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RESEARCH MEMORANDUM

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PROJECT RAND

RESEARCH MEMORANDUM

THE ATOMIC-HYDROGEN GUN

R. L. Bjork

RM-1707

May 15, 1956
Revised July 5, 1961

Assigned to ______________________

This research is sponsored by the United States Air Force under contract No. AF 49(638)-700 monitored by the Directorate of Development Planning, Deputy Chief of Staff, Development, Hq USAF.

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SUMMARY

In this memorandum, calculations are presented which indicate that light-gas guns powered by electrical discharges may be expected to achieve substantially higher projectile velocities than may be obtained by any present laboratory gun.

The thermodynamic properties of compressed, high-temperature hydrogen are calculated and presented, and it is found that for isentropic expansions in a region representative of gun operation, the gas closely follows an equation of state of the form $P \sim \rho^{\gamma}$, even though considerable dissociation occurs. Although γ is a function of entropy, this fact allows the application of a gun model used by Charters *et al.* of the Ames Aeronautical Laboratory in interpreting the performance of their light-gas gun which uses helium.[1]

Calculations made on this model show that to maximize muzzle velocity one should maximize two parameters:

- $P_o AL/m$, which is determined by the mechanical properties of the gun
- $\gamma^2 a_o/(\gamma-1)$, which is determined by the gas properties at the time of shear disc rupture

Factors which limit the increase of each of these parameters are discussed.

For the same gun parameter and gas temperature, hydrogen was found to give much higher muzzle velocities than helium.

Using values commensurate with the current state of the art, it was estimated that such a gun could deliver about 10 km/sec and would have a large potential for future improvement.

PREFACE

The body of this report was written in 1956, and the statements therein which refer to "the present time" and "the current state of the art" refer to that early date. The purpose of this preface is to note improvements in the arts which have occurred up to the present date (1961). The references used in the preface are added to the original list. It is clear from the following that the later work has demonstrated the feasibility of the electrically powered light-gas gun, and also that improvements in the operating breech pressure which were cited as possible in 1956 have now been realized, at least in the case of conventional light-gas guns.

It is now possible to give an unclassified reference to the work of Charters, et al.,[1] which was not originally the case.

Several laboratories have conducted small-scale feasibility studies on the electrically powered light-gas gun.[7-10] At the present time, two of these laboratories (Naval Research Laboratory and Avco-Everett) are engaged in the development of larger guns.[8,9] In their work, both agencies have made use of the thermodynamic properties of hydrogen which are calculated in Appendix A. They estimate that about 65 to 85 per cent of the theoretical velocities given in Fig. 7 are achieved experimentally, and they are seeking to understand why the full theoretical velocities are not obtained. In both places a major difficulty arises in estimating the fraction of electrical energy that is deposited in the working fluid, for it is difficult to measure the dynamic breech pressures accurately. Georgiev, using crusher gauges, estimates that pressures between 40,000 and 100,000 psi are attained, but notes that there is considerable scatter in the data.[9] Swift estimates that if half of the electrical energy is deposited in the gas, about 100,000 psi would be obtained.[8]

The velocities obtained with conventional-type light-gas guns have increased considerably. For example, workers at the Ames Research Center have shot polyethylene projectiles at 29,700 ft/sec (9.05 km/sec),[11] and at the Naval Research Laboratory a velocity of about 27,000 ft/sec (8.23 km/sec) was obtained with an aluminum projectile.[12]

Partridge and his workers at the Utah Research and Development Corporation[13] have developed a gun breech which in a static test has withstood about 750,000 psi. The breech was built up of concentric cylindrical shells which were heated before positioning and allowed to shrink on the inner shells. This method prestresses the interior to a high pressure. The same construction principle has been used by these people to design and construct a breech which will be operated at 400,000-psi dynamic loads.

Workers at Los Alamos[14] have used the pressures generated by hypervelocity impacts to obtain equation-of-state measurements up to about 2 megabars. In their experiments, a brass plate was accelerated by explosives to about 5 or 6 km/sec before impact. They estimate that there is an upper limit of 8 km/sec on the velocities attainable by this means, corresponding to the explosive escape velocity for Composition B explosive. Russian workers[15] have used hypervelocity impacts to generate pressures up to about 5 megabars and have measured the equation of state to that pressure. They are not specific about the acceleration technique used, but obtain particle velocities in iron of about 5 km/sec. To produce particle velocities of this magnitude by an impacting iron plate would require a plate velocity of 10 km/sec.

ACKNOWLEDGMENTS

The idea of powering a hydrogen gas gun by electrical means was first suggested to the author by E. P. Williams of The RAND Corporation. The author is indebted to F. R. Gilmore of RAND for helpful conversations relative to calculating the thermodynamic properties of hydrogen.

CONTENTS

SYMBOLS

A = cross-sectional area of launch tube

a_o = velocity of sound at shear disc rupture

e = specific internal energy

e_{20} = specific internal energy of molecular hydrogen at $0^{o}K$

H_2 = subscript referring to molecular hydrogen

H = subscript referring to atomic hydrogen

h = specific enthalpy

K = equilibrium constant in atmospheres

M = molecular weight

m = mass of bullet

n = number of moles per gm

r = P_H/P_{H_2}

P_o = pressure at shear disc rupture

P = pressure

S = entropy

s = specific entropy

T = temperature

T_o = temperature at shear disc rupture

T_s = standard temperature = $273.16^{o}K$

u = v/a_o

v = muzzle velocity

z = $P_o AL/m\alpha_o^{\ 2}$

α = $16\ \rho T/\rho_s T_s K$

α_o = $(\beta-1)a_o$

β = $(\gamma + 1)/(\gamma - 1)$

γ = $(\partial \ln P / \partial \ln \rho)_S$

η = $\frac{1}{2} mv^2 / P_o AL$ = $\frac{1}{2} u^2 / z$

ξ_o = $\gamma^2 \alpha_o$ = $2\gamma^2 a_o / (\gamma - 1)$

ρ = density

ρ_o = density at shear disc rupture

ρ_s = standard density of molecular hydrogen = 8.994×10^{-5} gm/cm^3

I. INTRODUCTION

In this paper a gun will be described and evaluated which gives promise of delivering hitherto unattainable bullet velocities. The propellant gas is partially dissociated hydrogen, hence the name "atomic-hydrogen gun."

With the advent of the space age, hypervelocity impact has become an increasingly interesting topic. Re-entering ICBM's strike even stationary objects at 23,500 ft/sec, satellites orbit at 26,000 ft/sec, and satellites in opposing orbits have relative velocities of 52,000 ft/sec. If one wishes to consider the effects of meteoroids on space vehicles, he must consider impacts in the 36,000 to 240,000 ft/sec range.

In addition, hypervelocity impact generates pressures on the order of millions of atmospheres, so that it may be used as a purely scientific tool to investigate the equations of state of materials in hitherto unexplored pressure regimes.

In order to investigate phenomena at these high velocities a great amount of effort has been devoted to attaining the highest possible velocity with a projectile of known mass and geometry, in which the light-gas gun has emerged as one of the most promising devices. Since even the best light-gas guns fall considerably short of the lowest interesting velocity, it is worthwhile to attempt to improve their performance.

In this memorandum, it is shown that light-gas guns powered by electrical discharges may be expected to propel projectiles considerably faster than has been possible in the past. In this application, hydrogen appears much superior to helium, whereas in previous applications hydrogen showed only a marginal superiority and generally bowed to helium in the interests of laboratory safety and expediency.

II. EXPLOSIVE-POWERED LIGHT-GAS GUNS

Conventional guns, which employ an explosive to propel a bullet, are not able to achieve muzzle velocities of more than about 8,000 ft/sec under normal firing conditions, and not more than about 10,000 ft/sec if the barrel is evacuated ahead of the bullet. It has been found experimentally that these limits may not be exceeded no matter how heavy the charge or how light the bullet. Internal ballisticians have long recognized that this velocity limitation is due to the comparatively high density of the gaseous explosive products. A large pressure drop is required to accelerate the dense propellant gases, resulting in low pressures on the base of the bullet, and consequently in relatively low bullet velocity. This fundamental limitation could be overcome in conventional guns if explosives were developed whose burned products had very low molecular weight and, therefore, low density. Such explosives would deliver and maintain high pressure at the bullet's base, resulting in high muzzle velocities. Such explosives, however, have not been developed.

A method of resolving the problem is embodied in the light-gas gun. This gun uses propelling gases of low molecular weight (e.g., hydrogen and helium) to achieve high pressures at the bullet, resulting in muzzle velocities greatly in excess of 10,000 ft/sec. The gases used are not explosive products in the conventional sense, but require external energy sources to effect the pressure build-up. One type of light-gas gun is illustrated schematically in Fig. 1.

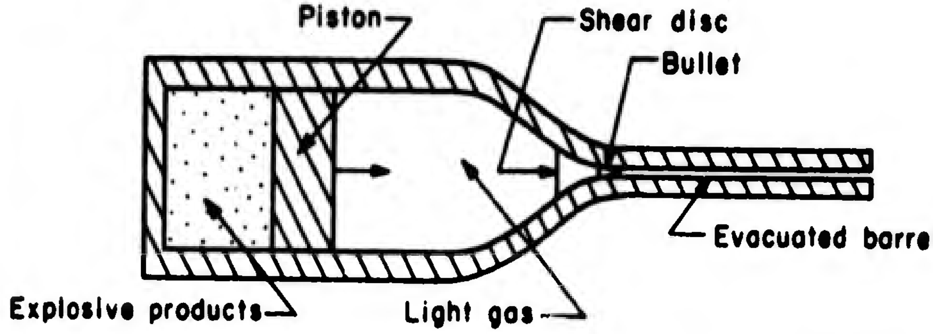


Fig. 1 — Schematic illustration of presently used light-gas gun

This gun uses an explosive to impart energy to the light gas by compressing it. A piston is fired into the light gas by an explosive charge. A shear disc isolates the bullet from the compressing light gas until a predetermined pressure is reached. Then the shear disc ruptures suddenly and the light gas, acting as the explosive products do in a conventional weapon, hurls the bullet through the barrel. In the following exposition, the conditions of the light gas just at shear disc rupture will be denoted by subscript zero; for example P_o, ρ_o, and a_o will denote the pressure, density, and sound velocity, respectively, at that particular time.

The fastest muzzle velocities achieved by guns of this type, which will be termed "explosive-powered" light guns, are 15,400 ft/sec by the gun at the Ames Aeronautical Laboratory,[(1)] and about 15,000 ft/sec by the gun at the Naval Ordnance Test Station, Inyokern. Velocities of this magnitude are the exception rather than the rule; most velocities from these guns and others of the same type are in the 7,000 to 13,000 ft/sec range.

III. PROPOSED MODIFICATION - THE ELECTRICALLY POWERED LIGHT-GAS GUN

The modification of the light-gas gun which is proposed is shown schematically in Fig. 2.

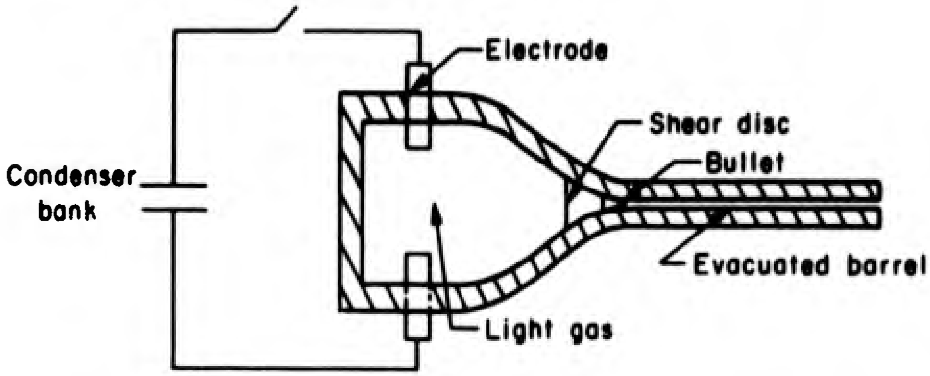


Fig. 2—Schematic illustration of proposed electrically powered light-gas gun

The main feature is that power is supplied to the light gas by an electrical discharge. That this is feasible has been shown experimentally by Perry.* For example, a sample of 60 cubic inches of air has been raised to a temperature of more than 10,000°K and a pressure of more than 20,000 psi by discharging a condenser bank through it. Of the 1,000,000 joules stored in the bank, about half were transferred to the gas in less than half a millisecond. The time could be drastically shortened by using condensers designed for this purpose.

The working cycle for such a gun consists of heating the gas at constant volume while the shear disc is intact, followed by an adiabatic expansion of the gas as it drives the bullet down the launch tube after shear disc rupture.

*Personal communication with R. Perry, Gas Dynamic Facility, Arnold Engineering Development Center, Tullahoma, Tennessee.

