The Johns Hopkins University
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A PASSIVE SYSTEM FOR DETERMINING THE ATTITUDE OF A SATELLITE

by

Harold D. Black

Applied Physics Laboratory
The Johns Hopkins University
8621 Georgia Avenue
Silver Spring, Maryland

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TABLE OF CONTENTS

SUMMARY

We demonstrate a simple, passive system to determine the attitude (orientation) of a satellite.

The system is based upon measurements of the direction cosines of two linearly independent vectors. The necessary measurements are performed in a satellite-fixed coordinate system and telemetered to a ground station. Knowledge of the satellite position (obtained from independent measurements) can then be used together with the telemetered results to compute the attitude matrix for the satellite.

A system designed and successfully implemented in the 1963 22A satellite is described. Some results are given.

I. INTRODUCTION

A group at the Applied Physics Laboratory has during the past several years developed "Solar Attitude" detectors (Ref. 1). In principle, these detectors are single solar cells placed on orthogonal axes of the satellite. From processing the telemetered voltage output from these cells we can obtain the direction cosines of the satellite-sun vector.

Relative to the satellite-fixed axes defined by the solar detectors, we can place another set of sensors to measure the direction cosines of another vector in body-fixed coordinates. We have used three orthogonally placed fluxgate magnetometers to measure the direction cosines of the earth's magnetic field. With the direction cosines of two vectors and independent knowledge of the satellite position, then, we derive the attitude matrix of the satellite.

II. ANALYSIS

We will define (in any convenient way) a set of orthogonal axes fixed in the satellite. Relative to this axis, then, we will measure at any particular instant, the required direction cosines.

Associated with the body-fixed axis, is a "reference" coordinate system (e.g., inertial coordinates) such that if the relationship of the body axis to the reference system is available, then, we can say that the attitude of the satellite is known. To make this statement explicit, we are given the components (r_1, r_2, r_3) of a vector of unit magnitude in body-fixed coordinates. This vector has a representation in another orthogonal coordinate system:

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \{A\} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \qquad (1)$$

wherein $\{A\}$ is an (orthogonal) rotational matrix which transforms from body-fixed to (say) inertial coordinates. If $\{A\}$ can be obtained explicitly then the attitude is known. In the system implemented by us, $\bar{r}$ is obtained from the solar attitude detectors, whereas $\bar{R}$ is computed from knowledge of the satellite orbit and the position of the sun.

It is intuitively clear that Eq. 1 is sufficient to determine the orientation of the satellite axes relative to the satellite sun-line <u>excepting</u> the degree of freedom involving rotation about this line. To determine all three degrees of freedom, it is sufficient to treat similarly any other linearly independent vector which can be resolved in satellite coordinates. For any vector, $\bar{b}$, linearly independent from $\bar{r}$, we write

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \{A\} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2}$$

In the system implemented by us, we compute $\bar{B}$ from the known position of the satellite and a mathematical model of the earth's magnetic field (Ref. 2). $\bar{b}$ is obtained from the fluxgate magnetometer telemeter results.

Equations 1 and 2 are then considered as a pair of simultaneous equations to be solved for the attitude matrix $\{A\}$. This solution is quite simple to achieve: Since $\{A\}$ transforms any vector in satellite coordinates and since $\bar{r}$ and $\bar{b}$ are linearly independent, we obviously have the relationship

$$\begin{pmatrix} (\bar{R} \times \bar{B})_1 \\ (\bar{R} \times \bar{B})_2 \\ (\bar{R} \times \bar{B})_3 \end{pmatrix} = \{A\} \begin{pmatrix} (\bar{r} \times \bar{b})_1 \\ (\bar{r} \times \bar{b})_2 \\ (\bar{r} \times \bar{b})_3 \end{pmatrix} \tag{3}$$

Equations 1, 2, and 3 are then combined into a single equation:

$$\begin{pmatrix} R_1 & B_1 & (\bar{R} \times \bar{B})_1 \\ R_2 & B_2 & (\bar{R} \times \bar{B})_2 \\ R_3 & B_3 & (\bar{R} \times \bar{B})_3 \end{pmatrix} = \{A\} \begin{pmatrix} r_1 & b_1 & (\bar{r} \times \bar{b})_1 \\ r_2 & b_2 & (\bar{r} \times \bar{b})_2 \\ r_3 & b_3 & (\bar{r} \times \bar{b})_3 \end{pmatrix}$$

As the matrix on the extreme right of Eq. 4 has columns which are the components of linearly independent vectors, its inverse necessarily exists. Thus, by inverting this matrix, we can solve for the attitude matrix, $\{A\}$.* A simple extension facilitates this inversion: If we use for the third column (rather than the components of $\bar{r} \times \bar{b}$) the components of $\frac{\bar{r} \times \bar{b}}{|\bar{r} \times \bar{b}|}$ and for the first column, the components of $\bar{b} \times \frac{\bar{r} \times \bar{b}}{|\bar{r} \times \bar{b}|}$, then the matrix we have to invert is an orthogonal matrix which can be inverted at sight.** These details, however, are only of computational interest as Eq. 4 embodies the crucial idea necessary to obtain the solution.

Once the attitude matrix is obtained, it is a trivial matter to obtain the orientation of the satellite relative to other known coordinate systems. A particularly convenient one that we have used is a coordinate system based on the position and angular momentum vectors of the satellite. Using this coordinate system we compute the angle that the satellite

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* The writer is indebted to his colleague, Dr. Stuart Haywood of APL/JHU for pointing out this method of solving Eqs. 1 and 2.

**The left side of Eq. 4 is, of course, replaced with the corresponding quantities in $\bar{R}$ and $\bar{B}$.

polar axis forms with the position vector to the center of the earth. Subsequently we resolve this angle into components along and orthogonal to the orbit plane.

Seemingly convenient for dynamic analyses would be to perform a formal transformation of the $\{A\}$ matrix to Eulerian angles. On the other hand, the work of Marguiles and Goodman (Ref. 3) has given us "pause for concern."

III. SOME RESULTS

The technique described above was successfully used aboard the 1963 22A satellite. This satellite contained a pioneering experiment to evaluate gravity-gradient stabilization. A description of this experiment is contained in Ref. 4. Sample results obtained are contained in Table I. These results confirmed that the gravity stabilization system was successful.

TABLE I

Angle Included Between Satellite Polar Axis and the Local Vertical

June 22, 1963

TIME			ALONG ORBIT	ACROSS ORBIT
(hr)	(min)	(sec)	(deg)	(deg)
11	58	53	7.0	-19.1
	59	15	6.3	-18.7
12	00	21	2.5	-16.9
	00	43	4.6	-17.5
	01	49	2.1	-15.6
	02	11	1.5	-14.5
	03	39	2.4	-13.0
	04	01	1.1	-11.9
	07	20	-1.5	- 5.6
	07	42	-1.7	- 5.4
	08	48	-2.2	- 2.9
	09	10	-2.5	- 1.9

IV. DISCUSSION

It is clear that measurements of any two linearly independent vectors in satellite coordinates are sufficient to determine the orientation--if these vectors have a known resolution in the reference coordinate system. Usually, the necessary conditions--because of available constraints--are less stringent. In our system we chose to measure all six quantities and use three available constraints

(e.g., $\sum_{i=1}^{3} r_i^2 = \sum_{i=1}^{3} b_i^2 = 1$, $\bar{r} \cdot \bar{b} = \bar{R} \cdot \bar{B}$) to minimize the effects of noise in the data. A discussion of this noise analysis would take us outside the scope of this paper.

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