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AN ANALYSIS OF PROGRESS CURVE
CONCEPTUAL ADVANCES AND
PROGRESS CURVE USES, SINCE 1956

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Vincent Colasuonno
Major USAF

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AN ANALYSIS OF PROGRESS CURVE CONCEPTUAL ADVANCES AND PROGRESS CURVE USES, SINCE 1956

THESIS

Presented to the Faculty of the School of Engineering of

the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Major USAF

Graduate Systems Management

September 1967

Preface

This paper is the result of my attempt to consolidate, under one cover, the state-of-the-art pertaining to progress curve theory. Since this topic does not contain a formal and unequivocal body of knowledge, the approach used was to review and evaluate the many reports and articles published on the subject so as to piece together that which appeared significant. The material was organized to lead from historical developments to present concepts and uses. From this vantage point, the factors which cause the progress curve to occur can be better understood, as can misuses of the concept.

The bibliography by no means exhausts the literature available. Much of the literature is repetitious; however, of the contemporary authors, the writings of R. W. Conway and A. Schultz, W. B. Hirschmann, R. P. Zieke, S. L. Young, and N. Baloff are noteworthy.

I would be delinquent if I did not acknowledge my debt to my thesis advisor, Dr. Hermann Enzer, faculty member of the Systems Management Department, Air Force Institute of Technology. His constructive critique was invaluable in shaping the final form of this paper.

I am particularly indebted to my wife for her encouragement and stoical understanding under very trying circumstances, and to my children, for their remarkable patience.

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Abstract

The progress curve concept is traced in historical developments through 1956. The paper then concentrates on extensions of concepts through review and evaluation of articles and reports. Non-linearity is discussed, as is capital-intensive production and new estimating relationships. Innovations in its use are examined. These include nonhomogeneous production, value engineering, second source procurements, and optimal lot size. Present uses of the curve, factors in its make-up, and misuse of the concept are discussed. While parts of the overall "learning" effect can be optimized and/or quantified, qualitative factors still exist. A quantitative-qualitative view of the progress curve is therefore recommended.

AN ANALYSIS OF PROGRESS CURVE CONCEPTUAL ADVANCES AND PROGRESS CURVE USES, SINCE 1956

I. Introduction

In 1936, Mr. T. P. Wright published an article entitled, "Factors Affecting the Cost of Airplanes". Wright hypothesized that the cumulative average labor cost for any quantity of airplanes produced decreases by a constant amount as that quantity of airplanes is doubled. This was the basic description of the phenomenon which is commonly called the learning curve. Other commonly used descriptors of this phenomenon are - progress curve, improvement curve, manufacturing progress curve/function, and experience curve, to name a few.

This report will use the term, "progress curve", rather than "learning curve". Learning curves imply worker learning, and although it cannot be denied that worker learning contributes to reduction in man-hours and/or cost during a production process, it is not at all clear just how much of the total contribution can be credited to worker learning. Since it is recognized that management innovations, engineering changes, and work simplification make a contribution to time/cost reduction, the term, progress curve, is felt to be a better descriptor of the total process.

Subsequent to Wright's article, a number of articles and reports were written on the phenomenon. Then, in 1956, Dr. Harold Asher published a research report entitled, "Cost-Quantity Relationships in the Airframe Industry" (Ref. 43). In it, Dr. Asher summarizes significant contributions to the development of the theory of progress curves. The bulk of his study is then primarily directed toward an examination of whether or not there exists "... sufficient empirical evidence to question the validity of the linear progress curve as applied to both the unit labor cost and the unit production cost for airframes" (Ref. 43:13). The meaning of a linear progress curve is explained in Chapter II.

Since Dr. Asher wrote his comprehensive study, there have been many articles and reports written on the progress curve. However, no known study has directed itself specifically to an investigation of advances in theoretical concepts concerning progress curves, or to an examination of the present use of progress curves in industry. Consequently, the primary purpose of this study is to survey the literature since 1956 and report on these two aspects of the subject. Auxiliary purposes include a look at some innovations in the use of progress curves, the use of the concept in capital intensive industries, factors which cause the phenomenon to occur, including worker learning, and misapplications of the progress curve.

It should be noted that although this study is a literature survey, industrial organizations were contacted for information on their use of the concept. It might be best said at this point that industrial organizations were very reluctant to discuss their specific usage, or application, of the progress curve concept. It was easy enough to secure generalized written material, however, company application was considered proprietary information and was unobtainable.

To the reader who is unfamiliar with this phenomenon, it is strongly recommended that Asher's report, at least, be studied, if not the original articles which he summarizes. As stated, the primary purpose of this report is to inquire into theoretical advances and uses of the progress curve. Although Chapter II deals with historical developments, it is treated in a cursory manner. Such cursory treatment should not be construed as a slight on its importance, however. It is introduced to give perspective for that which follows.

II. Historical Development of the Progress Curve Concept

Pre-1956

T. P. Wright. The earliest known work on the progress curve phenomenon was probably done by T. P. Wright who stated in his 1936 article, "Factors Affecting the Cost of Airplanes", that he "started his studies of the variation of cost with quantity in 1922" (Ref. 37:122). Wright hypothesized that the average labor cost (cumulative average) for any quantity of airplanes produced decreases by a constant amount as that quantity of airplanes is doubled. This can be expressed by the function

$$Y = aX^b$$

where "Y" is the cumulative average direct labor hours, "X" is the cumulative unit produced, "a" is the direct labor hours for the first unit, and "b" is the "slope" of the progress curve. When this exponential function is plotted on log-log paper, it becomes a straight line. This has prompted the layman's conceptualization of the progress curve as a straight line, when actually it is only the logarithmic form of the function which plots as a straight line. Throughout this study, linearity should be understood to mean linear on logarithmic grids.

Wright found that "Material also decreases in cost as quantity increases..." (Ref. 37:125), as well as overhead cost. He stated that his derivation was based on the assumption that no major changes are introduced during construction. Wright further indicated that the total cost curve changed its slope at unit 100, 1000 and 10,000. This would indicate that he considered the function as possessing four linear segments - a concept which will be discussed in more detail in Chapter III.

J. R. Crawford. Crawford's work was based on data from World War II aircraft production. In contrast to Wright, Crawford held that as quantity doubled, direct labor hours per unit (not cumulative average hours) decreased at a constant rate. Thus Crawford proposed the unit curve as indicative of the phenomenon rather than the cumulative average curve. He did, however, develop a cumulative average formula and a cumulative total man-hour formula as well as the asymptotic functions for these two relationships. He also attempted to simplify his cumulative total formula as well as to find new forms for the progress curve. Apparently, little of Crawford's simplifying relationships or his new forms have been accepted or employed by industry or the Air Force (Ref. 43:22).

A. B. Berghell. Berghell's work with progress curves led him to conclude that if cumulative average curves are plotted (for four different aircraft which he selected), the slopes of the curves do not differ significantly. He noted that labor hours required per particular aircraft are caused, in part, by aircraft weight. Thus, he divided direct labor hours by aircraft weight and worked with direct man-hours per pound. He concluded that a heavier airframe will generally require fewer man-hours per pound than a lighter aircraft at the same cumulative number of aircraft produced (Ref. 43:26). This would seem plausible. Broadly speaking, similar components are required to produce a heavy aircraft, as a light aircraft, and since in general, similar operations are required to produce and assemble these components, the hours per pound would be lower for a heavier airframe. This reasoning would hold only for aircraft of similar complexity however, for surely the complexity of the aircraft must affect labor hours. Such a concept seems to have sustenance in that K. A. Middleton found that "more man hours of labor are usually required to produce a hundred thousand pounds of combat planes than to produce a hundred thousand pounds of light training planes" (Ref. 43:27). Since combat aircraft are surely more complex than light training aircraft, it would follow that aircraft complexity affects labor hours per pound.

K. A. Middleton. Middleton's work does not support Berghell's contention that progress curve slopes do not differ significantly; "... contrary to Berghell's notion, the progress curves for the various aircraft chosen by Middleton did not exhibit the same slope" (Ref. 43:27). In an intensive review of the literature on learning curves, no other author was found to agree with Berghell's similar slope conclusion.

G. W. Carr. The linearity of the progress curve has been challenged by G. W. Carr (Ref. 11). He contends that costs are governed by aircraft type or performance, quantity, rate of delivery, and special considerations. Of these, Carr states that rate of delivery is probably the most important. With an increased rate of production, new crews are brought into the production process in sequence at different points in time. When each new crew is assigned, it is at its first unit of learning, whereas the first crew may be for example, on its fifteenth unit. This would cause an S-shape curve which starts out above a linear cumulative average line (which would presumably occur if all crews started at the same point in time) and then drops below the linear projection before reaching a "flat ... beyond which only negligible improvement may be expected" (Ref. 11:77). Carr cites an interesting example of two competing manufacturers who had almost identical overall costs in their respective plants, yet one used "high-priced labor with high man-hours and a low tool budget, while the other expended large amounts for tooling and had a labor-production rate 20% below that of his competitor" (Ref. 11:77). This is evidently the basis of Carr's concluding remark that the "only sound basis of final-cost comparison is found in dollars per airplane, or per pound of air-frame weight - not in man-hours of shop work." (Ref. 11:77). Thus, Carr not only disputes the linearity concept, he also infers (rather sketchily, though) that man-hours is not a suitable dependent variable.

W. Z. Hirsch. Hirsch did an empirical study on progress curve theory in 1952 (Ref. 17). He gathered departmental data for analysis, finding that the machining operation had an average slope of 87.1%, while the assembly operation's average slope was 75.4%. This reinforces the concept of a steeper slope for labor-intensive as opposed to capital-intensive production. In a subsequent article in 1956, (Ref. 16) Hirsch concluded that progress curve slopes vary enough that it would seem promising to separate manufacturing into machine and assembly work. He suggests a further breakdown of machine work based on whether a part was or was not machined for the first time. This suggestion was prompted by the apparent small uniformity in the various machine operation slopes, "a phenomenon which might find its explanation in the fact that the ratio between parts machined for the first time and parts machined many times previously varies from product to product" (Ref. 16:143).

P. Guibert. A comprehensive progress curve study was conducted by P. Guibert. Unlike most American authors, Guibert introduces the rate of production as a variable affecting unit labor cost. For a mathematical treatment of the effect of rate of production, see the original work of Asher (Ref. 43:32-34). He also holds that a horizontal asymptote is approached by the progress curve after a large number of units is approached. A Boeing Airplane Company study (Ref. 43:38) also reaches this conclusion. In addition, the Boeing report relates a rather important, and perhaps intuitively obvious argument that initial planning and tooling affect both the cost of the first unit and the slope of the progress curve. With intense initial planning and tooling, the man-hours of the first unit must certainly be reduced. In addition, the slope would be flatter since less improvement could be made. The converse would be true with inadequate pre-planning.

Crawford-Strauss. The Crawford-Strauss Study (Ref. 43:39) contributes additional factors which may cause lower man-hour per pound values in a progress curve. The authors show that the progress curve for bombers is lower than either fighters or transport aircraft. They observe that the bomber curve is lower because bombers enjoyed a high priority and its size permitted greater access in the assembly operation. Additionally, fighters were more complex and had more design changes, while transports never enjoyed a high-production priority.

Stanford-B Curve. The well-known Stanford Research Institute study concluded that early units of production do not tend to follow a straight line, rather they tend to form a convex (upward) curve. The equation proposed is

$$Y = a(B + X)^b$$

where "Y", "X", and "b" are the same as in the unit equation, "B" is an estimate of carry-over effects of progress gained on past production runs of similar products and is expressed as the number of equivalent units a new producer would have to complete to duplicate the performance of an experienced production line at unit one. The value of "a" is first unit cost only when "B" is zero, otherwise it is an estimate of what the first unit cost would have been without carry-over progress. Many authors claim that the Stanford - B curve is widely used. Two exceptions are Zieke who states that, "Stanford Research Institute no longer utilizes the function it developed because the evaluation of correct B values has proven difficult for current programs" (Ref. 39:30), and Reguero, who states that, "There are only a few samples of Learning Curve data that can be better described by the Stanford equation instead of by a straight line on log-log paper... The net result is that the Stanford equation is little used" (Ref. 60:219).

A. Alchian. Several studies of the progress curve were conducted by the Rand Corporation. Perhaps the most notable is that by Armen Alchian (Ref. 1). He reaches several interesting conclusions, one of which is that there was no evidence of any cessation of a decline in unit costs, based on the data he studied. The conclusion was qualified by his comment that it could not be determined whether or not the decline would cease for a substantially larger output than his data. Another finding was that the use of either an industry - wide average progress curve or a general airframe type curve (bomber, fighter, trainer) resulted in a 25% weighted average error of prediction for the first 1000 aircraft. Significantly, specific curves fitted to the past performance of a particular manufacturer resulted in 22% prediction errors - not a momentous improvement.

Harold Asher

Asher's work is broken into four major areas, that of progress curve data, the linear hypothesis, direct-labor progress curve, and production-cost progress curve. Significant ideas will be summarized below.

Concerning data (Ref. 43:Chap. 3), Asher deals mainly with direct labor, material, overhead, and subcontracting although, he also discusses engineering, tooling, and general and administrative expenses to some extent.

Asher asserts that airframe producers do not have a standardized method of collecting direct man-hour data. However, similarities do exist in that "work center" accounting as a basis for cost accumulation has been applied rather generally as well as "lot" costs rather than unit costs. As will be seen in a later chapter, lot data presents a problem in that if learning exists, the last unit in the lot takes less time to produce than the first one. Therefore, it would be theoretically incorrect to select the lot mid-point as representative of the average unit cost of the lot, especially in the early stages of production where the exponential curve changes shape rapidly. (See Chapter IV - Lot Mid-Points)

Concerning materials, he points out a unique problem exists in that, unlike labor which is measured in man-hours, materials cost is expressed in terms of dollars per unit or per pound, and "fluctuating materials prices will clearly affect the materials progress curve unless a suitable price index is used to deflate unit materials cost" (Ref. 43:52). In regards to overhead, Asher examined total overhead costs only, with no examination of its elements. He points out again on page 54, that since standardized accounting practices do not exist, each companies overhead charges will differ. (See Chapter IV - Overhead.)

Subcontracting data can be troublesome in that man-hours expended by subcontractors are usually not reported to prime contractors. Consequently, estimates of these hours are made by the prime contractor as a percentage of the total hours to build the aircraft (which is subject to progress curve reduction). It would seem that other errors, in addition to estimating errors, could become inherent in the data. Consider the case of a prime contractor building his tenth model of a four-engine aircraft for which the engines are subcontracted. The subcontractor has forty units of experience while the prime contractor has only ten. Yet, the prime contractor estimates engine man-hours based on his tenth unit of experience. If a large amount of subcontracting is done, a real problem may exist.

In addition, misinterpretation of the data may occur. For instance, airframe weight which is used to determine labor hours per pound, is defined to exclude the weight of government-furnished equipment and certain kinds of contractor-furnished equipment. However, "man hours include the time required to install certain equipment the weight of which is not included in airframe weight" (Ref. 43:59). Thus man-hours per pound is neither man-hours per pound of <u>airframe</u>

weight nor is it man-hours per pound of total aircraft weight.

Another problem which can assume major proportions in data handling, is that of changes to a model in production. Sudden increases in man-hours may appear when plotting unit or cumulative average data. If it is known that a model change occurred at that unit of production, the disturbance in man-hour reduction can be accounted for (See Chapter IV - Nonhomogeneous Production). Other causes of erratic disturbance may be such as excessive labor turnover or a strike. It would be prudent to be aware of such occurance so that order can be made of seemingly unreliable data. As to whether or not a regression equation should include the model change effect, Asher states that it is common practice to do so. However, he adds, "it is not clear that the regression equations for several different models are useful for the purpose of making comparisons or generalizations, since two models rarely experience the same number of changes" (Ref. 43:64).

In the introduction, it was noted that the main purpose of Asher's work was to examine the linear hypothesis of progress curves. He does this in Chapter IV of his text.

The powerful argument is advanced that if a progress curve is linear on logarithmic grids, and it is composed of several component curves, then the component curves must all have the same slope. Component curves would be the department curves which go into making the total product. It is easily envisioned that a department has several, if not many, production jobs toward building a product. The implication is apparent. All production jobs must have the same rate of learning if the product progress curve is linear. Such a conclusion may be questioned on intuitive grounds alone. Any number of empirical studies allow it to be rejected altogether. Even if it is allowed that department curves are linear, but of different slope, then the product progress curve still cannot be linear. (See Chapter III - Non-Linearity.)