The advantage gained by using condensers as a power source is the ability to produce higher gas temperatures at feasible breech pressures. In the case of helium, this results in increasing the sound velocity. In the case of hydrogen, the additional benefits of partial dissociation are realized, resulting in lower average molecular weight and lower values of γ. How these factors may be expected to improve the gun's performance is shown in the following sections.

MATHEMATICAL GUN MODEL

In analyzing the performance of their light-gas gun, Charters et al.[1] have found that the model illustrated in Fig. 3 accurately predicts their experimental results.*

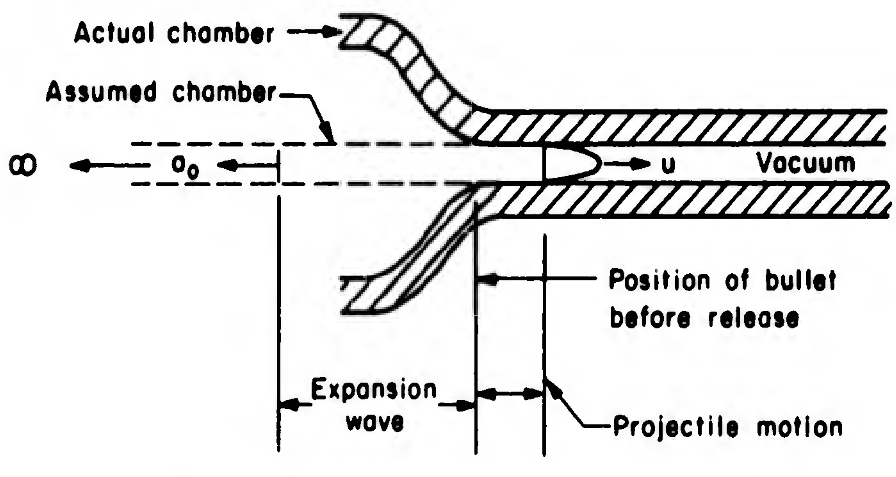


Fig. 3 — Mathematical gun model

In this model the pump tube is replaced by a hypothetical chamber of the same diameter as the launch tube, and extending to infinity. The gas

*They cite that this model was originally suggested by Dr. A. E. Siegel, Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland.

conditions in the hypothetical chamber are assumed to be initially uniform, and to correspond with those which exist in the real chamber at the instant of shear disc rupture. In the model, an expansion wave moves to the left with sound speed, a_0, after the shear disc is ruptured, and consequently the model predicts that the pressure will fall rapidly with time after the process begins. In the real case, the larger chamber is more effective in maintaining the pressure at the shear disc position, and this effect causes the model to consistently underestimate by about 10 per cent the velocity achieved. By taking 110 per cent of the estimated velocity, Charters obtains excellent agreement with the experimental velocities. This model will be used here without modification.

The assumptions of the model make it a simple one-dimensional problem which may be solved analytically by the method of characteristics. The result is

$$z = \frac{1}{\beta-1}\left[\frac{u - \frac{1}{\beta}}{(1-u)^{\beta}} + \frac{1}{\beta}\right] \qquad (1)$$

where

$z = P_0 AL/m\alpha_0^{\,2}$

$u = v/\alpha_0$

v = muzzle velocity

A = cross-sectional area of launch tube

L = launch tube length

m = mass of bullet

$\alpha_0 = (\beta-1)a_0$

$\beta = (\gamma+1)/(\gamma-1)$

Implicit in the derivation of Eq. (1) is the assumption that γ is a constant. Offhand, one would not expect this to be the case for a diatomic gas such as hydrogen in a region where temperature and pressure changes affect the degree of dissociation. Indeed, the definition of γ becomes somewhat cloudy in such a region. However, analysis shows that if one defines γ as

$$\gamma = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S \qquad (2)$$

over the entire region, and if this γ remains constant throughout an adiabatic expansion, then one is again led to Eq. (1). This definition of γ becomes equivalent to the common one in the regions where no change of composition takes place as the state is varied slightly.

The thermodynamic properties of hydrogen were calculated over the region of interest in gun operations (Appendix A) and the calculations showed that the γ so defined does remain constant to a remarkable degree along adiabats. Thus one is able to use Eq. (1) even for hydrogen.

The definition given in Eq. (2) implies that the adiabats will be straight lines on a log-log plot, the slope of the line being γ. In Fig. 4 several adiabats are shown to illustrate the degree to which the approximation of constant γ is satisfied. All of the adiabats shown begin at a pressure of 100,000 psi (6800 atm), which is a value commensurate with the current status of breech design. A summary of the thermodynamic values appropriate to each adiabat shown in the figure is given in Table 1.

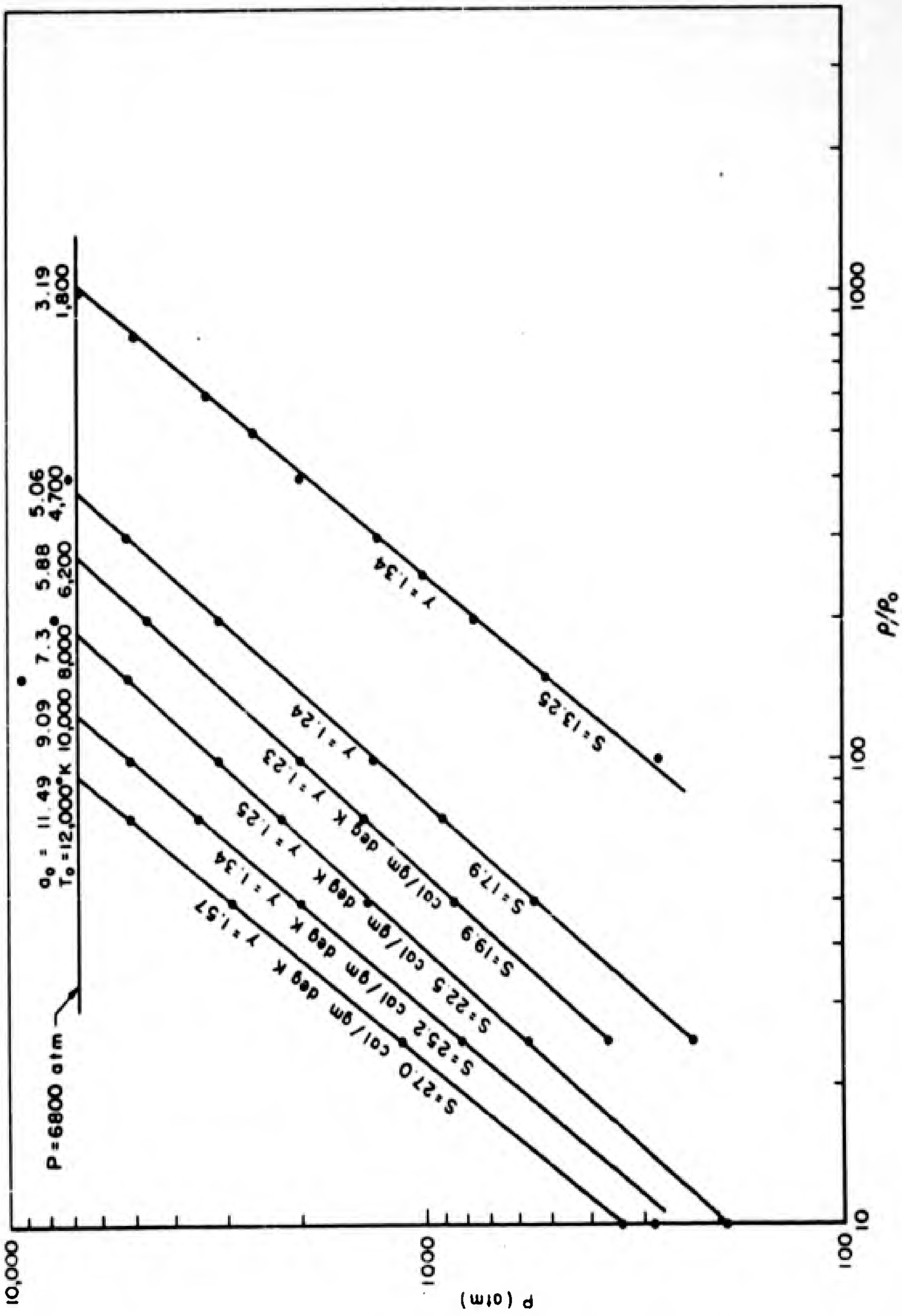


Fig. 4—Hydrogen adiabats

Table 1

s cal/gm deg	T_o deg K	γ	a_o km/sec
27.0	12,000	1.57	11.5
25.2	10,000	1.34	9.09
19.9	6,200	1.23	5.88
17.9	4,700	1.24	5.06
13.3	1,900	1.34	3.19

Physical Discussion of the Process

At the moment of shear disc rupture, the pressure on the base of the projectile is P_o. If the gas were able to maintain this pressure on the projectile during its flight down a barrel of length L, then the work done on the projectile would be P_oAL, and the work would appear as projectile energy, $1/2\ mv^2$. It is impossible for the gas to do more work than this. In actual cases, the gas is not able to follow the projectile sufficiently well to maintain the initial pressure on the bullet, particularly when the bullet attains high velocities. The gas will then perform only a fraction of the maximum allowable work on the bullet, which will be called the efficiency, expressed by

$$\eta = \frac{\frac{1}{2}mv^2}{P_oAL} \tag{4}$$

In terms of the dimensionless variables defined in Section IV,

$$\eta = \frac{1}{2}\frac{u^2}{z} \tag{5}$$

Figure 5 shows the efficiency as a function of muzzle "Mach number," defined as v/a_o, for four values of $\beta = (\gamma+1)/(\gamma-1)$. The curves were obtained by solving Eq. (1). It is evident that for small Mach numbers, the efficiency approaches unity, but falls off very rapidly for Mach numbers

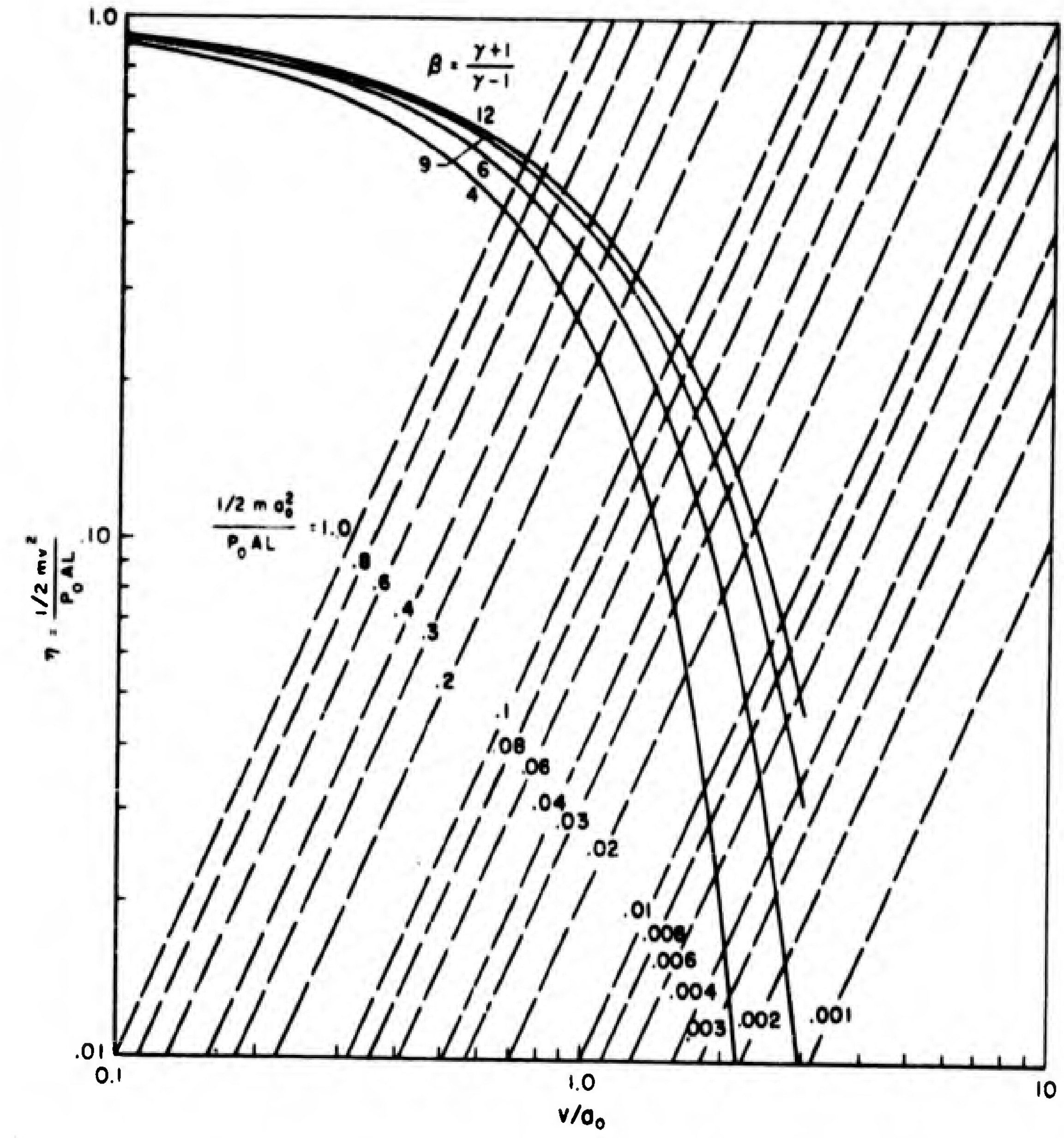


Fig. 5—Efficiency versus muzzle Mach number

greater than unity. Mach numbers greater than β-1 can never be obtained, for that corresponds to the limiting velocity of the gas, and efficiency must fall to zero there.

