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AN ANALYTICAL AND EXPERIMENTAL EVALUATION OF THE DAMPING CAPACITY OF SANDWICH BEAMS WITH VISCOELASTIC CORES

by

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Summary

A study is made of the free vibrations of sandwich beams with viscoelastic cores. The study, which is a generalization of a previous investigation by the authors (Ref. 1) includes formulation of the equations of motion and natural boundary conditions, derivation of expressions for the modal distribution of damping based upon "small damping" assumptions, numerical examples and a supporting test program.

Significant among the results were the high damping rates calculated for beams with steel facings and butyl rubber cores. The generally high values calculated for beams of various materials indicate that this type of construction is efficient for vibration damping applications.

It was found, however, that the calculated and test values were not in accord. This lack of agreement signifies the necessity for greater refinement in both analytical methods and test procedures.

Nomenclature

A, B, D	mode function coefficients
E, G	extensional storage modulus of facings, shear storage modulus of core
$\overline{E}, \overline{G}$	extensional loss modulus of facings, shear loss modulus of core
H_r	generalized damping coefficient
I_f	moment of inertia of facings about beam neutral axis
K_{rs}	generalized stiffness
M_r	generalized mass
T	kinetic energy
V	potential energy
U, W	displacement amplitudes
a, b	see equations (14a, b)
c	core depth
f_r	natural frequency
h	facing thickness
$\%(H/H_c)_r$	percentage of critical damping
i	$\sqrt{-1}$
ℓ	length of beam
m	mass per unit length of beam
p	modal characteristic
q	normal coordinate
t	time

u, w	displacement components in longitudinal and lateral directions respectively
x, z	coordinates in longitudinal and lateral directions respectively
α, β	see equations (17a, b)
δ_{rs}	Kronecker delta
η_r	amplitude of normal coordinate
ρ_f, ρ_c	density of facing, density of core
σ_x, τ_{xz}	axial stress in facing, shear stress in core
ϕ_r, ψ_r	mode functions
ω, ω_r, $\bar{\omega}_r$	frequency, undamped natural frequency, damped natural frequency

Part I: ANALYTICAL FORMULATION

Introduction

In Part I of this report, theoretical expressions are derived for the modal damping factors for a vibrating sandwich beam with both ends free. The damping in each mode is expressed first by an amplitude decay rate and then by a "modal percentage of critical damping". The natural frequencies and normal modes of a reference beam (the same beam but without damping) are also calculated and utilized in the damping analysis. The order of the derivation is as follows:

a. First, the mathematical representation of damping in the facing and core materials of the sandwich is formulated.

b. The equations of motion and natural boundary conditions are next derived for the free, damped vibration of the beam. This derivation parallels that in the book by N. J. Hoff (Ref. 3) for the static deflection of a cantilevered sandwich beam.

c. The damping terms are removed from the equations, free-free boundary conditions are specified, and the natural frequencies and normal modes of the undamped reference beam are calculated.

d. These two quantities, along with the original equations of motion are used to obtain an infinite set of "modal" equations of motion for free, damped vibration. The word modal is in quotes because the modes in which the motion is expressed are not the true natural modes of the damped beam itself but those of the undamped reference beam. These equations are thus not completely uncoupled. However, the coupling appears only in the damping terms and is in fact negligible under certain conditions. Rayleigh (Ref. 4) shows that if the damping is small (damping coefficient small compared to stiffness and inertia coefficients) and the response is only of concern near resonance, then the damping coupling terms can be neglected without serious error. The advantage of this approximate formulation of modal equations lies in the fact that the actual motion is more complicated and much more difficult to obtain; the modes themselves are in general complex functions (see Ref. 5).

e. With the assumption which uncouples the equations applied, the damping factor, damped natural frequency and percentage of critical damping are calculated for each mode from the "modal" equations.

Representation of Damping

The sandwich beams being analyzed herein are made up of metal facings which provide flexural stiffness and strength, and cores made of either rubbery or plastic materials which function both as shear resisting media and as damping layers. The damping being considered in the analysis is a type generally referred to as material damping, indicating that it occurs throughout the volume of the material. There may also be present, but usually to a negligible degree, friction damping due to slippage at the interface between facing and core.

The micro-structural mechanisms responsible for material damping in the metal facings and the visco-elastic core are quite different; metallic damping is generally associated with a number of complex phenomena whose total effect is sometimes called internal friction. Viscoelastic material damping, on the other hand, is associated with the curling and uncurling of long polymeric or elastomeric molecules. It is generally describable by a linear differential equation containing stress, strain and their time derivatives. The damping in metals is not as simply described mathematically; in the general case it is non-linear and dependent upon number of cycles. Fortunately for the analyst, these non-linearities are negligible at low stresses and both types of damping, although from different sources, can be represented as linear viscoelastic. (See Refs. 6 & 7 for detailed discussion).

The general constitutive equation for a linear viscoelastic material in shear can be written

$$\sum_n a_n \frac{\partial^n}{\partial t^n} \tau = \sum_n b_n \frac{\partial^n}{\partial t^n} \gamma \tag{1a}$$

Assuming the motion to be of the form

$$\tau, \gamma \sim e^{(-d + i\omega)t} \tag{1b}$$

where d is the rate of decay of the amplitude and ω the frequency, we find by substitution that

$$\tau = \frac{\sum b_n (-d + i\omega)^n}{\sum a_n (-d + i\omega)^n} \gamma \tag{1c}$$

which can be reduced to

$$\tau = G(d,\omega) + i\bar{G}(d,\omega) \gamma \tag{2a}$$

It has been shown experimentally that the real and imaginary coefficients of equation (2a) are not sensitive to rate of decay of amplitude as long as it is not too rapid. Thus we have

$$\tau = \left[G(\omega) + i\bar{G}(\omega)\right] \gamma \tag{2b}$$

Equation (2b) applies in the present case to the viscoelastically sheared core. The internal friction type of damping in the facings is neither frequency or decay rate sensitive at low stresses. Accordingly we have

$$\sigma = (E + i\bar{E})\epsilon \tag{3}$$

In the above equations E and $G(\omega)$ are storage moduli for extension of the facings and shear in the core respectively whereas $\bar{E}$ and $\bar{G}(\omega)$ are corresponding loss moduli.

There is usually also a strong temperature dependence in the properties of viscoelastic materials. We are assuming in this analysis a constant room temperature.

Equations of Motion and Boundary Conditions

The equations of motion for the free, damped vibration of a sandwich beam are herein derived by the use of Hamilton's Principle. The kinetic energy, the potential energy and the dissipated energy are all given herein without derivation since they have been previously derived by these authors in Ref. (1). The symbols used are defined in the Nomenclature section and are also shown in the schematic drawing of the beam (Figure 1).

$$T = \frac{1}{2}\int_{-\ell/2}^{\ell/2}\left[(2\rho_f h + \rho_c c)\,\dot{w}^2 + 2\rho_f h\dot{u}^2\right]dx \tag{4a}$$

$$W = \frac{1}{2} \int_{-\ell/2}^{\ell/2} \left[2E^{*}hu'^{2} + \frac{E^{*}h^{3}}{6} w''^{2} + G^{*}c \left(\frac{2u}{c + h} - w'\right)^{2} \right] dx \qquad (4b)$$

where

$$E^{*} = E + i\bar{E}, \quad G^{*} = G + i\bar{G} \qquad (4c)$$

The two terms of the kinetic energy expression, equation (4a) account for translatory and rotatory inertia respectively. Rotatory inertia can be neglected for the modes and beams considered herein. The three terms of the work corresponding to stored and dissipated energy (W) account for extension of the facings, bending of the facings and shear deformation in the core, respectively.

We apply Hamilton's Principle, which states

$$\delta \int_{t_1}^{t_2} (T + W)\, dt = 0 \qquad (5)$$

If we insert the expressions given in (4a, b and c) and carry out the integration by parts in the usual manner, we obtain the following equation.

$$\int_{t_1}^{t_2} \left\{ \left[2E^{*}hu'\delta u + \frac{E^{*}h^{3}}{6} w''\delta w' - \left(G^{*}c \frac{2u}{c + h} - G^{*}cw' - \frac{E^{*}h^{3}}{6} w'''\right)\delta w \right]_{-\ell/2}^{\ell/2} \right.$$

$$- \int_{-\ell/2}^{\ell/2} \left\langle \left[2E^{*}hu'' - G^{*}c \left(\frac{2u}{c + h} - w'\right) \right] \delta u + \right. \qquad (6)$$

$$+ \left[\frac{E^* h^3}{6} w^{iv} + G^* c \left(\frac{2u}{c + h} - w' \right)' + m\ddot{w} \right] \delta w \Big\rangle dx \Big\} dt = 0 \quad (6)$$

The variational equation of motion is thus:

$$\int_{-\ell/2}^{\ell/2} \Big\langle \left[2E^* h u'' - G^* c \left(\frac{2u}{c + h} - w' \right) \right] \delta u +$$

$$+ \left[\frac{E^* h^3}{6} w^{iv} + G^* c \left(\frac{2u}{c + h} - w' \right)' + m\ddot{w} \right] \delta w \Big\rangle dx = 0 \quad (7)$$

From equation (6) we extract the differential equations of motion

$$2E^* h u'' - G^* c \left(\frac{2u}{c + h} - w' \right) = 0 \quad (8a)$$

$$\frac{E^* h^3}{6} w^{iv} + G^* c \left(\frac{2u}{c + h} - w' \right)' + m\ddot{w} = 0 \quad (8b)$$

and the following natural boundary conditions:

At $x = \ell/2$ and $x = -\ell/2$:

Either $u' = 0$ or u is prescribed (9a)

Either $w'' = 0$ or w' is prescribed (9b)

Either $\frac{Eh^3}{6} w''' - \frac{2Gc}{c + h} u + Gcw' = 0$ or w is prescribed (9c)

Equations (8a,b) without the inertia and damping terms are given in Ref. (3).

