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RECTANGULAR-WIRE SPRING DESIGN

TECHNICAL REPORT

Henry Swieskowski

December 1969

SCIENCE & TECHNOLOGY LABORATORY

RESEARCH & ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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Mathematical development of a direct and simplified design method for rectangular wire springs is presented. Two numerical design examples are given. Case I, the initial load at assembled height, is the major load requirement. Case II, the energy capacity over the working stroke, is the major load requirement. The two basic rectangular wire configurations of the flat-wound and the edge-wound springs are considered separately. The analysis shows the variation of the final stress with respect to the compression ratio

$$I = \frac{F_2}{W} = \frac{\text{total deflection}}{\text{working stroke}}$$

for different load and configuration conditions.

FOREWORD

This report is a reprint of an article that was published in the technical magazine, "Machine Design".

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RECTANGULAR-WIRE SPRING DESIGN

...without tedious trial and error

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Rock Island, Ill.

THE TROUBLE with the usual rectangular-wire spring design method is that it is not direct. The approach presented in this article overcomes this obstacle in that it is based on the most basic design criteria—load and space requirements. As an additional refinement, this approach is developed for two common situations—when either the initial load or the energy capacity of the spring is specified. The two basic rectangular-wire spring configurations—flat-wound and edge-wound, Fig. 1—are treated separately. The usual design method is based on these equations:

$$R = \frac{P}{F} = \frac{K_1 G b t^3}{D^3 N} \qquad (1)$$

$$S = \frac{K_1 G t F}{K_2 D^2 N} \qquad (2)$$

where values for K_1 and K_2 are available for various valves of D/t and D/b.* This design technique involves a trial and error procedure and is cumbersome.

A more direct approach involves algebraic expressions that closely approximate the values of K_1 and K_2. By applying the method of least squares to curves for K_1 and K_2, the following expressions are obtained:

$$K_1 = 0.202 \left(\frac{b}{t}\right)^{0.451} \qquad (3)$$

$$K_2 = 0.416 \left(\frac{b}{t}\right)^{0.228} \qquad (4)$$

The following equations are used in the development of the design method:

Active solid height for edge-wound springs,

$$H = Nt \qquad (5)$$

Active solid height for flat-wound springs,

$$H = Nb \qquad (6)$$

Compression ratio (total deflection/working stroke),

$$I = \frac{F_2}{W} \qquad (7)$$

Spring load at minimum compressed height is

$$P_2 = \frac{P_1 I}{I - 1} \qquad (8)$$

*SAE *Manual on Design and Application of Helical and Spiral Springs*, SAE J795 July 1962

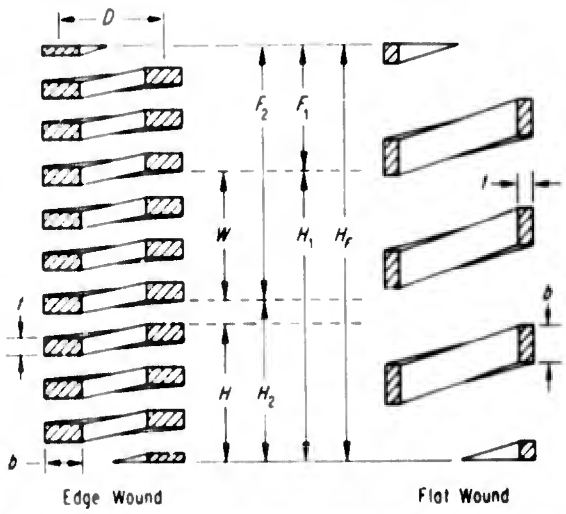

Fig. 1—Basic features of edge-wound and flat-wound rectangular wire springs.

Nomenclature

- b — Width of rectangular wire (long side), in.
- D — Mean coil diameter, in.
- D_h — Hole diameter (in which spring is installed), in.
- D_o — Outside coil diameter, in.
- E — Energy capacity of spring from assembled height to minimum compressed height, in. per lb
- F — Spring deflection, in.
- G — Modulus of torsion, psi
- H — Active solid height, in.
- H_f — Free height, in.
- H_1 — Assembled height, in.
- H_2 — Minimum compressed height, in
- I — Compression ratio
- K_1 — Deflection constant
- K_2 — Stress constant
- N — Number of active coils
- N_t — Total coils
- P — Spring load, lb
- P' — Mean load, lb $(P_1 + P_2)/2$
- R — Load-deflection rate, lb per in.
- S — Stress, psi
- t — Thickness of rectangular wire (short side), in.
- W — Working stroke (from assembled height to minimum compressed height), in

