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AEROSPACE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
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FOREWORD

This interim technical report was prepared by the Guggenheim Laboratories for the Aerospace Propulsion Sciences, Princeton University, Princeton, New Jersey, on Contract AF 33(657)-9962 for the Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force. The research reported herein was accomplished on Task 7063-03, "Energy Exchange Phenomena in Electric Arc Discharges" of Project 7063, "Mechanics of Flight" under the technical cognizance of Capt. Thiophilos of the Thermomechanics Research Laboratory of ARL.

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I. SUMMARY

A previous report (1)[*] described a series of total radiation surveys of a turbulent argon arcjet using a collimated radiation probe. The results, although based on admittedly "rough" experiments, indicated a total radiation loss considerably less than prior theoretical predictions. In the present report, these theories are reviewed in some detail, and the entire series of experiments is re-examined, particularly with reference to the effect of collimation on the results.

It was found that although the radiation distributions measured in (1) were accurate, the method used to analyze the data was incorrect. Using the same experimental data, the measured radiation loss was recalculated properly, taking into account the effect of the collimator. Results indicated a considerably larger loss than had been reported in (1), and were found to agree quite well with the theoretical estimates given in the present report.

II. INTRODUCTION

A. Purpose

The investigation described in this report was directed at two problems:

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*. Numbers in parentheses indicate references listed on page 29

(1) Analytical estimation of the radiation loss from the partly-ionized, collision-dominated argon arcjet.

(2) Detailed review of the collimated radiation probe technique, using a prior series of measurements both to evaluate the method and to correlate results with the analytical calculations.

B. Review of Previous Work

Previous experimental work, described in full in Ref. (1), was originally performed to provide a rough estimate of the radiation loss which might be encountered in the study of turbulent mixing (2).

These measurements were made in argon at about 13,000°K at one atmosphere pressure with the water-cooled collimated radiation probe of Figure 1. A lithium fluoride window was used on the vacuum thermopile (which measures total incident energy) in order to lower the cutoff frequency to approximately 1100 angstroms, and thereby include most of the ultraviolet contribution.

Three surveys were made as illustrated in Figure 2. The known temperature distribution (2) within the jet was then utilized to correlate the measurements. The radiated power per unit volume was assumed to obey the simple relation

$$P = A \left(T/T_{ref}\right)^n$$

It will be shown (see Section IV)that the radiant energy received by the thermopile when viewing the jet perpendicular to the axis is proportional to T^n, where T is the centerline temperature at that axial station.

The exponent "n" and the coefficient "A" were determined from the experiment (see Section IV). These values were then compared with theoretical estimates (see Section V).

III. CONCEPTUAL BACKGROUND

To properly evaluate the experimental observations, it was necessary to review existing theories for radiation loss mechanisms from collision-dominated plasmas. The three contibuting mechanisms are (a) bound-bound, (b) free-bound, and (c) free-free radiative electronic transitions. Bound-bound radiative electronic transitions are characterized by electronic transition from an excited electronic state to a lower electronic state. Free-bound radiative electronic transitions, i.e., radiative recombination, refer to the capture of a free electron by an ion. Free-free radiative electronic transitions (bremsstrahlung) refer to the interaction of a free electron with an ion in which the electron is decelerated but not captured, the change in kinetic energy appearing as radiation. Contributions due to electron cyclotron radiation do not appear because of the absence of an external magnetic field and the high collision rate.

This section presents a brief summary and evaluation of methods for determining radiation losses by the above-named mechanisms, together with numerical estimates applicable to the experiments.

A. Free-Bound and Bound-Bound Transitions

There are two classes of radiation, line radiation and continuum radiation. Line radiation occurs in transitions between clearly defined energy levels. In argon, line radiation would occur in a transition from an electronically excited state to a lower electronic state.

Continuum radiation occurs in transitions where the energy levels are not clearly defined. Obvious examples of continuum radiation are free-free and free-bound radiative transitions, since the upper energy levels in both cases are not quantized. Also, because of the disturbing effects of local electric fields in the plasma and other line broadening effects such as Doppler broadening and collision broadening, the energy levels near the ionization potential are smeared together, thereby producing another source of continuum radiation.

The intensity of radiation at the frequencies corresponding to the lines of argon will be much stronger than the background continuum. However, since the frequency intervals of the lines are small, the total contribution to the radiative flux should be small. Note that if the upper excited state population is far above equilibrium values, the line radiation could begin to be significant; however, in the present analysis the line radiation has been neglected.

In calculating radiation due to free-bound transitions, the most direct approach is to express the number of radiative recombinations in terms of radiative recombination cross sections to the various quantum levels in the atom. Making the usual assumptions that the ions are stationary and the electrons are in a Maxwellian distribution, we can integrate over the electron velocity distribution to obtain the number of radiative recombinations multiplied by the respective energy jumps and obtain the radiated power per unit volume. Unfortunately, no such detailed tables of radiative recombination cross sections exist for argon. The usual approach is to use the relation between the photoionization cross section and that of the inverse process, the radiative recombination cross section, derived from detailed balancing and/or quantum mechanics (3, 4).

The physical process may be written

$$e + A^{+} \rightleftharpoons A + h\nu$$

where e represents an electron

A represents an argon atom

A^{+} = represents an argon ion

h = Planck's constant

ν = frequency of emitted radiation

For transitions between the ith state of the atom and the jth state of the ion,

$$Q_{r_{j,i}} = \frac{h^2\nu^2}{2mc^2E}\,\frac{g_i}{\tilde{g}_j}\quad Q_{p_{i,j}}$$

where g_i = statistical weight of atom ith state

$\tilde{g}_j$ = statistical weight of ion jth state

m = electron mass

E = electron kinetic energy

$Q_{r_{j,i}}$ = recombination cross section for transition from jth to ith state

$Q_{p_{i,j}}$ = photoionization cross section for transition from ith to jth state

$$Q_{r_j} = \sum_i Q_{r_{j,i}}$$

$$Q_{p_i} = \sum_j Q_{p_{i,j}}$$

Above the threshold the photoionization cross section Q_{p_i} does not vary rapidly with frequency (6), while the recombination cross section Q_{r_j} is a strong function of electron energy. Lin (3) used this approach together with some further assumptions:

(a) The recombination cross section to the ground state is half the total recombination cross section,

(b) In each recombination all the kinetic energy, in addition to the recombination energy, is included in the radiation.

The first assumption is roughly true for hydrogen (5, 6). The second assumes that radiative decay of bound states subsequent to recombination is included in the sum. Lin obtained the radiation power loss P per unit volume as

$$P = N_e^{\,2} \left[\frac{8}{\pi m}\right]^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty e^{-E/kT}(I + E)\sum_j Q_{r_j} \, EdE$$

$$= \frac{N_e^{\,2}\left[\frac{8}{\pi m}\right]^{1/2} (kT)^{5/2}}{mc^2} \frac{g_1}{\tilde{g}_1} Q_{P_1} \left[\frac{1}{\theta^3} + \frac{3}{\theta^2} + \frac{6}{\theta} + 6\right]$$

where N_e = electron number density

I = ionization energy

$\theta = \frac{kT}{I}$

E = ion kinetic energy

k = Boltzmann constant

T = translational temperature

Using $Q_{P_1} = 0.36 \times 10^{-16}$ cm.^{2}, $\frac{g_1}{\tilde{g}_1} = 1/6$ (3, 5, 6), the radiation loss calculated by this approach is shown in Table I.

TABLE I

RADIATION LOSS DUE TO FREE-BOUND TRANSITIONS BY METHODS OF REFERENCE 3

Temperature (^{o}K)	Power Loss (erg/cm.^{3}-sec.)
15,000	19×10^{10}
14,000	12×10^{10}
13,000	7×10^{10}

Note that 10^{10} erg/sec is one kilowatt. These values for radiation loss are far greater than those observed experimentally. The discrepancy stems from the large number of transitions to the ground state from the continuum, so that in the corresponding spectral rang (786 to about 700 angstroms), the plasma is nearly opaque; i.e., the radiation has a very short mean free path and on the average is absorbed and re-emitted many times before it escapes. The same is also true in the spectral range 1080 to 1040 angstroms, corresponding to transitions from the critical potential to the ground state. These spectral regions are so far from the peak of the black-body curve that even if the outside of the jet were radiating as a black-body the total radiation would still be quite small; e.g., see Table II.