The muzzle velocity delivered by a gun having parameters m, A, and L, powered by a gas having parameters P_o, a_o, and γ, may be obtained by equating the curves of Fig. 5 with the equation

$$\eta = \frac{\frac{1}{2} m a_o^2}{P_o A L} \left(\frac{v}{a_o}\right)^2 \qquad (6)$$

In Fig. 5, Eq. (6) is represented by straight lines of slope 2, having intercepts $\frac{1}{2} m a_o^2 / P_o A L$ at $v/a_o = 1$.

Physical intuition leads one to expect that gases of high molecular velocity, or equivalently, high sound velocity, would be the most efficient in operating a light-gas gun. This is verified by Fig. 5. The η curves defined by Eq. (1) all have negative slopes on the log-log plot, and it is shown in Appendix B that this fact is sufficient to insure that increasing a_o while holding all other parameters constant always leads to higher muzzle velocities.

In addition, it may be seen from Fig. 5 that holding all other parameters constant and lowering γ increases the muzzle velocity. The physical reason for this is that a low value of γ implies a reservoir of internal energy which is delivered to the translational degrees of freedom upon adiabatic expansion, serving to maintain a higher pressure than the case of a high-γ gas which undergoes a similar volume increase.

One important energy reservoir for partially dissociated hydrogen is the atomic hydrogen itself, which partially recombines on adiabatic expansion with a large consequent release of energy.

Thus, to maximize the bullet velocity, one must

1) Maximize P_o

2) Maximize a_o

3) Minimize γ

The first condition gives a clear-cut dictum, viz. increase P_o. One will be limited in this direction by breech-design considerations, one of the most important of which is the material-strength problem.

The last two conditions will in certain cases compete with each other. If one is faced with this problem, a good rule of thumb to follow is to maximize

$$\xi_o = \frac{2\gamma^2 a_o}{\gamma - 1} = \gamma^2 \alpha_o$$

The degree of validity of this rule of thumb is illustrated in Fig. 6, where η is plotted versus v/ξ_o. On this plot the curves for various γ's are brought quite closely together for the range of γ from 1.25 to 1.67, which is the range of interest in designing guns.

In this instance the muzzle velocity is determined by the intersections of these curves and the lines

$$\eta = \frac{\frac{1}{2} m \xi_o^2}{P_o AL} \left(\frac{v}{\xi_o}\right)^2 \qquad (8)$$

a number of which are shown in the figure. One can see that for the same value of $P_o AL/m$, gases having the same ξ_o will yield the same muzzle velocity to within 7 per cent over the velocity range of 0 to $2a_o$, and the range of γ from 1.25 to 1.67. Below $v = a_o$, the spread is less than 5 per cent.

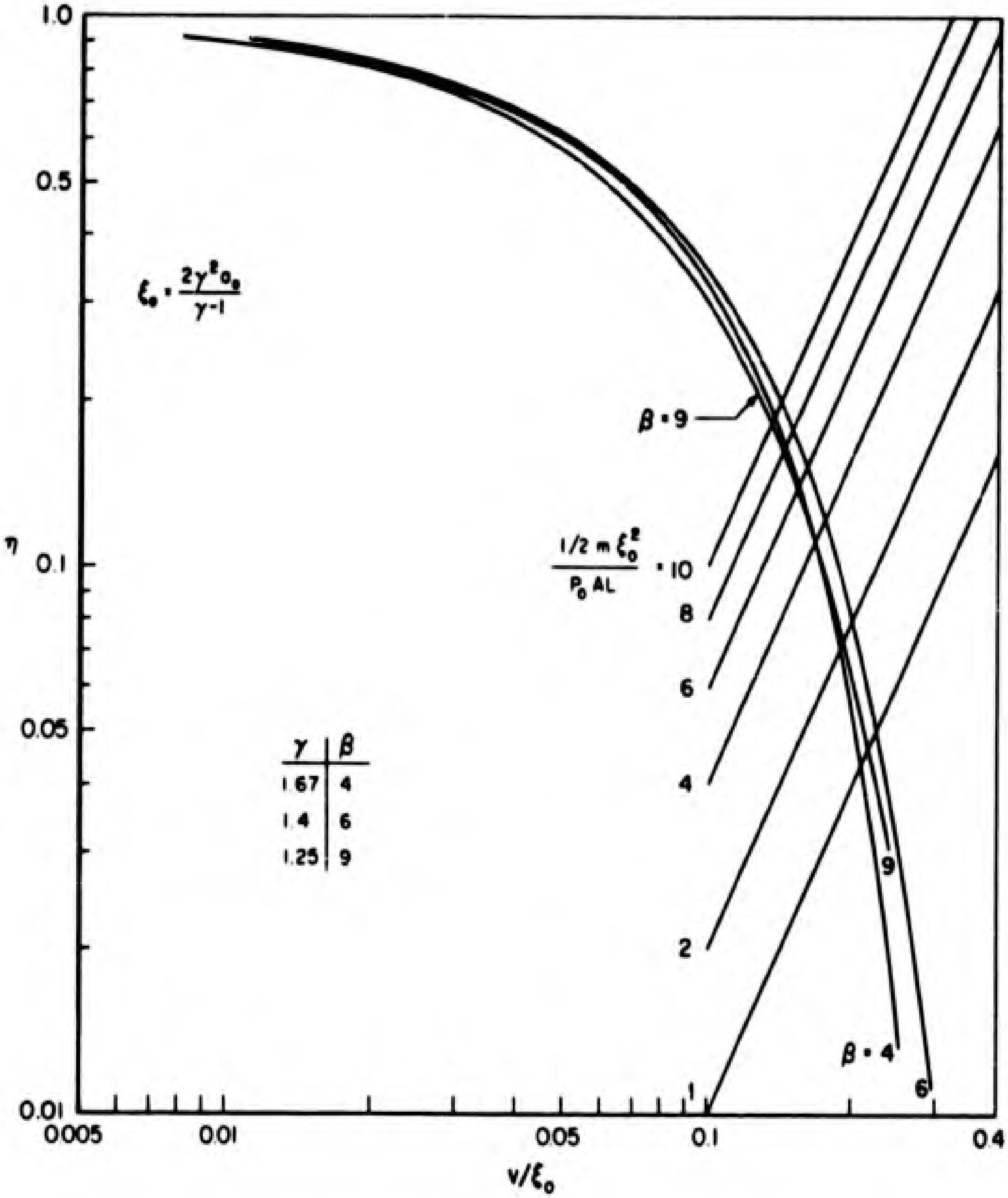


Fig. 6 — Efficiency as a function of v/ξ_0

Exemplary Calculations

For a number of gun parameters, muzzle velocities were calculated using both hydrogen and helium under a wide variety of initial conditions. The gas conditions used are tabulated in Table 2.

Table 2

Case	Gas	T_o deg K	s cal/gm deg	γ	a_o km/sec	ℓ_o km/sec
1	H_2	12,000	27.0	1.57	11.49	99.4
2	H_2	10,000	25.2	1.34	9.09	96.0
3	H_2	8,000	22.5	1.25	7.30	91.3
4	H_2	6,200	19.9	1.23	5.88	77.4
5	H_2	4,700	17.9	1.24	5.06	64.8
6	He	12,000	---	1.67	6.44	53.7
7	He	10,000	---	1.67	5.88	49.0
8	He	8,000	---	1.67	5.26	43.8
9	He	6,200	---	1.67	4.63	38.6
10	He	4,700	---	1.67	4.03	33.6

The hydrogen cases correspond with five of the adiabats shown in Fig. 4, and the helium cases were chosen so that the initial temperatures and pressures are the same as the hydrogen ones. In each case P_o was taken as 100,000 psi, or about 6800 atmospheres. This pressure is the one used in the Ames experiments,[1] and thus may be considered as presently accessible.

The gun parameters always occur in the combination P_oAL/m. The Ames experiments utilized the following values:

P_o = 100,000 psi

A = .266 cm^2 (the area of a circle .22 inch in diameter)

L = 134 cm

m = .211 gm

These values lead to P_oAL/m = 1.15×10^{12} cm^2/sec^2, which is one of the values used. The others were 2×10^{11}, 3×10^{12}, and 5×10^{12}. These combinations lead to the muzzle velocities shown in Table 3. The velocities are given in km/sec, and P_oAL/m in cm^2/sec^2.

Table 3

$\frac{P_oAL}{m}$ / Case	2×10^{11}		1.15×10^{12}		3×10^{12}		5×10^{12}	
	v	1.1v	v	1.1v	v	1.1v	v	1.1v
1	4.95	5.45	9.26	10.2	12.3	13.5	14.0	15.4
2	4.89	5.38	9.10	10.0	12.1	13.3	13.8	15.2
3	4.73	5.20	8.66	9.53	11.4	12.5	13.0	14.3
4	4.51	4.96	8.00	8.80	10.4	11.4	11.7	12.9
5	4.31	4.74	7.46	8.21	9.53	10.5	10.7	11.8
6	4.18	4.60	6.99	7.69	8.68	9.55	9.56	10.5
7	4.06	4.47	6.67	7.34	8.21	9.03	9.01	9.91
8	3.90	4.29	6.29	6.92	7.66	8.43	8.37	9.21
9	3.71	4.08	5.86	6.45	7.06	7.77	7.67	8.44
10	3.51	3.86	5.40	5.94	6.44	7.08	6.96	7.66

The values 1.1 v are the theoretical muzzle velocities as given by the model, and these are plotted versus ξ_o in Fig. 7. The figure shows in a striking manner that the muzzle velocity is essentially a function of only ξ_o for a given value of P_oAL/m. The figure is well suited for rough initial

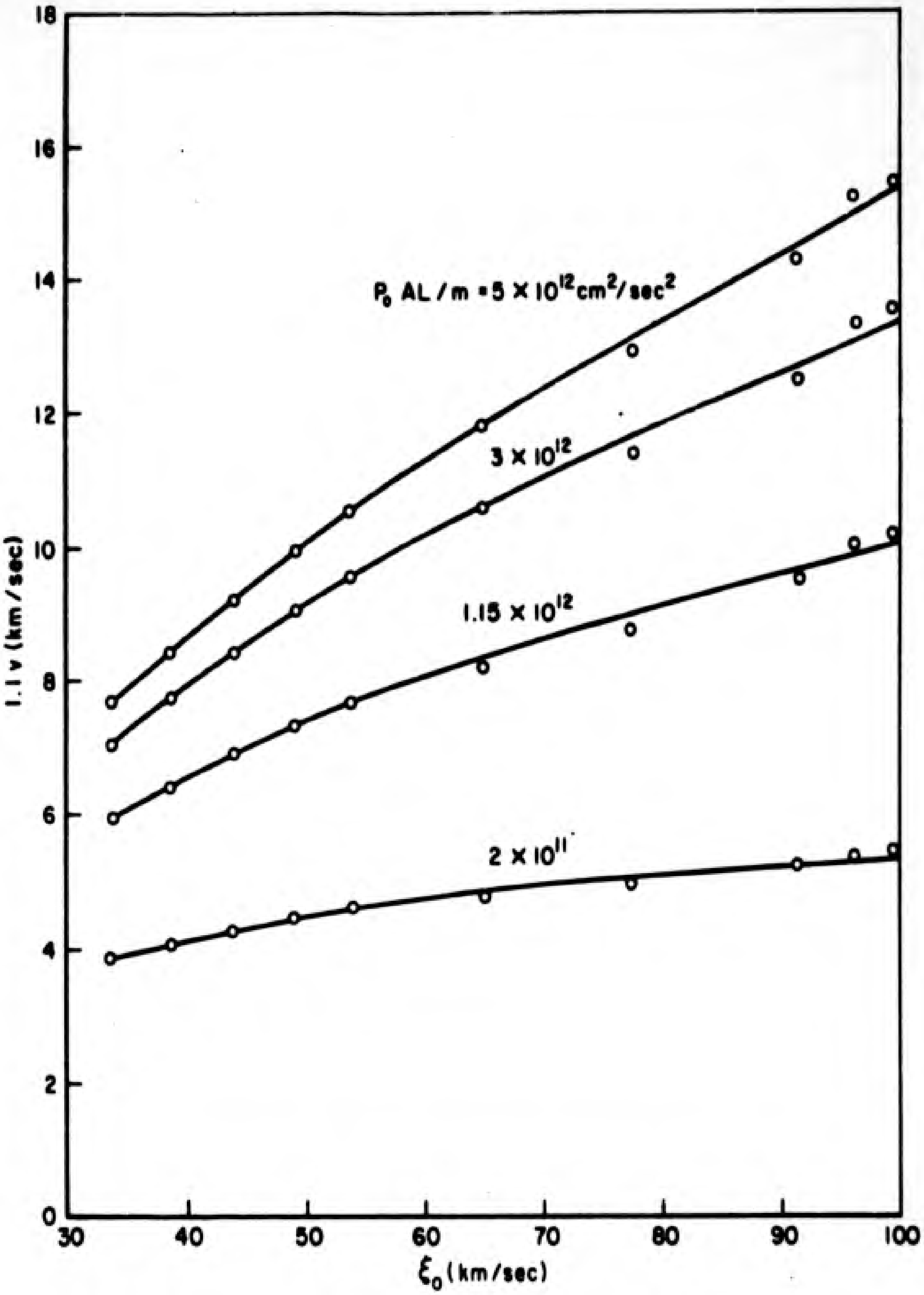


Fig. 7 — Theoretical muzzle velocity vs ξ_0

estimates of muzzle velocity by gun designers, since enough values of P_0AL/m are used that interpolation between them gives a good estimate.