Subscripts
- 1 Refers to value at assembled height
- 2 Refers to value at minimum compressed height

Edge-Wound Springs

Initial Load Required: From Equations 1, 3, and 5:

$$\frac{P}{F} = \frac{0.202\left(\frac{b}{t}\right)^{1.151} G t^5}{D^3 H} \qquad (9)$$

The combination of Equations 2, 3, 4, and 5 results in

$$S = \frac{0.485\left(\frac{b}{t}\right)^{0.223} G F t^2}{D^2 H} \qquad (10)$$

From Equations 7, 8, 9, and 10, it follows that

$$S_2 = \frac{0.921\, G^{0.6} W^{0.6} I P_1^{0.4}}{\left(\frac{b}{t}\right)^{0.357} D^{0.8} H^{0.6} (I-1)^{0.4}} \qquad (11)$$

Equation 11 shows the variations of final stress with values of torsion modulus, initial load, working stroke, and approximated values of mean coil diameter and active solid height.

It is important to determine the effect that compression ratio has on final stress, and in particular, what is the optimum compression ratio that gives the minimum final stress. By differentiation of Equation 10 with respect to I, and by setting the resulting expression equal to zero, minimum final stress is determined with $I = 5/3$.

A modified form of Equation 11 is plotted in Fig. 2 and shows the variation of final stress with compression ratio. Although final stress is a minimum when $I = 5/3$, Fig. 2 shows that values of $1.5 \leq I \leq 2.0$ is a favorable design interval because within this range final stress is less than 3 percent above the minimum value. Fig. 2 also shows how final stress varies with different values of b/t. In applications where diametral space is not limited, the ratio b/t should be as large as possible.

Example: These factors are typical of the information a designer has to work with:

Assembled height, H_1 .... 5.905 in.
Load at assembled height, P_1 .... 93 lb
Minimum compressed height, H_2 .... 3.500 in.
Modulus of torsion, G .... 11.5 x 10^6 psi
Hole diameter in which spring must fit, D_h .... 2.118 in.
Maximum final stress, S_2 .... 100,000 psi

Step 1: Estimate realistic values for mean coil diameter and active solid height.

Mean coil diameter is usually between $0.75D_h$ and $0.90D_h$ (which roughly corresponds with spring indexes of 3 to 9). For edge-wound springs, let $D = 0.85D_h = 0.85(2.118) = 1.800$ in. For ends closed and ground, $H = 0.8H_2$ is a good choice. Therefore, $H = 0.8(3.500) = 2.800$ in.

Step 2: Solve for b/t using Equation 11. For minimum final stress, let $I = 5/3$.

$$\left(\frac{b}{t}\right)^{0.357} = \frac{1.805(11.5\times10^6)^{0.6}(2.405)^{0.6}(93)^{0.4}}{100{,}000(1.800)^{0.8}(2.800)^{0.6}}$$

and $b/t = 1.263$.

Step 3: Determine the load deflection rate for $I = 5/3$ and $P_1 = 93$ lb ($P_2 = 232$ lb, from Equation 8). Load deflection rate, $R = (P_2 - P_1)/W = 58$ lb per in.

Step 4: Compute the thickness t from Equation 9.

$$t = \left[\frac{58(1.8)^3(2.80)}{0.202(1.263)^{1.151}(11.5\times10^6)}\right]^{1/5} = 0.196 \text{ in.}$$

and $b = 1.263\,(0.196) = 0.248$ in.

Step 5: Calculate the number of active coils, outside coil diameter, and free height. $N = H/t = 2.80/0.196 = 14.2$; $D_o = D + b = 1.8 + 0.248 = 2.048$ in.; and $H_f = H_2 + F_2 = 3.500 + 4.008 = 7.508$ in.

