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EXPLOSION PHYSICS

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Chapter XII

CUMULATION

63. General Information

The cumulative effect is an appreciable increase in the local action of the explosion. This effect is obtained by using charges which have a cavity on one of the ends — a cumulative cavity. If such a charge is initiated from the opposite end, then the explosive effect in the direction of the cavity axis is found to be much greater than if ordinary charges are used. It was established experimentally that if the surface of a cumulative cavity is covered with a relatively thin metal facing, then the armor-piercing action of the cumulative charge is increased by many times (Table 102).

The increased local action of charges with cavities has been known for more than 100 years. For a long time, however, this circumstance was not paid due attention and cumulative charges were not used in military or civilian technology.

The first systematic investigation of the phenomenon of cumulation was carried in 1923 — 1926 by Sukharevskiy, who established the dependence of the armor-piercing action of cumulative charges (without facing) on the shape of the cavity and on many other factors.

Cumulative charges have found extensive practical application only during the second world war. These charges were used in artillery and in explosives intended for fighting against tanks and fortification structures.

Serious experimental and theoretical investigations of cumulation began only during the second world war. The most outstanding work in this field has been done by Soviet scientists (Pokrovskiy, Lavrent'yev, and others).

A consistent hydrodynamic theory of cumulation, based on correct physical representations, was developed in 1945 by Lavrent'yev and independently of him by the American scientists Taylor, Reichelberger, and others.

On the basis of experience gained during the war it can be concluded that cumulative ammunition is an effective means of fighting against armored targets and engineering structures. They are also widely used in technology, particularly in oil extraction.

The study of the problem is best started from an examination of cumulative effect in pure form, i.e., in the absence of a metallic facing on the surface of the cavity.

If ordinary charges (without cavities) are used, we deal exclusively with propagation of explosion products and shock waves which diverge predominantly in a spherical direction. The characteristic feature of such a motion is the rapid decrease of the main parameters of the gas (pressure, velocity, density), principally as a result of the distribution of the explosion energy with motion of the detonation products and shock wave in a continuously increasing volume of a sphere.

To the contrary, in the motion of a converging stream of detonation products or converging shock waves, an appreciable increase takes place in the parameters of the medium. A specific feature of such motions is the sharp increase in the density of the gas energy, which in turn leads to a considerable increase in the local destructive action of the explosion. Motion of this kind is realised by exploding charges of special form — shaped charges.

Thus, the cumulation effect consists of an appreciable condensation of the detonation products, the increase of pressure in these products, and also a considerable increase in the energy density both in the scattering detonation products and in the shock waves produced by the explosion.

A classic example of cumulation is a spherical converging shock wave or detonation wave. At the center of convergence of such a wave, pressures can occur on the order of millions of atmospheres. This form of cumulation can be effected, in particular, by using charges shaped like hollow spheres, and simultaneously initiating the explosion over the entire outer surface of the sphere. However, the total cumulative effect will in this case be concentrated inside the cavity — in a zone adjacent to the center of the sphere. This form of purely radial cumulation, has a very limited application in practice. Nonetheless, it is of great scientific interest, since an analysis of this cumulation enables us to disclose certain laws which are common to all aspects of the cumulative effect as a whole.

Of greatest practical importance is directed axial cumulation. This form of cumulation can be realized by exploding charges containing cavities of various shapes (hemisphere, cone, parabola, hyperbola, etc.). Axial cumulation is due to condensation of the detonation products and their accelerated motion along the axis of the cavity. This form of cumulation, unlike radial cumulation, is always associated with a so-called cumulative jet, which has a directed motion.

64. SCATTERING OF EXPLOSION PRODUCTS FROM A CHARGE WITH AN OBLIQUE SURFACE

To determine the conditions of formation of the cumulative jet in the case of axial directed cumulation, it is necessary to consider first the principal laws of the scattering of detonation products from the inside of a cumulative cavity, which in turn reduces to an analysis of the problem of flow of detonation products from an oblique cut, i.e., to an investigation of detonation of a linear charge for the case when the detonation wave arrives at the charge surface at a certain angle.

Let us analyze the pattern of scattering of the surface layer of a charge in the case of an oblique detonation wave. In so doing, we can state that the principal part of the energy of this layer is radiated within a sufficiently small angle, the bisector of which makes an angle γ with the normal to the surface of the charge. The angle γ depends on the angle α between the front of the detonation wave and the surface of the charge (Fig. 157). On the average, more than 70% of the surface-layer energy is radiated within a 10° angle: $\gamma = 15^\circ$ when $\alpha = \pi/2$ and $\gamma = 10^\circ$ when $\alpha = \pi/4$.

We recall that in the calculation of the scattering of detonation products, starting from the Prandtl-Mayer solution, we are justified in speaking only of the scattering of the surface layer of the explosive. The scattering of the deeper layers will no longer obey this law. With increasing thickness of the charge, the thickness of the surface to which this solution is applicable also increases.

Both experiments and theory show that the surface layer of a shaped charge carries the main part of the energy consumed in destruction of the partition.

Let us analyze in greater detail the detonation of an elongated linear charge. The experiments of Pokrovskiy and Lokuchayev have shown, in full agreement with theory, that the maximum action is exerted on the partition by detonation products which make an angle of 7 — 14° with the normal to the charge surface. Inasmuch as the main part of the energy of a linear charge is concentrated within a small angle, it is possible to obtain a geometric construction for the fronts of the scattered detonation products for such a charge, specified in terms of any equation, i.e., a charge having the form of any arbitrary curve. To the contrary, it is possible to determine the equation of a linear charge, which produces on a specified distance a given detonation-product front surface.

Great interest attaches to a linear charge in the shape of a channel (Fig. 158). If such a charge is initiated on one of the ends, for example at the point O, the following will take place: the front of the detonation products, traveling from the line OA at an angle γ, and the front traveling from the line AB at the same angle, meet along the line O'A; with this, $\angle$ OAO' = $\pi/4 + \gamma$. Analogously we obtain a line O'B for AB and BC, and finally a line of encounter O'E for OA and BC. Obviously the coordinates of the point O' are

$$x = AB\cos^2(45^\circ - \gamma) = \frac{AB}{2}(1 + \sin 2\gamma);$$

$$y = OA - \frac{AB}{2}\cos 2\gamma.$$

If a charge of this kind is placed on a metallic plate, then the greatest deformation of the metal will take place precisely along the lines AO', O'B, and O'E. It is obvious that by measuring the angles OAO' and ABO' and the coordinates of the point O', it is possible to determine with high accuracy the angle γ, and also the value of $D/\bar{u} = w$, inasmuch as $\sin\gamma = \bar{u}/D$ (γ is the Mach angle in this problem).

Let the maximum action of the detonation products move with a velocity $\bar{u}$. Then in the detonation of a linear charge the surface of propagation of the maxima will have a straight-line form. Inasmuch as this surface is the envelope of a large number of individual waves, its angle of inclination to the surface of the charge will indeed be the Mach angle.

It is essential to note that here $\bar{u}$ is not the velocity of the particles, but the speed of motion of the maximum itself, which is several times smaller than the particle velocity. An analogous case takes place in one-dimensional scattering of gas escaping from a vessel, when the rate of motion ε_{max} is equal to $\bar{u} = c_i/\gamma$, and the maximum particle velocity is

$$u_{\max} = \frac{c_{in}}{\gamma}\,\frac{3\gamma-1}{\gamma+1} = \bar{u}\,\frac{3\gamma-1}{\gamma+1},$$

which yields $u_{max} = 2\bar{u}$ when $\gamma = 3$.

It is obvious that a very sharp and clear cut zone of action of the maximum ($\rho\bar{u}^n$) makes it possible to focus the streams of detonation products. This is possible under the following conditions. The detonation products traveling from different points of the detonating surface towards the focal point should converge simultaneously, and consequently the front of the converning wave of the detonation products should be spherical, and the angle between the tangent to the surface of the charge and the direction to the focus must be constant. Let us derive the equation for such a "cumulating" surface. We first consider the two-dimensional problem and derive the equation for a "cumulating" curve, which yields a convergent circular wave. We place the origin at the focus of this curve. Let the detonation begin at a certain point O (Fig. 159). Then from the tautochronism principle (the Fermat principle) we have

$$\frac{\overset{\frown}{OA}}{D} + \frac{AF}{\bar{u}} = \frac{OF}{\bar{u}} = \text{const.}$$

From this we obtain for the length of the arc OA the expression

$$\smile OA = \int_{\frac{\pi}{2}}^{\varphi} \sqrt{r^2 + \left(\frac{dr}{d\varphi}\right)^2}\, d\varphi = (OF - AF)\frac{D}{\bar{u}} = \frac{D}{\bar{u}}(r_0 - r), \quad (64,1)$$

where $\bar{u}$ is the velocity of motion of the maximum of action.

Differentiating the expression (64.1) with respect to φ, we obtain

$$\frac{dr}{r} = -\frac{d\varphi}{\sqrt{\frac{D^2}{\bar{u}^2} - 1}} \qquad (64,2)$$

Solving Eq. (64.2), we get

$$r = r_0 e^{\frac{\varphi + \frac{\pi}{2}}{\sqrt{\frac{D^2}{\bar{u}^2} - 1}}} \qquad (64,3)$$

This is the equation of a logarithmic spiral.

Forming a surface of revolution (about the axis of rotation OF), we obtain the cumulating surface, which yields a convergent spherical wave. As is well known, the logarithmic spiral satisfies the following condition: the angle between the tangent to any point A and the radius vector is a constant quantity.

It is clear from the construction that this angle is equal to $90° - \gamma$ and that $90° - \gamma = 180° - (\alpha + \varphi)$, where α is the angle of inclination of the tangent.

Hence

$$\gamma = \alpha - 90° + \varphi.$$

Inasmuch as

$$\operatorname{tg}(90° - \alpha) = \operatorname{ctg}\alpha = \frac{\frac{r'}{r} - \operatorname{tg}\varphi}{\frac{r'}{r}\operatorname{tg}\varphi + 1} = \operatorname{tg}\left[\operatorname{arctg}\frac{r'}{r} - \varphi\right] = \operatorname{tg}[-(\varphi + \gamma)].$$

we have

$$\operatorname{arctg}\frac{r'}{r} = -\gamma; \quad \frac{r'}{r} = -\operatorname{tg}\gamma,$$

But we have established that

$$\frac{r'}{r} = -\frac{1}{\sqrt{\frac{D^2}{\bar{u}^2} - 1}},$$

Therefore

$$\operatorname{tg}\gamma = \frac{1}{\sqrt{\frac{D^2}{\bar{u}^2} - 1}} \text{ and } \sin\gamma = \frac{\bar{u}}{D}. \qquad (64,4)$$

Thus, the fundamental relation for a straight-line charge is valid also in this case. The logarithmic spiral is the only curve which focuses the detonation products and simultaneously has the tautochronism property and the property of being able to direct towards the focus, from each element of its surface, detonation products having precisely identical parameters at precisely the same angle. Experiments with a detonating string (DS) give good agreement with theory. For a detonating string

$$\frac{\bar{u}}{D} = \frac{1}{5}, \quad \gamma = 14°.$$

The maximum deformation of a plate on which lies a spiral made of a detonating string occurs, accurate to several percent, at the theoretical focus of the spiral.

A three-dimensional charge made by rotating a logarithmic spiral around the OF axis, which yields a converging wave of detonation products, is of great principal interest, for when such a charge is sufficiently large it is possible to produce at its focus very high pressures, reaching a million atmospheres. The initial average value of the detonation products is 100,000 atm.

The surface formed by revolution of a logarithmic spiral will be the "cumulative" surface of charge, ensuring simultaneous focusing of the detonation products only when the detonator is located near the point O. If the layer of explosive between the detonator and the cumulative surface (at the point O) is sufficiently thick, then the form of the cumulative surface just considered no longer has this property, since it will no longer satisfy simultaneously the condition of tautochronismand the escape of detonation products at a specified angle to the surface of the charge (with a specified angle between the front of the detonation wave and the surface). By cutting off part of the charge formed by rotating a logarithmic spiral (for example, along the plane NN), we obtain a real cumulative charge, capable of ensuring very high pressure at the zone of convergence of the elementary jets.

However, the armor piercing ability of such a charge will be small; this is explained by the fact that no normal cumulative jet with axial action is produced when it is detonated. This problem will be considered in greater detail later on.

The theory of an unclosed cumulative charge, capable of ensuring directed axial cumulation, entails greater difficulties than the theory of a closed charge, since the escape of the detonation products will have no longer a central (point) symmetry, but an axial symmetry, and this increases the number of independent variables of the problem by one. However, the laws established above make it possible to draw several useful qualitative conclusions with respect to the nature and process and formation of the cumulative jet, and to make some approximate calculations of its parameters.

Process of formation of cumulative jet. It was established above that the main part of the energy of the detonation products, adjacent to the boundary of the exploded charge, is radiated inside a small angle, the bisector of which makes an angle γ with the normal to the surface of the charge. It follows therefore that when the detonation products escape through the surface of a cumulative cavity, they become deflected from their initial trajectory in such a way that the maximum action has a direction which is almost perpendicular to the surface. The detonation products and the shock wave front produced ahead of them experience a special kind of refraction. As a result of such a motion of the elementary jets, a detonation-product flow will be produced, converging along the axis of the cumulative cavity and having a greater density and velocity compared with the detonation products, which are scattered in other directions. The process of formation of the cumulative jet is shown schematically in Fig. 160.

Individual elementary jets will move normally to the surface of the cavity only near the cavity itself. Upon further movement of the jets will become rectified in accordance with the general laws, gas dynamics. At a certain distance from the base of the cavity, a maximum condensation of the cumulative jet will take place. This distance F indeed determines the place of location of the so-called cumulative focus. At a distance exceeding the focal distance, the cumulative jet degenerates rapidly, owing to the radial scattering of the detonation products which were compressed to high pressure.

It is known that the cumulative effect manifests itself sufficiently clearly only in the direct vicinity of the charge. With increasing distance to the charge, the cumulative effect decreases sharply and may even vanish completely. From this we can conclude that the action of the cumulative charge is due principally to the impact of the stream of detonation products (cumulative jet), which have a much greater density at short distances from the center of the explosion than the density of the air in the shock wave moving in front of them. Even in the usual escape of detonation products, as established in Chapter IX, the air density on the front of the shock wave is approximately 20 or 30 times less than the density of the stream of detonation products. The difference between the density of the air and of the cumulative stream is even greater.

We thus arrive at the conclusion that principal attention in the investigation of cumulation must be paid to an analysis of the motion of the explosion products, namely that part of the products which forms the cumulative jet proper. Problems connected with the motion of the shock wave are in this case of secondary significance.

Results of experiments have shown that the maximum velocity of a cumulative jet (the velocity of its frontal part) reaches 12 — 15 km/sec for charges made of high-explosive substances. The focal distance depends above all on the shape of the cavity: the smaller the curvature of the cumulative surface, the lesser the refraction that is experienced by the explosion products escaping through this surface, and consequently the greater the focal distance.

It is possible in principle to specify a cavity profile such that the cumulative focus is at a distance exceeding the zone of direct action of the explosion products.

In this particular case, the cumulative effect will be due principally to the convergence of the shock waves. However, the action of similar cumulative charges will be greatly inferior to the action of a normal cumulative charge.

For a specified cavity profile, the focal distance varies with the speed of detonation of the explosive. Let us illustrate this situation using as an example a charge with hemispherical cavity. If the detonation wave reaches the entire surface of the hemisphere simultaneously, the cumulative focus will be only somewhat farther than the center of the hemisphere. The reason the focus does not coincide fully with the center of the hemisphere is that in axial cumulation the elementary jets become straightened out as they approach the axis of the charge. A simultaneous approach of the front to the entire surface of the cavity is possible obviously only when the speed of detonation is infinite. It follows therefore that the less the speed of detonation of the explosive charge, the greater the corresponding focal distance. This is one of the explanations for the appreciable drop in the cumulative effect when cumulative charges made of low explosives (ammonites etc.) are set off.

For a specified cavity profile and for specified explosive properties, the focal distance can be changed by introducing inside the charge specially chosen "lenses" made of inert material or of another explosive. Such lenses make it possible to control the detonation process and, in particular, to ensure simultaneous approach of the detonation wave to the surface of the cavity. Charges with "lenses" are shown in Fig. 161. In case a the time of approach of the detonation wave to the surface of the cavity is regulated by changing the direction of the detonation front; in case b this is done by using a lens of an explosive in which the detonation velocity is smaller.

To explain the effect of the form and dimensions of the cavity on the destructive action of the cumulative charge, and also to estimate the energy and mass of the cumulative jet, it is necessary to establish first what part of the explosive charge produces the cumulative jet proper. We call this the active part of the cumulative (shaped) charge

65. ACTIVE PART OF SHAPED CHARGE

To estimate the active part of the shaped charge, we use the instantaneous-detonation scheme. In this case the rarefaction waves travel from all sides of the charge with equal velocity, so that it is very simple to determine the form of the surface of convergence of two rarefaction waves traveling from any two surfaces, in the case when these surfaces are specified (Fig. 162).

Let the equation of surface of revolution 1 be

$$y_1 = f_1(x_1). \qquad (65,1)$$

The equation of surface 2 is

$$y_2 = f_2(x_2). \qquad (65,2)$$

The sought equation is written in the form

$$y = f(x). \qquad (65,3)$$

It is obvious from the construction that

$$x = x_1 - z, \quad y = y_1, \quad x = x_2, \quad y = y_2 - z. \qquad (65,4)$$

Eliminating z from (65.4) we obtain

$$y - y_2 = x - x_1. \qquad (65,5)$$

Inasmuch as $x_1 = \varphi_1(y_1)$ and $y_1 = y$,

and $y_2 = f_2(x_2)$ and $x = x_2$,

relation (65.5) assumes the form

$$y+\varphi_1(y)=x+f_2(x). \qquad (65,6)$$

This equation determines the sought surface of encounter of two rarefaction waves. It is obvious that the line $y = f(x)$ should be equidistant from the lines $y_1 = f_1(x_1)$ and $y_2 = f_2(x_2)$. In principle it is easy to find the equation of this line. However, inasmuch as the stream of the detonation products is scattered more slowly inside the cavity than on the outside, a relatively larger part of the detonation products will escape precisely from the outer side. Therefore the line $y = f(x)$ shifts closer to the line $y_2 = f_2(x_2)$.

If the distance from the rear surface of the charge (Fig. 162) to the point O is not less than the distance from this surface to the vertex of the cavity (not less than r), then the line of encounter of the rarefaction waves will be approximately the line separating the masses of the explosion products, escaping in different directions. It is obvious that the volume of the active part of the charge, v_a, of the part moving in the direction of the cumulative cavity, is determined, disregarding the displacement of the line $y = f(x)$ due to the somewhat different conditions under which the detonation products escape from the outer and inner surfaces, by the integral

$$v_a=\pi\int_0^{r+h} y^2\,dx-\pi\int_h^{r+h} y_1^2\,dx_1. \qquad (65,7)$$

Here $\quad y=y_1;\quad dx=dx_1+dy_1-dy_2,$

which follows from (65.5).

We must know $\quad x_1=\varphi_1(y_1),$

and then $\quad dx_1=\frac{d\varphi_1}{dy_1}\,dy_1$

and

$$v_a = \pi \int_0^{r_0} y_1^2 \left(dy_1 + \frac{d\varphi_1}{dy_1} dy_1 - dy_2\right) - \pi \int_0^{r_0} y_1^2 \frac{d\varphi_1}{dy_1} dy,$$

which yields

$$v_a = \frac{\pi r_0^3}{3} - \pi \int_0 y_1^2 \, dy_2. \qquad (65,8)$$

It remains to determine y_2 as a function of y_1, an easy matter since

$$y_1 = f_1(x_1) \text{ and } y_2 = f_2(x).$$

To the contrary,

$$x = \varphi_1(y_1) = \varphi_2(y_2),$$

which yields

$$y_2 = \Phi(y_1)$$

and

$$v_a = \frac{\pi r_0^3}{3} - \pi \int_0^{r_0} y_1^2 \frac{d\Phi}{dy_1} dy_1. \qquad (65,9)$$

In the particular case when $y_2 = r_0 = \text{const}$

(cylindrical charge), Eq. (65.6) assumes the form

$$y + \varphi_1(y) = x + r_0$$

and the volume of the active part is determined by the formula

$$v_a = \frac{\pi r_0^3}{3}, \qquad (65,10)$$

inasmuch as the integrand of Eq. (65.9) vanishes, i.e., the active part depends only on the caliber of the charge. It is assumed here that the radius of the base of the cavity is equal to half the caliber of the charge. From this we must conclude that the active part of the charge decreases as the diameter of the base of the cavity is changed. Consequently, to obtain a large active part in a charge of given diameter, it is necessary to make the diameter of the base of the cavity as large as possible.

By way of example we consider a cylindrical charge with a hemispherical rear part and a cavity of arbitrary form (Fig. 163). The height of the charge is $2r_0 + h$. The volume of such a charge is

$$v_0 = \pi r_0^2 (r_0 + h) + \frac{2}{3}\pi r_0^3 - \pi \int_0^{r_0} \frac{y_1^2 dy}{y_1'} =$$

$$= \frac{5}{3}\pi r_0^3 + \pi r_0^2 h - \pi \int_0^{r_0} \frac{y_1^2 dy}{y_1'}. \qquad (65,11)$$

The ratio is

$$\frac{v_0}{v_a} = 5 + 3\frac{h}{r_0} - \frac{3}{r_0^3}\int_0^{r_0} \frac{y_1^2 dy}{y_1'}, \qquad (65,12)$$

where

$$y_1' = \frac{dy_1}{dx}. \qquad (65,13)$$

For a conical cavity we have

$$\pi \int_0^{r_0} \frac{y_1^2 dy}{y_1'} = \frac{\pi}{3} r_0^2 h, \quad v_0 = \frac{5}{3}\pi r_0^3 + \frac{2}{3}\pi r_0^2 h, \qquad (65,14)$$

$$\frac{v_0}{v_a} = 5 + 2\frac{h}{r_0}. \qquad (65,15)$$

From this it is obvious that for real charges ($h \approx 2r_0$) we have

$$\frac{v_0}{v_a} \approx 9,$$

i.e., the mass of the active part of the charge will be 11% of the mass of the entire charge. The charge shown in Fig. 163 has a minimum possible volume, at which use is made of its entire calculated active part.

The active part of a flat (non-cumulative) charge is determined as before by the relationship

$$v_a = \frac{\pi}{3} r_0^3.$$

The minimum volume of such a charge is

$$v_0 = \pi r_0^3 + \frac{2}{3}\pi r_0^3 = \frac{5}{3}\pi r_0^3. \qquad (65,16)$$

Therefore $v_0/v_a = 5$, something which can be derived directly from (65.4) by putting $h = 0$. Analogously, a cylindrical unrounded charge will have

$$v_0 = 2\pi r_0^3 \text{ and } \frac{v_0}{v_a} = 6. \qquad (65,17)$$

Stanyukovich investigated in detail the propagation of rarefaction waves in the scattering of detonation products under real conditions, and has shown that the relations derived above for the active part of any charge, for instantaneous detonation, are suitable with accuracy of 5% for the calculation of the active part in a real detonation.

Analyzing the results obtained, we can arrive at the following conclusions. The minimum height of the charge, at which its active part reaches its limiting value is for a cylinder $H_{lim} = 2r_0 + h$, corresponding to approximately two calibers in the case of real shaped charge with conical cavity. As the length of the charge decreases, the weight of the active part is reduced more slowly than the weight of the entire charge. This makes it possible to use charges with height $H < H_{lim}$ in shaped ammunition, without noticeable reduction in the cumulative effect.

It follows from (65.10) that as the diameter of the base of a cumulative cavity increases, the cumulative effect should increase appreciably, since the mass of the active part is proportional to the cube of the caliber.