One should note that for higher values of P_0AL/m, there is a greater payoff as one increases ξ_0.

POSSIBLE IMPROVEMENTS AND LIMITATIONS

The gun parameter which is probably most susceptible to improvement is P_0, since the factor which limits P_0 is the strength of the breech. At present, breeches are easily designed to withstand 100,000 psi. By clever design, it should be possible to design them for about 450,000 psi.* With this improvement, values of P_0AL/m on the order of the highest one used, viz. 5×10^{12}, may be obtained. However, increasing P_0 may be expected to cause a subtle experimental difficulty. It will increase the initial acceleration of the projectile by the same factor by which P_0 is increased, and so may lead to the problem of projectile breakup.

Current guns operating at 100,000 psi have experienced this problem, and only projectiles of strong materials can withstand the accelerations associated with this initial pressure. Soft materials, such as lead, invariably break up. However, it may be said that the severity of the problem should not be greatly increased by going over to the electrical gun, since the maximum acceleration will be the same for both guns operating at the same P_0.

Using only these theoretical results, one would conclude that increasing the barrel length would be fruitful. However, experiments with light-gas guns have shown that increasing the barrel length beyond 100 to

*Private communication with W. S. Partridge and S. S. Kistler, University of Utah.

200 calibers does not lead to increased muzzle velocities. The reasons for this phenomenon are not well understood, the explanation favored at present being that a boundary layer is formed next to the barrel walls which grows with time until the barrel is filled. Lengthening the barrel will probably not be favorable until someone correctly explains the difficulty and formulates a corrective measure.

The third possibility, that of minimizing m/A, also has inherent limitations. To minimize this quantity, one must use projectile materials of low density, or use very thin projectiles. One will be thwarted in this direction by projectile breakup and the fact that very thin projectiles might not be stable. The thickness of the Ames-group projectile, calculated on the assumption that it was aluminum of density 2.7 gm/cc, mass .211 gm, and area .266 cm^2, was .293 cm. This may be compared with its diameter of .559 cm. One would suppose that this projectile, which is only about one half a caliber long, is close to the smallest thickness allowable.

By condenser discharges, it should be possible to obtain temperatures on the order of 12,000°K in hydrogen rather easily. At a breech pressure of 100,000 psi, this leads to the conditions of case 1 in Table 2, where a_o = 11.49 km/sec and ξ_o = 99.4 km/sec. A theoretical muzzle velocity of 10.2 km/sec would then be given by the Ames parameter of $P_oAL/m = 1.15 \times 10^{12}$. This velocity is indicative of the immediate potential of such a device.

To realize the capabilities of this device will require a large bank of condensers, for reaching the breech conditions cited requires the addition of about 330,000 joules per gram of hydrogen used. Therefore, the cost of the condensers might constitute a practical limitation on the value of ξ_o. Although the initial expenditure for such a bank will be sizable (on the order of ten cents per joule installed), the bank will not deteriorate with use, and may be used for a variety of other interesting experiments; and most of the cost may eventually be recovered through resale. The parts of the gun which may be expected to deteriorate rapidly, the breech and adjacent sections of the barrel, are simple and relatively inexpensive, and may be designed to be replaced independently after a few shots.

It has been shown that hydrogen outperforms helium in the same gun starting at the same temperature and pressure. However, as one pushes to higher and higher temperatures in the future development of this device, it is conceivable that hydrogen may reach its limit in this direction before helium does. Factors which militate for this are 1) the higher thermal conductivity of hydrogen coupled with the finite time required to impart the energy to the gas, 2) recombination of atomic hydrogen on the walls of the gun, and 3) the lower ionization potential of hydrogen. Thus, at extremely high temperatures helium may again come into contention with hydrogen as the optimum working fluid for such a gun. It seems that the final disposition of this question must be settled by experiment.

IV. CONCLUSIONS

By using the light-gas-gun principle suggested here and designs commensurate with the current state of the art, it should be possible to achieve muzzle velocities on the order of 10 km/sec. Beyond this there is a potential for development of the device to yield even higher velocities.

Appendix A

THERMODYNAMIC PROPERTIES OF HYDROGEN

Under the operating conditions of the gun, the gas particles experience collisions at the rate of about 10^9 per second. Since the gun operating time is on the order of a millisecond, each particle will experience about 10^6 collisions in this period. For this reason it is assumed that the gas will remain in equilibrium. The properties of hydrogen were calculated on this assumption.

Only two species were deemed important, namely H and H_2.

For convenience in applying the results to gun calculations, the properties are not given in the usual thermodynamic terminology. Instead, they are given as follows:

$e - e_{20}$	specific internal energy of the mixture in joules/gm
$h - e_{20}$	specific enthalpy of the mixture in joules/gm
s	specific entropy of mixture in cal/gm deg K

The e_{20} denotes the fact that energies are referred to the ground state of the hydrogen molecule. In addition, the number of moles of each species per gram of mixture, the partial pressures of each species, and the total pressure are presented.

CALCULATION OF COMPOSITION

In the following, the subscripts H and H_2 will refer to the atomic and molecular species, respectively. Using the ideal gas law and the law of partial pressures, one can immediately write

$$\frac{M_{H_1}}{M_{H_2}} P_H + P_{H_2} = \frac{\rho T}{\rho_s T_s}$$

where M is the molecular weight, P_H and P_{H_2} are the partial pressures in atmospheres, ρ is the density of the mixture, $\rho_s = 8.994\ 10^{-5}$ gm/cm^3 is the density of molecular hydrogen at standard temperature and pressure, and $T_s = 273.16^\circ$K is the standard temperature.

From the definition of the equilibrium constant, one has

$$P_{H_2} = \frac{P_H^{\ 2}}{K}$$

Solving these two equations for r, the ratio of partial pressures, one has

$$r = \frac{P_H}{P_{H_2}} = \frac{4}{\alpha}\left(1 + \sqrt{1+\alpha}\right)$$

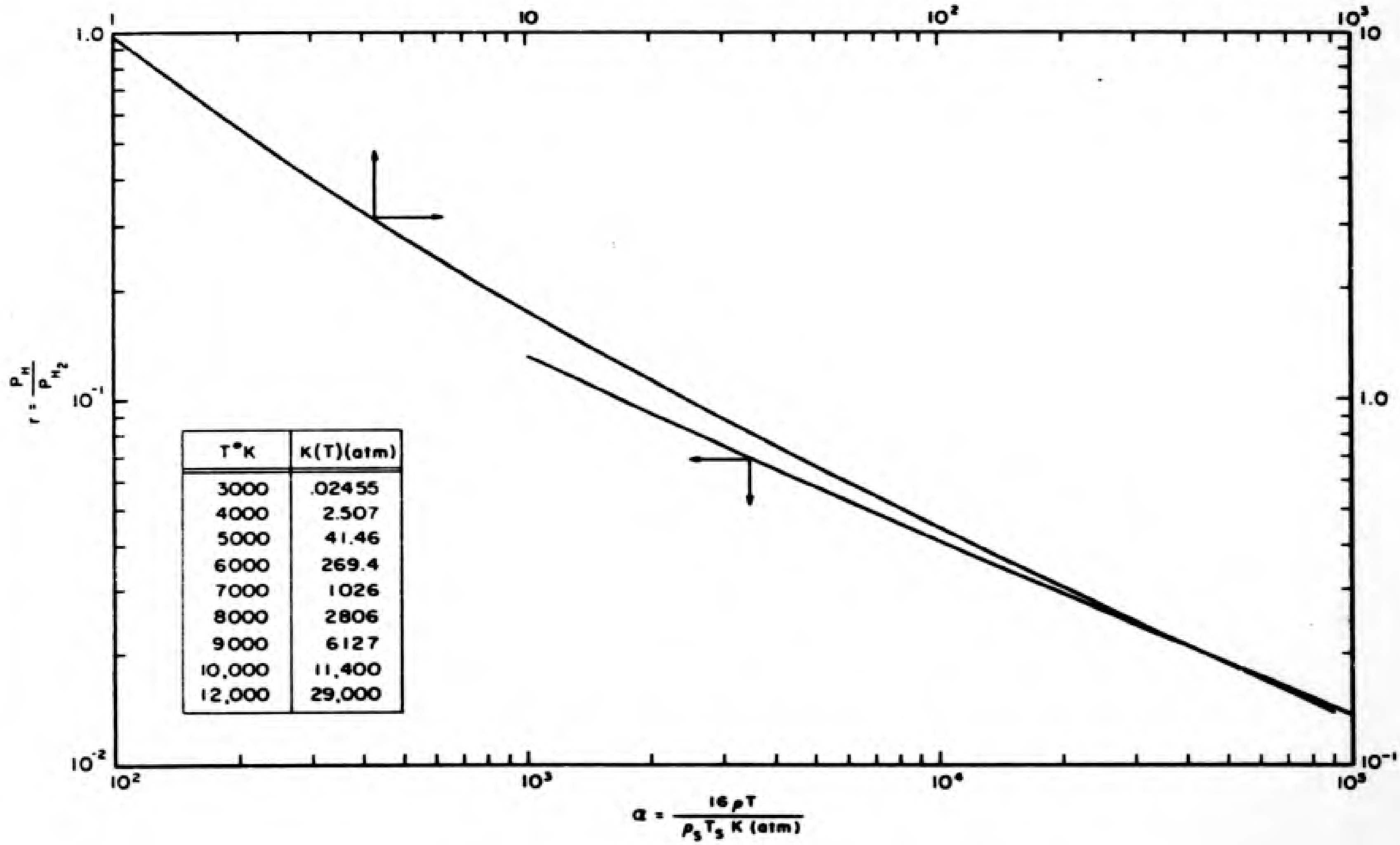
where $\alpha = \dfrac{16\ \rho T}{\rho_s T_s K(atm)}$. It is interesting that the composition is a function of the single variable, α. The equilibrium constant, K, was calculated from the input values tabulated in Table 4 and is presented there. The composition as a function of α is exhibited in Fig. 8.