TABLE II

RADIATION LOSS BY BLACKBODY RADIATION FROM JET SURFACE

Temperature (^{o}K)	Power Loss (erg/cm^2-sec) 700 - 786 A	1080 - 1040 A
15,000	3.3×10^7	12×10^7
14,000	1.3×10^7	6×10^7
13,000	$.5 \times 10^7$	3×10^7

Because of the inherent error introduced by Lin's assumptions in the above, a more appropriate method is that used in (7), which is the end result of a chain of contributions (8, 9, 10). First, absorption coefficients were calculated for "hydrogen-like" atoms. Unsöld (10) in particular used a number of simple assumptions in order to obtain an expression for the power loss per unit frequency interval per unit volume, which is given (7) by,

$$I_\nu = \frac{64\,\pi^{3/2} e^6}{3\sqrt{6}\, m^{3/2} c^3}\, Z_{eff}^2\, \frac{N_e^2}{(kT)^{1/2}} \exp(\Delta E/kT)$$

where e = electron charge

c = speed of light

Z_{eff} = effective nuclear charge (for atoms other than hydrogen)

ΔE = energy range over which the energy levels are "smeared" (i.e., near the ionization potential).

m = electron mass

Note that this quantity does not have any explicit frequency dependence. It is suggested (7) that this expression is valid down to 2000 A, below which there is a cutoff in continuum radiation. Radiation losses calculated from (7) are given in Table III.

TABLE III

LOSS BY CONTINUUM RADIATION USING METHOD OF REFERENCE 7

Temperature (^{o}K)	I (erg/cm.^{3})	P (erg/cm.^{3}-sec.)
15,000	5.0×10^{-6}	7.5×10^{10}
14,000	3.5×10^{-6}	5.2×10^{10}
13,000	1.9×10^{-6}	2.8×10^{10}
12,000	$.71 \times 10^{-6}$	1.1×10^{10}

In view of the questionable descent of the above formula, it has recently been criticized rather sharply (11). A new line of development (11, 12, 13, 14) produced a new expression for the radiated power loss, utilizing new quantum-mechanical calculations of the absorption cross sections. Experimental data are compared with this theory in (14), producing reasonably good agreement from 4,000 angstroms through the infrared. The results are expressed in a form similar to that of Unsöld (10), except that the nebulous concept of an "effective nuclear charge" Z^2_{eff} is replaced by a calculated function of the frequency ξ. For $\xi = Z^2_{eff}$ the two formulas agree; however, for argon ξ is about 0.8

in the infrared and rises to about 2.2 at 4000 angstroms (13). The result, given in a form similar to that of (7), is

$$I_\nu = \begin{cases} \dfrac{64\,\pi^{3/2}\,e^6}{3\sqrt{3}\,m^{3/2}\,c^3}\,\dfrac{N_e^{\,2}}{(kT)^{1/2}}\,e^{\Delta E/kT}\,\xi(\nu) & \nu \leq \nu_g \\[2ex] \dfrac{64\,\pi^{3/2}\,e^6}{3\sqrt{3}\,m^{3/2}c^3}\,\dfrac{N_e^{\,2}}{(kT)^{1/2}}\,e^{\Delta E/kT}\,\xi(\nu)\left\{\dfrac{e^{\alpha_g}-1}{e^{\alpha}-1}\right\} & \nu > \nu_g \end{cases}$$

where I_ν = free-bound radiation per unit volume per unit frequency interval

h = Planck constant

k = Boltzmann constant

ν = frequency of radiation

α = $h\nu/kT$

ν_g = 9.5×10^{14} (3160 angstroms)

The frequency ν_g corresponds to the limit of transitions where the lower level is the last of the closely spaced energy levels. Compared to the results of (7), the contribution to the total radiation is increased in the visible, but greatly diminished in the range 3000 - 2000 angstroms. The net effect is a reduction in the total radiated power.

Numerical data appear in Table IV, and predict losses by continuum radiation about half those of the Unsöld theory.

TABLE IV

LOSS BY CONTINUUM RADIATION USING METHOD OF REFERENCE 13

Temperature (^oK)	Power Loss (erg/cm^3-sec)
15,000	3.7×10^{10}
14,000	2.5×10^{10}
13,000	1.3×10^{10}
12,000	$.50 \times 10^{10}$

B. Bremsstrahlung

The energy loss of the arcjet due to bremsstrahlung, or free-free transitions, can be estimated with a fair degree of accuracy; at least the various published theories (15, 16, 17) are all in reasonably good agreement.

The bremsstrahlung power loss is calculated by computing first the power loss due to monoenergetic electrons. This is done in (16):

$$\mathcal{E}(\nu) = \frac{32\pi Z^2 e^6}{3m^2 v c^2} N_e N_i \frac{\pi}{\sqrt{3}} g(\nu)$$

where $\mathcal{E}$ = energy emitted per cm^3 per sec per unit frequency interval

v = electron velocity

g = "Gaunt factor" (a correction from quantum mechanics) $\approx$ unity

The intensity per unit frequency interval, I_ν, is obtained by integrating over a Maxwellian velocity distribution for the electrons:

$$I_\nu = \frac{32 \pi Z^2 e^6}{3 m^2 c^3} \left[\frac{2 \pi m}{3 kT}\right]^{1/2} N_e N_i \bar{g}(T,\nu) e^{-h\nu/kT}$$

where $\bar{g}$ = an average value of g

Assuming $\bar{g}$ is a constant, $Z = 1$, $N_e = N_i$, we obtain the power loss per unit volume

$$P_{brem} = 1.42 \times 10^{-27} N_e^{\,2} T^{1/2} \bar{g} \quad \text{ergs/cm}^3\text{-sec}$$

If the Gaunt factor is unity, the formula is identical to that given by Spitzer (15), in which the Born approximation was used. The numerical results of (17) are plotted in that reference, but unfortunately they only extend down to 4 E.V. The results seem to agree with those of (15, 16). Numerical results (using Spitzer's equation) appear in Table V.

TABLE V

BREMSSTRAHLUNG LOSS USING METHOD OF REFERENCE 15

Temperature ($^\circ$K)	Power Loss (erg/cm^3-sec)
15,000	5.7×10^9
14,000	3.6×10^9
13,000	1.9×10^9
12,000	$.5 \times 10^9$

C. Estimate of Total Radiation Loss

As indicated earlier, line radiation from the argon jet is neglected in the total radiation-loss estimate. Thus the only contributions to be considered are those due to continuum radiation, principally free-bound transitions, and bremsstrahlung. Summarizing the values of Tables IV and V, therefore, we obtain the total estimated loss given in Table VI.

TABLE VI

TOTAL ESTIMATED RADIATION LOSS

(SEE TABLE IV & V)

Temperature (^{o}K)	Power Loss (kw/cm^3)
15,000	4.27
14,000	2.86
13,000	1.49
12,000	.55

IV. EXPERIMENTAL STUDY

A. Apparatus

The experimental technique was based on the use of a simple collimated thermoelectric element (Figure 1) calibrated by a Bureau of Standards tungsten lamp. The element itself was vacuum-sealed, with a lithium fluoride window capable of transmitting all wavelengths down to 1060 angstroms. It was mounted in a water-cooled copper collimating tube as shown in Figure 1. The tube was supplied with a bleed to keep it filled with helium containing less than 1 ppm of impurity, in order to avoid any absorption within the tube itself. Sufficient cooling capacity was provided to permit operation of the probe inside the arcjet exhaust.

Three different types of surveys were made, as illustrated in Figure 2. The first consisted of a series of measurements in which the probe was oriented radially. These measurements were made at a number of axial locations, but with the probe tip just outside the jet and maintained at a fixed distance from the jet axis. The second series was made with the same probe orientation, but this time the probe was moved <u>radially</u> through the jet at one axial location to determine radiation characteristics in the interior of the jet. The third series was made with the probe oriented axially so as to also view the interior of the nozzle. Measurements were taken on the jet axis at several axial locations.

