63-1-3

AIR FORCE INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY
UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

WRIGHT–PATTERSON AIR FORCE BASE, OHIO

AF-WP-O-MAY 62 3,500

THE EFFECT OF FREQUENCY TRACKING, THE USE OF A PHASE-LOCK LOOP, AND PREDICTED TRACKING ON RECEIVER SENSITIVITY

GA/EE/62-2

EMIL G. RIEDEL JR
Captain, USAF

THESIS

Presented to the Faculty of the School of Engineering of

the Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

THE EFFECT OF FREQUENCY TRACKING, THE USE OF A PHASE-LOCK LOOP, AND PREDICTED TRACKING ON RECEIVER SENSITIVITY

GA/EE/62-2

EMIL G. RIEDEL JR
Captain, USAF

Graduate Astronautics

August 1962

Preface

This report is an evaluation of three special techniques used by radio receivers in the tracking of satellites. My formal training, both at the undergraduate level at the U.S. Naval Academy, and at the graduate level in the Astronautics program at AFIT, has been broad and lacking in specialization. I chose this thesis topic so that I might have the opportunity to study in detail at least one small portion of the field of Astronautics.

I wish to acknowledge the patient encouragement and helpful suggestions given to me by Professor T.L. Regulinski. I also wish to express my gratitude to my wife for her forbearance and help in typing this report. I should also like to acknowledge the begrudged quiet given by the four little men who make all work worthwhile.

Contents

List of Figures

List of Figures

List of Symbols

a	satellite acceleration (cm/sec^2)
AGC	Automatic Gain Control Circuit
B	bandwidth (cycles)
d	distance (Km)
f_d	Doppler frequency (cycles/sec)
f_o	Satellite transmitter frequency (cycles/sec)
F	receiver noise figure
G_t	gain of transmitter
G_r	gain of receiver
i	input condition to receiver
J	rate of change of acceleration (cm/sec^3)
K	Boltzmann's constant (1.38×10^{-23} joules/degree Kelvin)
$\mathcal{K}$	earth gravitational constant (3.987×10^{20} cm^2/sec^3)
P_r	required input signal power (watts)
P_R	power received over a line of sight transmission path (watts)
P_t	power transmitted by the satellite (watts)
o	output condition of receiver
$\frac{S}{N}$	signal to noise ratio (db)
T	noise temperature (degrees Kelvin)
V	satellite velocity (cm/sec)
V_r	satellite relative velocity (cm/sec)

List of Symbols

λ	wavelength of transmitted frequency (cm)
Θ_1	input signal (Phase-Lock Loop)
Θ_2	output signal (Phase-Lock Loop)

Abstract

The purpose of this study is to quantitatively compare frequency tracking, the phase-lock loop, and predicted tracking when used to improve the effectiveness of radio receivers in the tracking of earth satellites. The techniques are evaluated in terms of their effect on receiver sensitivity. Sensitivity is defined as the input signal power required to produce the output signal to noise ratio deemed necessary, by the system designer, for detection. The detection bandwidth for each technique is smaller than that of the conventional receiver. By using the smaller bandwidth much extraneous noise is removed and the output signal to noise ratio is improved. A lower value of input signal power is required to produce the output signal to noise ratio and the sensitivity is increased. The detection bandwidths for frequency tracking and the phase-lock loop are nearly the same. Because of the lower threshold of the phase-lock loop, the receiver sensitivity is greater by two orders of magnitude than that of frequency tracking. Prediction tracking is a form of correlation detection which makes more effective use of the reduced detection bandwidth. When used in conjunction with the other two techniques, the sensitivity is increased by another order of magnitude. The techniques, developed for tracking earth satellites,should prove of even greater value for tracking inter-planetary probes.

THE EFFECT OF FREQUENCY TRACKING, THE USE OF A PHASE-LOCK LOOP, AND PREDICTED TRACKING ON RECEIVER SENSITIVITY

I. Introduction

The purpose of this study is to quantitatively compare three techniques used to improve the effectiveness of radio receivers in the tracking of earth satellites. Since the power transmitted by a satellite is limited by weight considerations and the distances involved are comparatively large, the received power will be very small in magnitude. The most effective improvement any special technique could make would be to increase the sensitivity of the receiver. Therefore the techniques of frequency tracking, the use of a phase-lock loop and predicted tracking are evaluated in this study in terms of their effect on receiver sensitivity.

Background

The configuration considered in this study is that of a satellite, continuously transmitting an un-modulated, radio frequency signal to a fixed receiver on the earth. The relative motion of the satellite with respect to the receiver causes the received frequency to differ from the transmitted frequency by a varying value known as the Doppler frequency. The Doppler frequency for a particular orbit is unique and the accurate determination of this frequency is a primary method of tracking satellites.

The Doppler frequency for a transit from horizon to horizon will vary over a bandwidth of several kilocycles, while for a given increment of time the Doppler frequency may change only a few cycles. To accurately determine the orbit it is necessary to observe the entire pattern of the Doppler shift. The conventional receiver accomplishes this by being tuned to receive the entire possible shift. A detection bandwidth of several kilocycles is processed to extract an intelligence of a few cycles per second. The result is a signal buried in a sea of noise, requiring the signal strength to be much greater than the noise power of the entire Doppler shift bandwidth.

The problem is further complicated by the relatively small signal power transmitted by the satellite. Weight limitations placed upon the transmitter power supply limit the transmitted signal power to typical values of one watt or several watts at best. The power received over a line of sight transmission path is governed by the equation:

$$P_R = \frac{P_t\, G_t\, G_r \lambda^2}{16 \lambda^2 \; d^2} \qquad \text{(Ref 7:445)} \qquad (1.)$$

Assuming

$P_t = 1$ watt

$G_t = 1$

$G_r = 1$

$f_o = 1000$ Mcs and $\lambda = 0.3$ m

$d = 1000$ Km

For these conditions the power received would be 5.7×10^{-16} watts.

To clearly illustrate the problem this value is contrasted with the power required by a conventional receiver, using the entire Doppler shift bandwidth, to produce the output signal to noise ratio necessary for detection. It can be shown that the power required to produce a given output signal to noise ratio in a conventional receiver is governed by the equation:

$$P_r = F \frac{S_o}{N_o} K T B \qquad \text{(Ref 7:445)} \qquad (2.)$$

It can also be shown that the bandwidth for the entire Doppler shift is given by the equation:

$$B = \frac{2 V_r}{\lambda} \qquad \text{(Ref 6:90)} \qquad (3.)$$

For a distance of 1000 Km and a transmitting frequency of 1000 Mcs:

$V_r = 7.17 \times 10^5$ cm/sec

$B = 47.2$ Kcs

$\lambda = 0.3$ m

$F = 1$

$KT = 4 \times 10^{-21}$

$\frac{S_o}{N_o} = 26 \text{ db} = 400$

Under these assumptions the power required is 7.53×10^{-14} watts. Using a conventional receiver and the entire Doppler shift bandwidth the power required exceeds the power available by two orders of magnitude.

Assumptions

This study has been carried out utilizing the following assumptions. It is assumed that the motion of the satellite takes place in a plane defined at any instant by the satellite, the receiver, and the center of the earth. The term distance is assumed to mean the height of the satellite above the earth when over the receiver. The distances considered are 600, 800, 1000, and 1200 Km. These distances are arbitrary but have been chosen to be representative of the actual area of interest.

The observed velocity differs from the inertial velocity because of the earth's rotation. The observed velocity, denoted as V_r, is referred to as the relative velocity. The values of relative velocity for the distances considered were extracted from the Pickard and Burns Inc. report. (Ref 6:11) Golay states that the acceleration of a satellite is given by the equation:

$$a = \frac{\gamma}{d^2} \qquad \text{(Ref 3:191)} \qquad (4.)$$

and the rate of change of acceleration is given by the equation:

$$J = \frac{V\gamma}{d^3} \qquad \text{(Ref 3:191)} \qquad (5.)$$

The extracted values of velocity and the computed values of acceleration and rate of change of acceleration are shown in Table I.

Table I
Velocity, Acceleration, and Rate of Change of Acceleration

Distance (Km)	Vel. (cm/sec)	Acc. (cm/sec^3)	J (cm/sec^3)
600	7.58×10^5	1.11×10^5	1.38×10^3
800	7.37×10^5	6.24×10^4	5.76×10^2
1000	7.17×10^5	3.99×10^4	2.86×10^2
1200	6.96×10^5	2.77×10^4	1.60×10^2

All receiver noise factors are assumed to be unity. This is an optimistic assumption but this is a comparative study and the particular value of this constant will not affect the results. The temperature used for the calculation of noise power is 290° K. The transmitter frequencies considered are 100, 500, 1000, 1500, and 2000 Mcs. Again these are arbitrary, but representative of the actual span of frequencies used in satellite transmission.

Statement of the Problem

Two problems are proposed for solution in this study. The first is to develop a common basis for comparing the effect of the three techniques on receiver sensitivity. The second problem is to analyze each technique and to express the characteristics of the technique in terms relating to the common basis of comparison. The systems analyzed are; the second and third order systems of the frequency tracking technique, the second and third order systems of the phase-lock loop, and the effect of predicted tracking on all four systems.

II. Determining Receiver Sensitivity

Sensitivity is nominally defined as the input signal power required by a receiver to produce a standard output signal to noise ratio. In this study the standard output signal to noise ratio is that value deemed necessary, by the system designer, for detection.

The main purpose of frequency tracking and the phase-lock loop is to reduce the detection bandwidth. The techniques remove the small segment of bandwidth containing the intelligence while neglecting the broad bandwidth noise power. The output signal to noise ratio for the special techniques will be greater than for the conventional receiver which must pass the broad bandwidth noise power. The improvement in the signal to noise ratio caused by using the special technique is a function of the bandwidth reduction.

If one knows the required output signal to noise ratio, the detection bandwidth, and its effect on the signal to noise ratio, the required input signal to noise ratio can be determined. The noise power associated with the input signal to noise ratio is that contained in the input bandwidth, which is considered to be the entire Doppler shift. The noise power is given by the equation:

$$N = F\ K\ T\ B \qquad \text{(Ref 7:435)} \qquad (6.)$$

By multiplying the noise power by the required input signal to noise ratio one obtains the required input signal power. The required input signal power is a measure of the sensitivity.

The following example illustrates the procedure. At a distance of 600 km and using a transmitter frequency of 100 mcs the bandwidth for the entire Doppler shift is given by:

$$B = \frac{2V_r}{\lambda} = \frac{(2)\ (7.58 \times 10^5)}{(3 \times 10^2\ )} = 5.04 \times 10^3 \text{ cycles}$$

Under the assumption that the detection bandwidth is 25 cycles and the relationship between the bandwidths and the signal to noise ratios is:

$$\frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}} = \left(\frac{B_i}{B_o}\right)^{\frac{1}{2}} = \left(\frac{5.04 \times 10^3}{25}\right)^{\frac{1}{2}} = 14.2$$

Under the assumption that the required output signal to noise ratio is 26 db, which is the number 400, the required input signal to noise ratio is:

$$\frac{S_i}{N_i} = \frac{400}{14.2} = 28.1$$

The input noise power is given by:

$$N = F\ K\ T\ B$$

Under the assumptions that

$$F = 1$$

$$K = 1.39 \times 10^{-23} \text{ joules/degree K}$$

$$T = 290^{\circ}\ K$$

$$B = 5.04 \times 10^{3} \text{ cycles}$$

gives $N = 2.03 \times 10^{-17}$ watts.

Then

$$S_i = (28.1)\ (2.03 \times 10^{-17}) = 5.7 \times 10^{-16} \text{ watts}$$

In summary, the analysis of the techniques requires determination of the detection bandwidth, the required output signal to noise ratio and the relationship between the bandwidths and the signal to noise ratios.

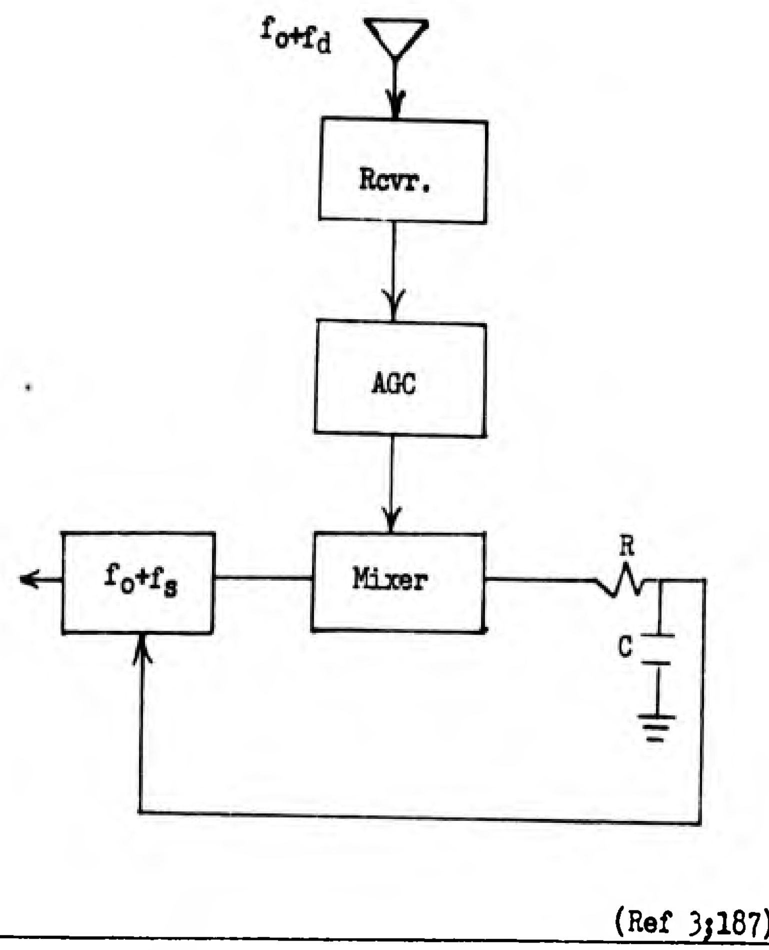
III. Frequency Tracking

The object of frequency tracking is to follow within narrow limits the incoming Doppler frequency. The basic procedure is to select a small element of bandwidth, for example 50 or 100 cycles, and maintain the center of this element aligned with the center of the intelligence bandwidth. The bandwidth element must be wide enough to include the entire change of the frequency per unit time and in normal practice will exceed this width several times to allow for noise or extraneous frequency modulation. The small bandwidth element becomes the detection bandwidth rather than the entire possible frequency change. Tracking can be done either manually or automatically.