Let us calculate now the effect of the shell and of the facing of a shaped charge on the magnitude of the active part. We consider the following one-dimensional problem: assume that two bodies of masses M_1 and M_2 move in an infinite non-deforming tube under the influence of an expanding gas (detonation product). Mass M_1 moves to the right and M_2 moves to the left. It is required to determine the velocities u_1 and u_2 of the masses, and also the masses of the detonation products that move to the right and to the left (m_1 and m_2). The following relations are obvious:

$$u = \frac{x}{t}; \quad m = m_1 + m_2; \quad \rho = \frac{m_1 + m_2}{t(u_1 - u_2)}. \qquad (65,18)$$

Assuming that the velocity of the gas is linearly distributed over its masses m_1 and m_2, we obtain from the law of conservation of momentum

$$\frac{m_1 u_1}{2} - \frac{m_2 u_2}{2} + M_1 u_1 - M_2 u_2 = 0. \qquad (65,19)$$

The law of conservation of energy yields

$$\frac{m_1 u_1^2}{6} + \frac{M_1 u_1^2}{2} + \frac{m_2 u_2^2}{6} + \frac{M_2 u_2^2}{2} = mQ = \frac{m\bar{c}_{in}^2}{6}. \qquad (65,20)$$

where $\bar{c}_{in}$ is the velocity of sound in the detonation product (for the case of instantaneous detonation). It is further obvious that inasmuch as $u = \frac{x}{t}$, then

we have

$$\frac{u_1}{m_1} = \frac{u_2}{m_2}. \qquad (65,21)$$

Solving (65.19), (65.20), and (65.21) simultaneously and taking into account the fact that $m = m_1 + m_2$, we obtain

$$m_1 = \frac{m}{2}\left(1 + \frac{M_2 - M_1}{M_1 + M_2 + m}\right), \quad m_2 = \frac{m}{2}\left(1 + \frac{M_1 - M_2}{M_1 + M_2 + m}\right); \qquad (65,22)$$

$$\left(\frac{u_1}{c_{in}}\right)^2 = \frac{2m(m + 2M_2)^2 (M_1 + M_2 + m)}{(m + 2M_2)^2 [m(m + 2M_2) + 6M_1(M_1 + M_2 + m)] + (m + 2M_1)^2} \times$$

$$\times \frac{1}{[m(m+2M_1)+6M_2(M_1+M_2+m)]}, \quad (65{,}23)$$

$$\left(\frac{u_2}{\bar{c}_{in}}\right)^2 = \frac{2m(m+2M_1)^2(M_1+M_2+m)}{(m+2M_2)^2[m(m+2M_2)+6M_1(M_1+M_2+m)]+(m+2M_1)^2} \times$$

$$\times \frac{1}{[m(m+2M_1)+6M_2(M_1+M_2+m)]}. \quad (65{,}24)$$

We now determine the unilateral momentum

$$I_1 = -I_2 = u_1\left[M_1 + \frac{m_1}{2}\right] = -u_2\left[M_2 + \frac{m_2}{2}\right]. \quad (65{,}25)$$

In the particular case when $M_2 = 0$

$$I_1 = -I_2 = -\frac{m_2 u_2}{2} = \frac{u_1}{2}[2M_1 + m_1]. \quad (65{,}26)$$

Inasmuch as for $M_2 = 0$ we have

$$M_1 = \frac{m^2}{2(m+M_1)}, \quad \frac{u_1}{\bar{c}_{in}} = \frac{m}{\sqrt{(m+4M_1)(m+M_1)}},$$

we obtain

$$I_1 = -I_2 = \frac{m\bar{c}_{in}}{4}\frac{(m+2M_1)^2}{(m+M_1)\sqrt{(m+M_1)(m+4M_1)}}; \quad (65{,}27)$$

When $M_1 \to 0$ we have $I_1 = m\bar{c}_{in}/4$, and when $M_1 \to \infty$ we have $I_1 = m\bar{c}_{in}/2$. These results are obvious, since the momentum doubles upon reflection from an absolutely rigid wall ($M_1 \to \infty$).

If $M_1 = M_2 = M$, we have

$$m_1 = m_2 = \frac{m}{2}, \quad (65{,}28)$$

$$u_1 = -u_2 = \bar{c}_{in}\sqrt{\frac{m}{m+3M}}. \quad (65{,}29)$$

$$I_1 = -I_2 = \frac{\bar{c}_{in}}{4}(m+2M)\sqrt{\frac{m}{m+3m}}. \quad (65{,}30)$$

If $M_1 \to 0$, then $I_1 = m\bar{c}_{in}/4$; when $M \to \infty$ we have $I_1 = \frac{m\bar{c}_{in}}{2}\sqrt{\frac{Mm}{3}} \to \infty$, which is quite natural, for when $M_1 = M_2 = M \to \infty$ the pressure on the wall acts for an unlimitedly long time.

A similar scheme can be used with a high degree of accuracy to investigate the scattering of the active part of a shaped charge, assuming m to be the mass of the explosive contained between the elements of the shell and the facing, while M_1 and M_2 are the masses of these elements. The distance between the elements of the facing and the shell is measured along the shortest straight line.

66. CUMULATION FOR A CAVITY WITH METALLIC FACING

In the presence of metallic facing on the surface of the cavity, as already noted, a very sharp increase in the cumulative effect is observed. In spite of the fact that in this case there are obtained the same physical features that characterise the explosion of a shaped charge without facing on the cavity. The picture of the phenomenon under consideration, however, changes considerably.

Experimental and theoretical researches have established that the intensification of the cumulative effect in the presence of facing is due to the very strong and unique redistribution of the energy between the explosion products and the material of the metallic facing, and the transfer of part of the metal into the cumulative jet. The main part of the energy of the active part of the cumulative charge is "pumped over" into the metal of the facing, in such a way that the energy is concentrated in a thin layer of metal, which forms the cumulative jet proper. As a result of this, a considerably greater energy density is obtained in the jet than when a charge is exploded without facing on the cavity. The maximum "condensation," determined by the ratio of the diameter of the cavity to the diameter of the jet, is four — five for a charge without facing. For a charge with metallic facing, the "condensation" is much higher, since the diameter of the cumulative jet is 1 — 3 mm.

All-out experimental investigations by methods of instantaneous x-ray photography, spark photography, etc., have made it possible to establish the nature of the cumulative jet and the mechanism of its formation. Particularly fruitful in the investigation of the phenomenon of cumulation in the presence of metallic facing is the method of instantaneous x-ray photography

The process has been investigated in greatest detail with charges having hemispherical and conical facings. As a result of all these investigations it was established that metallic facing becomes pinched under the action of the explosion products, and consequently its elements collapse successively with formation of a thin metallic jet, which has high velocity.

The overall picture of the process of deformation of the metallic facing and formation of the cumulative jet is shown on two series of x-ray pictures (Figs. 164 and 165). They record the process of pinching of the facing and the motion of the jet with time. It was found, in the processing of the experimental data, that the maximum rate of radial deformation of a steel cone with wall thickness 1 — 2 mm amounts to 1,000 — 2500 m/sec, depending on the type of explosive. So rapid a pinching of the facing causes it to become converted into a compact monolithic mass — pestle (Fig. 166) — which gives rise to the formation and subsequent development of the cumulative jet. As each element of the facing becomes pinched, its thickness increases, and the energy is concentrated for the most part in its internal layer. The jet is formed exclusively through flow of metal adjacent to the internal surface of the facing, this being the consequence of the fast collision between its elements at the instant of collapse.

The mass of the metal converted into a cumulative jet amounts on the average to 6 — 11% of the mass of the facing. A confirmation of the fact that the cumulative jet is connected with the flow of metal can be obtained not only from the foregoing results, but also from the following data. If a copper layer 0.05 mm thick is electroplated on the internal surface of a steel cone, no traces of copper are found in the pestle at all. On the other hand, if the copper is deposited on the outer surface of the cone, strips of oxidized copper are formed in the pestle. In tracing the pestle, one can detect along its axis a narrow channel, the presence of which indicates that the internal layers of the metal have much higher velocities than the external ones. Regarding the character of deformation of the facing metal during its pinching one can draw some conclusions also from results of metallographic investigations of pestles in sections that are sufficiently far from the axis.

All the microstructure photographs (Fig. 167) readily show the orientation and the drawing of the structural components in the axial direction.

The orientation and the drawing increase as the corresponding layers approach the axis.

The formation and motion of the cumulative jet can be broken up into two stages. These stages are greatly affected by the physical and mechanical characteristics of the facing metal.

The first stage characterizes the formation of the jet during the process of pinching of the facing. During this time the pestle and jet make up a single unit (see Figs. 164 and 165), but they move at different velocities.

The pestle moves relatively slowly (with a velocity 500 -- 1,000 meters per second). The jet, to the contrary, has a very high translational velocity. However, this velocity is different in different parts along the jet; the frontal part of the jet has the greatest velocity, while the velocity of the tail part is close to that of the pestle. Depending on the form and nature of the facing metal, the properties of the explosive charge, and other factors, the velocity of the frontal part of the jet can vary over a wide range. For aluminum facing with hyperbolic form, the velocity of the frontal part reaches, for example, approximately 11,000 meters per second.

Certain data on the velocity of the frontal part of the cumulative jet are given in Table 103.

Cumulative charges are made in all cases of an alloy of TNT with hexogene (D = 7600 m/sec).

The velocity gradients along the cumulative jets were established by direct experiments, namely with the aid of mirror scanning and successive cut off of individual elements of the jet by means of partitions of different thickness (Fig. 168). This method was first developed and used in 1946 by Baum and Shekhter. Later on it found wide use for the investigation of the cumulation process.

Second stage. Some time interval after the pinching of the facing, owing to the presence of velocity gradients, the jet breaks away from the pestle (Fig. 169). It can be considered that the most effective action of the cumulative charge is reached with the jet breaks away already after the metal is no longer fed to it from the pestle, the latter having served up to a definite instant as a reservoir for supply of the jet. This can proceed until the inertial forces, under the action of which the metal flows, are balanced by the forces of adhesion between the metal particles. From this point of view, the high plasticity of the material is a favorable factor. This factor is of particular importance for the normal process of pinching the facing. The facing should not be subject to brittle damage during the deformation process, or else the coefficient of transition of the metal into the jet will be sharply decreased, and its armor piercing action will be accordingly reduced. Fig. 170 shows facing during different stages of deformation, made of low-carbon and hardened steel. The pinching of the former, as can be seen from the figure, is not accompanied by brittle damage (Fig. 170a); facing made of hardened steel is damaged by pinching (Fig. 170b).

It is obvious that the conditions under which the jet breaks away from the pestle are determined by the velocity gradient and by the physical and mechanical characteristics of the facing metal, on which the maximum elongation of the jet depends. On the basis of the foregoing it can be concluded that the most effective action of the cumulative jet can be ensured only under a definite combination of physical-mechanical properties of the facing metal. It is necessary here to bear in mind that the properties of the metal under conditions of fast deformations can differ greatly from its properties determined under ordinary deformation velocities. For example, cast iron, which is brittle under ordinary conditions, behaves in the explosion of a shaped charge like a metal with relatively high plasticity.

As a result of investigations carried out by Baum and Sklyarov, the following was established.

The conditions under which a cumulative jet is shaped are determined by the microstructure of the facing metal an extent to which its structural components are capable of plastic deformation.

However, the plasticity of the metal under conditions of pinching is not determined uniquely under the influence of explosion by its standard characteristics. A relation has been noted between the tendency of metal to rapid pinching and the type of crystal lattice. The best pinching is observed in the case of facings made of metals with cubic lattice (aluminum, iron, or copper), while poor properties are exhibited by metals with hexagonal lattice (cadmium, cobalt, and magnesium). Best armor piercing action is reached when facing made of copper and iron is used.

By trapping the cumulative jet in some non-dense media and subsequent metallographic analysis it has been established that during the process of shaping the jet the metal does not melt. However, the temperature of the jet may reach in this case 900 — 1,000° C.

The motion of the jet in air is accompanied by a considerable oxidation of the metal, this being connected with the increased temperature of the surface layers of the jet due to friction against the air. As a result of this, one observes an intense glow of the cumulative jet, particularly in the case when the facing is made of duraluminum or aluminum. This makes it possible to photograph the motion of the jet in its own light, by means of streak photography, and thus determine its velocity of motion. A typical streak photograph of the motion of a jet is shown in Fig. 171.

The cumulative jet retains its monolithic properties only at the first stages of its motion. Soon the velocity gradients cause it to become dispersed into particles. The initial stage of destruction of the jet is shown in Fig. 172, which is a photograph obtained at an exposure on the order of 10^{-8} sec. Such an exposure was reached by using an electron-optical shutter, based on the Kerr effect. The apparatus with electron-optical shutter was developed by B. A. Ivanov. The photograph shows clearly the formation of necks, along which the jet becomes broken up into individual particles.

The use of this method inconjunction with microsecond x-ray photographs makes it possible to reproduce completely the picture of explosion of a shaped charge in the presence of a metallic cumulative cavity (Fig. 173).

The physical notions developed above concerning the phenomenon of cumulation in the presence of facing have served as a basis for an analytical description of this phenomenon. The classical theory of convergent jets was effectively used here.

This theory was first used by G. I. Pokrovskiy to describe the process of cumulation in the presence of conical facing. In the theory he assumes that in the pinching of the facing it is possible to neglect the elastic and viscous forces, compared with the inertial forces, under the influence of which the facing becomes compressed. The correctness of this assumption is justified in a paper by M. A. Lavrent'yev, who has developed the hydrodynamic theory of cumulation. Taking this circumstance into account, the facing metal can be likened during pinching to an ideal incompressible liquid.

To proceed to a discussion of the results of this theory, it is necessary to examine first the basic premises of the theory of converging jets.

67. ELEMENTS OF THE THEORY OF CONVERGING JETS

Let us consider the laws of motion of an incompressible liquid in the case of convergence of two identical flat jets (i.e., we consider the two-dimensional problem).

It is known that when two identical jets (of equal velocity and equal flow) converge at a certain angle (2α), two other jets are formed, in which the liquid moves in opposite directions, along the bisector of the convergence angle (Fig. 174). Here, as follows from the laws of conservation of mass, momentum, and energy, the velocities of the outgoing jets are equal in magnitude but opposite in sign, and are equal to the velocities of the initial converging jets. The masses of liquid, moving in the outgoing jets, are different. In the jet flowing in the same direction as the x-axis projection of the motion of the initial jets, the liquid mass is greater than in the jet flowing in the opposite direction. Let us prove the correctness of these statements.

It is obvious that we are considering a problem analogous to the problem of the outflow of a jet striking a perfectly solid surface at an angle α ; this surface coincides with the plane xOs.

Let the liquid flow per second in the incident jet be m_0; the velocity of the liquid in the jet is u_0. We note analogously for the jets flowing to the right and to the left, the rates of flow and the liquid velocities by m_1, m_2, u_1, and u_2, respectively. On the basis of the laws of conservation of mass, momentum, and energy, we arrive at the following relations:

$$m_1 + m_2 = m_0, \quad -m_1u_1 + m_2u_2 = m_0u_0, \tag{67,1}$$

$$m_1u_1 + m_2u_2 = -m_0u_0 \cos\alpha, \tag{67,2}$$

$$\frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2} = \frac{m_0 u_0^2}{2}. \tag{67,3}$$

It follows therefore that

$$-u_1 = u_0 = u_2, \tag{67,4}$$

$$\frac{m_1}{m_0} = \frac{1-\cos\alpha}{2} = \sin^2\frac{\alpha}{2}, \tag{67,5}$$

$$\frac{m_2}{m_0} = \frac{1+\cos\alpha}{2} = \cos^2\frac{\alpha}{2}. \tag{67,6}$$

It is assumed here that the initial jet flows from right to left and from top to bottom.

Inasmuch as the density of the liquid remains unchanged, the masses can be replaced by the transverse cross sections of the jet, with

$$\frac{\delta_1}{\delta_0} = \sin^2\frac{\alpha}{2}, \quad \frac{\delta_2}{\delta_0} = \cos^2\frac{\alpha}{2},$$

where δ_0, δ_1, and δ_2 are the transverse dimensions of the initial and of the outgoing jets ($m = \rho\delta|u|$, where ρ is the density of the liquid).

Let now the point of intersection between the jet and the plane xOz move along the x axis in a positive direction (to the right) with a certain velocity w, and then the velocity of the jet outgoing to the right (jet I) and to the left (jet II) will be

$$\left.\begin{aligned} w_1 &= u_1 + w, \\ w_2 &= u_2 + w. \end{aligned}\right\} \qquad (67,7)$$

Here and throughout, $u_2 < 0$. In this case, in the coordinate system in which the point of intersection is motionless, the resultant velocity w_0 and the direction (angle β) of motion of the initial jet are determined from the relations (Fig. 175)

$$w_0^2 = u_0^2 + w^2 - 2u_0 w \cos\alpha, \qquad (67,8)$$

$$\sin\beta = \frac{w}{w_0}\sin\alpha. \qquad (67,9)$$

In this system of coordinates we observe motion of a flat current, with limited thickness along the front and intersecting the plane xOz at a certain angle $\gamma = 180° - (\alpha + \beta)$.

If the velocity of the liquid w_0 and the angle β between the direction of motion and the front of the current moving in the plane xOz are specified, and also the angle α, we can change over to a coordinate system in which the point of intersection of the plane and the front of the liquid remains stationary.

It is interesting to consider three cases of liquid motion.

1. Let the angle $\beta = \pi/2$ (direction of motion of the liquid perpendicular to its front).

Then

$$w_0 = w\sin\alpha = \sqrt{w^2 - u_0^2} \qquad (67,10)$$

From this we get

$$w = -\frac{u_0}{\cos\alpha}, \quad w_0 = -u_0 \operatorname{tg}\alpha, \quad w_1 = -u_0\frac{1+\cos\alpha}{\cos\alpha}.$$

Taking (67.7) into account, we get

$$w_1 = w_0\frac{1+\cos\alpha}{\sin\alpha} = \frac{w_0}{\operatorname{tg}\frac{\alpha}{2}}, \quad w_2 = -u_0\frac{1-\cos\alpha}{\cos\alpha} \tag{67,11}$$

or

$$w_2 = w_0\frac{1-\cos\alpha}{\sin\alpha} = w_0 \operatorname{tg}\frac{\alpha}{2}. \tag{67,12}$$

2. Let the angle $\beta = \pi/2 - \alpha$ (direction of motion of the liquid is perpendicular to the x axis).

Then $$w_0 = w \operatorname{tg}\alpha = \sqrt{u_0^2 - w^2}. \tag{67,13}$$

Hence $w = u_0 \cos\alpha, \quad w_0 = -u_0 \sin\alpha, \quad w_1 = -u_0(1+\cos\alpha)$

or

$$w_1 = w_0\frac{1+\cos\alpha}{\sin\alpha} = \frac{w_0}{\operatorname{tg}\frac{\alpha}{2}}, \tag{67,14}$$

$$w_2 = -w_0\frac{1-\cos\alpha}{\sin\alpha} = -w_0 \operatorname{tg}\frac{\alpha}{2}. \tag{67,15}$$

3. Let $w_2 \equiv 0$, corresponding to a fully braked jet II. We then obtain from the conditions (67.7), (67.8) and (67.9)

$$-u_0 = w, \quad w_0 = -u_0\sqrt{2(1-\cos\alpha)} = -u_0 \sin^2\frac{\alpha}{2}, \tag{67,16}$$

$$w_1 = -2u_0 = \frac{w_0}{\sin\frac{\alpha}{2}}, \tag{67,17}$$

$$\sin\beta = -\sin\alpha\frac{u_0}{w_0} = \cos\frac{\alpha}{2}, \tag{67,18}$$

Hence

$$\beta = \frac{\pi-\alpha}{2}. \tag{67,19}$$

An analysis of these simple relations shows that when the stream crosses the plane xOz, a redistribution takes place of the mass and the energy of the initial stream, between the two resultant streams. With this, jet I, which moves to the right, has a small mass but a high energy, while jet II, to the contrary, has a large mass but low energy.

If the velocity w_0 and the angles α and β are specified, the values of u_0, w, w_1, and w_2 are determined in the general case from the relations

$$\left.\begin{aligned} -u_0 &= w_0 \frac{\sin(\alpha+\beta)}{\sin\alpha}, \\ w &= w_0 \frac{\sin\beta}{\sin\alpha}, \\ w_1 &= w_0 \frac{\sin\beta+\sin(\alpha+\beta)}{\sin\alpha}, \\ w_2 &= w_0 \frac{\sin\beta-\sin(\alpha+\beta)}{\sin\alpha}. \end{aligned}\right\} \tag{67,20}$$

The length of each jet, as is obvious, is equal to the length of the initial jet. This follows from the fact that in a stationary system of coordinates the velocities, and consequently, the lengths of all the jets, are the same. The ratio of the masses and energies of these streams, as shown by relations (67.5), (67.6), and (67.20) are determined by the formulas

$$\frac{m_1}{m_2} = \mathrm{tg}^2\frac{\alpha}{2}, \tag{67,21}$$

$$\frac{E_1}{E_2} = \frac{m_1 w_1^2}{m_2 w_2^2} = \left[\mathrm{tg}\frac{\alpha}{2}\cdot\frac{\sin\beta+\sin(\alpha+\beta)}{\sin\beta-\sin(\alpha+\beta)}\right]^2. \tag{67,22}$$

From this it follows that when $\alpha < \pi/2$,

$$\frac{m_1}{m_2} < 1, \quad \frac{E_1}{E_2} > 1.$$

When $\beta = (\pi - \alpha)/2$ and $w_2 \cong 0$ (Fig. 176), all the energy is transferred to jet I, and consequently the energy density in it, compared with the initial one, is greatly increased. Since the energy density per unit mass is $w^2/2$, we have

$$\frac{w_1^2}{w_0^2} = \frac{1}{\sin^2 \frac{\alpha}{2}}. \qquad (67,23)$$

When $\beta > (\pi - \alpha)/2$, the liquid in jet II moves to the right; when $\beta < (\pi - \alpha)/2$ it moves to the left; consequently the energy density in jet I decreases compared with the quantity given by (67.23).

When the length of the initial jet in the stationary system of coordinates, or the length of the front of the stream in the system of coordinates in which the point of intersection of the jet and the plane xOz is stationary, is small, the frontal part of the jet will not be described by the relations derived above, inasmuch as the liquid in this part will move in a non-stationary fashion, and will obey a more complicated law. An examination of the effect of collision at a certain angle of two unequal jets is somewhat more complicated than that of two equal jets. We shall not investigate this problem, for the principal interest in the cumulation phenomenon lies in the problem just considered.

For flat motion, a study of collisions of jets with allowance for the compressibility of the medium is relatively easy. The fundamental equations can be written in this case in the same form as for an incompressible medium. It is important, that in the case of supersonic jets striking and scattered in the xOz plane, there are formed one or several diagonal shocks. This leads to an increase in the entropy of the medium, and consequently, as it expands to the initial (atmospheric) pressure, it causes the gas density to be less than initial, and the temperature to be higher than initial. In the expansion of liquid jets, phenomena similar to cavitation can take place, i.e., the jet may become broken up.

68. ELEMENTS OF CUMULATION THEORY IN THE PRESENCE OF A METALLIC FACING

The theory of the cumulative effect in the presence of a metallic facing has been developed most completely for charges with cavities of conical form.

M. A. Lavrent'yev has examined in detail the following problem: the conical-facing elements, with constant wall thickness, instantaneously acquire a velocity normal to the generatrix of the cone. When the problem is so formulated, it can be reduced to analysis of the collision between jets in an axially-symmetrical steady-state flow of an ideal liquid. Fig. 177 shows a section through such a flow, obtained by approximate methods.

The velocity of the flow at the origin is zero. At $x \to -\infty$ the flow represents a cylindrical jet of radius r_1 and velocity $-u_0$. At $x \to \infty$ the flow is a cylindrical jet of radius r_0 and velocity u_0. The sections of the shroud boundary have a common asymptote

$$y = x \operatorname{tg} \alpha + a, \tag{68,1}$$

with

$$\operatorname{tg}\frac{\alpha}{2} = \frac{r_0}{r_1}. \tag{68,2}$$

From the condition that the velocity of the flow on the free surface of the shroud is u_0, the flow of liquid in the shroud is

$$\pi\left(r_0^2 + r_1^2\right) u_0 = 2\pi y\, \delta u_0, \tag{68,3}$$

where δ is the thickness of the shroud.

From this we can readily obtain an approximate expression for the thickness of the shroud δ as a function of the coordinate y and the radii of the jets r_0 and r_1:

$$\delta = \frac{r_0^2 + r_1^2}{2y} = \frac{r_0^2}{2y}\left(1 + \operatorname{ctg}^2\frac{\alpha}{2}\right). \tag{68,4}$$

Formula (68.4) is exact as $y \to \infty$.

To calculate the parameters of the cumulative jet, let us consider the motion of the liquid when jets collide in a coordinate system that moves uniformly to the right with velocity

$$v_0 = \frac{u_0}{\cos\alpha},$$

$$\xi = x - v_0 t.$$

In the new system of coordinates ξ, y, the conical shroud C will have a velocity w_0, which as $x \to \infty$ becomes orthogonal

to the generatrix of the asymptotic cone A, which indeed corresponds to the pinching case under consideration (the elements of the cone have a velocity normal to the generatrix). With this

$$w_0 = u_0 \operatorname{tg}\alpha, \qquad (68,5)$$

the velocity of the cumulative jet is

$$w_1 = w_0 \frac{1+\cos\alpha}{\sin\alpha}, \qquad (68,6)$$

the velocity of the pestle is

$$w_2 = w_0 \frac{1-\cos\alpha}{\sin\alpha}. \qquad (68,7)$$

It is easy to note that relations (68.6) and (68.7) are identical to the corresponding relations for the jet velocities, obtained in the examination of the flat problem ($\beta = \pi/2$). With this, as follows from the theory of collisions of flat jets, the length of the cumulative jet is equal to the length of the generatrix of the cone, and the radius of the jet r_0 is constant.

