.22993
      .80284     10.14514     -32.52007                                    .83402     10.13749       4.97993
      .80391     10.14445     -31.02007                                    .83467     10.13755       5.72993
      .80504     10.14379     -29.52007                                    .83532     10.13762       6.47993
      .80619     10.14316     -28.02007                                    .83597     10.13769       7.22993
      .80735     10.14256     -26.52007                                    .83662     10.13778       7.97993
      .80853     10.14199     -25.02007                                    .83727     10.13788       8.72993
      .80972     10.14145     -23.52007                                    .83791     10.13799       9.47993
      .81093     10.14094     -22.02007                                    .83856     10.13809      10.22993
      .81215     10.14047     -20.52007                                    .83920     10.13821      10.97993
      .81339     10.14003     -19.02007                                    .83984     10.13834      11.72993
      .81463     10.13962     -17.52007                                    .84048     10.13848      12.47993
      .81588     10.13924     -16.02007                                    .84112     10.13862      13.22993
      .81714     10.13889     -14.52007                                    .84176     10.13878      13.97993
      .81941     10.13858     -13.02007                                    .84239     10.13894      14.72993
      .81969     10.13830     -11.52007                                    .84302     10.13911      15.47993
      .82098     10.13807     -10.02007                                    .84365     10.13929      16.22993
      .82227     10.13785      -8.52007                                    .84428     10.13948      16.97993
```

NOT REPRODUCIBLE

FIGURE 9. (Continued)

NOT REPRODUCIBLE

```
SAMPLE CASE I                                                              VARIABLE CONE LIP [illegible]              PAGE [illegible]

                                              DATA FOR THE FLOW FIELD

 I  J        X            Y          MACH                        FLOW
          (INCHES)     (INCHES)     NUMBER     VELOCITY          ANGLE        PRESSURE        DENSITY    TEMPERATURE     STREAM
                                               (FT/SEC)        (DEGREES)       (PSIA)        (LBM/FT3)     (DEG-R)      FUNCTION.   IT

 1  1      0.0000      10.0000      1.0304     3545.949       -56.0000       270.[illegible]97[illegible]   .124[illegible]   534.77     0.000000     4
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       1.[illegible]

 2  1       .0748      10.0324      1.0482     3611.49[illegible]  -57.3935      264.[illegible]932     .127[illegible]3[illegible]   532[illegible].1      .1[illegible]21[illegible]    0
 2  2       .0247       9.9605      1.0675     3549.356       -56.0000       259.9402       .1259[illegible]     534.2[illegible]    0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.95[illegible]47

 3  1       .1565      10.0431      1.0640     3649.494       -56.7435       259.5273       .125[illegible]1     53[illegible]7.4[illegible]    .21071      0
 3  3       .0587       9.9366      1.0937     3730.[illegible]2   -56.0000       251.6590       .122[illegible]14     527[illegible].63    0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.9[illegible]33

 4  1       .2443      10.0527      1.0785     3693.327       -56.2935       255.4047       .124[illegible]4[illegible]   529[illegible]1.14    .32471      0
 4  4       .1004       9.8393      1.1147     3932.125       -56.0000       244.7513       .1[illegible]9[illegible]   524[illegible]9.54    0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.83927

 5  1       .3372      10.0915      1.0922     3734.495       -55.4047       251.4152       .122439      527[illegible].3[illegible]   .[illegible]43[illegible]   0
 5  5       .1491       9.7614      1.1369     3968.120       -56.0000       238.2752       .1172[illegible]     522[illegible].37    0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.76137

 6  1       .4350      10.0956      1.1052     3773.778       -54.7107       247.5410       .121[illegible]     525[illegible]     .56651      0
 6  6       .2644       9.5720      1.1577     3929.553       -56.0000       232.2424       .114[illegible]5     519[illegible].72    0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.67238

 7  1       .5373      10.1172      1.1179     3811.665       -54.0225       243.5137       .113[illegible]16    524[illegible].2[illegible]   .6[illegible]345     0
 7  7       .2662       9.5742      1.1774     3987.229       -56.0000       225.[illegible]32[illegible]   .112[illegible]39    517[illegible].[illegible]2   0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.57[illegible]1

 8  1       .6440      10.1344      1.1303     3846.460       -53.2816       240.2313       .115[illegible]7[illegible]   523[illegible].71    .42337      0
 8  8       .3343       9.4649      1.1961     4041.630       -56.0000       221.4239       .112[illegible]21    5[illegible]2.3[illegible]   0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.4[illegible]92

 9  1       .7548      10.1515      1.1424     3884.450       -52.5495       236.0015       .115[illegible]     522[illegible]     .90777      0
 9  9       .4039       9.3457      1.2138     4093.326       -56.0000       210.4930       .1[illegible]7     513[illegible]     0.000000     3
      REYNOLDS NO. =   0.              CF =    0.000000            TAU =    0.                    Y-DELTA =       9.34550
```

FIGURE 9. (Continued)

```
SAMPLE CASE I
                                                                                   VARIABLE COWL LIP RADIUS                              PAG.  11
 I   J          X              Y           MACH                       FLOW                                                                STREAM
             (INCHES)       (INCHES)      NUMBER     VELOCITY        ANGLE        PRESSURE      DENSITY     TEMPERATURE     FUNCTION    ITER
                                                     (FT/SEC)     (DEGREES)       (PSIA)      (LBM/FT3)      (DEG-R)