In order to correlate the results a simple analytical model of the jet was formulated, and previously-measured temperature profiles (2) were used to evaluate the temperature dependence of the radiated power per unit volume. These calculations are described later.

B. <u>Analysis of Collimator</u>

In order to evaluate properly the indication of the collimated probe, shown in Figure 1, it is necessary to examine in detail the geometry of the collimator. Consider the configuration and notation of Figure 3. The thermopile intercepts a solid angle Ω from an element of the radiating gas $r dr d\theta dy$, near the axis of the probe.

$$\Omega = \frac{\pi R_w^{\,2}}{(L + y)^2}$$

For radii between $r = r_{crit}$ and $r = r_{lim}$, only a fraction of the solid angle Ω is intercepted by the thermopile; for radii $r > r_{lim}$ no radiation from the element is intercepted. From Figure 3 we see that r_{crit} is given by,

$$\frac{r_{crit}}{R_o} = 1 - \frac{y}{L}\left[\frac{R_w}{R_o} - 1\right]$$

and r_{lim} is given by

$$\frac{r_{lim}}{R_o} = 1 + \frac{y}{L}\left[\frac{R_w}{R_o} + 1\right]$$

Inserting the appropriate numerical values for this experiment,

L = 3.0 inch

R_w = 0.093 inch

R_o = 0.060 inch

$0 \leq y \leq 0.75$ inch,

we find

$$0.87 \leq r_{crit}/R_o \leq 1$$

$$1 \leq r_{lim}/R_o \leq 1.63$$

Defining the quantity F as the fraction of the solid angle intercepted by the thermopile, F will be unity out to r_{crit}, zero beyond r_{lim}, and will take intermediate values between r_{crit} and r_{lim}. In the intermediate region it is necessary to evaluate F in detail. This is done in Appendix A. The result is

$$F = \frac{1}{\pi}\left\{\frac{\theta}{P^2} + \sin^{-1}\left(\frac{\sin\theta}{P}\right) - \frac{D\sin\theta}{P^2}\right\} \qquad r_{crit} \leq r \leq r_{lim}$$

where
$$\theta = \cos^{-1}\left(\frac{1 + D^2 - P^2}{2\,D}\right)$$

$$D = \frac{L}{R_o}\left[\frac{r}{y+L}\right]$$

$$P = \frac{R_w}{R_o}\left[\frac{y}{y+L}\right]$$

The quantity F is plotted in Figure 4. However, since the above expression for F would necessitate a numerical integration for the total radiant power intercepted, the function is approximated by the following simple relation

$$F = \frac{1 + \cos\pi\beta}{2}, \quad r_{crit} \leq r \leq r_{lim}$$

where $\beta = (r - r_{crit})/(r_{lim} - r_{crit})$

This approximation for F is also plotted in Figure 4 for comparison with the exact value.

Thus the solid angle of the thermopile window "seen" by the element is given by

$$\Omega = \frac{\pi R_w^2}{(y+L)^2} \times F$$

where

$$F = \begin{cases} 1, & 0 \leq r \leq r_{crit} \\ \dfrac{1 + \cos \pi \beta}{2}, & r_{crit} \leq r \leq r_{lim} \\ 0, & r \geq r_{lim} \end{cases}$$

This expression will now be used to determine radiation loss from the jet.

The energy received by the thermopile window from a volume element of the jet is given by

$$dE_{rec} = \frac{\Omega}{4\pi} Pdv$$

where E_{rec} = energy received by the thermopile window

P = volumetric rate of energy loss by radiation

v = volume

Thus $dE_{rec} = \left[\frac{R_w}{2(y+L)}\right]^2 P(x,y)\ F\ (r,y)\ 2\pi\, rdrdy$

where x = displacement of collimator axis (from arcjet nozzle exit) along arcjet axis.

and it has been assumed that $P \neq P(\theta,r)$ i.e., that the collimator "sees" only a small region in the r-direction.

We make the approximation that the radiant power per unit volume is given by the simple expression (see Section IV)

$$P = A\left[\frac{T}{T_{ref}}\right]^n$$

where A, n = constants

$T = T(x,y) = T(x)\theta(y)$

T_{ref} = arbitrary reference temperature

Inserting the proper values for F and the jet radius a, we obtain

$$E_{rec}(x) = \frac{\pi R_w^2 A}{2}\left[\frac{T(x)}{T_{ref}}\right]^n \int_0^{2a} \frac{\theta^n(y)dy}{(y+L)^2}\left\{\int_0^{r_{crit}(y)} rdr + 1/2 \int_{r_{crit}(y)}^{r_{lim}(y)} \left[1 + \cos\left(\pi\beta(r,y)\right)\right] rdr\right\}$$

where $\beta = \frac{L}{2yR_w}\left[r - R_o + \frac{y}{L}(R_w - R_o)\right]$

Evaluating the integrals within the bracket, this reduces to

$$E_{rec}(x) = \frac{\pi R_w^2 A}{2}\left[\frac{T(x)}{T_{ref}}\right]^n \int_0^{2a} \frac{(\alpha y^2 + \delta y + \gamma)\,\theta^n(y)dy}{(y+L)^2}$$

where $\alpha = \frac{R_w^2}{2L^2}\left[\frac{\pi^2 - 8}{\pi^2} + \frac{R_o^2}{R_w^2}\right]$

$\delta = \frac{R_o^2}{L}$

$\gamma = 1/2\, R_o^2$

For the region of highest radiation intensity; i.e., near the potential core of the turbulent jet, it was determined experimentally (2) that $\theta(y)$ was very nearly unity; i.e., that the radial jet temperature distribution was flat. Evaluating the above integral on this basis,

$$E_{rec}(x) = \frac{\pi R_w^2 A}{2} \; I \left[\frac{T(x)}{T_{ref}}\right]^n$$

where I is a function of geometry given by

$$I = 2a\alpha + (\delta - 2\alpha L) \log_e\left(\frac{2a+L}{L}\right) + \frac{2a(\alpha L^2 - \delta L + \gamma)}{L(2a + L)}$$

Note that the temperature dependence of the radiant energy received at the thermopile window is identical to the temperature dependence of the radiant power per unit volume, i.e.,

$$E_{rec} \sim T^n \quad \text{and} \quad P \sim T^n$$

C. Results

The quantities which are to be determined from experiment are the coefficient A and the exponent n.

1. Determination of Modulus "A"

The above analysis gives

$$A = \frac{2 E_{rec}(x)}{\pi R_w^2 I} \left[\frac{T(x)}{T_{ref}}\right]^{-n}$$

Taking T_{ref} at the jet nozzle exit centerline (x=o, r=o), we obtain

$$A = \frac{2 E_{rec}(o)}{\pi R_w^2 I}$$

Inserting numerical data,

$$a = 0.375 \text{ inch}$$

$$L = 3.00 \text{ inch}$$

$$R_w = 0.093 \text{ inch}$$

$$R_o = 0.060 \text{ inch},$$

we find

$$\alpha = 0.29 \times 10^{-3}$$

$$\delta = 1.20 \times 10^{-3} \text{ inch}$$

$$\gamma = 1.80 \times 10^{-3} \text{ inch}^2$$

$$I = 1.51 \times 10^{-4} \text{ inch}$$

The radiation intensity measured at $x = o$ in the survey of Figure 2A was 3.56×10^{-5} kw/cm^2 (1). Using the thermopile window area of πR_w^2 and the measured value $E_{rec}(o) = 1.02 \times 10^{-5}$ kw, we find

$$A = 0.31 \text{ kw/cm}^3$$

which gives

$$E_{rec}(x) = 1.5 \times 10^{-6} \left[\frac{T(x)\ ^oK}{12{,}600}\right]^n \qquad \text{kw}$$

or, using the value $n = 7.2$ from Section IV, the volumetric radiation power loss is

$$P(x) = 0.31 \left[\frac{T(x)^oK}{12{,}600}\right]^{7.2} \qquad \text{kw/cm}^3$$

From the temperature surveys of Ref. 2, the temperature behavior may be expressed by

(a) $T = 12{,}600^{o}K, \; 0 \leq x \leq 2$ cm

(b) $T = 12{,}600 \left(\frac{6.3 - x}{4.3}\right) , \; 2 \text{ cm} \leq x \leq 6.3$ cm

where radiation loss has been assumed negligible for $T < 7{,}000^{o}K$. The radius of the radiating portion of the jet ($T > 7{,}000^{o}K$) is then given by

$$a = a_o \left(\frac{6.3 - x}{6.3}\right) , \; 0 \leq x \leq 6.3 \text{ cm}$$

Thus the radiating portion is assumed to be a cone of base radius a_o ($\approx$ 1 cm) and height 6.3 cm.