Undamped Natural Frequencies and Modes

If the facing thickness is small with respect to the core depth, the resistance of the facings to bending about their own neutral axes can be neglected. This implies elimination of the first terms of equations (8b & 9c). This simplifying assumption is valid for the beams under consideration in this investigation. If we eliminate also the damping terms (i.e., let $E^* \longrightarrow E, G^* \longrightarrow G$) we can set

$$u(x,t) = Ue^{(px-i\omega t)}$$

$$w(x,t) = We^{(px-i\omega t)} \qquad (10a,b)$$

We obtain upon substitution of equations (10a, b) into equations (8a,b)

$$\left(2Ehp^2 - \frac{4Gc}{(c+h)^2}\right) U + \left(\frac{2Gc}{c+h} p\right) W = 0$$

$$\left(\frac{2Gc}{c+h} p\right) U + (-m\omega^2 - Gcp^2) W = 0 \qquad (11a,b)$$

For a non-trivial solution, the determinant of the coefficients of U and W must vanish. This leads to the following characteristic equation.

$$p^4 + \frac{m\omega^2}{Gc} p^2 - \frac{2m\omega^2}{Eh(c+h)^2} = 0 \tag{12}$$

The four roots of equation (12) are

$$p_{1,2,3,4} = \pm\sqrt{-\frac{m\omega^2}{2Gc} \pm \sqrt{\left(\frac{m\omega^2}{2Gc}\right)^2 + \frac{2m\omega^2}{Eh(c+h)^2}}} \tag{13}$$

If we let

$$a = \frac{m\omega^2}{2Gc}, \qquad b^2 = a^2 + \frac{2m\omega^2}{Eh(c+h)^2} \tag{14a,b}$$

we can rewrite the solution to equation (12) as

$$p_{1,2,3,4} = \pm\sqrt{-a \pm b} \tag{15}$$

or

$$p_{1,2} = \pm i\alpha \qquad p_{3,4} = \pm\beta \tag{16a,b}$$

where

$$\alpha = \sqrt{b+a}, \qquad \beta = \sqrt{b-a} \tag{17a,b}$$

If we denote the four solutions for Ue^{px} and We^{px} corresponding to the r^{th} value of ω as $\phi_r(x)$ and $\psi_r(x)$ respectively, then the general solution to equations (8a, b, modified) can be written

$$u(x,t) = \sum_{r=1}^{\infty} q_r \phi_r(x) \tag{18a}$$

$$w(x,t) = \sum_{r=1}^{\infty} q_r \psi_r(x) \tag{18b}$$

where q_r is the r^{th} normal coordinate and

$$\phi_r(x) = A_1 e^{i\alpha x} + A_2 e^{-i\alpha x} + A_3 e^{\beta x} + A_4 e^{-\beta x}$$

(19a,b)

$$\psi_r(x) = B_1 e^{i\alpha x} + B_2 e^{-i\alpha x} + B_3 e^{\beta x} + B_4 e^{-\beta x}$$

Notice that if we let the core shear storage modulus G approach finity, then we have

$$a = 0$$

(20a,b)

$$b^2 = \frac{m\omega^2}{EI_f} \text{ where } I_f = \frac{Eh(c + h)^2}{2}$$

and therefore

$$\alpha = \beta = \sqrt[4]{\frac{m\omega^2}{EI_f}} \tag{20c}$$

Thus equations (19a,b) reduce, as they should, to the equations for the natural modes of an elementary beam with a moment of inertia of I_f, which is the moment of inertia of the facings of the sandwich about the beam neutral axis.

If we rewrite equations (19a,b) and consider from now on only modes symmetrical about the centerline, we obtain

$$\phi_r(x) = D_1 \sin\alpha x + D_2 \sinh\beta x$$

(21a,b)

$$\psi_r(x) = D_3 \cos\alpha x + D_4 \cosh\beta x$$

The four coefficients D_1, D_2, D_3 and D_4 are not independent. We determine their relationships by substituting equations (21a,b) into equation (8b, modified). We obtain:

$$Gc\left[\frac{2}{c+h}(D_1\alpha\cos\alpha x + D_2\beta\cosh\beta x) + (D_3\alpha^2\cos\alpha x + D_4\beta^2\cosh\beta x)\right] -$$

(22)

$$-\frac{m\omega^2}{Gc}(D_3\cos\alpha x + D_4\cosh\beta x) = 0$$

Equating the coefficients of $\cos\alpha x$, etc., to zero and simplifying, we obtain

$$D_3 = -\frac{2}{c+h}\frac{\alpha}{\beta^2}D_1 \quad (23a)$$

$$D_4 = \frac{2}{c+h}\frac{\beta}{\alpha^2}D_2 \quad (23b)$$

The beams under consideration in the present report are free at both ends. Since we are considering only motion symmetric about the centerline we satisfy the boundary conditions at one end ($x = \ell/2$) and at the centerline ($x = 0$). The appropriate boundary conditions from equation (9, modified) are

$$\left[u\right]_{x=0} = 0; \qquad \left[w'\right]_{x=0} = 0 \tag{24a}$$

$$\left[\sigma_x\right]_{x = \frac{\ell}{2}} = 0 \text{ or } \left[u'\right]_{x = \frac{\ell}{2}} = 0;$$

(24b)

$$\left[\tau_{xz}\right]_{x = \frac{\ell}{2}} = 0 \text{ or } \left[\frac{2}{c + h} u - w'\right]_{x = \frac{\ell}{2}} = 0$$

The boundary conditions expressed in equations (24a,b) neglect the bending stiffness of the facings. They are therefore consistent with the simplified differential equations (equations 8a,b without the w^{iv} term).

It can be seen from equations (21a,b) and (24a) that the boundary conditions at $x = 0$ are satisfied. Imposition of the boundary conditions at $x = \ell/2$ leads to the frequency equation. Applying equations (21a,b) to equation (20b) gives

$$D_1\alpha\cos\bar{\alpha} + D_2\beta\cosh\bar{\beta} = 0 \tag{25a}$$

$$\frac{2}{c + h}(D_1\sin\bar{\alpha} + D_2\sinh\bar{\beta}) - (-D_3\alpha\sin\bar{\alpha} + D_4\beta\sinh\bar{\beta}) = 0 \tag{25b}$$

where

$$\bar{\alpha} = \alpha\ell/2, \qquad \bar{\beta} = \beta\ell/2 \tag{26}$$

Setting the determinant of the coefficients equal to zero after using equations (23a,b) yields

$$\alpha(1 - \frac{\beta^2}{\alpha^2})\cos\bar{\alpha}\sinh\bar{\beta} - (1 - \frac{\alpha^2}{\beta^2})\beta\sin\bar{\alpha}\cosh\bar{\beta} = 0 \tag{27}$$

Finally, equation (27) can be simplified to give the frequency equation as

$$\bar{\beta}/\bar{\alpha}\,\tanh\bar{\beta} + \tan\bar{\alpha} = 0 \tag{28}$$

This equation can be readily solved graphically for the roots $\bar{\alpha}$, $\bar{\beta}$ and corresponding ω^2. After calculation of, say, the r^{th} natural frequency, ω_r, we can determine the corresponding mode shape by substituting the r^{th} values of α, β, $\bar{\alpha}$, $\bar{\beta}$ into the following equation, obtained from equation (25a), after fixing the value of, say, D_1.

$$D_2 = -\frac{\alpha\cos\bar{\alpha}}{\beta\cosh\bar{\beta}}\,D_1 \tag{29}$$

The values of D_3 and D_4 are then determined from equations (23a,b).

