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MATHEMATICS IN NORTHERN AND SOUTHERN IRELAND

BY

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This report is based on a visit made to the cities of Belfast, Dublin, and Galway during the first week of June 1965. The following institutions were visited:

(a) Queen's University, Belfast

(b) University of Dublin

(c) National University of Ireland, Dublin Branch

(d) Dublin Institute for Advanced Studies

(e) National University of Ireland, Galway Branch

As will be brought out more fully below, the state of mathematical research and instruction (and undoubtedly this comment has some validity for many other disciplines as well) is much happier in Northern Ireland (Ulster) than in the independent Republic of Ireland (Eire). The reasons for this marked difference are evidently rooted in political, religious, and economic problems that have plagued the island for many centuries and, although the writer's primary objective is to describe Ireland's current mathematical activities, it seems quite essential to discuss the social problems insofar as they appear to affect the academic institutions.

The entire island has an area of some 30,000 square miles and 4.5 million people. Ulster, often referred to as "The Six Counties" in the Republican press, contains one-sixth of the area and one-third of the population, but it enjoys a far greater degree of industrialization, particularly ship-building and aircraft construction, than Eire, "The Twenty-six Counties." The presence in the Belfast area of a strong Scottish minority, with its tradition of intellectual and technical ability, has certainly played an important role in building a strong university and an economy far more viable than that of Eire. However, economic conditions in Ulster are not entirely satisfactory; this is due largely to the decline in shipbuilding, the area's leading heavy industry.

Most of the remainder of this report will be divided into two main portions, the first consisting of commentary on the impressions obtained by visiting the various institutions listed above, the second of a description of some of the mathematical research being conducted at these institutions.

General Discussion

(a) Queen's University, Belfast

This University was established in 1840, the "Queen" of the title being Victoria, and is an integral part of the British system of higher education, although there are some special administrative features connected with the fact that Ulster maintains its own ministry of education. The student body numbers about 4,000, and an increase to perhaps 7,000 by 1970 is anticipated. Building is going on at a hectic pace, but land appears to be scarce. This factor has played an important role in a recent government decision to establish a new university in the much smaller town of Coleraine, about thirty miles away. This decision has aroused much opposition at Queen's, on the grounds that Ulster's population and economy cannot adequately support two universities and that one well-supported institution is preferable to two which are not-so-well-supported. This observer, who tends to favor a single well-supported school, had the impression that the government's decision is perhaps not irrevocable.

There is a rather complete spectrum of faculties, and some of them have members of considerable distinction. Mathematics is divided into three departments: Pure Mathematics (Prof. S. Verblunsky, Chairman), Applied Mathematics (Prof. D.R. Bates, FRS, Chairman). and Engineering Mathematics (Dr. A.P. Roberts, Chairman). While there is controversy in many universities, both American and European, as to whether mathematics should be dispersed over several departments or consolidated into a single one, it would appear that at Belfast the current arrangement is quite effective and also satisfactory to the parties concerned.

The Department of Applied Mathematics is undoubtedly the strongest of the three. ("Mathematical Physics" would be a more accurate title, since practically all the research deals with mathematical problems of quantum physics, quantum chemistry, and other topics which are usually lumped together under the suggested heading, rather than such topics as fluid dynamics and elasticity which are ordinarily suggested by the title actually employed.) This supremacy seems to be due very largely to Bates and to his second-in-command, Prof. A.P. Dalgarno, who are both extremely capable and hard-working scholars with the ability to attract promising young people. I met a number of near-PhD's and recent PhD's, in some cases from other British universities, who were spending several years of very intense effort at Belfast and apparently enjoying it.

The visit to Pure Mathematics was rather brief and unsatisfying. Verblunsky, the only professor in the department, was away at the time, and I met only three less senior members: Dr. R. Cooper, who has been at Queen's for three decades and apparently quite inactive for most of that time, Dr. R. McC. Wilson, who recently took his degree under Verblunsky, and Dr. R. McFadden, also a very young man. Wilson gave the impression of being heavily dependent for guidance on Verblunsky, while McFadden had just accepted an appointment at the new University of Kent, apparently because he expects to find a more stimulating environment there. These three people, who constitute half of the department, left me with the impression that, in marked contrast to Applied Mathematics, there is little enthusiasm within their group, although Verblunsky himself, a man of fairly substantial accomplishments, is still active.