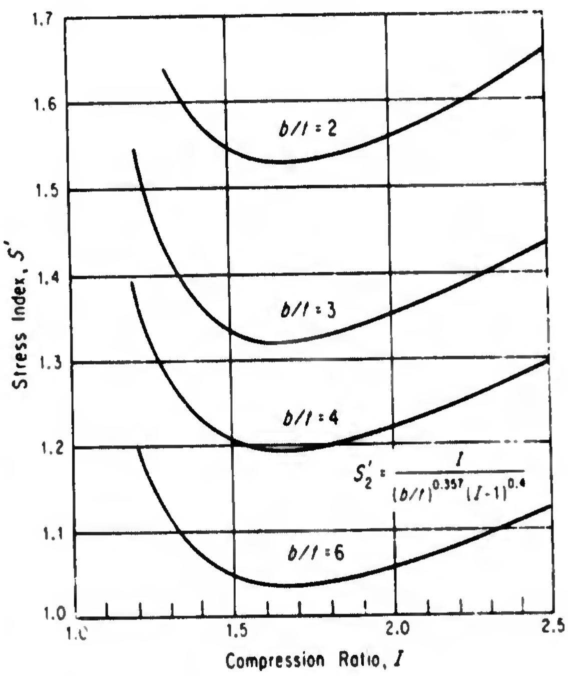

Fig. 2—Variation of stress and compression ratio for edge-wound springs for case where initial load is required. Recommended design region is values of I between 1.5 and 2.0.

Energy Capacity Required: Energy content of a spring (either edge-wound or flat-wound) is related to the initial load,

$$P_1 = \frac{(I-1)E}{(I-0.5)W} \qquad (12)$$

Using Equation 12, Equation 11 can be rewritten

$$S_2 = \frac{0.921\, G^{0.6} I W^{0.2} E^{0.4}}{\left(\frac{b}{t}\right)^{0.357} D^{0.8} H^{0.6} (I-0.5)^{0.4}} \qquad (13)$$

A nomograph based on Equation 13 is shown in Fig. 3. Stress reaches a minimum for a compression-factor value of $I = 1$. (This is in contrast to the previous example where $I = 5/3$, for the case where the initial load is required.) However, the range $I = 1$ to $I = 1.2$ can be considered favorable, because final stress within this range is not more than 5 percent above the minimum. Again, it is recommended that the ratio b/t be made as large as the available space permits because stress is inversely proportional to b/t.

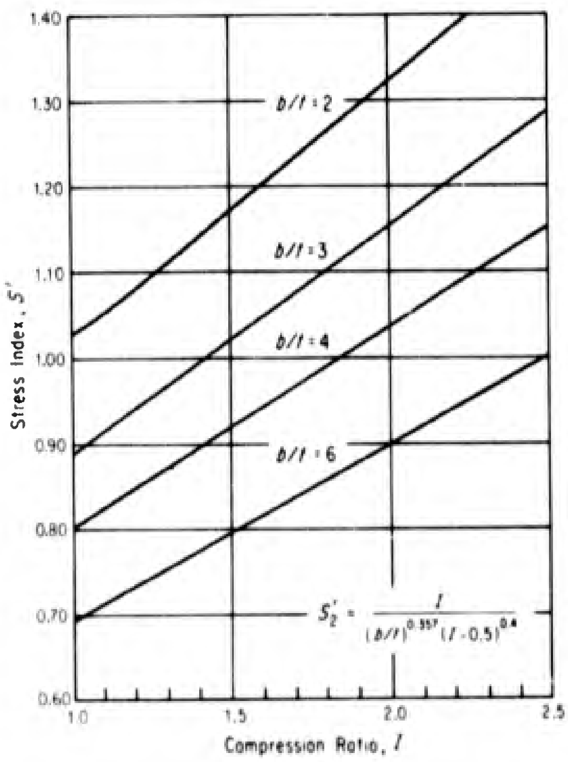

Fig. 3—Variation of stress and compression ratio when energy capacity is design requirement. Recommended design region is values of I between 1.0 and 1.2.

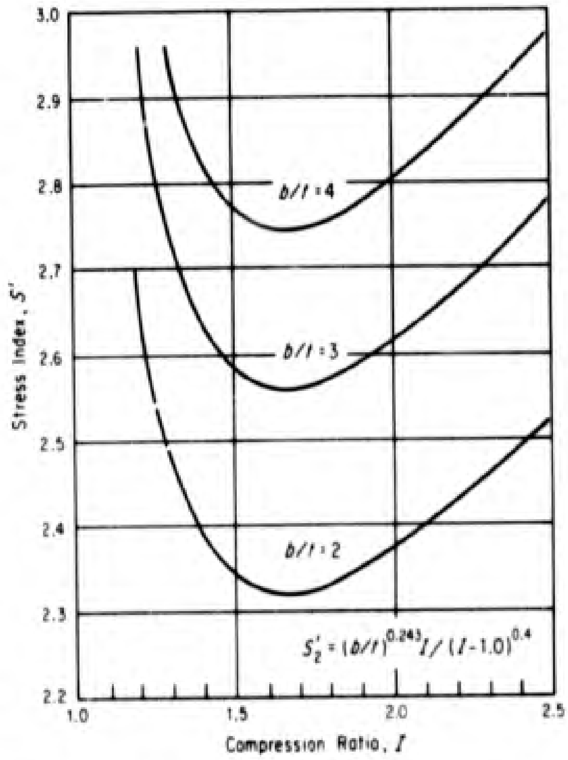

Fig. 4—Variation of stress and compression ratio for flat-wound springs for various b/t ratios for case where initial load is required.