An obvious retort is that if the progress curve does not depart significantly from linearity, it need not be rejected for practical reasons, especially if no alternative technique is better suited for predictive purposes. The simplicity of the linear curve is apparently a prime motivator for its continued use. However, with the advent of rapid computer solutions to tedious problems, the simplicity advantage may no longer be valid. Still, one would need to determine if the additional benefit of a more precise solution out-weighed the cost of its implementation.

Asher also treats the relationship between man-hours for the first unit and the slope of the resultant curve. He states that, "there is little doubt that these two variables (slope and "a" value) are related to one another" (Ref. 43:73). Apparently, pre-production planning and tooling have a definite effect on these variables. In addition, Asher contends that other factors which may affect slopes and the first unit cost are the producer's familiarity with a particular model, the percentage of parts of the new model which are common to the old model, and the producer's accumulated airframe experience. (See Chapter III - RAND.)

Concerning rate of production, Asher asserts that although it is agreed that rate of production does have an effect on man-hour cost or production cost, "... it is felt to be of minor importance, within a certain range of rates of production, and definitely subordinate to the effect of cumulative production" (Ref. 43:86). He argues that man-hour cost generally continue to decline even after peak rate is reached. Even with setup time as a cost factor, he contends that "after a certain number of pieces are produced with a given setup (the number was not identified) an increase in the rate of production will result in only negligible savings" (Ref. 43:87). This would seem to be an appropriate conclusion as qualified by the range of rates. If extreme examples of rates are viewed as not abnormal, a re-examination might be beneficial.

Asher examines shop group man-hour data in his fifth chapter. (See Chapter VII - Data Reliability.) He states that in general, a linear approximation is reasonable for initial quantities, however the slopes are unmistakably different which, of course, leads to a non-linear "sum of the shop group" curve. Furthermore, if a linear extrapolation is made from unit 100 to unit 1000, an error of over 25% can occur at unit 1000. If however, a linear projection are made "... between, say, units 600 and 1000; the resulting error would be negligible. If made at all, linear projections of the unit labor curve should be made for a relatively small quantity of aircraft" (Ref. 43:101).

If the linear hypothesis is used for scheduling and man-hour budgeting, it is felt by many that the resultant curve will be biased toward linearity. This presumably is due to workers not exceeding the schedule if it is easily met, or getting a revised schedule if it cannot be met. (See Chapter VII - Data Reliability.) Asher asserts however, that "... the available evidence certainly indicates that the linear bias is not nearly as tenacious as has been generally supposed. That is, even with a linear scheduling bias, the actual unit curve appears to depart from linearity" (Ref. 43:103).

Materials cost and overhead are treated next by Asher. Materials costs are found to decline with quantity. Although learning may allow materials to be used more efficiently, and ordered in sizes and shapes to reduce scrap, Asher contends that suppliers place a limit on volume price reductions, therefore "the theoretical existence of a minimum materials cost can hardly be disputed" (Ref. 43:114). This further reinforces his belief that the linearity concept is in error. However, he suggests that for outputs less than 1000 units, a linear materials curve might be satisfactory.

Overhead continues to plague planners, cost estimators, analysts and the like. Asher reports that overhead rates for his samples ranged from 107% to 165% of direct labor cost (Ref. 43:117). He further reports that conceivably overhead allocation per unit could increase if an unusual situation arose whereby total plantwide effort declined. Normally though, since overhead is allocated as a percentage of direct labor expended, it declines with, and is parallel to the direct labor curve. Asher suggests that further studies of overhead should be directed toward those overhead elements which are sensitive to plantwide effort, and toward those which are not. (See Chapter IV - Overhead.)

Lastly, Asher discusses production cost (in terms of dollars per pound), and tooling and engineering costs. As would be expected, the production cost curve is non-linear. Since the materials curve is much flatter, its percentage of production costs increases from 10%

at unit one to almost 40% at unit 1000 (Ref. 43:120). Regarding tooling, Asher presents some equations developed by the Stanford Research Institute. He differentiates between initial tooling and maintenance tooling costs, and shows that later aircraft models "benefit" from tooling inherited from an earlier model. Similarly, regression equations are presented for engineering costs. It was found that in general, heavier type airframes appear to require more engineering than do lighter airframes.

Asher concludes that the progress curve is not linear beyond certain values of cumulative output. His study shows linearity for up to 300 units for the fighter aircraft data used. He suggests that small projections beyond unit 300 are acceptable. In addition, if the allowable error for a given problem is large, a linear curve projection may be useful. If the allowable error is small, he suggests the use of a convex curve structured from summing component curves.

Asher then hypothesizes that the apparent linearity for many World War II models was caused by the (then) infant aircraft industry switch from job-lot production to mass production which resulted in many new and efficient improvements in methods. "These improvements continued throughout the war period" (Ref. 43:130), allowing a lowering of an assumed minimum cost asymptote. With each successive lowering of this assumed asymptote, a progress curve would appear linear rather than tending to level off.

This concludes the summary of that which is felt to be significant in progress curve theory developments through 1956. The remainder of this study will deal with post-1956 material.

III. New Concepts in Progress Curve Theory

It would be encouraging if it could be reported that promising headway was being made in the continued investigation and formulation of this phenomenon. However, there is an appalling dearth of material concerning new approaches or direction of study. Whether this is caused by the appealing simplicity of prior theory, or a general inability to predict (and therefore quantify) the causes of the phenomenon, is not at all clear. At any rate, the research for this report could uncover no major, or even minor theoretical advances. What has been discovered are extensions of prior concepts, and attempts to formulate better estimating relationships. A discussion and analysis of these extensions follows.

Non-Linearity

As we have seen in Chapter II, the linear curve was suspect even in Wright's original article, and was the main subject of Asher's work. However, many contemporary writers blandly assert linearity, while only a few even mention that linearity may not hold. Two notable exceptions are R. W. Conway and A. Schultz, and Nicholas Baloff.

In their 1959 article, Conway and Schultz (Ref. 13) emphasize that linearity cannot hold for a total cost curve if the component curves are linear and their slopes are not equal. "Strictly speaking then, this precludes the use of the simple 'linear' model for operations, departments, sections and total of the same project. Theoretically it can at best apply to only one level" (Ref. 13:41).

This theoretical consideration would hold only if the total cost curve was derived by summing the component regression linear curves. One could, of course, have a situation in which the raw cost data for each unit were summed to arrive at a total unit cost schedule. When each of these unit total cost data points are plotted, a linear regression line may show an adequate fit. At the same time, component regression curves may possess different slopes, because a regression line is merely a "best fit" line to a collection of data points. It is not inconceivable then, that the distribution of raw data points, when summed vertically for each unit, result in a relatively good linear regression fit for a total cost curve, and individual component curve data points possess good linear regression characteristics, with all of the linear curves having different regression slopes.

This particular point has not been discussed in any of the literature surveyed. There is the possibility that the total cost curve slope may not be as dependent on component curve slopes as has been generally thought, because of the characteristics of regression fitting of data points. Replotting empirical data from prior studies may well provide some insight into the dependence or independence of slopes.

Non-linearity occurred in some of the empirical studies of Conway and Schultz. They state that "... certain individual operations experienced a sudden and marked reduction in the rate of progress", while others "... continue with approximately a constant rate of progress to the end of the data. The reason for this ... was never determined with any degree of certainty" (Ref. 13:45-46). This interesting, if not frustrating "conflict" in the data apparently remained unresolved. The work of Nicholar Baloff in 1966 may have shed new light on this particular aspect of the subject.

In two articles concerning machine-intensive production (Ref. 3 and 4), Baloff hypothesizes that the existence of "learning" is not restricted to labor-intensive and labor-paced manufacturing. This in itself is an upheaval of traditional thought and will be discussed shortly. For the moment we are interested in Baloff's view of linearity, or rather, non-linearity.

Baloff states: "In many forms of continuous, machine-intensive manufacture, it is possible to divide the manufacturing history of a new product or process into essentially two distinct phases - a 'startup phase' and a 'steady-state phase' " (Ref. 3:26). His startup phase is characterized by steady increases in productivity. Steady-state has the characteristic whereby "... productivity of manufacture can vary unsystematically about an approximately constant level for some considerable period of time" (Ref. 3:26), assuming the absence of significant changes in technology. In short, increasing productivity per unit of time terminates in the steady-state phase, where productivity per unit of time becomes constant.

Baloff notes that steady-state may not occur if the production run is relatively short, or is distinguished by a series of discontinuous runs. He observes that many World War II aircraft had short production histories with no steady-state operation. He also notes that Asher's work suggested that some airframes have shown distinct indications of a steady-state condition.

Baloff's interpretation of past studies, in addition to his own empirical work may be a key extension to theory. If indeed, progress curves reach a steady-state condition, given a process of sufficient length, then a much improved estimating relationship can be devised. Baloff writes that in 20 of the 28 cases analyzed, a sharply delineated two-phase production history was displayed. Of the other eight, productivity increased "throughout the available production histories. The key word is available, since the absence of steady-state phases in these instances can be attributed to data availability constraints" (Ref. 3:29). By data availability constraints, Baloff evidentally means that data were not accumulated over a sufficient period of time.

The model which Baloff uses does not anticipate the production point at which steady-state might occur, nor does it describe the level of constant productivity which characterizes the steady-state phase. It describes only the increased productivity which occurs in the startup phase. Unfortunately, he offers no solution to this dilemma, except further research.

The model is of the form of the unit curve (not the cumulative average curve), with certain re-definitions. That is,

$$Y = aX^b$$

where "X" is cumulative output, and "a" and "b" are parameters. Here however, "b" is a positive exponent so that "Y" represents increasing productivity rather than decreasing labor hours per unit. This accounts for Baloff labeling "Y" as an index of process productivity. For example, if the productivity index, "Y", at the tenth unit of production is desired, the equation can be solved, if "a" and "b" are known. The solution for the twentieth unit (doubled quantity) would result in a larger value of "Y". This value of "Y" would be a constant percentage of the tenth unit, say 120%. Thus, as with the unit man-hour curve where unit man hours decrease at a constant percentage, here, productivity increases at a constant percentage. The value of "b" then, is the slope of the function, and is positive, representing increasing productivity per unit rather than being negative and representing decreasing man-hours per unit. It might be thought of as the "inverse" of the unit man-hour curve.

Estimation of the parameters remains a serious problem (as it is with the unit man-hour curve). This is so because the slope values derived from Baloff's empirical data vary greatly, even among processes of the same basic type. In addition, the inability to predict when constant productivity (steady-state) will be reached, limits the model to just a description of the startup phase. Before the concept can become useful, these problems will have to be overcome. However, they are the same problems which exist with the unit man-hours curve, assuming non-linearity. Recall the comment of Conway and Schultz, above, where they found that "... certain individual operations experienced a sudden and marked reduction in the rate of progress". Also, Asher's work showed that some airframe unit man-hour curves tapered off to an almost constant value of man-hours per unit. Both situations are akin to Baloff's steady-state phase. Additional research is sorely needed for Baloff does not even offer an explanation for steady-state, except for the comment that "... if the production life of the product or process is relatively long and continuous, it is quite likely that these increases in productivity will ultimately cease..." (Ref. 3:26).

Baloff's empirical work adds to the growing collection of data which disputes linearity. It is considered important, not only because it adds further evidence of non-linearity, but in addition, and significantly, because it pertains to capital-intensive production where progress curves are thought by many, not to exist. If non-linearity proves to be common to labor-intensive and capital-intensive production, such a link may aid in deriving suitable relationships to describe the phenomenon.

Concerning the liklihood that productivity increases will cease, a possible explanation is offered by the author of this thesis. A machine-intensive production process is characterized by a low ratio of direct labor and assembly operations. Machines do not "learn", and machine-intensive processes are paced by machine feed speeds, etc., therefore, worker improvement is inconsequential. However, significantly measurable increases in productivity do occur, as Baloff has shown. Therefore, "learning" does occur. Since it cannot be attributed to machines, and a worker's manual adaptation to a fixed task is of little consequence in these processes, it must be caused primarily by improvements and innovations in the production process. Such improvements may come from management, engineering, supervisors, or the relatively few workers themselves. Improvements may continue almost indefinitely if people actively seek ways to improve the process. However, regardless of the concept that improvement may be expected to continue almost indefinitely, it is conceivable that company management may shift emphasis from a particular process once it is convinced (rightly or wrongly) that the process performance is "satisfactory", however defined. The rationale might be found in the return on the investment of say, an engineering team assigned to improve a particular process. The return on the investment is expected to be high initially when rapid improvements can be made. As cumulative production doubles, productivity increases by a "set" percentage (the slope parameter). However, the <u>time</u> required for doubled quantities becomes ever greater. Therefore, the return on investment for improvements <u>per unit of time</u> diminishes rapidly (assuming a relatively constant percentage of improvement per doubled quantity). Other corporate problems may well demand solutions by the relatively sparce key personnel in an organization. A shift in personnel to solve new problems would seem to be an intuitively obvious policy. Thus, it may well be that <u>time</u> is a factor which must be examined (along with other aspects of the problems encountered) because of the economic implications of return on investment per unit of time for competing projects within a company.

Perhaps such a consideration is applicable only to competitive business. Consider the case of a company which is awarded a government contract. In the bidding, awarding and continuous government monitoring of such a contract, progress curves may well play an important role. Future contract awards may be based, in part, on past and present performance. Per unit time/cost reductions occurring as predicted by a progress curve might be considered as indicative of "good management", which too, may influence contract awards. Thus, progress over the length of a contract may carry high "political" weight for future awards. Management emphasis on continuous improvement may preclude moving key people to other jobs.

It would be an interesting exercise to study the real and unbiased data of an industrial concern to discover if tapering-off of productivity (or man-hours per unit reduction) can be coupled with shifts in key personnel. Caution would have to be exercised in selection case studies. For example, a high dollar value product with limited quantity production may not be amenable to steady-state conditions, via shifts in key personnel. However, research in this area may uncover meaningful information.

Analysis. The evidence of empirical work strongly suggests that linearity is a misnomer. Recall that the Stanford-B curve and the Boeing "humped-curve" show non-linearity during initial production. Recall also, that Carr hypothesized an S-shaped curve throughout production. Finally, Asher, Conway and Schultz, and Baloff show a tapering-off or steady-state condition.

In addition, during the research for this paper, verbal suggestions of curve "segments" were received. Unfortunately, details could not be procured. (Curve segments are nothing new. Indeed, they were suggested in Wright's article.) However, if they are being used in industry, as is suspected, the implication is strong that non-linearity may be recognized by some industrial concerns.

It is suggested that Carr's S-shaped curve may have been broken into three segments, with the segments being approximated by separate linear lines on log-log paper. In other words, initial production may be approximated by a 90-95% slope. With a build-up to "normal" production completed, another segment in the range of a 75% to 85% slope may be fitted to data, followed by a third segment with probably a flatter slope during the phase-down of production. Such a procedure might be the consequence of the difficulty of formulating Carr's S-shaped relationship.

Whether segments or any other non-linear relationship is used, there are obvious difficulties involved in formulating a suitable relationship because of the many factors involved (See Chapter VI). "Break-points" for segments, and segment slope values may well vary from case to case, depending on the accompanying circumstances peculiar to each manufacturer and each product. Experience with similar type products and production processes may provide indications. Conceivably, only generalized relationships may ultimately be developed, with specific values of parameters provided from each manufacturers experience. Whatever the difficulties, it must be re-emphasized that the weight of evidence is adding to the validity of non-linear progress curves.

Capital-Intensive Production

Past applications of progress curve theory have been generally limited to labor-intensive forms of manufacturing, especially where a high proportion of assembly operations is required. Some authors strongly imply, if not flat state, that the concept is applicable only to labor-intensive manufacturing. Three authors at least, refute this notion.

From their empirical studies, Conway and Schultz relate a case in which one particular operation was paced by the speed of a conveyor. When the speed was stablized, further progress was achieved "... by the elimination of operations by redesign or by improvements in quality and the consequent elimination of rework operations" (Ref. 13:45).

Baloff states emphatically that (Ref. 3:25), "The apparent omission of machine-intensive manufacturing from past applications of the learning curve concept is somewhat inexplicable, since pronounced and measurable learning phenomena do, in fact, accompany the introductions of new products or production processes in many mechanized forms of manufacture."