43  43         9.0003        5.2334       2.7739     7367.554      -15.2783       16.8591       .013619       3183.26       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        5.23336
   TRUNCATED AXIAL THRUST =    33320.2434 LBF                SPECIFIC IMPULSE =  225.0201 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38066.2849 LBF                 VACUUM SPECIFIC IMPULSE =  257.6713 LBF-SEC/LBM

44   1          .8255       10.1375       2.6137     7132.365       -4.7701       22.5156       .017230       3306.51        .99885       3
44  44         9.5694        5.0965       2.8126     7421.632      -14.8255       15.7161       .012863       3141.76       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        5.08351
   TRUNCATED AXIAL THRUST =    33356.4072 LBF                SPECIFIC IMPULSE =  225.2543 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38102.3385 LBF                 VACUUM SPECIFIC IMPULSE =  257.3151 LBF-SEC/LBM

45   1          .8262       10.1374       2.6391     7170.976       -4.0201       21.5078       .016606       3332.26        .99884       3
45  45        10.1879        4.9194       2.8522     7475.671      -14.3775       14.6300       .012135       3099.99       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        4.91943
   TRUNCATED AXIAL THRUST =    33393.1458 LBF                SPECIFIC IMPULSE =  225.5124 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38139.0804 LBF                 VACUUM SPECIFIC IMPULSE =  257.5629 LBF-SEC/LBM

46   1          .8268       10.1374       2.6647     7209.358       -3.2701       20.5374       .015969       3303.85        .99935       3
46  45        10.8019        4.7493       2.8925     7529.663      -13.9307       13.5934       .011436       3057.95       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        4.74933
   TRUNCATED AXIAL THRUST =    33430.1893 LBF                SPECIFIC IMPULSE =  225.7626 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38176.0850 LBF                 VACUUM SPECIFIC IMPULSE =  257.8128 LBF-SEC/LBM

47   1          .8275       10.1373       2.6905     7247.513       -2.5201       19.6035       .015395       3275.32        .99937       3
47  47        11.5985        4.5698       2.9337     7583.808      -13.4889       12.6194       .010761       3015.48       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        4.56977
   TRUNCATED AXIAL THRUST =    33467.2145 LBF                SPECIFIC IMPULSE =  226.0126 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38213.0792 LBF                 VACUUM SPECIFIC IMPULSE =  258.0626 LBF-SEC/LBM

48   1          .8281       10.1373       2.7164     7285.443       -1.7701       18.7049       .014819       3246.64        .93897       3
48  45        12.4646        4.3798       2.9759     7638.036      -13.0501       11.6902       .010112       2972.65       0.00000       3
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        4.37976
   TRUNCATED AXIAL THRUST =    33503.8614 LBF                SPECIFIC IMPULSE =  226.2601 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38249.7032 LBF                 VACUUM SPECIFIC IMPULSE =  258.3100 LBF-SEC/LBM

49   1          .8282       10.1373       2.7192     7289.516       -1.6893       18.6101       .014758       3243.54        .99886       2
49  49        12.4958        4.3585       2.9804     7643.836      -13.0021       11.5939       .010045       2958.6        0.00000       2
   REYNOLDS NO. =    0.                        CF =   0.00000           TAU =    0.                       Y-DELTA =        4.35851
   TRUNCATED AXIAL THRUST =    33507.7514 LBF                SPECIFIC IMPULSE =  226.2864 LBF-SEC/LBM
   TRUNCATED VACUUM AXIAL THRUST =    38253.5912 LBF                 VACUUM SPECIFIC IMPULSE =  258.3362 LBF-SEC/LBM
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FIGURE 9. (Continued)

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SAMPLE CASE I                                                                       VARIABLE COWL LIP RADIUS          PAG   12
                                          DATA FOR THE FLOW AND MULTIPLIER FIELDS
  I    J        X           Y           U            V           LAMBDA1      LAMBDA2      LAMBDA3     LAMBDA4      ERROR
             (INCHES)    (INCHES)    (FT/SEC)     (FT/SEC)

 49    1       .8242     10.1373     7286.348     -214.686       -.3130     -325.5563
 49   49     12.4958      4.3585     7447.861    -1719.765       -.1815     -571.1379                            -7.2463E-02

 48    1       .8251     10.1373     7251.966     -225.437       -.0875     -573.5513
 48   48     12.4045      4.3798     7440.769    -1724.089       -.1855     -578.2303                            -6.6487E-02

 47    1       .8275     10.1373     7240.554     -316.668       -.1283     -630.8202
 47   47     11.5985      4.5698     7374.612    -1768.968       -.2215     -644.3875                            -1.7053E-02

 46    1       .8268     10.1374     7197.619     -411.239       -.1725     -690.1383
 46   46     10.8619      4.7493     7308.198    -1812.751       -.2508     -710.8613                             2.2229E-02

 45    1       .8262     10.1374     7153.332     -502.727       -.2191     -751.2643
 45   45     10.1879      4.9194     7241.538    -1856.284       -.3034     -777.4614                             5.3802E-02

