AD NUMBER: AD0209139
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DAMPING OF BENDING VIBRATIONS IN THIN PLATES

I. INTRODUCTION

This memorandum presents a review of some of the aspects, both theoretical and experimental, of the damping of thin plates by applied damping layers.

Scope of this Memorandum

Much of the previous experimental and theoretical work has been concerned with damping of relatively simple elastic structures, such as a flat plate. However, many applications of interest involve complicated elastic structures such as may be found in aircraft, submarines or other marine vessels, automobiles, appliances, etc.

Nevertheless understanding the damping of flat plates is essential because:

1. A complete understanding of this probelm will provide qualitative and semi-qualitative understanding of the problem of damping of complicated structures, and
2. Before complete understanding of the problem of the damping of complicated structures can be obtained, complete understanding must be achieved for the undamped motion of the structures. To date such understanding has not been achieved for many complicated structures.

Thus one is faced with the necessity of investigating and developing new and improved damping materials largely in the context of the flat plate problem. Actually in spite of much previous work on flat plates, a great deal of work still remains. In this memorandum we restrict our discussion to the flat-plate problem.

Before we discuss the damping of plates in detail, let us consider the nature of damping and some of the reasons for using damping materials.

<u>What Is Damping</u>? Let us define damping as the extraction of power from a vibrating system. The vibrations in the system can be either travelling or standing waves in a distributed system (e.g., a bar or plate), or vibrations in a lumped-parameter system (e.g., a mass on a spring). The system vibrations may be steady-state or transient in nature.

<u>Why Damp</u>? The direct answer to this question is "to reduce the amplitude of steady-state vibrations or to hasten the decay of transient vibrations (or both)". By accomplishing these ends damping can:

a) limit the motion (e.g., at resonance) of structures subject to vibration excitation,

b) Attenuate structure-borne waves,

c) reduce the radiation of acoustic power from systems driven in vibration, (e.g., from ship hulls, machinery housings, air conditioning ducts, etc.)

d) increase, in certain cases, the transmission loss of structures subjected to acoustic or fluid-dynamic excitation,

e) limit fatigue and wear in vibrating structures,

f) inhibit hydro- or aero-dynamic flutter.

Adequate damping can be important. It can make the difference between acceptance and rejection of an appliance or vehicle by consumers, between adequate and unsatisfactory hearing conditions in a critical area, or possibly between safe passage and damage or destruction for a naval vessel on a mission.

<u>What Kinds of Damping Are There?</u> The type of damping that is of interest here can be described as linear, dynamic, mechanical hysteresis. Visco elastic materials exhibit such hysteresis. The term "visco-elastic" is descriptive of materials that have both elastic and viscous properties. The distortion of a sample of such a material is accompanied both by an elastic restoring force and by energy dissipation within the material.

By definition, the Young's modulus (modulus of elasticity) of a purely elastic material is the ratio of stress (force per unit area) applied to the material to the strain (fractional elongation) of the material in response to the applied stress. If we extend this definition to the case of a visco-elastic material, we find that the elastic modulus becomes a complex quantity. Thus, we define:

$$E_2 = E_2(1 + j\eta_2) \qquad (1)$$

where E_2 is the complex Young's modulus, E_2 is the real part of the modulus, and η_2 is defined as the loss factor. The subscript 2 is used here as applied to a damping material in anticipation of the notation required in Section II.

In general, both E_2 and η_2 are functions of frequency, temperature, and, in some temperature ranges, of moisture content. As we shall see later, <u>both</u> E_2 and η_2 are important in determining the effectiveness of a damping material.

When a periodic (sinusoidal) stress is applied to a visco-elastic material, a stress-strain relationship like that of Fig 1 is obtained. Here, because of the dissipation within the material, the strain lags behind the stress in time, resulting in the elliptical path shown. The energy dissipated during each cycle in a unit volume of the material is just the area within the stress-strain ellipse.

Other kinds of damping that may affect vibrating systems include the following:

a) friction damping
b) acoustical radiation damping
c) damping due to energy losses at boundaries, mounting points, etc.
d) magnetic or eddy-current damping
e) non-linear damping where the damping factor depends on the amplitude of displacement, or where the damping force varies with a power of velocity other than the first power.