W. B. Hirschmann write (Ref. 18:128-129): "Petroleum refining offers a good example of the type of industry to which the learning curve might be thought to be inapplicable. It is characterized by large investments in heavy equipment, and is so highly automated that learning is thought to be non-existent or too small to be of value." He shows that for an individual fluid catalytic cracking unit, a 90% slope occurs. Hirschman does not pursue capitol intensive production further, as the mainstream of his article is concerned with urging the use of the learning curve to industries other than the aircraft industry. We therefore return to Baloff and his empirical study.

Baloff's study was performed on production processes with the following characteristics (Ref. 3:27):

1. Sophisticated mechanization.
2. Synchronized mechanical pacing.
3. Large capital investments, generally exceeding $1,000,000 and frequently in excess of $1,000,000.
4. High rates of output, typically measured in tens of tons of product per hour of operation.
5. Relatively small direct-labor operating crews, usually numbering five or six employees.

He summarizes the results with the observations that a definite learning phenomenon does occur, it extends for various cumulative output ranges, and all but eight processes reached steady-state.

Furthermore, the regression lines are well fitted to the data points (regression results are shown in the article), however, the model parameters show considerable variation for the processes studied. A rather disturbing finding is that the "b" values of six basically similar types of electrolytic-tinning processes ranged from 0.174 to 0.713, or slopes of 88.6% to 61.0%. As Baloff suggests, further research is needed.

Concerning the causes of increased productivity in machine-intensive operations, Conway and Schultz have noted two as, the redesign of operations, and quality improvement which eliminates rework operation. Others would be such as liberalized tolerances, tooling improvements, plant layout redesign, and routing and handling of materials. A fuller discussion of factors contributing to improvements will be found in Chapter VI.

One would hesitate to make any sweeping conclusions on the basis of 28 cases in one study, especially in regard to parameter estimation, but the existence of the progress curve phenomenon is apparent in the machine-intensive processes examined. Baloff suspects that it will be found in many sectors of capital-intensive industry (recall Hirschmann's comments) because "... the manufacturing processes examined here bear some resemblance to mechanized processes in many other industries" (Ref. 3:32). Baloff's study could be a springboard for further research directions so necessary in progress curve theory.

New Estimating Relationships

As noted in Chapter II, the work of A. Alchian showed that errors of prediction of 25% could occur when using an aircraft industry-wide average progress curve, while curves fitted to the past performance of a particular manufacturer resulted in 22% prediction errors. Because of the inadequacies of these cost-quantity relationships, new ones have been sought. Three such relationships which do not rely on quantity as the independent variable are discussed next. As with most studies in the past, military aircraft production data has been used, presumably because of its availability as compared to missiles, or other forms of industrial production.

RAND. G. S. Levenson and S. M. Barro of the RAND Corporation have worked out new estimating relationships for aircraft cost elements (Ref. 54:Chap. IX). The estimating equations were derived primarily by statistical multiple regression techniques. Their approach is to relate airframe manhours or cost with aircraft physical and performance characteristics as the independent variables. Many potential explanatory variables were considered and tested by the authors, but they found that only aircraft gross weight, speed, and engine thrust were necessary for useful relationships for cost element estimation. Presumably, such factors as aircraft complexity are a function of speed and thrust. In addition, one constraint is found in the requirement that full production is necessary if the equations are to be valid.

"The estimating relationships were derived from data on post World War II cargo, tanker, fighter, bomber, and trainer aircraft that were produced in quantity for operational use by the Air Force or Navy. These aircraft, all of aluminum construction, range in speed from low subsonic to Mach 2.2... The sample includes production programs of ten different airframe contractors" (Ref. 54:IX-4).

Four major cost elements (engineering, tooling, manufacturing labor, and manufacturing material) are considered along with a number of subsidiary ones. Based on the data, a regression equation of exponential form is provided for each element so as to determine its cost at one particular production quantity - either the first or the 100th unit. For instance, where,

E_1 = initial engineering hours
W = gross takeoff weight (lb.)
S = maximum speed (knots)
P = maximum sea level thrust (lb.)

the initial engineering hours are estimated by (Ref. 54:IX-15),

$$E_1 = (8.0)S^{.55}\,P^{.88}$$

and the manufacturing labor hours at the 100th unit are estimated by (Ref. 54:IX-48)

$$H_{100} = (1.45)W^{.74}\,S^{.43}$$

Such equations provide one point to plot - for either the first or the 100th unit for a cost element. A log-linear cost quantity curve is then passed through this point. (One wonders about the validity of a linear assumption, especially after the previous discussion. However, the data of the authors presumably supports linearity. With post World War II aircraft as samples, the possibility exists that production runs were not of sufficient length to "allow" tapering-off.) The slope of this curve is obtained from an average of slopes observed for individual sample aircraft. Levenson and Barro found that slope values were not correlated to any significant extent with aircraft physical and performance characteristics, or to any other characteristic of the development or production program (Ref. 54:IX-53). They concluded that variations in slope were merely random fluctuations which led to their use of average values for each cost element. For example, the average manufacturing labor hours slope was 75%, while materials was 89%. (See Analysis portion of this section.)

The reader may wonder why this approach is considered worthy of note since we are back to a cost-quantity relationship with familiar values of slope. Recall that parameter estimation has been, and still is, a significant problem in progress curve work. The slope value has been the subject of argument more often than the "a" value assigned in any specific equation. Perhaps this has been due to the assumption that one merely uses the "actual" cost or hours of the first unit, leaving only the "b" value open to question. The inadequacy of such a procedure has been explained succinctly by Conway and Schultz (Ref. 13, 44).

> Even if, as seems doubtful, one is able to determine with any accuracy the cost of the first unit, its use as the basic parameter to locate the function is not necessarily advisable. As a matter of fact, in only one of the many hundreds of sets of data were the authors able to determine the first piece cost of any item, assembly, part, or component. In addition, it seems unlikely that the first piece cost can be determined soon enough to enable its use as a predictor. To be useful for production decisions relating to engineering effort, production planning, manpower planning, or design changes, estimates far in advance of production are necessary.

Levenson and Barro appear to have circumvented this problem by deriving estimating relationships (rather than judgmental opinion) to establish the height of the function. If their procedure provides a more accurate estimation, then it can be considered a significant contribution. Mr. Barro notes in a separate paper (Ref. 45:3) that since the procedure is new, insufficient data is available to test the model.

In discussing the limitations of this procedure, they warn of the dangers of extrapolation beyond the sample boundaries; the cost effect of titaninum and stainless steel in construction as opposed to aluminum; and that the equations are valid only for a full production effort. Significantly, they note that, "Some information suggests that manufacturing material costs may level off at some quantity on the order of 1000 airframes; something similar may occur for other manufacturing costs. The available data are not sufficient to either confirm or deny this possibility" (Ref. 54:IX-73). The reader is referred back to the discussion of non-linearity in this chapter.

Planning Research Corporation. In February, 1965, Planning Research Corporation (hereafter referred to as PRC) published a report entitled, "Methods of Estimating Fixed-Wing Airframe Costs, PRC R-547". That report was revised in April, 1967 (Ref. 59).

The PRC approach is similar in some respects to the RAND study. PRC uses three cost elements - direct manufacturing labor, manufacturing materials, and engineering tooling. Estimating relationships are provided from regression analysis of sample data for each cost element. However, different variables are used and there are four equations for each cost element, which provides the cost or hours at units 10, 30, 100 and 300. These four point estimates are then used to derive a cost-quantity relationships of the familiar form, $Y = aX^b$, or its logarithmic form.

For example, for manufacturing direct labor, the cost equation is (Ref. 59:III-12),

$$Y_{n1} = a_{0n} + a_{1n} S_s^2 + a_{2n} P_n^{-1/2} + a_{3n} W_a^{-1/2} + a_{4n} D_n^{1/2}$$

where,

Y_{n1}	=	estimated labor cost for unit "n" in man-hours per pound of airframe weight
a_{jn}	=	estimated value of the coefficients at unit "n"
S_s	=	maximum mach number at sea level
P_n	=	monthly delivery rate at unit "n"
W_a	=	AMPR weight of the first unit
D_n	=	percentage change in airframe weight from unit one at unit "n"

Values are provided for the coefficients for quantities 10, 30, 100 and 300. The four points generated are plotted on logarithmic paper and a straight line of "best fit" is drawn through them. The slope and first unit cost are obtained from this line.

Similarly, the manufacturing materials cost equation is,

$$Y_{n2} = b_{0n} + b_{1n} S_a + b_{2n} T + b_{3n} W_a^{1/2} + b_{4n} P_n^{-1/2}$$

where,

Y_{n2}	=	estimated materials cost for unit "n" in 1963 dollars per pound of unit one airframe weight
S_a	=	maximum Mach number at altitude
T	=	time factor = (Calendar year of first delivery minus 1940)

Finally, equations are given for tooling and engineering, for recurring as well as non-recurring costs.

Center for Naval Analysis. A third study was done by J. V. Yance in June 1965 for the Center for Naval Analysis (CNA).

Unfortunately it is a classified document (confidential). However, in an unclassified paper by Barro (Ref. 45), the CNA procedure is briefly explained.

The CNA procedure uses a single regression equation for unit airframe production costs,

$$\frac{C}{W} = KW^a S^b N^c e^{dT}$$

where a, b, c and d are parameters

$\frac{C}{W}$ = cost per pound of airframe weight

W = airframe weight

S = maximum speed

N = cumulative production quantity

T = time: the year in which the airframe was produced

Comparison of Procedure. Barro undertakes to compare the three studies in his paper mentioned above (Ref. 45). His comparison included the PRC report of February 1965, not their revision of April 1967. A few of his observations are highlighted below.

Barro notes (Ref. 45:12) that there is a great temptation to include too many explanatory variables as determinants of cost in an estimating relationship. He asserts that by the usual statistical tests, some of the variables of the PRC model turn out not to be significant. They were included, apparently, because they seem to contribute to an understanding of the determinants of cost. Barro points out that not only are they nonsignificant, but they are undesirable for a very practical reason: their inclusion in a numerical computation is almost as likely to move the result away from an accurate estimate as toward it.

The slope used in the three procedures are obtained by different methods. In the RAND procedure, the slope is estimated separately as an average of values observed for individual sample aircraft. In the PRC method, the slope value is determined by fitting a log-linear curve through the four points obtained from regression equations. In the CNA method, the slope is one of the parameters in the regression equation itself. It is estimated along with the other parameters.

Another comment by Barro concerns overhead. He states that the whole overhead cost issue is one of the major unresolved problems of cost analysis. (See Chapter IV - Overhead, and Chapter VIII - Conclusions).

Finally, as Barro notes (Ref. 45:3): "... the ultimate criterion for comparison of different methods is success in predicting costs of new aircraft, outside of the samples from which estimating relationships were derived. Since all three procedures are recently published, there has not been much new data generated since the work was performed, and there has not been sufficient time to perform a comparative analysis of the reliability of predictions for new aircraft."

Analysis. The reason for new estimating relationships has been discussed. The validity and value of these new relationships cannot be determined until they are tested, as noted above. This, unfortunately, leaves the possible user perplexed as to whether or not to attempt to use them, and if so, which one. No answer can be given here, but a few comments need to be made.

First, the CNA procedure does not appear to offer much help, in that, now four parameters plus a constant (K) must be estimated, whereas in the simpler cost-quantity relationships, only two parameters were estimated.

Second, the RAND procedure uses an average of the sample slopes. With as many different type aircraft as there are in the sample, one wonders about the consequences of such a procedure. Barro had noted (Ref. 45:19) that when they attempted to relate slope values in terms of aircraft characteristics, their results were negative. However, during a personal telephone conversation with Mr. Barro, he remarked that Mr. Levenson intended to continue working on this problem and hopefully, uncover a realistic and usable relationship. If one can be discovered, a more accurate methodology may be in the offing, especially if the height of the function can also be correlated with aircraft type and characteristics. At present, their procedure establishes the height from the entire mixed aircraft sample. It would appear that new research may well uncover some meaningful relationships.

Third, the elaborate methodology of the PRC procedure to establish their four points does not necessarily add precision to the function. Once again, a multitude of parameters must be estimated. Then, from the generated points, a log-linear line of best fit establishes the slope of the final cost-quantity equation.

One can only speculate about the usefulness of such methodologies until some empirical testing confirms or refutes their accuracy. However, the approach of establishing curve height by a relationship, rather than by judgmental comparison to similar models, has merit.

(See Chapter V - Some Airframe Manufacturer's Applications of Progress Curves, for a fuller discussion of this point.) If RAND can relate slope and height with aircraft type and characteristics (ignoring non-linearity problems for the moment because of the full production effort assumption which may preclude non-linearity) such an approach would appear to be the most promising. Finally, since Barro and Levenson noted that costs may level off at some quantity on the order of 1000 aircraft, (possibly the beginning of production phase-down), the non-linearity aspect of the problem may have to be incorporated into the model.

IV. Innovations in the Use of Progress Curves

The value of progress curve theory is found in its use as a predictive device. Known reduction in cost or labor hours over cumulative output for example, is invaluable information for costing, pricing, production scheduling, manpower requirements and the like. Application of the theory is not difficult, especially if certain simplifying assumptions hold for the problem at hand. However, in many sectors of industrial production, unique situations arise which compound the problem solution. In this chapter we shall examine several techniques or innovations in the use of progress curve theory for the solution of unique problems.

Nonhomogeneous Production

One simplifying assumption which is certainly not characteristic of the aircraft industry in the present age, is that of sequential production of a homogeneous product. In the past, military aircraft were usually built for one branch of the service. In addition, when a model was significantly changed, production of the modified aircraft caused discontinuance of the old model. In effect, simultaneous production of similar aircraft was rare. In the present however, joint-service aircraft and multi-model production is common. The F-4 and F-111 are prime examples. Even in commercial aviation, special customer requirement in domestic and foreign sales cause multi-model production.

Garg and Milliman. Anand Garg and Pierce Milliman (Ref. 14) devised a procedure and a modified progress curve formula to handle the unique costing problem for multi-model production. They note that some portions of the production work will be common to all models, some portions will be common to some models, but not others, and some portions may be unique to just one model. Also, at any arbitrary point in production, common components will have a larger cumulative total produced than will unique components. This is significant in that the fifteenth unit of a late model aircraft has fifteen units of experience for its unique components only. It would be inaccurate to compute man-hours or cost at the fifteenth unit for common components, for they will have a much larger number of units of experience.

The authors build their model from a two version aircraft case and extend it to "n" versions (Ref. 14:24-25). For the two version case (models I and II) we have work,

1. Common to I and II
2. Unique to I
3. Unique to II

We will use the convention of attaching a star (*) to all equation symbols for work common to I and II, a prime (') for work unique to I, and a double prime (") for work unique to II. Thus, the man-hours of work for model I, i.e. Y(I), is the sum of man-hours for work components common to I and II, (Y^*), and the man-hours for work components unique to I, (Y'). That is,

$$Y(I) = Y^* + Y'$$

$$Y(II) = Y^* + Y''$$

Garg and Milliman utilize the Stanford-B equation as a unit man-hour estimating device ($Y = a(X + B)^b$). Therefore, unit man-hours can be represented by

$$Y^* = a^* (X^* + B^*)^{b^*}$$

$$Y' = a' (X' + B')^{b'}$$

$$Y'' = a'' (X'' + B'')^{b''}$$

where,

$$X^* = X(I) + X(II)$$

$$X' = X(I)$$

$$X'' = X(II)$$

For a three aircraft case, we have work

1. Common to I, II, & III
2. Common to I & II
3. Common to II & III
4. Common to I & III
5. Unique to I
6. Unique to II
7. Unique to III

with corresponding equations as above.

When extending to "n" versions, a summation equation must be used, in which each of the work groups is assigned a unique number, i.e., 1, 2, ..., i, ...

The equation and subsequent derivations as shown by the authors is,

$$Y_j = \sum_{i=1}^{2^n - i} a_i \delta_i^j (X_i + B_i)^{b_i}$$

where δ_i^j = 1 for work groups which contain the jth version, and equals zero otherwise. It is important to note that X_i, designating the number of units of experience for work category "i", is independent of the sequence in which earlier units were produced.