In real shaped charges the rate of pinching of the facing is not constant, since the momentum which the facing acquires during the explosion of the charge is likewise not constant along the generatrix, and this leads to the appearance of velocity gradients along the cumulative jet and to derangement of the jet. In addition, a change in the angle α takes place during the pinching of the elements of the facing and the formation of the corresponding jet elements from the

Lavrent'yev calculated the parameters of a cumulative jet for charges with conical cavities and nearly conical cavities, with allowance for these factors.

Let us give the solution of this problem for the particular case of a conical cavity with constant facing thickness.

We consider the motion of a conical element which has at a fixed instant of time t an abscissa x (Fig. 178).

In this case

$$y = x \operatorname{tg} \alpha. \qquad (68,8)$$

We put

$$u = 2\delta y = 2\delta x \operatorname{tg} \alpha. \qquad (68,9)$$

In addition, assuming a linear distribution of the momentum along the generatrix of the cone, we obtain for the velocity w of the pinching of the facing

$$w = w_0 (1 - kx). \qquad (68,10)$$

The length of the normal between the element A of the facing and the axis is $\eta = y/\cos \alpha$.

At an instant of time $t_{in} > t$, the facing element under consideration, whose abscissa is x, forms a jet element at a point with abscissa x_0, equal to

$$x_0 = x + y \operatorname{tg} \alpha. \quad (68,11)$$

It is obvious that

$$t_{in} = \frac{\eta}{w}. \quad (68,12)$$

By virtue of the fact that the velocity w is not constant, the element will turn within the time $t_{in} - t$ through an angle $\Delta\alpha$, with

$$\Delta\alpha = -\frac{\eta}{w}\frac{dw}{dx}. \qquad (68,13)$$

Hence, putting

$$\bar{\alpha} = \alpha + \Delta\alpha \qquad (68,14)$$

and using (68.6) and (68.10) for the velocity of the elements of the jet, we obtain

$$w_1 = w_0 (1 - kx) \frac{1 + \cos \bar{\alpha}}{\sin \bar{\alpha}}. \qquad (68,15)$$

The angle is

$$\bar{\alpha} = \alpha + \Delta\alpha = \frac{\alpha + kx \operatorname{tg}\alpha}{1 - kx}. \qquad (68,16)$$

For the radius of the jet we have, in accordance with formula (68.4)

$$r_x^2 = \frac{2\delta y}{1 + \operatorname{ctg}^2 \frac{\bar{\alpha}}{2}} = \frac{2\delta x \operatorname{tg}\bar{\alpha}}{1 + \operatorname{ctg}^2 \frac{\bar{\alpha}}{2}}, \qquad (68,17)$$

which yields after simple transformations

$$r_x = \sqrt{\delta x (\operatorname{tg}\bar{\alpha} - \sin\bar{\alpha})}, \qquad (68,18)$$

where x is the distance from the frontal part of the jet.

In the theory developed we did not establish relations for the rate of pinching of the cumulative facing as a function of the parameters of the explosive charge. Without this dependence we can determine numerically the values of the basic parameters of the cumulative jet. In addition, Lavrent'yev's theory does not take into account the strength characteristic of the facing metal, which in many cases may exert an influence on the conditions of formation of the cumulative jet.

Let us examine a method for a theoretical determination of the parameters of the cumulative jet, with allowance for the mass and energy of the active part of the cumulative charge. This method has been developed by Baum and Stanyukovich, who considered also the question of the limiting conditions of formation of cumulative jet as a function of the strength characteristics of the facing metal.

We first consider the motion of the shell as a whole (the motion of the center of gravity of the shell) under the action of the scattering detonation products, without account of the pinching. This problem is solved readily on the basis of the general theory of the sweeping of bodies by the detonation products.

The equation of conservation of energy for one-dimensional unilateral escape of detonation products, under the assumption that the detonation is instantaneous, with simultaneous sweeping of some body of mass M, can in the case of total expansion of the explosion products be written in the form

$$\frac{Mu_m^2}{2} + \frac{s}{2}\int_0^{u_m} \rho u^2 \, dx = mQ_v. \qquad (68,19)$$

Here u_m is the limiting velocity of the swept body, s is the area of the transverse cross section of the body, and m is the mass of the explosion product.

Expression (68.19) can be readily derived by recognizing that the energy remaining in the detonation products is

$$\frac{1}{2}\int_0^{u_m} u^2\,dm,$$

where $dm = s\rho dx$.

As shown by the theory of one-dimensional escape of detonation products, in the case of complete expansion of the explosion products, i.e., when $p = p_a$, where p_a is atmospheric pressure, we have

$$u = \frac{x}{t}, \quad \rho = \frac{m}{su_m t}. \qquad (68,20)$$

Therefore

$$\frac{1}{2}\int_0^{u_m} u^2\,dm = \frac{m}{2}\int_0^{u_m} \frac{x^2\,dx}{u_m t^3} = \frac{mu_m^2}{6}; \qquad (68,21)$$

Hence, taking account of the fact that $D^2 \approx 16\, Q_v$ when the isentropic exponent γ has a value 3, we obtain from (68.19)

$$\frac{u_m}{D} = \frac{1}{2\sqrt{2\left(\frac{M}{m} + \frac{1}{3}\right)}}. \qquad (68,22)$$

This is the limiting velocity of the swept body and explosion products on the border of the swept body.

The scattering of the explosion products, as we already know, is not uniform, but, by introducing (see Section 65) the concept of the mass of the active part of the charge, $m = m_a$, we can assume that the explosion products of the active part move along the axis of the charge. Consequently, we must take m in (68.19) — (68.22) to mean the mass m_a of the active part of the charge. Strictly speaking, relations (68.19), and consequently also (68.22), are valid for the case of instantaneous detonation.

Since the detonator is established in that part of the charge, which is opposite the cumulative cavity, the energy density in the case of a real detonation, calculated per unit mass of the active part of the charge, is greater than Q_v. We denote this energy density by

$$Q^* = \beta^2 Q_v, \tag{68,23}$$

where $\beta > 1$. With this, relation (68.22) assumes the form

$$\frac{u_m}{D} = \frac{\beta}{2\sqrt{2\left(\frac{M}{m_a} + \frac{1}{3}\right)}}. \tag{68,24}$$

Let us calculate the value of β.

The energy density on the front of the detonation wave for $\gamma = 3$ is equal to

$$Q_{in} = \frac{1}{2}\left[u_{in}^2 + \frac{p_{in}}{\rho_{in}}\right]. \tag{68,25}$$

Inasmuch as

$$u_{in} = \frac{D}{4}, \quad p_{in} = \frac{\rho_0 D^2}{4}, \quad \rho_{in} = \frac{4}{3}\rho_0,$$

we have

$$Q_{in} = \frac{D^2}{8} = 2Q_v. \tag{68,26}$$

Let us calculate the energy density Q_0 of that part of the charge in which $u \equiv 0$ (prior to the scattering). It is obvious that in this case

$$Q_0 = \frac{p_k}{\rho_k(\gamma - 1)} = \frac{p_{in}}{6\rho_0} = \frac{D^2}{24} = \frac{2}{3}Q_v, \tag{68,27}$$

for when $\gamma = 3$ we have $p_k = (8/27)p_{in}$ and $\rho_k = (8/9\rho_0)$.

We can put with a sufficient degree of accuracy

$$Q^* = \frac{1}{2}(Q_{in} + Q_0) = \frac{4}{3}Q_v. \tag{68,28}$$

There is no sense in calculating more accurately the distribution of the energy of the active part of the charge, although this can be readily done if the outline of this part is known.

From the theory of one-dimensional scattering we know that 4/9 of the entire mass of the charge is scattered inside of the active part, and that the energy of this part is 16/27 of the entire energy of the charge, while 5/9 of the mass and 11/27 of the energy go in the opposite direction. Consequently, in the first case the energy density will exceed the average value by 4/3 times. On the basis of our calculations, we assume $\beta^2 = 4/3$, from which we obtain as a final expression

$$\frac{u_m}{D} = \frac{1}{\sqrt{6\left(\frac{M}{m_a} + \frac{1}{3}\right)}} \approx \frac{0{,}41}{\sqrt{\frac{M}{m_a} + \frac{1}{3}}}. \qquad (68,29)$$

The center of gravity of the facing will move at this velocity, if the pinching is disregarded. In a real detonation, the average density of the explosion products of the active part ρ^* is greater than the initial density ρ_0 of the shaped charge. The value of ρ^* can be taken to be, with sufficient practical accuracy,

$$\rho^* = \frac{1}{2}\left(\frac{4}{3}\rho_0 + \frac{8}{9}\rho_0\right) = \frac{10}{9}\rho_0. \qquad (68,30)$$

Relation (68.30) should be used in the calculation of the active mass m_a. For a steel conical facing of a 76-mm shell 2 mm thick we obtain, for h/d = 1.58,

$$M = 130\,g, \quad m_a = \frac{\pi r_0^3}{3}\rho_0 \approx 45\,g.$$

For D - 7600 m/sec (TG alloy) we have u_m = 1750 m/sec.

Let us consider now the motion of the facing with simultaneous pinching. To explain the basic laws observed in this case, let us consider the following scheme.

Let the detonation occur instantaneously. A flat plate of mass M moves under the influence of the expanding detonation products in such a way, that its lower part glides along the symmetry axis (Fig. 179). It is obvious that in the instantaneous-detonation scheme, the velocity of the plate will be normal to its surface, i.e., $u_m = w_{0max}$.

This velocity corresponds to the maximum velocity of collapse of the facing. However, different points of the plate (facing) will actually have different velocities of motion relative to the symmetry axis.

This is explained by the fact that the distances to the symmetry axes will be different for different facing elements, inasmuch as anybody acquires its limiting velocity not instantaneously, but along its motion. In addition, the angle of inclination of the facing to the symmetry axis changes during its pinching (increases). Therefore as the average rate of pinching of the facing will be

$$\bar{u} = \bar{w}_0 = \eta \frac{0{,}41D}{\sqrt{\frac{M}{m_a} + \frac{1}{3}}}. \quad (68{,}31)$$

The coefficient η takes into account the fact that not all the energy of the active part is taken into account, and includes also the correction for the average increase in the angle of inclination of the facing to the axis, which takes place as the facing is pinched. We shall consider below the same problem in greater detail, and will then calculate the coefficient $\bar{\eta}$.

Assuming in the present case that the direction of motion of the facing is perpendicular to its generatrix ($\beta = \pi/2$), we finally obtain in the ideal-liquid scheme the following approximate relation for the velocity of motion of the jet

$$\bar{w}_1 = \bar{\eta} \frac{0{,}41D}{\sqrt{\frac{M}{m_a} + \frac{1}{3}}} \frac{1}{\operatorname{tg} \frac{\alpha}{2}}. \quad (68{,}32)$$

As will be shown below, $\bar{\eta} = 0.67$ (for this example); therefore $w_1 \approx 7300$ m/sec, which is close to the experimentally established value.

It follows from the theory that as the angle of the cone decreases, the velocity of the jet should increase. This agrees with the experimental data (Table 103).

It also follows from (68.32) that if geometric similarity is observed with respect to the forms of the charge and the cavity, with respect to the constancy of the ratio M/m_a, and other conditions being equal (quality of the explosive, material of the facing), the velocity of the cumulative jet should be independent of the diameter of the charge or of the cavity, something actually observed in experiment (Table 104).

If the thickness of the facing, δ, is not too small (and otherwise no normal jet is formed), then as δ is decreased the velocity of the jet should increase up to a certain limit, something confirmed by the data of Table 105.

Let us consider qualitatively the effect of compressibility of a metal of the facing and its resistance in strength on the process of pinching and formation of the cumulative jet.

Inasmuch as the height and thickness of the layer of the active part of the charge decreases from the axis to the base of the cavity, the rate of pinching also decreases for the peripheral parts of the facing, compared with the internal ones. As soon as the pressure developed during the collision of the corresponding parts of the facing drops to a value p_{st}, i.e., to a certain limiting value, the process of jet formation terminates. Here p_{st} corresponds to the internal pressure, proportional to the cohesion forces.

As a result of the lack of constancy of the rates of pinching of the different elements of the facing, a velocity distribution is formed also along the cumulative jet, wherein the leading parts of the jet acquire velocities greater than the rear parts. The considerable velocity gradients lead to a stretching of the jet and to its breakup into a series of individual parts.

Let us cite a few considerations pertaining to the described phenomenon. The maximum pressure in the impact between two identical bodies, as is well known, is described, independently of angle of collision, by the following relation

$$p_x = \frac{\rho_a u_0^2}{4\left(1 - \frac{\rho_a}{\rho_x}\right)}, \qquad (68,33)$$

where ρ_a and ρ_x are the densities of the bodies prior to impact and on the front of the shock wave produced in (compression wave) upon impact, and u_0 is the velocity of the shock.

Relation (68.33) is a special case of a more general relationship (see Section 73).

Using the law of compressibility

$$p = f(\rho) \approx A\left[\left(\frac{\rho_x}{\rho_a}\right)^n - 1\right], \qquad (68,34)$$

we can exclude ρ_x from (68.33) and (68.34), and determine $p_x = \varphi(\rho_a, u_0^2)$.

Inasmuch, as we have already indicated, u_0 has different values for each facing element, the quantity $p_x \sim u_0^2$ will also vary.

Let us assume approximately that the active mass of the explosive per facing element depends on the distance r of the given element from the symmetry axis:

$$m_{ar} = m_{a0}\left(1 - b\frac{r}{r_0}\right), \qquad (68,35)$$

where r_0 is the radius of the base of the cumulative cavity, and m_{a0} is the element of the mass of the active part of the charge and the symmetry axis, and $b < 1$.

We then have approximately

$$\frac{p_{xr}}{p_{x0}} = \frac{u_{mr}^2}{u_m^2} = \frac{m_{ar}}{m_{a0}} = 1 - b\frac{r}{r_0}. \qquad (68,36)$$

Here p_{xr}, p_{x0}, u_{mr}, and u_m are the pressures and velocities on the axis and at a distance r from it. It follows therefore, that actually for any charge there is always such a radius $r = r_{st}$, at which $p_{xr} = p_{st}$. The material of the facing elements, becoming pinched, will no longer produce a cumulative jet if $r > r_{st}$

It is obvious that the limiting pinching velocities should be less than the velocity of armor piercing of the jet.

The conditions for the formation of a cumulative jet greatly influence the laws of its motion and its action on a partition. We note that when the angle α approaches $\pi/2$, the cumulative jet will no longer be formed inasmuch as the speed of pinching, which is proportional to cos α, is small and the pinching pressure is less than p_{lim}. In view of the fact that the pressure developed upon impact of the detonation wave against the facing,

$$p_{sp} = p_{in}(1 + 1{,}4\sin^2\alpha) \qquad (68{,}37)$$

increases with increasing angle α, the facing should break up into a series of fragments. This fragment, scattered at an angle $\alpha \leqslant \pi/2$, will have a directed motion with a velocity

$$u_{mr} = u_m \sin\alpha \sqrt{1 - b\frac{r}{r_0}} = 0{,}41 D \sin\alpha \sqrt{\frac{1 - b\frac{r}{r_0}}{\frac{M}{m_a} + \frac{1}{3}}}. \qquad (68{,}38)$$

Only the fragments traveling from the central portion of the facing will have in this case a considerable velocity.

Let us determine approximately the optimal thickness of the cumulative facing. We carry out the calculation for a cone. Let the length of the jet be $l = l_0 \psi$, where ψ is the coefficient of elongation of the jet under conditions of armor piercing.

The mass of the jet is $M_1 = \pi l_0 \psi \bar{r}_0^2 \rho_0,$ (68,39)

where $\bar{r}_0$ is the average radius of the jet.

According to (67.5), the mass of the facing is

$$M = \frac{M_1}{\sin^2\frac{\alpha}{2}} = \frac{\pi l_0 \psi \bar{r}_0^2 \rho_0}{\sin^2\frac{\alpha}{2}} = \frac{2}{3}\pi R_c h \delta\rho_0. \qquad (68{,}40)$$

Here R_0 is the radius of the base of the cone, h is the height of the cone ($h = l\cos\alpha$, where l is the generatrix of the cone, approximately to the initial length of the jet l_0).

From (68.40) we obtain $\bar{r}_0 = \sqrt{\frac{2}{3}\frac{R_c \delta\cos\alpha}{\psi}} \sin\frac{\alpha}{2}.$ (68,41)

Bearing in mind that the maximum effective length of the jet, at which the maximum armor piercing is reached, is

$$l_{ef} = l_0 \psi_{ef}$$

(ψ_{ef} is the maximum elongation at which the jet still remains monolithic), and solving Eq. (68.41) with respect to δ, we can approximately determine the optimal thickness of the facing:

$$\frac{\delta_{opt}}{R_c} = \frac{3\psi_{ef} \bar{r}_0^2}{2R_c^2 \cos\alpha \sin^2 \frac{\alpha}{2}}. \qquad (68,42)$$

Assuming, on the basis of experiment, for a steel facing $\psi_{ef} \approx 3$ and $\bar{r}_0 = 0.80$ mm ($\bar{r}_0$ changes little with varying δ), we obtain for the case which we are considering (R = 30 mm, $2\alpha = 35°$), $\delta_{opt} = 2.20$ mm, which is close to the experimentally established value (2 — 2.5 mm).

In conclusion let us establish the limiting ratio of the mass of the facing to the mass of the active part of the cumulative charge, at which the formation of the jet will cease.

We shall assume here (see (68.33)) that the limiting pressure at which the jet formation ceases is

$$\bar{p}_{st} \leqslant \frac{\rho_a \bar{u}_{0st}^2}{4\left(1 - \frac{\rho_a}{\rho_x}\right)}. \qquad (68,43)$$

If we know $\bar{p}_{st}$ for the given material, and the corresponding value of ρ_a/ρ_x, we can determine u_{st}.

At values $u_0 < u_{st}$, as we have already indicated, further flow of the metal ceases and the cumulative jet no longer forms.

Using (68.31) with account of the coefficient η, we obtain

$$\left(\frac{M}{m_a}\right)_{st} = \left(\frac{0{,}41 D\eta}{\bar{u}_{0\,st}}\right)^2 - \frac{1}{3}. \qquad (68{,}44)$$

$(M/m_a)_{st}$ can be determined from experiment. In experiments carried out by Baum, with a 56-mm charge of ammotol 90/10 of density 1.2 g/cm^3 (D = 3400 m/sec) and a copper conical facing (d_{base} = 45 mm, cone angle $2\alpha = 37°$, δ = 3 mm, M = 429 grams), no jet was formed at all. Here $(M/m_a)_{st} \approx 4.57$ and $\bar{u}_{0\,st}$ = 420 m/sec. Using (68.43), we obtain $p_{st} \approx 70{,}000$ kg/cm^2.

Knowing the value of $\bar{u}_{0\,st}$, we can determine $(M/m_a)_{st}$ and the limiting thickness of facing for a cumulative charge made of any explosive.

For a charge made of the TG alloy, $(M/m_a)_{st}$ = 0.21, and the limiting thickness of the copper cone δ_{st} = 16.0 -- 17.0 mm.

In conclusion we note that the quantity p_{st}, established for a given material under the conditions of pinching the facing, should be considerably less than the quantity p_{st}, characteristic of the same material under the conditions of armor piercing by a cumulative jet. This is explained by the fact that the strength characteristics of the materials are not constant, but depend appreciably on the character of the load applied to the partition and on the conditions of its deformation.

For example, for facing made of steel, the limiting velocity of the cumulative jet w_{1cr}, acting on a steel partition, is 2,000 meter per second, which corresponds to a value $p_{lim} \approx 4.8 \times 10^5$ kg/cm^2. When $w_1 < w_{1cr}$, the armor piercing almost completely disappears. The large value of p_{lim} in this case is explained by the instability of the cumulative jet and the fact that it is continuously destroyed during the armor-piercing process.

69. EFFECT OF NON-UNIFORMITY OF PINCHING OF THE FACING ON THE DISTRIBUTION OF THE VELOCITIES IN A CUMULATIVE JET

Starting from the theory of the active part of the cumulative (shaped) charge, we can establish the character of compression of the facing and the distribution of the velocities along the cumulative jet, a character which is of great interest.

In view of the fact that the ratio $(M/m_a)_i$ for individual facing elements is not constant, but decreases from the vertex towards the base of the cumulative cavity, the rate of compression of the individual elements of the facing will also change, in accordance with (68.31). It is necessary to take into account here the fact that the angle α_i of inclination of the various facing elements to the charge axis will also change during the compression process. The joint influence of these factors (the fact that both u and α are not constant) will result in an uneven distribution of velocities w_{1i} along the cumulative jet. The magnitudes of the active parts of the different facing elements can be determined geometrically (Fig. 180). For this purpose it is sufficient to know the shape of the facing and of the charge, and also the weight of the facing and the weight of the body of the charge.

Inasmuch as we are considering the action of the explosion products from the active part on the facing, the expansion of these products can, as before, be assumed one-dimensional. In this case, for instantaneous detonation the following relation will hold true:

$$M_i \frac{du_i}{dt} = M_i u_i \frac{du_i}{dn_i} = M_i \frac{du_i^2}{2dn_i} = Sp, \qquad (69,1)$$

where M_i is the mass of the i-th element of the facing, u_i -- its velocity, S -- area, and n_i -- portion of the path of the i-th element to the symmetry axis.

Since the mass of the i-th element of the active part of the charge is $m_i = SL\,\rho_0$, and

where

$$p=\bar{p}_{in}\left(\frac{L}{L+n}\right)^3, \qquad (69,2)$$

we have

$$\bar{p}_{in}=\frac{\rho_0 D^2}{8},$$

Hence

$$M_i=\frac{du_i^2}{2dn_i}=\frac{S\rho_0 D^2}{8}\left(\frac{L}{L+n}\right)^3=\frac{m_i D^2}{8L}\left(\frac{L}{L+n}\right)^3. \qquad (69,3)$$

$$\frac{d\left(\frac{u_i}{D}\right)^2}{d\left(\frac{n}{L}\right)}=\left(\frac{m}{M}\right)_i\frac{1}{4\left(1+\frac{n}{L}\right)^3}. \qquad (69,4)$$

Integrating (69.4) subject to the condition that $u=0$ when $n=0$, we arrive at

$$\frac{u_i}{D}=\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{m}{M}\right)_i\left[1-\left(\frac{L}{L+n}\right)^2\right]}. \qquad (69,5)$$

Inasmuch as

$$\frac{L}{L_0}=\frac{m}{m_0}=f\left(\frac{\lambda}{l}\right), \qquad (69,6)$$

where m_0 and L_0 are the mass and the length of the element of the active part directly adjacent to the charge axis, we have

$$\frac{u_i}{D}=\frac{1}{2}\sqrt{\frac{f\left(\frac{\lambda}{l}\right)}{2}\frac{m_0}{M_i}\left[1-\left(\frac{L}{L+n}\right)^2\right]}. \qquad (69,7)$$

The function $f(\lambda/l)$ can be readily determined geometrically for any specified form of cavity. The mass of any element of the facing is also known; in the most general case we have

$$M_i=M_0\varphi\left(\frac{\lambda}{l}\right).$$

Assuming the angle under which any element of the facing approaches the charge axis remains unchanged (this angle is equal to $90^\circ - \alpha_0$), we readily determine the rate of collapse of an arbitrary facing element.

In fact

$$n_i = \lambda \operatorname{tg} \alpha_0, \tag{69,8}$$

Therefore Eq. (69.7) can be rewritten as

$$\frac{u_i}{D} = \frac{1}{2}\sqrt{\frac{m_0}{2M_0}\frac{f}{\varphi}\left[1-\left(\frac{L}{L+\lambda \operatorname{tg}\alpha_0}\right)^2\right]}. \tag{69,9}$$

In the case of a conical cavity with facing of constant thickness

$$f\left(\frac{\lambda}{l}\right) = 1 - \frac{\lambda}{l}, \quad \varphi\left(\frac{\lambda}{l}\right) = 1, \quad L = L_0\left(1 - \frac{\lambda}{l}\right).$$

Here

$$\frac{m_0}{M_0} = \frac{L_0}{\delta}\frac{\rho_0}{\rho_1}\sin\alpha_0, \quad l = \frac{R_0}{\sin\alpha_0} \text{ and } L_0 = R_0,$$

where δ is the thickness of the facing, ρ_0 the density of the explosive, ρ_1 the density of the facing metal, and R_0 the radius of the base of the cone.

Now Eq. (69.5) assumes the form

$$\frac{u_i}{D} = \frac{1}{2}\sqrt{\frac{R_0\rho_0\sin\alpha_0}{2\delta\rho_1}\left(1-\frac{\lambda}{R_0}\sin\alpha_0\right)}\times$$

$$\times\sqrt{1-\left[\frac{R_0\left(1-\frac{\lambda}{R_0}\sin\alpha_0\right)}{R_0\left(1-\frac{\lambda}{R_0}\sin\alpha_0\right)+\lambda\operatorname{tg}\alpha_0}\right]^2}. \tag{69,10}$$

The value of w_{1i} is again given by

$$w_{1i} = \frac{u_i}{\operatorname{tg}\frac{\alpha_0}{2}}. \tag{69,11}$$

For the specific example given above (R_0 = 30 mm, δ = 2 mm, α_0 = 17°30', ρ_0 = 1.6 g/cm^3, ρ_1 = 7.8 g/cm^3, and D = 7600 m/sec), using Eqs. (69.10) and (69.11), we determine the corresponding values of u_1 and the distribution of the velocities w_{1i} in the cumulative jet. The results of the calculations are shown in Fig. 181 (curve 1).