<u>Damped Free Vibration</u>

With the undamped natural frequencies and corresponding mode functions determined, we now go back and replace the damping terms in the equations of motion. The dynamic characteristics of the undamped reference beam can be used to form an infinite set of modal equations which are uncoupled except for the damping terms.

Substituting the solutions given by equations (18a,b) into the variational equation of motion, equation (7), and considering now only half the beam,

$$\int_0^{\ell/2}\left[\left\{2(E^* + i\bar{E})h \sum_s q_s \phi_s'' - \frac{2(G + i\bar{G})c}{c + h}\left(\frac{2}{c+h}\sum_s q_s\phi_s - \sum_s q_s\psi_s'\right)\right\}\phi_r \delta q_r + \right. \tag{30}$$

$$\left. + \left\{\frac{2(G + i\bar{G})c}{c + h}\sum_s q_s\phi_s' - (G + i\bar{G})c\sum_s q_s\psi_s + m\sum_s \ddot{q}_s\psi_s\ \psi_r\delta q_r\right]dx = 0$$

We must bear in mind that both components of $G^* = G(\omega) + i\bar{G}(\omega)$ take on values corresponding to the frequency satisfying the equation in which the term appears. Since the frequency associated with equation (30) is $\bar{\omega}_r$, the r^{th} damped natural frequency, E^* and G^* in that equation corresponds to $\bar{\omega}_r$. Now let

$$K_{rs}(\bar{\omega}_r) = \int_0^{\ell/2}\left[\left\{2Eh\phi_s'' - \frac{2Gc}{c + h}\left(\frac{2}{c + h}\phi_s - \psi_s'\right)\right\}\phi_r + \right. \tag{31}$$

$$\left. + \left\{\frac{2Gc}{c + h}\phi_s' - Gc\psi_s''\right\}\psi_r\right] dx$$

and let the similar term with E, G replaced by $\bar{E}$, $\bar{G}$ be denoted

$H_{rs}(\omega_r)$. In accordance with the previously discussed assumption we neglect the coupling terms. Therefore

$$H_{rs}(\bar{\omega}_r) = H_r(\bar{\omega}_r)\,\delta_{rs} \tag{32}$$

Let

$$M_r\delta_{rs} = \int_0^{\ell/2} m\psi_r\psi_s dx \text{ where } \delta_{rs} = \begin{cases} 1 & r = s \\ 0 & r \neq s \end{cases} \tag{33}$$

Using the orthogonality relations between the normal modes of the undamped reference beam we can show that

$$K_{rs}(\omega_r) = \omega_2^2 M_r\delta_{rs} \tag{34}$$

Assuming that the frequency $\bar{\omega}_r$ differs only slightly from ω_r we set

$$\begin{aligned} K_{rs}(\bar{\omega}_r) &= K_{rs}(\omega_r) = \omega_r^2 M_r\delta_{rs} \\ H_r(\bar{\omega}_r) &= H_r(\omega_r) \end{aligned} \tag{35a,b}$$

Now applying equations (31) through (35) to equation (30) we arrive at the following modal equation.

$$\ddot{q}_r + \omega_r^2 q + i\,\frac{H_r(\omega_r)}{M_r}\,q_r = 0 \tag{36}$$

$$r = 1, \ldots \infty$$

where

$$H_r(\omega_r) = \int_0^{\ell/2} \left(2\bar{E}h\phi_r''\phi_r - \frac{4\bar{G}c}{(c+h)^2}\phi_r^2 + \frac{2\bar{G}c}{c+h}\psi_r\phi_r + \right. \tag{37}$$

$$\left. + \frac{2\bar{G}c}{c+h}\phi_r\psi_r - \bar{G}c\psi_r''\psi_r\right) dx$$

Formulas for calculating M_r and H_r are given in the Appendix.

If we assume $H_r/K_r << 1$ (which implies $H_r/M_r << \omega_r^2$) and let q_r be proportional to

$$e^{(-d_r + i\bar{\omega}_r)t} \tag{38}$$

we obtain the rate of amplitude decay as

$$d_r = \frac{H_r(\bar{\omega}_r)}{2\omega_r M_r} \tag{39}$$

and the damped natural frequency as

$$\bar{\omega}_r = \omega_r \left[1 - \frac{H_r(\bar{\omega}_r)}{2\omega_r^2 M_r}\right] \tag{40}$$

The concept of critical damping is not as clear in the general case of viscoelastic damping as in the special case of viscous damping, to which it is usually applied. If we define critical damping as the amount of damping which causes the motion to become non-oscillatory, and if we then attempt to determine this amount of damping by letting $\bar{\omega}$ approach zero, a complication arises from the fact that the damping is not expressed by a constant, as is viscous damping, but by a function of frequency. Thus, in defining critical damping, the manner in which this function varies with increased damping must be specified. An expression can be written however, for percentage of critical damping, although its interpretation is not the same as in the simpler case:

$$\left[\frac{H}{H_c}\right]_r = \frac{H_r(\omega_r)}{2\omega_r^2 M_r} \tag{41}$$

Part II: EXPERIMENTAL PROGRAM

Specimen Preparation

To provide a counterpart for the evaluation of the theory developed in Part I and to establish its practicality, a test program was carried out in which a set of sandwich beam test specimens were constructed and tested at the Naval Applied Science Laboratory, U.S. Naval Base, Brooklyn, New York. The specimens were prepared in accordance with Table 1 and Figure (3). The (14) facing material on the specimens was mild steel plate of 0.040 inch and 0.080 inch nominal thicknesses. Core materials were a representative elastomer and a representative thermoplastic. The core materials selected were butyl rubber and polyvinyl-chloride as the elastomer and thermoplastic respectively.

The butyl rubber was obtained in a nominal 1/8 inch thickness and the polyvinyl-chloride was obtained in nominal thicknesses of 0.250 inches and 0.375 inches. The 1/8 inch thick butyl rubber was laminated into 0.250 and 0.375 inch thicknesses to correspond with the polyvinyl-chloride material by the use of solvent-based neoprene rubber adhesive. A thin brush coat of the adhesive was applied to the mating surfaces allowed to dry to an aggressive tack and immediately bonded.

Upon completion of the fabrication of the core materials, one side of the steel face material was disc sanded to remove all rust and mill scale, and the sandwich beams were prepared as follows:

a. Elastomeric Beams. The mating surface of the steel face and laminated butyl rubber core was given a thin brush coat of the solvent-based neoprene rubber adhesive, allowed to dry to an aggressive tack and immediately bonded into a sandwich beam.

b. Thermoplastic Beams. The mating surfaces of the steel face and the polyvinyl-chloride core were given a thin brush coat of an epoxy adhesive and immediately bonded into a sandwich beam. All specimens were lightly weighed down to assure intimate contact of the mating surfaces, and were held in a jig to prevent slippage of the component parts during curing of the adhesives. Specimens were allowed to cure for 96 hours prior to damping tests.

Description of Materials

a. The 1/8 inch thick butyl rubber had a Shore durometer hardness ranging between 60 and 80.

b. The solvent based neoprene rubber adhesive was U.S. Rubber Company adhesive No. 6244.

c. The 0.250 and 0.375 inch thick polyvinyl-chloride material was Boltaron (PVC) 6200 Normal impact Type 1.

d. The epoxy adhesive was Epibond 126 adhesive consisting of Epibond 126 base resin and Hardener 9816. Prior to use, the adhesive was prepared by mixing 100 parts of the Epibond 126 base resin with 12 parts of weight by the Hardener 9816.

Test Method

The modal distribution of damping was determined by measurement of the decay rate of free vibration in each of the first few modes for each specimen. This method was successfully used previously by the Laboratory in damping evaluations of disks and beams coated with damping material on one face.

The test beams were suspended horizontally by means of long nylon cords to provide minimum restriction to vibratory motions, thus simulating a "free-free" boundary condition. The support points were located a half inch from each end of the beam. An electrodynamic vibration exciter suspended by cords was attached to the test beam at the center of one face and a piezoelectric accelerometer was attached at the corresponding position on the opposite face. In this manner, the beam, exciter and pick-up acted as an integral unit with no external influence. Figure (4) is a photograph of the test set-up.

The damping capacity was determined as follows:

(1) The beam was excited and each resonant frequency was determined by observing the peak accelerometer voltages throughout the frequency range of interest.

(2) The source of excitation was cut off instantaneously; the logarithm of the exponential free decay curve was recorded

on a storage oscilloscope (memo-scope), and the slope of the curve envelope was measured.

(3) The following equations were used (see Ref. (8)).

$$D = \frac{C.F}{T/D} \times \tan\alpha \tag{42}$$

$$\%\ H/H_{cr} = \frac{1.83\ D}{f_n} \tag{43}$$

where:

D	= decay rate, db/sec.
C.F	= calibration factor of vertical axis of scope, db/division.
T/D	= horizontal axis calibration of scope, sec/division.
$\tan\alpha$	= slope of envelope of logarithmically converted exponential curve.
f_n	= resonant frequency.
$\%\ H/H_{cr}$	= percentage of critical damping.