In this memorandum, however, we are concerned only with the damping in visco-elastic materials.

II. ANALYSIS OF A SINGLE DAMPING LAYER ON A FLAT BAR

In this section we discuss bending waves in a bar, the effect of a damping layer applied to the bar, and methods for optimizing this damping.

A. Bending Waves in a Bar

Figure 2 shows an element of a bar subjected to pure bending. Bending moments, M, acting at each end of the element give rise to the stress distribution shown for the cross section. If the bar cross section is rectangular or otherwise symmetrical about an axis in the plane of bending, the neutral plane, at which the longitudinal stress is zero, passes through the center of the bar cross section. The strain of a fiber of the beam is proportional to the distance from the fiber to the neutral plane.

The bending stiffness, B_1, of a bar is defined as the ratio of the bending moment, M, to the curvature of the element. In terms of the properties of the bar

$$B = \int_{y=0}^{d} w\, E\, (y - y_0)^2\, dy \tag{2}$$

where

y = coordinate measured from the bottom of the bar cross section,

y_0 = coordinate of the neutral plane,

d = thickness of the bar,

w = width of the bar at y, and

E = modulus of elasticity of the bar material.

Note that in general both w and E may be functions of y.

For a bar of rectangular cross section and of one material only, Eq (2) becomes

$$B = Ew \int_{y=0}^{d} (y - y_0)^2 \, dy = E I_0 \qquad (3)$$

where I_0 is the area moment of the beam cross section taken about the neutral axis of the cross section.

The differential equation governing the propagation of free bending waves on a bar (whose thickness is small relative to the wavelength of bending waves) is

$$\frac{\partial^4 z}{\partial x^4} + \frac{1 - s^2}{B/m} \frac{\partial^2 z}{\partial t^2} = 0 \qquad (4)$$

where

z = displacement of the bar normal to its length,

m = mass of the bar per unit length, and

s = Poisson's ratio for the bar material. Usually, as we shall do hereafter, s^2 may be neglected relative to 1.

From Eq (4) we find the velocity, c_B, of bending waves in the bar to be

$$c_B = \left(\frac{B}{m}\right)^{\frac{1}{4}} \sqrt{\omega} \qquad (5)$$

where

$\omega = 2\pi f$, the angular frequency of the bending vibration.

If we say the bar material has no inherent losses, then E, B, and c_B are real quantities, and the waves travel without attenuation.

B. A Bar With an Applied Damping Layer

Consider a bar with a homogeneous layer of damping material applied to one of its sides. An element of such a bar is shown in Fig 3 with bending moments, M, applied at its ends. In general, the damping material would be considerably less stiff than the bar material. Accordingly, the stress-distribution diagram shows relatively smaller stresses in the damping layer.

The neutral plane moves up slightly from its position in the undamped bar. For a rectangular cross section the new coordinate of the neutral plane is

$$y_0 = d_1 - \frac{1}{2}\,\frac{E_1 d_1^2 - E_2 d_2^2}{E_1 d_1 + E_2 d_2} \qquad (6)$$

The bending stiffness of the composite bar is calculated by applying Eq (2) as follows:

$$B = \int_{y=0}^{d_1+d_2} w\, E\, (y-y_0)^2\, dy$$

$$= w \left\{ E_1 \int_{y=0}^{d_1} (y-y_0)^2\, dy + \overline{E}_2 \int_{y=d_1}^{d_2} (y-y_0)^2\, dy \right. \qquad (7)$$

Here we use subscripts 1 and 2 for the bar and the damping layer respectively. Quantities without subscripts, e.g., $\overline{B}$, refer to the composite bar. A rectangular cross section is assumed. Note that $\overline{B}$ will be a complex quantity because $\overline{E}_2$ for the damping material is complex (see Eq (1)).

By analogy with our earlier results for the undamped bar, we may define for the composite (damped) bar

$$\overline{B} = \overline{E}\, I_0 = B(1 + j\eta) \qquad (8)$$

Here the loss factor, η, for the composite bar is related directly to the complex bending stiffness.