It is clear from the figure that the maximum velocity gradients in the cumulative jet due to the facing elements adjacent to the vertex and to the base of the cavity. This should result in a rapid detachment of the jet from the pestle and an intense dispersion of the frontal part of the jet. The foregoing agrees with the results of the experimental investigations.

In real charges the detonation does not occur instantaneously, and we must take into account the time during which the detonation wave covers the distance from the vertex to the base of the cone. During that time the vertex of the facing covers a certain distance x_0, and consequently the angle of inclination of the facing changes (Fig. 182).

Inasmuch as for the vertex of the facing we have

$$\frac{u}{D}=\frac{1}{2}\sqrt{\frac{m_0}{2M_0}\left[1-\left(\frac{x}{L_0+x}\right)^2\right]}=\frac{1}{D}\,\frac{dx}{dt}, \qquad (69,12)$$

where

$$\frac{m_0}{M_0}=\frac{L_0\rho_0\sin\alpha_0}{\delta\rho_1},$$

we get

$$\int\limits_0^{x_0}\frac{dx}{\sqrt{1-\frac{x^2}{(L_0+x)^2}}}=\frac{D\tau}{2}\sqrt{\frac{1}{2}\,\frac{L_0\rho_0\sin\alpha_0}{\delta\rho_1}}. \qquad (69,13)$$

Here τ is the time during which the detonation wave covers the path from the vertex to the base of the cone. It is obvious that

$$\tau=\frac{l\cos\alpha_0}{D}.$$

Inasmuch as

$$\int_0^{x_0} \frac{dx}{\sqrt{1-\frac{x^2}{(L_0+x)^2}}} = \int_0^{x} \frac{(L_0+x)\,dx}{\sqrt{L_0^2+2L_0x}} \approx$$

$$\approx \frac{L_0}{6}\left(1+2\frac{x_0}{L_0}\right)^{\frac{3}{2}} + \frac{L_0}{2}\left(1+2\frac{x_0}{L_0}\right)^{\frac{1}{2}} - \frac{2}{3} = f(x_0) \quad (69,14)$$

(when $x_0/L_0 < 1$ this solution is sufficiently accurate), we get

$$f(x_0) = \frac{l\cos\alpha_0}{2}\sqrt{\frac{1}{2}\frac{L_0\rho_0}{\delta\rho_1}\sin\alpha_0}. \quad (69,15)$$

We determine x_0 from (69.14) and (69.15).

Knowing x_0, we can readily determine $\bar{\alpha}$ -- the angle of inclination of the facing to the axis of the charge at the instant of termination of the detonation:

$$\frac{x_0}{l} = \frac{\sin(\bar{\alpha}-\alpha_0)}{\sin\bar{\alpha}} = \cos\alpha_0 - \sin\alpha_0 \operatorname{ctg}\bar{\alpha}, \quad (69,16)$$

where $\bar{\alpha} - \alpha_0$ is the increment in the angle.

In solving the averaged problem, we can assume that the average value of the angle is

$$\alpha_{av} = \frac{\bar{\alpha}+\alpha_0}{2} = \alpha_0 + \frac{\Delta\alpha}{2}. \quad (69,17)$$

When solving the problem by elements, we can assume with sufficient accuracy that

$$\frac{\Delta\alpha_i}{\Delta\alpha} = 1 - \frac{\lambda}{l}. \quad (69,18)$$

Therefore, when calculating u_i and w_{1i}, we must replace the angle α_0 in formulas (69.10) and (69.11) by the angle

$$\alpha_i = \alpha_0 + \Delta\alpha_i.$$

For the example considered above we have $m_0/M_0 \approx 1$ and $f(x_0) =$

$0.23\,l$, hence $x_0/L_0 = 1$, $x_0/l = 0.3$, and $\Delta\alpha \approx 6°$.

The correction for the angle yields for the solution of the average problem (in the calculation of w_{1av}) a coefficient

$$\eta_{1_{av}} = \frac{\operatorname{tg}\frac{\alpha_0}{2}}{\operatorname{tg}\left(\alpha_0 + \frac{\Delta\alpha}{2}\right)}. \tag{69,19}$$

In our case $\alpha_0 = 17°30'$ and $\eta_{1av} \approx 0.70$.

In order to find the distribution of the velocities along the jet, it is necessary, after specifying λ, to calculate the values of $\Delta\alpha_i$ from (69.18).

In our example,

$$\Delta\alpha_i = 6°\left(1 - \frac{\lambda}{l}\right).$$

The character of the velocity distribution, takes into account the change in the angle, is shown in Fig. 181 (curve 2). When $\lambda/l = 0.5$ we get $w_1 = 7200$ m/sec, which is very close to the maximum cumulative-jet velocity established for a similar charge experimentally.

Let us calculate now the coefficient η as applied to formula (68.32). The volume of the active part of the charge is $v_a = (\pi/3) \times l^3 \sin^2\alpha_0$; the limiting volume in which the explosion products of the active part expand during the process of compression of the facing is $v = \frac{\pi}{3} l^3 \frac{\sin^2\alpha_0}{\cos\alpha_0}$ (see Fig. 180). Considering that

$$\frac{v_a}{v_a + v} = \frac{\bar{\rho}}{\rho_a} = \frac{\bar{c}}{c_H}, \tag{69,20}$$

where $\rho \sim \bar{c}$, where $\bar{c}$ is the average velocity of sound in the explosion products, we obtain

$$\frac{v_a}{v_a + v} = \frac{\cos\alpha_0}{1 + \cos\alpha_0}, \tag{69,21}$$

$v_a + v$ is the limiting volume occupied by the explosion products from the active part of the charge at the end of compression of the facing.

The ratio of the energy that still remains at that instant in the explosion products, to the initial energy, is

$$\frac{E}{E_{in}} = \left(\frac{\bar{c}}{c_{in}}\right)^2 = \frac{\cos^2 \alpha_0}{(1+\cos\alpha_0)^2}. \qquad (69{,}22)$$

The kinetic energy acquired by the facing is

$$E_c = E_{in} - E = E\left[1 - \frac{\cos^2\alpha_0}{(1+\cos\alpha_0)^2}\right],$$

$$\frac{E_c}{E_{in}} = 1 - \frac{\cos^2\alpha_0}{(1+\cos\alpha_0)^2} = \frac{1+2\cos\alpha_0}{(1+\cos\alpha_0)^2} = \eta_2^2, \qquad (69{,}23)$$

where η_2 is a coefficient that takes into account the incomplete utilisation of energy in the determination of $\bar{w}_1$ by Eq. (68.32).

For the example under consideration, $\eta_2 \approx 0.95$. Inasmuch, as was established above, an allowance for the correction for the average change of the angle α leads for this case to $\eta_1 = 0.70$, we obtain in the determination of $\bar{w}_1$ by formula (68.32) $\bar{\eta} = 0.95 \times 0.70 \approx 0.67$.

Inasmuch as the velocity in the cumulative jet increases from the frontal part towards the elements behind, and then decreases again towards the end of the jet, a velocity distribution will take place, namely, the frontal part of the jet will accelerate while the central part will be retarded. This causes a slight increase in the diameter of the jet.

To describe the final distribution of the velocities in the jet and to determine its diameter, we make use of the conservation laws.

Let us consider a simple case. Let the velocity of the jet on the interval $0 < l \leq \bar{l}$ increase linearly from zero to w_{1max} (when $x = l - \bar{l}$), and let it decrease to zero linearly over the interval $\bar{l} \leq x \leq l$ (it vanishes at $x = l$). The origin is made to coincide with the end of the jet. After a certain time, a new set of conditions is established, wherein the velocity of the frontal part of the jet becomes equal to w_{1max}. The distribution of the velocities will be linear (from w_{1max} to zero at $x = 0$). The length of the jet becomes equal to $l - \bar{l}$, and the ratio of the average areas of the jet sections before and after redistribution will be $(l - \bar{l})/l$.

Let us prove this. Before the redistribution of the velocities we have

$$m_1 = s_1 \rho \bar{l}, \quad m_2 = s_1 \rho (l - \bar{l}), \quad m = m_1 + m_2 = s_1 l \rho.$$

Averaging the velocity in the jet, we obtain

$$I_1 = \frac{m_1 w_{1\,max}}{2}, \quad I_2 = \frac{m_2 w_{1\,max}}{2},$$

$$I = I_1 + I_2 = \frac{m w_{1\,max}}{2},$$

$$E_1 = \frac{m_1 w_{1\,max}^2}{6}, \quad E_2 = \frac{m_2 w_{1\,max}^2}{6},$$

$$E = E_1 + E_2 = \frac{m w_{1\,max}^2}{6}$$

(ρ is the density of the jet, s_1 its transverse average cross section, l — length of the jet).

After redistribution of the velocities, we have

$$m = s_2 \rho (l - \bar{l}),$$

$$I = \frac{m w_{1\,max}}{2}, \quad E = \frac{m w_{1\,max}^2}{6},$$

Thus proving our assumptions.

It follows from these relations that

$$\frac{s_2}{s_1} = \frac{l}{l - \bar{l}}$$

or

$$\frac{r_2}{r_1} = \sqrt{\frac{l}{l - \bar{l}}}. \qquad (69,24)$$

The time during which the redistribution takes place can be estimated by the relation

$$\tau_0 = \frac{2l}{c_0}, \tag{69,25}$$

where c_0 is the velocity of sound in the material of the jet. This relation takes into account the travel of the compression and rarefaction waves in both directions.

By the end of the compression of the facing, the length of the jet is

$$l_\tau = \bar{l}_0 + w_{1\,\max}\tau_0, \tag{69,26}$$

where $\bar{l}_0 = l - l_0$.

Actually, the initial distribution of the velocity in a cumulative jet is not linear (see Fig. 181). Therefore the exact solution of the problem is more complicated. One can always, however, break up the jet into several intervals and assume the velocity distribution to be linear in each.

70. THEORY OF ARMOR-PIERCING ACTION OF CUMULATIVE JET

The theory of armor-piercing action of the cumulative jet was first developed by Lavrent'yev. He started from the assumption that in a collision between a jet and the armor high pressures are developed, at which one can neglect the strength resistance of the metal and the armor can be regarded as an ideal incompressible liquid. In this connection, Lavrent'yev analysed in detail the following problem.

Assume that the jet has the form of a cylinder of radius r_0; the velocity of all its elements is the same and is equal to w_1. In addition, we assume that the jet penetrates into a cylinder of radius r_1, coaxial with the jet. In this formulation this problem is equivalent to the problem considered above, concerning the collision of two jets; the change in signs of the velocities of the jet reduces the scheme of formation of a jet by compression of the facing to the scheme of the operation of the jet as it penetrates a medium with the same density.

In this case Fig. 177 can be regarded as a scheme for the penetration of the jet A into an obstacle B, if it is assumed that the obstacle B (pestle) has at $x \to -\infty$ a velocity equal to zero. From this it follows that the equation for the velocity of penetration of the jet (armor-piercing velocity) is

$$u = \frac{w_1}{2}. \qquad (70,1)$$

It follows from (70.1) that when the jet penetrates a depth L, there is consumed also a part of the jet equal to L, i.e., the maximum depth of armor piercing is equal to the length of the cumulative jet.

If the jet and the armor have different densities, then the velocity of armor piercing is determined by the formula

$$u = w_1 \frac{1}{\sqrt{\frac{\rho_2}{\rho_1}} + 1}, \qquad (70,2)$$

and the depth of armor piercing is given by

$$L = l \sqrt{\frac{\rho_1}{\rho_2}}, \qquad (70,3)$$

where ρ_1 and ρ_2 are the densities of the jet metal and of the armor, and l is the length of the jet, equal to the length of the generatrix of the cone.

Lavrent'yev indicates that the initial scheme that he has assumed is valid if the pressure upon collision between the jet and the armor exceeds 2×10^5 kg/cm^2, i.e., if $w_1 > w_{cr} \approx 4 \times 10^3$ m/sec.

Results of a verification have shown that the velocities and depth of armor piercing as calculated by Lavrent'yev differ in many cases from the experimental values.

The main reason for the discrepancy between the theory and experiment is the neglect of the compressibility of the metals at high pressures, particularly neglect of the strength resistance of the material of the partition.

The strength resistance of the metals, as is well known, increases in general with increasing dynamics of the load, and as will be shown below, under certain conditions it becomes commensurate with the pressure produced by a cumulative jet.

In this case the depth of armor piercing should depend not only on the length of the jet and on the ratio of the densities of the metals, but also on the velocity of the jet and on the strength characteristics of the armor.

The strength resistance p_{st} of metals under the action of dynamic loads cannot be determined with sufficient accuracy by theory, in view of the lack of reliable data on the variation of the parameters of the crystal lattice of metals at high pressures.

The value of p_{st} can be established, however, on the basis of experimental data on the limiting velocity of the cumulative jet w_{lcr}, at which the armor piercing ceases.

It is obvious that at this velocity the pressure of the jet will be balanced by the summary forces of resistance of the partition, which are made up of inertia forces p_{iner} and strength resistance forces p_{st}.

However, inasmuch as the velocity of motion of the partition is negligibly small near the limit of armor piercing, we can with full justification neglect the quantity p_{iner} and determine p_{st} from the theoretical relation between the pressure and the velocity of the cumulative jet.

The results of determination of w_{lcr} for certain metals have been obtained by Baum and Shekhter and are listed in Table 106.

In order to determine w_{lcr} for each material, the limiting thickness of armor piercing was established, accurate to 3 — 5 mm, and streak photography was used to measure the exit velocity of the cumulative jet at this thickness.

Fig. 168 shows a photograph of the motion of the tail part of a cumulative jet.

It follows from Table 106 that w_{lcr} depends on the relation between the densities of the jet and partition metals and their physical-mechanical characteristics.

It can thus be concluded that armor-piercing ability is possessed not by the entire jet, but only by some part of it, which we shall call the effective length of the jet l_{ef}.

The magnitude of l_{ef} is determined by the character of the distribution of the velocities along the cumulative jet, as it shown schematically in Fig. 183.

Naturally, in the determination of l_{ef} it is necessary to consider the jet at the instant of its maximum elongation, at which it still retains its monolithic nature. Obviously, in this state it will have the maximum armor-piercing action. The quantity l_{ef} can be calculated if we know the law of motion of the cumulative jet.

The theory of armor-piercing action of jets, with allowance for the compressibility of the jet and partition metals and of the strength characteristics of the latter, has been developed by Baum and Stanyukovich.

71. MOTION OF CUMULATIVE JET

Let us consider first the motion of a cumulative jet in air. It is obvious that at relatively small distances from the charge (up to several meters), which are the ones of practical interest, we can neglect the air resistance and consider the motion of the jet in vacuum. It is furthermore also obvious that the internal pressure at different parts of the jet is close to atmospheric and that the pressure gradient $(\partial p/\partial x)_t$ is small compared with the velocity gradient $(\partial u/\partial x)_t$. It can be assumed that $(\partial p/\partial x)_t = 0$.

In this case we can use for the description of the motion of the jet the Euler equations for unsteady one-dimensional motions of liquid:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0. \qquad (71,1)$$

When $\partial p/\partial x = 0$ (in this case $u = v_1$) the general solution of (71.1) is written in the form

$$x = ut + F(u). \tag{71,2}$$

Knowing the law of distribution of the velocity u over the coordinate x at any definite instant of time, for example at $t = 0$, we can readily determine the arbitrary function $F(u)$ (the origin of coordinates and the starting time are quite arbitrary, since in (71.1) t and x are under the integral sign).

Assume that when $t = 0$ we have $u = f(x)$ or $x = \varphi(u)$, where (u) is a specified velocity function. It is then obvious that

$$F(u) = \varphi(u),$$

i.e.

$$x = ut + \varphi(u). \tag{71,3}$$

Motion of this type is inertial; each particle of the jet has a constant velocity, determined by the initial conditions, independent of the time. It follows therefore that

$$\left(\frac{\partial x}{\partial t}\right)_{x_0} = u = u_0(x_0), \tag{71,4}$$

where u_0 describes the distribution of the velocities as a function of the Lagrangian coordinate x_0. The value of x_0 determines the position of the particle at the instant of time $t = 0$.

At any arbitrary instant of time $t > 0$ the position of the particle is determined by the expression

$$x = x_0 + u_0(x_0)\, t. \tag{71,5}$$

Assume that when $t = 0$ the length of a certain part of the jet is

$$l_{0\,(1,\,2)} = x_{2,\,0} - x_{1,\,0}.$$

Then when t > 0 the length of this part of the jet is determined by the expression

$$l_{t\,(1,2)} = l_{0\,(1,2)} + [u_{2,0}(x_{2,0}) - u_{1,0}(x_{1,0})]\,t. \tag{71,6}$$

Let us consider two possible cases of motion of the jet. In the first case we assume that the velocity depends linearly on the coordinate, i.e., that when t = 0 we have

$$u = u_0\left(1 - \alpha\frac{x_0}{l_0}\right), \tag{71,7}$$

where u_0 is the velocity of the frontal part of the jet, α is a dimensionless coefficient depending on the velocity gradient which can be determined from experiment by using data on the velocity distribution along the jet, and l_0 is the initial effective length of the jet.

When $x = l_0$ we have

$$u = u_0(1 - \alpha) = u_{cr},$$

where $u_{1cr} = w_{1cr}$.

It follows therefore that

$$\alpha = 1 - \frac{u_{cr}}{u_0}$$

and

$$\frac{u}{u_0} = \left[1 - \left(1 - \frac{u_{cr}}{u_0}\right)\frac{x_0}{l_0}\right]. \tag{71,8}$$

From expression (71.8) we obtain

$$\frac{x}{x_0} = \frac{1 - \frac{u}{u_0}}{1 - \frac{u_{cr}}{u_0}}. \tag{71,9}$$

Using (71.3) and (71.8), we obtain the relationship

$$\frac{x}{l_0} = \frac{ut}{l_0} + \frac{1 - \frac{u}{u_0}}{1 - \frac{u_{cr}}{u_0}} \qquad (t > 0).$$

From this we determine u = u(x, t):

$$\frac{u}{u_0} = \frac{-\frac{x}{l_0} + \frac{u_0}{u_0 - u_{cr}}}{-\frac{u_0 t}{l} + \frac{u_0}{u_0 - u_{cr}}}. \qquad (71,10)$$

The position of each particle of the jet at $t > 0$ is determined by the relation

$$x = x_0 + u_0 t\left[1 - \left(1 - \frac{u_{cr}}{u_0}\right)\frac{x_0}{l_0}\right]. \qquad (71,11)$$

The effective length of the jet is determined by the relation

$$l_{ef} = l_0 + (u_0 - u_{cr})\,t. \qquad (71,12)$$

In the second case we assume that when $t = 0$ the distribution of the velocities along the jet is determined by the law

$$u = u_0\left[\left(1 - \alpha\frac{x_0}{l_0}\right) + \beta\left(\frac{x_0}{l_0}\right)^2\right]. \qquad (71,13)$$

Assume that when $x = l_0$ we have $u = u_{cr}$. Then

$$\frac{u_{cr}}{u_0} = 1 - \alpha + \beta,$$

Hence

$$\beta = \frac{u_{st}}{u_0} + \alpha - 1,$$

which finally yields

$$\frac{u}{u_0} = 1 - \alpha\frac{x_0}{l_0} + \left(\alpha - 1 + \frac{u_{st}}{u_0}\right)\left(\frac{x_0}{l_0}\right)^2, \qquad (71,14)$$

and the parameter α must be determined by experiment. From (71.14) we readily determine the value of x when $t > 0$:

$$x = x_0 + u_0 t\left[1 - \frac{\alpha x_0}{l_0} + \left(\alpha - 1 + \frac{u_{cr}}{u_0}\right)\left(\frac{x_0}{l_0}\right)^2\right]. \qquad (71,15)$$

The total effective length of the jet is determined in this case as before by the relation (71.12).

During the process of motion the jet stretches and after a certain time t_{st} it loses its monolithic nature. Let us determine t_{st}.

In the case when the velocity along the jet varieslinearly, the relative elongation of the jet can be directly expressed by the relation

$$\bar{\varepsilon} = \frac{u_0 - u_{cr}}{l_0} t_{st}. \quad (71,16)$$

If the process of stretching of the jet is limited only to the region of elastic deformations, then the determination of t_{st} can be made by means of the well known relation $\sigma_u = E\varepsilon$,

where σ_u is the ultimate resistance of the metal, ε is the relative elongation, and E is the modulus of elasticity.

However, the deformation of the jet takes place beyond the elastic region, and consequently Hooke's law cannot be used in this case to calculate the relative elongation of the metal. For this purpose one should use the experimentally-determined relationships, establishing the connection between the destructive load and the relative elongation of the corresponding metal. To obtain these relationships one can use the true stress diagrams (Fig. 184), which include both the elastic region and the region of plastic deformation, for which the growth of the stresses is characterized by the equation

$$\sigma - \sigma_0 = A(\varepsilon - \varepsilon_0).$$

Replacing this equation by the approximate equation

$$\sigma = D\varepsilon + \text{const}, \quad (71,17)$$

where D is the strengthening modulus, knowing the value of S_k and neglecting the elastic deformation, we can determine from the diagram approximately the relative elongation

$$\bar{\varepsilon} \approx \frac{S_k - \sigma_u}{D}, \quad (71,18)$$

where S_k is the resistance of metal to rupture.

From (71.16) and (71.18) we get

$$t_{st} = \frac{l_0}{u_0 - u_{st}} \frac{S_k - \sigma_u}{D}. \tag{71,19}$$

Substituting the value of t_{st} into (71.12) we obtain finally a relation for the limiting effective length of the jet

$$l_{ef} \approx l_0\left(1 + \frac{S_k - \sigma_u}{D}\right). \tag{71,20}$$

This formula can as yet not be employed directly for numerical calculations, since the available values of S_k and D are valid only for ordinary temperatures and low metal deformation velocities. As the temperature and the deformation velocities are increased, these quantities change. The values of S_k and D at the temperatures and tremendous deformation velocities which are reached in the movement of a cumulative jet are still unknown.

However Eq.(71.20) leads us to the conclusion that to increase l_{ef}, and consequently also the armor-piercing effect, it is necessary to ensure such a combination of physical-mechanical properties of the metal of the cumulative facing, at which the ratio $(S_k - \sigma_u)/D$ reaches a maximum possible value.

On the basis of the data on the armor-piercing ability of cumulative charges and x-ray photographys of cumulative jets, we can assume that in the case of facing made of soft steel

$$1 < \frac{S_k - \sigma_u}{D} \leqslant 2,$$

i.e., l_{ef} amounts to approximately $3l_0$, where l_0 is of the order of the length of the generatrix of the cumulative cavity.

72. PHYSICAL PRINCIPLES OF THE THEORY OF ARMOR-PIERCING ACTION OF A CUMULATIVE JET

In the construction of a theory of the armor-piercing action of a cumulative jet, one cannot get along without having clear cut ideas of the mechanism of destruction of the armor.

Without a preliminary analysis it is difficult to ascertain whether we deal in this case with brittle destruction of the armor material or with sone other phenomenon.

An elementary calculation shows in general that even near the armor-piercing limit the energy of the cumulative jet is sufficient to ensure melting of the armor metal.

Let us determine the limiting value of the jet, at which total melting of the steel can still be attained, assuming that the facing and consequently the jet are made up of the same metal.

The kinetic energy per unit mass of the jet is

$$\varepsilon_1 = \frac{u_{st}^2}{2}. \qquad (72,1)$$

The energy necessary to melt a unit mass of partition is

$$\varepsilon_2 = q. \qquad (72,2)$$

For steel under ordinary pressures q = 0.3 kcal/g.

Comparing (72.1) and (72.2) we obtain for the case under consideration u_{st} = 1600 m/sec.

The actual value of u_{st} should be somewhat greater, since at high pressures (on the order of 10^6 kg/cm^2), which the metal of the partition experiences during the instant of impact, the melting temperature and the specific heat of the metal will be considerably higher than at normal pressure, i.e., u_{st} will approach the value u_{cr}, which for ordinary steel is 2050 m/sec.

It is difficult to assume, however, that under the impact conditions, which prevail for approximately a millionth of a second, it is possible to excite all the degrees of freedom which determine the conditions under which the partition metal melts.

We arrive at the same conclusion also as a result of the experimental fact that the depths of armor-piercing for metals which are quite similar in nature (with almost identical values of q and equal density) depend appreciably on their strength characteristics.

For example, for steel with hardness $H_B = 125$ we have $u_{cr} =$ 2,000 m/sec, and for hardened steel with $R_C = 50$ we have $u_{cr} = 2200$ m/sec.