The parameter must be determined, of course, and the authors make the simplifying assumption that "B" and "b" remain constant for a family of aircraft. The parameter "a" however would vary with the amount of work in each work group. They associate this with the number of engineering drawings, "D", such that

$$a_i = k\, D_i$$

where "k" is chosen so that a unit of any version would require a fixed number of man-hours, Y_f, if it were the first airplane in the program. Thus,

$$Y_I = Y_{II} = \ldots = Y_f = \sum_{i=1}^{2^n-1} a_i\, \delta_i^j\, (1 + B)^b$$

and

$$Y_f = k(1 + B)^b \sum_{i=0}^{2^n-1} \delta_i^j\, D_i$$

where D_i is the number of drawings for work group "i". Therefore,

$$Y_f = k(1 + B)^b\, D_j$$

where D_j is the total number of drawings for version "j".

Since the total number of drawings may vary with each version, "k" will take different values with different versions. From the preceeding equation,

$$k_j = \frac{Y_f}{(1 + B)^b D_j}$$

where "k" has been subscripted to show its dependence on a version of the aircraft. Substituting for "a" and then "k" into the original summation equation yields the model:

$$Y_j = \frac{Y_f}{(1 + B)^b D_j} \sum_{i=1}^{2^n - 1} \delta_i^j (X_i + B)^b$$

The authors state that the parameters "Y", "B" and "b" may be estimated from actual experience, if a program is in progress. Otherwise they can be selected by methods ordinarily used to estimate parameters in the Stanford-B formula (Ref. 14:25).

The assumption of constant slope and "B" value for a family of airplanes may cause some consternation. Also, relating engineering drawing count to the parameter "a" may be questionable. However, the authors state that any other available measure may be chosen - if it provides satisfactory results. It is inferred from this comment that the authors found the engineering drawing count to be their best measure.

A formidable amount of classification and computation is evident. (A computer solution would be an invaluable aid). Although formidable, in this day of mult-model production of multi-million aircraft, one may ask: What alternative technique is available?

The results of a test of the model on assembly shop direct man-hours shows significant improvement over the Stanford-B curve. (See table, below, (Ref. 14:26). For instance, the Stanford-B curve estimated only 18% of the aircraft within 5% of the actual cost, while the Modified Curve formulation estimated 48% of the aircraft within 5% of the actual cost.

Error Per Cent*	Per Cent of Airplanes With Estimates Within the Indicated Error Using, Modified Curve	Stanford-B Curve
5	48	18
10	86	48
15	92	74
20	96	80
30	98	90
40	100	94
100	100	100

$$*\text{Error Per Cent} = \frac{(\text{Actual man-hours} - \text{estimated man-hours})\,100}{\text{Actual man-hours}}$$

Baker and Silver. W. W. Baker and J. Silver of the Cost Analysis Division, Aeronautical Systems Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, co-authored a paper (Ref. 44) with similar intent to the one discussed above. Rather than the Stanford-B curve, Baker and Silver use the unit curve upon which to build their model. They neither assume a constant slope for a family of aircraft, nor do they estimate the "a" parameter via engineering drawing count or any other measure. The authors state that direct labor data was available from aircraft component labor accounts, that it was available by aircraft unit number, and that it could be segregated as common or unique. Thus, they would obtain parameter values for components from basic data, and summation of component costs is all that is required.

The authors include a section on computer logic required for their model. Apparently the model was not tested - at least, no results are offerred. However, given the kind of data claimed, the approach is straightforward and promising.

<u>Value Engineering Cost Reduction Proposals</u>

During the course of production of government contracted work, contractors often discover ways to reduce costs. Design changes and materials substitution are two such means. If the product's performance characteristics are not reduced, "value" is added to the product. Cost reduction proposals are of interest to this paper.

Contractor motivation for submitting cost-reduction proposals is found in contractor sharing of the generated savings from the change. Herein lies a problem. "The problem is that it is difficult to estimate the amount of money a value engineering change actually saves as production continues, and, as a result, it is difficult to arrive at the amount of money the contractor should receive for making the value engineering change" (Ref. 53:2).

Part of the difficulty lies in estimating the savings generated per unit. Once determined (it is negotiable), the Armed Services Procurement Regulation (ASPR 1-1703.2) specifies that one of three basic plans of sharing savings with the contractor will be used. These are (Ref. 53:2).

1. Contractor participation in cost savings under the instant contract only (production contract).
2. Contractor participation in cost savings under the instant contract plus royalty payments on actual future procurements within a stated period.
3. Contractor participation in cost savings under the instant contract plus estimated savings on additional quantities expected to be procured subsequently, the estimated additional quantities being stated in the instant contract.

Returning now to the problem of estimating the amount of money a cost reduction idea actually saves, a grossly oversimplified example will be offerred for illustrative purposes. Suppose, for example, that the cost of the 75th unit of production is $20. Assume that the progress curve phenomenon shows the unit cost of the 1000th unit will be $10. Now, if a cost-reduction change initiated at the 75th unit reduces the cost of the 75th unit by $10, what will be the cost of the 1000th unit? If the cost reduction of $10 is assumed to be constant over production, the 1000th unit will cost nothing to produce! This is, of course, absurd. Reasoning would suggest that if the cost of the 75th unit is reduced by one-half, the cost of the 1000th unit will have been reduced by one-half also, or $5.

Thus, it should be apparent that if costs decline in accordance with a progress curve slope, the value of a cost-reduction proposal will also decline over future production. Since the contractor shares in the cost savings generated by his cost-reduction proposal, his share of the savings per unit cannot be computed as a constant dollar value per unit. The contractor's share of the savings must therefore decline in accordance with a progress curve slope. This is, in essence, the problem of determining the total amount of money the contractor should receive for his value engineering change proposal.

If a linear progress curve is assumed, the problem solution is straightforward. Slope value to be used is, as always, an issue. An agreed upon slope value may be written into the contract. Alternately, the slope may be determined from production data. However determined, it becomes the basis for computation.

If careful analysis is not applied, cost-reduction proposals may increase the total cost of the contract of the government. More money may be paid to the contractor as his share of the savings generated than is saved by the change. Two simplified examples of this possibility are offered from the U. S. Army Missile Command publication, "Value Engineering and Experience Curve Predictions" (Ref. 53). The author of the referenced publication has stated that his examples are based on cases with which he has come in contact.

The first example is similar to the simplified example, above. Assume an ASPR type-3 contract, as defined above, which calls for a 50% contractor share of value engineering cost reductions under the instant contract, and a 30% share of savings on future production quantities. Suppose a $500 savings per unit can be achieved from a change proposal, and 90 units remain to be produced on the instant contract, with 2400 units expected to be procured subsequently. If savings per unit is considered constant over all production, rather than declining in value, the contractor would be awarded $382, 500 ($500 x 50% x 90 units plus $500 x 30% x 2400 units). Assuming a 73% progress curve, the contractor's share is computed to be $113, 306 (Ref. 53:4), which is far removed from the above figure. Even more significant, the declining value of $500 per unit would produce a <u>total</u> savings of $347, 688. If the progress curve analysis were not applied to the contractor's share of savings, he would receive $382, 500. With the <u>total</u> actual saving at $347, 688, the government would suffer a net loss of $34, 812 from the cost reduction proposal.

In the second example, a situation may occur in which a change proposal not only reduces unit cost, but also changes the slope of subsequent production. This could conceivably occur where the change allows automation of a labor-intensive process. Automated processes characteristically have a flatter slope than labor-intensive processes. As can be seen in Figure 1 (next page), the change reduces unit cost, but the flatter sloped curve crosses the projection of the steeper cost curve at some future point in production. Thus, the paradox arises that, given sufficient production, a cost reduction idea may result in higher unit costs and eventually, in higher total costs. Of course, if the change not only reduces unit cost, but results in a steeper sloped curve, total cost reduction is enhanced. The referenced publication provides several other situations for the interested reader.

The preceeding material is, admittedly, a simplification of the evaluation of cost-reduction proposals. Many other factors may compound the problem. One which comes to mind is: what is the cost of modifying the units already produced, if a design change necessitates modification? Another factor may be that the production delivery schedule is so critical, that delays caused by some changes cannot be accepted. However, the purpose of this discussion has been to indicate possible applications of the progress curve concept with cost reduction proposals.

Second Source Procurements

There is the possibility that if a contractor is the only source of supply for a particular item, that contractor will charge a premium for his service. With the introduction of a second source (or more),

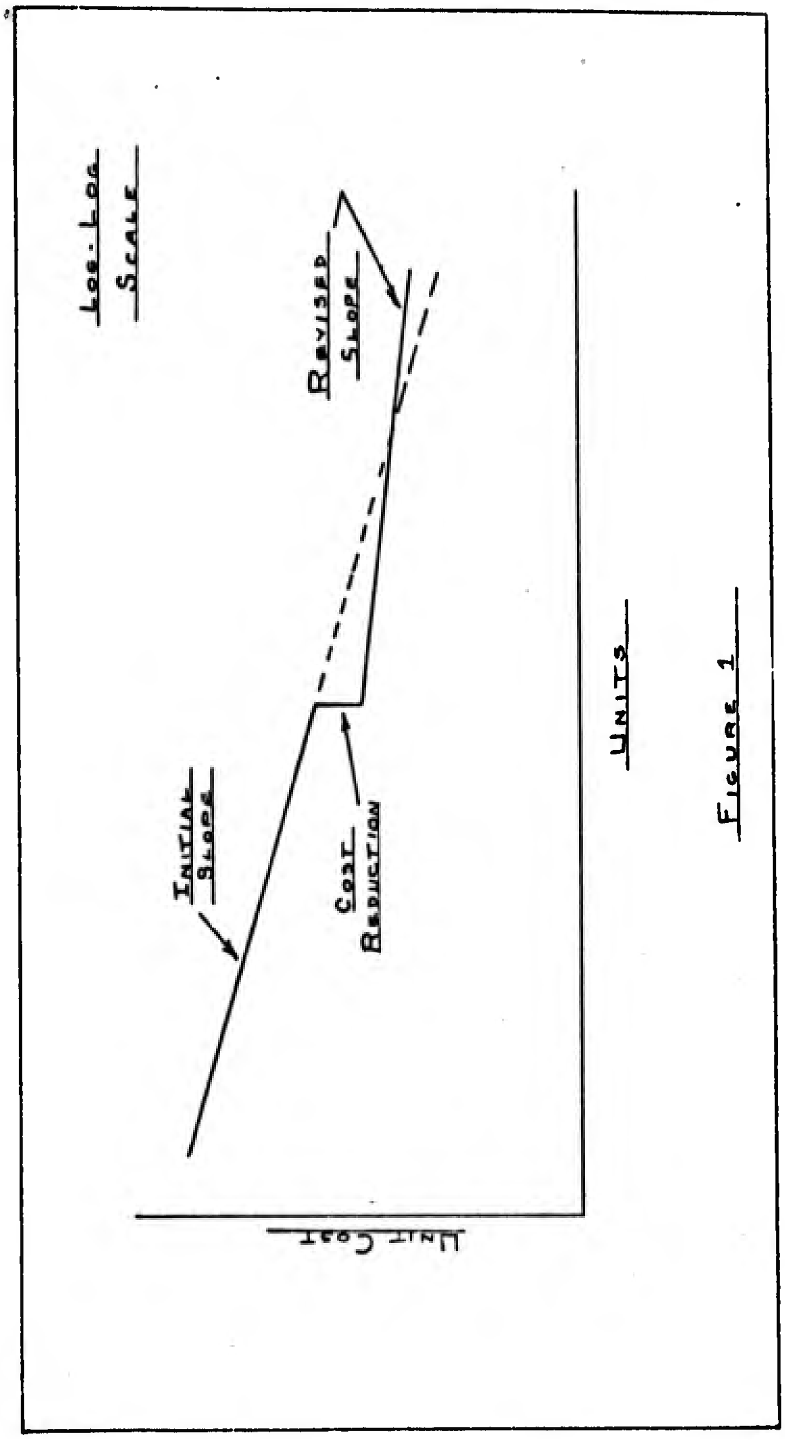
Log-Log Scale
Revised Slope
Initial Slope
Cost Reduction
Units
Unit Cost

Figure 1

competition will presumably force the price down, assuming no collusion. This is certainly no startling discovery. There may exist however, unique and rather interesting situations in which progress curve theory can be utilized to solve cost minimization problems via optimization techniques.[1] An initial statement of the problem will be a simplified case before more complexities are suggested.

Assume a one-time procurement for which only two producers can supply a particular item. Neither one has sufficient plant capacity to fulfill government requirements, but collectively they have capacity in excess of requirements.[2] Assume also that their production follows the <u>unit</u> progress curve hypothesis, where Y_1 and Y_2 are the unit costs of the two producers, i.e.,

$$Y_1 = a\,X^b \qquad a \neq c$$
$$Y_2 = c\,X^d \qquad b \neq d$$

The problem now becomes one of finding the optimum quantity to order from each producer so as to minimize the total cost to the government for quantity "N". A solution may be found by an iterative process, or by the calculus so as to minimize total cost (TC), which is the sum of the unit costs.

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[1] The problem formulations and concepts of solution were obtained from personal communications with Miss Claire Seelbinder, a mathematician, employed by the U. S. Army Missile Command, Redstone Arsenal, Alabama.

[2] This particular type of problem is similar to the standard economic Multiple-plant Monopolist problem. The progress curve functions would, of course, be used as the plants' cost functions. See, for example, J. M. Henderson and R. E. Quandt, <u>Microeconomic Theory</u>, New York: McGraw Hill Book Company, 1958, p. 172.

$$TC = \sum_{x=1}^{i} aX^b + \sum_{x=1}^{j} cX^d \text{ ; where } i + j = N$$

The summation process may be replaced by integration, however this would be an approximate solution since the function is a discrete function and not a continuous one.

The problem can be vastly complicated by changing the problem bounds from a one-time procurement to a series of yearly contracts of differing quantities. This could occur in munitions procurement subject to budgetary constraints and/or the international situation. (Presumably a 5 or 7 year plan would anticipate requirements by year.) In addition, plant capacity may be assumed to vary by year. Thus, let

R_j ($j = 1, \ldots, n$) = requirements for year "j"
p_j ($j = 1, \ldots, n$) = first plant's capacity for year "j"
q_j ($j = 1, \ldots, n$) = second plant's capacity for year "j"
x_{1j} ($j = 1, \ldots, n$) = first plant's assigned production for year "j"
x_{2j} ($j = 1, \ldots, n$) = second plant's assigned production for year "j"

such that

$$X_{1j} \leq p_j$$
$$X_{2j} \leq q_j$$
$$X_{1j} + X_{2j} \geq R_j$$

The total cost function (allowing an approximate solution by integration) becomes,

$$TC = \int_{p_{j-1}}^{p_j} a X_{1j}^{b}\, dx + \int_{q_{j-1}}^{q_i} c X_{2j}^{d}\, dx$$

To minimize this function, one would use a non-linear programming technique. It is far beyond the scope of this paper to even suggest an outline of such a solution since it involves some rather rigorous mathematics. It is sufficient to state that a minimum total cost solution can be found.

The problem can be shaded differently by introducing other factors. Consider the case where only one producer has the technical know-how, and his capacity allows him to fulfill yearly requirements. This sole source has been utilized for several years. The government suspects it is paying prime prices and elects to "sponsor" the development of a second source through perhaps and R & D contract, or a cost-plus fixed-fee production contract. The question arises as to when is the appropriate time to introduce the second source. Also, what quantity should the second source be allowed to produce for experience before competitive bidding between both sources is introduced. If competition is introduced too early, the second source may not be able to act as a threat to the first source. If introduced late, the second source may be gaining excess experience at government expense. These are managerial factors which, if quantified, could be incorporated into a cost model.

Such situations do exist. Historical pricing data on a particular missile system purports (Reference: footnote 1) that after a first source producer had been in production for at least three years, a government announced intent to introduce a second source caused a

5% drop (approximately) in the prime's price. When the second source contract was let, the first source dropped his price by another 5%. When the first and second source bid against each other, the first contractor dropped his price another 15%, but lost the bid anyway by a narrow margin.

In this section, it has been shown that progress curve theory has been combined with Operations Research techniques. From the viewpoint of Operations Research, perhaps nothing new has been introduced, for the progress curve merely becomes the cost function in a cost minimization problem. However, from the viewpoint of progress curve application, Operations Research techniques may be considered new, and could prove to be a very useful tool for these types of decision problems.