There are many possible schemes that can be used for manual frequency tracking. Only one scheme will be discussed in this study and it is included only to convey the basic idea of the procedure. Manual tracking has two severe limitations, one of which is the inability of the human operator to precisely deal with such small quantities as parts of seconds or several cycles per second. The second limitation is that the signal power must exceed the noise power of the entire Doppler shift bandwidth in order for the signal to be detected.

In one manual scheme the Doppler audio frequency signal, obtained from the receiver detector output , is fed into one pair of deflection plates (x-axis) of an oscilloscope. The other plates (y-axis) are connected to an audio oscillator output. The oscillator is adjusted to give a 1:1 Lissajous pattern, nominally a circle. The audio oscillator also intensity modulates the oscilloscope beam (z-axis), so the circular pattern appears as a "half-moon". If the frequency of the audio oscillator is low the "half-moon" rotates in one direction. If the frequency is high it rotates in the opposite direction. The operator tries to keep the "half-moon" stationary by varying the output frequency of the audio frequency oscillator. The result is that the weak, varying amplitude Doppler signal is replaced by a strong, constant amplitude signal from the audio oscillator. As previously pointed out the accuracy of this system is limited to the human operator's ability to track the signal. To be competitive with other systems it must be accurate to one-fourth of a cycle and this is beyond human capability. Also the Lissajous figure is subject to fading and collapsing because of the weak Doppler signal.

Figure 1 shows a schematic representation of an elementary automatic frequency tracking circuit. This circuit represents the basic principles involved in any automatic frequency tracking system. A localoscillator generates a signal of frequency $f_o + f_s$ which closely approximates $f_o + f_d$. The servo circuit causes the frequency $f_o + f_s$ to follow $f_o + f_d$ as closely as the limitations of the circuit will allow. The useful output of the circuit is the constant amplitude signal $f_o + f_s$.



An Elementary Automatic Frequency Tracking Circuit

Figure 1

There are two disadvantages of this circuit. First the signal power must be greater than the noise power within the receiver bandwidth so that the signal controls the action of the AGC circuit. Secondly, at the optimum damping ratio the allowable frequency excursion is too small to accurately track the changing Doppler frequency. (Ref 3:188)

The elementary system can be modified and refined to overcome these disadvantages. A detailed description of one configuration presented by Golay is included in Appendix A. This system will accurately track the frequency to one-fourth of a cycle. The noise can exceed the signal by several orders of magnitude and the system can track a rapidly varying signal. The significant result is that the detection bandwidth for a second order system, as shown by Golay is given by the equation:

$$B = 1.275\left(\frac{a}{\lambda}\right)^{\frac{1}{2}} \tag{7.}$$

and the detection bandwidth required by a third order system is given by the equation:

$$B = 1.747\left(\frac{J}{\lambda}\right)^{\frac{1}{3}} \qquad \text{(Ref 3:187-191)} \tag{8.}$$

The bandwidths for the entire Doppler shift , for the distances and frequencies considered are shown in figure 2. The bandwidths for these conditions range from 4.64 kcs to 101.2 kcs. The bandwidths for the second and third order frequency tracking systems are shown in figures 3 and 4 respectively. The bandwidths for the second order system range from 12.5 cycles to 109 cycles, and for the third order system from 1.4 cycles to 7.8 cycles. The second order system provides a decrease of bandwidth by two orders of magnitude and the third order system reduces the bandwidth by three orders of magnitude.

It can be shown that the improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}} = \left(\frac{B_i}{B_o}\right)^{\frac{1}{2}} \qquad \text{(Ref 1:27)} \qquad (9.)$$

The improvement in the signal to noise ratios , for the second and third order systems are shown in figure 5. The second order system gives an improvement varying from 11.5 db to 16.2 db. The third order system gives an improvement varying from 16.2 db to 21.9 db.

Golay states that the output signal to noise ratio required by both the second and third order systems is 26 db. (Ref 3:192) All of the required information for computation of the required input signal power has now been stipulated. The results of the computations are shown graphically in figure 6. The input signal power for the second order system varies from 3.82×10^{-16} watts to 51.1×10^{-16} watts. The power required by the third order system varies from 1.25×10^{-16} watts to 14.2×10^{-16} watts. It should be noted that for the illustrative conditions in Section I, i.e. a distance of 1000 km and transmitting frequency of 1000 mcs that the power available exceeds the power required by the third order system but is less than that required by the second order system.

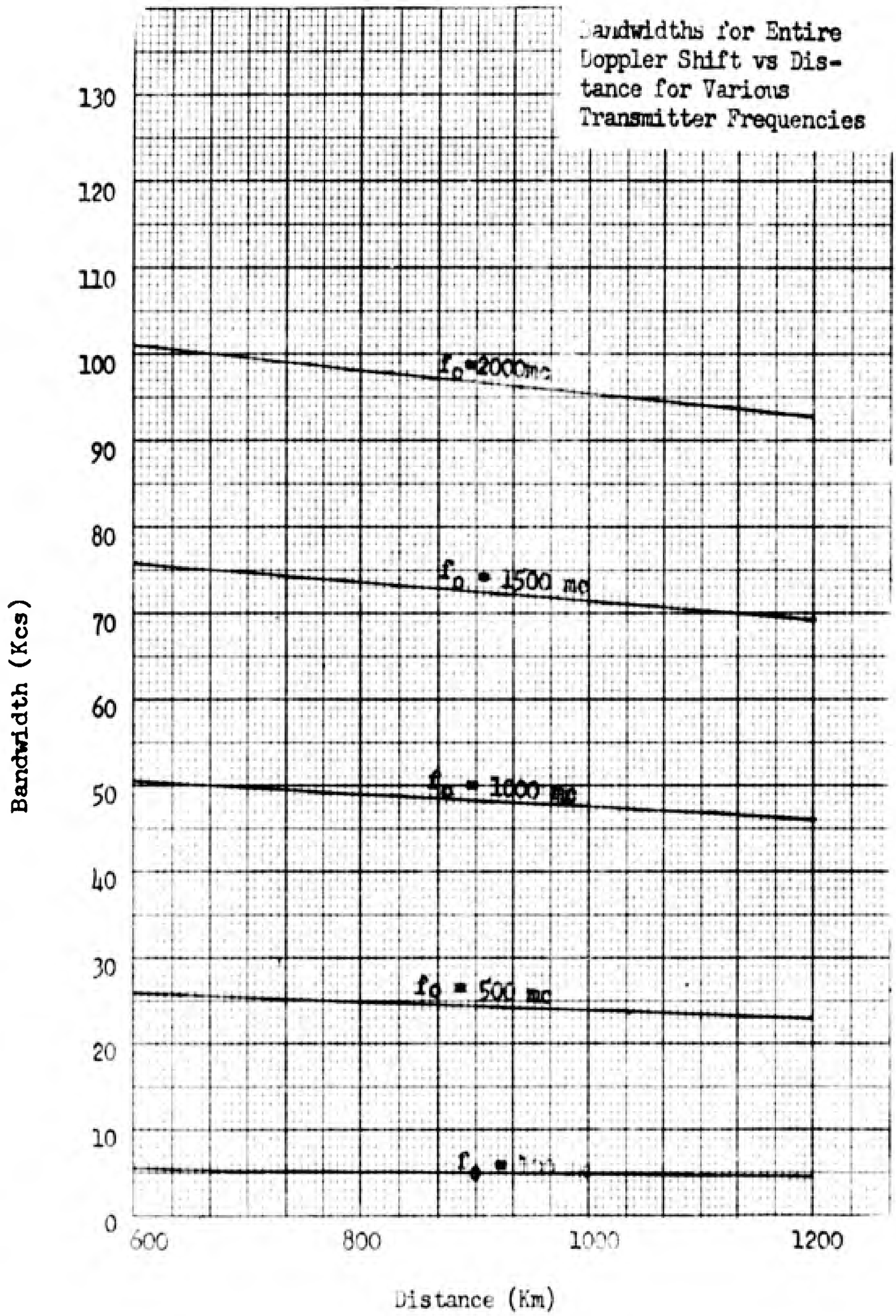

Bandwidths for Entire
Doppler Shift vs Dis-
tance for Various
Transmitter Frequencies
Bandwidth (Kcs)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
f_o=2000mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
600
800
1000
1200
Distance (Km)

Figure 2

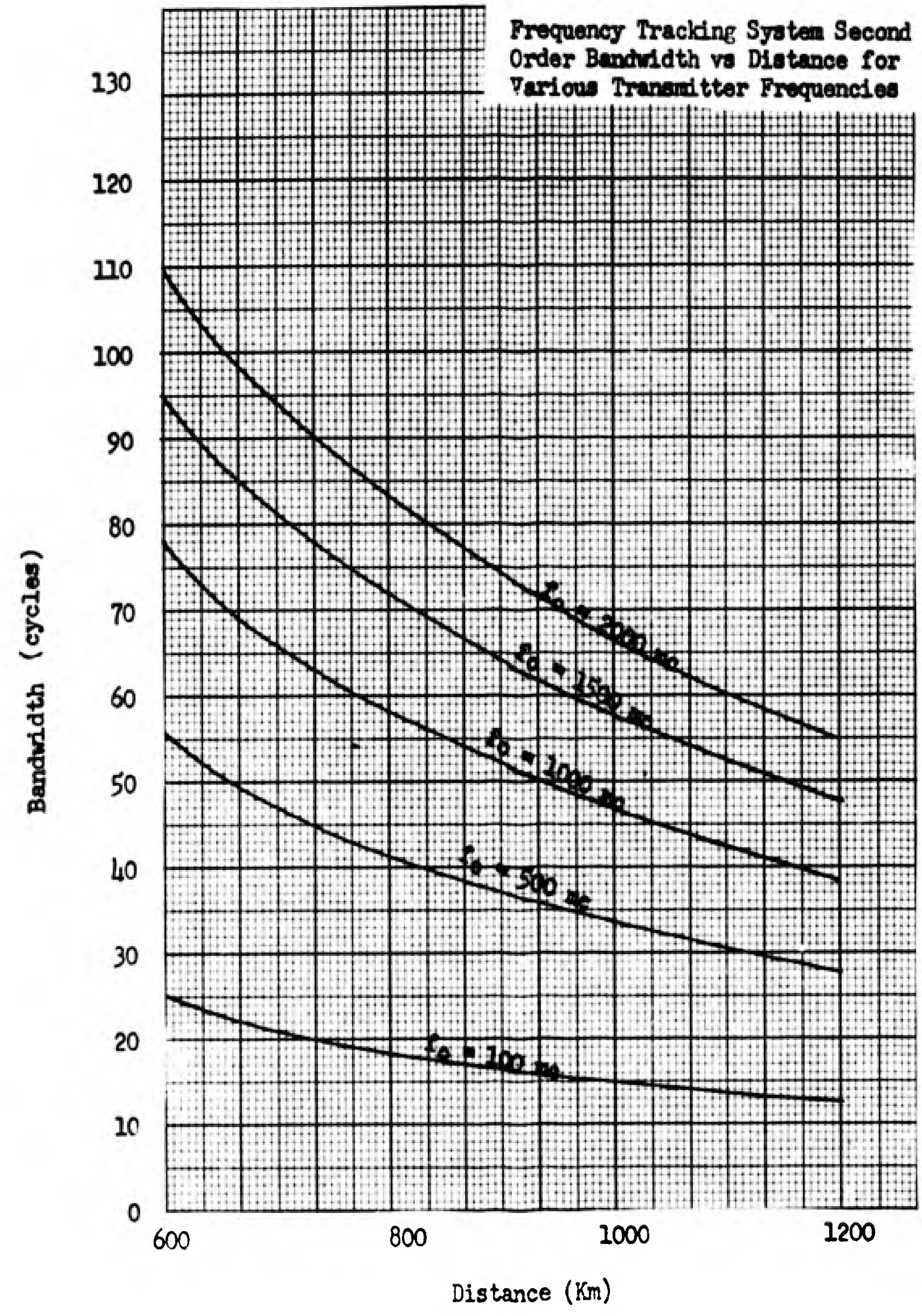

Frequency Tracking System Second Order Bandwidth vs Distance for Various Transmitter Frequencies
130
120
110
100
90
80
70
60
50
40
30
20
10
0
Bandwidth (cycles)
600
800
1000
1200
Distance (Km)
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc

Figure 3

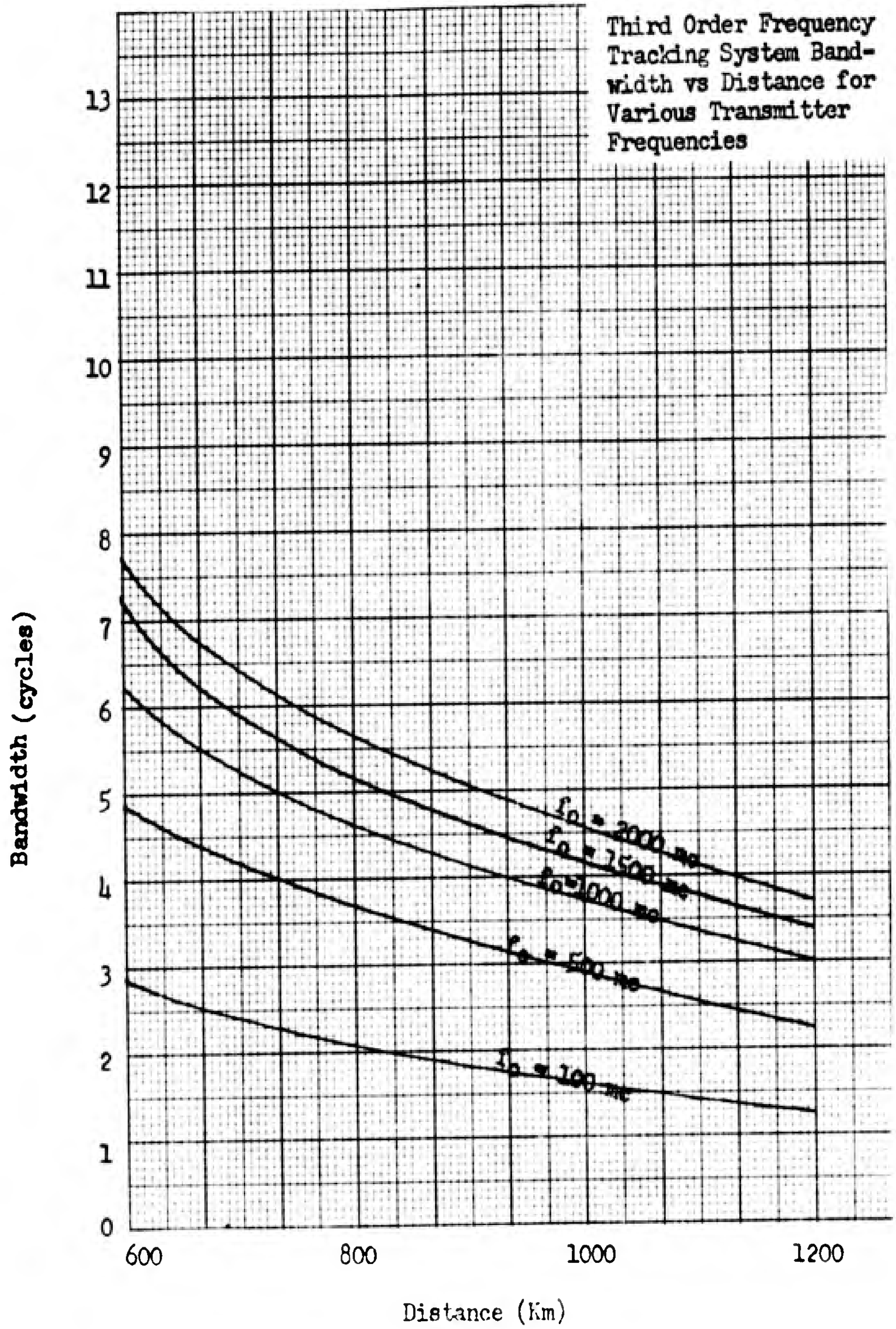
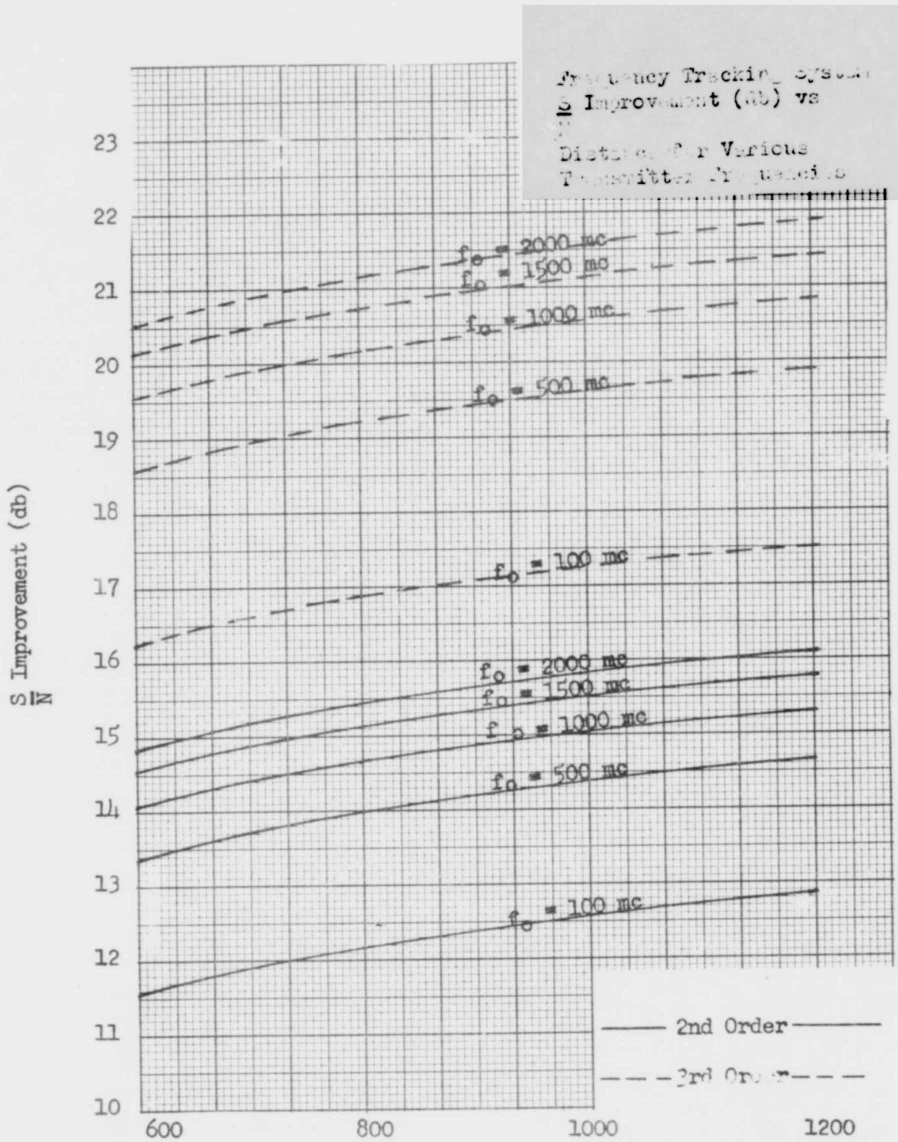

Third Order Frequency Tracking System Bandwidth vs Distance for Various Transmitter Frequencies
Bandwidth (cycles)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
600
800
1000
1200
Distance (Km)
f_o = 2000 mc
f_o = 1500 mc
f_o=1000 mc
f_o = 500 mc
f_o = 100 mc

Figure 4



vs
Various
23
22
21
20
19
18
17
16
15
14
13
12
11
10
S/N Improvement (db)
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
2nd Order
3rd Order
600
800
1000
1200
Distance (Km)

Figure 5

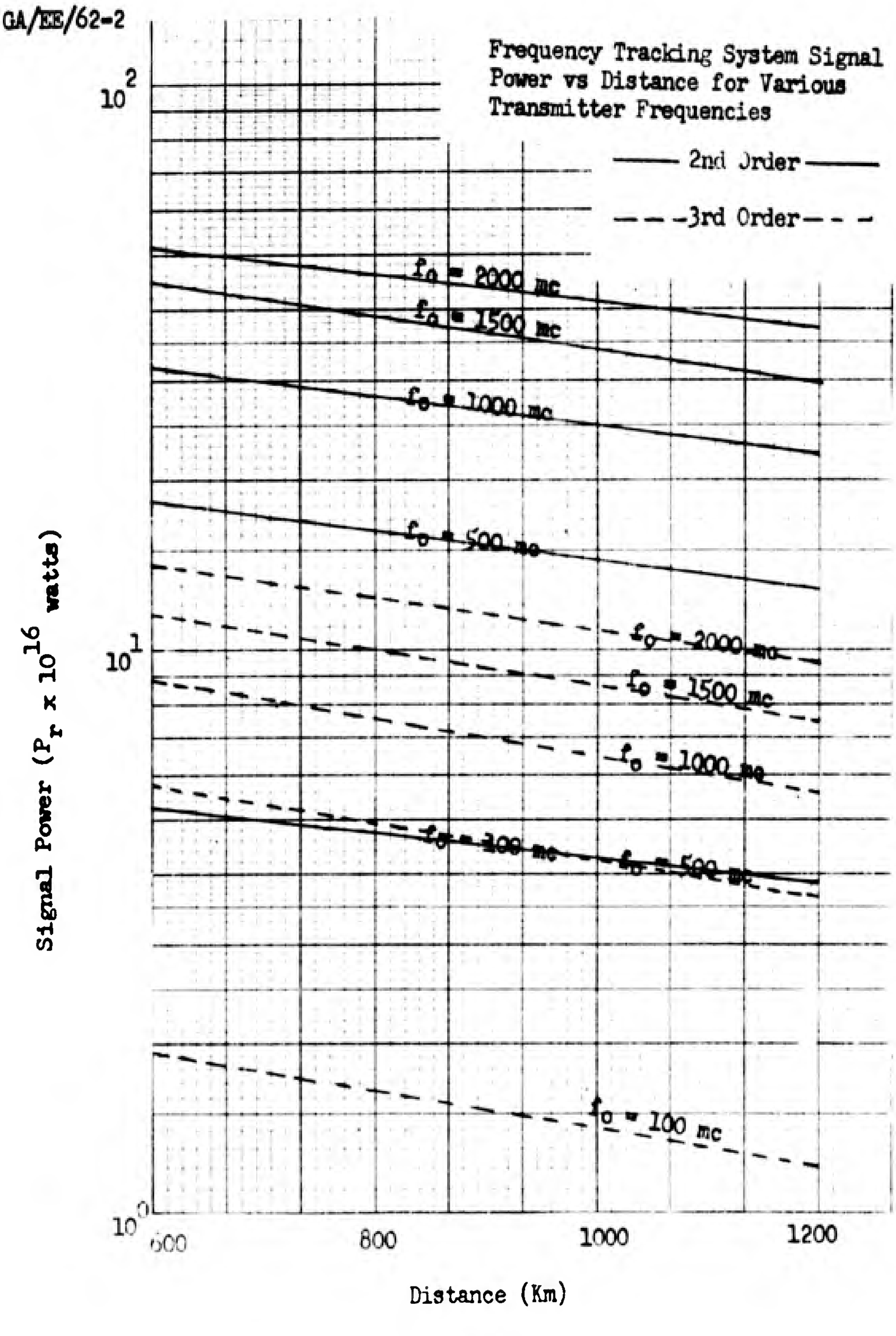
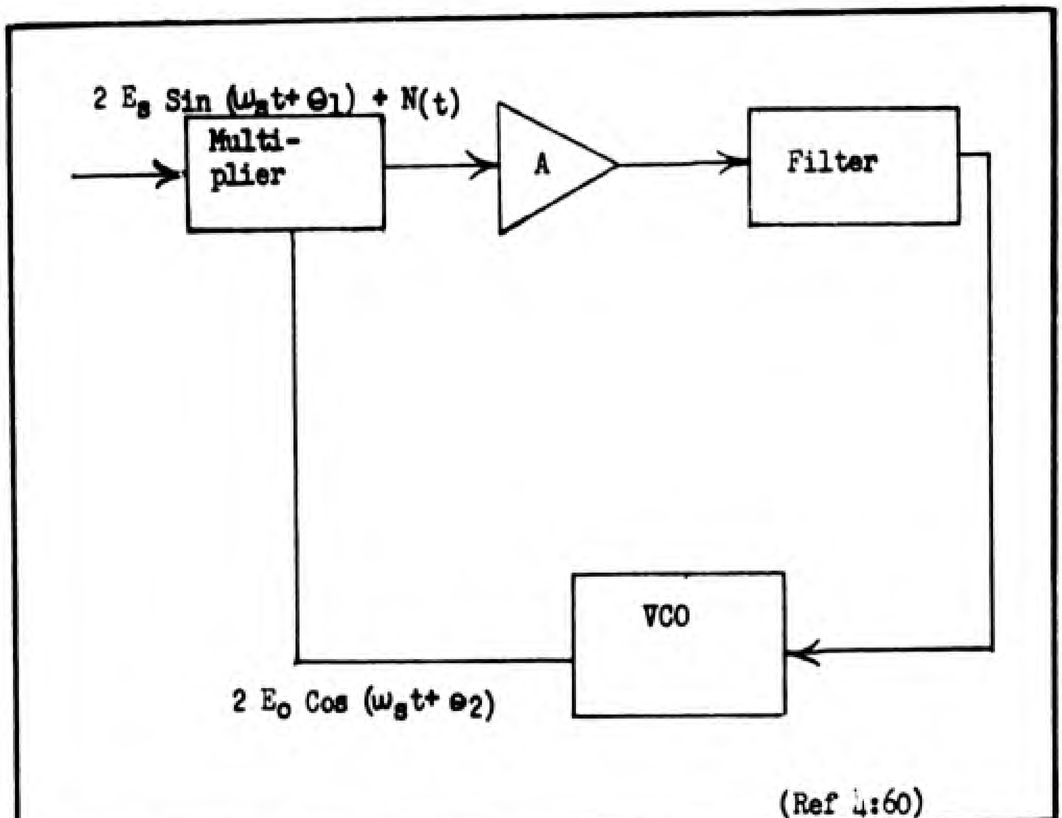