The Department of Engineering Mathematics is housed in part of an attractive new ten-story building, Belfast's "skyscraper," which was opened in May 1965. This Department provides the mathematical instruction for all students in the various branches of engineering, and it has acquired a number of junior staff members who are working on research problems which should lead soon to the first doctorates conferred by this Department. The members of the staff are on the youngish side; and while there is probably no one possessing at present the stature of Bates, Dalgarno, or Verblunsky, there seems to be, both on the part of the chairman and his colleagues, a determination to build up an active program of research activities. Within a few years there should be some fairly definite indication as to whether there will be some measure of success.

(b) University of Dublin

This is the oldest university on the island, having been established in 1591 under Protestant English auspices. It was intended to be a second Cambridge, with its system of colleges and fellows. This objective was emphasized by giving the first college of the new institution the name Trinity. Now, almost four centuries later, Trinity remains the sole college of the University, but the theoretical and often quite confusing distinction between the College and the University, and the corresponding distinction between fellows and non-fellows, apparently corresponding quite closely to the "ins" and the "outs," are still maintained.

The Mathematical Institute consists (1964-65) of eight members, including two professors, G.A. Dirac in pure mathematics (particularly interested in graph theory) and

J.S.R. Chisholm in applied mathematics (particularly quantum field theory). The general atmosphere appeared to me to be a rather unhappy one. While salaries appear to be somewhat below those in British universities, perhaps three-quarters of the latter, this does not seem to be a significant cause of discontent. Much more serious is the failure of the administration to provide adequate financial support for book purchases, travel to meetings, and visiting lecturers. I was told that the Institute's total annual allowance for new books, departmental stationery, postage, visitors, etc., is £100 (!). This figure apparently does not include the purchase of journals, and the book situation is helped somewhat by the fact that the Trinity library has retained its traditional status as one of the six British Copyright institutions, despite the political separation of Eire from the UK. Another mitigating factor is that the theoretical physicists (Chisholm, in particular) have available the collection of the Dublin Institute for Advanced Studies, which, while not completely adequate, is the only reasonably satisfactory mathematical library in Eire. Nevertheless, these small plus factors do not outweigh the minuses, even if the picture that was drawn for me may be somewhat exaggerated. To go into a bit more detail, the University's total holdings, while impressive in numbers, and also qualitatively in some fields, are poorly housed in cramped quarters; books are stacked according to size rather than subject matter, and reading room is virtually non-existent. The basement stacks, which include much of the mathematical collection, were not electrified until last year. A new library building is now under construction, but it will evidently be too small, space and serviceability having been sacrificed for elegance of facade and design.

I also heard the complaint that the fellows of Trinity dominate the politics of the University and that the pursuit of a leisurely academic existence receives preference over the pursuit of new knowledge. Even taking everything with the appropriate number of grains of salt, I could not help feeling depressed over the whole state of affairs. A few final remarks may be added, namely that neither Dirac nor Chisholm appears deeply attached to the University, and that one of the younger staff members indicated that his fondness for Dublin, rather than satisfaction with conditions at the University, keeps him from seeking to leave.

(c) National University of Ireland, Dublin Branch

The National University was founded in 1909, although the separation of Eire from the United Kingdom was still some

decades in the future. It consists of three "University Colleges," located at Dublin, Cork, and Galway, each being a university in itself with enrollments of about 6000, 2000, and 1500, respectively, and a small "Recognized College" in the town of Maynooth. Despite the officially secular nature of the National University, a conflict exists in a very real sense in Irish higher education, with the National University being heavily under the influence of the Catholic hierarchy while the University of Dublin represents, at least in the view of its opponents, the former Protestant English oppressors. A few items may be listed which throw some light on this conflict.

(1) Observant Catholics must receive a bishop's dispensation to attend the University of Dublin. In fact, this dispensation is apparently granted upon request, and about one-third of the students are Catholics, but the practice is maintained as a means of warning the faithful who attend the University to remain aware of the moral pitfalls which they may encounter.

(2) A large number of ordained priests hold positions on the National University's faculty; while one may accept that no favoritism is shown to them in the competition for appointments, it does appear that a very strong effort is made to place academically trained priests on the faculty. To quote a clerical faculty member whom I met: "The National University is a secular institution, but, of course, nothing would be taught that runs counter to Catholic moral principles." The wording may not be exact, but the content is.