Lot Mid-Points

Quite often, cost or man-hour data is available only for production lots. Cost or man-hours per unit is not provided. This presents a problem in plotting data - what unit should be used to represent the entire lot? Recall that a progress curve is an exponential function which is not linear on arithmetic grids, although it approaches linearity (on arithmetic grids) after a large number of units have been produced. Use of the arithmetic mid-point for plotting lot averages is theoretically incorrect. It would make little practical difference where the function approaches linearity (on arithmetic grids) but appreciable errors may be introduced where it changes shape rapidly in the early phase of production. The smaller the lot, the smaller the error, of course. However, large lot quantities with huge total production quantities is not uncommon. It can be shown (Ref. 64:49) that for a 75% slope curve, with lots of 1000 units, the algebraic lot mid-point for the first lot is unit 283, which is significantly removed from the arithmetic mid-point of unit 500. Thus, with large lot sizes, appreciable errors can be introduced.

Several methods are available for computing algebraic lot mid-points (Ref. 64:49). These include manual calculation via a mid-point equation (which proves to be quite tedious), the use of standard progress curve tables, and specially designed tables. However, these three methods require that the slope be known. A dilemma then arises with lot data. Given the slope, algebraic lot mid-points can be computed. Conversely, given algebraic lot mid-points, the data can then be used to compute the slope. However, algebraic mid-points cannot be found without knowing the slope. Since "something" must be done, the approach which seems to be prevalent is to estimate the slope from rough plotting of whatever data is available. If no data is available, presumably the estimate is based on "experience and judgment".

"Alpha and Omega and the Experience Curve" (Ref. 64), briefly relates that a computer program is available[3] which provides as output, " . . . the input data (the actual data points and the algebraic lot mid-points if lot data is used), the exponent of the slope, the

[3]Copies of a punched computer program deck or a printed program are available. Free copies are available to government agencies only. Also, a magnetic tape program can be obtained free of charge, if a blank magnetic tape is sent with the request. These programs are for an IBM 7094 computer. This information was obtained from Mr. Jim Fleener, U. S. Army Missile Command, Redstone Arsenal, Alabama.

per cent slope, the correlation coefficient, first upper and lower standard error ratios, the second upper and lower standard error limits, and the unit and cumulative average values for Units 1, 10, 1000, and 10,000" (Ref. 64:65). The text does not specify whether the slope must be known. However, it has been verified[3] that an estimate of the slope is inputed into the computer, and through an iteration process, the computer converges on a solution of the lot mid-points and the actual slope as determined from the data.

Another computer program (written in Fortran computer language for an IBM 1410 computer) is available. It allows input data of either of the following forms (Ref. 61:2).

1. Non-lot data, where cost is specified as cumulative or non-cumulative.
2. Lot data, where cost is specified as average unit per lot, non-cumulative total by lot, or cumulative total by lot.

   The program also permits the addition or subtraction of a constant "z" from each unit number. This permits the adjustment of the unit numbers by a constant in a fashion similar to the Stanford-B technique.

   Segments of a learning curve which have different slopes must be input separately, with a title card for each.

With the availability of computer programs, rapid and accurate solutions to a vexing problem are possible. A convergence solution may be computed manually of course, but it would probably prove to be very tedious and prone to arithmetic mistakes.

Optimal Lot Size

Students of Operations Research are well familiar with problems concerning the determination of optimal lot sizes for production runs. In the simplist traditional formulation of the cost model, total costs are seen to vary by lot size because of set-up cost per lot and inventory carrying cost per item. This assumes that demand is known over a specified time period, there are no costs associated with shortages or overstocked inventories, and set-up costs are independent of the number of pieces in the lot. With a series of small production lots to satisfy a known demand over a specified time period, set-up cost can be seen to increase. If set-up cost are reduced by producing in larger lots, or just one batch, inventory carrying cost per item increases. The optimal lot size to minimize total cost is found by "balancing" the various costs in any particular cost model. Traditional formulation, however, assumes no "learning" over cumulative output.

E. C. Keachie and R. J. Fontana have innovated with theory by incorporating the learning phenomenon into traditional cost models. Their article, "Effects of Learning On Optimal Lot Size", deals with the simple model described above so as to "... isolate and understand better the influence of the learning curve and the transmission of learning for intermittent production" (Ref. 22:B-104). They pose three cases concerning transfer of learning from period to period. The reader is referred to the article for the details of each specific model.

Case I assumes total transfer of learning from lot to lot. With this assumption, it is clear that there is no advantage in large lots to take advantage of lower costs caused by learning. A series of small lots would have the same learning characteristic as would one continuous production run. Thus, the optimal lot size is dependent only on set-up and storage costs. Manufacturing cost decrease (learning effect) is not a factor and is omitted.

Case II assumes no transmission of learning between lots. Obviously, the longer the manufacturing period (large lots), the lower the manufacturing costs. The optimal lot size is thus dependent on three costs - set-up, manufacturing, and storage. Manufacturing costs are represented by the progress curve function. The minimum total cost is found by setting the first derivative of the total cost function equal to zero and solving for the number of lots, and thereby, the lot size.

Case III assumes partial transmission of learning between lots, and is probably the most realistic situation. A further assumption is made that the slope remains constant, whereas the "a" parameter decreases from lot to lot. Since manufacturing costs decrease from lot to lot, the total cost model includes a term for the sum of the lot manufacturing costs. The solution is as in Case II, above.

The results show that the learning effect significantly affects optimum lot size. Even with the assumption of no transfer of learning between lots and a slope as flat as 97%, the optimal lot size increased by approximately 500%. It would seem that progress curve theory has an important place in the theory of optimum lot size.

<u>Overhead</u>

Some authors have naively stated that overhead follows a progress curve function. However, it only appears to do so. This is because it is usually allocated as percentage of direct labor hours. Since direct labor hours follow a progress curve function, certainly a percentage of that function will be parallel to it. Other than this, there is no apparent connection between overhead and progress curves.

The subject could therefore be dropped here, but since it is so inextricably interwoven into costing, and cost is sometimes the dependent variable in progress curve studies, it would seem appropriate to comment on it.

Since the inception of accounting, no issue has probably provoked more discussion and dissenting opinions than has allocation of overhead. Variable overhead presents no especial problem for costing in that it varies directly with production and can be equitably charged to a particular unit (or lot) of production. Fixed overhead allocation however, is the blight of costing.

One method of handling the problem for single-product firms is to allocate fixed costs evenly over the units of output in a particular time period. (The problem is not so easily handled in multi-product firms.) This would be equitable if production were at, or close to capacity output. As such, the total costs of operating a business would be discharged impartially. However, if output drops to 50% of capacity, each unit would be charged with twice its former fixed overhead. In an absolute sense, it seems preposterous to assert that a unit of production costs more to produce this month than last month when fixed costs have not changed at all. (Here, fixed costs are broadly defined as those costs which are incurred in operating the plant and are invariant with production. In other words, whether ten units are produced, or a thousand, fixed cost remain constant.) If the production costs remain the same and the fixed costs remain the same, does the product cost more, or does the product not cost the same, with the plant now considered to be operating at a loss? This viewpoint is the essence of the proposal for overhead allocation as presented by

S. G. Sturmey in his article, "Cost Curves and Pricing In Aircraft Production". His particularly lucid explanation (Ref. 32:963) is quoted in part below. The underlining has been added for emphasis.

> In practice, a product which utilizes 10% of the capacity all of the time, or all of the capacity 10% of the time, is expected to bear all its variable overheads and one-tenth of the total of fixed overheads. In a cost schedule relating to a particular product, therefore, the share of the overheads borne by each unit produced should be the same as the share of the capacity, both managerial and physical, allocated to that unit. The cost schedule of the plant should average all the overheads over the units produced, so that for a plant working at normal capacity the summation of the cost schedules of the individual products at the quantities actually produced will give the same fixed overhead coverage as will the plant curve, whereas if the plant is operating below capacity some part of the overheads will not be costed to the units produced. <u>This method of allocating overheads means that a company with idle capacity may make a loss, even though each individual product is apparently profitable. This seems a more reasonable picture of the situation than that obtained by costing all overheads to the products and so making one or all of the products appear as money losers.</u> Product costing, as opposed to plant costing, is unusual in the aircraft industry, as in many others.

Sturmey then, would establish an accounting system in which a percentage of fixed costs is allocated to a unit of production, based on that unit's percentage of plant capacity utilization. As he notes, this would preclude the appearance of a product being a money loser during periods of idle capacity operation, and a profit maker during periods of full capacity operation. It might prevent the business concern from eliminating a product line because it appears to be unprofitable.

Whether the fixed costs associated with idle capacity are inputed to the product, or to the plant as an operating loss, is a moot point. If the loss is to be made up by raising prices, it is immaterial whether the motivation is product "cost", or plant operating loss. The end result will be the same in either case, and the advantage of equitable costing is maintained with Sturmey's system. In addition, and more important to the purpose of this paper, it would be a giant stride toward standardizing a costing system in which "actual" cost computation remained constant rather than varying by accounting system used, as well as plant capacity. This particular point of "actual" cost, against which estimating relationships are judged, is considered important and will be more fully developed in the conclusion chapter of this paper.

Analysis

Innovations in the use of the progress curve, which have been considered worthy of note, have been outlined. It has been seen that progress curve theory can be effectively combined with mathematical techniques in allied fields. Operations Research programming techniques look to be especially fruitful for cost minimization problems, as noted in the sections on second source procurement and optimal lot size.

A costing problem is not simple. This has been recognized even by those who prefer simplified relations so that solutions, however gross, may be calculated. Gross solutions have been defended when the problem is a comparison of alternative choices. Even here, however, the argument contains the implicit assumption that errors in solutions are all in the same direction. In other words, the inaccuracies of the relationship either undercost or overcost all alternatives. The possibility exists that due to peculiarities in each case, one alternative may have its error fall in the overcosting direction, while another alternative is undercosted. With multi-million dollar contracts being commonplace, the consequences of such errors may be indeed, grim.

To continue to seek other simple relationships, because the present ones have failed to fit reality, is akin to seeking the proverbial pot-of-gold at the end of the rainbow. A multi-variate problem exists (see Chapter VI). Seemingly insolvable problems are being solved by the application of analytical tools. Computers stand ready to do the tedious work. The increased cost of research into, and the use of, correct relationships is a pittance when compared to the sunk cost of an advanced defense system which is realized all too late, to cost more than it returns in value.

V. Application of Progress Curves in Industry

There are many uses made of progress curve theory in industry. Its use in estimating as well as projecting man-hours and/or costs in the airframe industry will be covered in the second section of this chapter. The first section will note some of the common uses in industry in general.

Before discussing progress curve applications, it would be appropriate to state that the progress curve phenomenon has not been "universally" accepted in industry. W. S. Hirschmann (Ref. 18:127-128) lists the following as some possible reasons.

a. Lack of awareness that "learning" can be reasonably well quantified. The author points out however, that its reliability is comparable to that experienced with engineering construction estimates.

b. Skepticism that improvement can continue. After a period of time, there is a natural inclination to feel that the last source of betterment has been wrung out of an operation.

c. Many companies believe it is not applicable to their particular type of business - capital intensive or highly automated, for instance. Hirschmann points out however, that a method can always be improved by effectively trying to improve it.

d. Shallow sloped curves may not be recognized as improvement, or may be attributed to other causes. Also inflation may obscure the slope if dollar costs are plotted and a price deflator is not used.

e. Lack of awareness that the curve can describe group as well as individual performance. Hirschmann notes that the curve embraces more than the increasing skill of an individual operator; it includes the collective efforts of many people in line and staff positions (presumably engineering and management personnel).

Characteristic Applications

Price Negotiations. As many industrial concerns have adopted the progress curve for costing and pricing in government contracting, so too have many applied its principle when subcontracting. Often the subcontractor must be indoctrinated in its use, and convinced of its applicability. (In some cases the prime contractor will "prove his point" from his own records). Once convinced, the subcontractor may draw up proposals in a similar manner as the prime contractor does with the government.

One method employed is to compute man-hours only as subject to learning. The total labor hours are then multiplied by some average labor hour rate. Factory overhead and general and administrative expense may be allocated as a percentage of man-hours (Ref. 2:93), with material and component parts costs per unit added on (Ref. 31:73). Sometime, special tooling costs may also be included.

Negotiation may be introduced (under a price redetermination clause, for example) after a predetermined number of units of a lengthy production run are completed. It may also be introduced for a follow-on contract to an initial contract. In either case, the actual man-hours employed on the completed units may be considered as cumulative average hours to establish one point on the curve. If a slope can be mutually agreed upon (or obtained from production data), total man-hours are easily calculated. Alternately, a mutually agreeable man-hour figure may be negotiated for the remaining contract (or follow-on contract), expressed as average hours for that portion remaining, or total cumulative average hours for all work. From this, a progress curve may be constructed for future work. If additional contracts do not run sequentially, a new starting point on the curve must be negotiated due to less than total transfer of learning during breaks in production experience.

The above has, of necessity, been kept general. Many other methods may be employed. Drawbacks in slope and curve height determination are evident. In addition, during negotiations each party may well attempt to gain the most favorable position possible for his interests. Each may enter negotiations with an optimistic figure and a no-lower (higher) - than figure. There are no rules governing the play between starting and ending quotes, especially where internal records are considered proprietary.

Planning. Many areas of planning provide opportunities for application of progress curves. One such use is in facilities planning (Ref. 39:77). Where rate of production is not specified in a contract, increased and predictable production rates may require additional floor space. Conceivably, space requirements can be programmed via expected productivity increases. An excellent use of progress curves is found in personnel planning (Ref. 39:77). Hiring and layoffs are sensitive to production rates. In addition, at capacity output, decreasing manpower needs (due to learning) can be balanced with natural attrition, and minimum hire and training programs can be based on expected reduction in manpower needs. Perhaps the best application is found in long-range planning (Ref. 39:77-78). Errors of projection may easily satisfy accuracy requirements. In addition, and most important, long-range planning is not concerned as much with absolute values as it is with comparing the value of alternative choices. Computational ease permits future programs to be evaluated without an expensive detailed examination of each project element. Long range personnel requirements can be concurrently evaluated. Scheduling is another obvious application (Ref. 39:76). With an increasing rate-of-output requirement, increasing productivity may keep pace, precluding additional production facilities or manpower needs. With constant rate-of-output requirements, decreasing manpower needs will allow excess labor to be shifted to other activities on a planned basis. S. B. Smith offers a simplified example of scheduling with the curve (Ref. 31:75). Make or Buy decisions can be improved via progress curves (Ref. 2:93). A company may estimate lower initial costs for in-house work. If however, its slope is flatter than a subcontractor's slope, long-run costs may be higher. The intersection of the "make" curve with the "buy" curve would indicate the quantity at which a decision would reverse itself. Progress curve applicability is also found in financial planning (Ref. 2:94). Cost may exceed an established price in the initial phase of production. Plotting a decreasing cost curve with a horizontal price line would indicate the required financial assistance needed for a predictable time period before receipts exceed costs.

The preceeding has been a quick sketch of characteristic applications of progress curves in industry. As can be envisioned, an entire paper can easily be written on price negotiations and/or planning alone. Since curve height and slope, as well as non-linearity, have been previously discussed, and will be commented on again in subsequent chapters, we shall move directly to the subject of control.

Control. It is to be expected, and desired, that management establish controls, else planning is for naught. However, authors are divided in opinion as to whether or not the progress curve should be used as a control device. Therefore a discussion of the controversy would seem more appropriate than to cite examples of progress curve application to control.

Some authors reason that since the phenomenon exists, it should be used as a budgetary device to control production. One such author, Ronald Brenneck, states (Ref. 8:59 - underlines added for emphasis)

> The learning curve has proved to be an invaluable tool for pricing purposes. It is of equal importance in conveying to production personnel what is expected of them... When management has committed itself to manufacture a product for a given sales price, the agreement must be translated into a workable measure of production task, namely, direct labor hours allowed per unit... Profit measurement in the form of break-even analysis is required. It provides information to management as to how the shop must be budgeted in order to achieve the overall goal of a normal profit. This budget should be presented to the manufacturing division of a company in the form of a learning curve.

These comments are representative of the type of thinking of those who would use the progress curve as a control device.

The opposing viewpoint is crystallized best by the taut words of Conway and Schultz (Ref. 13:50). "It has also been used as a means of pacing operators on an assembly line... Carrying the function into use as a detailed labor control device seems to be stretching experience too far... Applying it to individual operations as a control is justified by no data which the authors have seen." The comments of R. P. Zieke add coherence: "The successful utilization of the progress function depends to a large degree on the confidence and acceptance by everyone within the organization of its fairness, relative correctness, and nonarbitrary nature. The progress function should be a mirror of production, not a goad" (Ref. 39:34).

Analysis. It seems that a clear understanding of the management process is necessary for a resolution of this conflict.