Frequency Tracking System Signal Power vs Distance for Various Transmitter Frequencies
2nd Order
3rd Order
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 100 mc
f_o = 500 mc
f_o = 100 mc
10^2
10^1
10^0
Signal Power ($P_r \times 10^{16}$ watts)
600
800
1000
1200
Distance (Km)

Figure 6

IV. The Phase-Lock Loop

A second technique for reducing the detection bandwidth and the required input signal power is by the use of a phase-lock loop. A schematic representation of the loop is shown in figure 7. The multiplier beats the signal input and the output of the voltage con-



A Typical Phase-Lock Loop

Figure 7

trolled oscillator together giving several resultant terms , including a low frequency output proportional to $(\Theta_1 - \Theta_2)$. The loop filter passes only the low frequency term , which is applied to the voltage controlled oscillator and which forces the output signal, Θ_2, to be equal to the input signal Θ_1. The bandwidth required by the phase-lock loop must be large enough to pass the difference between the input and output signals. Since this difference varies less than the signal frequency the bandwidth is reduced and the amount of noise passed into the loop is reduced. (Ref 8:66)

The signal proportional to $(\theta_1 - \theta_2)$ is known as the transient phase error and is necessary to develop the voltage required to keep the VCO at the same frequency as the signal. However the phase transient cannot exceed 90^o for at this point the sign of the open loop gain will change and the loop will become unstable. The value of θ_1 can fluctuate very rapidly because of random noise added to the signal, therefore it is necessary to limit the peak value of $(\theta_1 - \theta_2)$ to 90^o, and the mean value to less than 90^o. (Ref 1:27)

It is shown in Appendix C that when the maximum mean phase transient error, hereafter called the phase error, is limited to 30^o the bandwidth required by a second order system is given by the equation:

$$B = 1.760 \left(\frac{a}{\lambda}\right)^{\frac{1}{2}} \qquad (10.)$$

and thebandwidth for a third order system is given by the equation:

$$B = 1.170 \left(\frac{J}{\lambda}\right)^{\frac{1}{3}} \qquad (11.)$$

If the phase error is limited to 30^o the output signal to noise ratio necessary for maintaining lock is 3 db. (Ref 1:27)

As the phase error is increased the ability of the loop to follow a varying signal is increased and the required detection bandwidth is decreased. If the phase error is increased to 50° the detection bandwidth, as shown in Appendix B, for the second order system is given by the equation:

$$B = 1.367\left(\frac{a}{\lambda}\right)^{\frac{1}{2}} \tag{12.}$$

and for the third order system by the equation:

$$B = 0.968\left(\frac{J}{\lambda}\right)^{\frac{1}{3}} \tag{13.}$$

The detection bandwidths for the second order system for both the 30° and 50° phase errors are shown in figure 8. For the 30° phase error the bandwidths vary from 17 cycles to 150 cycles and for the 50° error from 13.2 cycles to 117.1 cycles. The detection bandwidths for the third order system for both the 30° and 50° phase error are shown in figure 9. For the 30° phase error the bandwidths vary from 0.946 cycles to 5.26 cycles and for the 50° phase error the bandwidths vary from 0.785 cycles to 4.36 cycles. The detection bandwidths are reduced, from the entire Doppler shift, by two orders of magnitude for the second order system and by three orders of magnitude for the third order system.

The improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}} = \left(\frac{B_i}{B_o}\right)^{\frac{1}{2}} \qquad \text{(Ref 1:27)} \qquad (9.)$$

The improvement in the signal to noise ratio for the 30° phase error is shown in figure 10 and for the 50° phase error in figure 11. The second order system shows an improvement of 10.8 db to 15.4 db for the 30° phase error and 11.4 db to 16.0 db for the 50° phase error. The third order system shows an improvement of 17.1 db to 22.8 db for the 30° phase error and 11.4 db to 23.2 db for the 50° phase error.

As the maximum phase error is increased, the filter bandwidth required to pass the difference signal is increased and the internal noise power is increased. As a result of this increase the threshold required to remain inlock is increased. The relationship between the maximum phase error and the system threshold is non-linear and is best determined by experiment. Weaver shows that if the phase error is increased from 30° to 50° the threshold, expressed in decibels, doubles. (Ref 7:64) Thus the output signal to noise ratio necessary for maintaining lock in the 50° phase error system is 6 db.

All of the information for computation of the required input signal power is stipulated. The results of the computations are shown graphically in figures 12 and 13. The second order system requires power ranging from 2.24×10^{-18} watts to 3.12×10^{-18} watts for the 30° phase error and from 39.1×10^{-18} watts to 54.2×10^{-18} watts for a phase error of 50°. The signal power required by the third order system varies from 0.529×10^{-18} watts to 5.82×10^{-18} watts for a 30° phase error and from 9.47×10^{-18} watts to 100×10^{-18} watts for a 50° phase error.

Both the second and third order systems, using both a 30° and 50° phase error, are capable of detecting the signals discussed in the illustrative example of section I. Also it is to be noted that even though the 50° phase error system requires a smaller detection bandwidth than the 30° phase error system , it is less sensitive because of the increased threshold.

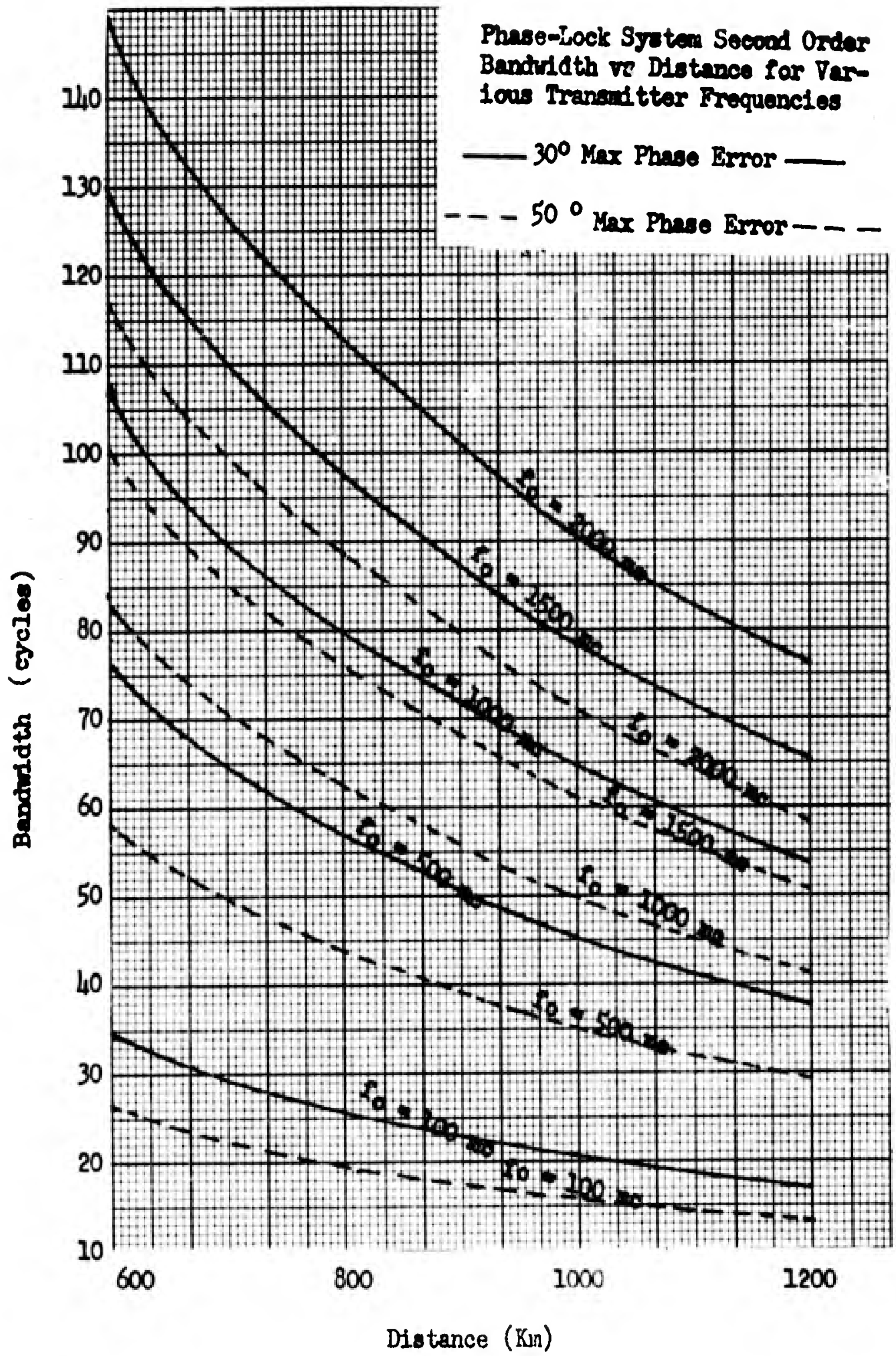

Phase-Lock System Second Order Bandwidth vs Distance for Various Transmitter Frequencies
30° Max Phase Error
50 ° Max Phase Error
Bandwidth (cycles)
Distance (Km)
600
800
1000
1200
10
20
30
40
50
60
70
80
90
100
110
120
130
140
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc

Figure 8

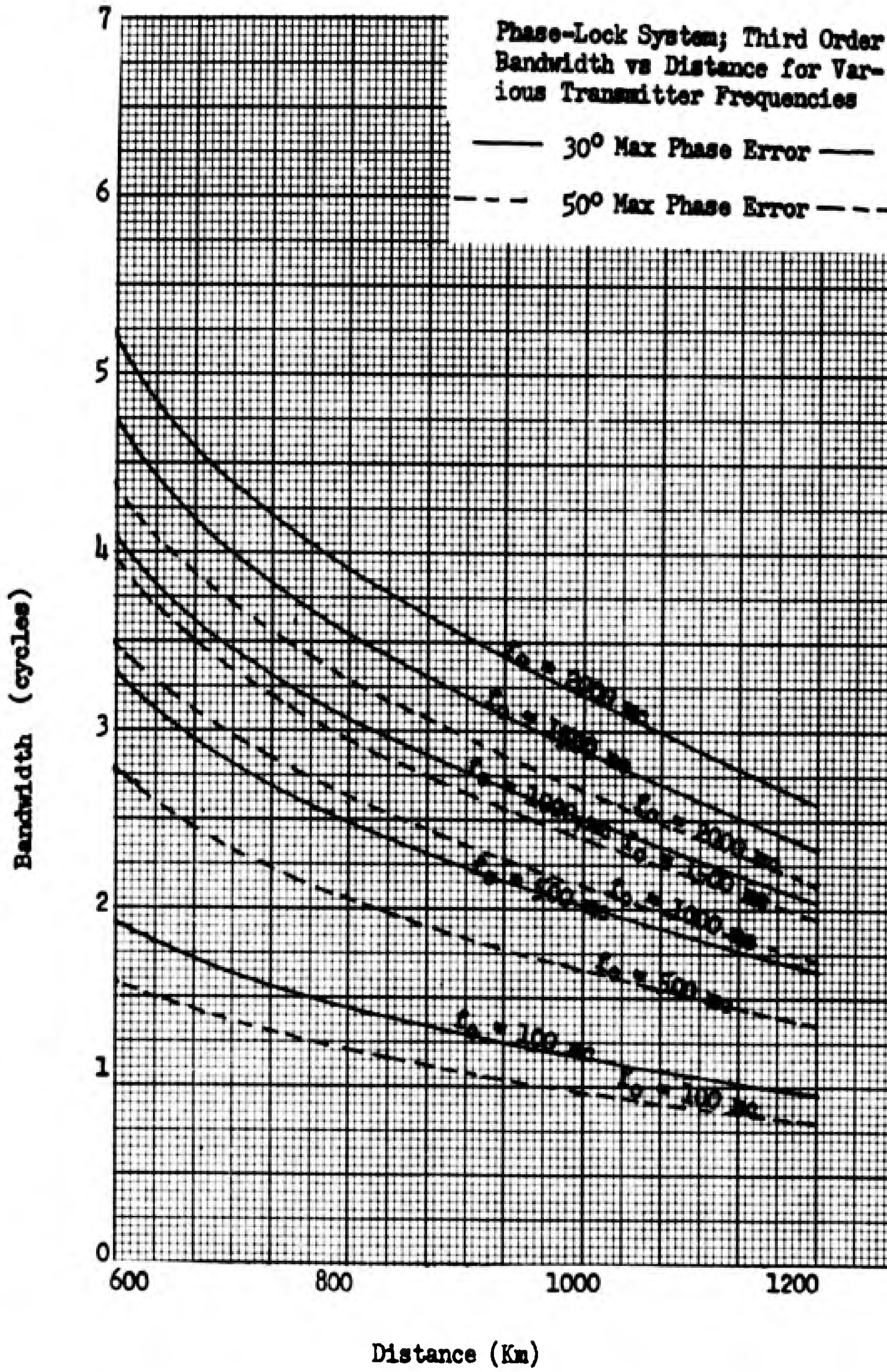

Phase-Lock System; Third Order Bandwidth vs Distance for Various Transmitter Frequencies
30° Max Phase Error
50° Max Phase Error
Bandwidth (cycles)
7
6
5
4
3
2
1
0
600
800
1000
1200
Distance (Km)