The substitution of Eq (1) into Eq (7) gives an expression for δ in terms of E_1, E_2, η_2, d_1, and d_2. If we make the assumption that $\eta_2^2 << 1$, we find the following:

$$\frac{\eta}{\eta_2} = \frac{a\xi}{1 + a\xi} \frac{3 + 6\xi + 4\xi^2 + 2a\xi^3 + a^2\xi^4}{1 + 2a(2\xi + 3\xi^2 + 2\xi^3) + a^2\xi^4} \qquad (9)$$

and

$$\frac{B}{B_1} = \frac{1 + 5a\xi + 2a(3 + 2a)\xi^2 + 2a(2 + 3a)\xi^3 + 5a^2\xi^4 + a^3\xi^5}{1 + 2a\xi + a^2\xi^2} \qquad (10)$$

where for convenience we have written

$$a = \frac{E_2}{E_1}, \qquad \xi = \frac{d_2}{d_1}. \qquad (11)$$

The analysis leading to Eqs (9) and (10) is the same as that first reported by Oberst in 1952. Eq (9) is exactly as he presented it. In re-deriving the results, however, we find that Oberst's equation for the quantity B/B_1 should be corrected to agree with Eq (10). Numerically, the correction is small, and it should have no great effect on his conclusions.

The relation expressed by Eq (9) has been plotted by Oberst, and is repeated here as Fig 4. We have not yet had the opportunity to plot up Eq (10) for comparison with Oberst's results.

It follows from the conventional definition of wavelength that

$$\lambda_B = \frac{c_B}{f} \tag{12}$$

Thus direct measurements of frequency and of the wavelength of bending waves determine c_B. The ratio c_B/c_{B_1} of bending-wave velocities after and before damping is related, through the use of Eq (5), to the quantity B/B_1. Through an auxiliary calculation (see Oberst) one may use measurements of the natural frequencies of the bar instead of measuring λ_B.

Now we see that by making two measurements one may determine η and B/B_1 (c_{B_1} may be calculated). Presumably, d_2/d_1 is measurable and E_1 is known. Therefore, Eqs (9) and (10) may be used to determine E_2 and η_2, the basic properties of the damping material.

C. Interpretation of Results

Inspection of Fig 4 shows that the maximum obtainable value of η is of the order of η_2, the loss factor of the damping material. This result is reasonable; and, as one would expect, it is approached with thick damping layers.

The real importance of Fig 4 lies in the quantitative guidance it can provide in the specification of damping layers. We may write the following approximate relation, valid for values of

η/η_2 less than about 0.5 and for values of E_2/E_1 between 10^{-5} and 10^{-2}:

$$\eta \doteq \xi^2 a \eta_2 = \left(\frac{d_2}{d_1}\right)^2 \frac{E_2}{E_1} \eta_2 \qquad (13)$$

This relation applies for many cases of practical interest. Values of E_2/E_1 for most damping materials (the reference is E_1 for steel) do lie in the range 10^{-5} to 10^{-2}. Also thickness ratios, d_2/d_1, are often of the order 1 or greater, and thus fall within the range of interest. Some applications of Eq (13) are discussed in the following section.

For reference some typical values of η are given in the following table:

LOSS FACTOR, η, FOR SOME COMMON MATERIALS AND STRUCTURES

Material	Approximate Loss Factor η
Most metals	10^{-5} - 10^{-4}
Wood	10^{-2}
Automobile - body panels (Germany)	3×10^{-3} - 10^{-1}
Soft rubber	10^{-2} - 10^{-1}
Well-damped panels with $m_2/m_1 = 0.2$	up to 10^{-1}
Maximum obtainable with very thick coatings	up to 1

D. Optimizing Single-Layer Damping

It follows from Eq (13) that for a fixed ratio of damping-material thickness to plate thickness and for a fixed plate material, the maximum value of η results from maximizing the product $E_2\eta_2$. Thus for good damping in this case one requires not simply a large value of η_2, but a large $E_2\eta_2$ product. This fact, first pointed out by Oberst, is a very important contribution to damping technology. It is important to note that the <u>weight</u> of the damping material is unimportant in determining its damping effectiveness. This is in direct contrast to the sometimes-proposed intuitive requirement that "in order to damp well, a damping material must be lossy and heavy."