Instrumentation

The instrumentation employed in this investigation is given below. Figure (5) is a schematic arrangement of the instrumentation.

a. Hewlett Packard Oscillator, Model 202B.

b. McIntosh Amplifier, Model MC-60.

c. Hewlett Packard Electronic Counter, Model 521E.

d. Goodman Vibration Generator, Model 390A.

e. Endevco Accelerometer, Model 2213.

f. Massa Lab. Preamplifier (Cathode Follower), Model M-114-B.

g. Massa Lab. 60 DB Amplifier, Model M-185.

h. General Radio Sound and Vibration Analyzer (Filter) Type 1554-A.

i. Ballantine Lab. Voltmeter, Model 643.

j. Audio Instrument Company Logger, Type 122B.

k. Hughes Aircraft Memo-scope, Model 104.

ℓ. Tectronic Dual Beam Oscilloscope, Type 502.

Part III: CORRELATION AND DISCUSSION OF RESULTS

General

Calculations of the first few modal damping factors were made for the test beam specimens described in Part II. Calculations were also made for the natural frequencies and natural modes of the corresponding undamped reference beams. These quantities were used in calculating the damping factors in accordance with the assumptions outlined in Part I. In the present part of the report, these numerical results are correlated and discussed.

The specimens were designed with typical sandwich beam proportions; i.e., length-to-depth ratio and facing thickness-to-core depth ratio. The materials selected were a typical structural metal for the facings (mild steel) and representative elastomeric (butyl rubber for specimens 2 and 3) and polymeric (polyvinyl-chloride for specimens 5 and 6) materials for the cores. Two different facing thicknesses were applied to beams of each core material; specimens 2 and 5 having 0.040 inch facings and specimens 3 and 6 having 0.080 inch facings. The geometric properties of the beams are summarized in Table I.

The storage and loss moduli for the facing material ($E,\overline{E}$) were assumed to be independent of frequency as shown in Table I. These properties were obtained from Ref. (7). The moduli for the core materials $(G,\bar{G})$ are room temperature properties given as functions of frequency in Fig. (6). They were obtained from Ref. (9) for butyl rubber and Ref. (10) for polyvinyl-chloride.

Natural and Resonant Frequencies

Preliminary to calculating the modal damping factors and as input for that calculation, we determined the natural frequencies of the undamped reference beams. A graphical solution of the frequency equation, equation (28), in which the two terms $(\bar{\beta}/\bar{\alpha}\tanh\bar{\beta})$ and $(-\tan\bar{\alpha})$ were plotted as functions of $(\bar{\alpha})$, provided two sets of curves whose intersections gave the roots of the equation. Figure (7) shows the solution for beam #2. The roots $\bar{\alpha}$ are the solid circles and the corresponding natural frequencies obtained from a separate plot of frequency versus $\bar{\alpha}$ are shown as rectangles. With the frequencies known, the mode coefficients were calculated from equations (23a, b) and (29). This data is listed in Table II for all four specimens. The number of modes calculated for beams #5 and 6 was taken as the number observed in the tests.

The undamped frequencies can be compared in Table II with the resonant frequencies observed in the tests for the polyvinyl-chloride core beams, numbers 5 and 6. Although the experimental program included tests of some butyl rubber core beams, their characteristics could not be calculated because of unavailability of the material properties. It can be seen that the calculated and observed frequencies are generally about as close as may be expected, inasmuch as the amount of damping present in the beams produces a "frequency-shift" which is not negligible as in conventional, slightly damped structures.

To ascertain the effect of shear deformation on frequency, we calculated from elementary beam theory the natural frequencies of an undamped beam with the same properties as beams 2 and 5 except that the shear modulus was assumed to be infinitely large. The values in the last column below were obtained for the first four modes:

Mode	Natural	Frequency	
No.	Beam #2	Beam #5	$G = \infty$
1	31.8	66	71.2
3	75.4	330	385
5	121	760	950
7	170	1060	1767

Comparing these with the corresponding frequencies for beam #2 and beam #5, we see that shear deformation has tremendously influenced all of the frequencies of the soft-core beam #2 while the influence has been far less in the case of the stiffer core beam #5 and almost negligible in the first mode. As expected, the influence of shear deformation can be seen to become greater the higher the mode.

The mode shapes (Table I) exhibit the expected tendency in the case of the stiffer beams, numbers 5 and 6, namely, that in the higher modes the "sinh" and "cosh" contributions become negligible. However, in the case of the softer core beams, numbers 2 and 3, where most of the deflection is due to shear, it is observed that the ratio of u to w displacement is much smaller, indicating little rotation of normal elements, and further, that the "sin" component of the u displacement as well as the "cosh" component of the w displacement become negligible in the higher modes so that for higher mode approximations we could use:

Soft beams (#2,3): $\phi(x) = D_2 \sinh \beta x$

$\psi(x) = D_3 \cos \alpha x$

Stiff beams (35,6): $\phi(x) = D_1 \sin \alpha x$

$\psi(x) = D_3 \cos \alpha x$

Damping Capacity

Prior to calculating the quantities which measure the damping in each mode, and as an input for that calculation, we determined the generalized modal masses, stiffnesses and damping coefficients. The equations given in the Appendix were used and the numerical values are shown in Table III. The calculations were performed on an electronic computer. Of significance here is the fact that the damping coefficient H_r is smaller than the stiffness coefficient K_r, but not much smaller in all cases, especially in beams #2 and 3. Since an important assumption in the analysis was that $H/K << 1$, a certain amount of discrepancy between analytical and observed damping quantities was expected.

The amount of damping in a given mode can be expressed by any of a number of quantities. The choice of a damping parameter for any particular problem depends upon the application intended. For example, in some cases such as acoustic deadening, the rate of decay of amplitude with time for a freely vibrating damped structure is the most important quantity, while in other cases like fatigue reduction the rate of decay of amplitude per cycle, which is associated with both the logarithmic decrement and the percentage of critical damping, may be of most interest.

In the present problem both the time rate of amplitude decay and the percentage of critical damping are calculated. Comparison with test values is made only for the former quantity since this was the quantity which was directly measured in the tests. The last three columns of Table III give this data. A wide discrepancy is apparent between the calculated and test values for the decay rate, d_r for beams #5 and 6. No reason for this discrepancy can be stated with certainty as there are a number of factors which could account for it. There are indications, however, that the invalidity of the "small damping" assumption inherent in both the analytical and test procedure is responsible. There is justification for accepting the calculated values, at least as an approximate solution, based on the good agreement obtained between calculated and observed natural frequencies. This agreement indicates that (1) the structural theory and the material properties (except loss modulus) are satisfactory and (2) that the effect of damping on frequency is rather small. Thus the results of the damping calculation, barring numerical errors, should be acceptable as an approximation. A check was made on the damping calculations which gave substantiating results. It consisted of assuming the damping to be "proportional" in the sense that the loss tangent is uniform throughout the beam. When this is true the damping expression reduces to a very simple form as follows.

If the loss tangents of the facings and core are the same, i.e., if

$$\frac{\overline{E}}{E} = \frac{\overline{G}}{G} = \eta \tag{44}$$

then the modal equations of motion are <u>precisely</u> uncoupled (not by approximation) and a much simpler expression for the decay rate results, which is

$$d_r = \eta \frac{\omega_r}{2} \tag{45}$$

Although the loss tangents are not the same for the facings and core in the present problem, we can still apply equation (45) inserting the loss tangent of the core material as an approximate calculation for beams #2 and 5. The justification for this is that the core material accounts for much (82%) of the volume and much of the strain energy, so that <u>its</u> loss tangent rather than that of the facing should have the predominate effect on d_r. This calculation performed for beam #2 gave the following results which provide something of a check on the more complex calculation.

Mode No.	d_r(eqn. 39)	$d_r = \pi f_r \frac{\bar{G}}{G}$ (eqn. 45)
1	41.9	31.0
3	120	100
5	217	185
7	337	300
9	478	431

While the values calculated within the limitations of the so-called "small damping" (equation 39) and "proportional damping" (equation 45) theories are in reasonably close agreement, the actual beams tested do not fall precisely into either of these categories. It is apparent from the modal coefficients that these theories have been extended somewhat beyond their ranges of applicability. However, it is not felt that this accounts totally for the disparity between the calculated and test data. Both a more refined theory and test method appear to be desirable goals for future work in the area of vibration damping research. Such refinements have not been needed in the past because material damping of such large magnitudes has not long been attained in rigorously analyzed structural components.