 44    1       .8255     10.1375     7107.661     -593.107       -.2683     -814.1992
 44   44      9.5694      5.0905     7174.532    -1899.136       -.3493     -844.4675                             7.9775E-02

 43    1       .8249     10.1375     7060.625     -682.356       -.3263     -878.8075
 43   43      9.0003      5.2334     7107.176    -1941.367       -.3995     -911.8234                             1.0134E-01

 42    1       .8236     10.1377     6962.554     -857.369       -.4330    -1012.2270
 42   42      7.9695      5.5180     6971.554    -2024.962       -.5084    -1047.4448                             1.3418E-01

 41    1       .8223     10.1378     6859.277    -1027.591       -.5576    -1150.9761
 41   41      7.1229      5.7772     6834.458    -2106.536       -.6327    -1184.5414                             1.5744E-01

 40    1       .8214     10.1381     6750.968    -1192.815       -.6947    -1294.5134
 40   40      6.3747      6.0144     6595.779    -2186.676       -.7724    -1323.2244                             1.7407E-01

 39    1       .8197     10.1383     6537.804    -1352.696       -.8449    -1442.3202
 39   39      5.7239      6.2326     6555.471    -2263.516       -.9281    -1463.5285                             1.8592E-01

 38    1       .8184     10.1386     6519.966    -1507.657      -1.0082    -1593.6673
 38   38      5.1549      6.4345     6413.555    -2339.314      -1.0998    -1605.4445                             1.9404E-01

 37    1       .8171     10.1389     6397.639    -1656.931      -1.1954    -1748.2040
 37   37      4.6546      6.6219     6269.990    -2413.356      -1.2580    -1749.0089                             1.9925E-01

 36    1       .8159     10.1392     6271.012    -1800.555      -1.3767    -1905.5632
```

FIGURE 9. (Continued)

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SAMPLE CASE I
                                                                VARIABLE COWL LIP RADIUS                  PAG   10
                                 NEW WALL COMPUTED,     ITERATION =  1
         X             Y          THETA
     (INCHES)      (INCHES)     (DEGREES)                    X             Y           THETA
                                                         (INCHES)      (INCHES)     (DEGREES)
      0.00000       9.03495     -53.89581
       .02470       8.99244     -53.79581                 1.69091       6.91157     -39.75564
       .05875       8.93402     -53.73094                 1.84510       6.74944     -37.87124
       .10043       8.86270     -53.65964                 2.01423       6.65032     -36.67324
       .14910       8.77969     -53.58439                 2.20020       6.52463     -35.46527
       .20440       8.68563     -53.50575                 2.40324       6.35147     -34.28711
       .26518       8.58088     -53.42789                 2.63196       6.23371     -33.09612
       .33434       8.46568     -53.34876                 2.88343       6.07023     -31.89556
       .40097       8.34012     -53.27025                 3.16220       5.92112     -30.68147
       .43333       8.29899     -53.24626                 3.47354       5.72380     -29.46514
       .44598       8.27777     -53.15357                 3.82222       5.52375     -28.22471
       .47075       8.23645     -53.96544                 4.21333       5.32434     -26.96192
       .50704       8.17672     -53.47053                 4.65465       5.10557     -25.67274
       .56724       8.08267     -53.28130                 5.15467       4.87219     -24.34954
       .62906       7.99326     -54.39170                 5.72392       4.62329     -22.98413
       .69410       7.90528     -52.67129                 6.37466       4.35571     -21.58366
       .76321       7.81724     -51.06408                 7.12293       4.07221     -20.09336
       .83696       7.72839     -49.54352                 7.98933       3.75821     -18.54479
       .91589       7.63817     -48.09341                 9.00027       3.44513     -16.90741
      1.00145       7.54513     -46.69926                 9.56942       3.27878     -15.84905
      1.09397       7.44923     -45.35994                10.15767       3.11447     -15.15436
      1.19407       7.35015     -44.05532                10.85139       2.92722     -14.23466
      1.30277       7.24729     -42.77834                11.59847       2.74741     -13.27074
      1.42106       7.14021     -41.52526                12.40461       2.65436     -12.26191
      1.55304       7.02846     -40.29247                12.49595       2.54447     -12.15462
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NOT REPRODUCIBLE

FIGURE 9. (Continued)

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                                FINAL SOLUTION FOR THE OPTIMUM PLUG CONTOUR                                            PAGE 102

      X            Y           THETA                                      X            Y           THETA
   (INCHES)     (INCHES)    (DEGREES)                                  (INCHES)     (INCHES)    (DEGREES)

    0.00000      8.13672    -60.94742                                  1.60538      6.01197    -41.35677
     .02336      8.09491    -60.76245                                  1.76383      5.87635    -39.94655
     .05133      8.04504    -60.67761                                  1.93779      5.73429    -38.54026
     .08419      7.98664    -60.59065                                  2.12974      5.58522    -37.13006
     .12210      7.91351    -60.50362                                  2.34202      5.42954    -35.73534
     .16515      7.84356    -60.40920                                  2.57733      5.25359    -34.33504
     .21335      7.75864    -60.31342                                  2.83972      5.08909    -32.94458
     .26572      7.66537    -60.23053                                  3.12975      4.90511    -31.55571
     .31639      7.57963    -60.13159                                  3.45451      4.71209    -30.16940
     .34027      7.53724    -59.96354                                  3.81795      4.50671    -28.75564
     .38120      7.46700    -59.52535                                  4.22590      4.29939    -27.40533
     .43354      7.37349    -58.77505                                  4.69529      4.05735    -26.02804
     .51110      7.25508    -57.33191                                  5.20444      3.81205    -24.65269
     .58509      7.14302    -55.79198                                  5.79357      3.55031    -23.27708
     .65975      7.03678    -54.27337                                  6.45545      3.27059    -21.90017
     .73725      6.93142    -52.79595                                  7.23607      2.97197    -20.51620
     .81317      6.82634    -51.33756                                  8.12591      2.65146    -19.11048
     .90444      6.72235    -49.91431                                  9.10166      2.30749    -17.67296
     .99946      6.61246    -48.44426                                 10.37926      1.93754    -16.12291
    1.10034      6.50038    -47.02984                                 11.07127      1.74296    -15.24353
    1.21102      6.38554    -45.60778                                 11.82807      1.54382    -14.23684
    1.33100      6.26600    -44.19958                                 12.49449      1.37936    -13.25509
    1.46214      6.14167    -42.77079
```