Once r has been determined, the number of moles of each species per gram of mixture is determined from the relations

$$n_H = \frac{.9921\ r}{r+2}\ ,\ n_{H_2} = \frac{.9921}{r+2}\ .$$

DETERMINATION OF THERMODYNAMIC PROPERTIES

Once the composition is determined to the extent of knowing the number of moles of each species per gram, one can use the molal quantities given in the table of inputs to determine the specific properties of the mixture. One must correct the free energy and enthalpy of atomic hydrogen given in Table 4 so that they are referred to the same energy scale as the molecular species. To this end the value of dissociation energy of the hydrogen molecule found



T°K	K(T)(atm)
3000	.02455
4000	2.507
5000	41.46
6000	269.4
7000	1026
8000	2806
9000	6127
10,000	11,400
12,000	29,000

Fig. 8 — Composition of hydrogen

Table 4

THERMODYNAMIC INPUTS

(Data from 300°K to 4000°K is taken from Ref. 2; molecular hydrogen data from 5000°K to 12,000°K is taken from Ref. 3; atomic hydrogen data from 5000°K to 12,000°K is taken from Ref. 4)

Molecular Hydrogen				Atomic Hydrogen			
T °K	$\frac{H-H_o^o}{RT}$	$\frac{S^o}{R}$	$-\frac{F^o-F_o^o}{RT}$	$\frac{H-H_o^o}{RT}$	$\frac{S^o}{R}$	$-\frac{F-F_o^o}{RT}$	K (atm)
300	3.416	15.73	12.31	2.500	13.80	11.30	
600	3.464	18.16	14.69	2.500	15.53	13.03	
1000	3.506	19.98	16.47	2.500	16.81	14.31	
2000	3.694	22.65	18.96	2.500	18.54	16.04	
3000	3.897	24.39	20.49	2.500	19.56	17.06	.02455
4000	4.069	25.71	21.64	2.500	20.26	17.78	2.507
5000	4.172	26.72	22.55	2.500	20.83	18.33	41.46
6000	4.274	27.60	23.32	2.500	21.29	18.79	269.4
7000	4.361	28.35	23.99	2.500	21.67	19.17	1026
8000	4.434	29.02	24.58	2.500	22.00	19.51	2806
9000	4.506	29.62	25.11	2.500	22.30	19.80	6127
10000	4.568	30.16	25.59	2.500	22.57	20.07	11,400
12000	4.686	31.13	26.44	2.506	23.01	20.52	29,300

by Beutler[5] was used, namely 36,116 ± 6 cm^{-1}. The values of constants used are as follows:

$$T_s = 273.16^\circ K$$

$$R = 1.98717 \text{ cal/mole deg} = 8.31433 \text{ joules/mole deg}$$

$$hc/k = 1.4388 \text{ cm deg}$$

$$\rho_s = 8.994 \times 10^{-5} \text{ gm/cm}^3$$

The steps used in the calculation are as follows:

For a given value of $\frac{\rho}{\rho_s}$ and T

1. $$K = \exp\left\{-2\left(\frac{F-F_o^{\,o}}{RT}\right)_H + \left(\frac{F-F_o^{\,o}}{RT}\right)_{H_2} - \frac{51,964}{T}\right\}$$

2. $$\alpha = .058574 \frac{\rho}{\rho_s} \frac{T}{K}$$

3. $$r = \frac{4}{\alpha}\left(1 + \sqrt{1+\alpha}\right)$$

4. $$P_H = \frac{K}{r}$$

5. $$P_{H_2} = \frac{P_H}{r}$$

6. $$P = P_H + P_{H_2}$$

7. $$n_{H_2} = \frac{.9921}{r+2} \; ; \; n_H = \frac{.9921r}{r+2}$$

8. $$s = \left[n_{H_2}\left(\frac{S^o}{R}\right)_{H_2} + n_H\left(\frac{S^o}{R}\right)_H - n_H \ln P_H - n_{H_2} \ln P_{H_2}\right](1.98717)$$

9. $$h - e_{20} = 8.31433\, T \left[n_{H_2}\left(\frac{H^o - H_o^{\,o}}{RT}\right)_{H_2} + n_H\left(\frac{H^o - H_o^{\,o}}{RT}\right)_H + n_H \frac{25,982}{T}\right]$$

10. $e - e_{20} = h - e_{20} - \frac{P}{\rho/\rho_s} (1.12653 \times 10^3)$

F. R. Gilmore(6) of RAND has independently calculated these properties by a somewhat different method in the temperature range from 5000°K to 12,000°K. Good agreement exists between his values and those presented here.

The properties presented in Table 5 are:

Column 1. Temperature in degrees Kelvin. Each page contains values pertinent to a given value of ρ/ρ_s.

Column 2. n_H, number of moles of atomic hydrogen per gram of mixture.

Column 3. n_{H_2}, number of moles of molecular hydrogen per gram of mixture.

Column 4. P_H, partial pressure of atomic hydrogen in atmospheres.

Column 5. P_{H_2}, partial pressure of molecular hydrogen in atmospheres.

Column 6: P, total pressure in atmospheres.

Column 7. $e - e_{20}$, internal energy per gram of mixture in joules per gram. (e_{20} is the internal energy of one gram of molecular hydrogen at zero temperature and pressure.)

Column 8. s, specific entropy of mixture in calories per gram degree.

Column 9. $h - e_{20}$, specific enthalpy of mixture in joules per gram.

The second column under each heading gives the power of ten by which the first column should be multiplied. The data are plotted in Figs. 9 and 10.

USE OF FIGURES IN DESIGNING THE LIGHT-GAS GUN

Figures 9 and 10 can be used to achieve a rough design of an electrically powered light-gas gun.

The first stage of gun operation involves an electrical deposition of energy in the gas at constant volume. Under these conditions, the energy

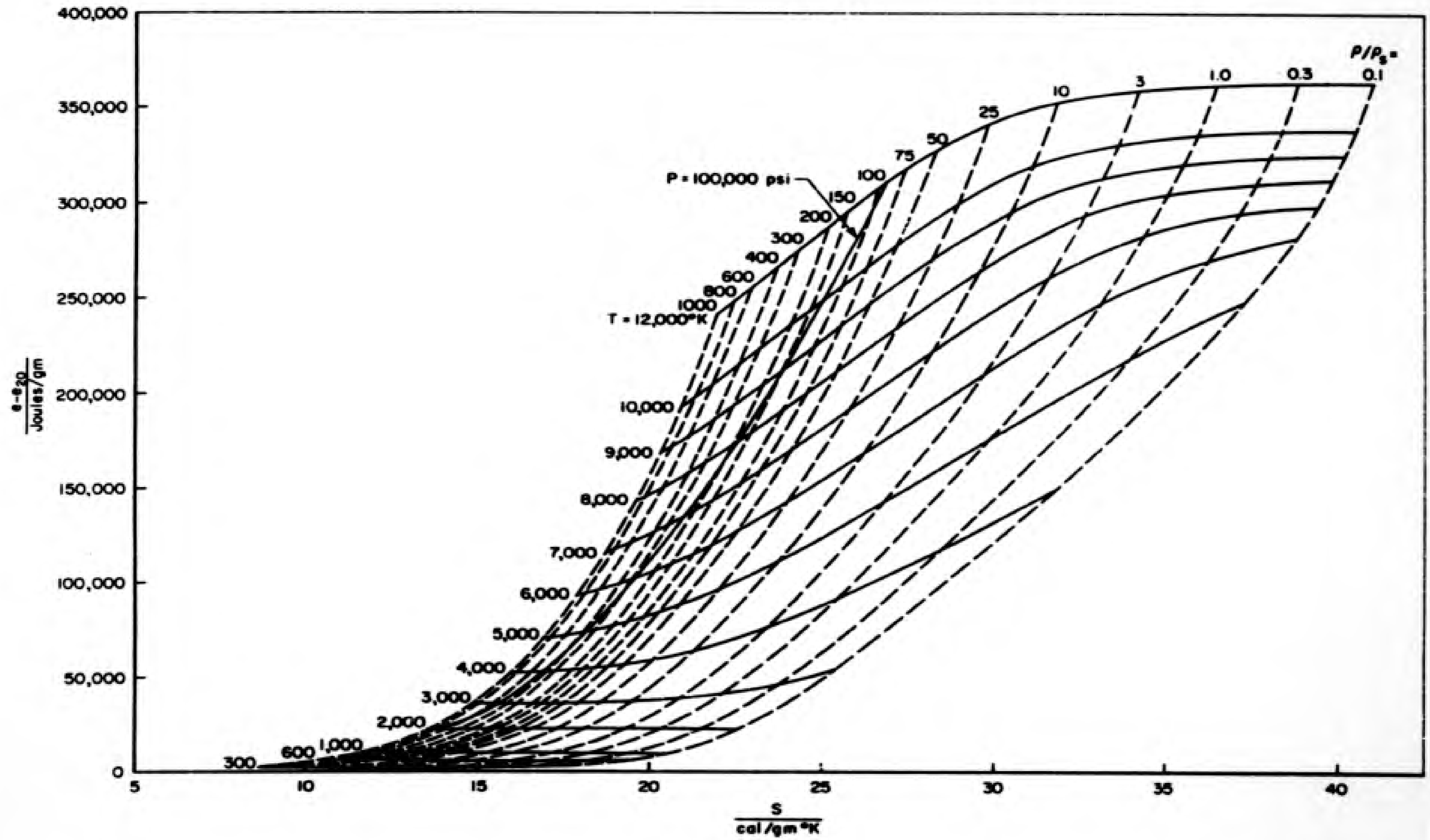


Fig. 9 —Internal energy versus entropy for hydrogen in equilibrium

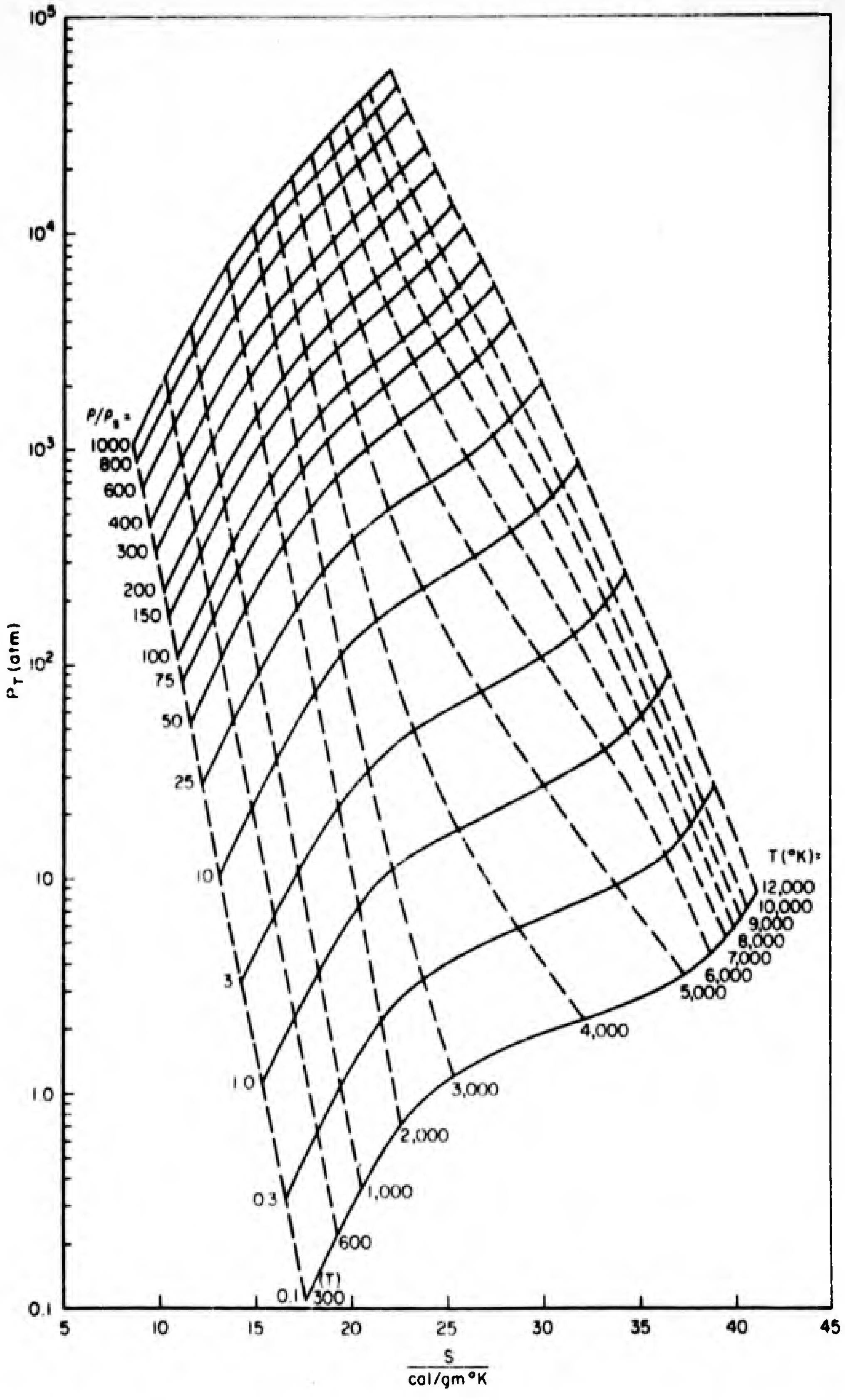
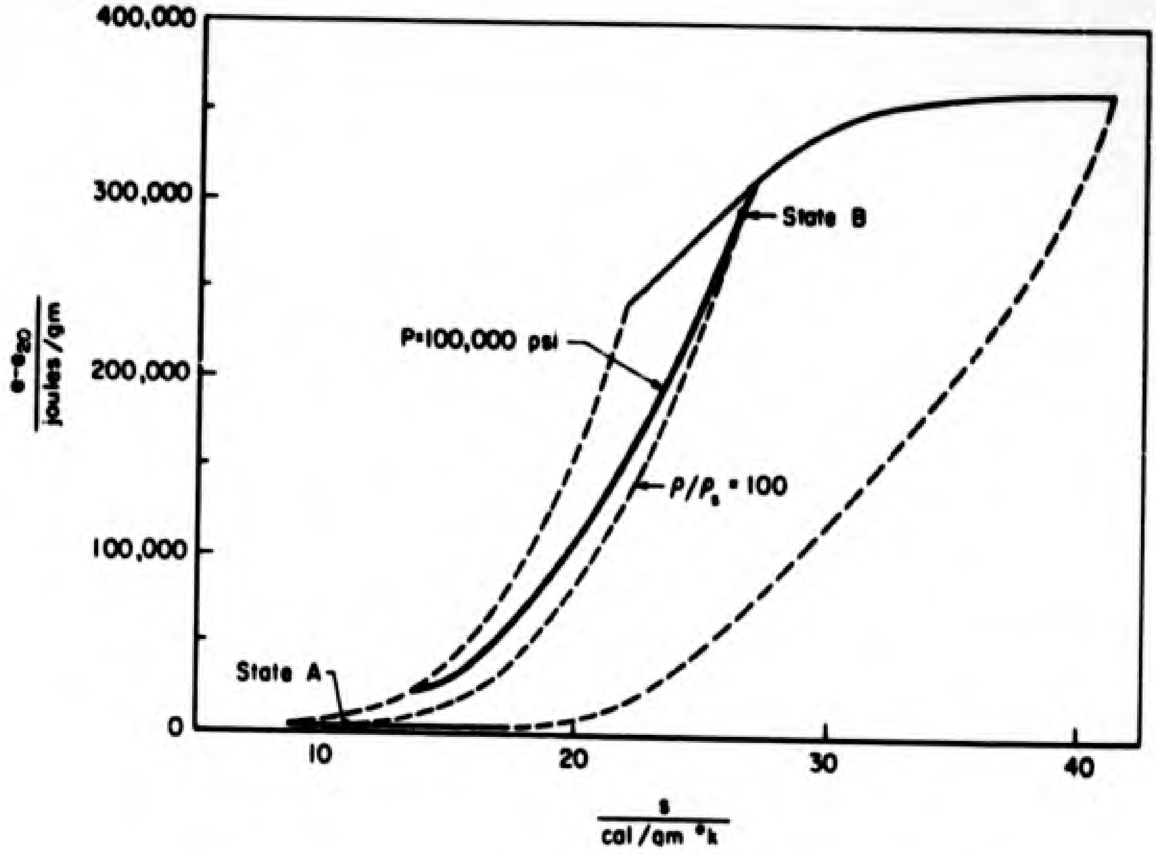