(3) The "Recognized College" at Maynooth appears to exist solely to provide the academic portion of the training of candidates for the priesthood.

(4) An additional complication is provided by the small but effective group of "linguistic nationalists." Although everyone in Ireland understands and speaks English, and practically all lectures at the National University are given in that language, the use of Gaelic is encouraged by granting a bonus on any examination written in that language. A board, established by the Ministry of Education, is engaged in the arduous task of developing a Gaelic terminology for the sciences. While one may sympathize with the ardor of the nationalists, one must question whether such a poor country can afford to indulge this ardor.

(5) Somewhat related to (4) is the insistence on spreading the nation's meager resources among the three

major branches of the National University. The national government also provides most of the budget of the University of Dublin. While the claim is made that many promising young people from the southern and western sections of the country would be unable to go to Dublin to study, one also must ask whether the cost of duplicating facilities is not greater than that of providing a single strong and comprehensive center of higher education. The fact that Gaelic is more popular at the two smaller branches than in Dublin is undoubtedly an important factor. In any case, when I raised this question, both in Dublin and in Galway, I was told, in virtually the same words, that there is no point to discussing such a change because it is a political impossibility.

My visit to the Dublin branch of the National University was rather brief, and occurred near the end of the examination period, when faculty members were under pressure to complete their grading duties as promptly as possible. I did not meet the only professor in the department, P.G. Gormley (there is also a Department of Mathematical Physics, with its own professor), but I enjoyed a fairly long discussion with R. Ingram, S.J., who has been on the faculty for some years and holds a PhD from Johns Hopkins. The discussion dealt mostly with the difficulties of education and research in Ireland, rather than with any specific mathematical topics.

The Dublin branch is at present severely crowded in its location near the heart of the city, but an extensive building program is under way in the suburban district of Belfield; a new campus for the science faculties is well on the way to completion and is already partially occupied. I was not able to visit this campus, which should certainly represent a vast improvement in teaching and research facilities, both for the faculties to be located there and for those faculties which will now have more room available in the older location.

In contrast to this encouraging development, I gained the impression, from the few mathematicians whom I met, that conditions are very difficult -- teaching assignments are heavy and library facilities are limited. This is reflected in a statement that was made to me to the effect that, while the University is authorized to confer the PhD in mathematics, the faculty tends to encourage promising young men to go to England or the United States -- with the hope that a place can then be found for them in Ireland upon the completion of their studies.

(d) Dublin Institute for Advanced Studies

This Institute has a most remarkable history. Briefly, in 1938 the Irish prime minister, Eamonn de Valera, who had as a young man studied some mathematics and had contemplated a career in that field before deciding on politics, was introduced in Geneva to Erwin Schrödinger, who had been wandering from place to place in Europe since he had left Berlin in 1935. Evidently de Valera saw an opportunity to "acquire" Schrödinger and build a center of theoretical physics around him. He promptly introduced a bill into the Irish parliament to establish an Institute for Advanced Studies. With great political acumen, he suggested the establishment of two schools within the Institute -- one devoted to Celtic studies and the other to theoretical physics; in connection with the latter school he freely invoked the name of William Rowan Hamilton. His bill was approved in 1940, and the two schools of the Institute were opened, late that year, in an inter-connected pair of 18th-century Georgian residences which were named "Hamilton House."

Schrödinger, who had come to Dublin late in 1939, assumed the title of Senior Professor of the newly established Institute, and he remained until 1956. An active group of senior and junior theoretical physicists gathered around him. Outstanding among these people was Werner Heitler, who spent the years 1941-49 at the Institute; during the second half of this period he served as Director of the School of Theoretical Physics, a position which Schrödinger had relinquished, although he retained his title of Senior Professor.

The distinguished Hungarian cosmic ray physicist L. Janossy, who had spent the years 1939-1947 at Bristol, accepted an invitation to come to the Institute as head of a third school, devoted to studies in cosmic rays. He remained for only three years, returning to Hungary in 1950, but the School of Cosmic Ray Studies continues its activities. It is located separately from the other two Schools, and I did not have an opportunity to visit it.