NOT REPRODUCIBLE

FIGURE 9. (Concluded)

2. SAMPLE CASE II

Figure 10 is a listing of the data pack for Sample Case II. This sample case illustrates the rotational flow (MODEL = 2) analysis option (OPTION = 1). The flow field is axisymmetric (NU = 1) and tabular thermodynamic data are given for three streamlines (NSL = 3). The plug contour is input in tabular form. Point I is located at y = 8.15466 inches with a flow angle of -60.69423 degrees, and a Mach number of 1.03. The input data listing in conjunction with the Input Section, Section IV, is self explanatory.

Figure 11 is selected output from Sample Case II. Page one is a program abstract and selected portions of the input data. Page two is the tabular thermodynamic data, and page three is the tabular wall contour data with calculated slopes at each point. Page four is the computed data on the initial-value surface (left characteristic IK in Fig. 2) and pages six and sixteen are the first and last pages of the computed flow field.

Sample Case II requires 60 seconds of computational time on Purdue's CDC 6500 machine.

```
  SAMPLE CASE II
 FLOW FIELD ANALYSIS ONLY
  2    1    1    0
  11   21   1    0    1    5    1    0    0    0    0    3   11
1.03        -60.69423 8.15466    0.05       1.5
14.7        148.077   12.5       0.0        0.0       0.0
0.0         0.0001    0.0        0.0        0.0
0.0
  1.0000000F+00  3.4533237E+03  1.3349049E-01  5.3811659E+03  2.7935229E+02
  1.2000000E+00  4.0530429E+03  1.1006453E-01  5.1475635E+03  2.2033051E+02
  1.4000000E+00  4.6117294E+03  8.8547512E-02  4.8963604E+03  1.6860687E+02
  1.6000000E+00  5.1281469E+03  6.9781267E-02  4.6353523E+03  1.2579029E+02
  1.8000000E+00  5.6024144E+03  5.4072426E-02  4.3712662E+03  9.1919709E+01
  2.0000000E+00  6.0357090E+03  4.1344295E-02  4.1095890E+03  6.6075356E+01
  2.2000000E+00  6.4299692E+03  3.1292548E-02  3.8545548E+03  4.6907328E+01
  2.4000000E+00  6.7876306E+03  2.3511131E-02  3.6092397E+03  3.3000064E+01
  2.6000000E+00  7.1114062E+03  1.7578051E-02  3.3757173E+03  2.3076095E+01
  2.8000000E+00  7.4041152E+03  1.3104837E-02  3.1552377E+03  1.6080118E+01
  3.0000000E+00  7.6685591E+03  9.7590786E-03  2.9484029E+03  1.1189771E+01
0.5
  1.0000000E+00  3.4533237E+03  1.3349049E-01  5.3811659E+03  2.7935229E+02
  1.2000000E+00  4.0530429E+03  1.1006453E-01  5.1475635E+03  2.2033051E+02
  1.4000000E+00  4.6117294E+03  8.8547512E-02  4.8963604E+03  1.6860687E+02
  1.6000000E+00  5.1281469E+03  6.9781267E-02  4.6353523E+03  1.2579029E+02
  1.8000000E+00  5.6024144E+03  5.4072426E-02  4.3712662E+03  9.1919709E+01
  2.0000000E+00  6.0357090E+03  4.1344295E-02  4.1095890E+03  6.6075356E+01
  2.2000000E+00  6.4299692E+03  3.1292548E-02  3.8545548E+03  4.6907328E+01
  2.4000000E+00  6.7876306E+03  2.3511131E-02  3.6092397E+03  3.3000064E+01
  2.6000000E+00  7.1114062E+03  1.7578051E-02  3.3757173E+03  2.3076095E+01
  2.8000000E+00  7.4041152E+03  1.3104837E-02  3.1552377E+03  1.6080118E+01
  3.0000000E+00  7.6685591E+03  9.7590786E-03  2.9484029E+03  1.1189771E+01
1.0
  1.0000000E+00  3.4533237E+03  1.3349049E-01  5.3811659E+03  2.7935229E+02
  1.2000000E+00  4.0530429E+03  1.1006453E-01  5.1475635E+03  2.2033051E+02
  1.4000000E+00  4.6117294E+03  8.8547512E-02  4.8963604E+03  1.6860687E+02
  1.6000000E+00  5.1281469E+03  6.9781267E-02  4.6353523E+03  1.2579029E+02
  1.8000000E+00  5.6024144E+03  5.4072426E-02  4.3712662E+03  9.1919709E+01
  2.0000000E+00  6.0357090E+03  4.1344295E-02  4.1095890E+03  6.6075356E+01
  2.2000000E+00  6.4299692E+03  3.1292548E-02  3.8545548E+03  4.6907328E+01
  2.4000000E+00  6.7876306E+03  2.3511131E-02  3.6092397E+03  3.3000064E+01
  2.6000000E+00  7.1114062E+03  1.7578051E-02  3.3757173E+03  2.3076095E+01
  2.8000000E+00  7.4041152E+03  1.3104837E-02  3.1552377E+03  1.6080118E+01
  3.0000000E+00  7.6685591E+03  9.7590786E-03  2.9484029E+03  1.1189771E+01
```