It is perhaps trivial to observe that for a given damping material (fixed E_2, η_2) [redacted] the loss factor for the damped plate increases with increasing thickness of the damping layer. However, Eq (13) tells us quantitatively that the loss factor varies with the square of the thickness ratio, a fact which allows quantitative specification of how much additional material is needed in a given case to obtain, say, a factor of two increase in η.

When the weight of the damping material is the limiting factor in a given application, $E_2\eta_2$ is no longer the quantity to be maximized. Noting that the surface mass, m_2, of the damping material is equal to the product $d_2\rho_2$ where ρ_2 is the density of the damping material, we may rewrite Eq (13) as follows:

$$\eta \doteq \left(\frac{m_2}{m_1}\right)^2 \left(\frac{\rho_1}{\rho_2}\right)^2 \frac{E_2}{E_1}\,\eta_2 \qquad (14)$$

Now from Eq (14) we see that for a given plate (fixed m_1, E_1, ρ_1) and for a fixed value of m_2, we maximize the loss factor η by maximizing the quantity $E_2\eta_2/\rho_2{}^2$.

There is another class of problems, for example, the treatment of stationary machinery, in which neither the weight nor the thickness of the damping material is particularly critical. Under these circumstances we may build up the thickness of the damping material until we reach the upper limiting values indicated in Fig 4. There we see that the maximum obtainable value of η is simply η_2, the loss factor for the damping material itself. Obviously, the material appropriate for such a case is that with the maximum value of η_2, more or less without regard to the value of E_2.

E. Recent Developments

Recognizing that a high $E_2\eta_2$ product and a low density are desirable qualities in a damping material, workers in Germany have guided their development work accordingly. The results have been gratifying. Several filled high-polymer materials with very good characteristics have been developed. Values of η_2 are moderate, about 0.2 at room temperature over the frequency range 20 - 4000 cps. In addition, the materials are very stiff, having values of E_2 somewhat greater than 10^{10} dynes/cm^2. (For steel $E_1 \doteq 2 \times 10^{12}$; thus $E_2/E_1 \doteq 0.5 \times 10^{-2}$, a relatively high value.)

In this country considerable attention has been given recently to damping tapes for application to thin plates. These tapes consist of a layer of adhesive damping material applied to a metal foil (e.g. aluminum) backing layer. The damping layer adheres to the surface to which the tape is applied, with the foil backing exposed on the outside.

Some tapes comprise two tape-plus-adhesive layers, and in some applications a number of layers of tape are applied to give the desired damping. At this time we have not yet been able to make a direct comparison between the damping tapes and the new filled high-polymer materials. Tests (of which we are aware) to date have not led to a determination of equivalent values of E_2 and η_2 for a given thickness of damping tape. This type of problem deserves further study. as we point out in Section IV.

III. MEASURING THE PROPERTIES OF DAMPING MATERIALS

A. Measures of Damping Effectivness

There are a number of different parameters that may describe the damping of a structure. Among these are the following:

η = loss factor, as defined in Eq. (1)

Q = "Factor of Merit", indicative of the sharpness of a of a resonance of the plate. $Q = f_o/\Delta f$, where f_o is the resonance frequency and Δf is the band width between "half power" (0.7 - amplitude) points of the resonance.

T = reverberation time of free bending vibrations on the plate, i.e., time for [redacted] amplitude to drop to 0.001 of its initial value.

D_t = decay rate, or rate of decrease of squared amplitude of free bending vibrations on the plate. (db/sec).

D_λ = attenuation of a progressive bending wave in a distance equal to 1 wavelength. (db/wavelength).

D_ℓ = attenuation of a progressive bending wave in a unit distance. (db/unit distance).

All of these parameters describe the same dissipation phenomenon. They are uniquely inter-related as follows:

$$Q = \frac{1}{\eta}$$

$$T = 2.20/\eta f_o \text{ (sec)}$$

$$D_t = 60/T = 27.3\eta f_o \text{ (db/sec)}$$

$$D_\lambda = 13.6\eta \text{ (db/wavelength)}$$

$$D_\ell = 13.6\ \eta/\lambda \text{ (db/unit length)}$$

Thus it is possible to discuss only one of these parameters, e.g., η, without loss of generality.