Thus, the Institute now consists of three schools, and I think that their existence reflects considerable credit upon Ireland. However, it appears that the Institute's future is somewhat dubious. On the one hand, plans are under way to provide a suitable new home for the two original Schools, which share their rather uncomfortable quarters with one of the ministries of the national government. On the other hand, there appears to be a strong feeling that the

Institute constitutes a luxury which the nation can ill afford. (I believe, but am not certain on this point, that the annual budget for all three schools runs to about £250,000 /$700,000/.) Evidently the nation's great esteem for the aged de Valera, who now holds the figurehead position of President, has enabled him to protect the Institute from its opponents; whether the Institute can survive without his protection remains to be seen.

Returning to the School of Theoretical Physics, its permanent staff currently consists of two Senior Professors, J.L. Synge and C. Lanczos; the former serves as Director of the School. About twenty young people ("Scholars") spend periods of a year or two at the School; some of them already have doctorates and others work on their dissertations. The School does not confer degrees, but evidently a dissertation approved by one of the senior members of the School is unhesitatingly accepted by the student's university. Both Synge, who spent some years at the University of Toronto, Ohio State University, and Carnegie Institute of Technology, and Lanczos, who served as Einstein's assistant in Berlin during the 1920's, are primarily interested in rather specialized mathematical problems of relativity theory, which seem (to me at least) to be out of the mainstream of significant research. On the other hand, these two men appear to keep the younger people busy and enthusiastic, and they conduct seminars and colloquia which serve to make life more pleasant than it would otherwise be for the theoretical physicists at the National University and at Trinity.

(e) National University of Ireland, Galway Branch

Galway, a town of about 25,000 population, is the "metropolis" of a beautiful but rather thinly populated and poorly developed section of west-central Ireland. The Galway campus of the National University has a remarkably broad class of faculties, including a splendid medical school which has the advantage of maintaing close contact with a new and excellently equipped hospital, located immediately adjacent to the University, which was established by the national government to serve the needs of the entire area.

The Mathematics Department is badly understaffed. The only professor, Sean Tobin, who is a competent algebraist, has been unable to make an appointment to a position that has been authorized for some time. This has compelled him to shoulder a very heavy teaching load, and his research has suffered accordingly. The remainder of the department consists of a number of very young men, several of whom are working on dissertations under the supervision of advisers

at other schools. Thus, while Galway offers satisfactory instruction in mathematics, its significance as a center of research is evidently rather minor, and it is doubtful whether any improvement is likely to occur.

Research Activities

As stated in the introductory remarks, this section is devoted to a brief description of some of the research being conducted at the various institutions which I visited. Needless to say, no attempt has been made to list all topics, or even to give a detailed discussion of those few topics which have been selected.

It may be remarked that each of the academic institutions in Ireland has a computer center. Belfast has a DEUCE, provided and maintained with ONR support, while Trinity College and the three branches of the National University each have an IBM 1620. Belfast's computer group conducts a program of instruction at the graduate level and appears to keep very busy in serving the entire university, especially Bates' group. It is hoped to replace the DEUCE soon with a more modern machine. The computing groups in Eire do not appear to be engaged in more than small-scale operations, and I was not informed of any plans for expansion of these facilities.

(a) Queen's University, Belfast

Department of Pure Mathematics

Verblunsky's published work in the last few years has centered around "Cauchy exponential series," a generalization of Fourier series. If $Q(z)$ denotes the function $e^z/(e^z-1)$ and $f(t)$ denotes any function (Lebesgue) integrable on $[0,1]$, then it is easily seen that the series

$$\sum c_n \exp(z_n x), \quad c_n = \operatorname{res}_{z=z_n} \left\{ Q(z) \int_0^1 f(t) \exp(-zt)\,dt \right\}$$

coincides with the Fourier series of $f(t)$. Here the quantities z_n denote the poles of $Q(z)$, indexed in a suitable manner with the set of integers. More generally, if we take for $Q(z)$ the function $e^z/(e^z-k)$, where k is any non-zero constant, the above series now becomes k^x times the Fourier series of $k^{-t}f(t)$. The above series may be constructed with any meromorphic function $Q(z)$ possessing infinitely many poles; if any of the poles are non-simple, a slight modification of the coefficients c_n is employed. Verblunsky and his pupil Wilson have investigated the behavior of these series for various choices of the function $Q(z)$. Verblunsky has devoted