FIGURE 10. DATA DECK FOR SAMPLE CASE II

```
0.12986    7.92535
0.29744    7.63378
0.43750    7.39724
0.58986    7.16220
0.74255    6.95073
0.91302    6.73806
1.10705    6.51948
1.33781    6.28423
1.61363    6.02970
1.94679    5.75144
2.35284    5.44486
2.85197    5.10497
3.47112    4.72602
4.24712    4.30126
5.23195    3.82235
6.50178    3.27873
8.17435    2.65670
10.44426   1.94057
11.90366   1.54703
12.49267   1.40368
```

FIGURE 10. (Concluded)

NOT REPRODUCIBLE

```
SAMPLE CASE II                                            FLOW FIELD ANALYSIS ONLY                   PAG    1

                  PLUG NOZZLE OPTIMIZATION FOR MAXIMUM THRUST INCLUDING VARIABLE INLET GEOMETRY


ABSTRACT

   THIS PROGRAM WAS WRITTEN BY GEAROLD R. JOHNSON AT THE PURDUE UNIVERSITY, JET PROPULSION CENTER,
   AS PART OF THE REQUIREMENTS OF THE UNITED STATES AIR FORCE CONTRACT NUMBER F33615-67-C-1068 WHICH WAS
   SPONSORED BY THE AERO PROPULSION LABORATORY, WRIGHT-PATTERSON AIR FORCE BASE, OHIO.  PROFESSORS H. DOYLE
   THOMPSON AND JOE D. HOFFMAN WERE THE PRINCIPLE INVESTIGATORS FOR PURDUE UNIVERSITY.


   THE GAS DYNAMIC MODEL IS BASED ON THE ASSUMPTIONS THAT THE FLOW IS A CONTINUUM, STEADY, ISENTROPIC,
   ROTATIONAL, AND EITHER FROZEN OR EQUILIBRIUM CHEMICAL COMPOSITION.  THE GAS THERMODYNAMIC MODEL
   MAY BE EITHER CALORICALLY AND THERMALLY PERFECT OR INPUT IN TABULAR FORM.

   THE TYPE OF NOZZLE (TWO-DIMENSIONAL OR AXISYMMETRIC), THE NOZZLE MASS FLOW RATE, THE GAS PROPERTIES,
   AN INITIAL ESTIMATE OF THE PLUG CONTOUR AND AN INITIAL ESTIMATE OF THE INLET GEOMETRY MUST BE PROVIDED.
   THE FLOW FIELD AND DESIGN EQUATIONS ARE SOLVED BY A NUMERICAL METHOD OF CHARACTERISTICS.  THE PLUG
   CONTOUR AND THE INLET GEOMETRY ARE SYSTEMATICALLY ADJUSTED UNTIL THE CONVERGENCE CRITERIA ARE
   SATISFIED.

   THIS PROGRAM IS CAPABLE OF EXECUTING ONE OF THE FOLLOWING FOUR OPTIONS...

        (1)  ANALYSIS ONLY, SPECIFIED BY SETTING OPTION = 1

        (2)  OPTIMIZATION WITH FIXED COWL LIP RADIUS, SPECIFIED BY SETTING OPTION = 2

        (3)  OPTIMIZATION WITH VARIABLE COWL LIP RADIUS, SPECIFIED BY SETTING OPTION = 3

        (4)  OPTIMIZATION WITH FIXED INLET GEOMETRY, SPECIFIED BY SETTING OPTION = 5
```

• • •

```
   FOR THIS CASE OPTION (1) HAS BEEN SPECIFIED IN THE INPUT

   THE AXISYMMETRIC PLUG NOZZLE IS DESIGNED FOR THE FOLLOWING CONDITIONS

        MASS FLOW RATE =   148.077 LBM/SEC

        PLUG LENGTH =   12.500 INCHES

        AMBIENT PRESSURE = 14.70 PSIA
```