Figure 9

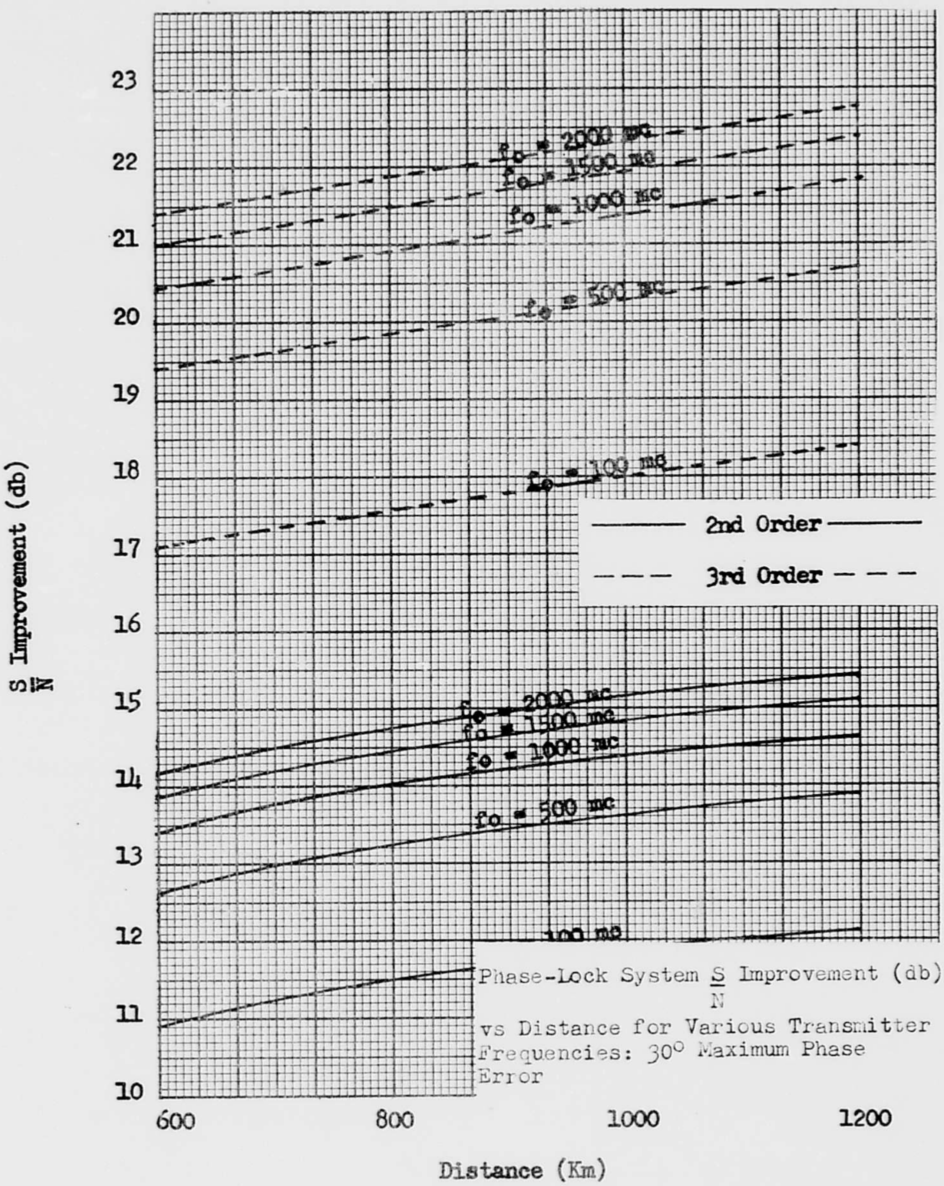
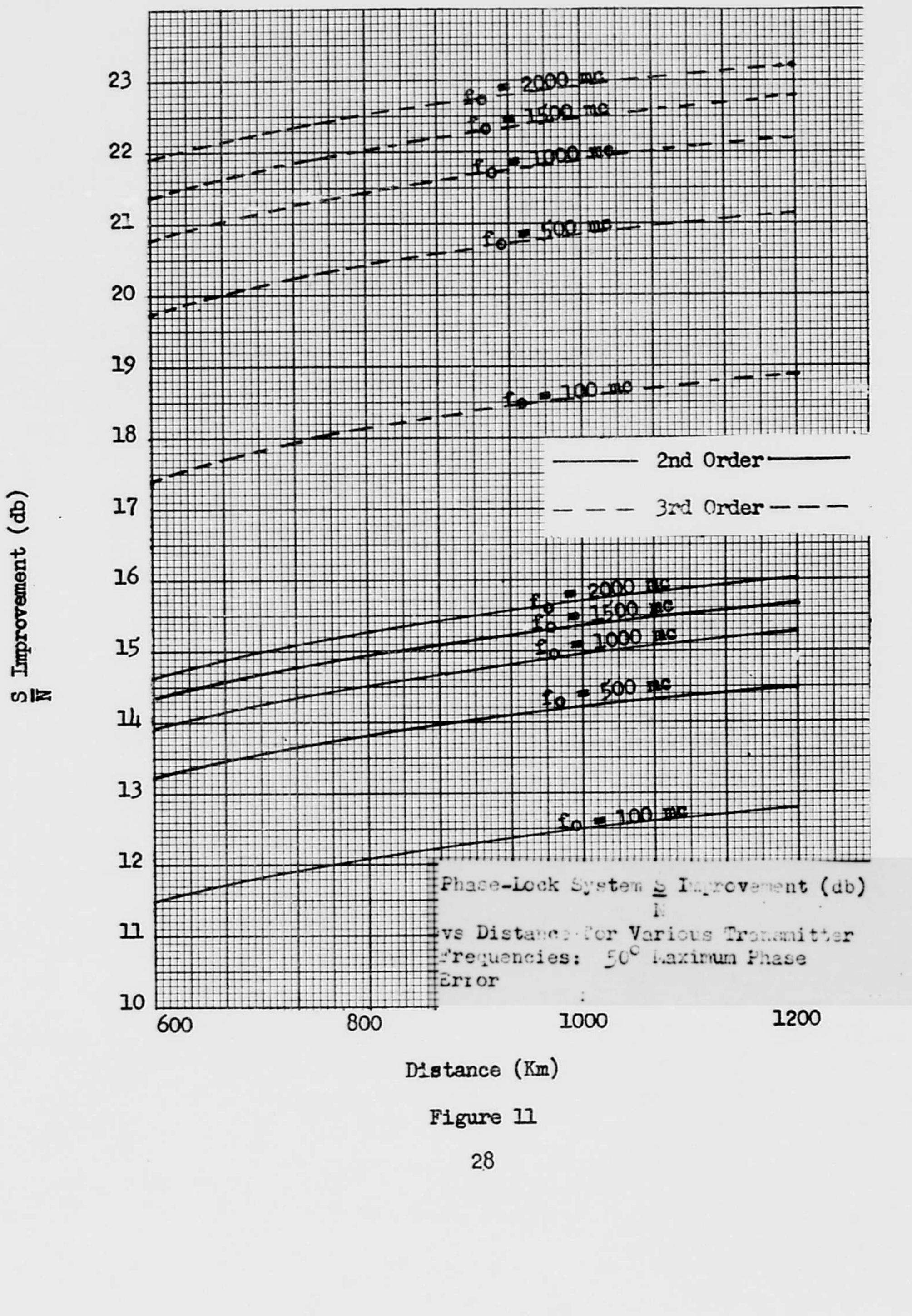
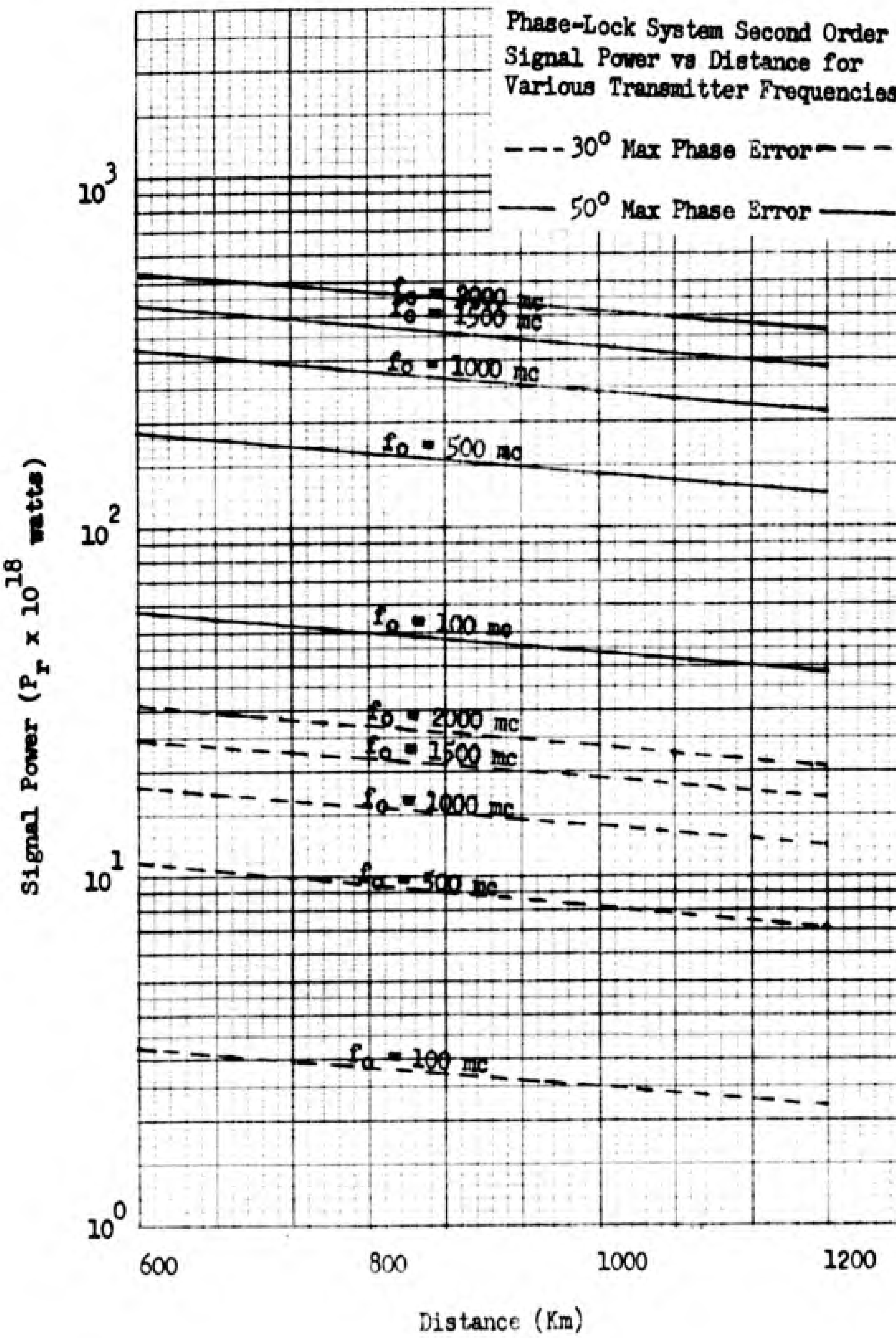

23
22
21
20
19
18
17
16
15
14
13
12
11
10
$\frac{S}{N}$ Improvement (db)
fo = 2000 mc
fo = 1500 mc
fo = 1000 mc
fo = 500 mc
fo = 100 mc
2nd Order
3rd Order
fo = 2000 mc
fo = 1500 mc
fo = 1000 mc
fo = 500 mc
100 mc
Phase-Lock System $\frac{S}{N}$ Improvement (db)
vs Distance for Various Transmitter Frequencies: 30° Maximum Phase Error
600
800
1000
1200
Distance (Km)

Figure 10



23
22
21
20
19
18
17
16
15
14
13
12
11
10
600
800
1000
1200
$\frac{S}{N}$ Improvement (db)
Distance (Km)
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
2nd Order
3rd Order
Phase-Lock System $\frac{S}{N}$ Improvement (db)
vs Distance for Various Transmitter
Frequencies: 50° Maximum Phase
Error

Figure 11



Phase-Lock System Second Order
Signal Power vs Distance for
Various Transmitter Frequencies
30° Max Phase Error
50° Max Phase Error
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
10^3
10^2
10^1
10^0
Signal Power ($P_r \times 10^{18}$ watts)
600
800
1000
1200
Distance (Km)