These assumptions may now be incorporated into an integration of the total power loss

$$P_{tot} = \int_v P(x)dv = .31 \pi \left[\int_0^2 \left(\frac{6.3-x}{6.3}\right)^2 dx + \left(\frac{4.3}{6.3}\right)^2 \int_2^{6.3} \left(\frac{6.3-x}{4.3}\right)^{9.2} dx \right]$$

or $\quad P_{tot} = 1.48$ kw

This is approximately 10% of the net gas power ($\approx$ 15 kw for this experiment). Note, however, that in order to determine the total radiation loss, it is necessary to add the power radiated from the interior of the nozzle outward through the nozzle plane, and subtract the power radiated from the external jet back into the nozzle. These two contributions are considered later.

2. Determination of Temperature Exponent "n"

We now seek to determine experimentally the value of the exponent "n". As stated previously,

$$E_{rec}(x) = \frac{\pi R_w^2 \, A \, I(x)}{2} \left[\frac{T(x)}{T_{ref}}\right]^n$$

Using the expression for I(x), we may now determine n. Since the width of the jet varies with axial position, we may set

$$a = a_o - kx \qquad k = \frac{1}{6.3}, \qquad x \text{ in cm.}$$

Also the first term in the expression for I(x) is dominant, and we may thus set, to good approximation,

$$I \approx \text{const} \cdot (a_o - x/6.3)$$

then

$$\frac{E(x_1)}{E(x_2)} = \left[\frac{a_o - x_1/6.3}{a_0 - x_2/6.3}\right]\left[\frac{T(x_1)}{T(x_2)}\right]^n$$

from which

$$n = \frac{\log 10\left[E(x_1)/E(x_2)\right] - \log 10\left[\dfrac{a_o - x_{1/6.3}}{a_o - x_{2/6.3}}\right]}{\log 10\left[T(x_1)/T(x_2)\right]}$$

Using experimental data from (1, 2),

At $x_1 = 0.0$ cm,

$T_1 = 12{,}600^oK$

$E_1 = 3.56 \times 10^{-5}$ kw/cm^2

At $x_2 = 3.2$ cm

$T_2 = 9{,}500^{\circ}K$

$E_2 = 2.20 \times 10^{-6}$ kw/cm^2

Thus $n = 7.2$

3. Determination of Other Radiation Contributions

(a) Upstream Radiation from Gas into Nozzle

Considering the radiating portion of the jet to be a cone of length ℓ and base radius a_o, with radius a at axial distance x, the power P_{rr} reradiated back into the nozzle may be written

$$P_{rr} = \int_0^{\ell} \pi a^2(x)\ P(x)\ \ f(x)\ dx$$

where $a(x) = a_o \dfrac{\ell - x}{\ell}$ $(a_o \approx 1 \text{ cm})$

$$P(x) = \begin{cases} A \ , & 0 \leq x \leq 2 \text{ cm} \\ A\left[\dfrac{\ell - x}{\ell - 2}\right]^{7.2} , & 2 \text{ cm} \leq x \leq \ell \end{cases}$$

f(x) = fraction of energy radiated from transverse slab of thickness dx which reaches nozzle.

For two axially-opposed circles with radiation from one side,(18) provides the expression

$$f(x) = \frac{1}{4}\left[\frac{x^2+a^2+1}{a^2} - \sqrt{\left(\frac{x^2+a^2+1}{a^2}\right)^2 - \frac{4}{a^2}}\ \right]$$

or, introducing the above values for $a(x)$ and a_o,

$$f(x) = \frac{1}{4}\left[1 + \frac{\ell^2(x^2+1)}{(\ell - x)^2} - \sqrt{\left[1 + \frac{\ell^2(x^2+1)}{(\ell - x)^2}\right]^2 - \frac{4\ell^2}{(\ell - x)^2}} \right]$$

Since this expression is singular at $x = \ell$, it is approximated in the present analysis by

$$f_1(x) \approx \frac{1}{2}\left[\frac{\ell - x}{\ell}\right]^{6.7}$$

A comparison between the approximate and exact values of $f(x)$ is shown in Figure 5, from which it is clear that the error introduced by the approximation is negligible.

Integrating to find the re-radiated power, we obtain

$$P_{rr} = \frac{\pi A \ell a_o^2}{19.4}\left\{ 1 + \left(\frac{\ell - 2}{\ell}\right)^{8.2}\left[0.588 - \left(\frac{\ell - 2}{\ell}\right)^{1.5} \right]\right\} \quad \text{kw}$$

Substituting the numerical values

$$A = 0.31 \text{ kw/cm}^3$$

$$a_o = 1 \text{ cm}$$

$$\ell = 6.3 \text{ cm},$$

we obtain $P_{rr} = 0.32$ kw

Thus the power radiated from the external jet to the surroundings is

$$P_{gas} = P_{tot} - P_{rr} = 1.48 - 0.32 \text{ kw}$$

or $P_{gas} = 1.16$ kw

(b) Radiation Loss from Interior of the Nozzle

Inside the nozzle radiation is being emitted by the gas column, the arc, and the hot cathode. Most of the radiation leaving the nozzle exit will come from the gas column, since the arc and the cathode intercept a much smaller solid angle. For simplicity, the gas in the nozzle is assumed to be at a uniform temperature, with no reflected radiation leaving the exit plane.

Then

$$dP_{N.E.} = P(x)\, f(x)\, dv$$

where $P(x)$ = radiant power per unit volume

$f(x)$ = fraction of radiant energy from a thin slab of width dx, at a distance x from the nozzle exit plane, which passes through the nozzle exit

dv = $\pi a_o^2 dx$

$P_{N.E.}$ = radiant power emitted from the nozzle exit

As in the previous section $f(x) \approx \frac{1}{2}\left(\frac{\ell - x}{\ell}\right)^{6.7}$

where ℓ = length of gas column (nozzle exit to cathode)

Assuming the temperature within the nozzle to be 12,600°K, the measured peak temperature in the exterior jet,

P = 0.31 kw/cm^3

a_o = 1 cm.

ℓ = 10.2 cm

and

$$P_{N.E.} = \frac{0.31\, \pi a_o^2}{2} \int_0^{\ell} \left[\frac{\ell - x}{\ell}\right]^{6.7} dx = 0.64 \text{ KW}$$

An independent experimental check on this determination is provided by the test series of Figure 2c, in which the probe was oriented axially on the jet center-line. It was observed that with this probe orientation (2), the total measured radiation rate was approximately 314 times greater than that measured radially (Figure 2A).

Comparing this with the above estimate; i.e., assuming that the nozzle is filled with uniform-temperature gas at 12,600°K, the axially-oriented probe would have indicated a total radiated power 147 times that of the radial measurement. The two determinations may be brought into agreement by assuming an interior nozzle gas temperature given by

$$\frac{T}{12,600} = \left[\frac{314}{147}\right]^{\frac{1}{7.2}}$$

or T = 14,000°K, instead of 12,600°K.

Because of the degree of uncertainty of many of the assumptions and the relative crudeness of the experimental observations, this agreement between the two determinations is considered to be reasonable. The nozzle loss is therefore estimated at 0.64 kw, as calculated above.