FIGURE 11. SELECTED OUTPUT FROM SAMPLE CASE II

```
SAMPLE CASE II                                                    FLOW FIELD ANALYSIS ONLY          PAGE  2

EQUILIBRIUM OR FROZEN FLOW WITH VARIABLE PROPERTIES        NO. STREAMLINES =  3     NO. OF POINTS PER LINE = 11

                                   STREAMLINE NO.   1     STREAMFUNCTION =    0.0000

                 MACH          VELOCITY          DENSITY        TEMPERATURE       PRESSURE

             1.000000E+00   3.453324E+03   1.334905E-01   5.381156E+03   2.793523E+02
             1.200000E+00   4.053043E+03   1.100645E-01   5.147563E+03   2.233055E+02
             1.400000E+00   4.611729E+03   8.854751E-02   4.895350E+03   1.685059E+02
             1.600000E+00   5.128147E+03   6.978127E-02   4.635352E+03   1.257903E+02
             1.800000E+00   5.602414E+03   5.407243E-02   4.371266E+03   9.191971E+01
             2.000000E+00   6.035709E+03   4.134430E-02   4.109589E+03   6.607536E+01
             2.200000E+00   6.429969E+03   3.129255E-02   3.854555E+03   4.690733E+01
             2.400000E+00   6.787631E+03   2.351113E-02   3.609240E+03   3.300006E+01
             2.600000E+00   7.111406E+03   1.757805E-02   3.375717E+03   2.307610E+01
             2.800000E+00   7.404115E+03   1.310484E-02   3.155238E+03   1.609012E+01
             3.000000E+00   7.668559E+03   9.759079E-03   2.945403E+03   1.115977E+01

                                   STREAMLINE NO.   2     STREAMFUNCTION =     .5000

                 MACH          VELOCITY          DENSITY        TEMPERATURE       PRESSURE

             1.000000E+00   3.453324E+03   1.334905E-01   5.381156E+03   2.793523E+02
             1.200000E+00   4.053043E+03   1.100645E-01   5.147563E+03   2.233055E+02
             1.400000E+00   4.611729E+03   8.854751E-02   4.895350E+03   1.685059E+02
             1.600000E+00   5.128147E+03   6.978127E-02   4.635352E+03   1.257903E+02
             1.800000E+00   5.602414E+03   5.407243E-02   4.371266E+03   9.191971E+01
             2.000000E+00   6.035709E+03   4.134430E-02   4.109589E+03   6.607536E+01
             2.200000E+00   6.429969E+03   3.129255E-02   3.854555E+03   4.690733E+01
             2.400000E+00   6.787631E+03   2.351113E-02   3.609240E+03   3.300006E+01
             2.600000E+00   7.111406E+03   1.757805E-02   3.375717E+03   2.307610E+01
             2.800000E+00   7.404115E+03   1.310484E-02   3.155238E+03   1.609012E+01
             3.000000E+00   7.668559E+03   9.759079E-03   2.945403E+03   1.115977E+01

                                   STREAMLINE NO.   3     STREAMFUNCTION =    1.0000

                 MACH          VELOCITY          DENSITY        TEMPERATURE       PRESSURE

             1.000000E+00   3.453324E+03   1.334905E-01   5.381156E+03   2.793523E+02
             1.200000E+00   4.053043E+03   1.100645E-01   5.147563E+03   2.233055E+02
             1.400000E+00   4.611729E+03   8.854751E-02   4.895360E+03   1.685059E+02
             1.600000E+00   5.128147E+03   6.978127E-02   4.635352E+03   1.257903E+02
             1.800000E+00   5.602414E+03   5.407243E-02   4.371266E+03   9.191971E+01
             2.000000E+00   6.035709E+03   4.134430E-02   4.109589E+03   6.607536E+01
             2.200000E+00   6.429969E+03   3.129255E-02   3.854555E+03   4.690733E+01
             2.400000E+00   6.787631E+03   2.351113E-02   3.609240E+03   3.300006E+01
             2.600000E+00   7.111406E+03   1.757805E-02   3.375717E+03   2.307610E+01
             2.800000E+00   7.404115E+03   1.310484E-02   3.155238E+03   1.609012E+01
             3.000000E+00   7.668559E+03   9.759079E-03   2.945403E+03   1.115977E+01
```

NOT REPRODUCIBLE

FIGURE 11. (Continued)

NOT REPRODUCIBLE

```
SAMPLE CASE II
                                                          FLOW FIELD ANALYSIS ONLY                  [illegible]
                                    FIRST GUESS FOR THE OPTIMUM PLUG CONTOUR

      X            Y           THETA                                 X            Y           THETA
   (INCHES)     (INCHES)     (DEGREES)                            (INCHES)     (INCHES)     (DEGREES)

   0.00000      8.15466     -53.59423                             2.35234      [illegible]  [illegible]
    .12996      7.92535     -53.30132                             2.85197      [illegible]  [illegible]
    .29744      7.63379     -53.90257                             3.47112      4.72602      [illegible]
    .43750      7.39724     -53.45951                             4.24712      4.30126      [illegible]
    .58386      7.16223     -53.48652                             5.23195      3.82230      [illegible]
    .74255      6.95073     -52.74756                             6.40178      3.27873      -21.95509
    .91342      6.73506     -49.95410                             8.17435      2.65076      [illegible]
   1.10763      6.51948     -47.00774                            10.44426      1.94037      [illegible]
   1.37791      6.28423     -44.15491                            11.40306      1.54703      -13.94224
   1.51563      6.02970     -41.31586                            12.49257      1.40366      -13.37542
   1.64279      5.75144     -38.49599
```