Figure 12

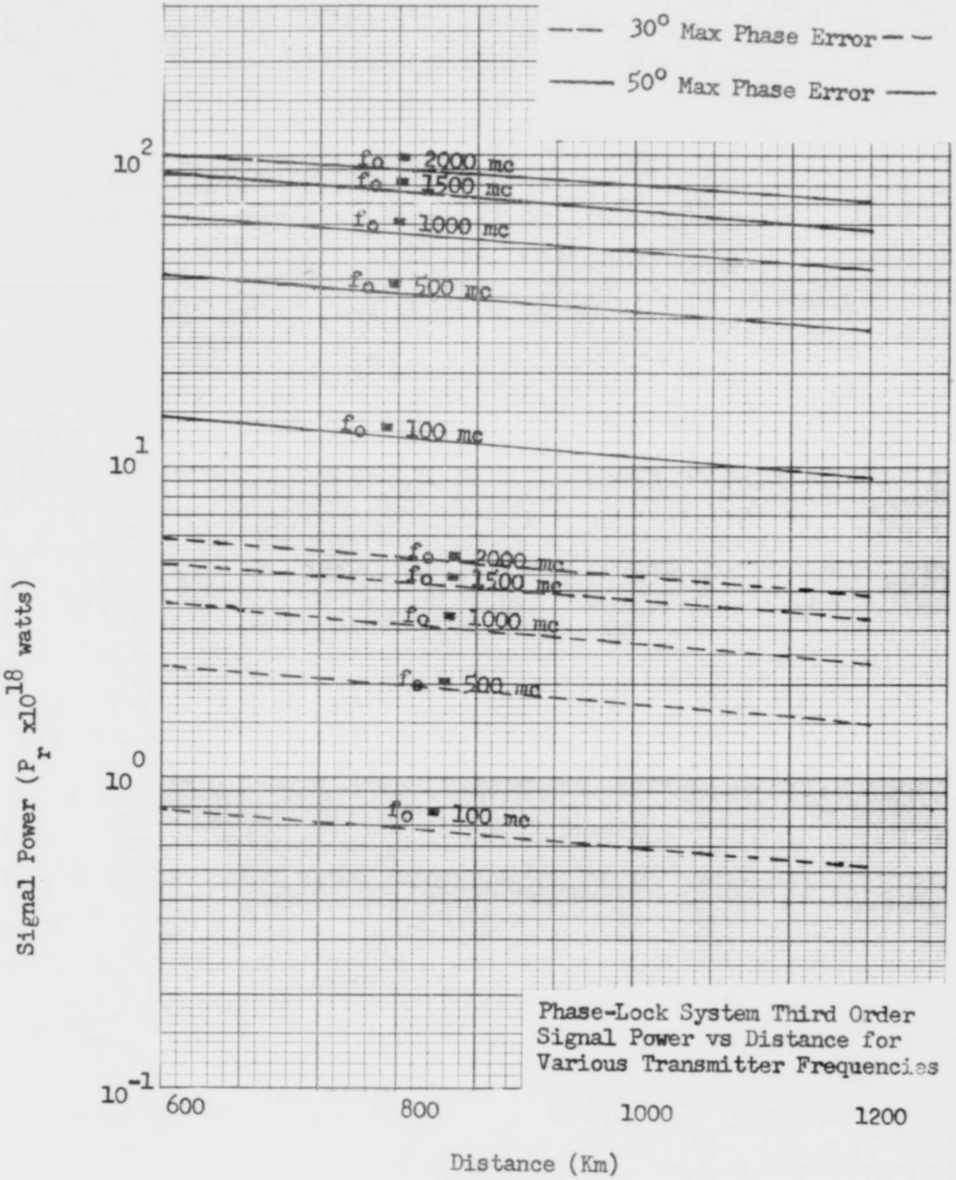

30° Max Phase Error
50° Max Phase Error
10^2
10^1
10^0
10^{-1}
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
Signal Power (P_r x10^{18} watts)
600
800
1000
1200
Distance (Km)
Phase-Lock System Third Order
Signal Power vs Distance for
Various Transmitter Frequencies

Figure 13

V. Predicted Tracking

The third techniquefor reducing the required input signal power is predicted tracking. The orbit of a satellite is a relatively stable ellipse. The orbit decays over long periods of time but the decay is very slow and can be accurately predicted. The orbit will change position with respect to a fixed receiver because of the earth's rotation. This motion is also well known and can be accurately predicted. Thus by accounting for the orbit decay and the rotation of the earth, the position and motion of the satellite with respect to a fixed receiver can be accurately predicted. If the predicted motion and the known transmitter frequency are combined, the Doppler frequency can be predicted. The use of predicted Doppler frequency in the detection process is an example of correlation detection. Golay describes predicted tracking as, "Cranking into the circuit everything which can be predicted about the flight of the vechicle, as well as the presumably well-known motion of the ground station with respect to the center of gravity of the solar system." (Ref 3:191) In this study the uses of correlation detection will be limited to its effect upon frequency tracking and the phase-lock loop.

In actual practice the inserting of available data is done by computer and can be performed in various ways. The significant result is that the receiver sensitivity is increased in direct proportion to the reduction in bandwidth. (Ref 2:42)

The improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}} = \frac{B_i}{B_o} \qquad \text{(Ref 2:42)} \tag{14.}$$

Applying this relationship to the detection bandwidths for the frequency tracking and phase-lock loop techniques gives even greater improvement in the signal to noise ratio. The improvement in the signal to noise ratio for the frequency tracking system is shown in figure 14. The second order system improvement ranges from 23.1 db to 32.3 db. The third order system improvement ranges from 32.4 db to 43.7 db. The improvement for the phase-lock technique is shown in figure 15. The second order system shows an improvement ranging from 21.7 db to 30.8 db. The third order system shows an improvement ranging from 38.7 db to 45.5 db. For both techniques, predicted tracking doubles, when expressed in decibels, the improvement in the signal to noise ratio.

The input power required is computed in the same manner as in the previous two sections. The power required by the frequency tracking system is shown in figure 16. The second order system required power ranges from 15.5×10^{-18} watts to 172×10^{-18} watts. The third order system required power ranges from 2.3×10^{-18} watts to 12.7×10^{-18} watts. The power required by the phase-lock technique is shown in figure 17. The second order system required power ranges from 13.8×10^{-20} watts to 121×10^{-20} watts. The third order system required power ranges from 0.76×10^{-20} watts to 4.2×10^{-20} watts. The lower values for the third order system is not shown in figure 17 but has been separately calculated. The required input signal power, for both techniques, is reduced by at least one order of magnitude by the addition of predicted tracking.

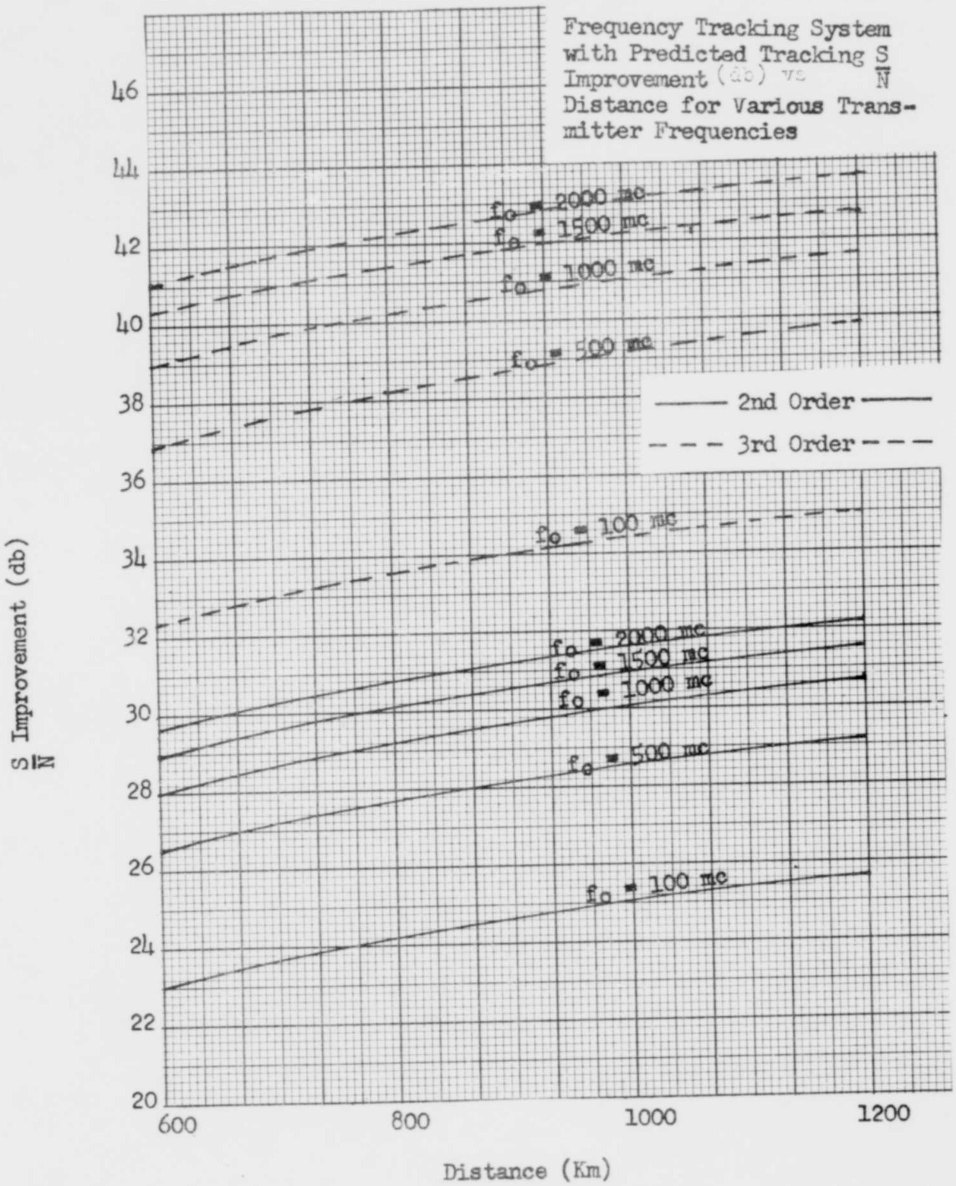

Frequency Tracking System with Predicted Tracking S/N Improvement (db) vs Distance for Various Transmitter Frequencies
46
44
42
40
38
36
34
32
30
28
26
24
22
20
S/N Improvement (db)
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
2nd Order
3rd Order
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
600
800
1000
1200
Distance (Km)

Figure 14

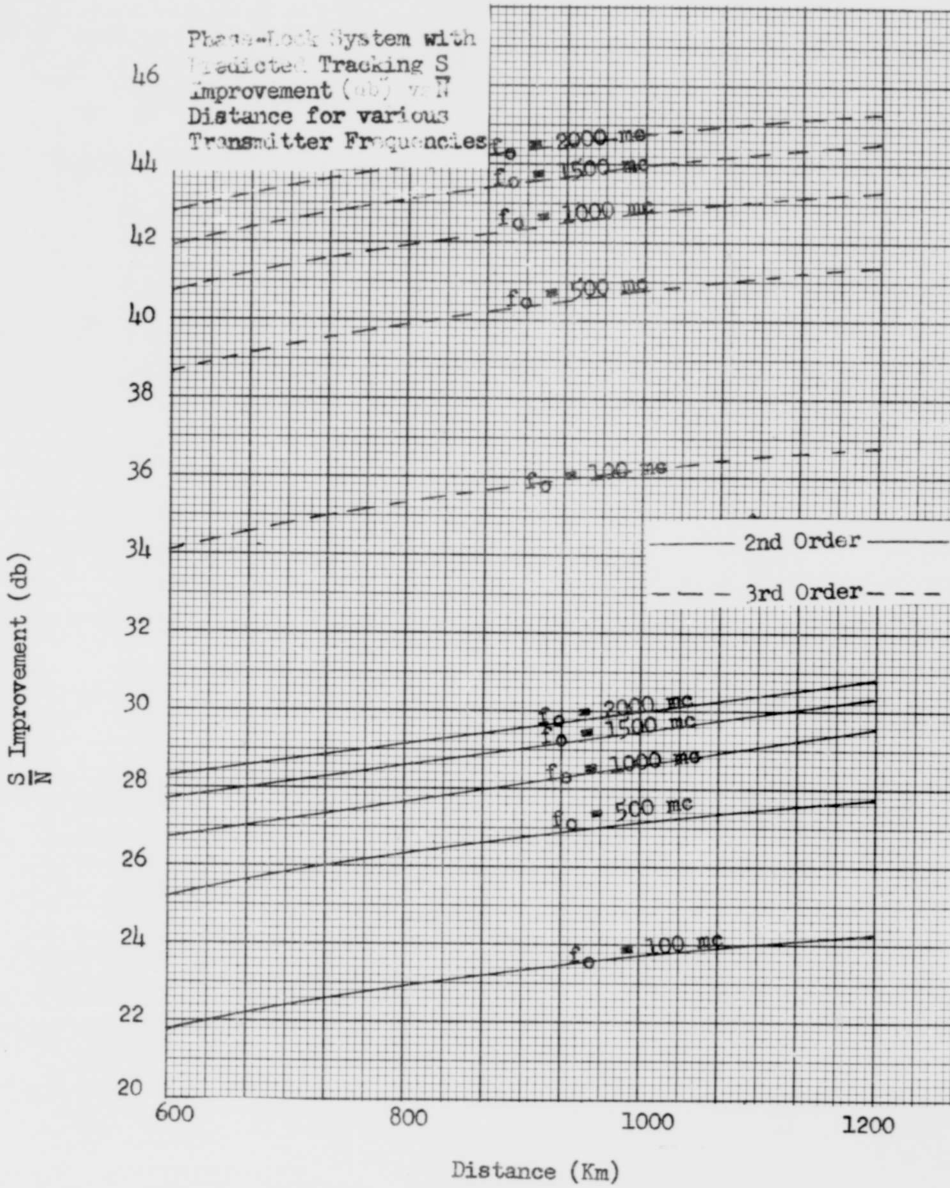

Phase-Lock System with
Predicted Tracking S
Improvement (db) vs N
Distance for various
Transmitter Frequencies
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
2nd Order
3rd Order
$\frac{S}{N}$ Improvement (db)
46
44
42
40
38
36
34
32
30
28
26
24
22
20
600
800
1000
1200
Distance (Km)