two papers [Rend. Circ. Mat. Palermo 10 (1961) and Proc. London Math. Soc. 12 (1962)] to the case $Q(z)=ze^z/(ze^z+c)$, c being any non-zero constant. A typical result is that if f(t) is of bounded variation on [0,1], then the Cauchy exponential series (CES) is uniformly equiconvergent with the Fourier series (FS) of f(t) on any compact set interior to the open interval (0,1). (That is, the differences of the corresponding partial sums of the CES and the FS converge to zero, uniformly on compact subsets.) Another result is that, if the FS converges in the C1 sense at a given point x in the open interval (0,1), then a slightly modified version of the CES also converges in the C1 sense at x, to the same limit as the FS. A more intricate relationship between CES and FS, involving Cesaro summation of order k, is also established. Wilson [Proc. London Math. Soc. 15 (1965)] extends Verblunsky's results to the function $Q(z) = e^z/\{e^z-r(z)\}$, where r(z) is a rational function; he demonstrates that, under suitable hypotheses, Cesaro summability of order m of the FS of f(t) at a point x of the open interval (0,1) implies the same type of summability (to the same limit) of the CES; here m denotes the difference of the degrees of the numerator and denominator of r(z).

D.B. McAllister, one of the young members of the department, presents an interesting and rather detailed study of a special class of "multilattice groups" in Proc. Camb. Phil. Soc. 61 (1965). This concept was introduced by M. Benado in 1954. A multilattice is a partially ordered set possessing the following property and its dual (i.e., with $\leq$ replaced by $\geq$): if a, b, and x are elements satisfying $a \leq x$, $b \leq x$, then there exists an element y, satisfying $a \leq y \leq x$, $b \leq y \leq x$, such that there is no element u other than y satisfying $a \leq u \leq y$, $b \leq u \leq y$. (Briefly, if a pair of elements a, b possess an upper bound x, they possess a minimal upper bound y which is "under" x; in contrast to a lattice, it is not assumed that x and y have an upper bound in common.) A multilattice group is, then, a partially ordered group which is a multilattice under the prescribed partial ordering.

Department of Applied Mathematics

A vast and steady stream of publications pours out of this department, much of it bearing the names of Bates and Dalgarno. Frequently one or the other name appears as joint author with a junior member of the department.

Many papers in the Proceedings of the Physical Society present the results of computations concerned with collision, capture, and dissociation processes. These papers demonstrate

an excellent grasp of the mathematical principles of quantum mechanics and an ability to develop effective computational techniques. Some idea of the nature of the problems under consideration may be given by citing, almost at random, the titles of a few papers out of several dozen from this department that have appeared during the past year in the aforementioned journal:

Bates and Kingston, Collisional-radiative recombination at low pressures and deviation (33, 43-47 (1964)); Bates and Williams, Low energy collisions between hydrogen atoms and protons (83, 425-433 (1964)); Bell and Stewart, The fine structure of the ^2P term of the lithium isoelectronic sequence (83, 1039-1043 (1964)); Chen and Dalgarno, Error estimates for atomic and molecular quantities (85, 399-402 (1965)); Bates and Holt, Dissociation of hydrogen molecular ions by fast protons (85, 691-702 (1965)); Khare and Moiseiwitsch, The angular distribution of electrons elastically scattered by helium atoms and by hydrogen molecules (85, 821-839 (1965)).

Among Dalgarno's many fields of interest we may mention the study of ionospheric and stratospheric phenomena. Among his many publications along these lines we may cite a paper on corpuscular radiation in the upper atmosphere /Annales de Geophysique 20, 65-74 (1964)/, two papers on photoionization of atomic oxygen /Journal of the Atmospheric Sciences 21, 463-474 (1964)/, and Planetary and Space Science 12, 235-246 (1964)/.

While the mathematical level of the work of this department is often vigorous rather than rigorous, I was indeed deeply impressed with the caliber of the staff. Certainly this is the strongest group of mathematicians on the island, and one of the strongest in Britain. It is a source of great satisfaction to know that ONR is providing some of the support for this group!

Department of Engineering Mathematics

The principal research interests of this department lie in the fields of optimal control theory, stability of fluid flows, heat flow, and vibrations.

A.P. Roberts has published some half-dozen papers, several of them essentially expository in nature, during the past few years, most of them in Transactions of the Society of Instrument Technology (Sept. 1959, Sept. 1961, June 1963,

March 1965). He appears to possess a good command of the techniques and ideas due primarily to Bellman and Pontryagin, and he gets quite tangible results on problems of optimal regulation in the presence of noise, self-optimization, etc. It would be difficult to present even highly condensed versions of these papers here, but they do impress me as very competent work, well worth careful study.