Conclusions

The results of this analysis in which a free-free sandwich beam with visoelastic core has been studied, extend those of a previous analysis by the authors (Ref. 1) in which an infinite (or simply supported) beam was investigated. In the present report, as in the previous, a large amount of damping for a given weight of beam has been shown to be attainable by the use of the visco-elastic core. In the present case a more general formulation has been given including all admissible boundary conditions. Further, an experimental back-up program has verified the applicability of the structural theory for predicting natural frequencies but indicated that a more refined theory is required to accurately predict the damping when it is of such a large magnitude.

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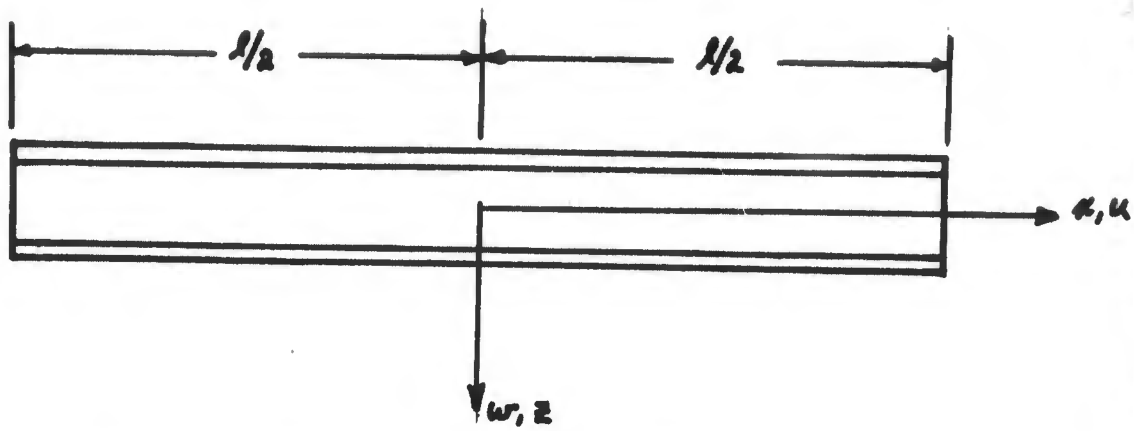
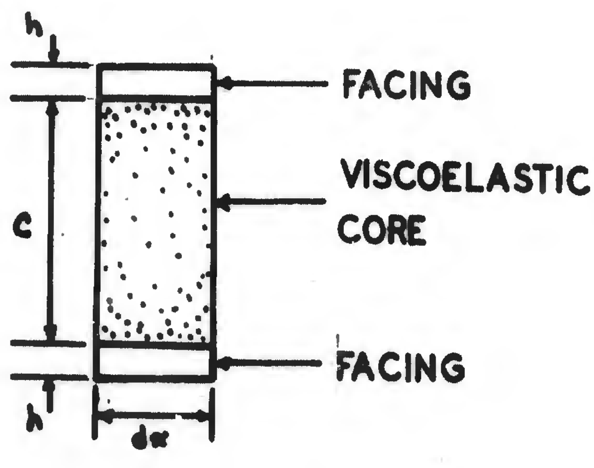
List of Illustrations and Tables

Illustrations

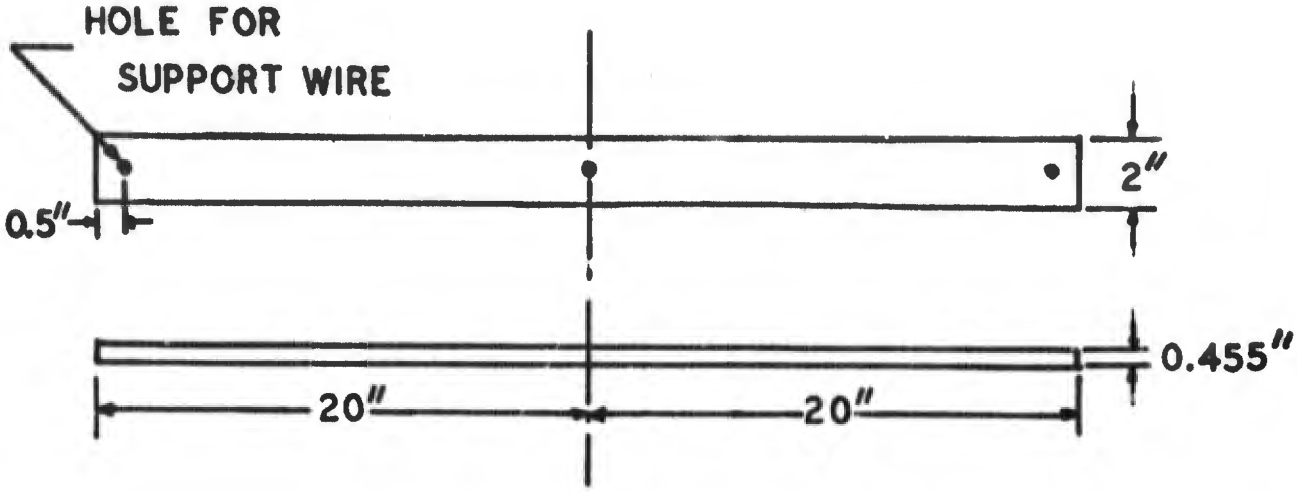
Tables



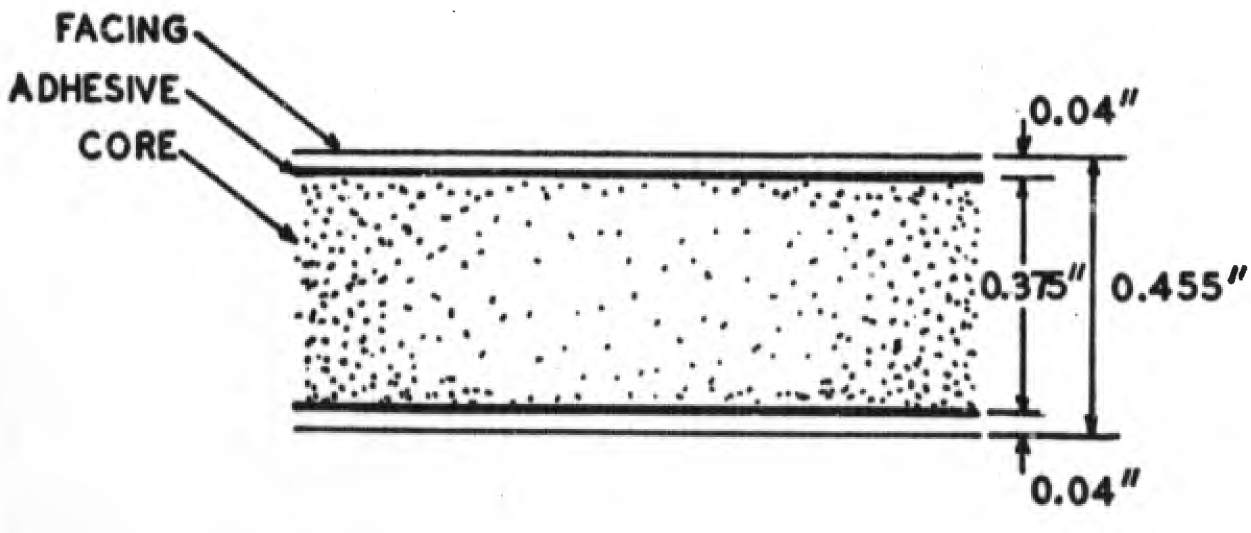
SANDWICH BEAM AND COORDINATES
FIGURE (1)



SANDWICH BEAM ELEMENT
FIGURE (2)