Flat-Wound Springs

Initial Load Required: Equations 1, 3, and 6 can be combined to give

$$\frac{P}{F} = \frac{0.202\left(\frac{b}{t}\right)^{2.451} G t^5}{D^3 H} \tag{14}$$

From Equations 2, 3, 4, and 6,

$$S = \frac{0.485\left(\frac{b}{t}\right)^{1.223} G F t^2}{D^2 H} \tag{15}$$

The relationship between final stress and initial load, derived from Equations 7, 8, 14, and 15, is:

$$S_2 = \frac{0.921\left(\frac{b}{t}\right)^{0.243} G^{0.6} W^{0.6} I P_1^{0.4}}{D^{0.8} H^{0.6} (I - 1)^{0.4}} \tag{16}$$

In the case of flat-wound springs, the final stress is directly proportional to b/t, Fig. 4. Optimum design range for the compression ratio remains the same as for edge-wound springs, $1.5 \leq I < 2.0$.

Energy Capacity Required: Substitution of Equation 12 into Equation 16 results in

$$S_2 = \frac{0.921\left(\frac{b}{t}\right)^{0.243} G^{0.6} W^{0.2} I E^{0.4}}{D^{0.8} H^{0.6} (I - 0.5)^{0.4}} \tag{17}$$

Again, minimum final stress is obtained when $I = 1$; however, the range from $1 < I \leq 1.2$ is considered favorable for spring design.

Example: Typical design parameters are:

Assembled height, H_1 .... 8.830 in.
Minimum compressed height, H_2 .... 8.200 in.
Energy capacity, E .... 524 in. per lb
Modulus of torsion, G .... 11.5×10^6 psi
Hole daimeter, D_h .... 1.437 in.
Maximum final stress, S_2 .... 90,000 psi

From given requirements: working stroke $W = 8.830 - 8.200 = 0.630$ in.

Step 1: Estimate values for the mean coil diameter and active solid height. For flat-wound springs, a reasonable choice for mean coil diameter is $D = 0.8D_h = 0.8(1.437) = 1.150$ in.

For ends closed and ground, let $H = 0.8H_2 = 0.8(8.200) = 6.560$ in. To have some precompression at assembled heights, let $I = 1.1$.

Step 2: Solve for b/t using Equation 17.

$$\left(\frac{b}{t}\right)^{0.243} = \frac{90{,}000(1.15)^{0.8}(6.560)^{0.6}}{1.243(11.5 \times 10^6)^{0.6}(0.630)^{0.2}(524)^{0.4}}$$

and $b/t = 2.953$

Step 3: Determine the load-deflection rate. For $I = 1.1$ and $F_2 = 0.693$ from Equation 7, $F_1 = F_2 - W = 0.693 - 0.630 = 0.063$ in.; for $E = 524$ in. per lb, the mean load equals $E/W = 524/0.630 = 832$ lb.

Load deflection rate $R = P'/(F_1 + W/2) = 832/(0.063 + 0.630/2) = 2{,}200$ lb per in.

Step 4: Calculate thickness t with Equation 14.

$$t = \left[\frac{2{,}200(1.15)^3 6.56}{0.202(2.953)^{2.451}(11.5 \times 10^6)}\right]^{1/5} = 0.232 \text{ in.}$$

and $b = 2.953(0.232) = 0.685$ in.

Step 5: Compute the number of active coils, outside coil diameter, and free height. $N = H/b = 6.56/0.685 = 9.6$; $D_o = D + t = 1.150 + 0.232 = 1.382$ in.; $H_f = H_2 + F_2 = 8.200 + 0.693 = 8.893$ in.

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Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U. S. Army Weapons Command Research & Engineering Directorate Rock Island, Illinois 61201	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

RECTANGULAR-WIRE SPRING DESIGN (U)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Henry Swieskowski

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1969	12	1

8a. CONTRACT OR GRANT NO. b. PROJECT NO. DA 1W56260A607 c. AMS Code 5525.11.80700.03 d.	9a. ORIGINATOR'S REPORT NUMBER(S) RE-TR 70-125 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U. S. Army Weapons Command

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Rectangular 2. Spring 3. Design 4. Optimum 5. Edge and flat wound						

Unclassified
Security Classification

AD ____________ Accession No. ____________

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