Figure 15

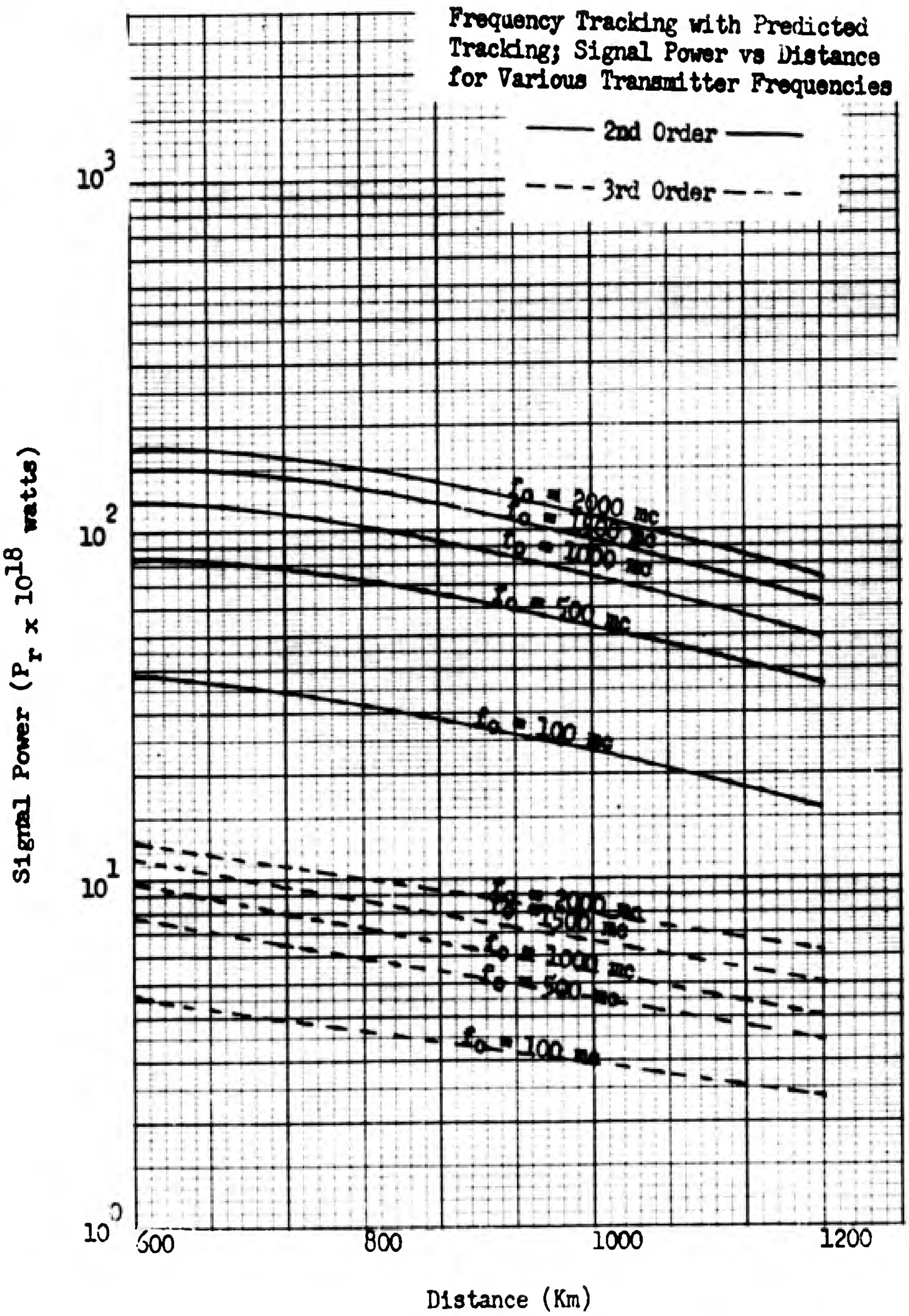

Frequency Tracking with Predicted Tracking; Signal Power vs Distance for Various Transmitter Frequencies
2nd Order
3rd Order
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
10^3
10^2
10^1
10^0
Signal Power ($P_r \times 10^{18}$ watts)
600
800
1000
1200
Distance (Km)

Figure 16

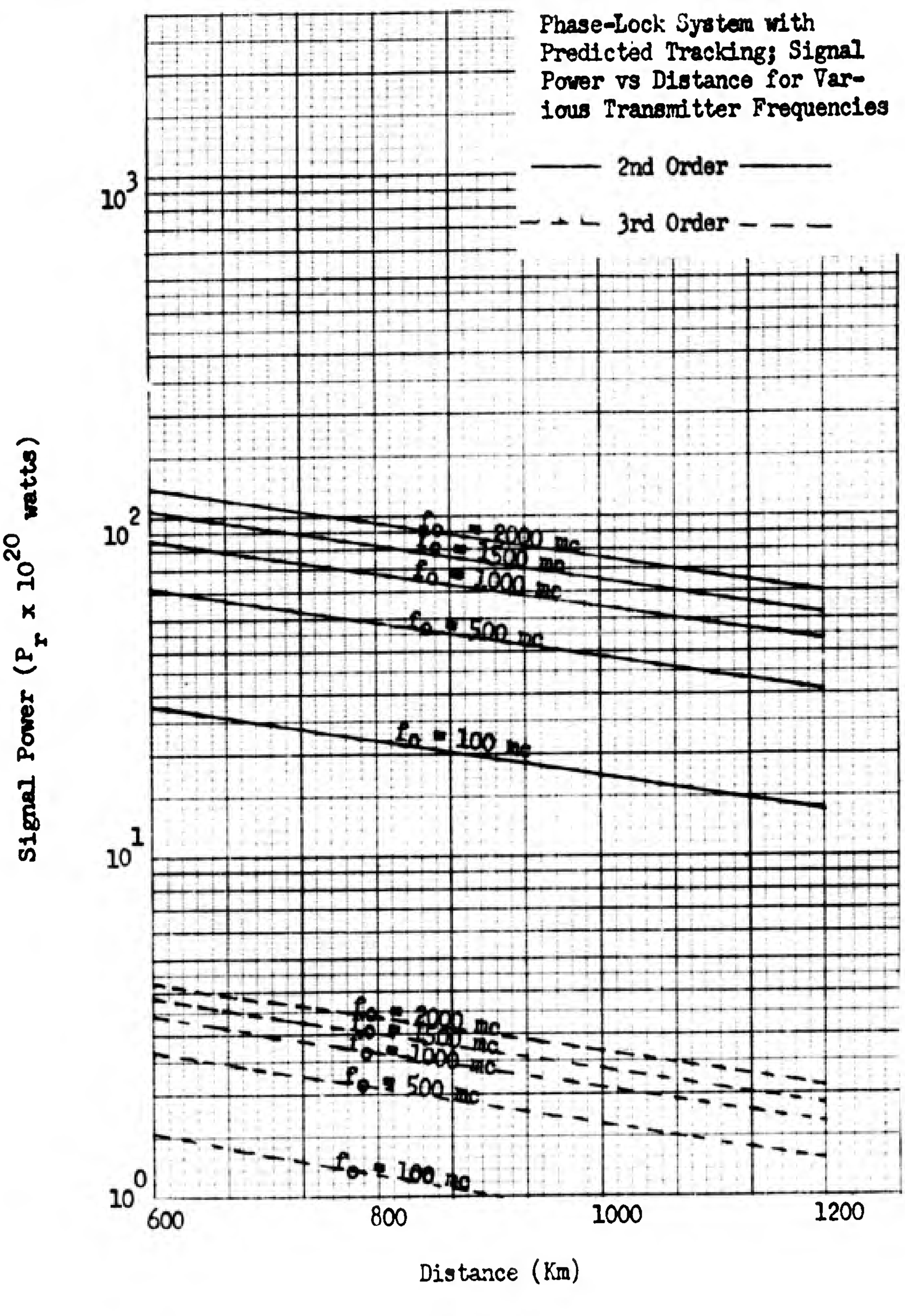

Phase-Lock System with Predicted Tracking; Signal Power vs Distance for Various Transmitter Frequencies
2nd Order
3rd Order
Signal Power ($P_r \times 10^{20}$ watts)
10^3
10^2
10^1
10^0
f_o = 2000 mc
f_o = 1500 mc
f_o = 1000 mc
f_o = 500 mc
f_o = 100 mc
600
800
1000
1200
Distance (Km)

Figure 17

VI. Comparison and Summary

It has been shown that by using the techniques of frequency tracking and the phase-lock loop that the detection bandwidth for a receiver processing a varying frequency can be considerably reduced. As a result of this reduction the output signal to noise ratio is greater than that of a conventional receiver. The conventional receiver must process the entire range of possible frequencies whereas the special techniques can select only the bandwidth containing the intelligence and a major portion of the noise power is eliminated from the system. The end result of the bandwidth reduction is that a weaker signal can be detected because the sensitivity of the receiver is improved.

In the previous three sections the three techniques have been compared to a conventional receiver, processing the entire Doppler shift bandwidth. In this section the techniques are compared to each other. Only the phase-lock system using the 30° phase error will be considered in this comparison as it has been shown to be more sensitive than the 50° phase error system.

Figure 18 shows the power required for the second order system of the four possible combinations of the techniques considered. In figures 18 and 19 the values of the signal power required for three transmitter frequencies 100, 1000, and 2000 mcs are plotted. The values for 500 and 1500 mcs are omitted to avoid excess cluttering of the plot but they would fall between the immediate higher and lower values of the transmitted frequency. The phase-lock technique requires less input signal power than the frequency tracking system by an order of two magnitudes. The use of predicted tracking reduces the required power of each system by an order of magnitude. Figure 19 shows the input signal power required for the third order system of the four possible combinations. The phase-lock system requires less power than the frequency tracking system by an order of three magnitudes. The use of predicted tracking reduces the power required by each system by two orders of magnitude. It is also to be noted that the third order system is generally one order of magnitude better than the second order system in all comparable systems.

The detection bandwidths for the frequency tracking system and the phase-lock loop system are very close in magnitude. By appropiate choice of system parameters the bandwidths for a given order could be made to be the same. It must be concluded that detection bandwidth is not the critical criterion in determining system sensitivity. The determining factor is system threshold and the low threshold of the phase-lock loop system makes it more sensitive than the frequency tracking system. The predicted tracking technique improves both system in equal magnitude and thus does not alter the basic conclusion.

One conclusion that cannot be drawn from this study is the optimum frequency for the satellite transmitter. Of the frequencies considered in the study it would appear that 100 mcs would be the most desirable. However there are many factors involved in this choice. Among the factors are the effect on antenna gain, the effect on noise temperature, atmospheric and ionospheric attenuation and system fabication considerations. The determination of the optimum frequency is certainly a study by itself.

The techniques considered in this study were developed for use in satellite tracking. However they do not exploit their full potentialities in this limited area. The techniques are of even greater value when used in interplanetary tracking. As the velocity of the vehicle increases to reach the necessary escape and trajectory velocities the Doppler shift will become very large. It is under these circumstances that the sensitive phase-lock loop and the use of predicted tracking will make the best utilization of the available power. This study has been carried out from the viewpoint of the effects of the technique on receiver sensitivity. Very little adjustment would be required to determine the effect on the useful range for a given transmitted power. It is obvious that the techniques considered extend the useful range considerably.

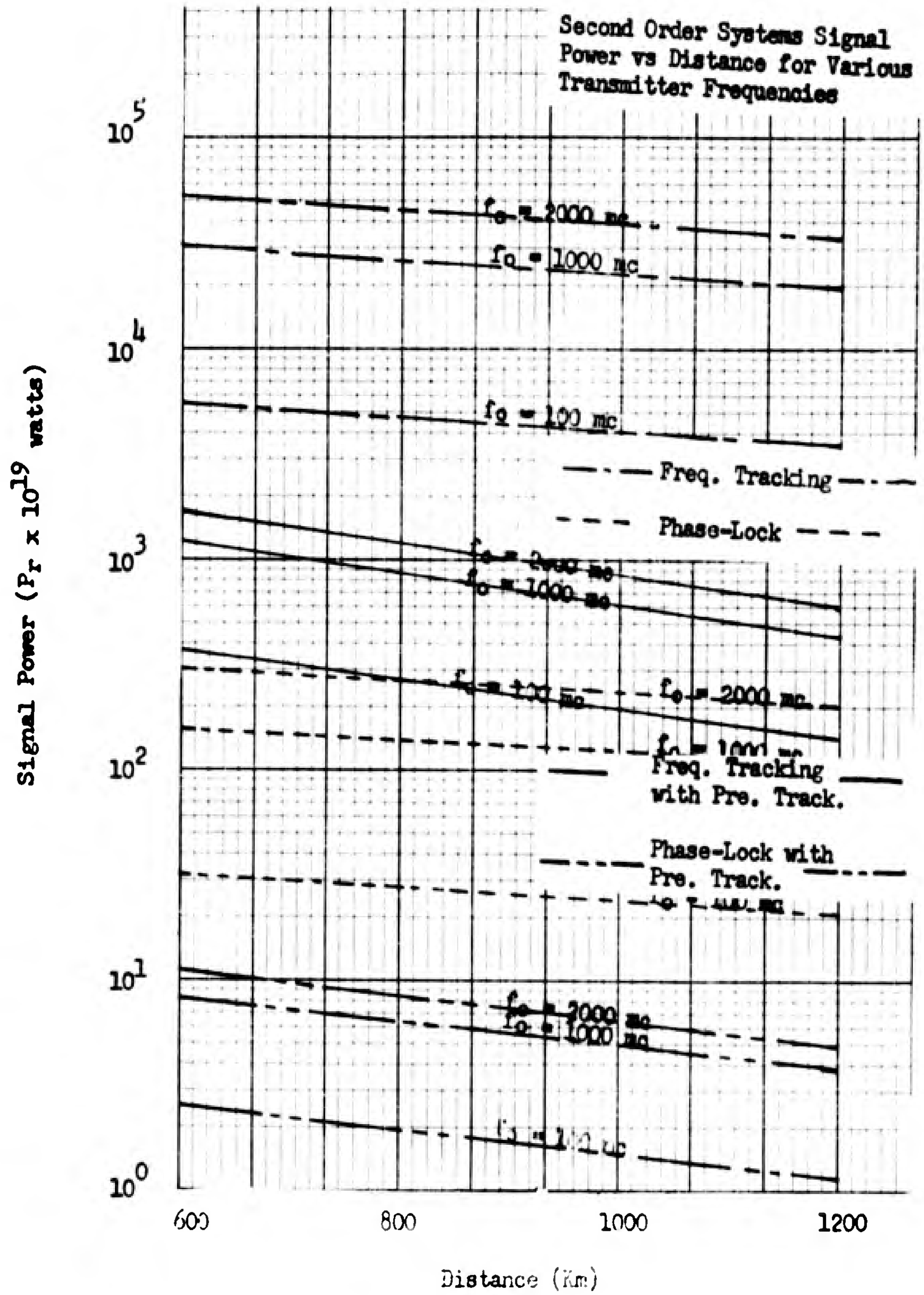

Second Order Systems Signal Power vs Distance for Various Transmitter Frequencies
Signal Power ($P_r \times 10^{19}$ watts)
10^5
10^4
10^3
10^2
10^1
10^0
f_o = 2000 mc
f_o = 1000 mc
f_o = 100 mc
Freq. Tracking
Phase-Lock
Freq. Tracking with Pre. Track.
Phase-Lock with Pre. Track.
600
800
1000
1200
Distance (Km)