Fig. 10 — Pressure versus entropy for hydrogen in equilibrium

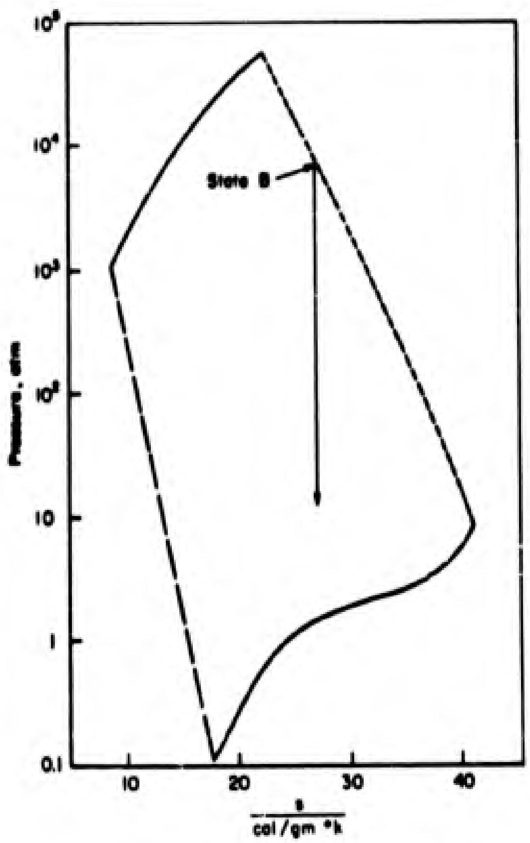


deposited appears entirely as internal energy, and the energy required to reach a given state at shear disc rupture may be found from Fig. 9 as is illustrated in the above diagram.

One first chooses the initial state, A, defined by a certain temperature and density ratio. In this state, the gas has internal energy e_A per gram. One then follows the line of constant density until the desired pressure is reached. To facilitate the present calculations, the 100,000 psi isobar was drawn in Fig. 9 by the use of Tables 4 and 5. The intersection of the constant-density line and the isobar defines state B, which is the state at shear disc rupture. The difference in specific internal energy between states A and B is the amount which must be added to the gas.

The preceding figure illustrates the case where ρ/ρ_s is 100. At state B, where the pressure is 100,000 psi, the gas has a specific internal energy of about 300,000 joules per gram. Thus, starting at state A, where the gas is at room temperature and has a specific internal energy of about 3,000 joules per gram, one must add about 297,000 joules per gram to reach the desired gas conditions.

The next phase of the gun operation is assumed to be an isentropic expansion of the gas as it propels the projectile down the barrel. To follow the gas conditions, one identifies state B in Fig. 10, and proceeds straight downward in that figure at constant entropy. This is illustrated in the following figure:



As one proceeds down the isentrope in the preceding figure, he identifies the pressure-density coordinates of several points and plots them on log-log paper. The slope of the resultant line gives the value of γ to be used in calculating the performance of the gun.

Table 5

THERMODYNAMIC PROPERTIES

(The right-hand number within each column--except the stub--is the power of ten by which the left-hand number in that column should be multiplied.)

T ($^\circ K$)	n_H (moles/gm)	n_{H_2} (moles/gm)	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{\circ}$ (j/gm)	s (cal/gm deg)	$h-e_{20}^{\circ}$ (j/gm)
ρ/ρ_s = 0.1								
300	---	4.96 -1	---	1.10 -1	1.10 -1	2.99 3	1.77 1	4.23 3
600	---	4.96 -1	---	2.20 -1	2.20 -1	6.10 3	1.94 1	8.57 3
1,000	---	4.96 -1	---	3.66 -1	3.66 -1	1.03 4	2.07 1	1.45 4
2,000	---	4.96 -1	---	7.32 -1	7.32 -1	2.22 4	2.26 1	3.05 4
3,000	7.14 -2	4.60 -1	1.58 -1	1.02	1.18	5.13 4	2.53 1	6.46 4
4,000	4.71 -1	2.61 -1	1.39	7.70 -1	2.16	1.52 5	3.21 1	1.76 5
5,000	8.60 -1	6.59 -2	3.17	2.43 -1	3.42	2.48 5	3.73 1	2.87 5
6,000	9.62 -1	1.52 -2	4.26	6.73 -2	4.33	2.82 5	3.88 1	3.31 5
7,000	9.82 -1	4.86 -3	5.08	2.51 -2	5.10	2.99 5	3.94 1	3.56 5
8,000	9.88 -1	2.05 -3	5.83	1.21 -2	5.85	3.12 5	3.99 1	3.78 5
9,000	9.90 -1	1.06 -3	6.58	7.06 -3	6.58	3.25 5	4.02 1	3.99 5
10,000	9.91 -1	6.36 -4	7.31	4.69 -3	7.32	3.38 5	4.06 1	4.20 5
12,000	9.92 -1	2.97 -4	8.78	2.63 -3	8.78	3.63 5	4.11 1	4.62 5
ρ/ρ_s = 0.3								
300	---	4.96 -1	---	3.29 -1	3.29 -1	2.99 3	1.66 1	4.23 3
600	---	4.96 -1	---	6.59 -1	6.59 -1	6.10 3	1.83 1	8.57 3
1,000	---	4.96 -1	---	1.10	1.10	1.03 4	1.96 1	1.45 4
2,000	---	4.96 -1	---	2.20	2.20	2.22 4	2.16 1	3.05 4
3,000	4.19 -2	4.75 -1	2.78 -1	3.16	3.43	4.49 4	2.37 1	5.78 4
4,000	3.11 -1	3.41 -1	2.75	3.02	5.77	1.17 5	2.85 1	1.39 5
5,000	7.17 -1	1.37 -1	7.94	1.52	9.46	2.18 5	3.39 1	2.53 5
6,000	9.10 -1	4.09 -2	1.21 1	5.43 -1	1.26 1	2.71 5	3.63 1	3.19 5
7,000	9.64 -1	1.40 -2	1.49 1	2.18 -1	1.52 1	2.95 5	3.72 1	3.52 5
8,000	9.80 -1	6.06 -3	1.74 1	1.07 -1	1.75 1	3.11 5	3.77 1	3.76 5
9,000	9.86 -1	3.16 -3	1.96 1	6.30 -2	1.97 1	3.24 5	3.81 1	3.98 5
10,000	9.88 -1	1.90 -3	2.19 1	4.20 -2	2.19 1	3.37 5	3.84 1	4.20 5
12,000	9.90 -1	8.89 -4	2.63 1	2.36 -2	2.63 1	3.63 5	3.89 1	4.62 5

Table 5 (continued)

T (^oK)	n_H (moles/gm)	n_{H_2} (moles/gm)	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^o$ (j/gm)	s (cal/gm deg)	$h-e_{20}^o$ (j/gm)
				ρ/ρ_s = 1.0				
300	---	4.96 -1	---	1.10	1.10	2.99 3	1.54 1	4.23 3
600	---	4.96 -1	---	2.20	2.20	6.10 3	1.71 1	8.57 3
1,000	---	4.96 -1	---	3.66	3.66	1.03 4	1.84 1	1.45 4
2,000	---	4.96 -1	---	7.32	7.36	2.22 4	2.04 1	3.05 4
3,000	2.32 -2	4.84 -1	5.13 -1	1.07 1	1.12 1	4.08 4	2.21 1	5.35 4
4,000	1.85 -1	4.03 -1	5.46	1.19 1	1.74 1	9.01 4	2.54 1	1.10 5
5,000	5.17 -1	2.38 -1	1.91 1	8.77	2.78 1	1.75 5	3.00 1	2.07 5
6,000	7.88 -1	1.02 -1	3.49 1	4.52	3.94 1	2.46 5	3.31 1	2.90 5
7,000	9.09 -1	4.16 -2	4.70 1	2.15	4.91 1	2.84 5	3.45 1	3.39 5
8,000	9.54 -1	1.91 -2	5.63 1	1.13	5.74 1	3.06 5	3.52 1	3.70 5
9,000	9.72 -1	1.02 -2	6.45 1	6.80 -1	6.52 1	3.22 5	3.56 1	3.95 5
10,000	9.80 -1	6.21 -3	7.23 1	4.59 -1	7.28 1	3.36 5	3.60 1	4.18 5
12,000	9.86 -1	2.94 -3	8.73 1	2.60 -1	8.76 1	3.62 5	3.65 1	4.61 5
				ρ/ρ_s = 3.0				
300	---	4.96 -1	---	3.29	3.29	2.99 3	1.43 1	4.23 3
600	---	4.96 -1	---	6.59	6.59	6.10 3	1.60 1	8.57 3
1,000	---	4.96 -1	---	1.10 1	1.10 1	1.03 4	1.73 1	1.45 4
2,000	---	4.96 -1	---	2.20 1	2.20 1	2.22 4	1.93 1	3.05 4
3,000	1.34 -2	4.89 -1	8.93 -1	3.25 1	3.34 1	3.87 4	2.09 1	5.12 4
4,000	1.12 -1	4.40 -1	9.89	3.90 1	4.89 1	7.43 4	2.33 1	9.27 4
5,000	3.47 -1	3.22 -1	3.85 1	3.57 1	7.41 1	1.39 5	2.67 1	1.67 5
6,000	6.17 -1	1.88 -1	8.19 1	2.49 1	1.07 2	2.10 5	2.98 1	2.50 5
7,000	7.99 -1	9.65 -2	1.24 2	1.50 1	1.39 2	2.61 5	3.17 1	3.13 5
8,000	8.92 -1	5.02 -2	1.58 2	8.89	1.67 2	2.93 5	3.27 1	3.56 5
9,000	9.35 -1	2.84 -2	1.86 2	5.67	1.92 2	3.14 5	3.33 1	3.87 5
10,000	9.57 -1	1.78 -2	2.12 2	3.93	2.16 2	3.31 5	3.37 1	4.12 5
12,000	9.75 -1	8.62 -3	2.59 2	2.29	2.61 2	3.60 5	3.43 1	4.58 5

Table 5 (continued)