A.P. Gallagher and A. McD. Mercer have written a series of three papers on small disturbances in plane Couette flow /Journal of Fluid Mechanics 13 (1962) and 18 (1964); Proceedings' of the Royal Society, A 286 (1965)/. In the first two papers they obtain an explicit solution of a linearized version of the Navier-Stokes equation and they obtained strong, but not decisive, evidence that Couette flow is stable at all Reynolds numbers. In the third paper they consider the effect of heating the lower plate; in particular they obtain the interesting conclusion that the Reynolds number at which instability sets in is independent of the presence of shear.

It may be added that Mercer has just transferred from Roberts' department to the "pure" department. In recent volumes of the Quarterly Journal of Mathematics he has published a number of very competent papers on integral transform pairs and a generalized Poisson summation formula. He appears to possess a very good analytical technique.

N.T. Dunwoody has published two papers dealing with boundary valve problems for elliptic regions (Quarterly Journal of Mechanics and Applied Mathematics 13, August 1960; Journal of Applied Mechanics 84E, 1962). In these papers he seeks to by-pass separation of variables and the use of Mathieu functions through an approximation device which appears to permit the use of multiple trigonometric series instead, with a considerable reduction in the amount of numerical work needed to obtain reasonable numerical results. While the method appears to work well in a few specific problems, no satisfactory analysis is presented. More recently he has published a few papers on viscous fluid flows (Journal of Aerospace Science 29, 1962; Journal of Fluid Mechanics 20 1964). The latter paper deals with the stability of an electrically conducting viscous fluid flowing between parallel vertical planes, a magnetic field being applied at right angles to these planes. However, I have not studied this paper in any detail.

(b) University of Dublin

Dirac appears to confine himself very strictly to the theory of graphs; he is almost certainly one of the most active and competent workers in this field. A number of his recent papers [Wiss. Zeitschrift Univ. Halle 13 (1964); Jour. Reine u. Angew. Math. 214/5 (1964); Jour. London Math. Soc. 39 (1964)] deal with "chromatic" graphs. A graph is "k'-colorable" if its vertices can be partitioned into k' mutually disjoint classes (color groups) such that no two points in the same class are joined by an edge. The "chromatic number" k of a graph is the minimum possible value of k'. If the "valency" of a vertex is defined as the number of edges issuing from the vertex, and the "valency-variety" w of a graph as the number of distinct valencies of its vertices, while the order n is the number of vertices, then various results appear in the aforementioned papers concerned these three quantities. For example, Dirac shows that $k \leq n - [\frac{1}{2}w]$, and that equality can hold if $n \geq 2$ and $1 \leq w \leq n-1$.

Dirac does not appear to concern himself with possible applications of his results, to circuit analysis, for example. I think that this is regrettable; nevertheless, he impresses me as a very capable and useful worker in this field, which is still very incompletely developed.

B.H. Murdoch, who took his PhD at Princeton, is strongly interested in classical analysis and in probability theory. In Vol. 39 (1964) and Vol. 40 (1965) of Jour. London Math. Soc. he has published interesting papers on harmonic and preharmonic functions. In the first of these papers he proves the following striking theorem: If $p(x_1, x_2, \ldots, x_k)$ and $f(x_1, x_2, \ldots, x_k)$ are harmonic for all values of $x_1, x_2, \ldots, x_k$, if p is a homogeneous polynomial, and if p and f agree in sign (in the sense that $pf \geq 0$ everywhere), then p and f are constant multiples of each other. This generalizes a theorem of Loomis and Widder, which is restricted to two variables and the case $p \equiv x_1$ (or $p \equiv x_2$). In the two papers covered by the second reference he obtains a number of analogues of the aforementioned theorem for preharmonic functions; these are, roughly speaking, functions defined on discrete sets and possessing a mean-value property analogous to that of harmonic functions. Such functions arise in the solution of boundary-value problems by finite-difference methods. A paper entitled "Preharmonic functions on groups" should appear soon, as well as some papers on random walks and on uniformly distributed sequences.

I was unable to acquaint myself with the work of a young algebraic topologist, D.J. Simms, or with that of

Chisholm and the applied mathematicians who collaborate with him.