V. THEORETICAL-EXPERIMENTAL CORRELATION

The comparison of experiment and theory may be made on the basis of the previously-stated experimental model for volumetric radiant power loss

$$P = A \left[\frac{T}{T_{ref}}\right]^n$$

At the test temperature of 12,600°K (measured independently by a radiation-insensitive device as described in Reference 2), the data of Table VI provide theoretical values

$$A = 0.97 \qquad \text{kw/cm}^3\text{-sec}$$

$$7.0 < n < 10.5$$

The experimental data discussed in the previous section produced the results

$$A = 0.31 \qquad \text{kw/cm}^3\text{-sec}$$

$$n = 7.2$$

In view of the theoretical uncertainties described in Section III and the relative crudeness of the experimental technique, the agreement appears to be reasonably good.

Note also that the total measured radiation loss from the arcjet is given by

$$\begin{aligned} P &= P_{tot} - P_{rr} + P_{N.E.} \\ &= 1.48 - 0.32 + 0.64 \text{ kw} \\ &= 1.80 \text{ kw} \end{aligned}$$

Since the net power delivered to the gas was approximately 15 kw, the total radiation loss (at 12,600°K peak argon temperature) represented 12% of the jet power.

REFERENCES

1. Jacobs, P. F., and Grey, J., "Measurements of Total Energy Radiated from an Argon Arcjet", Princeton University Aeronautical Engineering Report No. 621, August, 1962.

2. Grey, J., and Jacobs, P. F., "Turbulent Mixing in a Partially Ionized Gas", Princeton University Aeronautical Engineering Report No. 625, October, 1962.

3. Lin, S. C., "On the Limiting Velocity for a Rotating Plasma," AVCO Research Report No. 101, April, 1961.

4. Fowler, R. H., "Statistical Mechanics", Cambridge University Press, 1955, Chap. XIX, pp. 724-726.

5. Massey, H. S. W., and Burhop, E. H. S., "Electronic and Ionic Impact Phenomena", Oxford University Press, 1952, Chap. VI, pp. 330-342.

6. Bates, D. R., "Atomic and Molecular Processes", Academic Press, London, 1962, Chap. 7, pp. 245-258.

7. Petschek, H. E, Rose, P. H., Glick, H. S., Kane, A., and Kantrowitz, A. R., "Spectroscopic Studies of Highly Ionized Argon Produced by Shock Waves", Journal of Applied Physics, Vol. 26, January 1955, p. 144.

8. Kramers, H. A.,"On the Theory of X-Ray Absorption and of the Continuous X-Ray Spectrum", Philosophical Magazine, Vol. 46, 1923, p. 836.

9. Gaunt, J. A., "Continuous Absorption", Proceedings of the Royal Society, London, Vol. 33, 1938, p. 654.

10. Unsöld, V. A., "Uber das kontinuierliche Spektrum der Hg - Hochdrucklampe, des Unterwassfunkens, und ahnliche Gasentladungen", Annals der Physik, Vol. 33, 1938, p. 607

11. Biberman, L. M., and Norman, G. E., "On the Calculation of the Photoionization Absorption", Optics and Spectroscopy, Vol. 8, 1960, p. 230.

12. Burgess, A., and Seaton, M. F., "A General Formula for the Calculation of Atomic Photo-Ionization Cross-sections," Monthly Notices, Royal Astronomical Society, Vol. 120, 1960, p. 121.

13. Biberman, L. M., Norman, G. E., and Ulyanov, K. N., "On the Calculation of Photoionization Absorption in Atomic Gases," Optics and Spectroscopy, Vol. 10, 1961, p. 297.

14. Schluter, D., "Der kontinuierliche Absorptions koeffizient des Argons," Zeitschrift fur Astrophysik, Vol. 56, 1962, p. 43.

15. Spitzer, L., "Physics of Fully Ionized Gases," Interscience Publishers, London, 1956, Chap. 5., pp. 88-90.

16. Brussaard, P. J., and van de Hulsf, H. C., "Approximation Formulas for Nonrelativistic Bremsstrahlung and Average Gaunt Factors for a Maxwellian Electron Gas," Reviews of Modern Physics, Vol. 34, July 1962, p. 507.

17. Janes, G. S., and Koritz, H. E., "Numerical Calculation of Absolute Bremsstrahlung Intensity for a Fully Ionized, Fully Dissociated Hydrogenic Gas," AVCO Research Report No. 70, September 1959.

18. Eckert, E. R. G., and Drake, R. M. Jr., "Heat and Mass Transfer," 2nd Ed., McGraw-Hill, N. Y., 1959, p. 398.

APPENDIX A : CALCULATION OF FRACTION F OF SOLID ANGLE INTERCEPTED BY COLLIMATED THERMOPILE

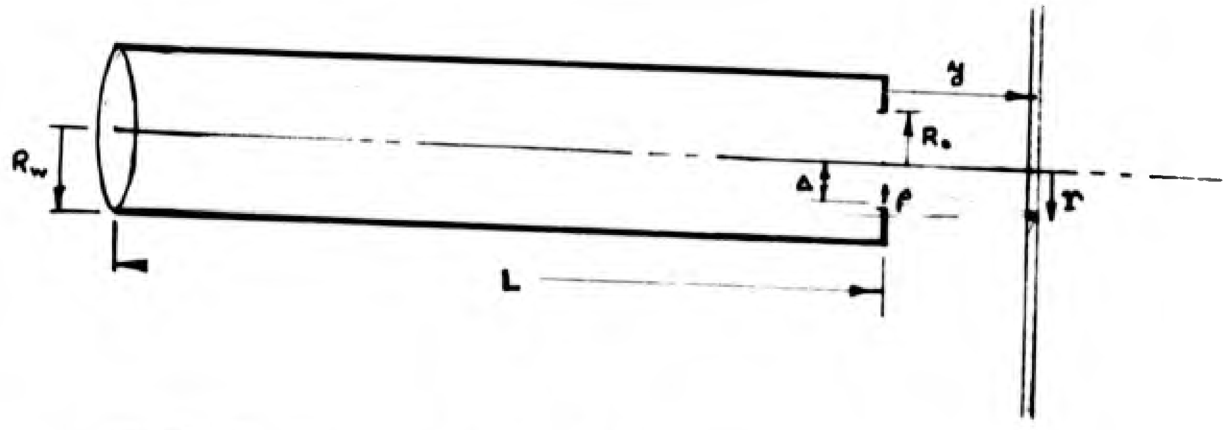


FIGURE A-1a

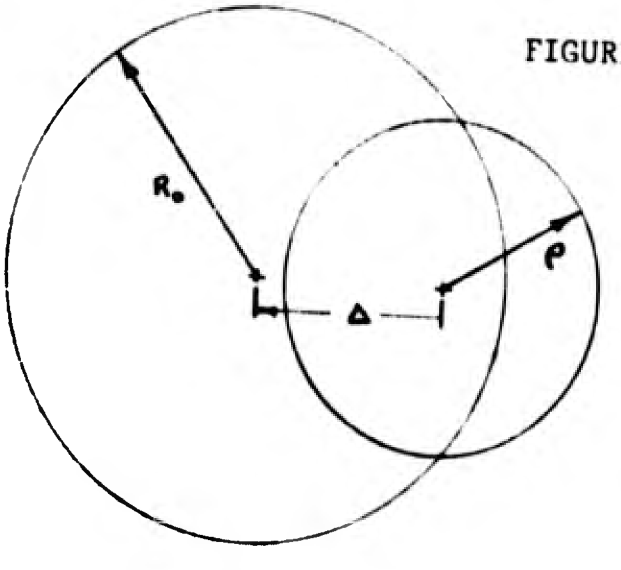


FIGURE A-1b

Define R_o = radius of orifice

ρ = radius of circular section of "radiation-cone", at y=o

Δ = center-to-center displacement of "radiation-cone" circular section, at y = o

From Figures A-1a and A-1b,

$$\frac{\Delta}{L} = \frac{r}{y+L} \quad \text{or} \quad \Delta = L\left[\frac{r}{y+L}\right]$$

and to very close approximation

$$\frac{\rho}{y} = \frac{R_w}{y+L} \quad \text{or} \quad \rho = R_w \left[\frac{y}{y+L}\right]$$

When $r = r_{crit}$

$$\Delta_{crit} = \frac{L}{y+L}\left[R_o - \frac{y}{L}(R_w - R_o)\right] = R_o - R_w(y/y+L)$$

or $\quad \Delta_{crit} + \rho = R_o,$

which corresponds to the internal tangency case, in which all light passing through the orifice reaches the thermopile window.