Even more instructive are data on the effect of the strengths of the corresponding metals on the depth of armor-piercing. These data were obtained by Baum and Skalyarov.

The results of the experimental determinations are listed in Table 107.

Experiments were carried out with charges made of TG 50/50 alloy 42 mm in diameter and 84 mm high, with the cumulative cavity of hyperbolic form and a facing made of aluminum alloy 2 mm thick.

On the basis of the foregoing we can conclude that the metal of the armor when under the influence of the cumulative jet is apparently in a particular quasi-liquid state, in which the transition conditions depend considerably on its strength characteristic.

By way of the main strength characteristic of the metal we shall henceforth use the quantity p_{st}, which can be readily determined by experiment.

The basis for the construction of the more refined theory of armor-piercing action, as shown by the results of the investigations performed, should be the following physical factors:

1. At high pressures, which occur upon interaction of a cumulative jet with a partition, it is necessary to take into account the compressibility of the metals, which becomes appreciable.

2. The strength characteristics of the partition metal

influence greatly the depth of the armor piercing. The destruction of the armor is connected with overcoming the strength forces in the metal and its transition into a quasi-liquid state.

3. Taking the foregoing circumstance into account, the speed of armor piercing can be calculated at the speed of motion of the separation boundary between the metal of the jet and the metal of the armor.

4. The depth of armor piercing, other conditions being equal, is determined by the effective length of the cumulative jet ℓ_{ef}, at which its velocity becomes equal to u_{cr}. After this velocity is reached in the jet, the armor piercing ceases.

73. VELOCITY OF ARMOR PIERCING AND PRESSURE DURING THE ENCOUNTER BETWEEN THE JET AND THE PARTITION

When the jet strikes the partition, a shock wave is produced both in the jet and in the partition. To determine the pressure that develops when the jet penetrates into the partition and to determine the velocity of the armor piercing, we can use the well known relationships from the theory of shock waves:

$$u_x = u - \sqrt{p_x (v_{1,0} - v_{1,x})} = \sqrt{p_x (v_{2,0} - v_{2,x})}, \qquad (73,1)$$

where u is the velocity of the corresponding jet element, u_x is the velocity of motion of the separation boundary between the two media (the speed of armor piercing), p_x is the pressure on the boundary between the media at the instant of the shock, $v_{1,0}$ and $v_{1,x}$ -- the specific volumes of the striking body (jet) at the initial instant of time and at the instant of shock, respectively; $v_{2,0}$ and $v_{2,x}$ are the specific volumes of the body receiving the shock (partition).

Let us put $v_x = v_0(1 - \alpha)$. After simple transformations of (73.1) we arrive at the following formulas, which determine p_x and u_x

$$p_x = \frac{\rho_{1,0} u^2}{\left(\sqrt{a_1} + \sqrt{a_2 \frac{\rho_{1,0}}{\rho_{2,0}}}\right)^2}, \qquad (73,2)$$

$$u_x = \frac{u}{1+\sqrt{\frac{\alpha_1}{\alpha_2}\frac{\rho_{2,0}}{\rho_{1,0}}}}, \tag{73,3}$$

where $\rho_{1,0}$ and $\rho_{2,0}$ are the initial densities of the striking and struck bodies.

Unlike (70.2), a formula established by Lavrent'yev, Eqs. (72.3) and (72.3) take into account the influence of the compressibility (α) of the colliding bodies on the parameters of the armor piercing.

It follows from (73.3) that in the case when the jet and the partition are made of the same metal ($\alpha_1 = \alpha_2$ and $\rho_{1,0} = \rho_{2,0}$) the armor piercing velocity is $u_x = u/2$.

In this particular case formulas (73.3) and (70.2) lead to an identical result. It was shown in Chapter IX that at pressures on the order of 10^5 kg/cm^2 and higher the connection between p and ρ is established by the law

$$p_x = A\left[\left(\frac{\rho_x}{\rho_0}\right)^n - 1\right]. \tag{73.4}$$

Expressing ρ_x/ρ_0 in terms of α and using (73.4), we find

$$\frac{\rho_x}{\rho_0} = \frac{1}{1-\alpha},$$

$$\alpha = 1 - \frac{1}{\left(1+\frac{p_x}{A}\right)^{\frac{1}{n}}}. \tag{73,5}$$

Substituting the resultant relation for α into (73.2) we obtain

$$p_x = \frac{\rho_{1,0}u^2}{\left\{\sqrt{1-\frac{1}{\left(1+\frac{p_x}{A_1}\right)^{\frac{1}{n}}}} + \sqrt{\frac{\rho_{1,0}}{\rho_{2,0}}\left[1-\frac{1}{\left(1+\frac{p_x}{A_2}\right)^{\frac{1}{n}}}\right]}\right\}^2} \tag{73,6}$$

The rate of motion of the separation boundary u_x, calculated from formula (73.3) corresponds for a medium whose strength is disregarded (water, lead, etc.).

Under real conditions of armor piercing, it is necessary to take into account the strength resistance of the armor metal p_{st}, and consequently the actual velocity of motion of the separation boundary, equal to the velocity of armor piercing, will be less than u_x. We denote this velocity by $u_{x,0}$.

The value of $u_{x,0}$ can be determined from the formula

$$u_{x,0}=\sqrt{(p_x-p_{st})(v_{2,0}-v_{2,x})}. \quad (73.7)$$

Here p_{st} is interpreted as the initial "internal" pressure in the armor metal.

Transforming (73.7), we can write for $u_{x,0}$ the relation

$$u_{x,0}=\sqrt{\left[\frac{\rho_{1,0}u^2}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2\frac{\rho_{1,0}}{\rho_{2,0}}}\right)^2}-p_{st}\right]\frac{\alpha_2}{\rho_{2,0}}}=$$

$$=u\sqrt{\frac{\alpha_2\rho_{1,0}}{\rho_{2,0}}\left[\frac{1}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_1\frac{\rho_{1,0}}{\rho_{2,0}}}\right)^2}-\frac{p_{st}}{\rho_{2,0}u^2}\right]},$$

or finally

$$u_{x,0}=u\sqrt{\frac{1}{\left(1+\sqrt{\frac{\alpha_1}{\alpha_2}\frac{\rho_{2,0}}{\rho_{1,0}}}\right)^2}-\frac{p_{st}\alpha_2}{\rho_{2,0}u^2}}. \quad (73.8)$$

When $\rho_{2,0}u^2 \gg \alpha_2 p_{st}$, relation (73.8) goes into relation (73.2), and if the jet and the partition consists of one and the same metal, then $u_{x,0} = u/2$. At velocities u of cumulative jets, with which we usually deal in practice, we have $u_{x,0}/u < 1/2$.

When $p_x = p_{st}$ we have $u_{x,0} = 0$, something attained when $u = u_{cr}$. The results obtained correspond to the real condition of armor piercing by cumulative charges.

Knowing the values of u_{cr} from experiment for the corresponding metals, we can readily calculate their values of p_{st}. For this purpose we use relation (73.6), which after substitution of $u = u_{cr}$ and $p_x = p_{st}$ assumes the form

$$p_{st} = \frac{\rho_{1,0} u_{cr}^2}{\left[\sqrt{1 - \frac{1}{\left(1 + \frac{p_{st}}{A_1}\right)^{\frac{1}{n}}}} + \sqrt{\frac{\rho_{1,0}}{\rho_{2,0}}\left(1 - \frac{1}{\left(1 + \frac{p_{st}}{A_2}\right)^{\frac{1}{n}}}\right)}\right]^2} = \theta \rho_{1,0} u_{cr}^2. \quad (73,9)$$

It is convenient to calculate p_{st} graphically. For this purpose one specifies different values of p_x for which the corresponding values of u are calculated. The results of the calculations are plotted in the form of graphs $p_x = f(u)$, which can be used to determine p_{st} from the experimentally obtained values of u_{cr}.

The relation between the pressure p and the velocity u of a cumulative jet is found in Table 108 and in Fig. 185.

As can be seen from Fig. 185, for steel with hardness $H_B = 125$ we have $p_{st} = 4.8 \times 10^5$ kg/cm^2, and for duraluminum with hardness $H_B = 115$ we have $p_{st} = 2.8 \times 10^5$ kg/cm^2.

We note that for steel p_{st} amounts to about 20 — 25% of the maximum (initial) pressure which is produced in the partition at the instant that the frontal part of the cumulative jet strikes it.

The velocity of armor piercing $u_{x,0}$ is calculated in the following fashion. Using the plot (Fig. 185), one determines from the jet velocity u the corresponding value of p_x, after which formula (73.4) is used to determine α_1 and α_2. The resultant values of these quantities are substituted in relation (73.8).

The results of certain calculations, carried out as applied to partitions and cumulative jets made of steel and duraluminum, are listed in Table 109, where the dependence of $u_{x,\,0}$ on the jet velocity u is given.

Table 110 compares the values of the average velocities of armor piercing, established experimentally and calculated from Lavrent'yev's formula and from formula (73.8) for the initial stages of armor piercing (armor thickness 22 — 30 mm). In the calculations, use was made of the average velocity (u_{av}) of the jet element consumed during the process of armor piercing.

As can be seen from the table, the results of the theoretical calculation by formula (73.8) and those of the experiment are in full agreement.

The results of calculation by Lavrent'yev's formula lead in all cases (even for the initial stages of armor piercing) to excessive values of the armor-piercing velocity. Naturally, at large depths of armor piercing this discrepancy will increase noticeably.

74. DETERMINATION OF DEPTH OF ARMOR PIERCING

Let us examine the problem of the motion of the boundary between the media (armor piercing) in general form. The velocity of penetration of the jet is

$$u_{x,\,0}=\frac{dx}{dt}=u\sqrt{\frac{1}{\left(1+\sqrt{\frac{\alpha_1}{\alpha_2}\,\frac{\rho_{2,\,0}}{\rho_{1,\,0}}}\right)^2}-\frac{p_{st}\alpha_2}{\rho_{2,\,0}u^2}}, \qquad (74{,}1)$$

where u is a function of x and t.

Converting the right half of (74.1), we obtain

$$\frac{dx}{dt}=u_0\sqrt{\frac{u^2}{u_0^2\left(1+\sqrt{\frac{\alpha_1}{\alpha_2}\,\frac{\rho_{2,\,0}}{\rho_{1,\,0}}}\right)^2}-\frac{p_{st}\alpha_2}{\rho_{2,\,0}u_0^2}}, \qquad (74{,}2)$$

where u_0 is the initial velocity of the frontal part of the jet.

Introducing the notation

$$1+\sqrt{\frac{a_1}{a_2}\frac{\rho_{2,0}}{\rho_{1,0}}}=\beta_1, \quad \frac{p_{st}a_2}{\rho_{2,0}u_0^2}=\frac{\beta_2}{\beta_1^2}, \tag{74,3}$$

we rewrite (74.2) as

$$\frac{d\frac{x}{l_0}}{d\frac{u_0 t}{l_0}}=\frac{1}{\beta_1}\sqrt{\left(\frac{u}{u_0}\right)^2-\beta_2}, \tag{74,4}$$

where l_0 is the effective length of the jet at $t = 0$.

Solving this differential equation subject to the initial conditions $x = 0$, $t = 0$, $u = u_0$, and $f(0, 0) = 1$, we obtain $x = F(t)$. Knowing that by the end of the armor piercing we have $u/u_0 = u_{cr}/u_0$, we obtain from the simultaneous solution of the equations $x_c = F(t_c)$ and $f(x_c, t_c) = u_{cr}/u_0$, we can determine $x_c = L$, where L is the depth of the armor piercing and t_c is the time required to complete the armor piercing.

A solution of this problem leads to a cumbersome expression, from which the value of L can be determined by successive approximation or by using tables.

However, the depth of the armor piercing can be determined in a simpler fashion. For this purpose it is necessary to average the velocity in the jet in such a way that the conservation of energy is fulfilled for the jet. For a linear distribution of velocities in the jet, as follows from simple geometrical considerations (Fig. 186), we can write

$$\frac{u_0-u}{u_0-u_{cr}}=\frac{\bar{l}_0-x}{\bar{l}_0},$$

or

$$u=u_0-\left(1-\frac{x}{\bar{l}_0}\right)(u_0-u_{\text{кр}}) \tag{74,5}$$

($\bar{l}_0$ is the length of the jet at the instant of arrival at the armor); the energy of the jet is

$$E=\frac{s\bar{l}_0\rho_0\bar{u}^2}{2}=\frac{sl_0\rho_0}{2}\int_0^1 u^2 d\frac{x}{l_0}=\frac{sl_0\rho_0}{2}\int_0^1\left[u_{cr}+\frac{x}{l}(u_0-u_{cr})\right]^2 d\frac{x}{l_0}=$$

$$=\frac{sl_0\rho_0}{2}\left[u_{cr}^2+\frac{(u_0-u_{cr})^2}{3}+u_{cr}(u_0-u_{cr})\right], \tag{74,6}$$

and from (74.5) and (74.6) we obtain

$$\frac{\bar{u}}{u_0}=\sqrt{\frac{1+\frac{u_{cr}}{u_0}+\frac{u_{cr}^2}{u_0^2}}{3}}, \tag{74,7}$$

where $\bar{u}$ is the averaged jet velocity.

To calculate the depth of armor piercing, we obtain the equation

$$u_{x,0}=\frac{dx}{dt}=\frac{\bar{u}}{\beta_1}\sqrt{1-\beta_2}.$$

Integrating this expression, we get

$$L=\frac{\bar{u}t_c}{\beta_1}\sqrt{1-\beta_2}, \tag{74,8}$$

where L is the depth of armor piercing, and t_c is the time at the end of the armor piercing. Obviously,

$$t_c=\frac{\bar{l}_0+L}{\bar{u}}, \tag{74,9}$$

Hence

$$\frac{L}{\bar{l}_0}=-\frac{1}{\dfrac{1+\sqrt{\frac{\alpha_1}{\alpha_2}\frac{\rho_2}{\rho_1}}}{\sqrt{1-\frac{p_{st}\,\alpha_2}{\rho_2\bar{u}^2}\left(1+\sqrt{\frac{\alpha_1}{\alpha_2}\frac{\rho_2}{\rho_1}}\right)^2}}-1}. \tag{74,10}$$

As $p_{st}\to 0$ we have

$$\frac{L}{\bar{l}_0}=\sqrt{\frac{\alpha_2}{\alpha_1}\frac{\rho_{1,0}}{\rho_{2,0}}}. \tag{74,11}$$

If the coefficients of compressibility of the metals used for the jet and for the partition are the same ($\alpha_1=\alpha_2$), we arrive at the Lavrent'yev formula

$$L=\bar{l}_0\sqrt{\frac{\rho_{1,0}}{\rho_{2,0}}}. \tag{74,12}$$

We can conclude from the equations obtained that the armor-piercing ability of a cumulative jet, other conditions being equal, is proportional to the length of the generatrix of the facing. Therefore a facing of hyperbolic form is better than a conical one of the same height. The use of a facing with a generatrix in the form of a branch of a logarithmic spiral is not advisable, since it does not form a normal jet but a spherically shaped bullet.

To obtain maximum armor-piercing effect ($L = L_{max}$) it is necessary to choose the quantity ℓ_0, in such a way, that during all the time of its succeeding penetration of the jet in the armor the jet remains continuous.

The optimum value $\overline{\ell_0} = \ell_0 + R_f$ (ℓ_0 is the initial length of the jet and R_f is the focal distance) is equal to the optimal distance from the charge to the armor, at which the cumulative jet has the maximum piercing ability.

Consequently, in this case the concept "focal distance" has an entirely different physical meaning than for a cumulative charge without facing.

Other conditions being equal, the focal distance will be greater for a charge in which the metal of the cumulative facing has a greater ability to become stretched without breaking.

For modern cumulative charges with conical cavity and with facing made of iron we have

$$\frac{l_{ef}}{l_0} \approx 3,$$

where ℓ_0 is approximately equal to the length of the generatrix of the cone.

75. DETERMINATION OF THE DIAMETER OF THE SHELL HOLE

An exact determination of the diameter of the shell hole is quite difficult. An approximate relationship for the calculation of the diameter of the shell hole can be obtained, however, from the following considerations.

It can be assumed that after the frontal part of the jet has struck the surface of the partition, high pressures p_x of short duration (which we have calculated above) are formed. These pressures are rapidly removed, and consequently the radial motion of the material of the partition is under the influence of the inertial forces. Bearing in mind that the lateral shock wave is almost cylindrical, we can write the following relation

$$\frac{p_l}{p_x} = \sqrt{\frac{r_0}{R_h}}, \tag{75,1}$$

where p_l is the lateral pressure, p_x the pressure in the axial direction, r_0 the average radius of the jet, and R_h is the running radius of the shell hole. Assuming the radial motion of the metal to have terminated when $p_l = p_{st}$, we obtain from (75.1) the radius of the shell hole.

$$(R_h)_{entr} = r_0\left(\frac{p_x}{p_{st}}\right)^2. \tag{75,2}$$

At the instant of termination of the armor piercing, i.e., when $p_x = p_{st}$ then $R_h = r_0$. For this example, the maximum values are $p_x = 2.0 \times 10^6$ kg/cm^2 and $p_{st} \approx 5.0 \times 10^5$ kg/cm^2. The average radius of the jet $\bar{r}_0$ is determined from the formula

$$\bar{r}_0 = \sqrt{\frac{2}{3}\frac{R_c \delta \cos\alpha}{\psi}} \sin\frac{\alpha}{2}.$$

Substituting for our specific case the values $2R_c = 60$ mm, $\delta = 2$ mm, and $\alpha = 35°$, and also taking account of the fact that the maximum coefficient of elongation for modern steel shells is $\psi \approx 3$, we obtain $\bar{r}_0 = 0.76$ mm.

Using (75.2) we find the maximum diameter of the shell hole to be $d_h = 2R_h = 24$ mm, which is in good agreement with experiment.

When the charge is exploded under conditions such that the process of normal formation of the cumulative jet is disturbed, and the jet is not effectively stretched (for example, if the charge is placed directly on the armor), then the diameter of the shell hole will increase considerably, something rarely observed in practice. Naturally, the depth of the armor piercing (inasmuch as the length of the jet is $l < l_{ef}$) is decreased thereby.

76. EFFECT OF FAST ROTATION ON THE STABILITY OF A CUMULATIVE AND ITS ARMOR-PIERCING ACTION

It is known that armor-piercing action of rotating cumulative ammunition is much less than that of non-rotating ones. With increasing angular velocity of the cumulative charge, the harmful influence of the rotary action becomes intensified. Systematic investigations of the effective rotation on the cumulative effect, as a function of the diameter and of the shape of the cavity, of the angular velocity of the charge, and of its distance to the partition, have been carried out by Baum. Some experiments in this direction were also carried out under the leadership of Lavrent'yev.

Effect of caliber (diameter of the base of the cavity) of the charge on the armor piercing effect during rotation. In the investigation of the influence of this factor, the charges used were made of TG 50/50 alloy in steel shells with conical cavity. The cumulative funnels (facings) were made of steel. The tests were carried out at a speed of rotation of 20,000 — 30,000 rpm. The charges were exploded at the focal distance from the armor. The results of these experiments are listed in Table 111.

It is seen from the table that with increasing caliber of the charge, the harmful influence of rotation becomes stronger, and it manifests itself to a greater degree for charges with deep conical cavity. This is explained by the fact that because of the law of conservation of angular momentum the elements of the jet will have an angular velocity determined by the angular momentum of the corresponding element of facing relative to the instantaneous axis of its rotation during collapse. The maximum angular velocity of the jet is determined by the expression

$$\omega_j = \omega_0 \left(\frac{R_0}{R_c} \right)^2, \qquad (76,1)$$

where ω_0 is the angular velocity of the charge, R_0 the radius of the base of the cone, and R_c the radius of the base of the cone (pestle) after its complete compression.

Here R_c pertains to the pestle and not to the jet, for at the instant of formation the pestle and the jet make up a single entity, and consequently, have the same angular velocity.

It follows from (76.1) that with increasing caliber of the charge the angular velocity of the jet increases noticeably, and this, as will be shown below, is a harmful effect on its stability.

Influence of the cone angle on the armor piercing effect during rotation. In order to investigate the influence of this factor, tests were made of 76-mm cumulative charges, with conical cavities and with aperture angle of 60, 35, and 27°. The results of the tests are given in Table 112.

On the basis of the data of Table 112, we can make the following conclusions.

1. A high cone with stationary explosion ensure a greater armor-piercing effect than a low cone. This is explained principally by the fact that in the case of a high cone a longer length and a higher speed of cumulative jet are attained.

2. Other conditions being equal, the harmful influence of rotation becomes stronger with decreasing aperture angle of the cone. This is explained by the fact that a long jet becomes bent under rotation conditions and is therefore less stable.

The effect of the speed of rotation on the armor piercing ability is illustrated in Table 113.

Table 114 shows data on the influence of rotation as a function of the distance of the charge to the armor. The tests were made on 76-mm charges with conical cavity.

It is seen from the table that in the explosion of charges with conical cavities ($h/d_{base} = 1$) under stationary conditions the cumulative jet has a sufficiently stable armor-piercing ability when the distance from the charge to the armor is equal to two calibers. However, the harmful influence of rotation on the armor piercing ability of a cumulative jet begins to manifest itself even at very short distances from the charge to the armor, this being evidence of the fast derangement and destruction of the jet under rotation conditions.

In studies of the question of the structure and causes of derangement of a cumulative jet upon rotation, it is particularly interesting, from the principle point of view, to determine the behavior and operating conditions of the jet at relatively large distances from the point of explosion of the charge, at which one observes very highly pronounced symptoms of derangement of the jet even in ordinary stationary explosion.

77. ON THE STABILITY OF A CUMULATIVE JET

The results of the investigations enable us to conclude that the most important initial forms of derangement of the jet manifest themselves in the following manner:

1. In a destruction of the monolithic nature of the jet under the influence of the velocity gradients, as a result of which the jet break up into a larger or smaller number of elements. At large gradients an intense dispersion of the jet may take place, and it may consequently change into a stream of minute particles of metal.

The decrease in the armor piercing ability with distance is in this case a consequence of the reduction in the density and radial divergence of the jet, owing to the air resistance. Fig.187 shows the zone of damage (shell holes and dents) of a 20-mm armor produced by a 76-mm cumulative charge at a distance of 4.5 meters from the point of explosion, (this zone has the form of an ellipse with semi-axes $a = 18$ cm and $b = 38$ cm).

2. In the broadening of the jet with subsequent radial disruption under the influence of the elastic compression energy accumulated in the jet during the period of its formation. A derangement of this type is clearly observed in jets made of lead, which has a large volume compressibility. The character of a jet made up from lead facing is illustrated in Fig 188.

In bending of the jet as a result of the asymmetry of the explosion momentum or the cumulative facing, which leads to a deviation of the individual elements of the jet from their normal trajectory and to a reduction in the armor-piercing ability (Fig. 189). This form of instability can lead under certain conditions (in the case of relatively large bending and breakup of the jet) to the formation of two or more holes in the armor.

Under real conditions of application of cumulative charges, one must not exclude the possibility of simultaneous appearance of all the foregoing factors.

Under conditions of stationary explosion, the derangement of the jet manifests itself sufficiently noticeably only at relatively large distances from the point of explosion. However, in the case of rapid rotation of the charge, the jet immediately experiences under the influence of the centrifugal forces ever increasing derangement connected with the degree of bending of the jet and the radial scattering of its individual elements. These phenomena should lead at relatively close distances to the charge to an increase in the diameter of the hole, to a breakup of the jet with simultaneous reduction in the depth of armor piercing, while at large distances it leads to an almost complete vanishing of the armor-piercing ability.

The foregoing considerations are in full agreement with experimental results. Thus, for example, Fig. 190 shows the result of action of a cumulative jet on a partition under conditions when the charge is exploded with rotation at 20,000 rpm. In this case, owing to the derangement of the jet, several dents, scattered over a relatively large area, were produced in the armor. The results of action on armor, produced by an analogous charge with stationary explosion and for the same distance from the partition, are shown in Fig. 191.

A steel shield 20 mm thick was placed along the path of motion of cumulative jets of a 76-mm charge, a distance 135 cm from the charge. The character of the damage to the shield in the case of stationary explosion,is shown in Fig. 192, while the case of rotation is shown in Fig. 193.

On the basis of the data given, we must conclude that the observed differences in the loss of stability of the jet under rotation and under conditions of stationary explosion are only quantitative in character. The derangement of the jet observed in stationary explosion at relatively large distances from the charge, occurs at much lesser distances in the case of rotation.

Deformations of the cumulative charge under possible asymmetries of the explosion impulse or of the facing were investigated theoretically by Kreyn. From the results he obtained it follows that even in the case of slight asymmetry of the cumulative facing or of the explosion impulse, a shift takes place in the center of formation of the jet relative to the initial axis of the charge. In this connection and also as a result of the time change in the direction of the initial velocities of the jet elements, the jet becomes curved.

According to Lavrent'yev, this type of derangement of the cumulative jet is encountered most frequently in practice and is the main reason for the harmful influence of rotation on the armor piercing ability of the cumulative charges. He starts here from the following considerations.