First, it is no secret that parochial attitudes exist in a business organization. Each department head would like other departments to "stand still long enough to be counted"; in other words, to be predictable, so that his own action can be varied for the optimization of his department's tasks. It appears that Brenneck has seized upon the progress curve as a means of predicting and thereby reducing production costs to a "given" in solving pricing and profit problems. Such an approach is not warranted. The progress curve is definitely not an irrefutable natural law. Furthermore, there is considerable debate as to whether its simple mathematical expression is either descriptively accurate or adequately quantifies the phenomenon. There are numerous articles which hail its simplicity, glibly suggesting ranges of slope values while solemnly warning of dire consequences from inaccurate parameter estimation. Neither is mention made of the existence of non-linearity, nor the possibility that gross errors might be caused by non-linearity, no matter the parameters. Such articles are fraught with danger for the uninitiated.

Second, optimization of a single department is nearly always at the expense of other departments. This is the futility of parachialism - it does not seek the best solution for the business as a whole. A "systems approach" would seek optimization at the highest level possible, if the organizational entity itself cannot be optimized. It would seem that production and pricing at least, should work together to maximize profits.

Third, if inference is correctly drawn, such statements as, "...conveying to production personnel what is expected of them..." and, "...direct labor hours allowed per unit..." are cardinal managerial errors and potentially disasterous to an organization. Production function personnel would certainly bristle from such dictatorial edicts, countering logically that pricing should be based on production costs, not establish them to meet a profit goal.

A solution is best found in the tenets of systems analysis: optimization at the highest possible level rather than parochial suboptimization. This presumes that organizational members understand that the progress curve is not a precise, unfailing description of reality. It also presumes that production goals are established with the participation and acceptance of production personnel. Finally, the progress curve must not be used as a weapon to prod production but rather as a reflection of expectations of an intelligent management team.

Some Airframe Manufacturer's Applications

As noted in the introduction, generalized material concerning progress curve theory was easily obtained from industrial organizations, but specific application was considered proprietary information and was unobtainable directly from any company. A study was secured however, in which techniques used by seven airframe manufacturers to estimate and project man-hours and/ or costs are discussed (Ref. 52). The study itself is undated, but several footnotes carry a 1958 date, implying that the study was done during, or after 1958. Unfortunately, the information may be outdated. Still, after many years of experimenting, a company may well settle one a particular scheme, and conceivably is still using it today.

The report notes that certain specific figures, percentages and formulae have been disguised to protect the proprietary interests of a particular company. As an added precaution, the material has been "...designed for use by government personnel only" with the further stipulation that "...personnel must not reveal any of the information on a particular company's learning curve methods and techniques to industry personnel representing other than that manufacturer" (Ref. 52:76). In order to discuss the material at all in this thesis, company names will not be used and information will not be detailed. It is felt that this is sufficient to preclude any possible proprietary infringements.

Company A. Estimates for the number of man-hours required to fabricate unit one of a new aircraft are made by comparing major sections of the proposed aircraft with sections of past aircraft which they most clearly resemble. Estimating personnel in conjunction with engineering and production representatives consider such factors as design, construction methods, types and quantities of material required, and similarity of proposed manufacturing processes. In addition, a ratio may be established to estimate man-hours for the new major section as follows,

$$\frac{\text{weight of old}}{\text{estimated weight of new}} = \frac{\text{man-hours of old}}{\text{"x" man-hours for new}}$$

However, this weight relationship is frequently modified by a subjectively determined complexity factor.

The first unit is that one which the company actually fabricates, not necessarily the first production model. The company feels man-hours used in experimental models bears a relation to subsequent manufacturing because whenever possible, the same personnel working on experimental models switch over to production models; in addition, production type tooling is fabricated for use on experimental models. The introduction of production tooling is dependent upon monthly rate of production and total quantity of aircraft called for by the contract.

The slope of the progress curve is estimated by considering the complexity, quantity, and rate of production of the new model plus the type of tooling to be used. Historically, Company A experienced a 90% slope for experimental units as well as the first production units. When production tooling is introduced (presumably at "normal" production), the historical average slope was approximately 80%, however these slopes may be subjectively modified when estimating new models.

A unit average curve is plotted (based on lot data), rather than a cumulative average curve. The company feels that the unit average curve indicates a trend or slope of production more accurately than does the cumulative average. By revealing the fluctuation in man-hours between lots, the unit average curve provides the estimator with an opportunity to determine the cause of fluctuations, discounting those which will not affect future production and giving weight to those which will. The company notes that for <u>any given aircraft</u>, the curve tends to flatten out at about the 250th unit. In the company's opinion, this empirical characteristic is the result of stabilization of tooling and production methods at or near this point in production. From this point on, the rate of learning is largely a function of worker improvement in performing an established job.

Major changes to an aircraft are estimated via an engineering estimate of the hours involved in each manufacturing operation required for the changed work. The changed work is then plotted on a separate learning curve apart from the curve for the unchanged aircraft. The slope is determined as previously done.

Company B. This company has developed a master cumulative average improvement curve which is used as an index for comparison and estimating purposes. The slope has been determined from historical data of the company. The verticle position of the master curve is determined from historical experience as the optimum cumulative average which it could expect to reach at unit 1000 (Ref. 52:88). The justification given for this technique is that the slope has been substantiated by production experience, and the verticle position, uncorrected for weight or complexity, is not an average of production experience, but rather a standard based on optimum efficiency which provides a goal to shoot for and an internal control over production (Ref. 52:92). (One wonders whether the "goal and control" biases the production effort to fit the master curve, thereby reinforcing the "accuracy" of the master curve.) When new models are introduced, adjustments to the master curve are made for two major factors - weight and complexity.

Unlike Company A, Company B does not use a direct relationship for weight changes. They reason that when weight doubles, man-hours do not double, but rise in some lesser proportion of increased weight. The weight correction factor has been determined from historical experience. This concept seems to be much more realistic an approach than is the direct ratio of Company A.

The master curve is predicated upon aircraft having a specific level of complexity. Each new model is subjectively evaluated by estimating personnel along with engineering and production personnel, and assigned a complexity factor.

The new model man-hours are then estimated from the master curve for the number of units being bid on - say 250 units. The cumulative average man-hours for 250 units from the master curve are adjusted by the weight and complexity factors. The master slope may then be adjusted for such factors as the rate of production and the number of contracted aircraft. These adjustments result in a progress curve for the new model.

Major changes are estimated by an analysis of the changes in major components. Company B develops separate progress curves during production for each major component of the aircraft. If for example, the major change occurs at unit 75, and involves just one major component, the analysis is localized to that component. A new component curve is derived with that component being considered now as unit one, while the remainder of the unchanged components are at unit 75. Immediate and subsequent effects are thus calculated by summing component curves at any desired production quantity.

Company C. This company does not include experimental models in their progress curves because these models are built in a separate shop and by other than production personnel. In addition, it claims an extremely low turnover rate among its shop personnel allowing for higher retained learning to be transferred to new production. Consequently, their curves start at a lower unit one value and experience a flatter slope. They have experienced different rates of learning in each of the major categories of production costs - machined parts at 95%, sub and major assembly at 85%, and final assembly at 80%. The slopes are said to vary with rate of production and complexity of tooling, becoming steeper as the optimum rate is approached and/or more complex tooling used. Their composite slope for estimating purposes is approximate 85%.

The first unit man-hour estimate is calculated by subjectively comparing the new design with prior aircraft. Factors considered important are such as, size and weight of the aircraft, type of equipment to be installed, production processes to be used, rate of production, and type of tooling. No weight or complexity formulae are used.

Company C uses the unit average curve (based on lot data) as being more sensitive and a better indicator of trends than the cumulative average. In estimating follow-on procurement, it projects from the last several lots, reasoning that the most recent data are most representative of the company's current production experience.

Major changes are estimated in much the same way as in the original aircraft estimate.

<u>Company D.</u> A slope of 90% has been derived from historical data for experimental models and production aircraft at low production rates, with an 81% slope for normal production runs. The first unit estimate is made by comparative analysis with prior aircraft for factors such as weight, complexity, production methods, and type of tooling. Adjustments may be made for example, for the amount of metal cutting or painting and sealing. The unit average curve is used for the same reasons, and in basically the same manner as Company C, above.

Major changes are estimated in a different manner by this company. Detailed estimates of the effect of the change are made, but the estimate is forecast at unit 1000. An "appropriately" sloped line is drawn towards unit one. The changed aircraft is not considered as restarting at unit one but at some subjectively determined unit after unit one because of learning carry-over from the just changed model. Why the change estimate is based on unit 1000 when the original aircraft is not, is unknown.

Company E. This company builds its estimate from curves based on production operations, i.e., detailed work, sub-assembly and major assembly, each with different slopes subject to adjustment in individual cases. It finds also, that a slow monthly rate of production will flatten a slope substantially as compared to capacity production.

The first unit estimate is made, once again by comparative analysis with prior aircraft for weight, complexity, production methods, tooling, etc. This figure may be adjusted, dependent upon the amount of learning it is felt has been gained in building the experimental models.

The unit average curve (based on lot data) is used for projection purposes. For major change proposals, it projects the unit curve to the mid-point of the anticipated changed aircraft lot, and adjusts this figure.

Company F. Subjective slope determination based primarily on rate of production and type of tooling involved in the methodology of Company F. It too finds that a flatter slope occurs for low rate production. In the company's opinion, this is caused by contract forced high rate tooling with low rate production. Individual workers must then be assigned broader jobs with less opportunity to specialize on a particular operation (which is characterized by steeper improvement rates).

Two interesting reasons are offerred for company experienced "toeing-up" of its progress curves. The first is that last lots tend to be smaller, and as a result, set-up and other fixed costs (Ref. 52:117 - presumably man-hours rather than plant fixed costs) must be allocated over a smaller number of units. Second, the company feels that certain amount of parts "stealing" from future lots by shop personnel, to replace defective or rejected items on a current lot, is inevitable. At the last lot, delays and inefficiencies occur while replacement parts are secured, thus "toeing-up" the curve.

Unit one estimates are based on similar factors as Company E, and others. However, estimates are made by first summing aircraft components, and then comparing this estimate with one made independently for the aircraft as a whole.

Curve projections are based on the unit average curve primarily, but the cumulative average curve is used for "visualizing" an over-all trend for the contract. Major changes are estimated in a manner very similar to Company E.

Company G. Slope and unit one estimate are determined by comparison with similar past aircraft. Similar factors as above are considered.

For projection purposes, actual accumulated data is examined for abnormal non-recurring fluctuations which will not affect future improvement. These data are then disregarded and a curve is fitted to the remaining data. Its slope is then projected from the last available cumulative average data.

This company believes that the slope is affected mainly by rate of production and production build-up, and the way such costs as mock-up costs are allocated. Consequently, it allocates mock-up expenses equally to all aircraft on the initial production contract. This generally results in a flatter slope than if allocated only to the initial unit(s). Additionally, no leveling off of progress had been experienced by this company even for one production run of 6000 aircraft. Major changes are estimated for the same technique employed by Company B.

Summary and Analysis. Most of the companies reviewed used unit average curves generated from actual production lot data to project follow-on production. Unit average curves are also used as an aid in estimating costs of major changes, as are weight changes and other subjective determinants.

Determining slope values and unit one estimates in advance of production is considered a highly subjective matter. Some companies find different slopes are applicable for experimental models as opposed to production models. Some find definite leveling-off of progress, whereas Company G did not experience it. The most important factors claimed to affect the slope are rate of production, tooling, and complexity.

It is apparent from the different methods employed that estimates are best viewed as estimates only. Any figure desired can be reached by the contractor by changing any of the many subjective adjustment factors. Estimates are, of course, both necessary and useful whether in a competitive environment or a monopsonistic environment. Under competition, a bad product cost estimate may result in financial disaster. In the world of government contracting, contractor bids, the type of contract involved (for example, cost-plus types as opposed to fixed-price), and price negotiation (and re-negotiation) are three of the many factors which may "influence" cost estimates and determine financial outcome.

With many controversial issues still unresolved after thirty one years of progress curve application, one is drawn to the conclusion that progress curve theory is more an "art" than it is a science. Before improvements in its predictive ability can be realized, intense investigation of the basic causes of the phenomenon is an apparent prerequisite to its accurate quantification. If accurate quantification is possible (see Chapter VIII - Conclusion), standardization of the definitions and categorization of factors involved, as well as a systematic method of application, should produce better, if not highly accurate, estimates.

VI. Factors Contributing to Time/Cost Reduction

As with everything else about the progress curve, its cause factors, and the contribution of each cause factor to the total effect, is still being deliberated. Indeed, there are those who assert that the existence of the curve is predicated on its use. It is held that without such conditioning, the phenomenon would not occur, at least not with the regularity which is presumed in the mathematical model. However, the weight of evidence strongly indicates that it does in fact occur, and that it occurs with a uniformity which can be measured. In a study by S. A. Billon, the question is answered directly: "Analysis of empirical data strongly suggests that the existence of the time reduction curve phenomenon is not predicated on the use of the curve for purposes of planning and control of man-hours required in production. The firms, from which the data was obtained, did not employ a preconceived model of time reduction, yet a definite regularity in time reduction was observed in a majority of cases."[1]

We shall proceed under the assumption that the phenomenon does exist and that its expected use is not a condition necessary for its occurrence. An examination into the major factors which contribute to time/cost reductions follows. The purpose of such an examination is to explore the "stratification" of the curve to gain additional insight into the phenomenon. Of the factors involved, many authors assert that worker learning is the major contributor to the total effect.

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[1] Billon, S. Alexander. "Industrial Time Reduction Curves as Tools for Forecasting." Dissertation Abstracts, XXIII: 1215, Ann Arbor, Michigan: University Microfilms, Inc., October, 1962.

<u>Worker Learning</u>. Numerous studies have been conducted concerning the psychology of learning. As noted by S. A. Konz (Ref. 68:13-15), pioneering studies started in 1885, resulting in countless theories. Studies include those which attempt to restrict the influences on learning to just worker improvement of a manual operation. Methods and/or process improvements by management and engineering personnel were not a factor. One concept which emerges is that, in general, except for initial learning which is slow, improvement increases at a decreasing rate over cumulative trials, approaching a presumed asymptote. Many studies show that a plateau occurs in the curve after some undefined point in the learning process before resuming its general shape. This would result in as "S" shaped curve. Konz states that, "Some industrial studies support the existence of a plateau and some do not. Whether it exists in most learning situations is open to question" (Ref. 68:15).

One objective of worker learning studies is the establishment of standards for operator performance. Standards were traditionally set by time and motion studies, however these are static type studies which do not account for continuous improvement. The present trend appears to favor the use of worker learning curves to set standards, as well as to establish incentive programs. F. J. Powers (Ref. 26) proposes such a scheme based on a "standard reference curve" which is "S" shaped. However, even in this plan, a normal or 100% performance is presumed to occur at the end of a calculated learning period. The period is based on the length of a standard cycle time (standard time for one complete execution of the fixed task), and manual complexity of the job. This plan was devised for short run production, however, and no suggestion is presented for an application to long run production.

Another study, which is not restricted to short run production is titled, "The Prediction of Learning Rates for Manual Operations", written by Walton M. Hancock (Ref. 15). This study, too, is concerned with predicting the number of cycles (repetitions of the fixed task) necessary to attain a predetermined standard time. However, the function used is the familiar exponential form. Specifically (Ref. 15:46),

$$y = k X^{-a}$$

where,
- "y" is the cycle time
- "k" is a constant determined by multiplying 2.5 times the standard time for the operation
- "X" is the cycle number
- "a" is a constant determined by substitution at the point where "X" equals the number of cycles to standard, and "y" equals the standard time

It should be noted that the "slope" of this function is determined (by substitution) at a point - standard time and the number of cycles to achieve that standard. It is not a regression slope. In addition, the constant, "k", is a function of standard time. However, results show a reasonably good fit between predicted and actual learning rates. Overnight and longer breaks caused a loss in learning, as would be expected. Although cycle time increased after a break, subsequent time reduction appears to follow a parallel slope. Also, the more experienced the operator, the less the effect of the break.

Hancock does not extend the results to imply a log-linear relationship, although he professes a knowledge of aircraft industry progress functions. He concludes: "At the present time, the variation in ability between individuals has been recognized as important but is not included in the methodology due to the need to develop measures of the abilities involved...the prediction of human performance is extremely complex ..." (Ref. 15:46).