B. Types of Tests for Damping Materials

In recent years in this country it has been popular to measure and intercompare the characteristics of different damping materials with an apparatus known as a Geiger Plate. In this test a damping material is applied to one side of a steel plate approximately 20 x 20 x 1/4 in. The effectiveness of the damping material is characterized by the rate (db/sec) at which the fundamental mode of vibration of the plate decays after having been excited to steady-state motion. Measurements with this apparatus are often carried out over a range of temperatures.

The Geiger-Plate method of testing suffers from several disadvantages. First of all, measurements are made at only one frequency, although the characteristics of a damping material are known to vary with frequency. (See the work of Nolle, Oberst, and many others).

In addition, because the test plate is not large with respect to the wavelength of bending waves in the plate, edge effects become quite important. Under these circumstances the results are not quantitatively useful in predicting the damping to be expected when the same damping material is applied to other structures. That is, the method is limited to determining the decay rate or, equivalently, the effective damping, of the Geiger plate. Furthermore, it does not determine both the real and imaginary parts of the elastic modulus of the damping material. As such it is an incomplete test of damping materials.

A more significant and useful scheme has been developed by Oberst in Germany, and has been used successfully by BBN and other workers. In this test the damping material is applied to a strip or bar of metal. Both the real and imaginary parts of the elastic modulus of the damping material can be determined by measurements of bending vibrations in the coated strip. (See Section II-B.) Furthermore the dimensions of such a strip are chosen so that the wavelength of bending waves is appreciably smaller than the length of the strip. Thus end effects are minimized.

For small loss factors, say, in the range 10^{-3} to about 3×10^{-2}, the test strip is suspended. (See Fig 5.) Then η is determined either from Q-measurements or decay-rate (D_t) measurements.

For larger loss factors, however, it is probably best to measure the attenuation (D_ℓ or D_λ) and wavelength of progressive bending waves. An appropriate apparatus is shown in Fig 6 where a sand termination is used to reduce reflections from the end of the test strip.

We expect that future experimental work on damping materials will lead to a more complete understanding of these various testing techniques and their limitations.

IV. COMPOSITE DAMPING TREATMENTS

Up to this time only a very limited amount of work has been done on composite damping treatments. By composite treatments we mean those involving several different materials. These materials may be damping materials with different characteristics or some may be damping materials and others may be structural or stiffening materials.

One example of a composite structure is the metal-backed damping tape which has been used, for example, on the inside of the skin of an aircraft fuselage. (See Section II-E.) In this particular case, the damping tape is in effect a layer of damping material backed up with a stiffening layer of aluminum foil. A similar damping structure was recommended by BBN for use in damping large steel plates on Naval vessels. In this case, the damping structure comprised a layer of visco-elastic damping material applied to the plate and backed up with a lighter steel plate.

Figure 5 shows several possible composite damping structures. Figure 5-a shows a general 3-layer structure, layer 1 being the plate to be damped, and layers 2 and 3 being the composite damping structure. Figure 5-b shows what has been termed a "checkerboard" composite structure. The treatment applied to the plate is envisioned as a series of strips of two different damping materials, with the strips taking alternate positions adjacent to and spaced away from the plate. To our knowledge, such a structure has not yet been tested. The purpose of such a structure would be to take advantage of the properties of the two different damping materials. For example, it might be possible to obtain relatively high damping over a broad temperature range by using materials whose loss factors peaked at different temperatures.

The same type of thinking might well motivate tests of structures like that in Fig 5-a. However, the structure of Fig 5-b has the advantage of placing each of the materials "far" from the plate over half of the area covered. Our earlier theoretical work has shown that the outer layers of a damping material are under a greater strain than the inner layers, and therefore are more effective in damping.

Still other composite structures can be conceived and should be evaluated and perhaps tested in the future. Such structures might include filled or reinforced structures. For example, it has been suggested that glass fiber cloth or mat might be impregnated with a high polymer material and used as a damping layer. Initial tests made by others have shown some promise for this type of structure. In any case, we see that considerable work remains to be done in the field of damping of flat plates.