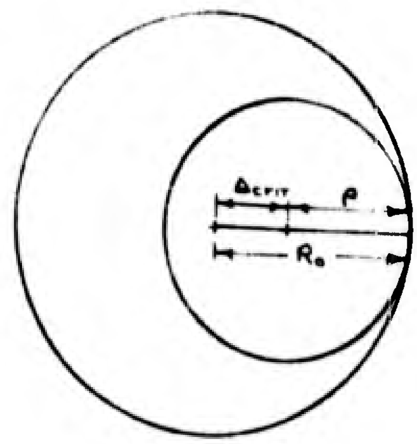


FIGURE A-2a

Also, when $r = r_{lim}$

$$\Delta_{lim} = \frac{L}{y+L}\left[R_o + \frac{y}{L}(R_w + R_o)\right] = R_o + R_o\left[\frac{y}{y+L}\right]$$

or $\quad \Delta_{lim} - \rho = R_o,$

which corresponds to the case of external tangency in which no light reaches the thermopile window.

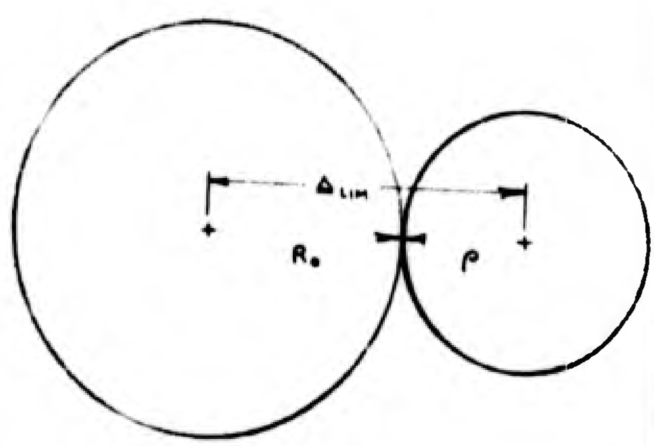


FIGURE A-2b

We now define

F = fractional area of circle of radius ρ through which light passes.

$$= \frac{S}{\pi \rho^2} \quad \text{(see Figure A-3)}$$

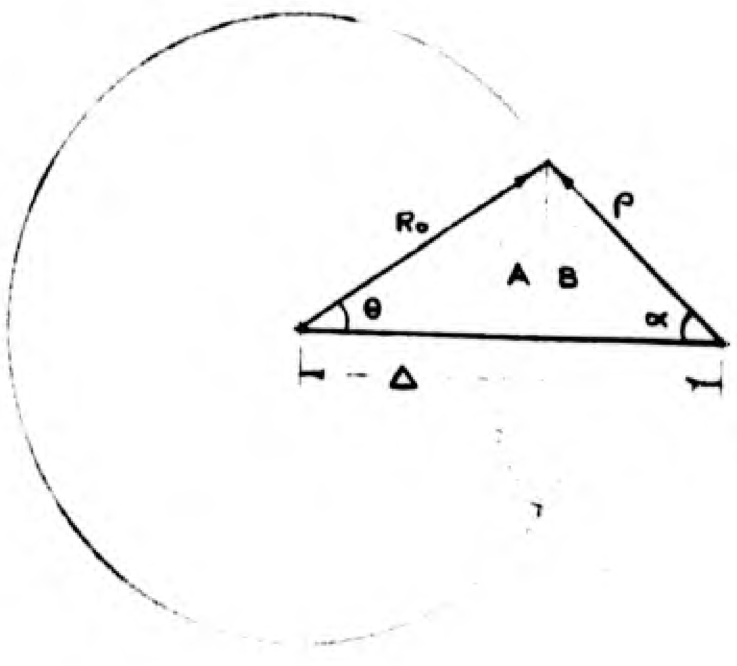


FIGURE A-3

where S = Shaded Area = $2A + 2B$,

$$A = \frac{\alpha \rho^2}{2} - \frac{\rho \sin\alpha \; \rho\cos\alpha}{2}$$

$$B = \frac{\theta R_o^2}{2} - \frac{R_o \sin\theta \; R_o \cos\theta}{2}$$

hence $S = \alpha \rho^2 - \rho^2 \sin\alpha \cos\alpha + \theta R_o^2 - R_o^2 \sin\theta \cos\theta$

From the law of ines $R_o \sin\theta = \rho \sin\alpha$

and also $\rho\cos\alpha = \Delta - R_o\cos\theta$

Then $$F = \frac{S}{\pi\rho^2} = \frac{\alpha}{\pi} - \frac{R_o\sin\theta\,(\Delta - R_o\cos\theta)}{\pi\rho^2}$$

$$+ \frac{\theta R_o^2}{\pi\rho^2} - \frac{R_o^2\sin\theta\cos\theta}{\pi\rho^2}$$

Thus $$F = \frac{1}{\pi}\left[\alpha + \theta\left(\frac{R_o}{\rho}\right)^2 - \frac{R_o\,\Delta\sin\theta}{\rho^2}\right]$$

From the law of cosines

$$\rho^2 = R_o^2 + \Delta^2 - 2R_o\Delta\cos\theta$$

or $$\cos\theta = \frac{R_o^2 + \Delta^2 - \rho^2}{2R_o\Delta}$$

or $$\theta = \cos^{-1}\left\{\frac{R_o^2 + \Delta^2 - \rho^2}{2R_o\Delta}\right\}$$

Defining

$$D \equiv \frac{\Delta}{R_o} = \frac{L}{R_o}\left[\frac{r}{y+L}\right] = D(r,y)$$

$$P \equiv \rho/R_o = \frac{R_w}{R_o}\left[\frac{y}{y+L}\right] = P(y)$$

then $$F = \frac{1}{\pi}\left[\frac{\Theta}{P^2} + \sin^{-1}\left\{\frac{\sin\Theta}{P}\right\} - \frac{D\sin\Theta}{P^2}\right]$$