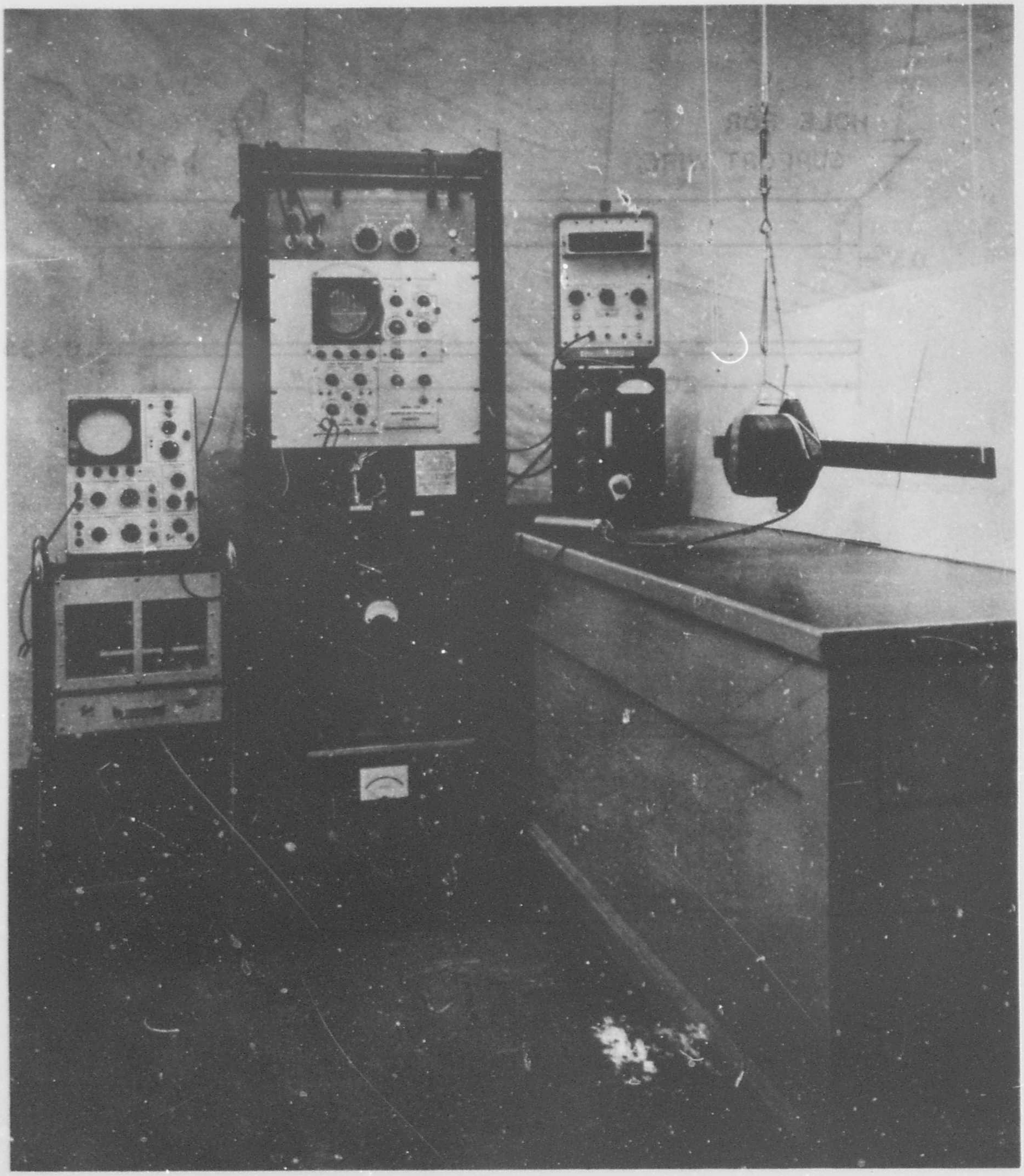


BEAM DIMENSIONS



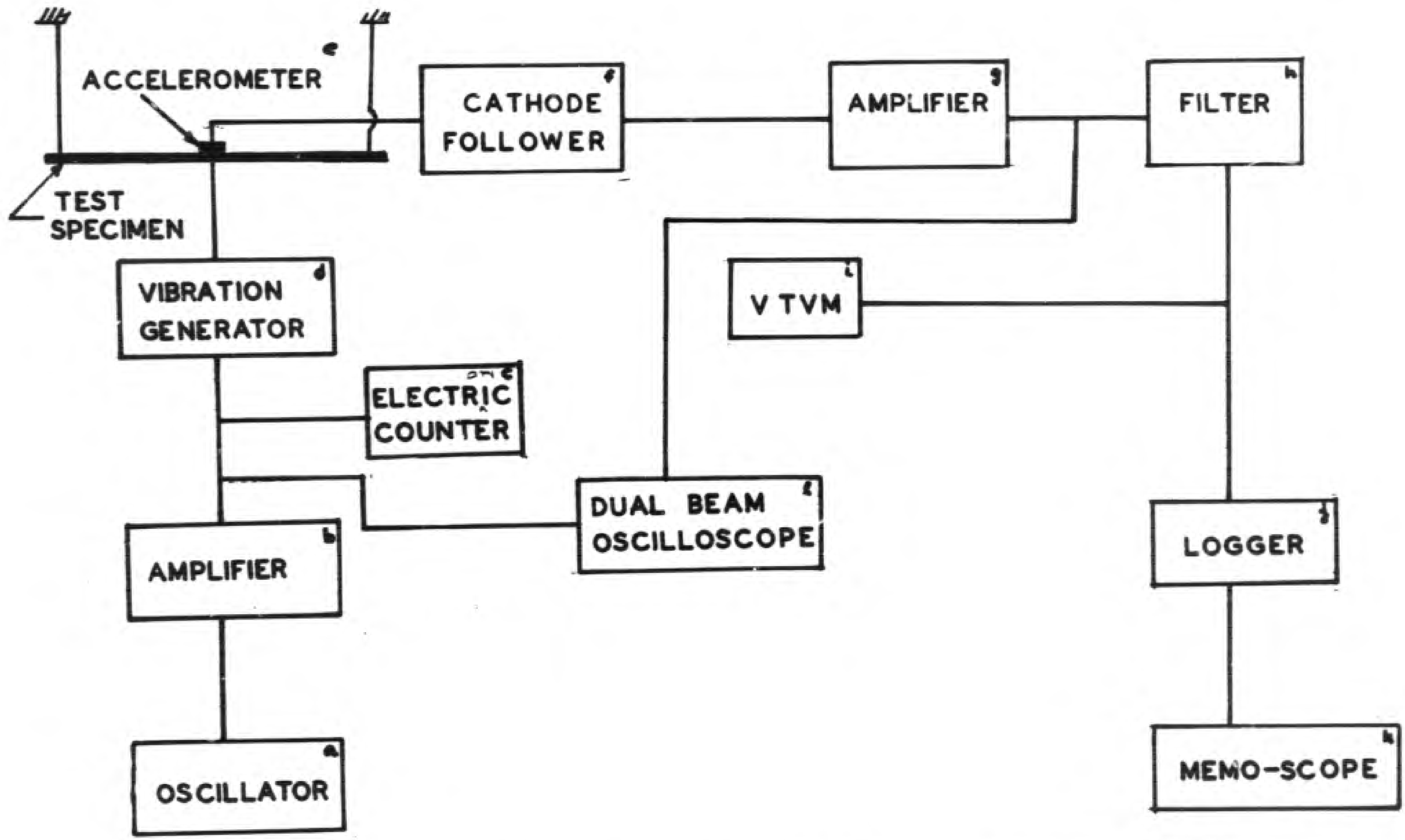
CROSS-SECTION, BEAMS 2,5

FIGURE 3



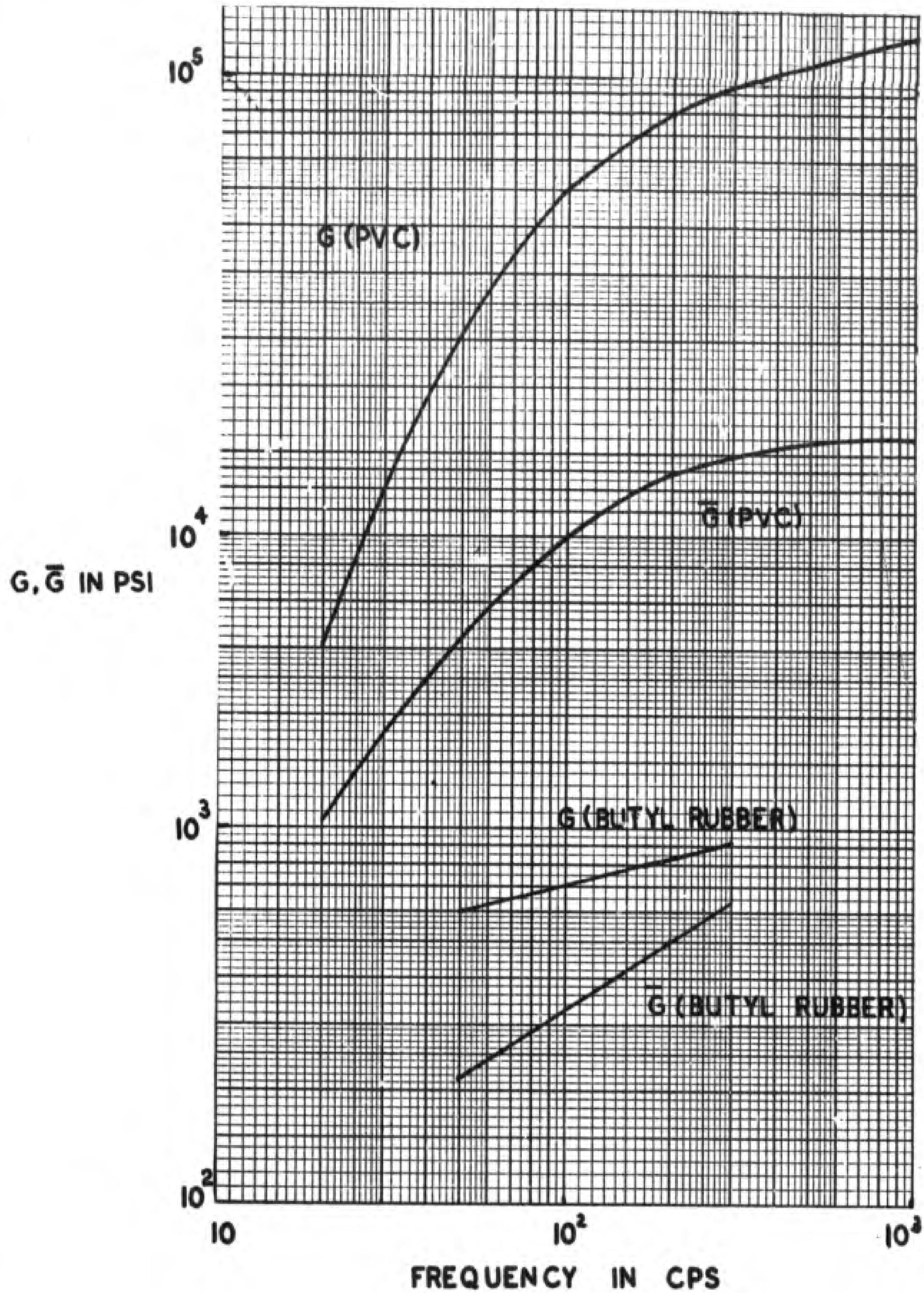
TEST SET-UP

FIGURE (4)



SCHEMATIC ARRANGEMENT OF INSTRUMENTATION

FIGURE (5)



SHEAR STORAGE MODULI (G) AND LOSS MODULI ($\bar{G}$)
VS. FREQUENCY

FIGURE (6)

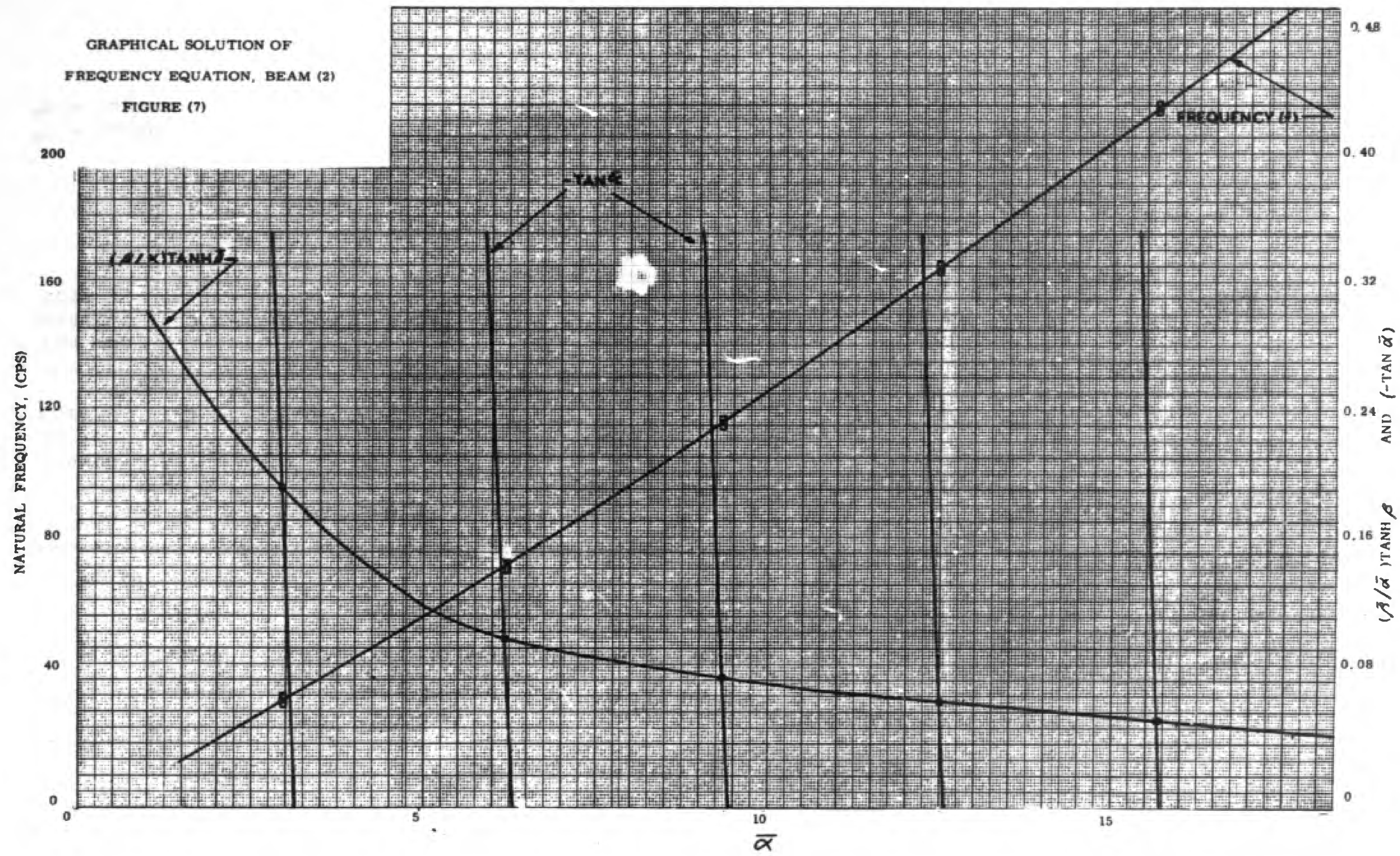

GRAPHICAL SOLUTION OF
FREQUENCY EQUATION, BEAM (2)
FIGURE (7)
NATURAL FREQUENCY, (CPS)
200
160
120
80
40
0
0
5
10
15
$\bar{\alpha}$
0.48
0.40
0.32
0.24
0.16
0.08
0
$(\bar{\beta}/\bar{\alpha})\tanh\bar{\beta}$ AND $(-\tan\bar{\alpha})$
$-\tan\bar{\alpha}$
$(\bar{\beta}/\bar{\alpha})\tanh\bar{\beta}$
FREQUENCY (f)

Table I

Sandwich Beam Test Specimen Details

	Specimen Geometry				Specimen Material Properties							
					Facing				Core			
	Specimen #	Facing thickness h	Core Depth c	Beam Length ℓ	Material	Density ρ_f	Storage Modulus E	Loss Modulus $\bar{E}$	Material	Density ρ_c	Storage Modulus $G(\omega)$	Loss Modulus $\bar{G}(\omega)$
Elastomeric Core	2	0.040"	0.375"	40"	Mild Steel	7.29×10^{-4}	30×10^6	30×10^3	Butyl Rubber	1.14×10^{-4}	C(3)* F(6)*	C(4) F(6)