Figure 18

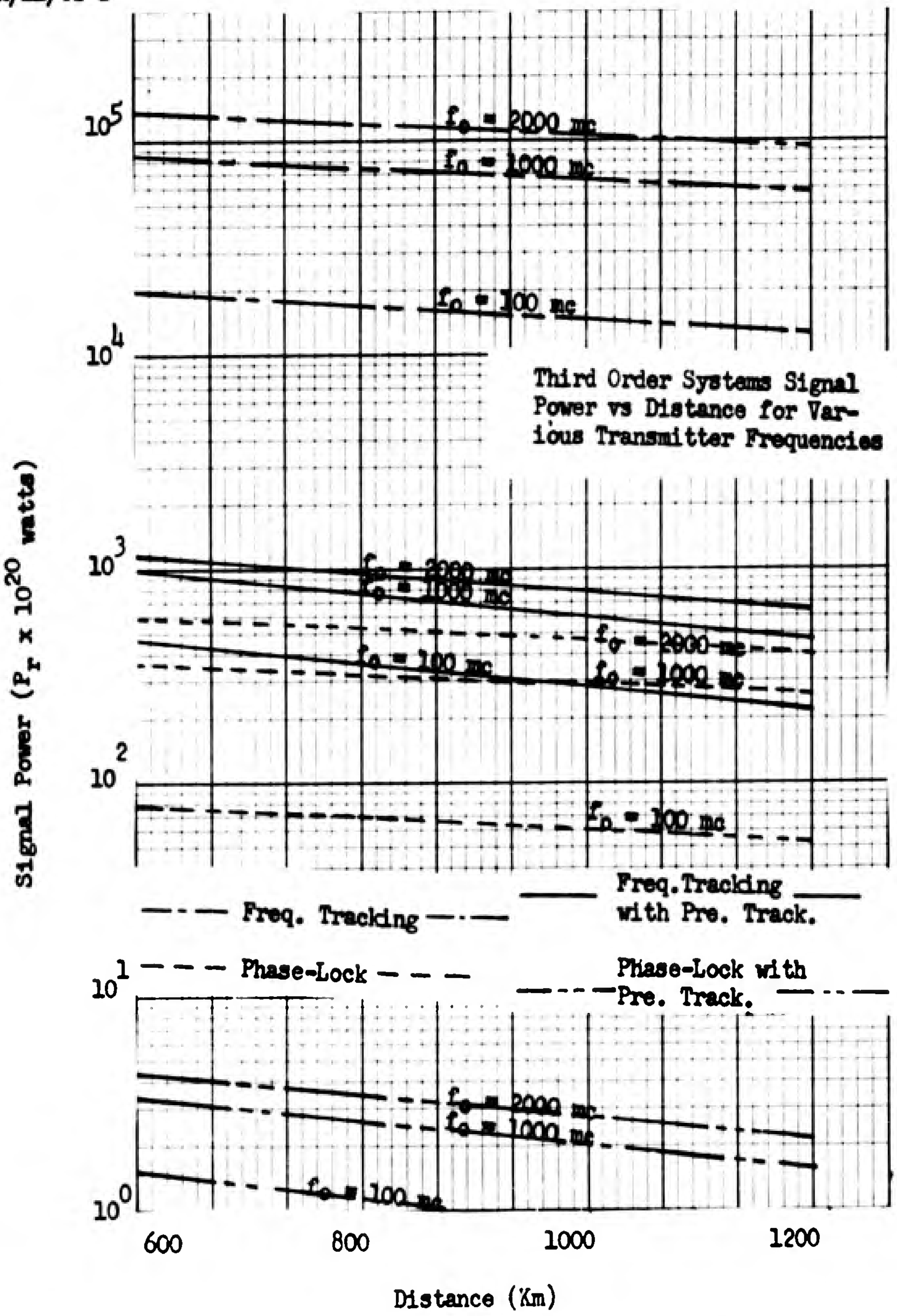

Signal Power ($P_r \times 10^{20}$ watts)
10^5
10^4
10^3
10^2
10^1
10^0
f_o = 2000 mc
f_o = 1000 mc
f_o = 100 mc
Third Order Systems Signal Power vs Distance for Various Transmitter Frequencies
Freq. Tracking
Freq. Tracking with Pre. Track.
Phase-Lock
Phase-Lock with Pre. Track.
600
800
1000
1200
Distance (Km)

Figure 19

Bibliography

1. Anderson, Roy E. et al "Satellite Tracking by Radio" ASTIA AD 263168 (August 1961)

2. Ashbrook, F.M. and Stevenson, D.D. "The Navy's Portable Satellite Tracking Stations" IRE-Transactions on Space Electronics and Telemetry v SET-6, n.1:41-45 (March 1960)

3. Golay, Marcel J.E. "The Application of Radio Interferometry to Extraterrestial Metrology" IRE-Transactions on Space Electronics and Telemetry v SET-5 n.4:186-193 (December 1959)

4. Jaffee, R. and Rechtin E. "Design and Performance of Phase-Lock Circuits Capable of Near-Optimum Performance Over a Wide Range of Input Signals and Noise Levels" IRE-Transactions on Information Theory v IT-1 n. 1:66-76 (March 1955)

5. Kamm, J.M. Jr The Effect of Phase-Lock Circuitry on Range of Communication Systems (unpublished) Dayton, Ohio :Air Force Institute of Technology (August 1961)

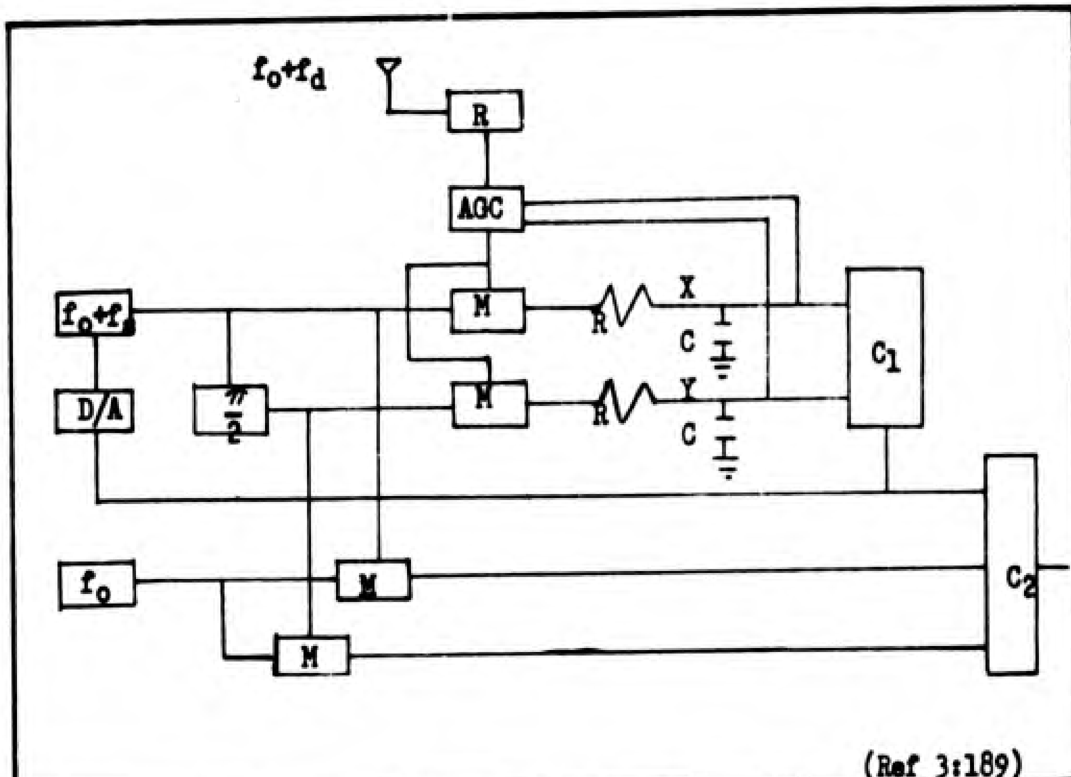
6. Pickard and Burns Inc. "Scientific Report #1 on Radio Tracking of Earth Satellites" ASTIA AD 160846 (June 1958)

7. Rideout, Vincent C. Active Networks New York: Prentice Hall Inc., 1954

8. Weaver, C.S. "Thresholds and Tracking Ranges in Phase-Locked Loops" IRE-Transactions on Space Electronics and Telemetry v SET-7 n. 3:60-70 (September 1961)

Appendix A

Automatic Frequency Tracking Circuit



Automatic Frequency Tracking Circuit

Figure 20

The incoming signal, f_o+f_d, is mixed with two quadrature CW signals of the servoed frequency $f_o + f_s$. The mixer outputs, x and y, are filtered in two separate RC filters characterized by the same constant, 2RC. The filter outputs, x and y, are the two components of a voltage vector and the angular coordinate of the voltage vector is the phase difference between the incoming and servoed signals.

The x and y signals constitute the input of an n-stage bidirectional counter, C_1, without short time internal memory, which registers every quarter turn of the xy vector. The output of the C_1 counter is connected to a digital-to-analog (D/A) converter which, starting with the count zero produces uniform step-wise voltage increases for every $\pi/2$ change of relative phase of f_d and f_s. The counter C_2 is designed to count the total number of 1/4 cycles of the Doppler shift f_d. It does this by adding to the 1/4 cycles of f_s the 1/4 cycles of $f_d - f_s$ obtained from the xy vector and counted by C_1. Thus the count registered by C_2 is the number of $\lambda/8$'s traveled by the vehicle. (Ref. 3:188-189)

Appendix B

Calculation of Bandwidth for Phase-Lock Technique

Kamm has shown that the maximum phase error for a second order system is given by the equation:

$$E_{(t)\ max} = \frac{\Delta\alpha}{(B_2)^2}\left(1 + \sqrt{2}\, e^{-\pi} \sin \frac{\pi}{4}\right) \quad \text{(Ref 5:15)}$$

where

$$\Delta\alpha = \dot{f}_d = \frac{a}{\lambda}$$

$$B_2 = \text{Bandwidth Parameter} = 2B \text{ (Detection Bandwidth)}$$

When $E_{(t)\ max} = 30^{o} = 0.0833$ cycles the equation becomes:

$$E_{(t)\ max} = \frac{a}{\lambda (B_2)^2}(1.04) = 0.0833$$

solving for B_2 gives:

$$B_2^{\ 2} = \frac{1.04}{0.0833}\,\frac{a}{\lambda} = 12.5\,\frac{a}{\lambda}$$

$$B_2 = 3.53\left(\frac{a}{\lambda}\right)^{\frac{1}{2}}$$

$$B = 1.765\left(\frac{a}{\lambda}\right)^{\frac{1}{2}}$$

If $E_{(t)\ max} = 50^{o} = 0.139$ cycles then:

$$B_2 = 2.735\left(\frac{a}{\lambda}\right)^{\frac{1}{2}}$$

$$B = 1.367\left(\frac{a}{\lambda}\right)^{\frac{1}{2}}$$

The maximum phase error for a third order system is given by the equation:

$$E_{(t)\,max} = 1.081\,\frac{\Delta\beta}{(B_3)^3} \qquad \text{(Ref 5:23)}$$

where B_3 = Bandwidth Parameter = 2B

$$\Delta\beta = \ddot{f}_d = \frac{J}{\lambda}$$

for $E_{(t)\,max} = 30^{\circ} = 0.0833$ cycles

$$B_3^{\,3} = 13.0\left(\frac{J}{\lambda}\right)$$

$$B_3 = 2.35\left(\frac{J}{\lambda}\right)^{\frac{1}{3}}$$

$$B = 1.175\left(\frac{J}{\lambda}\right)^{\frac{1}{3}}$$

For $E_{(t)\,max} = 50^{\circ} = 0.139$ cycles

$$B_3 = 1.937\left(\frac{J}{\lambda}\right)^{\frac{1}{3}}$$

$$B = 0.968\left(\frac{J}{\lambda}\right)^{\frac{1}{3}}$$

Vita

Emil George Riedel Jr. was born on [redacted], the son of Mr. and Mrs. Emil George Riedel. After completing his work in 1950 at [redacted], he enrolled at Ohio University. He entered the United States Naval Academy in 1951 from which he was graduated in 1955 with the degree of Bachelor of Science. His military assignment prior to coming to the Institute of Technology was as navigator in Air Refueling Operations of the Strategic Air Command.

Permanent address: [redacted]

This thesis was typed by Mrs. E.G. Riedel Jr.

UNCLASSIFIED

UNCLASSIFIED