The asymmetry of an explosion impulse occurs almost always, in one degree or another, in real charges, as a result of a deflection of the axis of the cavity from the axis of the facing, incorrect position of the detonator, and many other factors. However, under conditions of stationary explosion the initial curving of the jet will be reduced by two stabilizing factors: the action of the air and of stresses in the jet, occurring as a result of its stretching, and consequently the jet becomes rectified.

Confining ourselves to an analysis of the second factor, we can consider in first approximation the jet to be a string under a stress p_0 in plastic flow. The frequency of oscillation of the string, and consequently of the jet, is determined, as is well known, from the formula

$$\tau = \frac{\pi}{l} \sqrt{\frac{p}{\rho}}, \qquad (77,1)$$

where l is the length of the jet, p is the tension, and ρ is the density.

If we assume $p = 4 \times 10^3$ kg/cm^2 and $l = 3$ cm (the jet element with the most clearly pronounced antinode), then we obtain from formula (77.1) that the time necessary to eliminate the antinode is

$$t_0 < 5 \cdot 10^{-5}\ sec,$$

i.e., when the rear part of the jet has a velocity $w_1 = 4 \times 10^3$ m/sec, the antinode will be eliminated if the considered jet element covers a path s = 2 cm.

Thus, this stabilizing factor can appreciably reduce the amplitude of the jet along the path of free flight of the tail part of the jet up to the armor. Lavrent'yev notes that for the asymmetries existing in real charges, at angular velocities of the facing of 5,000 -- 15,000 rpm, the centrifugal forces exert large stabilizing forces due to the tension of the jet.

When a rotating jet moves, its antinodes will grow; the tail part of the jet will not fall into the hole punched by its frontal part; the energy of the tail part will be expended in punching a new hole.

In conclusion we note that this factor cannot be considered to be the only cause of the harmful influence of rotation on the cumulative effect. Along with the bending of the jet, no less important a derangement (as shown by instantaneous photographs) is the breakup of the jet into separate particles.

A simple calculation shows that a jet can be considerably scattered in a radial direction under the influence of the centrifugal forces during the process of its motion, if the jet rotates.

In fact, the maximum centrifugal forces occurring in a jet are determined by the formula

$$p = \rho_0 r_0 \omega^2 = \rho_0 r_0 \omega_0^2 \left(\frac{R_c}{r_0}\right)^2, \qquad (77,2)$$

where r_0 is the initial diameter of the cumulative jet, and ρ_0 is the initial density of the jet.

The centrifugal forces calculated by means of this formula, produced in a jet when a 76-mm cumulative shell is fired from a 15-caliber gun, is approximately 18,000 kg/cm^2.

Under the influence of the centrifugal forces, the divergence of the jet should be a function of the time. The acceleration acquired by the particles of the jet is determined by the expression

$$\frac{d^2 r}{dt^2} = \omega^2 r, \qquad (77,3)$$

where r is the radius of the jet, t is the time, and ω is the angle of velocity of the jet.

Integrating this differential equation, we obtain

$$\frac{r}{r_0} = \frac{1}{2}\left(e^{\omega t} + e^{-\omega t}\right). \qquad (77,4)$$

Knowing the value ω_0 = 300 rpm, we can readily calculate the ratio r/r_0 for any instant of time.

The total time of motion of the elements of the jet in the air (along a 50 mm path) and penetration of 30 mm into the armor amounted to $t = 3 \times 10^{-5}$ sec for the case considered here.

The results of the calculations show that by that instant of time the diameter of the jet increases by 25%, while the area of its damage increases by 56%. The average density of the jet is accordingly decreased by 50%. This result agrees with the reduction in armor piercing ability (by 32%) which was found in our experiments on rotation.

78. SUPERSPEED CUMULATION

Of great interest in experimental physics is the production of gas and metal jets, moving with velocities on the order of many tens of kilometers per second. In addition to using strong electric discharges, which lead to such plasma velocities, it has been shown by I.V. Kurchatov and others that to obtain such high velocities one can use cumulation methods.

Analyzing the main relation of the theory of cumulation, namely

$$w_1 = \frac{w_0}{\operatorname{tg}\frac{\alpha}{2}}, \qquad (78,1)$$

we can readily verify that the velocity of the cumulative jet increases with decreasing angle α. It follows therefore that if the velocity w_0 of the collapse of the facing elements is sufficiently large (something that is attainable by suitable choice of the explosive, of the metal facing, or of other parameters of the cumulative charge), then at sufficiently small values of α it is possible to reach in principle quite high cumulative-jet velocities. Such velocities can be realized, in particular, under conditions when cylindrical cumulation is produced.

In many of the works of American scientists it was shown that when cylindrical tubes of light metals are used for the facing, the lateral surface of which is surrounded with a sufficiently thick layer of explosives, it is possible to produce in the frontal part of the cumulative jet velocities on the order of several tens of kilometers per second. The greatest velocity of cumulative jets, equal to 90 km per second, was reached by using tubes of beryllium, the specific gravity of which is 1.5. A schematic arrangement of a cumulative charge is shown in Fig.194.

Owing to the fact that the detonation wave does not reach various elements of the facing simultaneously, the collapse of the facing, in spite of its cylindrical form, still takes place under a certain angle that increases as the detonation wave moves along the tube. By regulating in some manner the time of approach of the detonation wave to the different elements of the tube, we can change within certain limits the angle α and therefore ensure a specified distribution of velocities in the cumulative jet with a maximum possible velocity of its frontal part.

The parameters of the cumulative jet (CJ) for the case of cylindrical cumulation can be very easily calculated. We first estimate the angle at which the elements of the facing converge towards the axis. We shall give the solution for the plane case, obtained by Baum and Stanyukovich.

For calculation purposes we take a scheme, wherein the detonation wave, bent by a "lens," begins to excite the active part of the charge at a distance y_0 from the metallic facing (Fig. 195). The front of the detonation wave, beginning at the point y_0, reaches a point x_0 within a time

$$t_0 = \frac{\sqrt{x_0^2 + y_0^2}}{D}, \tag{78,2}$$

The motion of this element of the facing begins precisely at that instant of time. We shall assume that each facing element moves along the y axis, and then the law of motion of the facing element is determined by the relations

$$\frac{M}{s}\frac{d^2y}{dt^2} = p, \tag{78,3}$$

where $M = s\delta h$ is the mass of the given facing element, s is its area, h is its thickness, and δ is the density of the facing material. Hence

$$\frac{d^2y}{dt^2} = \frac{p}{\delta h}, \tag{78,4}$$

Integrating (78.4) with respect to the time, we find that the velocity of motion of this element of facing is

$$w_0 = \frac{dy}{dt} = \frac{1}{\delta h}\int_{t_0}^{t} p\,dt = \frac{I(t)}{\delta h}, \tag{78,5}$$

where $I = I(t)$ is the momentum transferred to the given facing element, calculated per unit area of the facing.

The pressure acting on the given facing element can be approximate by the relation

$$p = p_0\left(\frac{t_0 - t_1}{t - t_1}\right)^3, \tag{78,6}$$

where the quantities p_0 and t_1 depend on the coordinate x_0 of the given facing element. It can be assumed here that the momentum transferred by a given explosive element to a given facing element is constant and independent of x_0. The momentum developed by the detonation products in the given direction (without the attached masses), the products having a mass m, are determined, as we already know, by the expression

$$I_0 = \frac{4}{27} mD. \tag{78,7}$$

Here half the mass of the explosion products moves in one direction, and half moves in the other. The total momentum is zero, inasmuch as the forces acting during the explosion are internal forces. Upon reflection from an absolutely rigid wall

$$I_{ref} = 2I_0 = \frac{8}{27} mD. \tag{78,8}$$

If the explosive is connected in a metallic case, and the mass of the wall of the case, corresponding to the mass of the linear explosive element, is M_2, while the mass of the facing element is M_1, we can calculate the corresponding momentum developed upon detonation and sweeping of the attached masses of the facing and of the wall.

The corresponding relations are given in Section 65.

In the particular case when $M_2 = 0$, the formula becomes much simpler:

$$I = I_0 \frac{(m + 2M_1)^2}{\sqrt{(m + M_1)^3 (m + 4M_1)}}. \tag{78.9}$$

If $M_2 = M_1 = M$, then $$I = I_0\left(1 + \frac{2M}{m}\right)\sqrt{\frac{1}{1 + \frac{3M}{m}}}. \tag{78,10}$$

The last case is simple for calculation; we shall therefore use it. In the general case we put

$$I = \eta I_0. \tag{78,11}$$

In particular, η is determined from (78.9) or from (78.10). Inasmuch as $M = sh\delta$, we have $m = sH\rho_0$ and $M/m = h\delta/H\rho_0$, where ρ_0 is the density of the explosive. Then

$$I = I_0\left(1 + \frac{2h\delta}{H\rho_0}\right)\sqrt{\frac{1}{1+\frac{3h\delta}{H\rho_0}}} = \eta I_0. \tag{78,12}$$

Starting from relation (78.6), we get

$$I = \int_{t_0}^{\infty} p\,dt = \frac{p_0}{2}(t_0 - t_1). \tag{78,13}$$

In the case of normal reflection, the quantity p_0 is determined by the relation

$$p_0 = \frac{64}{27} p_{in}, \tag{78,14}$$

where $p_{in} = \rho_0 D^2/4$, hence

$$p_0 = \frac{16\rho_0 D^2}{27}. \tag{78,15}$$

In the case of a gliding detonation wave

$$p_0 = p_{in}. \tag{78,16}$$

In the general case we can put

$$p_0 = p_{in} + \left(\frac{64}{27} - 1\right) p_{in} \cos^2\beta, \tag{78,17}$$

where β is the angle between the front of the detonation wave and the facing. Expression (78.17) is best written in the form

$$p_0 = p_{in}\left[1 + \frac{37}{27}\cos^2\beta\right]. \tag{78,18}$$

Thus, inasmuch as

$$\cos^2\beta = \frac{y_0^2}{x_0^2 + y_0^2}, \quad t_0 = \frac{\sqrt{x_0^2 + y_0^2}}{D}, \text{ бу-}$$

we have

$$p_0 = p_{in}\frac{64y_0^2 + 27x_0^2}{27\,(x_0^2 + y_0^2)}. \tag{78,19}$$

Using relations (78.7), (78.11), and (78.19), we obtain

$$I = \eta I_0 = \frac{4s\rho_0 HD\eta}{27} = \frac{s\rho_0 D}{8\cdot 27}\,\frac{64y_0^2 + 27x_0^2}{x_0^2 + y_0^2}\left(\sqrt{x_0^2 + y_0^2} - Dt_1\right), \tag{78,20}$$

from which it follows that

$$Dt_1 = \sqrt{x_0^2 + y_0^2} - 32\eta H\frac{x_0^2 + y_0^2}{64y_0^2 + 27x_0^2}, \tag{78,21}$$

Thus, we know all the constants (p_0, t_0, t_1) in the approximate law of decrease of pressure with time on the boundary between the explosion products and the facing. In particular, if $x_0 = 0$, $y_0 = H$, $M_2 = 0$, and $M_1 \to \infty$, then $\eta = 2$ and $Dt_1 = 0$, as follows from the theory of reflection of detonation waves. Since

$$p = p_0\left(\frac{t_0 - t_1}{t - t_1}\right)^3,$$

where $p = p_0(x_0)$ and $t_1 = t_1(x_0)$ (for specified y_0), we find by using (78.3) that

$$w_0 = \frac{dy}{dt} = \frac{p_0}{\delta h}\int_{t_0}^{t}\left(\frac{t_0 - t_1}{t - t_1}\right)^3 = \frac{p_0}{2\delta h}\left(t_0 - t_1\right)^3\left[\frac{1}{(t_0 - t_1)^2} - \frac{1}{(t - t_1)^2}\right]. \tag{78,22}$$

Integrating (78.22) with respect to the time, we obtain the law of motion of the given facing element

$$y = \frac{p_0}{2\delta h}\,(t_0 - t_1)\left[t + t_1 - 2t_0 + \frac{(t_0 - t_1)^2}{t - t_1}\right]. \tag{78,23}$$

Since

$$p_0 = p_{in}\frac{64y_0^2+27x_0^2}{27(y_0^2+x_0^2)}, \quad t_0 - t_1 = \frac{32H\eta}{D}\frac{x_0^2+y_0^2}{64y_0^2+27x_0^2},$$

Eq. (78.23) assumes the form

$$y = \frac{16}{27}\frac{p_{in}H\eta}{D\delta h}\frac{(t-t_0)^2}{t-t_1}, \tag{78,24}$$

or

$$y = \frac{4}{27}\frac{H\eta p_0}{h\delta}\frac{D(t-t_0)^2}{t-t_1}. \tag{78,25}$$

Knowing $y = y(t, x_0)$ we readily determine the angle α at which the given element of the facing approaches the symmetry plane.

It is obvious that

$$\operatorname{tg}\alpha = \left(\frac{\partial y}{\partial x_0}\right)_t. \tag{78,26}$$

The velocity which the given facing element has at that time is

$$w_0 = \left(\frac{\partial y}{\partial t}\right)_{x_0}. \tag{78,27}$$

Since the jet velocity is $w_1 = w_0/\tan(\alpha/2)$ and since $\tan(\alpha/2) \approx (1/2)\tan\alpha$ at small angles (and to obtain high velocities we are interested precisely in small angles), we have

$$w_1 \approx \frac{2w_0}{\operatorname{tg}\alpha} = 2\frac{\left(\frac{\partial y}{\partial t}\right)_{x_0}}{\left(\frac{\partial y}{\partial x_0}\right)_t} = 2\frac{\partial(y;x_0)}{\partial(t;y)} = -2\left(\frac{\partial x_0}{\partial t}\right)_y. \tag{78,28}$$

Using the expressions (78.25), we find that

$$t = t_0 + Ay \pm \sqrt{2Ay(t_0-t_1)+A^2y^2}, \tag{78,29}$$

where

$$A = \frac{27}{8}\frac{\delta}{p_0}\frac{h}{H\eta D},$$

Rewriting (78.28) in the form

$$w_1 = -\frac{2}{\left(\frac{\partial t}{\partial x_0}\right)_y}, \tag{78,30}$$

we find that

$$w_1 = \frac{2}{\frac{dt_0}{dx_0} \pm \frac{Ay\left[\frac{dt_0}{dx_0} - \frac{dt_1}{dx_0}\right]}{\sqrt{A^2y^2 + 2Ay(t_0 - t_1)}}}; \tag{78,31}$$

$$\operatorname{tg}\alpha = \frac{4}{27}\frac{H}{h}\frac{\rho_0 D}{\delta}\frac{t - t_0}{(t - t_1)^2}\left[(t - t_0)\frac{dt_1}{dx_0} - 2(t - t_1)\frac{dt_0}{dx_0}\right], \tag{78,32}$$

where

$$\frac{dt_0}{dx_0} = \frac{x_0}{D\sqrt{x_0^2 + y_0^2}},$$

$$\frac{dt_1}{dx_0} = \frac{x_0}{D}\left[\frac{1}{\sqrt{x_0^2 + y_0^2}} - \frac{64 \cdot 37\eta H y_0^2}{[64y_0^2 + 27x_0^2]^2}\right]. \tag{78,33}$$

It is obvious that when t = 0 we also have $\alpha = 0$, but the mass scattered in this case is also zero. In this case both opposing parts of the facing come in contact, and the process of jet formation does not begin at all. With increasing time, the angle α and the scattering mass increase, and the velocity of the jet decreases. If considerable masses go into the jet, tremendous velocities can be obtained, on the order of 100 km/sec.

Let us find the limiting expressions for small angles, corresponding to small values of x_0:

$$t_0 = \frac{y_0}{D}, \quad t_1 = \frac{y_0}{D} - \frac{\eta H}{2D}, \quad \frac{dt_0}{dx_0} = \frac{x_0}{Dy_0}, \quad \frac{dt_1}{dx_0} = \frac{x_0}{Dy_0}\left(1 - \frac{37\eta H}{64 y_0}\right),$$

$$\frac{w_0}{D} = \frac{4}{27}\frac{\rho_0}{\delta h}(\eta H)^3\left[\left(\frac{1}{\eta H}\right)^2 - \left(\frac{1}{2Dt - 2y_0 + \eta H}\right)^2\right], \tag{78,34}$$

$$y = \frac{4}{27}\frac{\rho_0 HD}{\delta h}\frac{\left(t - \frac{y_0}{D}\right)^2}{\left(t - \frac{y_0}{D} + \frac{\eta H}{2D}\right)}, \tag{78,35}$$

$$w_1 = -\frac{2Dy_0}{x_0}\frac{1}{1 + \frac{Ay\frac{37}{64}\frac{\eta H}{y_0}}{\sqrt{A^2y^2 + Ay\frac{\eta H}{D}}}}, \tag{78,36}$$

$$\operatorname{tg}\alpha=\frac{4}{27}\frac{H}{h}\frac{\rho_0}{\delta}\frac{x_0}{y_0}\frac{t-\frac{y_0}{D}}{\left(t-\frac{y_0}{D}+\frac{\eta H}{2D}\right)^2}\times$$
$$\times\left[\left(t-\frac{y_0}{D}\right)\left(1-\frac{37}{64}\frac{\eta H}{y_0}\right)-2\left(t-\frac{y_0}{D}+\frac{\eta H}{2D}\right)\right]. \quad (78,37)$$

It is obvious that the forming jet will have the greatest velocities at small values of x_0. The maximum velocity will occur when $x_0 = 0$. The energy of a given jet element is practically equal to the energy of motion of the given facing element; this energy increases with time and as $t \to \infty$ it reaches its maximum value. When $t \to \infty$ we have

$$\frac{w_0}{D}=\frac{4\eta\rho_0 H}{27\delta h},\quad \frac{w_1}{D}=-\frac{2y_0}{x_0},\quad \operatorname{tg}\alpha=-\frac{4H\rho_0 x_0}{27h\delta y_0}\left(1+\frac{37}{64}\frac{\eta H}{y_0}\right). \quad (78,38)$$

The energy of a given jet element is

$$\overline{E}_1=\frac{1}{2}\overline{m}_1\overline{w}_1^2, \quad (78,39)$$

where $\overline{w}_1$ is the velocity of the given jet element.

Inasmuch as the mass of the given jet element is

$$\overline{m}_1=\overline{m}_0\sin^2\frac{\alpha}{2}\approx\frac{\overline{m}_0}{4}\operatorname{tg}^2\alpha, \quad (78,40)$$

where $\overline{m}_0$ is the mass of the given facing element, we have

$$\overline{E}_1=\frac{8\overline{m}_0 D^2}{27\cdot 27}\frac{H^2\rho_0^2}{h^2\delta^2}\left(1+\frac{37}{64}\frac{\eta H}{y_0}\right)^2. \quad (78,41)$$

This energy is independent of the value of x_0.

The momentum of the given jet element is

$$\overline{I}_1=\overline{m}_1\overline{w}_1=-\frac{8\overline{m}_0 D H^2\rho_0^2 x_0}{27\cdot 27 h^2\delta^2 y_0}\left(1+\frac{37}{64}\frac{\eta H}{y_0}\right)^2. \quad (78,42)$$

The momentum increases with increasing x_0.

Since in practice the maximum facing velocity w_0 is reached within relatively small time intervals elapsed from the start of the motion, there is no need to move the shell from the plane, or in the real case from the symmetry axis, to a large distance. It is sufficient to choose for this distance a quantity on the order of (2 — 3) y_0.

The difference between the axially symmetrical case and the plane case will be insignificant.

We can develop an analogous elementary theory for facing of arbitrary curved profile. The general relations will remain in force in this case. It is necessary to take account of the fact that in the integration of (78.5) we have

$$t_0 = t_0(x_0), \quad y = f(x_0), \tag{78,43}$$

i.e., y is a specified function of x_0.

Apparently it is best to choose the shell in linear form, but deflecting it somewhat towards the symmetry axis in a direction opposite to the compression (Fig. 196).

Let us proceed to determine the pressures and temperatures whenever the CJ strikes a rigid partition.

Upon collision of two CJ having velocities on the order of 100 km/sec, exceedingly high pressures should arise and rather high temperatures, which exceed greatly those in collisions of ordinary CJ.

In order to ascertain what equation of state should be used in this case to describe the processes of collision, let us make first a rough estimate of the pressures that are produced thereby.

The problem of collision between two cumulative jets is equivalent to the problem of collision against an absolutely rigid partition. When striking a partition, a shock wave is produced in the cumulative jet, traveling from the wall, in which the initial pressure, as is well known, is determined by the relation

$$p_{sh} = \frac{\rho_0 u_0^2}{1 - \frac{\rho_0}{\rho_{sh}}} = \frac{\rho_0 u_0^2}{\alpha}, \tag{78,44}$$

where u_0 and ρ_0 are the velocity and density of the cumulative jet, p_{sh} and ρ_{sh} are the pressure and density on the front of the resultant shock wave.

It is obvious that the pressure p_{sh} will have a minimum when $\alpha = 1$. The quantity α is determined by the equation of state of the given material.

If we assume in the range of high pressures of interest to us that the material of the jet obeys an equation of state of the form $p = A\rho^n$, then the limiting density ρ_{sh} will be determined by the relation

$$\frac{\rho_{sh}}{\rho_0} = \frac{n+1}{n-1}.$$

Here

$$\alpha_{st} = \frac{2}{n+1}, \quad p_{sh} = \frac{n+1}{2}\rho_0 u_0^2. \tag{78,45}$$

The Eq. (78.45) yields the maximum pressure p_{sh} produced during the shock. At very high pressures (on the order of 10^8 kg/cm^2), any medium is converted into an electronic gas, and then the equation of state $p = A\rho^n$ indeed applies, with $n = 5/3$ (as in the case of a monatomic gas) and relation (78.45) assumes the form

$$p_{sh} = \frac{4}{3}\rho_0 u_0^2. \tag{78,46}$$

When $u_0 = 100$ km/sec and $\rho_0 = 1.5$ g/cm^2 (beryllium), we obtain from (78.46) that $p_{sh} = 1.5 \times 10^8$ kg/cm^2, confirming the correctness of our estimate of the pressure, and giving grounds for regarding the jet material as a degenerate electronic gas.

As is well known from statistical physics, the equation of state of this gas is

$$p = \frac{1}{5}\left(\frac{6\pi^2}{g}\right)^{\frac{2}{3}} \frac{h^2}{4\pi^2 m_e}\left(\frac{N}{V}\right)^{\frac{5}{3}}, \tag{78,47}$$

where g is the statistical weight of the particles, N is the number of particles in a single mole, m_e is the electron mass, h is Planck's constant, equal to 6.558×10^{-27} erg-sec, V is the molar volume of the substance at a pressure p. For the case under consideration $g = 2$, $N = ZN_a$, where N_a is Avogardo's number and Z is the

number of electrons per atom of the given metal (for beryllium $Z = 4$), $V = \mu/\rho$, where μ is the atomic weight of the metal, ρ its density at a pressure p (for beryllium $u = 9$ g/mole). Thus, for beryllium

$$p = \frac{1}{5}(3\pi^2)^{\frac{2}{3}} \frac{h^2}{4\pi^2 m_e} \left(\frac{4N_a}{9}\right)^{\frac{5}{3}} \rho^{\frac{5}{3}}. \qquad (78,48)$$

Let us calculate now the temperature occurring during the impact of a cumulative jet against a partition.

We can obtain a first rather crude estimate of the temperature from the following considerations. The internal energy of the medium on the front of the shock wave is

$$E = \frac{p}{2}(v_0 - v) = \frac{pv}{n-1}. \qquad (78,49)$$

For a degenerate electron gas, inasmuch as $n = 5/3$, we have

$$E = \frac{3}{2} pv. \qquad (78,50)$$

We now rewrite the Eq. (78.50) in the form

$$E = c_v T = \frac{3}{2} p \frac{\mu}{\rho}. \qquad (78,51)$$

As is well known, at high pressures, the atomic specific heat of solids tends to a value of 6 cal/g-atom-deg. If in this case we assume for the jet material $c_v = 6$ cal/g-atom-deg, we have

$$T = \frac{3 \cdot 9 \cdot 1{,}35 \cdot 10^{11} \cdot 2{,}34 \cdot 10^{-5}}{2 \cdot 6 \cdot 6} \approx 1{,}1 \cdot 10^6 \,^\circ\mathrm{K}. \qquad (78,52)$$

This temperature must, however, be conceded to be too high, since the specific heat of substances in the state of an electronic gas is an increasing function of the temperature.

Accordingly, in order to obtain a more exact estimate of the temperature of interest to us, we start out from the formula for the specific heat for a degenerate electronic gas

$$c_v = \beta N T \left(\frac{V}{N}\right)^{\frac{2}{3}}, \qquad (78,53)$$

where

$$\beta = \left(\frac{9\pi}{6}\right)^{\frac{2}{3}} \frac{m_e k^2 4\pi^2}{h^2}$$

and k is Boltzmann's constant, equal to 1.38×10^{-16} cal/deg.