FIGURE 11. (Continued)

```
SAMPLE CASE II                                                          FLOW FIELD ANALYSIS ONLY              523

                                              DATA FOR THE INITIAL VALUE LINE

                                     MACH                       FLOW
     X               Y              NUMBER       VELOCITY       ANGLE        PRESSURE       DENSITY      TEMPERATURE
  (INCHES)        (INCHES)                       (FT/SEC)     (DEGREES)       (PSIA)       (LBM/FT3)       (DEG-R)

  0.00000         6.15466          1.03000      3545.94881    -60.59423     270.09712       .12989      5346.75224
   .07567         6.17513          1.03493      3560.24601    -60.17695     268.30982       .12937      5341.91715
   .15698         6.19654          1.03942      3573.73380    -59.62538     267.45085       .12884      5336.81373
   .24367         6.21874          1.04412      3588.04677    -59.04341     266.03209       .12826      5331.44794
   .33546         6.24187          1.04901      3602.92817    -58.43217     264.55600       .12771      5326.05596
   .43214         6.26575          1.05408      3618.33344    -57.79477     263.03426       .12711      5320.44612
   .53375         6.29039          1.05932      3634.23296    -57.13796     261.45949       .12649      5314.63414
   .63983         6.31578          1.06472      3650.60381    -56.45224     259.83609       .12586      5308.[illegible]3
   .74933         6.34191          1.07027      3667.42465    -55.75179     258.17229       .12519      5301.80755
   .86160         6.36877          1.07597      3684.68411    -55.03484     256.46337       .12452      5294.92275
   .91783         6.38111          1.07856      3692.56331    -54.73[illegible]9   255.64339       .12421      5291.4[illegible]
```

NOT REPRODUCIBLE

FIGURE 11. (Continued)