	3	0.080"	0.250"	"	"	"	"	"	"	"	"	"
Thermoplastic Core	5	0.040"	0.375"	"	"	"	"	"	Polyvinyl Chloride	1.38×10^{-4} #sec^2/in^4	C(1)* F(6)*	C(2) F(6)
	6	0.080"	0.250"	"	"	"	"	"	"	"	"	"

*C = Curve
*F = Figure

Table II

Calculated and Observed Dynamic Characteristics

	Specimen No.	Mode No.	Undamped Natural Frequency f cps	Resonant Frequency $\bar{f}$ cps		Mode Function Coefficients			
						D_1	D_2	D_3	D_4
			Calculated	Calculated	Test	Calculated			
Elasto-meric Core	2	1	31.8			1	2.46	- 394	23.2
		3	75.4			1	- 4.27	- 646	-10.4
		5	121			1	5.83	- 866	6.56
		7	170			1	- 7.16	-1053	- 4.72
		9	220			1	8.44	-1235	3.67
	3	1	22.1			1	4.16	- 991	33.3
		3	50.1			1	- 7.48	-1664	-15.7
		5	79.4			1	10.5	-2272	10.2
		7	110			1	-13.2	-2797	- 7.53
		9	142			1	15.8	-3286	5.93
		11	176			1	-18.2	-1370	- 4.87
		13	210			1	20.5	-4160	4.12
Thermo-plastic Core	5	1	66		70	1	0.160	- 450	6.07
		3	330		353	1	- 0.0130	- 22.7	- 0.201
		5	760		811	1	0.00151	- 16.6	0.0137
	6	1	61		66	1	0.190	- 62.3	8.48
		3	290		337	1	- 0.0245	- 35.1	- 0.415
		5	631		766	1	0.00522	- 28.6	0.0498