(c) National University, Dublin

The only member of the faculty with whose research I have become acquainted is D. Judge, of the Department of Mathematical Physics. He is primarily interested in rigorous development of the fundamental principles of quantum mechanics. A good indication of his interests is furnished by three brief notes in Physics Letters 5, 189 and 190 (1963); 13, 138 (1964). The first two items seek to remedy previous proofs, which the author claims are defective, of the uncertainty relation $\Delta L_z \Delta\phi \geq \hbar/2$; here L_x and ϕ denote, of course the z-component of angular momentum and the co-latitude, respectively. The third item sketches a proposed procedure for overcoming the difficulty involved in using "eigenfunctions," such as exp(ikx), which are not quadratically integrable. Judge proposed the embedding of the "usual" Hilbert space L_2, which is separable, in a non-separable extension, L_2', which contains, in some sense, eigenfunctions of the position and momentum operators, x and $-i\hbar\frac{\partial}{\partial x}$, respectively. A detailed exposition of these ideas has been submitted to the Journal of Mathematics and Mechanics.

(d) Dublin Institute for Advanced Studies

As stated earlier, the main effort of the Institute's mathematical staff, both permanent and transient, appears to be concerned with mathematical questions arising in relativity theory. Insofar as I can evaluate the publications that have come out of the Institute during the past few years, it would appear that nothing in the nature of a really major contribution has been made. Nevertheless, the results are, I believe, not without value, and it is to be hoped that the Institute will continue to receive the needed financial support.

Lanczos has been trying valiantly to eliminate the mathematical difficulties that beset general relativity theory as a result of the appearance of a hyperbolic metric -- i.e., a quadratic differential form in the four variables of space-time which assumes both positive and negative values. Lanczos proposes a positive definite metric $g_{ij}dx^i dx^j$ whose metrical coefficients are periodic in each of the four coordinates. The period is the same for each coordinate, and is to be extremely small, so that, in some sense, the metric appears to be almost Euclidean when looked at macroscopically, while, on the other hand, extremely large curvatures are introduced.

Lanczos claims to demonstrate the possibility of signal propagation in such a universe. His arguments impress me as being heuristic and highly speculative, far from convincing.

These ideas are presented in Journal of Mathematical Physics 4 (July 1963) and Physical Review 134 (April 1964). Other papers by Lanczos on relativity will be found in Reviews of Modern Physics 29 (July 1957) and 34 (July 1962).

It may be added that during a part of the American phase of his career (1931-1954) Lanczos established a solid reputation for his work in numerical analysis, and he has published several excellent texts in this field, as well as one on the variational principles of mechanics. Since coming to Ireland in 1954, however, he appears to be completely devoted to relativity theory.

The other Senior Professor, J.L. Synge, also is very much concerned with relativity theory, but apparently not so exclusively as Lanczos. During the past decade he has published a two-volume exposition of his approach to the subject, and a considerable number of papers. These papers, of which Proceedings of the Royal Socity, A. 280 (1964) appears to be typical, center about attempts to obtain particular solutions of the field equations which demonstrate desirable properties (behavior at infinity and at singular points). Again, I feel that these contributions, while of some interest, are not in the nature of major contributions. A recent (1964) series of lectures at the Institute, in which a detailed exposition was given A.Z. Petrov's classification of gravitational fields, was issued as No. 15 of "Communications of the Dublin Institute for Advanced Studies, Series A."

Conclusions

Northern Ireland's single university is an active and significant mathematical center. Its Department of Applied Mathematics, in particular, impresses me as being one of the most active and competent groups in the UK.

Eire, on the other hand, does not appear likely to develop any major centers of mathematical research, even though the Institute for Advanced Studies might, given favorable conditions, rise in significance. While the nation's resources are rather limited, it is my feeling that more could be accomplished, within the bounds imposed by these limited resources, if bitterness over injustices perpetrated in the past were sublimated in the form of an intense effort to improve the nation's political, economic, and educational structure.

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This report, which deals with the current state of mathematical activities in Northern Ireland (Ulster) and Southern Ireland (Eire), is based on visits which the author has made to most of Ireland's universities and to the Dublin Institute for Advanced Studies. In addition to brief discussions of some of the research problems currently under investigation, the report contains some commentary on the political and social problems which, in the author's opinion, have a considerable effect on education and scholarship in Eire.

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Ulster	Control Theory						
Eire	Cauchy exponential series						
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