The appeal of a log-linear relationship has not gone unnoticed. Our old friends, Conway and Schultz, have reported that where some basic data of worker learning were replotted on log-log paper, they conform to a log-linear relationship. The slopes of six set of such data varied between 88.9% and 91.2% (Ref. 13:47). Although they accede that worker learning contributes to the progress curve phenomenon, Conway and Schultz call attention to an important consideration. Studies are either performed in a laboratory environment or in a highly artificial atmosphere created by the very process of setting up a controlled experiment. In addition, the authors are "... convinced that the extent of the production workers' contribution to progress is as much a function of motivation and sociological factors as of physical capability. There can be considerable difference between the progress workers <u>will</u> contribute in an industrial environment and the progress they <u>can</u> achieve in a controlled experiment" (Ref. 13:47).

In a similar vein, Hirschmann notes that a "ceiling psychology" may well affect improvement (Ref. 18:135). He describes a case in which a new machine with a given rated capacity, operated initially at only half its rated capacity. (Although this does not concern worker improvement of a fixed task, its reinforcement of the above remarks is considered sufficient reason to introduce the case here.) Engineers were assigned to bring the machine's output up to "normal". When this was reached, efforts were about to cease on the presumption that rated output is the limit of capacity. However, a "learning" pattern was noticed and efforts were continued which resulted in achieving triple the rated capacity in four months. Hirschmann does not note whether further improvement continued, or whether efforts were discontinued at that point. One wonders whether his ceiling psychology took hold at triple rated capacity. One also wonders whether the economic returns from the engineering team efforts were considered to be diminishing to the point where alternative projects appeared more fruitful. (See discussion of this concept in Chapter III - Non-Linearity).

Hancock's article was published in January of 1967. It was based on a research project of the Industrial Engineering Department of The University of Michigan. Because of its recent vintage and apparent breadth, the hesitant assumption is made that no further work on the exponential function previously described has been conducted.

It would seem beneficial to progress curve theorists to be able to segregate worker contributions. However, this may not be possible on an individual basis because of the many, as yet, unquantified psychological and sociological variables which affect a given worker's motivation to improve in any given time period. Perhaps industrial studies on group performance would result in better prediction capabilities by tending to average-out individual idiosyncrasies and abilities. Even with this approach, worker learning is not independent of its environment. Management emphasis, at least, is an interrelated factor. As inferred by many authors and stated by J. L.

Kottler (Ref. 23:177) "Depending only on the worker, and without efficient program management, it would indeed be fortunate if the improvement was of the amount that should be expected and this improvement continued for any sustained length of time." It would seem then, that much work remains to be done if reliable quantification is to be accomplished.

Management. Management may include a gamot of items, depending on how it is defined. Most definitions would include first line supervisors as well as higher echelon managers and related staffs. How well managers plan and control has an effect on the curve. How well proper coordination of effort is integrated, and solutions to problems devised, also has an effect. Corporate policies and work environment will influence productivity, as well as utilization of employees' suggestion. Personnel hiring, training, and motivation are factors. Incentive plans may well cause increased productivity. M. S. Tittleman reports that the introduction of an incentive program changed the slope of production in one study, from 93% to 70% (Ref. 35:37). The slope then leveled out to a constant after a period of time. It can be argued that incentives may not increase learning, but merely increase the work pace. This however, seems to be a moot point for industrial psychologists to investigate. Since the progress curve measures decreasing labor hours per unit, and as long as reject and/or scrap rates do not increase, learning (by the progress curve definition) occurs.

Management contributions through planning, coordination, control and miscellaneous items as suggested above, are seen to be unquantifyable in the main. How would one begin to estimate the effect of a decision or policy change, except to qualitatively surmise that it will improve or hinder productivity?

Engineering. W. Z. Hirsch (Ref. 16:138) would remind us that, "The more complicated the manufacturing process the more important the contribution of the engineer". Design engineers may specify tight tolerances initially to insure performance. As experience is gained, liberalized tolerances and re-design will allow less expensive and faster production. Tooling improvements, both in quality, and geared to production rate and quantity, can return handsome dividends in labor hours and total costs. Industrial engineering efforts in plant layout, routing and materials handling are dimensions of improvement. Others are methods improvements via time and motion studies, systems and procedures improvement, and optimum sequencing. If constant rate of output is not required, Sturmey notes (Ref. 32:973) that costs per unit of output will fall as the rate is increased up to the capacity of the line. Further increases will cause unit costs to rise because of additional equipment required, plus higher labor rates for shift and overtime work. Lastly, lot size has been seen to affect costs (see Chapter IV - Optimal Lot Size).

Some of these engineering innovations are quantifyable - others are not. In a complex manufacturing process, the contributions of engineering may well overshadow worker learning.

Materials. Since materials are a significant part of total cost, a progress curve expressed in terms of material cost, or total cost per cumulative output, bears observation. Cost improvement can be realized through correct material quality specifications, sized to minimize cutting and scrap. Development of reliable, as well as alternative source of supply, to preclude shortages and delays can greatly enhance the materials cost outlook. Also, quantity discounts balanced with storage costs is another factor.

Worker learning, management, engineering, and materials have been used as titles to group the major factors which contribute to time/cost reductions. Other factors can undoubtedly be named, but the above are considered sufficient. One other aspect needs to be considered. Our discussion has centered on improvements. Crawford and Strauss have listed factors which affect the level, or height, of the curve. Some of these are (Ref. 43:42),

1. The length of the production run.
2. Whether or not the model was engineered for mass production.
3. Whether or not proven engineering was available when production was started.
4. Whether or not high production was started from low production tools.
5. Introduction of design changes.
6. Whether or not all tools were available when production was started.
7. Availability of experienced manpower.
8. Relative priority attached to a given model.

Analysis. With such a staggering list, any attempt to "build" a progress curve from its component factors would be a formidable and monumental task, even if they all could be quantified. In addition, it would probably be found that while many separate factors improve erratically, their cumulative effect will generally produce a fairly smooth curve. The aggregate approach has been used, then, for good reason. T. P. Wright did show separate curves for man-hours, materials, and overhead. No attempts have been made to show the "progress" of management or engineering directly, with some possible exceptions perhaps, as in Hirschmann's machine productivity case.

Still, the problem with an aggregate approach is that it may hide, or cover up, inefficiencies. It is conceivable that engineering changes to a production process will allow continuation of overall improvement by compensating for a leveling-off, or even a toeing-up of worker learning. Satisfaction with expected man-hour per unit decreases, without knowledge of its causes, may not allow the opportunity to capitalize on possibly greater decreases in man-hours per unit. For example, if an engineering change simplified, or eliminated a segment of the production process, and worker learning is assumed to continue at some regular rate, then the overall man-hours per unit slope should become steeper by virtue of the engineering change.

Such reasoning suggests an approach which could prove to be valuable. If further research results in a reasonably accurate measurement and prediction model of individual or group worker learning, such learning can be historically plotted throughout a particular production process. Assuming no management innovations at the time an engineering change is introduced, the effect of that change is conceivably measurable. Conversely, assuming the other variables are held constant, a management change can be "measured" in terms of productivity. In short, preoccupation with expected log-linear reductions may mask opportunities for greater reductions. It may also prejudice an investigation of the phenomenon.

While some factors are quantifyable in terms of a progress curve model, others are amenable to quantification for optimization purposes. The optimal lot-size model discussed in Chapter IV is an example, as is the model of second source procurement. If then, those factors which can be optimized, are optimized, they may be treated as a constant in the aggregate progress curve, rather than a variable. In such a manner, isolation and observation of those factors which are true variables will be an easier task. Concurrently, the analyst has some assurance that slope and height changes are not caused by the optimized factors.

Lastly, it would be foolhardy to attempt to quantify management and engineering innovations as an expectation which will conform to a progress curve. Whereas repitious work may allow learning measurement, originality is uniquely unpredictable. Its sporadic nature implies this. Given a large organization, innovations may "average-out" at some measurable rate. They also may not; at least not in terms of a regular magnitude over time. A "programmed innovation rate" would be preposterous, and would undoubtedly tend to stifle that which it intends to stimulate.

VII. Misuses of the Concept

When a concept, devised as an estimating tool, is "cast in concrete" and inherits the dignity of a natural law, as it has in some quarters, it will inevitably be misused. Persons not familiar with its characteristics can be easily led astray. Those individuals who are intimate with its characteristics may find advantageous uses for it. In this chapter we will examine a limited number of misunderstandings and/or misuses of the progress curve. They may prove to be enlightening.

Illusion of Progress. One of the well known drawbacks of some statistical measures is that they tend to hide the significance of the data. Cumulative averages are a prime example. Although not widely used, the cumulative average curve is sometimes employed because it permits a linear fit, while the same data plotted as a unit curve (or a unit average curve from lot data) do not follow linearity. This would seem to be all the more reason not to use it.

Conway and Schultz (Ref. 13:48) warn that "The cumulative average is a smoothing process whose power increases as the production quantity increases ... it becomes so insensitive to changes in the unit data that catastrophic events could take place in the job with an imperceptible effect on the cumulative average plot."

Other authors also take up the cry. Samuel L. Young (Ref. 38:412) offers a dramatic example, with some hypothetical data designed to emphasize the point, as follows,

Unit Number	Actual Hours/Unit	Cumulative Total Hours	Cumulative Average Hours/Unit
1	1,000	1,000	1,000
2	700	1,700	850
3	700	2,400	800
4	700	3,100	775
5	700	3,800	760
6	700	4,500	750
.	.	.	.
.	.	.	.
.	.	.	.
300	700	210,300	701

As can be seen, no improvement actually occurs beyond Unit 2, whereas the cumulative average column shows continuous improvement. As Young states, "This creates the illusion of progress, when in fact nothing is happening. Management may be lulled into a false sense of achievement and assurance that all is well" (Ref. 38:412).

Data Reliability. Most knowledgable authors are seriously concerned about the reliability of data. The experience of Conway and Schultz lead them to believe that "actual labor hours are seldom accurately recorded, and considerable doubt exists as to the validity of operator times charged to direct vs. indirect labor accounts" (Ref. 13:43). S. L. Young underscores the latter point in his article (Ref. 38:412), as does Frank J. Andress (Ref. 2:90). R. P. Zieke cites an example (Ref. 39:79) of time "juggling" where a shop foreman moved workers who were surplus on other jobs, to a job with a steep sloped schedule. However, the time charge cards were not changed. The records would show, of course, that the scheduled slopes were met on all jobs, thereby reinforcing their "accuracy". Meanwhile, in the shop, the foreman was shifting workers as necessary to meet schedules, while charging time cards as necessary to meet the scheduling sections assigned slopes. This "technique" may be expedious for short run jobs where it is considered more convenient to complete the work than it is to condemn the slope. For a long production run, such a "speed-up" schedule can be challenged, with considerable pressure applied through labor unions, or through shop grevience procedures. Alternately, as Zieke notes (Ref. 39:76), "When the slope selected is too flat, production units will meet the schedule with ease, but almost never exceed the schedule by more than a small amount until large quantities of spare time become difficult to hide." Young concurs with this thought. He comments (Ref. 38:411), "Basic motivations for job security will prevent men from finishing the job early and then loafing or charging their time to overhead or idle-time accounts. Hence, 'work will expand and fill the time available', and the planned time allowance will become a minimum time to be charged to the job."

If such practices are widespread in a particular company, its progress curves may be standing on an unreliable data base.

<u>Slope.</u> Slope values are almost always quoted in integer values. The difference in spread between 79 1/2% and 80% at 500 units is 6%, becoming 7% at 1000 units (Ref. 39:78). If a negotiated firm-fixed-price contract with a government agency is based on a proposal with an agreed-upon slope for costs and a profit add-on, one need not stretch his imagination too far to recognize which in direction rounding off the slope value will occur. Zieke states: "Aerospace estimators exhibit an almost unbelievable nonchalance in this matter, while computing million dollar programs down to the penny" (Ref. 39:79).

Another characteristic misunderstanding decried by the above author is that of relating slope values to contractor efficiency. The inference is made (Ref. 39:76) that with the Department of Defense and especially the United States Air Force, steep sloped progress curves are considered indicative of efficient production and are favored over shallow sloped curves. Consider the following situation in which it is assumed that bidding contractors are required to submit progress curve estimates with their bids, but need not maintain curves during production. This is not unreasonable for "small" contractors and/or firm-fixed price bids. If two contractors submit bids of nearly the same total price, the steeper sloped bid may well have an edge and be awarded the contract. One possible misuse of this out-look can be seen by examining Figure 2 (next page). Assume that two replies to a contract being let for 1000 units resulted in identical bids of "x" dollars per unit. Company A's bid contains a steeper slope than Company B. All other things being equal, the following reasoning might be used: Since the cost is the same in either case, not only is Company A more efficient (completely fallacious reasoning), but we can get additional quantities at a cheaper price in an extension to this contract, or a follow-on contract. On the surface, the latter reason is appealing. However, Zicke notes that most changes in quantity are cut-backs, and, "Financially painful extensions (to the contractor) in order quantities offer only a small risk because of the numerous engineering changes that permit ... relief on the basis of new requirements that change the product and require new bids" (Ref. 39:32). With a cut-back, the differential price paid by the government can be extensive. In addition, if Contractor A

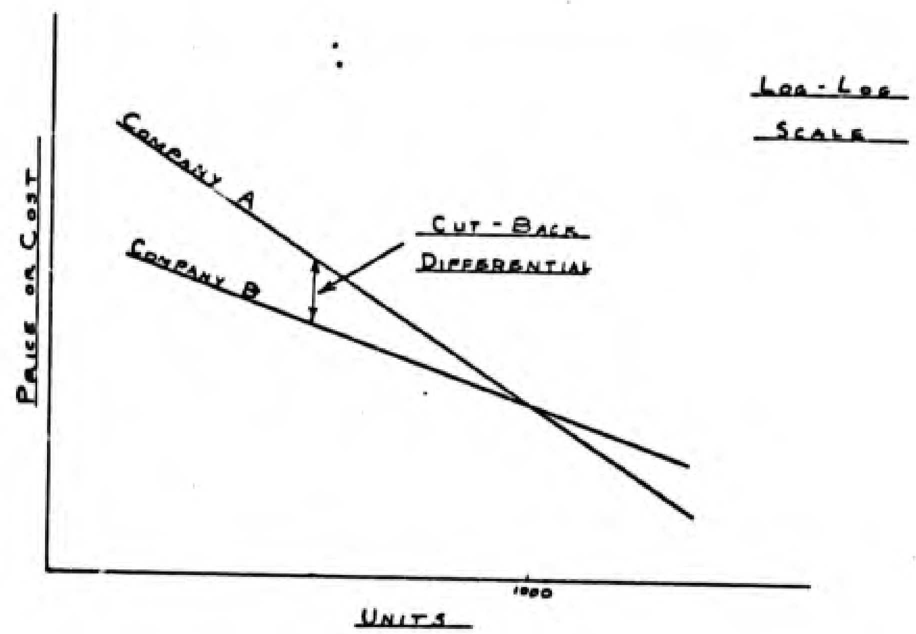

Figure 2

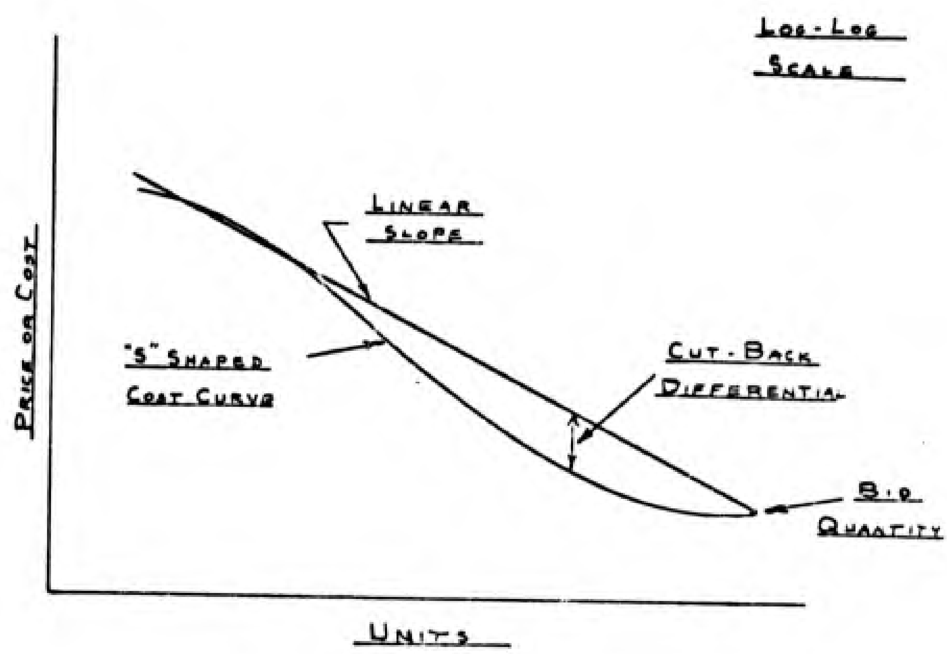

Figure 3

actually has a flatter slope than he shows in his bid, additional profits are seen to occur by recalling the effects of a half degree change in slope at 500 units. Finally, costs incidental to the cut-back are usually recoverable on defense contracts.