T (°K)	n_H $\left(\frac{\text{moles}}{\text{gm}}\right)$	n_{H_2} $\left(\frac{\text{moles}}{\text{gm}}\right)$	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{o}$ $\left(\frac{\text{j}}{\text{gm}}\right)$	s $\left(\frac{\text{cal}}{\text{gm deg}}\right)$	$h-e_{20}^{o}$ $\left(\frac{\text{j}}{\text{gm}}\right)$
				ρ/ρ_s = 10				
300	---	4.96 -1	---	1.10 1	1.10 1	2.99 3	1.31 1	4.23 3
600	---	4.96 -1	---	2.20 1	2.20 1	6.10 3	1.49 1	8.57 3
1,000	---	4.96 -1	---	3.66 1	3.66 1	1.03 4	1.62 1	1.45 4
2,000	---	4.96 -1	---	7.32 1	7.32 1	2.22 4	1.81 1	3.05 4
3,000	7.39 -3	4.92 -1	1.64	1.09 2	1.11 2	3.74 4	1.96 1	4.98 4
4,000	6.28 -2	4.65 -1	1.85 1	1.37 2	1.56 2	6.38 4	2.13 1	8.14 4
5,000	2.10 -1	3.91 -1	7.74 1	1.44 2	2.22 2	1.10 5	2.38 1	1.35 5
6,000	4.18 -1	2.87 -1	1.85 2	1.27 2	3.12 2	1.68 5	2.63 1	2.04 5
7,000	6.13 -1	1.89 -1	3.17 2	9.78 1	4.15 2	2.23 5	2.83 1	2.70 5
8,000	7.53 -1	1.19 -1	4.45 2	7.05 1	5.15 2	2.65 5	2.97 1	3.23 5
9,000	8.39 -1	7.64 -2	5.58 2	5.07 1	6.08 2	2.96 5	3.05 1	3.64 5
10,000	8.90 -1	5.12 -2	6.57 2	3.78 1	6.94 2	3.18 5	3.11 1	3.97 5
12,000	9.39 -1	2.66 -2	8.31 2	2.36 1	8.55 2	3.54 5	3.19 1	4.50 5
				ρ/ρ_s = 25				
300	---	4.96 -1	---	2.75 1	2.75 1	2.99 3	1.22 1	4.23 3
600	---	4.96 -1	---	5.49 1	5.49 1	6.10τ 3	1.40 1	8.57 3
1,000	---	4.96 -1	---	9.15 1	9.15 1	1.03 4	1.53 1	1.45 4
2,000	---	4.96 -1	---	1.83 2	1.83 2	2.22 4	1.72 1	3.05 4
3,000	4.68 -3	4.94 -1	2.59	2.73 2	2.76 2	3.68 4	1.86 1	4.92 4
4,000	4.02 -2	4.76 -1	2.97 1	3.51 2	3.81 2	5.90 4	2.01 1	7.61 4
5,000	1.38 -1	4.27 -1	1.28 2	3.94 2	5.21 2	9.48 4	2.20 1	1.18 5
6,000	2.92 -1	3.50 -1	3.23 2	3.88 2	7.11 2	1.42 5	2.41 1	1.74 5
7,000	4.60 -1	2.66 -1	5.94 2	3.44 2	9.38 2	1.92 5	2.59 1	2.34 5
8,000	6.06 -1	1.93 -1	8.94 2	2.85 2	1.18 3	2.35 5	2.73 1	2.89 5
9,000	7.15 -1	1.39 -1	1.19 3	2.30 2	1.42 3	2.71 5	2.83 1	3.35 5
10,000	7.90 -1	1.-1 -1	1.46 3	1.86 2	1.64 3	2.99 5	2.90 1	3.73 5
12,000	8.76 -1	5.80 -2	1.94 3	1.28 2	2,07 3	3.42 5	2.99 1	4.35 5

Table 5 (continued)

T ($^\circ$K)	n_H $\left(\frac{\text{moles}}{\text{gm}}\right)$	n_{H_2} $\left(\frac{\text{moles}}{\text{gm}}\right)$	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{\circ}$ $\left(\frac{\text{j}}{\text{gm}}\right)$	s $\left(\frac{\text{cal}}{\text{gm deg}}\right)$	$h-e_{20}^{\circ}$ $\left(\frac{\text{j}}{\text{gm}}\right)$
$\rho/\rho_s = 50$								
300	---	4.96 -1	---	5.49 1	5.49 1	2.99 3	1.16 1	4.23 3
600	---	4.96 -1	---	1.10 2	1.10 2	6.10 3	1.33 1	8.57 3
1,000	---	4.96 -1	---	1.83 2	1.83 2	1.03 4	1.46 1	1.45 4
2,000	---	4.96 -1	---	3.66 2	3.66 2	2.22 4	1.56 1	3.05 4
3,000	3.31 -3	4.94 -1	3.67	5.47 2	5.51 2	3.65 4	1.79 1	4.89 4
4,000	2.86 -2	4.82 -1	4.22 1	7.11 2	7.53 2	5.65 4	1.92 1	7.34 4
5,000	1.00 -1	4.46 -1	1.85 2	8.23 2	1.01 3	8.67 4	2.08 1	1.09 5
6,000	2.17 -1	3.87 -1	4.81 2	8.58 2	1.34 3	1.26 5	2.26 1	1.57 5
7,000	3.56 -1	3.18 -1	9.18 2	8.22 2	1.74 3	1.70 5	2.42 1	2.09 5
8,000	4.89 -1	2.52 -1	1.44 3	7.43 2	2.19 3	2.12 5	2.55 1	2.61 5
9,000	6.01 -1	1.96 -1	2.00 3	6.50 2	2.65 3	2.49 5	2.65 1	3.08 5
10,000	6.87 -1	1.53 -1	2.53 3	5.63 2	3.10 3	2.79 5	2.73 1	3.49 5
12,000	7.99 -1	9.65 -2	3.54 3	4.27 2	3.97 3	3.28 5	2.84 1	4.18 5
$\rho/\rho_s = 75$								
300	---	4.96 -1	---	8.24 1	8.24 1	2.99 3	1.12 1	4.23 3
600	---	4.96 -1	---	1.65 2	1.65 2	6.10 3	1.29 1	8.57 3
1,000	---	4.96 -1	---	2.75 2	2.75 2	1.03 4	1.42 1	1.45 4
2,000	---	4.96 -1	---	5.49 2	5.49 2	2.22 4	1.61 1	3.05 4
3,000	2.70 -3	4.95 -1	4.49	8.21 2	8.26 2	3.64 4	1.75 1	4.88 4
4,000	2.34 -2	4.84 -1	5.18 1	1.07 3	1.12 3	5.53 4	1.88 1	7.22 4
5,000	8.25 -2	4.55 -1	2.28 2	1.26 3	1.49 3	8.29 4	2.02 1	1.05 5
6,000	1.81 -1	4.05 -1	6.02 2	1.35 3	1.95 3	1.19 5	2.18 1	1.48 5
7,000	3.02 -1	3.45 -1	1.17 3	1.34 3	2.51 3	1.59 5	2.33 1	1.97 5
8,000	4.24 -1	2.84 -1	1.88 3	1.26 3	3.14 3	1.99 5	2.45 1	2.46 5
9,000	5.32 -1	2.30 -1	2.65 3	1.15 3	3.80 3	2.35 5	2.56 1	2.92 5
10,000	6.19 -1	1.86 -1	3.43 3	1.03 3	4.46 3	2.66 5	2.64 1	3.33 5
12,000	7.42 -1	1.25 -1	4.93 3	8.30 2	5.76 3	3.18 5	2.75 1	4.04 5

Table 5 (continued)

T (^{o}K)	n_H (moles/gm)	n_{H_2} (moles/gm)	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{o}$ (j/gm)	s (cal/gm deg)	$h-e_{20}^{o}$ (j/gm)
ρ/ρ_s = 100								
300	---	4.96 -1	---	1.10 2	1.10 2	2.99 3	1.09 1	4.23 3
600	---	4.96 -1	---	2.20 2	2.20 2	6.10 3	1.26 1	8.57 3
1,000	---	4.96 -1	---	3.66 2	3.66 2	1.03 4	1.39 1	1.45 4
2,000	---	4.96 -1	---	7.32 2	7.32 2	2.22 4	1.58 1	3.05 4
3,000	2.34 -3	4.95 -1	5.19	1.10 3	1.10 3	3.63 4	1.72 1	4.87 4
4,000	2.03 -2	4.86 -1	6.00 1	1.43 3	1.49 3	5.47 4	1.84 1	7.15 4
5,000	7.19 -2	4.60 -1	2.65 2	1.70 3	1.96 3	8.07 4	1.98 1	1.03 5
6,000	1.59 -1	4.16 -1	7.05 2	1.84 3	2.55 3	1.14 5	2.13 1	1.43 5
7,000	2.68 -1	3.62 -1	1.39 3	1.87 3	3.26 3	1.52 5	2.27 1	1.89 5
8,000	3.81 -1	3.06 -1	2.25 3	1.80 3	4.05 3	1.90 5	2.39 1	2.36 5
9,000	4.84 -1	2.54 -1	3.22 3	1.69 3	4.90 3	2.26 5	2.49 1	2.81 5
10,000	5.71 -1	2.11 -1	4.21 3	1.56 3	5.77 3	2.57 5	2.57 1	3.22 5
12,000	6.98 -1	1.47 -1	6.18 3	1.30 3	7.48 3	3.10 5	2.68 1	3.94 5
ρ/ρ_s = 150								
300	---	4.96 -1	---	1.65 2	1.65 2	2.99 3	1.05 1	4.23 3
600	---	4.96 -1	---	3.29 2	3.29 2	6.10 3	1.22 1	8.57 3
1,000	---	4.96 -1	---	5.49 2	5.49 2	1.03 4	1.35 1	1.45 4
2,000	---	4.96 -1	---	1.10 3	1.10 3	2.22 4	1.54 1	3.05 4
3,000	1.91 -3	4.95 -1	6.35	1.64 3	1.65 3	3.62 4	1.68 1	4.86 4
4,000	1.66 -2	4.88 -1	7.36 1	2.16 3	2.23 3	5.39 4	1.80 1	7.07 4
5,000	5.91 -2	4.66 -1	3.27 2	2.58 3	2.91 3	7.80 4	1.93 1	9.98 4
6,000	1.32 -1	4.30 -1	8.77 2	2.86 3	3.73 3	1.09 5	2.06 1	1.37 5
7,000	2.25 -1	3.83 -1	1.75 3	2.97 3	7.72 3	1.43 5	2.19 1	1.79 5
8,000	3.25 -1	3.34 -1	2.88 3	2.95 3	5.83 3	1.79 5	2.30 1	2.23 5
9,000	4.20 -1	2.86 -1	4.18 3	2.85 3	7.03 3	2.13 5	1.40 1	2.66 5
10,000	5.02 -1	2.45 -1	5.56 3	2.71 3	8.27 3	2.44 5	2.48 1	3.06 5
12,000	6.31 -1	1.81 -1	8.38 3	2.40 3	1.08 4	2.98 5	2.59 1	3.79 5

Table 5 (continued)

T ($^{\circ}$K)	n_H (moles/gm)	n_{H_2} (moles/gm)	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{\circ}$ (j/gm)	s (cal/gm deg)	$h-e_{20}^{\circ}$ (j/gm)
ρ/ρ_s = 200								
300	---	4.96 -1	---	2.20 2	2.20 2	2.99 3	1.02 1	.423 3
600	---	4.96 -1	---	4.39 2	4.39 2	6.10 3	1.19 1	8.57 3
1,000	---	4.96 -1	---	7.32 2	7.32 2	1.03 4	1.32 1	1.45 4
2,000	---	4.96 -1	---	1.46 3	1.46 3	2.22 4	1.52 1	3.05 4
3,000	1.66 -3	4.95 -1	7.34	2.19 3	2.20 3	3.61 4	1.65 1	4.85 4
4,000	1.44 -2	4.89 -1	8.51 1	2.89 3	2.94 3	5.34 4	1.77 1	7.01 4
5,000	5.14 -2	4.70 -1	3.79 2	3.47 3	3.85 3	7.63 4	1.89 1	9.80 4
6,000	1.15 -1	4.38 -1	1.02 3	3.88 3	4.90 3	1.05 5	2.01 1	1.33 5
7,000	1.99 -1	3.97 -1	2.05 3	4.10 3	6.15 3	1.38 5	2.13 1	1.72 5
8,000	2.89 -1	3.52 -1	3.41 3	4.15 3	7.56 3	1.72 5	2.24 1	2.14 5
9,000	3.77 -1	3.08 -1	5.00 3	4.09 3	9.09 3	2.04 5	2.33 1	2.56 5
10,000	4.55 -1	2.68 -1	6.72 3	3.96 3	1.07 4	2.35 5	2.41 1	2.95 5
12,000	5.82 -1	2.05 -1	1.03 4	3.63 3	1.39 4	2.89 5	2.53 1	3.67 5
ρ/ρ_s = 300								
300	---	4.96 -1	---	3.29 2	3.29 2	2.99 3	9.79	4.23 3
600	---	4.96 -1	---	6.59 2	6.59 2	6.10 3	1.15 1	8.57 3
1,000	---	4.96 -1	---	1.10 3	1.10 3	1.03 4	1.28 1	1.45 4
2,000	---	4.96 -1	---	2.20 3	2.20 3	2.22 4	1.48 1	3.05 4
3,000	1.35 -3	4.95 -1	8.99	3.29 3	3.30 3	3.61 4	1.61 1	4.85 4
4,000	1.18 -2	4.90 -1	1.04 2	4.34 3	4.45 3	5.28 4	1.72 1	6.95 4
5,000	4.22 -2	4.75 -1	4.67 2	5.26 3	5.72 3	7.44 4	1.84 1	9.59 4
6,000	9.54 -2	4.48 -1	1.27 3	5.96 3	7.22 3	1.01 5	1.95 1	1.28 5
7,000	1.65 -1	4.13 -1	2.56 3	6.41 3	8.97 3	1.31 5	2.06 1	1.65 5
8,000	2.44 -1	3.74 -1	4.31 3	6.63 3	1.09 4	1.62 5	2.16 1	2.04 5
9,000	3.21 -1	3.35 -1	6.40 3	6.68 3	1.31 4	1.93 5	2.25 1	2.43 5
10,000	3.93 -1	3.00 -1	8.70 3	6.63 3	1.53 4	2.23 5	2.33 1	2.80 5
12,000	5.14 -1	2.39 -1	1.36 4	6.36 3	2.00 4	2.76 5	2.44 1	3.51 5