Table III

Modal Inertia, Stiffness and Damping Characteristics

	Specimen No.	Mode No.	Modal Mass M_r	Modal Stiffness K_r	Modal Damping Coeff. H_r	Rate of Amplitude Decay d_r(1/sec)		Fraction, Critical Damping
			Calculated			Calculated	Test	Calculated
Elastomeric Core	2	1	610	2.44×10^{7}	8.81×10^{6}	41.9		0.210
		3	1671	3.75×10^{8}	1.58×10^{8}	120.0		0.253
		5	3016	1.74×10^{9}	8.86×10^{8}	217.4		0.286
		7	4470	5.10×10^{9}	2.92×10^{9}	337.0		0.316
		9	6157	1.18×10^{10}	7.46×10^{9}	478.0		0.346
	3	1	5.62×10^{3}	1.08×10^{8}	3.29×10^{7}	25.3		0.152
		3	1.60×10^{4}	1.59×10^{9}	5.78×10^{8}	57.3		0.182
		5	2.99×10^{4}	7.22×10^{9}	3.13×10^{9}	104.8		0.210
		7	4.54×10^{4}	2.17×10^{10}	1.05×10^{10}	165.9		0.240
		9	6.26×10^{4}	4.98×10^{10}	2.64×10^{10}	234.6		0.263
		11	8.07×10^{4}	9.87×10^{10}	5.69×10^{10}	319.5		0.289
		13	1.00×10^{5}		1.08×10^{11}	406.0		0.308
Thermoplastic Core	5	1	8.98	1.54×10^{6}	2.71×10^{8}	36.1	18.5	0.0871
		3	2.24	9.63×10^{6}	1.41×10^{6}	152.0	27.7	0.0735
		5	1.20	2.74×10^{7}	3.38×10^{6}	296.0	43.3	0.0613
	6	1	23.5	3.45×10^{6}	5.98×10^{5}	32.6	14.9	0.0851
		3	7.33	2.43×10^{7}	3.46×10^{6}	130.0	23.3	0.0712
		5	4.86	7.64×10^{7}	9.14×10^{6}	237.0	40.9	0.0597

Appendix

Calculation of Generalized Mass and Damping Coefficient

Determination of the damping capacity of a beam vibrating in a single mode, say the r^{th}, requires the evaluation of the generalized mass (M_r) and damping coefficient (H_r). The expressions for these quantities are given in equations (33 & 37) respectively. Equation (37) can be rewritten in summation form as follows. (The r subscript can be dropped hereafter).

$$H = \sum_{i=1}^{5} f^{(i)} g^{(i)} \tag{A-1}$$

where

$$\begin{aligned}
f^{(1)} &= 2\bar{E}h & g^{(1)} &= \int \phi''\phi\,dx \\
f^{(2)} &= -\frac{4\bar{G}c}{(c+h)^2} & g^{(2)} &= \int \phi^2\,dx \\
f^{(3)} &= \frac{2\bar{G}c}{c+h} & g^{(3)} &= \int \psi'\phi\,dx \\
f^{(4)} &= \frac{2\bar{G}c}{c+h} & g^{(4)} &= \int \phi'\psi\,dx \\
f^{(5)} &= -\bar{G}c & g^{(5)} &= \int \psi''\psi\,dx
\end{aligned} \tag{A-2}$$

where

$$\int = \int_0^{\ell/2}$$

Noting that ϕ and ψ are linear functions of D_1, D_2, D_3 and D_4, we can write the $g^{(i)}$ as

$$g^{(i)} = \sum_{j=1}^{4} \sum_{x=1}^{4} h_{jk}^{(i)} D_j D_k \tag{A-3}$$

Expanding the integrands of equations (A-2) using equations (22), and performing the integration, we arrive at the following expansions for the $h_{jk}^{(i)}$.

$$h_{11}^{(1)} = -\frac{\alpha^2}{2}\left[\frac{\ell}{2} - \frac{\sin\alpha\ell}{2\alpha}\right]$$

$$h_{22}^{(1)} = -\frac{\beta^2}{2}\left[\frac{\ell}{2} - \frac{\sin\beta\ell}{2\beta}\right]$$

$$h_{12}^{(1)} = h_{21}^{(1)} = -\frac{(\beta^2 - \alpha^2)}{2(\beta^2 + \alpha^2)}\left[\beta\sin\frac{\alpha\ell}{2}\cosh\frac{\beta\ell}{2} - \alpha\cos\frac{\alpha\ell}{2}\sinh\frac{\beta\ell}{2}\right]$$

$$h_{11}^{(2)} = \frac{1}{2}\left[\frac{\ell}{2} - \frac{\sin\alpha\ell}{2\alpha}\right] \tag{A-4}$$

$$h_{22}^{(2)} = -\frac{1}{2}\left[\frac{\ell}{2} - \frac{\sinh\beta\ell}{2\beta}\right]$$

$$h_{12}^{(2)} = h_{21}^{(2)} = \frac{2}{2(\alpha^2 + \beta^2)}\left[\beta\sin\frac{\alpha\ell}{2}\cosh\frac{\beta\ell}{2} - \alpha\cos\frac{\alpha\ell}{2}\sinh\frac{\beta\ell}{2}\right]$$

$$h_{13}^{(3)} = h_{31}^{(3)} = -\frac{\alpha}{2}\left[\frac{\ell}{4} - \frac{\sin\alpha\ell}{4\alpha}\right]$$

$$h_{24}^{(3)} = h_{42}^{(3)} = -\frac{\beta}{2}\left[\frac{\ell}{4} - \frac{\sinh\beta\ell}{4\beta}\right]$$

$$h^{(3)}_{14} = h^{(3)}_{41} = \frac{\beta}{2(\alpha^2 + \beta^2)} \left[\beta \sin\frac{\alpha l}{2} \cosh\frac{\beta l}{2} - \alpha\cos\frac{\alpha l}{2} \sinh\frac{\beta l}{2}\right]$$

$$h^{(3)}_{23} = h^{(3)}_{32} = -\frac{\alpha}{2(\alpha^2 + \beta^2)} \left[\beta \sin\frac{\alpha l}{2} \cosh\frac{\beta l}{2} - \alpha\cos\frac{\alpha l}{2} \sinh\frac{\beta l}{2}\right]$$

$$h^{(4)}_{13} = h^{(4)}_{31} = \frac{\alpha}{2}\left[\frac{l}{4} + \frac{\sin\alpha l}{4\alpha}\right]$$

$$h^{(4)}_{24} = h^{(4)}_{42} = \frac{\beta}{2}\left[\frac{l}{4} + \frac{\sinh\beta l}{4\beta}\right]$$

$$h^{(4)}_{23} = h^{(4)}_{32} = \frac{\beta}{2(\alpha^2 + \beta^2)} \left[\beta \cos\frac{\alpha l}{2} \sinh\frac{\beta l}{2} + \alpha\sin\frac{\alpha l}{2} \cosh\frac{\beta l}{2}\right]$$

$$h^{(4)}_{14} = h^{(4)}_{41} = \frac{\alpha}{2(\alpha^2 + \beta^2)} \left[\beta \cos\frac{\alpha l}{2} \sinh\frac{\beta l}{2} + \alpha\sin\frac{\alpha l}{2} \cosh\frac{\beta l}{2}\right]$$

$$h^{(5)}_{33} = -\frac{\alpha^2}{2}\left[\frac{l}{2} + \frac{\sin\alpha l}{2\alpha}\right]$$

$$h^{(5)}_{44} = \frac{\beta^2}{2}\left[\frac{l}{2} + \frac{\sinh\beta l}{2\beta}\right]$$

$$h^{(5)}_{34} = h^{(5)}_{43} = \frac{\beta^2 - \alpha^2}{2(\beta^2 + \alpha^2)} \left[\beta \cos\frac{\alpha l}{2} \sinh\frac{\beta l}{2} + \alpha\sin\frac{\alpha l}{2} \cosh\frac{\beta l}{2}\right]$$

From equations (A-1 & A-3) the following expression is written for H:

$$H = \sum_{i=1}^{5} \sum_{j=1}^{4} \sum_{k=1}^{4} f^{(i)} h_{jk}^{(i)} D_j D_k \tag{A-5}$$

The generalized mass can be written in a similar manner.

$$M = m \sum_{j=1}^{4} \sum_{k=1}^{4} P_{jk} D_j D_k \tag{A-6}$$

where

$$P_{33} = \frac{1}{2}\left[\frac{\ell}{2} + \frac{\sin\alpha\ell}{2\alpha}\right]$$

$$P_{34} = P_{43} = \frac{1}{\alpha^2 + \beta^2}\left[\beta\cos\frac{\alpha\ell}{2}\sinh\frac{\beta\ell}{2} + \alpha\sin\frac{\alpha\ell}{2}\cosh\frac{\beta\ell}{2}\right] \tag{A-7}$$

$$P_{44} = \frac{1}{2}\left[\frac{\ell}{2} + \frac{\sinh\beta\ell}{2\beta}\right]$$

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