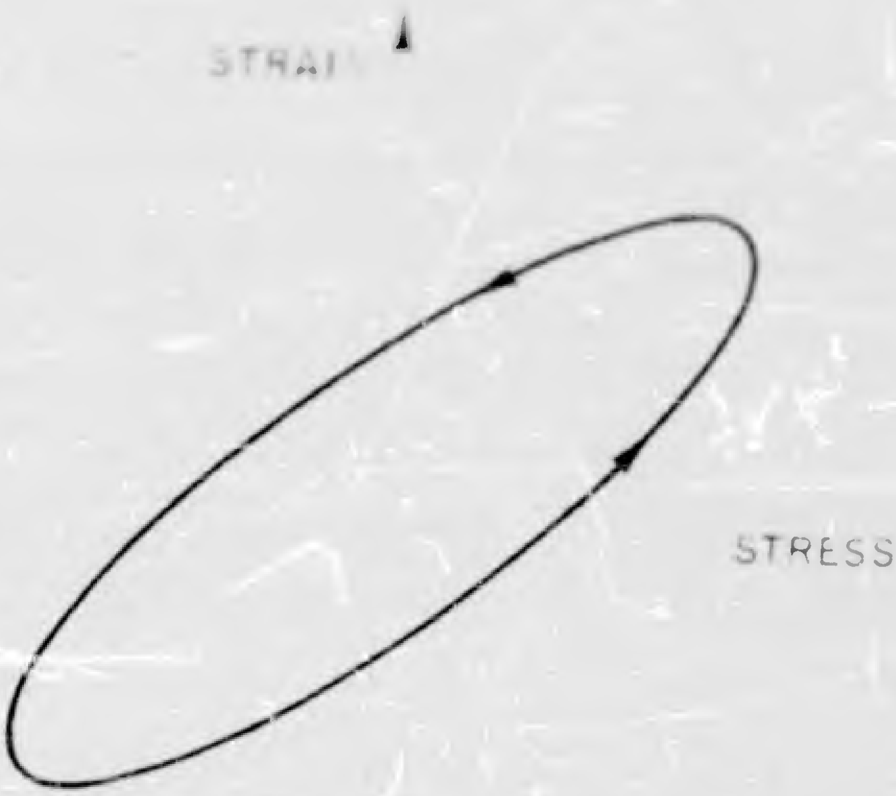


FIG. 1 STRESS-STRAIN ELLIPSE FOR SINUSOIDAL DEFORMATION OF A DAMPING MATERIAL

ENERGY LOSS PER CYCLE IS PROPORTIONAL TO THE AREA WITHIN THE ELLIPSE