For the case which we are considering, relation (78.53) becomes

$$c_v = \left(\frac{2\pi}{6}\right)^{\frac{2}{3}} \frac{m_e k^2 \cdot 4\pi^2}{h^2} 4N_a T \left(\frac{\mu}{4\rho N_a}\right)^{\frac{2}{3}}.$$

Substituting into this expression the numerical values of the corresponding quantities, we obtain

$$c_v = 2{,}87 T \frac{\text{эсм}}{\text{град}}.$$

Starting from this, we transform (78.51). We obtain

$$c_v T = 2{,}87 T^2 = \frac{3}{2} \cdot 1{,}35 \cdot 10^{11} \cdot \frac{9}{6} = \frac{9}{4} \cdot 1{,}35 \cdot 10^{11},$$

Hence

$$T = 3{,}25 \cdot 10^6\ {}^\circ\mathrm{K}.$$

We have determined this temperature under the assumption that the material of the jet behaves like a degenerate electronic gas under conditions of impact.

Let us verify now the correctness of this assumption.

As is well known, the criterion for strong degeneracy is $\theta \ll 1$, where

$$\theta = \frac{2^{\frac{5}{3}} \pi m_e k V^{\frac{2}{3}} T}{h^2 N^{\frac{5}{3}}}. \tag{78,54}$$

Substituting in (78.54) the numerical values of the corresponding quantities, we obtain

$$\theta = 4{,}73 \cdot 10^{-8} T, \tag{78,55}$$

from which we see that at temperatures on the order of $10^5 - 10^6$ deg K we have $\theta << 1$, i.e., the material of the jet indeed must be regarded as a degenerate electronic gas during the instant of impact.

Thus, the second version which leads at u_0 = 100 km/sec to an shock temperature on the order of 300,000° K, is apparently sufficiently accurate.

It follows from this that at impact velocities on the order of several tens of kilometers per second, the cumulative jet will have not only armor piercing but also strong armor igniting action.

At jet impact velocities on the order of 100 km/sec, we can expect the appearance not only of ordinary radiation, but also of harder radiation.

Table 102

Effect of ordinary and cumulative charges on armor

(1) Характеристика заряда	(2) Преграда	(3) Характер деформации преграды
Сплошной цилиндрический, высота — 180 *мм*, диаметр — 65 *мм* (4)	Бронеплита толщиной 200 *мм* (5)	Вмятина (6)
То же, с конической выемкой без облицовки (7)	То же (8)	Кратер глубиной 22 *мм* (9)
То же, выемка имеет стальную облицовку толщиной 2 *мм* (10)	То же (8)	Сквозная пробоина (11)

1) Characteristic of charge, 2) Partition, 3) Character of deformation of partition, 4) Continuous cylindrical, height 180 mm, diameter 65 mm, 5) Armor plate 200 mm thick, 6) Dent, 7) The same, with conical cavity without facing, 8) The same, 9) Crater 22 mm deep, 10) The same, cavity has steel facing 2 mm thick, 11) Hole pierced through.

Table 103

Dependence of velocity of frontal part of cumulative jet on certain factors

Размеры заряда 1		Характер облицовки 2		3	4 Параметры выемки			
диаметр, *мм* 5	высота, *мм* 6	материал 7	толщина, *мм* 8	Форма выемки	9 диаметр основания, *мм*	10 угол раствора конуса, *град*	11 Скорость головной части струи, *м/сек*	12 Скорость головной части струи, *м/сек* *)
30	70	13 Сталь	1	14 Полусферическая	28	—	3000	6050
30	70	»	1	Конус 15	27,2	60	6500	7650
30	70	»	1	»	27,2	35	7300	8500
30	70	»	1	»	27,2	27	7400	9000
30	70	»	1	Гипербола 16	27,2	—	9500	—
42	85	»	4	»	37,2	—	7150	10700

17 *) Данные для зарядов с дюралюминиевой облицовкой толщиной 1 *мм*.

1) Dimensions of charge, 2) Character of facing, 3) Form of cavity, 4) Parameters of cavity, 5) Diameter, mm, 6) Height, mm, 7) Material, 8) Thickness, mm, 9) Diameter of base, mm, 10) Angle of cone, degrees, 11) Velocity of frontal part of jet, m/sec, 12) Velocity of frontal * part of jet, m/sec, 13) Steel, 14) Hemispherical, 15) Cone, 16) Hyperbola.

17) — * Data for charges with duraluminum facing 1 mm thick.

Table 104

Effect of M/m_a on the velocity of the frontal part of a cumulative jet (form of cavity — cone, aperture angle 35°)

	① Материал облицовки	② Диаметр заряда, *мм*	③ Скорость головной части струи, *м/сек*
④	Сталь	22	7400
	›	30	7300
	›	42	7400
⑤	Дюралюминий.	22	8800
	› .	30	8500
	› .	42	8600

1) Material of facing, 2) Diameter of charge, mm, 3) Speed of frontal part of jet, m/sec, 4) Steel, 5) Duraluminum.

Table 105

Effect of thickness of facing on the velocity of frontal part of cumulative jet

(Dimensions of charge: h = 85 mm, d = 42 mm)

	① Материал облицовки	② Толщина облицовки, *мм*	③ Скорость головной части струи, *м/сек*
④	Сталь	2	6700
	›	3	6050
	›	4	5250
⑤	Дюралюминий .	1	8600
	›	2	7600
	›	3	7500
	›	4	7150

See Table 104.

Table 106

Critical velocities of armor piercing

① Материал преграды	② Твердость по Бринелю	③ Материал кумулятивной струи	④ Критическая скорость струи, *м/сек*
⑤ Дюралюминий	115	Дюралюминий ⑤	2900
⑥ Сталь	125	Дюралюминий ⑤	3300
Сталь	125	Сталь ⑥	2050
⑦ Сталь закаленная . . .	$R_c = 50$	Сталь ⑥	2200

1) Material of partition, 2) Brinnel hardness, 3) Material of cumulative jet, 4) Critical velocity of jet, m/sec, 5) Duraluminum, 6) Steel 7) Hardened steel.

Table 107

Dependence of armor piercing action of jet on hardness of armor

① Материал преграды	② Твердость по Бринелю	③ Глубина бронепробивания, *мм*
④ Сталь	100	111
Сталь	350	80
⑤ Алюминиевый сплав	50	327
Алюминиевый сплав	200	256

1) Material of partition, 2) Brinnel hardness, 3) Depth of armor piercing, mm, 4) Steel, 5) Aluminum alloy.

Table 108

Dependence of pressure on velocity in the penetration of a cumulative jet

① p, кг/см²	② u, м/сек (дюралюминиевая струя по дюралюминиевой преграде)	③ u, м/сек (дюралюминиевая струя по стальной преграде)	④ u, м/сек (стальная струя по стальной преграде)
$1 \cdot 10^5$	1300	900	510
$2 \cdot 10^5$	2320	1640	970
$3 \cdot 10^5$	3200	2270	1380
$4 \cdot 10^5$	4000	2870	1760
$5 \cdot 10^5$	4670	3390	2120
$6 \cdot 10^5$	5310	3870	2450
$7 \cdot 10^5$	5930	4350	2785
$8 \cdot 10^5$	6520	4775	3070
$9 \cdot 10^5$	7060	5180	3350
$1 \cdot 10^6$	7570	5580	3630
$1{,}2 \cdot 10^6$	8540	6340	4110
$1{,}4 \cdot 10^6$	9450	7040	4640
$1{,}6 \cdot 10^6$	10250	7660	5110
$1{,}8 \cdot 10^6$	11100	8290	5550
$2 \cdot 10^6$	11850	8890	5980

1) p, kg/cm², 2) u, m/sec (duraluminum jet on duraluminum partition), 3) Duraluminum jet on steel partition, 4) Steel jet on steel partition.

Table 109

Dependence of velocity of penetration on the velocity of the jet

① Материал струи — дюралюминий ③ ② Материал брони — сталь ④		① Материал струи — сталь ④ ② Материал брони — сталь ④	
u, м/сек ⑤	$u_{x,0}$ м/сек	u, м/сек	$u_{x,0}$, м/сек
3300	0	2050	0
4000	610	3000	940
5000	1060	4000	1540
6000	1470	5000	2080
7000	1890	5500	2360
8000	2260	6300	2770
9000	2650	7000	3340

1) Jet material, 2) Armor material, 3) Duralmminum, 4) Steel, 5) m/sec.

Table 110

Experimental and calculated velocities of armor piercing

① Толщина преграды, *мм*	② Материал преграды	③ Материал струи	④ Средняя скорость элемента струи $u_{ср}$, *м/сек*	$\frac{u_{x,0}}{u_{av}}$		
				⑤ из эксперимента	⑥ по формуле (73,8)	⑦ по формуле Лаврентьева
30	⑧ Сталь	⑧ Сталь	6300	0,43	0,44	0,50
20	⑨ Дюралюминий	⑨ Дюралюминий	8000	0,43	0,44	0,50
30	⑨ Дюралюминий	⑧ Сталь	7100	0,57	0,60	0,63

1) Thickness of partition, mm, 2) Material of partition, 3) Material of jet, 4) Average velocity of jet element, u_{av}, m/sec, 5) From experiment, 6) From formula (73.8), 7) From Lavrent'yev's formula, 8) Steel, 9) Duraluminum

Table 111

Effect of rotation on armor piercing action of cumulative charges

① Диаметр заряда, *мм*	② Параметры конуса		④ Толщина стенок облицовки, *мм*	⑤ Глубина бронепробивания, *мм*		⑧ Понижение бронепробивного эффекта, %
	③ диаметр основания выемки, *мм*	$\frac{h}{d}$		⑥ без вращения	⑦ $n = 20\,000$ *об/мин*	
32	26	1	1,0	45 ± 5	37 ± 4	20
55	44	1	1,5	77 ± 1	57 ± 2	26
76	56	1	2,0	132 ± 3	90 ± 5	32
32	26	2	2,0	74 ± 5	44 ± 5	31
76	56	2	2,0	205 ± 5	82 ± 2	60

1) Charge diameter, mm, 2) Cone parameters, 3) Diameter of base of cavity, mm, 4) Thickness of facing walls, mm, 5) Depth of armor piercing, mm, 6) Without rotation, 7) n = 20,000 rpm, 8) Reduction in armor piercing effect, percent.

Table 112

Effect of angle of aperture of the cone on armor-piercing action of rotating cumulative charges

Форма выемки (1)	Параметры выемки (2)			Бронепробивной эффект, *мм* (3)		Понижение бронепробивного эффекта при вращении, %
	α	$d_{осн}$, *мм* (4)	$\frac{h}{d_{осн}}$	без вращения (5)	$n = 20\,000$ *об/мин* (6)	
(8) Конус	27	56	2,0	205 ± 5	82 ± 2	60
»	35	56	1,2	160 ± 5	86 ± 8	46
»	60	56	1,0	130 ± 3	90 ± 5	32
(9) Гипербола . .	—	56	2,0	160 ± 5	85 ± 5	47

1) Form of cavity, 2) Parameters of cavity, 3) Armor piercing effect, mm, 4) d_{base}, mm, 5) Without rotation, 6) rpm, 7) Reduction in armor-piercing effect upon rotation, percent, 8) cone, 9) hyperbola

Table 113

Effect of speed of rotation on armor piercing action of cumulative charges

Диаметр заряда, *мм* (1)	Форма выемки (2)	Параметры выемки (3)		Глубина бронепробивания, *мм* (4)				
		$d_{осн}$, *мм* (5)	$\frac{h}{d}$	без вращения (6)	$n = 5000$ *об/мин* (7)	$n = 10\,000$ *об/мин*	$n = 15\,000$ *об/мин*	20 000 *об/мин*
76	(8) Конус	56	2	205	120	115	98	82
76	(9) Гипербола	56	2	160	150	130	100	85

1) Diameter of charge, mm, 2) Form of cavity, 3) Parameters of cavity, 4) Depth of armor piercing, mm, 5) d_{base}, mm, 6) Without rotation, 7) rpm, 8) Cone, 9) Hyperbola.

Table 114

Dependence of armor piercing action (depth of armor piercing, mm) on the distance between the charges and the armor.

Скорость вращения	Расстояние между зарядом и броней, *мм*			
	10	40	76	152
Без вращения	100	132	—	120
20 000 *об/мин*	70	90	70	40

1) Speed of rotation, 2) Distance between charge and armor, mm, 3) Without rotation, 4) rpm.

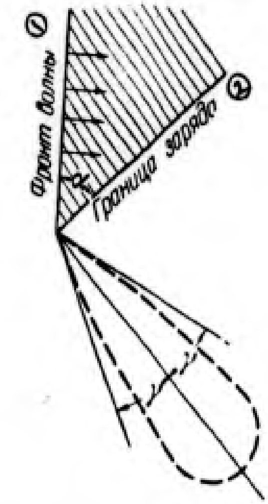


Fig. 157. Scattering of detonation products from an oblique cut. 1) Wave front, 2) Boundary of charge.

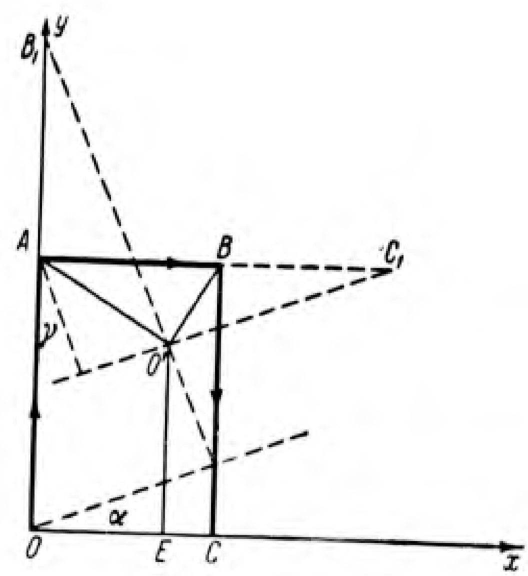


Fig. 158. Detonation of Π-shaped charge.

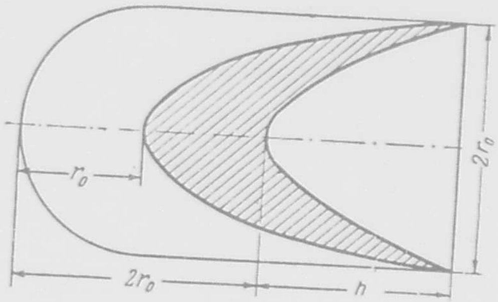


Fig. 163. Active part of cylindrical cumulative charge.

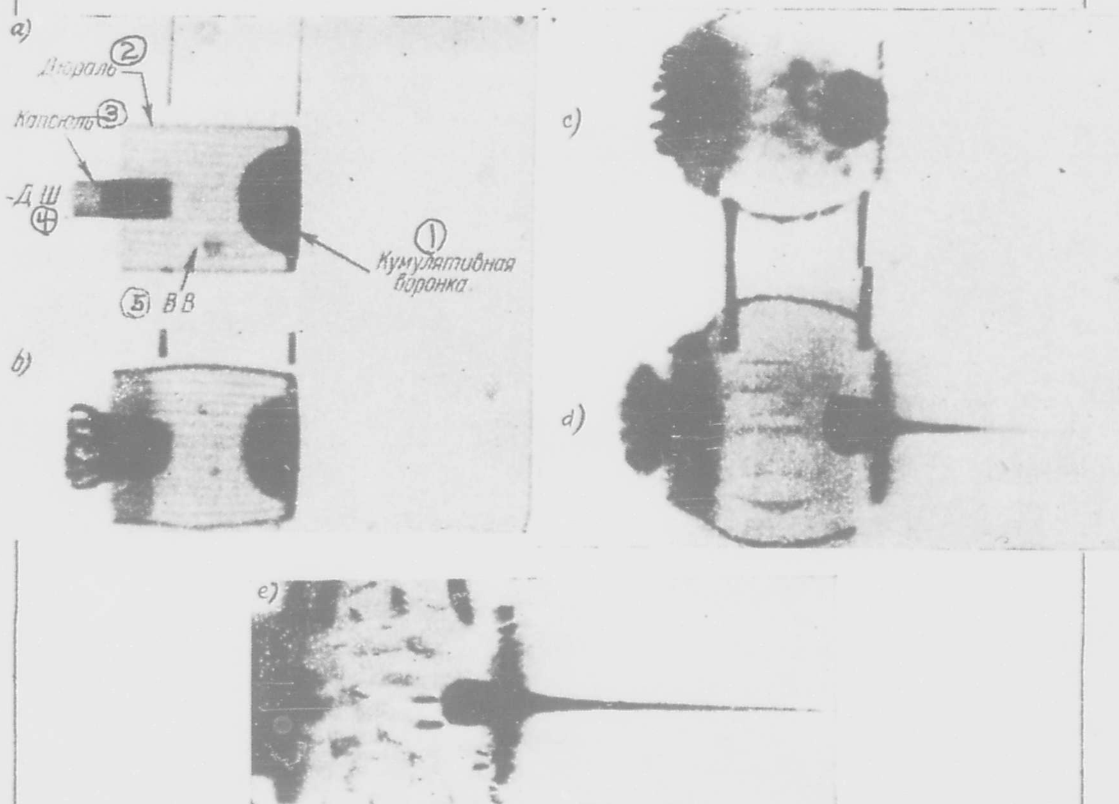


Fig. 164. Shaping of cumulative jet upon compression of hemispherical facing: a — charge prior to explosion, b — charge 6 microseconds after explosion, c — 8 microseconds after, d — 12 microseconds after, e — 24 microseconds after.

1) Cumulative funnel, 2) Duraluminum, 3) Capsule, 4) Detonating pin, 5) Explosive.

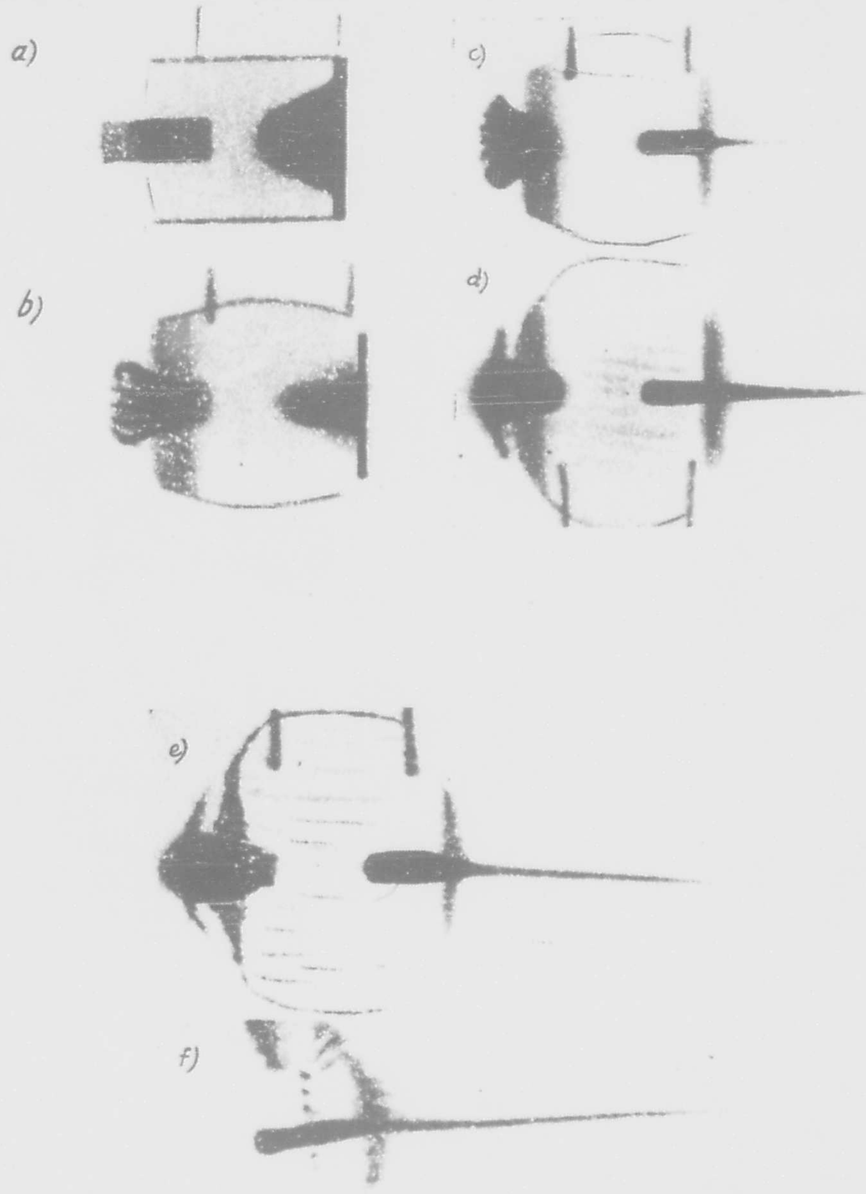


Fig. 165. Formation of cumulative jet upon compression of conical facing: a— prior to explosion, b -- charge 6 microseconds after explosion, c -- 12 microseconds after, d -- 50 microseconds after, e -- 70 microseconds after, f -- 24 microseconds after.

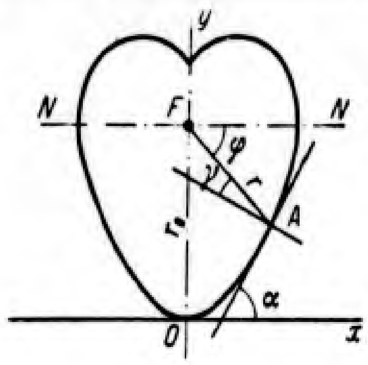


Fig. 159. Logarithmic spiral.

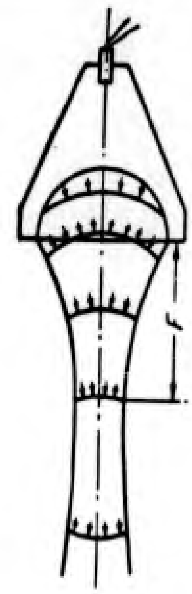


Fig. 160. Shaping of the cumulative jet.

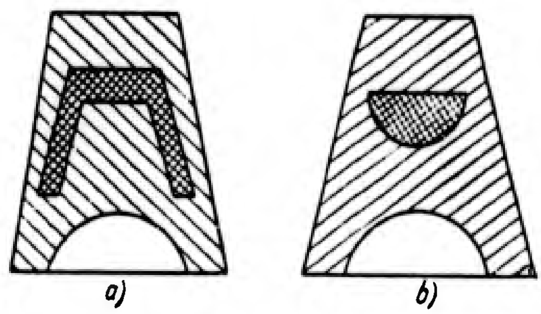


Fig. 161. Cumulative charges with "lenses."

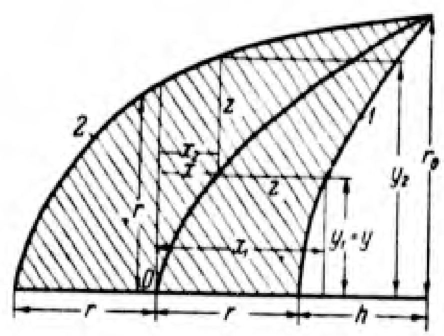
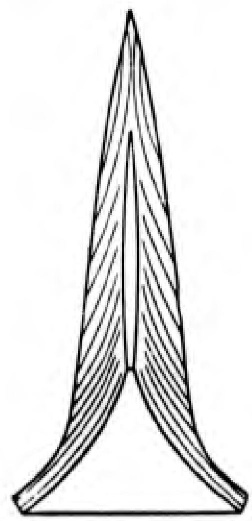


Fig. 162. Active part of cumulative charge.



Fig, 166. Pestle.