Another aspect of possible government over-payments can be seen in Figure 3. The hypothetical situation is as follows: Assume a cost-plus-incentive-fee contract, requiring reports of production costs. A large contractor submits production data which reasonably follows the linear slope shown, and which the linear-conscious government agency expects. If the contractor's actual costs follow an "S" shaped curve as shown in Figure 3(and hypothesized by some authors), a cutback results in extra profits in addition to cut-back compensation. Not only this, but if the proposal quantity is, say 1000 units, and the progress curve submitted with the bid is based on an estimate of unit cost at an arbitrarily high unit number, and a linear curve "backed-in" to unit one, the net effect is the same as a cut-back. The contractor will be operating on the "S" curve and is compensated from the linear curve.

One last comment on slope "errors" will be noted; this comment in regard to intra-company use of progress curves. If a company believes in the linearity of a progress curve at the total production effort level, it conceivably can impose that slope value on all its departments as a control device, via schedules. (If it allows that department slope are different, then the total cannot be linear, except as hypothesized in Chapter III.) If alert management recognizes that department slopes must be different - assembly vs. machinery, for example - then different slopes may be assigned to departments.

Even more astute management may recognize that sections within a department will probably have different slopes, and so assign them. Managers with deep insight will note that different type jobs mean different slope value. The really clairvoyant manager will recognize that individual jobs must require individual slope values - thus, schedules are set. But, argues the foreman, the slope is too steep (rarely too shallow), and the "in-fighting" begins.

As we have seen from the data reliability section of this chapter, and the factors which contribute to the progress curve effect, control devices and preoccupation with expected results may mask opportunities for greater results. One reason for this is that emphasis is primarily placed on meeting the schedule, rather than generating improvement. As Conway and Schultz have argued, "...once a control or quantitative objective is imposed on an organization, there are strong forces created to make the performance fit the objective" (Ref. 13:41). Emphasis on the wrong objective detracts from, and biases the primary objective.

Curve Height. It is the contention of Samuel Young (Ref. 38:410) that historically high "cost-plus-fixed-fee" production contracts (as well as development contracts) in a monopsonistic environment, established a base which fosters high cost curves estimates. This is an interesting thought in that, as seen in Chapter V, airframe manufacturers estimate the first unit man-hours/ cost of a new model by comparing it to similar prior models, then project a slope from the first unit height. If historical man-hours/ cost were inflated to take advantage of a fee based on a percentage of cost, then it follows that estimates based on historical data will be high. Couple this with

Young's comment: "...'work will expand and fill the time available', and the planned time allowance will become a minimum time to be charged to the job" (Ref. 38:411). If maintaining high estimates with "actual" production data is not difficult, then the cycle is complete. Historically high costs perpetuate present high costs. As Young points out, the Department of Defense is currently stressing incentive type contracts, however, "...at best it will be a long time before this trend creates a significant amount of cost-effective history" (Ref. 38:410).

Perhaps this thought is applicable in a monopsonistic environment only. Presumably, competition will weed out the inefficient producer. However, the former environment is by no means insignificant and within it, the progress curve originated and thrives today.

Other misunderstandings and misuses can be found to supplement the ones outlined above. As with any concept, instances of its perversion is not cause to discard its use, nor should the assumption be made that all users are tainted.

VIII. Summary and Conclusions

This paper has covered a multitude of ideas. Following a review of significant contributions to progress curve theory through 1956, subsequent writings were examined. The major issues were, and still remain, slope linearity, curve height and the validity of the relationship itself. In addition, the interrelationship between first unit cost and slope value is of significant importance.

Concerning the last remark, there seems to be no doubt that the man-hours or cost of the first unit is a function of, at least, pre-production planning and tooling, the producer's accumulated experience, and his familiarity with the particular product through, perhaps, its similarity with former models. The more intensive the pre-production planning and tooling, the lower will be the first unit cost and the flatter will be the slope. As reported by airframe manufacturers (Chapter V), slope is also affected by rate of production, product complexity, tooling and production methods, to name a few factors. Each particular manufacturer is confronted with these, and more variables when he must decide at what point to end preparatory work and start production. There is a cost associated with each factor. It would appear that each manufacturer's decision on the amount of preparatory work to be done would be based on his appraisal of its effect on production costs. In other words, more preparatory work (and its associated higher costs) equates with lower production costs. Under the assumption that minimization of total project cost is desired, one can envision a solution in cost minimization techniques, which would balance preparatory and production costs to achieve the lowest total cost.

Formulating this problem would be no easy matter. As always, definitions are important. Pre-production or preparatory work would seemingly be defined as all work up to the point when the first unit is started. The functional relationship for production costs would be, of course, the progress curve, whether the simple cost-quantity relationship, or a more descriptive formulation. Although the first unit cost (which establishes the height of the simple cost-quantity relationship) is a function of preparatory work, the first unit cost is part of production costs. Therefore, in the interrelationship between preparatory costs and production costs, the value of the "a" parameter will vary in accordance with preparatory work. But how does it vary? As noted, formulating the problem will be no easy matter.

Functional notation may be employed to outline the scope of the problem. Total cost is the sum of preparatory costs and production costs. Production costs, however, are a function of preparatory costs, quantity, and other factors which may define the progress curve. Functionally, then,

$$TC(P, Q, A) = P + VC(P, Q, A)$$

where,
TC = total cost
P = preparatory costs
Q = quantity
A = all other factors affecting the progress curve
VC = production costs

Cost minimization is found by setting the partial derivatives of the function with respect to P, Q, and A, equal to zero. (The proper second order conditions would have to be verified, of course, to insure that the minimum point is obtained.) Thus,

$$\frac{\partial TC}{\partial P} = 1 + \frac{\partial VC}{\partial P} = 0$$

$$\frac{\partial TC}{\partial Q} = \frac{\partial VC}{\partial Q} = 0$$

$$\frac{\partial TC}{\partial A} = \frac{\partial VC}{\partial A} = 0$$

The interpretation to be placed upon this "solution" is that total cost minimization is invariant with respect to Q and A. It varies only with respect to preparatory costs. Thus, the obvious conclusion is that one would continue to invest a dollar into preparatory costs as long as total costs are decreased by more than a dollar.

Specific functional relationships would have to be established before a specific solution could be obtained. Formidable problems can be seen in the quantification of a producer's accumulated experience and his familiarity with the model being produced. If his experience and/or familiarity are extensive, less preparatory expense would be required than for an inexperienced producer. The question is - how much less? A subjective weighting factor to "adjust" for the experience factor may be the only solution to this dilemma. Although the impreciseness of such a procedure is undeniably undesirable, one may ask - what alternative is available if total cost minimization is desired? Even a gross solution may be preferable to an arbitrary rule of thumb.

A solution would, by no means, be identical for all manufacturers.

Each would have a unique solution dependent on the parameter values associated with each firm. However, a range of solutions may be able to be specified. For example, for a sample of manufacturers, the optimum position for preparatory work might be found to vary between say, seven and twelve per cent of total cost. If such a result could be shown to be valid, it could be of some consequence in terms of dollars saved on multi-million dollar contracts.

Curve height has been described as a major issue. The credibility of historical costs cast a stigma of doubt upon this parameter. Estimates based on prior production models in the aircraft industry, at least, may be misleading because of a possible "inflated" data base (see Chapter VII - Curve Height). In addition, future aircraft costs may present a geometric departure from the past. For these reasons, the newer techniques of RAND and Planning Research Corporation may prove to be of much superior quality because these relationships include aircraft physical and performance characteristics to establish curve height, rather than just a comparison with former models. Once data is accumulated with which to test these relationships, their worth will be established. Still, these new models retain the linear assumption for slope.

As with curve height, so too with linearity. The basic tenet of linearity has been under constant assault. Indeed, even Wright suggested that the total cost curve changes slope. The decomposition of component curves suggest that they too, may not be linear. In addition, the credibility of data and the propensity to meet schedules adds doubt as to the reliability of that which supports linearity. As a practical expedient, Asher suggests that the curve is close enough to linearity to use up to certain cumulative outputs, but warns of projecting beyond this point, except in small increments. Another expedient, the use of linear segments, may be an acceptable approximation to the "S" shaped hypothesis of Carr, with the center section corresponding to Ash 's suggestion of approximate linearity.

Approximate solutions aside, however, simple relationships have failed to fit reality. Limited success (with gross estimates resulting in many cases), has apparently been ignored by some who have imputed a scientific respectability to the progress curve. A precise mathematical solution, using imprecise estimates of parameters, and an imprecise functional relationship, hardly adds credibility to the concept.

If the newer techniques prove to establish the height of the function more precisely, intensive research into worker learning may be the approach to solve the slope dilemma. The exponential learning rate displayed in a manual fixed task has promise. Although the parameters are in terms of standard time, other relationships can undoubtedly be devised in terms of productivity. Extensions beyond manual fixed tasks would be desirous. This would allow segregation of worker learning in the complex production process, permitting (hopefully) the isolation of other effects. As suggested in the analysis section of Chapter VI, some component factors, if optimized, may be reduced to constants so as to allow more precise study of those factors subject to learning. Other factors which are qualitative, perhaps may also be isolated, but prediction of qualitative factors does not appear to be possible. However, standardization of the definitions and categorization of factors involved, combined with a systematic method of approach to the problem, should allow more accurate relationships to be formulated. If more accurate component curves can be constructed, the summation of component curves is felt to be the basis for a more accurate indication of total costs, even if not a simple linear relationship. It is suggested that the result will conform to Carr's S-shaped hypothesis.

The preceeding may be described as a micro approach. It would have the advantage of allowing a measure of regulation to the many variables involved while relationships are being formulated. Without this kind of "regulation", irregularities in results would go unexplained. Another approach, a macro approach, may be productive in confirming the shape of the total cost curve. As suggested in the non-linearity section of Chapter III, a total cost curve may be obtained by summing the costs for each unit. If such a procedure independently produces an S-shaped curve, the evidence will be overpowering that linearity is a misnomer. It is suggested, once again, that the results will tend to conform to Carr's S-shaped curve.

In testing hypotheses, relationship predictions are compared to actual data to check the "fit" obtained. In addition to data reliability problems, the situation is compounded by cost accumulation problems. "Actual" cost is that cost which is accumulated from accounting records. There are probably as nearly as many accounting systems as there are companies using accounting systems. Where some may identify a particular type of labor as indirect, others may call it direct. This would certainly affect direct man-hour compilation - against which hypotheses are checked. The illusive nature of "actual" costs is even more nondescript when overhead is introduced. Are we not then, comparing a relationship against a variable standard? It would seem that one point of objectivity which can be established is to somehow, some way, standardize an accounting system so that the "actual" costs of various companies are at least comparable. This is not to suggest that companies be forced to use a standard accounting system. It is merely a recommendation that for hypothesis testing, "actual" costs in any company are actually "actual" by some consistent and standardized definition. Without such standardization, one will never be quite sure whether a derived relationship is either valid or reliable.

In summary, many problems were seen to exist through 1956. Many proposals have been reviewed subsequent to that time. A pattern appears to have emerged in that small parts of the complex problem now have suggested solutions, as evidenced by the work on nonhomogeneous production, lot mid-points, and optimal lot size, for example, The larger problems of non-linearity and curve height are still in the formative stages of solution. This paper has attempted to draw together isolated works which indicate a trend toward final solution. That trend appears to point toward a non-linear solution, possibly S-shaped, and the establishment of curve height by means other that subjective comparisons to previous models.

It is apparent that more is known now (at the micro level), than was known eleven years ago. What is apparently needed at the present time is a concerted effort at drawing together all the pieces to form a composite whole. Such an effort, drawing upon advanced analytical and statistical tools, is capable of greatly advancing the "state-of-the-art" concerning progress curves. It will not, however provide a final solution, because the progress curve phenomenon is not a "natural law". Certain intractable problems prevent total quantification of all factors involved. These intractable problems are found in the inability to adequately predict human behavior.

It is obvious that the human element is the one responsible for learning. The phenomenon occurs not because some mystical force shapes progress, but rather because people learn to be more productive. Increased productivity comes from simple motor skill improvement. It comes also from work simplification and process improvement devised by the human brain. Engineers ease tolerances, redesign to ease production snags, and redesign to eliminate unnecessary components or complexity. Engineers are human. Management aids productivity by less quantifyable means. The psychological and sociological environment established by management may well be more meaningful to progress than all the other factors. At least in the extreme of bad management, productivity may become static. Managers are people. Materials improvement come about because people involved in the function use their human intellect to reduce unit cost. And so, the sum total of improvement can be attributed to people.

It is obvious that the human element is the one responsible for learning. The human element may also end increased productivity. It may be for the arbitrary reason of "that's good enough", or by design, because the economic return on the investment is higher on alternative work. It may be due to dissatisfaction with the work environment, either at the worker or professional level. Alternately, a decision may be made to continue to seek improvement on high dollar value items on programs running for several years. Also, such a decision may be forced because of external pressures imposed by contract or competition. However, whatever the decision, it is made by people.

It is obvious that the human element is the one responsible for learning. If a mathematical model used to describe a phenomenon does not describe it accurately, the model should be changed. People can't be forced to fit into a pattern. Even if the model describes what a "rational" man should do, it may not be describing what a "rational" man does do, because the concept of rational action may be incorrect. At best, it is extremely difficult to predict the human element, even if irrational action is assumed not to occur.

It is obvious that the human element is the one responsible for learning. At one time it was thought impossible to quantify factors which were "clearly" qualitative. Judgment and experience were the prerequisites to entry into management. However, the quantification concept persisted and met with successes - some great, some small. The pendulum has swung. Quantitative analysis is "in", qualitative analysis is "out". Unfortunately, some converts have not recognized that the human element cannot be accurately quantified with present knowledge.

It is therefore held that the progress curve dilemma has its solution in a combination quantitative-qualitative approach. That which can be optimized, should be - at the highest organizational level possible. That which can be quantified by, hopefully, more accurate progress curve relationships, should be. The results should reflect the expectations of progress, not the desires. If the expectations are not fulfilled, the reasons may best be found in qualitative factors.

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Appendix A

Information Sources

Some, but not all of the references researched were:

a. Technical Abstracts Bulletin - published by the Defense Document Center, Defense Supply Agency, Department of Defense - an index of government reports, including reports from research agencies and industrial organizations under contract with the Department of Defense.

b. Rand Index - an index of the publications of the Rand Corporation, Santa Monica, California. The Rand Corporation is an independent, non-profit organization engaged in a program of research concerned with the security and public welfare of the United States. It has done extensive work in the area of progress curves.

c. Applied Science and Technology Index - an index of books, pamphlets, and public documents as well as periodical articles in the field of applied science.

d. Dissertation Abstracts - an index to doctoral dissertations prepared in the United States and Canada.

e. Masters Theses and Doctoral Dissertations in the Pure and Applied Sciences - an index of theses accepted by Colleges and universities in the United States.

f. Defense Logistics Bibliography (U. S. Army Logistics Management Center) - an index and abstracts of logistic studies and related material.

g. International Abstracts in Operation Research - contain a subject and author index, as well as abstracts.

h. International Journal of Abstracts. Statistical Methods in Industry - contains abstracts of papers reporting applications of statistical methods to industrial situations.

i. The Journal of Economic Abstracts - an international journal covering articles published in sixteen countries and eight languages.

j. Engineering Index - an index of articles appearing in technical magazines, government documents and engineering college bulletins.

k. Business Periodical Index - an index covering periodicals in the fields of accounting, general business, labor and management, marketing and purchasing, industries, and trades.

In addition, numerous telephone calls were placed to cost analysis agencies of the Department of Defense, the United States Air Force, Army, and Navy, industrial organizations such as Boeing Corporation, McDonnell Corporation, and North American Corporation, and research corporations such as Aerospace Industries Associates and Rand Corporation.

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