where $$\Theta = \cos^{-1}\left\{\frac{1 + D^2 - P^2}{2D}\right\}$$

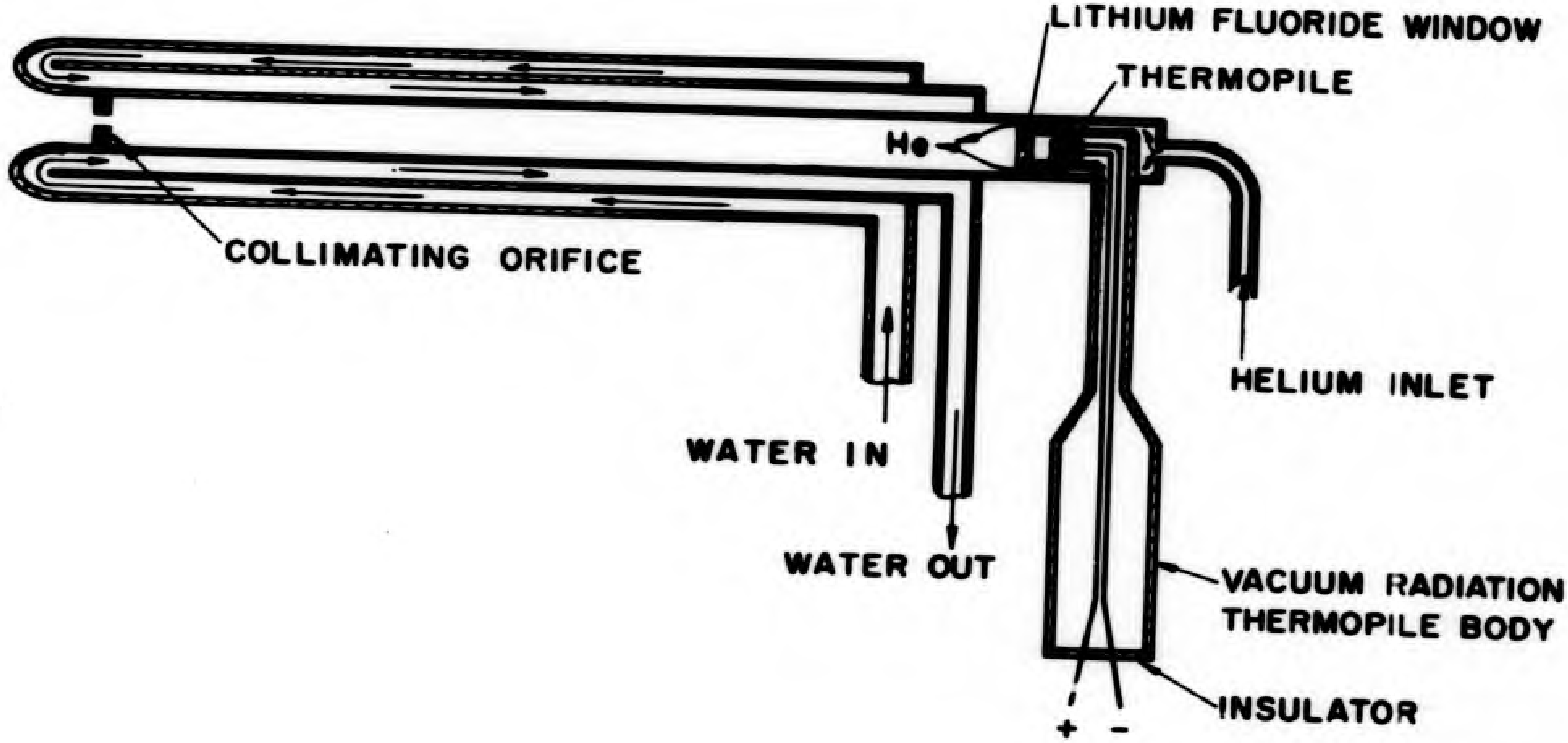
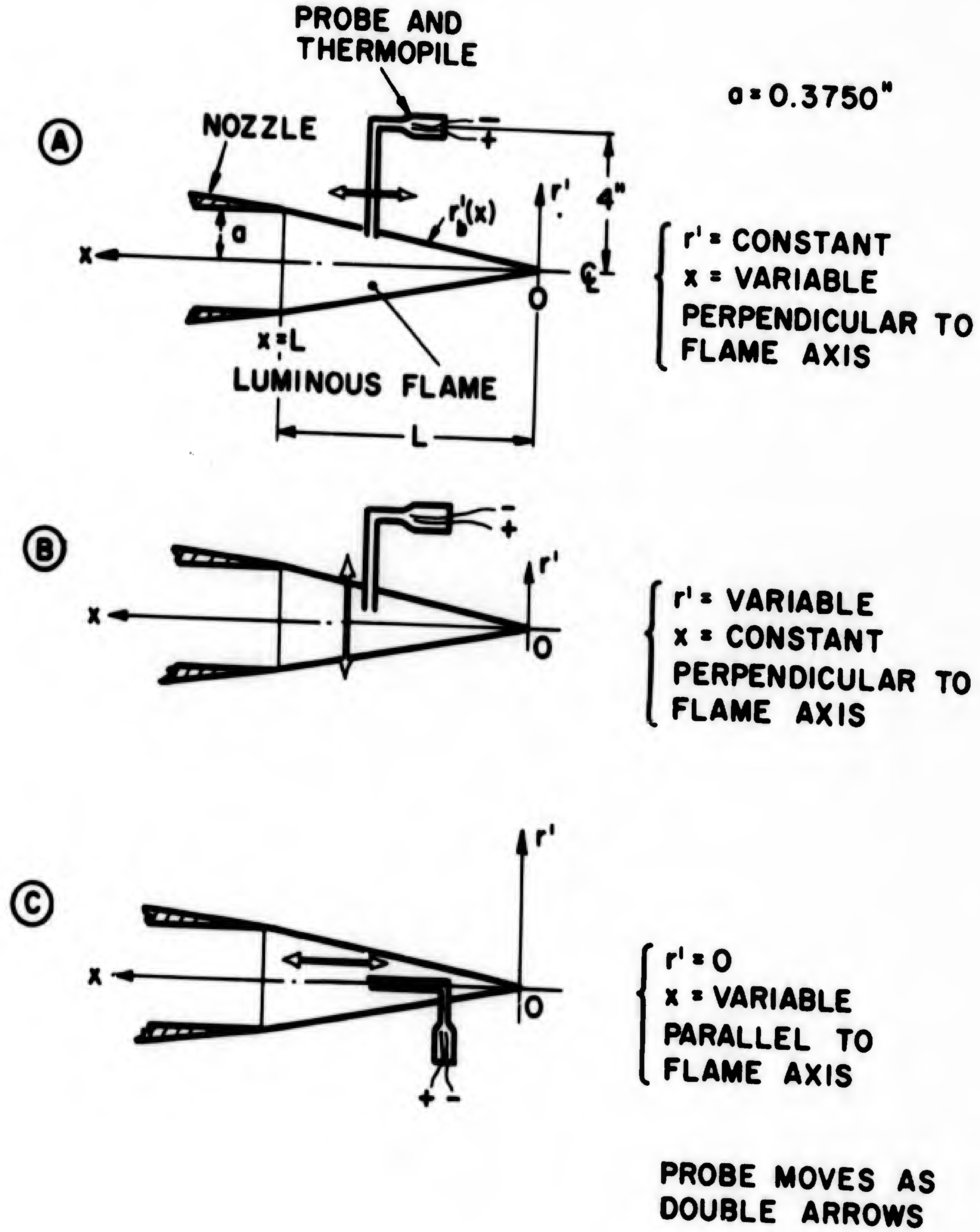