NOT REPRODUCIBLE

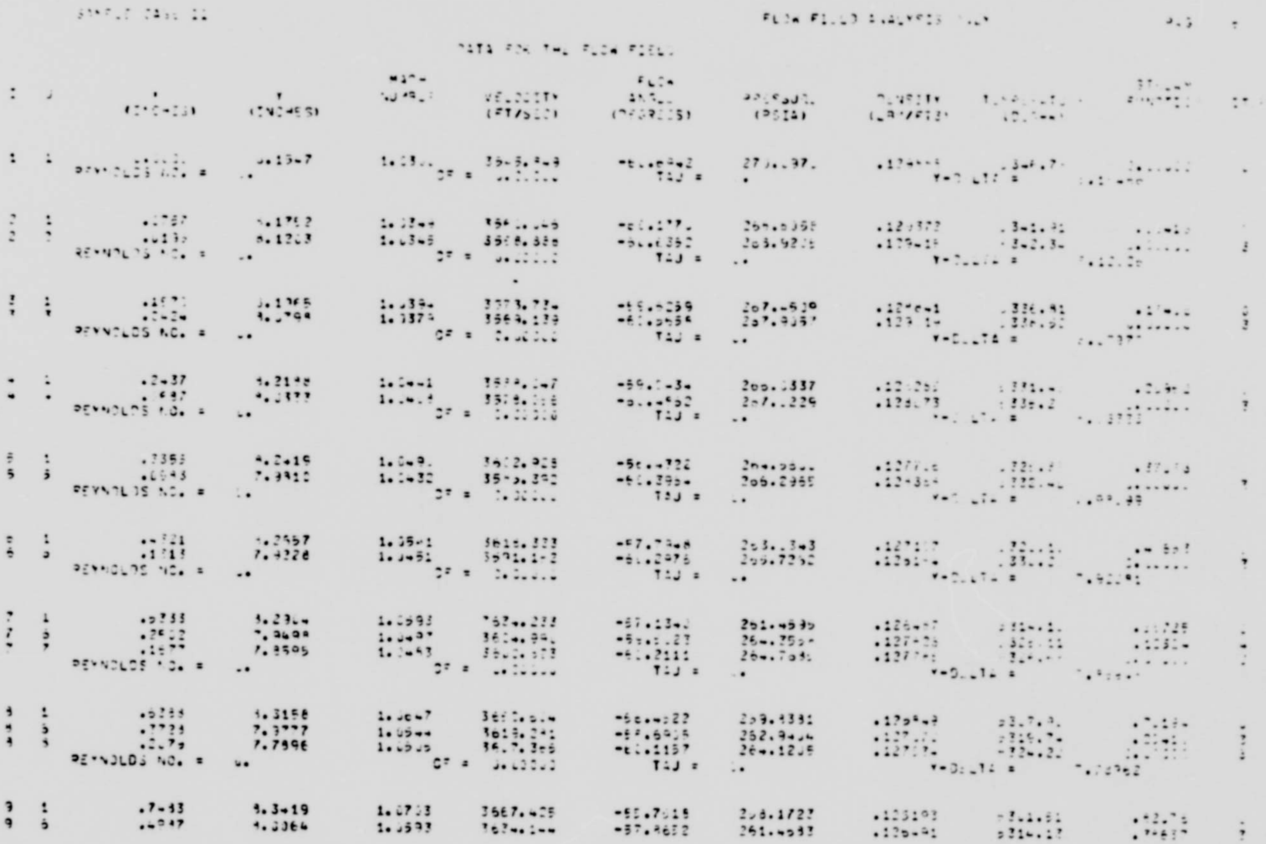

FIGURE 11. (Continued)

```
        SAMPLE CASE II                                                                    FLOW FIELD ANALYSIS ONLY                 PAGE  16

                                                MACH                        FLOW                                                          STREAM
 I   J         X              Y              NUMBER      VELOCITY         ANGLE          PRESSURE       DENSITY      TEMPERATURE       FUNCTION   ITER
            (INCHES)       (INCHES)                      (FT/SEC)       (DEGREES)        (PSIA)        (LBM/FT3)      (DEG-R)

47   6      1.3907         8.1213         2.5990        7109.929        -5.9311         23.1132        .017504       3376.67          .93523       3
47  11      2.0344         7.7719         2.5861        7093.174        -5.0624         23.5479        .017932       3391.41          .85518       3
47  16      2.6797         7.3132         2.5991        7110.160        -6.3392         23.1110        .017560       3376.64          .79250       3
47  21      3.7029         6.8495         2.6003        7112.211        -6.7159         23.0540        .017504       3375.13          .65676       3
47  26      4.6083         6.3394         2.5693        7095.353        -6.9007         23.5075        .017843       3367.38          .55870       3
47  31      5.7564         5.5322         2.5833        7085.825        -7.2149         23.7652        .019005       3344.61          .44341       3
47  36      7.2100         4.7746         2.5675        7061.374        -7.7475         24.4395        .018413       3412.60          .32493       3
47  41      9.5636         3.3991         2.5410        7019.859        -9.1975         25.0144        .019137       3442.93          .14069       3
47  46     12.1870         1.6344         2.4624        6893.054       -12.2417         29.4695        .021447       3534.59          .01235       4
47  47     12.4959         1.4328         2.4657        6898.385       -13.6759         29.2931        .021346       3530.76         0.00000       3
        REYNOLDS NO. =    0.                           CF =   0.00000           TAU =   0.                     Y-DELTA =      1.40231
        TRUNCATED AXIAL THRUST =   34275.6000 LBF          SPECIFIC IMPULSE =  231.4783 LBF-SEC/LBM
        TRUNCATED VACUUM AXIAL THRUST =   37504.3941 LBF           VACUUM SPECIFIC IMPULSE =  253.2759 LBF-SEC/LBM
```

NOT REPRODUCIBLE

FIGURE 11. (Concluded)

3. SAMPLE CASES III, IV, V and VI

Figures 12, 13, 14 and 15 are listings of the data packs for Sample Cases III, IV, V and VI, respectively. A variety of options are represented as summarized in Fig. 7. The data listings in conjunction with Section IV, INPUT, are self explanatory. The computational times for all six sample cases on Purdue's CDC 6500 are as follows:

CASE	COMPUTATIONAL TIME (SECONDS)
I	199
II	60
III	230
IV	66
V	109
VI	108

```
  SAMPLE CASE III
  FIXED COWL LIP RADIUS
 1    0    2   20
11    0    1    0    1  100    1    1    0    1    0    0    0
500.0     6000.0    56.0      1.23
1.03      -58.0     8.2       0.05      1.5
0.0       50.0      12.5
0.01      0.0001    1.0
8.5
-13.0
```

FIGURE 12. DATA DECK FOR SAMPLE CASE III

```
  SAMPLE CASE IV
  FIXED INLET OPTION
 1    1    5   20
15    0    1    0    1  100    1    1    0    1    0    0    0
500.0     6000.0    56.0      1.23
1.08      -53.0     9.5       0.05      2.0
0.0       148.077   10.0      0.0       0.0       0.0
0.005     0.0001    1.0       0.0       0.0
6.0       -60.0
-16.0      3.5
```

FIGURE 13. DATA DECK FOR SAMPLE CASE IV

```
     SAMPLE CASE V
CONSTANT LENGTH WITH BOUNDARY LAYER
    1     1     3    20
   11     0     1     0     1   100     1     1     0     1     0     0     0
500.0        6000.0     56.0       1.23
1.03         -58.0      10.0       0.05      2.0
14.7         148.077    12.5       0.125     1.3    E-14 5.0  E-06 6000.0
0.0075       0.0001     1.0        0.0       0.0
-13.0        4.0
```

FIGURE 14. DATA DECK FOR SAMPLE CASE V

```
     SAMPLE CASE VI
     CONSTANT SURFACE AREA
    1     1     3    10
   11     0     1     0     1   100     1     1     0     1     0     0     0
500.0        6000.0     56.0       1.23
1.03         -58.0      10.0       0.05      2.0
14.7         148.077    12.5       0.0       0.0         0.0
0.0075       0.0001     0.0        1.0       0.0
-13.0        4.0
```

FIGURE 15. DATA DECK FOR SAMPLE CASE VI

REFERENCES

1. Johnson, Gearold R., Thompson, H. Doyle, and Hoffman, Joe D., "Design of Maximum Thrust Plug Nozzles with Variable Inlet Geometry. Vol. I, Theoretical Development and Results", Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Technical Report No. AFAPL-TR-70-75, Vol. I, October 1970.