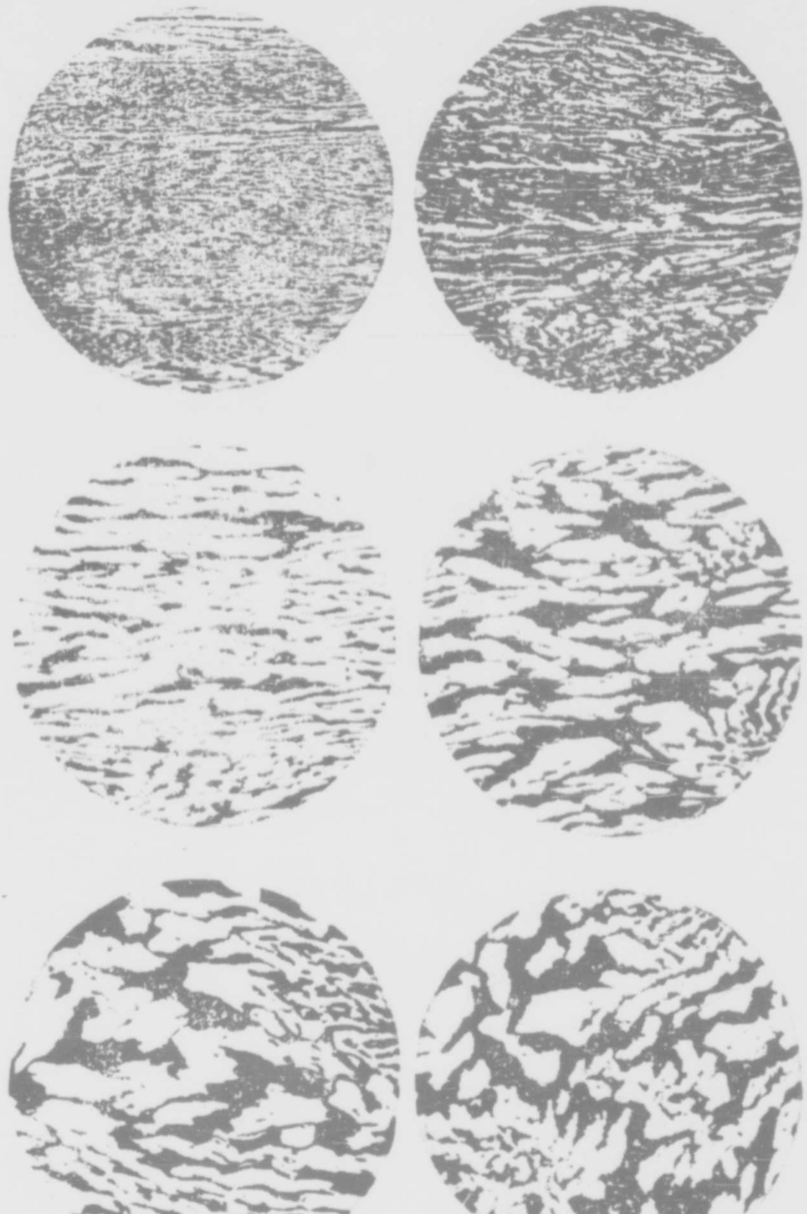


Fig. 167. Microphotographs of polished sections of the pestle (brass).

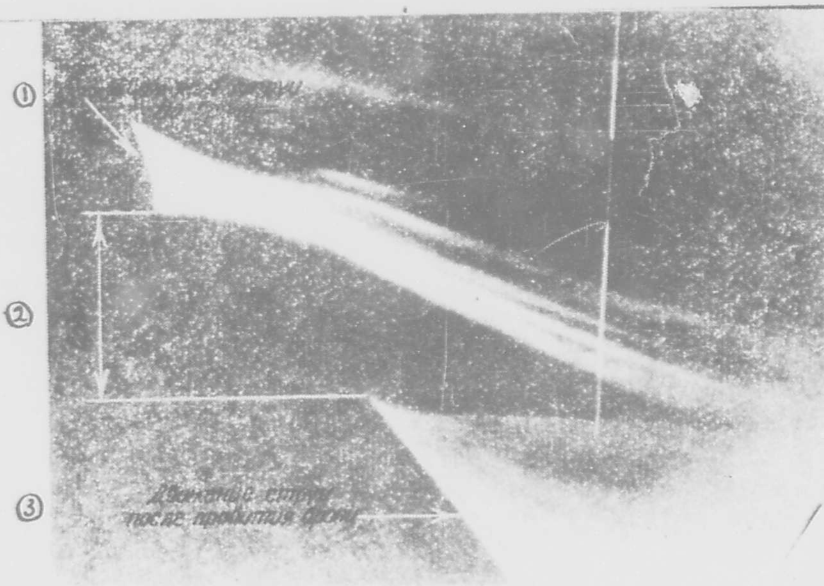


Fig. 168. Motion of cumulative jet before and after piercing the armor.

1) Motion of jet before reaching armor, 2) Time, 3) Motion of jet after piercing the armor.

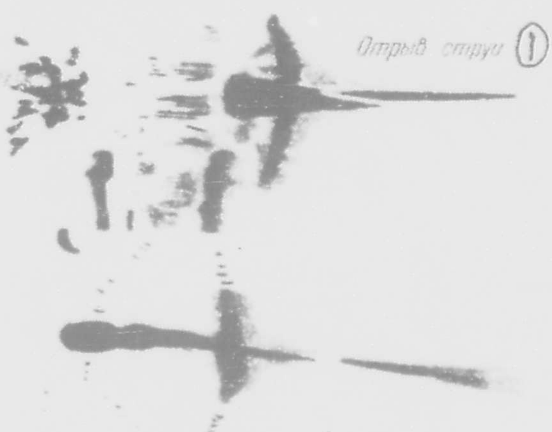
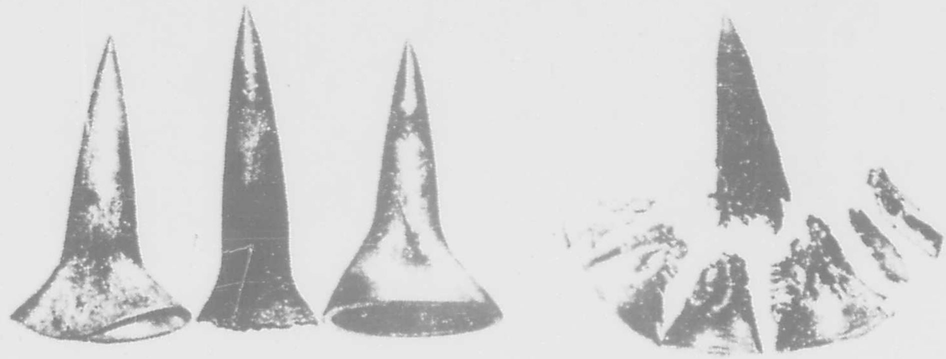


Fig. 169. Detachment of jet from pestle.

1) Detachment of jet.



a)

b)

Fig. 170. Compressed facing made of soft (a) and hard (b) steel.

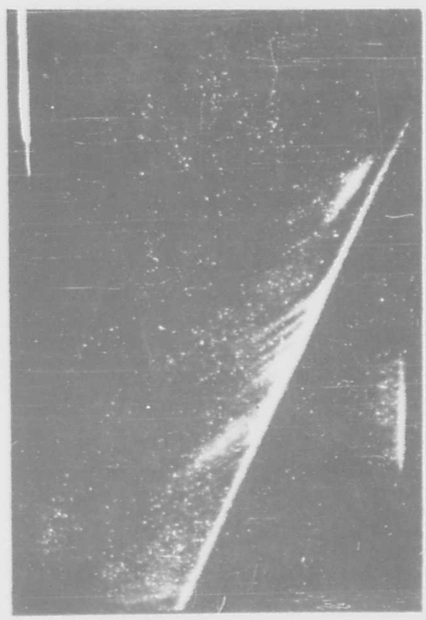


Fig. 171. Streak photograph of motion of frontal part of cumulative jet.

Fig. 172. Spark photograph of dispersed cumulative jet (45 microseconds after start of explosion).

1) Cumulative jet, 2) Ballistic wave.

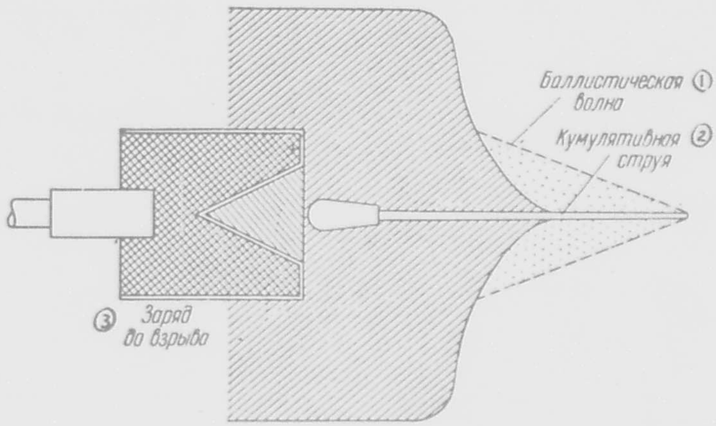


Fig. 173. Explosion of cumulative charge (scheme).

1) Ballistic wave, 2) Cumulative jet, 3) Charge prior to explosion.

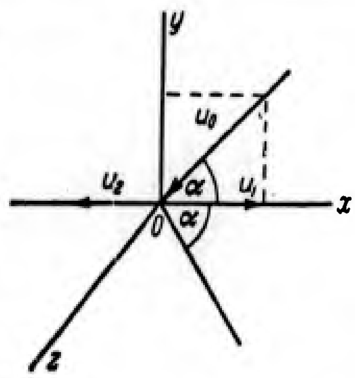


Fig. 174. Collision of jets.

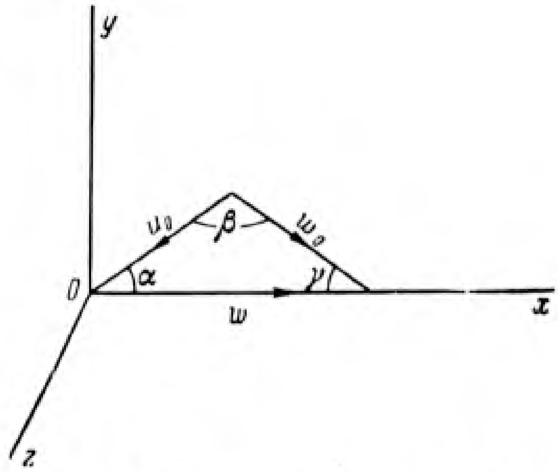


Fig. 175. Collision of jets. Point of intersection of jet with plane moves along the plane).

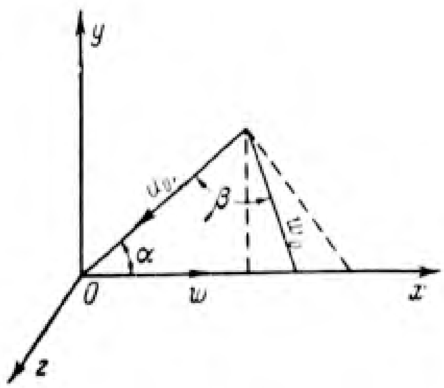


Fig. 176. Collision of jets. Case of formation of one jet after collision ($\beta = (\pi - \alpha)/2$).

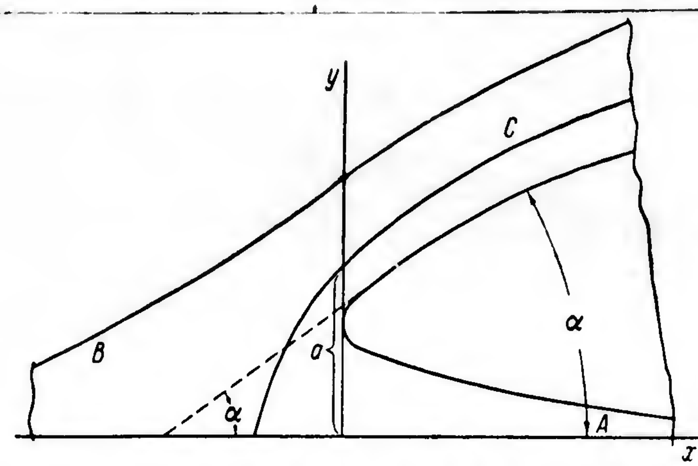


Fig. 177. Convergence of jet (steady-state flow of ideal liquid). C -- initial jet, A and B -- diverging jets.

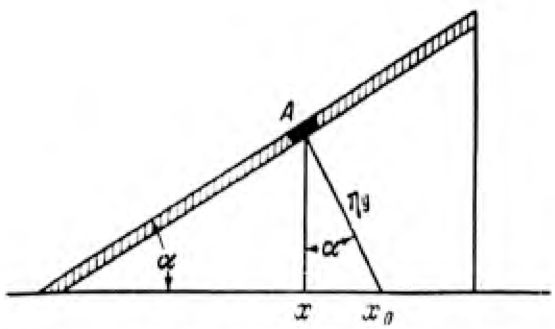


Fig. 178. Motion of elements of conical facing.

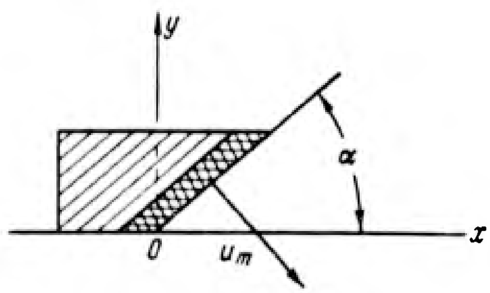


Fig. 179. Motion of flat plate under the action of a detonation product.

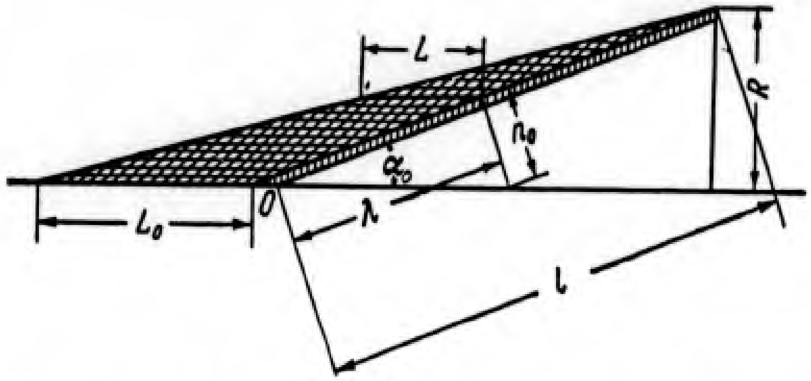


Fig. 180. Compression of conical facing of cumulative charge.

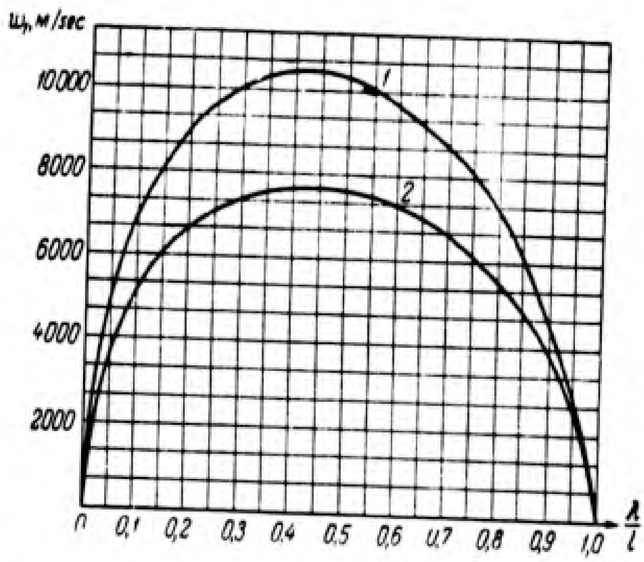


Fig. 181. Distribution of velocities along cumulative jet.

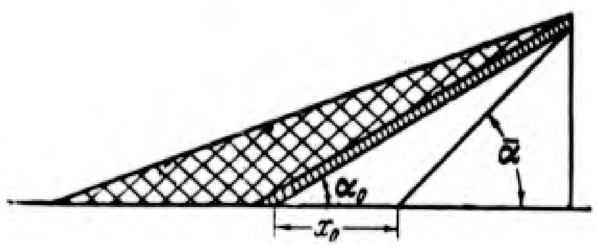


Fig. 182. Compression of conical facing with allowance for non-uniformity of detonation of the charge.

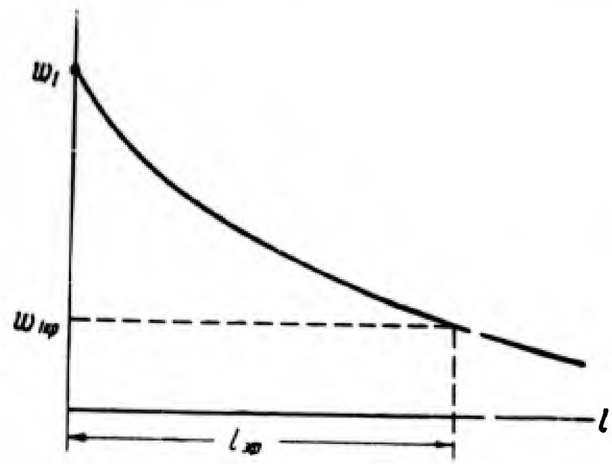


Fig. 183. Distribution of velocities along cumulative jet.

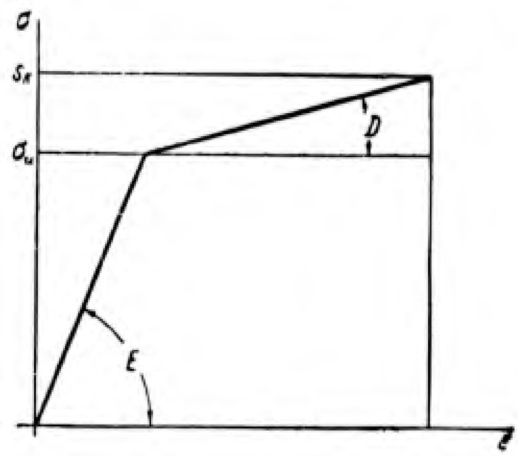


Fig. 184. Diagram of true stresses.

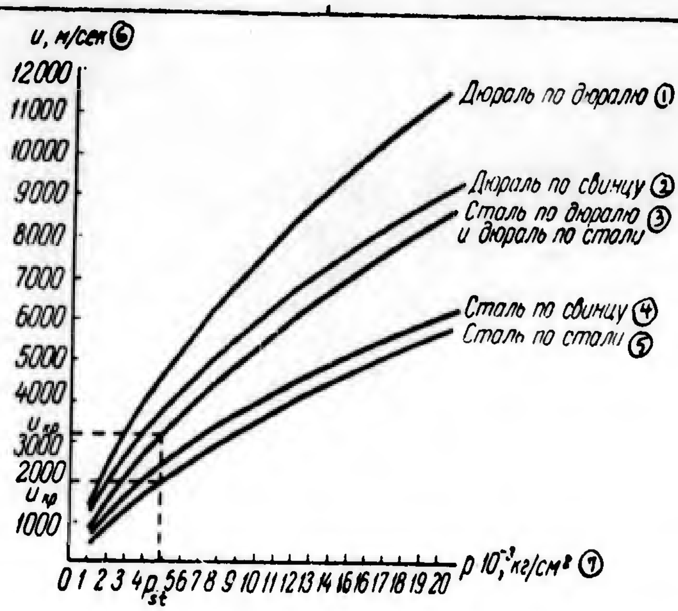


Fig. 185. Connection between the velocity and pressure upon interaction of cumulative jets with different partitions.

1) Duraluminum with duraluminum, 2) Duraluminum with lead, 3) Steel with duraluminum and duraluminum with steel, 4) Steel with lead, 5) Steel with steel, 6) u, m/sec, 7) kg/cm^2.

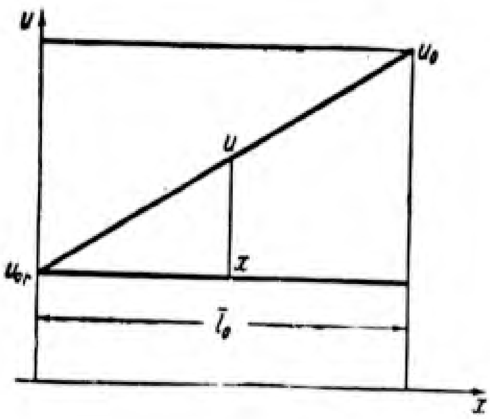


Fig. 186. Linear distribution of velocities along cumulative jet.

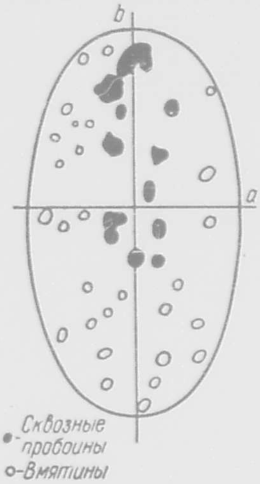


Fig. 187. Effect of 76-mm cumulative charge on a plate (distance between charge and armor is 4.5 meters).

● — pierced holes, ○ — dents.

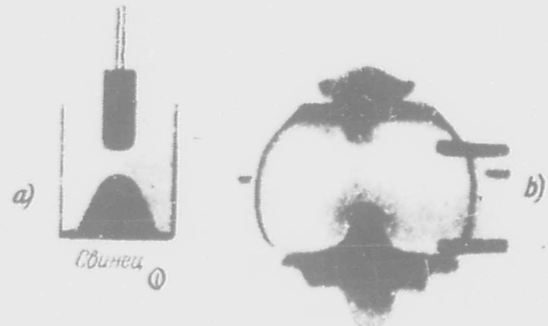


Fig. 188. Lead cumulative: a) charge prior to explosion, b) formation of lead cumulative jet after 12 microseconds from the explosion.

1) Lead.

Fig. 189. Disturbance of stability of cumulative jet.

1) Bending of jet.

Fig. 190. Action of rotating cumulative charge on a partition (n = 20,000 rpm).

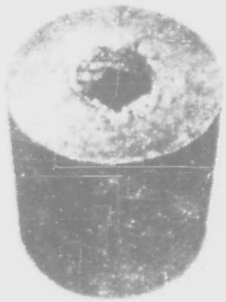


Fig. 191. Action of non-rotating cumulative charge on a partition.

Fig. 192. Action of non-rotating cumulative charge on a partition.

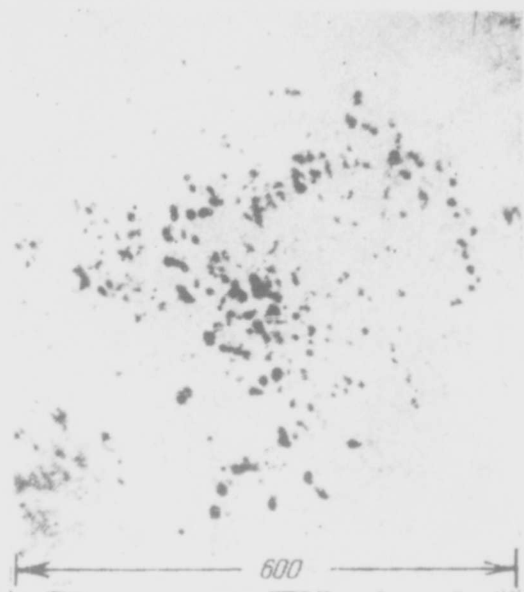


Fig. 193. Action of rotating cumulative charge on a partition.

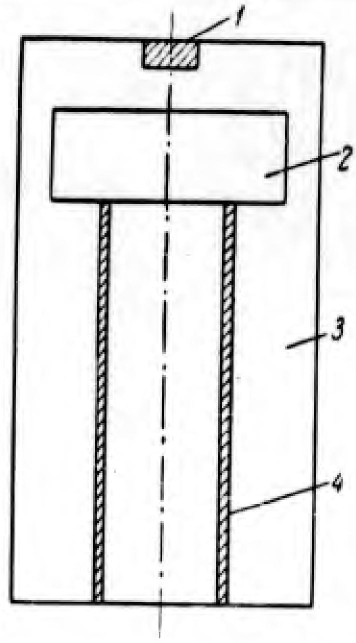


Fig. 194. Cumulative charge: 1 -- detonator, 2 -- bulky lens, 3 -- charge of explosive, 4 -- cumulative facing.

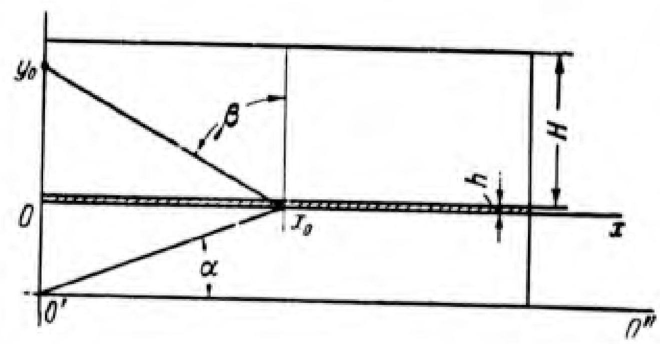


Fig. 195. Used in derivation of relationships for cumulative charge with cylindrical facing.

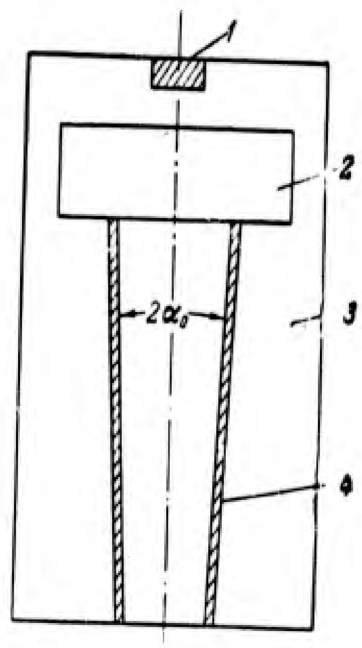


Fig. 196. Cumulative charges: 1 -- detonator, 2 -- bulky lens, 3 -- explosive charge, 4 -- cumulative facing.

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2. Explosions — Mathematical analysis

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