Table 5 (continued)

T (^oK)	n_H (moles/gm)	n_{H_2} (moles/gm)	p_H (atm)	p_{H_2} (atm)	p (atm)	$e-e^o_{20}$ (j/gm)	s (cal/gm deg)	$h-e^o_{20}$ (j/gm)
				ρ/ρ_s = 400				
300	---	4.96 -1	---	4.39 2	4.39 2	2.99 3	9.51	4.23 3
600	---	4.96 -1	---	8.79 2	8.79 2	6.10 3	1.12 1	8.57 3
1,000	---	4.96 -1	---	1.46 3	1.46 3	1.03 4	1.25 1	1.45 4
2,000	---	4.96 -1	---	2.93 3	2.93 3	2.22 4	1.45 1	3.05 4
3,000	1.17 -3	4.95 -1	1.04 1	4.39 3	4.40 3	3.60 4	1.58 1	4.84 4
4,000	1.02 -2	4.91 -1	1.21 2	5.80 3	5.92 3	5.25 4	1.69 1	6.92 4
5,000	3.66 -2	4.78 -1	5.41 2	7.05 3	7.59 3	7.32 4	1.80 1	9.46 4
6,000	8.31 -2	4.54 -1	1.47 3	8.05 3	9.52 3	9.84 4	1.91 1	1.25 5
7,000	1.45 -1	4.24 -1	3.00 3	8.75 3	1.17 4	1.27 5	2.02 1	1.60 5
8,000	2.15 -1	3.89 -1	5.07 3	9.18 3	1.43 4	1.57 5	2.11 1	1.97 5
9,000	2.85 -1	3.53 -1	7.58 3	9.39 3	1.70 4	1.86 5	2.20 1	2.34 5
10,000	3.52 -1	3.20 -1	1.04 4	9.45 3	1.98 4	2.15 5	2.27 1	2.71 5
12,000	4.66 -1	2.63 -1	1.65 4	9.31 3	2.58 4	2.67 5	2.38 1	3.40 5
				ρ/ρ_s = 600				
300	---	4.96 -1	---	6.59 2	6.59 2	2.99 3	9.11	4.23 3
600	---	4.96 -1	---	1.32 3	1.32 3	6.10 3	1.08 1	8.57 3
1,000	---	4.96 -1	---	2.20 3	2.20 3	1.03 4	1.21 1	1.45 4
2,000	---	4.96 -1	---	4.39 3	4.39 3	2.22 4	1.41 1	3.05 4
3,000	9.57 -4	4.96 -1	1.27 1	6.58 3	6.60 3	3.60 4	1.54 1	4.84 4
4,000	8.34 -3	4.92 -1	1.48 2	8.71 3	8.86 3	5.21 4	1.65 1	6.87 4
5,000	3.00 -2	4.81 -1	6.65 2	1.07 4	1.13 4	7.18 4	1.75 1	9.30 4
6,000	6.84 -2	4.62 -1	1.82 3	1.23 4	1.41 4	9.53 4	1.85 1	1.22 5
7,000	1.20 -1	4.36 -1	3.72 3	1.35 4	1.72 4	1.22 5	1.95 1	1.54 5
8,000	1.79 -1	4.06 -1	6.36 3	1.44 4	2.07 4	1.50 5	2.04 1	1.89 5
9,000	2.40 -1	3.76 -1	9.58 3	1.50 4	2.46 4	1.78 5	2.12 1	2.24 5
10,000	2.99 -1	3.47 -1	1.32 4	1.54 4	2.86 4	2.05 5	2.19 1	2.58 5
12,000	4.03 -1	2.95 -1	2.14 4	1.57 4	3.71 4	2.56 5	2.30 1	3.26 5

Table 5 (continued)

T (^{o}K)	n_H (moles/gm)	n_{H_2} (moles/gm)	P_H (atm)	P_{H_2} (atm)	P (atm)	$e-e_{20}^{o}$ (j/gm)	s (cal/gm deg)	$h-e_{20}^{o}$ (j/gm)
ρ/ρ_s = 800								
300	---	4.96 -1	---	8.79 2	8.79 2	2.99 3	8.82	4.23 3
600	---	4.96 -1	---	1.76 3	1.76 3	6.10 3	1.05 1	8.57 3
1,000	---	4.96 -1	---	2.93 3	2.93 3	1.03 4	1.18 1	1.45 4
2,000	---	4.96 -1	---	5.86 3	5.86 3	2.22 4	1.38 1	3.05 4
3,000	8.29 -4	4.96 -1	1.47 1	8.78 3	8.79 3	3.60 4	1.51 1	4.83 4
4,000	7.23 -3	4.92 -1	1.71 2	1.16 4	1.18 4	5.19 4	1.62 1	6.85 4
5,000	2.60 -2	4.83 -1	7.69 2	1.43 4	1.50 4	7.09 4	1.72 1	9.21 4
6,000	5.95 -2	4.66 -1	2.11 3	1.65 4	1.86 4	9.35 4	1.82 1	1.20 5
7,000	1.05 -1	4.44 -1	4.34 3	1.83 4	2.27 4	1.19 5	1.91 1	1.51 5
8,000	1.57 -1	4.17 -1	7.44 3	1.97 4	2.71 4	1.45 5	2.00 1	1.83 5
9,000	2.12 -1	3.90 -1	1.13 4	2.07 4	3.20 4	1.72 5	2.07 1	2.17 5
10,000	2.65 -1	3.64 -1	1.57 4	2.15 4	3.71 4	1.98 5	2.14 1	2.50 5
12,000	3.61 -1	3.15 -1	2.56 4	2.23 4	4.79 4	2.48 5	2.25 1	3.16 5
ρ/ρ_s = 1000								
300	---	4.96 -1	---	1.10 3	1.10 3	2.99 3	8.60	4.23 3
600	---	4.96 -1	---	2.20 3	2.20 3	6.10 3	1.03 1	8.57 3
1,000	---	4.96 -1	---	3.66 3	3.66 3	1.03 4	1.16 1	1.45 4
2,000	---	4.96 -1	---	7.32 3	7.32 3	2.22 4	1.36 1	3.05 4
3,000	7.41 -4	4.96 -1	1.46 1	1.10 4	1.10 4	3.59 4	1.49 1	4.83 4
4,000	6.47 -3	4.93 -1	1.91 2	1.45 4	1.47 4	5.17 4	1.60 1	6.83 4
5,000	2.33 -2	4.84 -1	8.61 2	1.79 4	1.87 4	7.04 4	1.69 1	9.15 4
6,000	5.34 -2	4.69 -1	2.37 3	2.08 4	2.31 4	9.22 4	1.79 1	1.18 5
7,000	9.44 -2	4.49 -1	4.88 3	2.32 4	2.81 4	1.16 5	1.88 1	1.48 5
8,000	1.42 -1	4.25 -1	8.39 3	2.51 4	3.35 4	1.42 5	1.96 1	1.80 5
9,000	1.92 -1	4.00 -1	1.28 4	2.66 4	3.93 4	1.68 5	2.04 1	2.12 5
10,000	2.41 -1	3.76 -1	1.78 4	2.77 4	4.55 4	1.94 5	2.10 1	2.45 5
12,000	3.31 -1	3.31 -1	2.93 4	2.93 4	5.86 4	2.43 5	2.21 1	3.09 5

Appendix B

PROOF THAT INCREASING ONLY a_o RAISES MUZZLE VELOCITY

For a given value of $\beta = (\gamma+1)/(\gamma-1)$, Eq. (1) has the form $z(u)$. Since the efficiency is given by $\eta = 1/2\ u^2/z$, Eq. (1) gives implicitly $\eta = f(v/a_o)$. It was noted that the plot of $\log \eta$ versus $\log (v/a_o)$ always has a negative slope. The object is to show that this fact is sufficient to insure that if the quantities m/P_oAL and β are held fixed, then increasing a_o always leads to increasing v.

Since the logarithm is a monotonic increasing function of its argument, the negative slope of the log-log plot implies that $f'(v/a_o)$ is also negative.

The quantity, v, is the solution of the equation

$$f\left(\frac{v}{a_o}\right) = \frac{1}{2}\frac{mv^2}{P_oAL} \tag{9}$$

Taking the derivative of Eq. (9) with respect to a_o, one has

$$f'\left(\frac{v}{a_o}\right)\left[\frac{1}{a_o}\frac{dv}{da_o} - \frac{v}{a_o^2}\right] = \frac{mv}{P_oAL}\frac{dv}{da_o}, \tag{10}$$

which leads to

$$\frac{dv}{da_o} = \frac{f'\left(\frac{v}{a_o}\right)\frac{v}{a_o^2}}{\frac{f'\left(\frac{v}{a_o}\right)}{a_o} - \frac{mv}{P_oAL}} \tag{11}$$

Since m/P_oAL, v, and a_o are all positive quantities, it is clear that if $f'(v/a_o)$ is negative, then the right-hand side of Eq. (11) is always positive. Therefore, under these conditions

$$\frac{dv}{da_o} > 0 \tag{12}$$

REFERENCES

1. Charters, A. C., B. P. Denardo, and V. J. Rossow, Development of a Piston-compressor Type Light-gas Gun for the Launching of Free-flight Models at High Velocity, NACA TN 4143, November 1957.

2. Hilsenrath, J., et al., Tables of Thermal Properties of Gases, NBS Circular 564, 1955.

3. Fickett, W., and R. D. Cowan, J. Chem. Phys., Vol. 23, 1955, p. 1349.

4. Wooley, H. W., Thermodynamic Functions for Atomic Hydrogen and the Negative Atomic Ion of Hydrogen, NBS Report 3988, May 1, 1955.

5. Beutler, H., Z. Physik. Chem., Vol. B29, August 1935, p. 315.

6. Gilmore, F. R., Approximate Thermodynamic Properties of Compressed Hydrogen Gas from 5000° to 12,000° K, The RAND Corporation, Research Memorandum RM-1650, March 7, 1956.

7. Bloxsom, D. E., Jr., "Use of Capacitor Discharges to Produce High-Velocity Pellets," J. App. Phys., Vol. 29, No. 7, July 1958, pp. 1049-1051.

8. Swift, H. F., Proceedings of the Fourth Symposium on Hypervelocity Impact, Air Proving Ground Center, Eglin Air Force Base, 1960.

9. Georgiev, S., Avco-Everett Research Laboratory, personal communication.

10. Partridge, W. S., Utah Research and Development Corporation, personal communication.

11. Charters, A. C., Ames Research Center, personal communication.

12. Atkins, W. W., Naval Research Laboratory, personal communication.

13. Partridge, W. S., et al., Proceedings of the Symposium on Hypervelocity Techniques, Denver Research Institute, October 1960, p. 20.

14. McQueen, R. G., and S. P. Marsh, "Equation of State for Nineteen Metallic Elements from Shock-Wave Measurements to Two Megabars," J. App. Phys., Vol. 31, No. 7, July 1960, pp. 1253-1269.

15. Al'tshuler, L. V., et al., "Dynamic Compressibility and Equation of State of Iron Under High Pressure," Soviet Phys. JETP, Vol. 34(7), No. 4, October 1958, pp. 606-614.

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