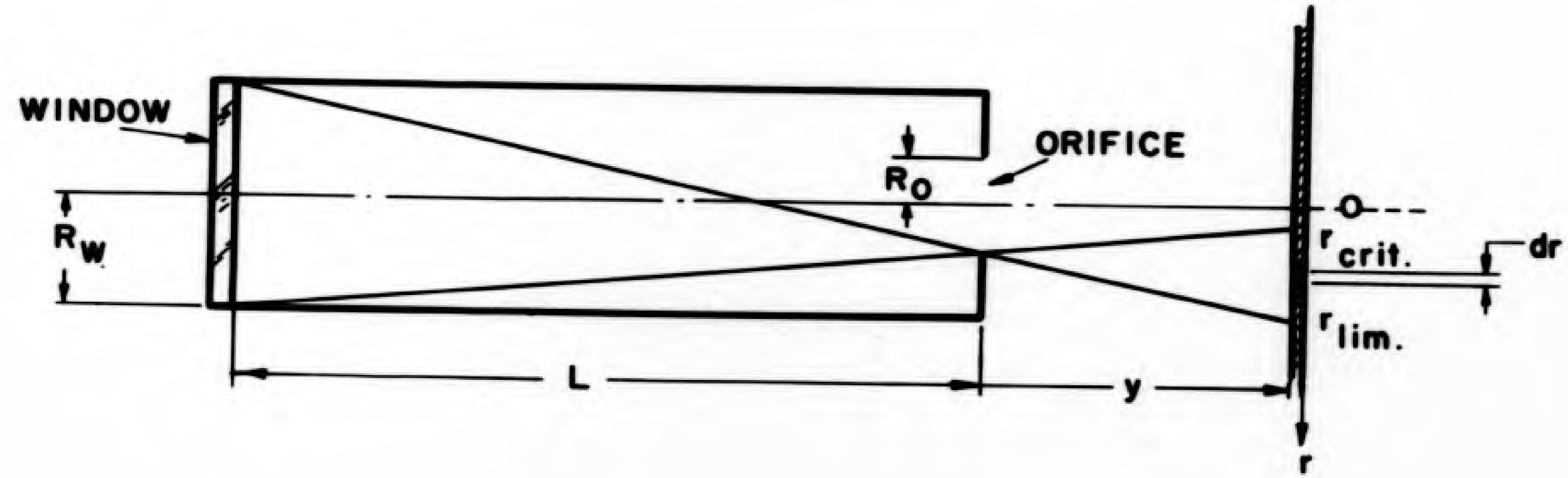
DIAGRAM OF RADIATION PROBE

FIGURE 1



PROBE ORIENTATIONS AND TRAVERSE MODES

FIGURE 2



COLLIMATOR GEOMETRY

FIGURE 3

F VERSUS REDUCED RADIUS

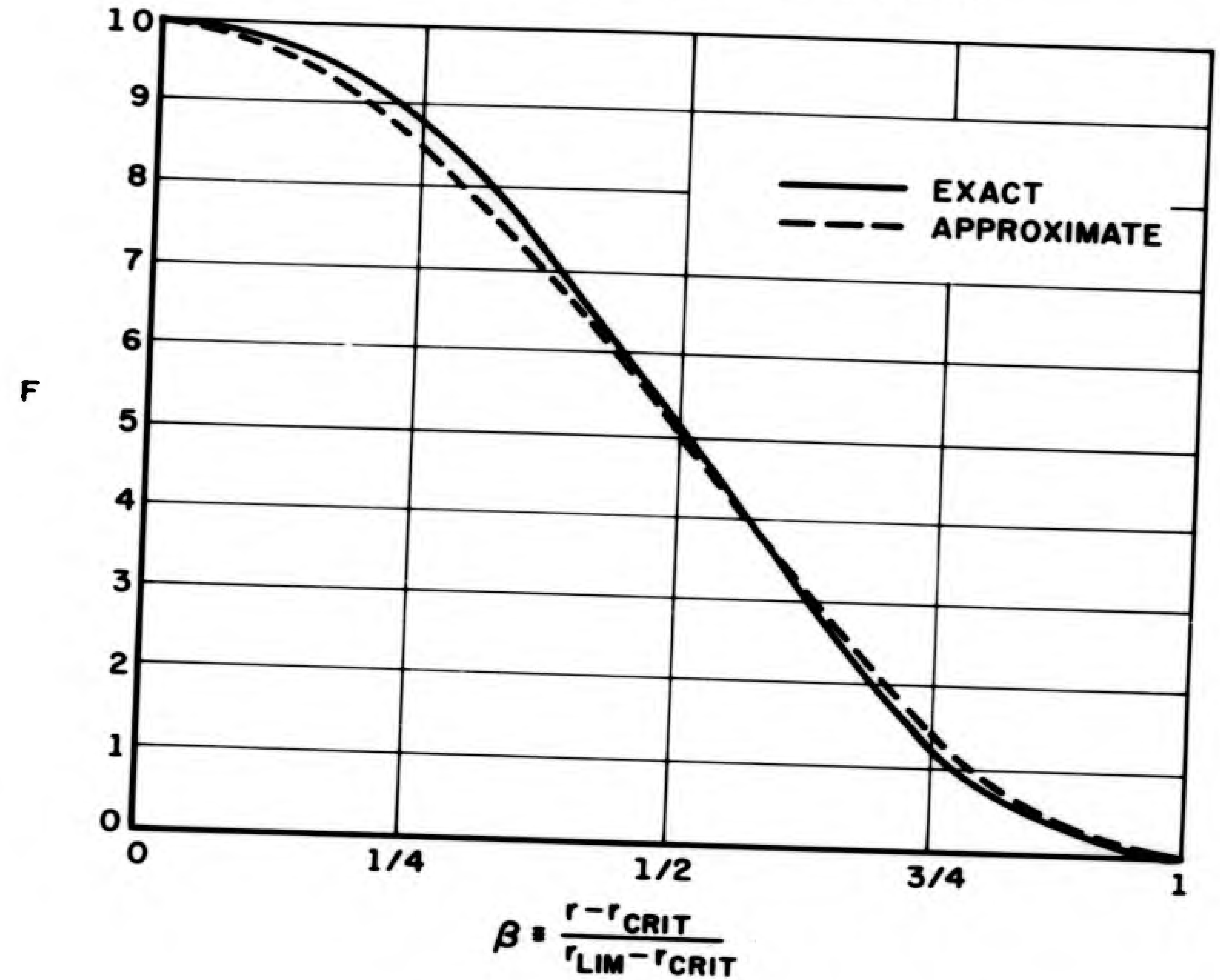


FIGURE 4

VIEW FACTOR (CIRCLE–CIRCLE) VERSUS AXIAL POSITION

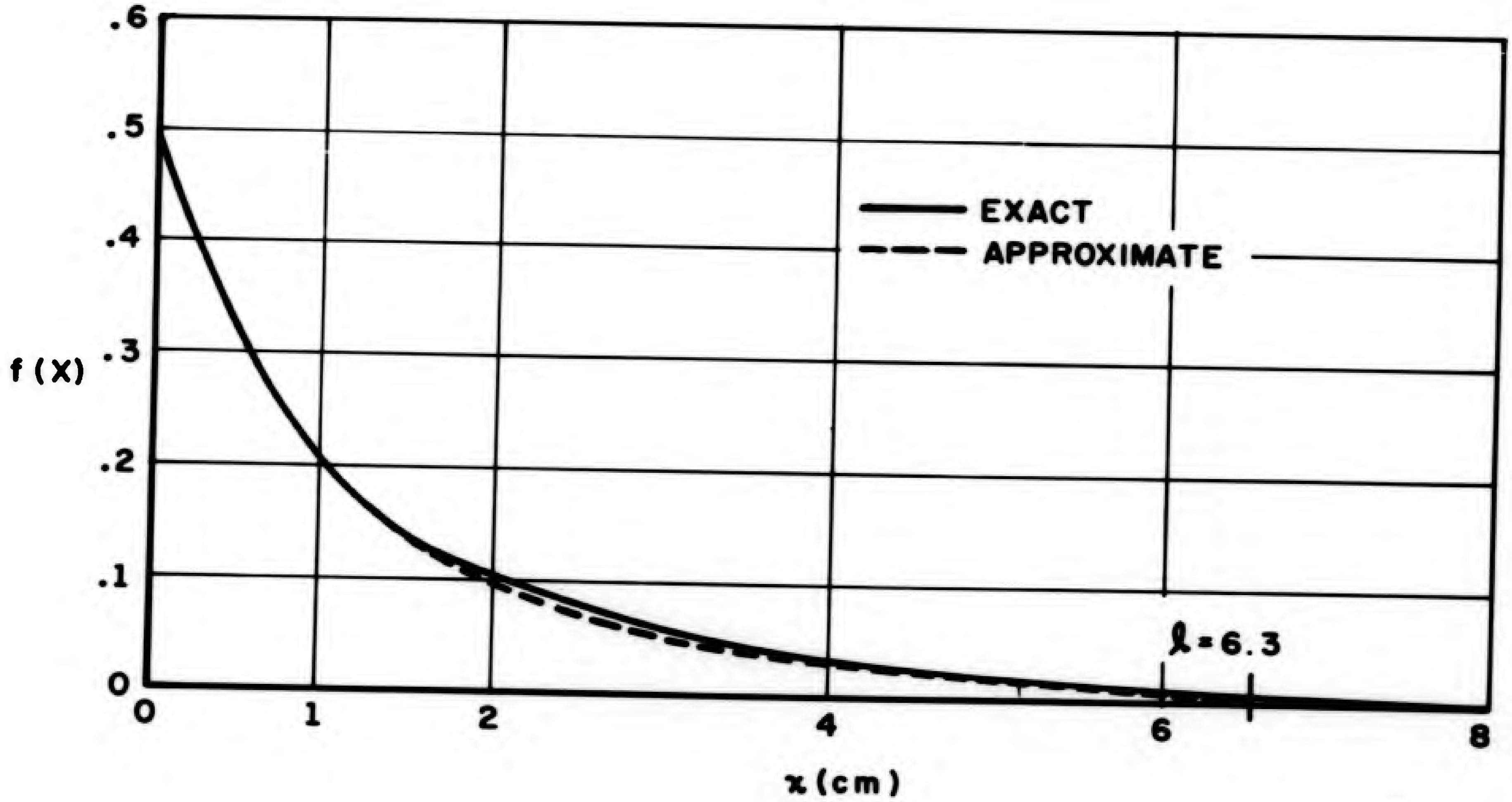


FIGURE 5

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