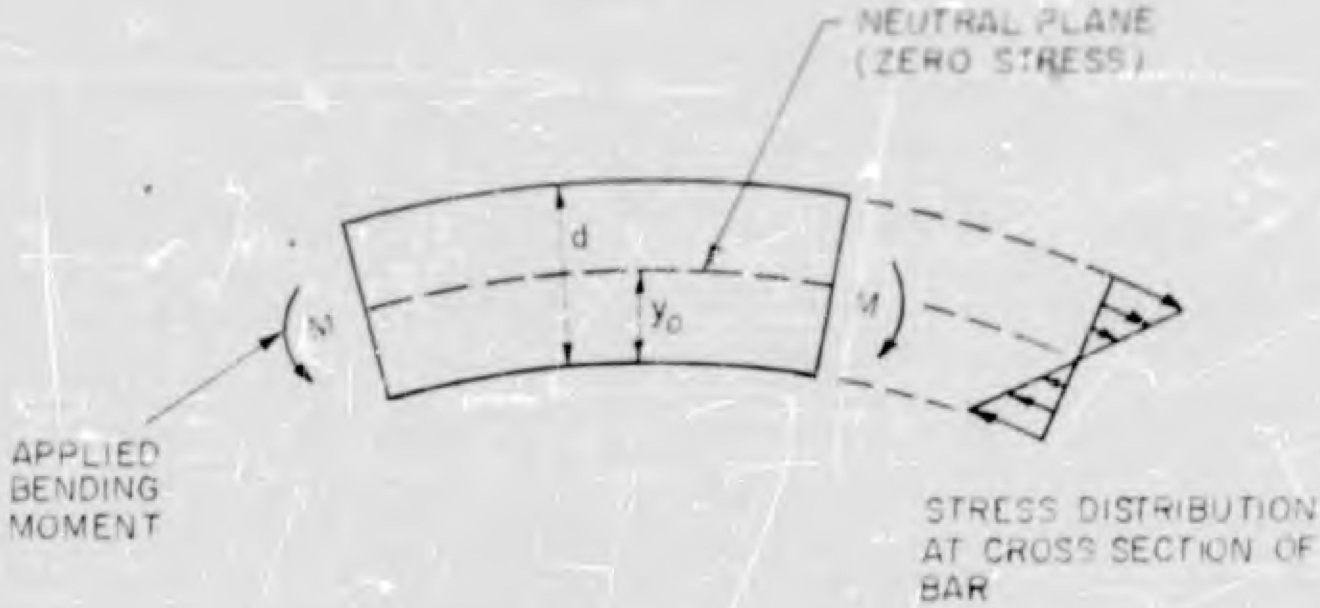


FIG. 2 ELEMENT OF A BAR IN PURE BENDING

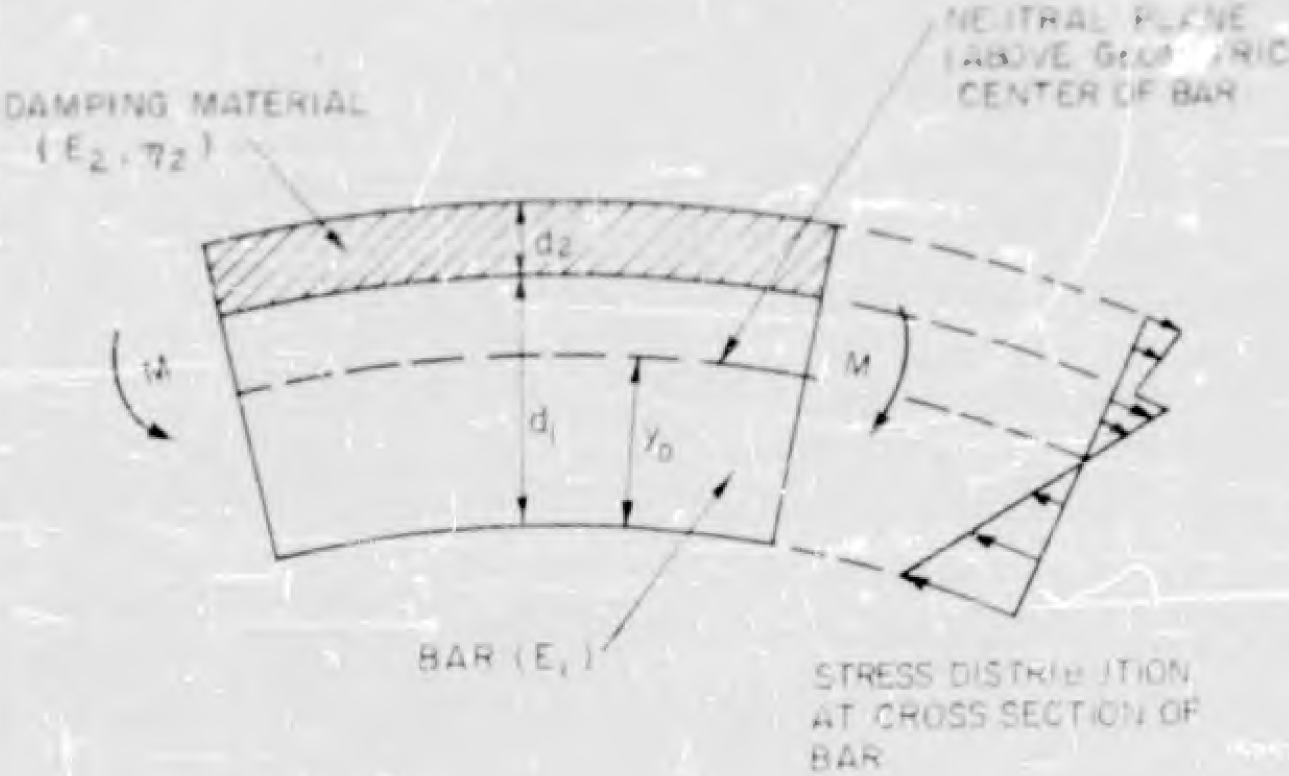


FIG 3 ELEMENT OF A DAMPED BAR IN PURE BENDING

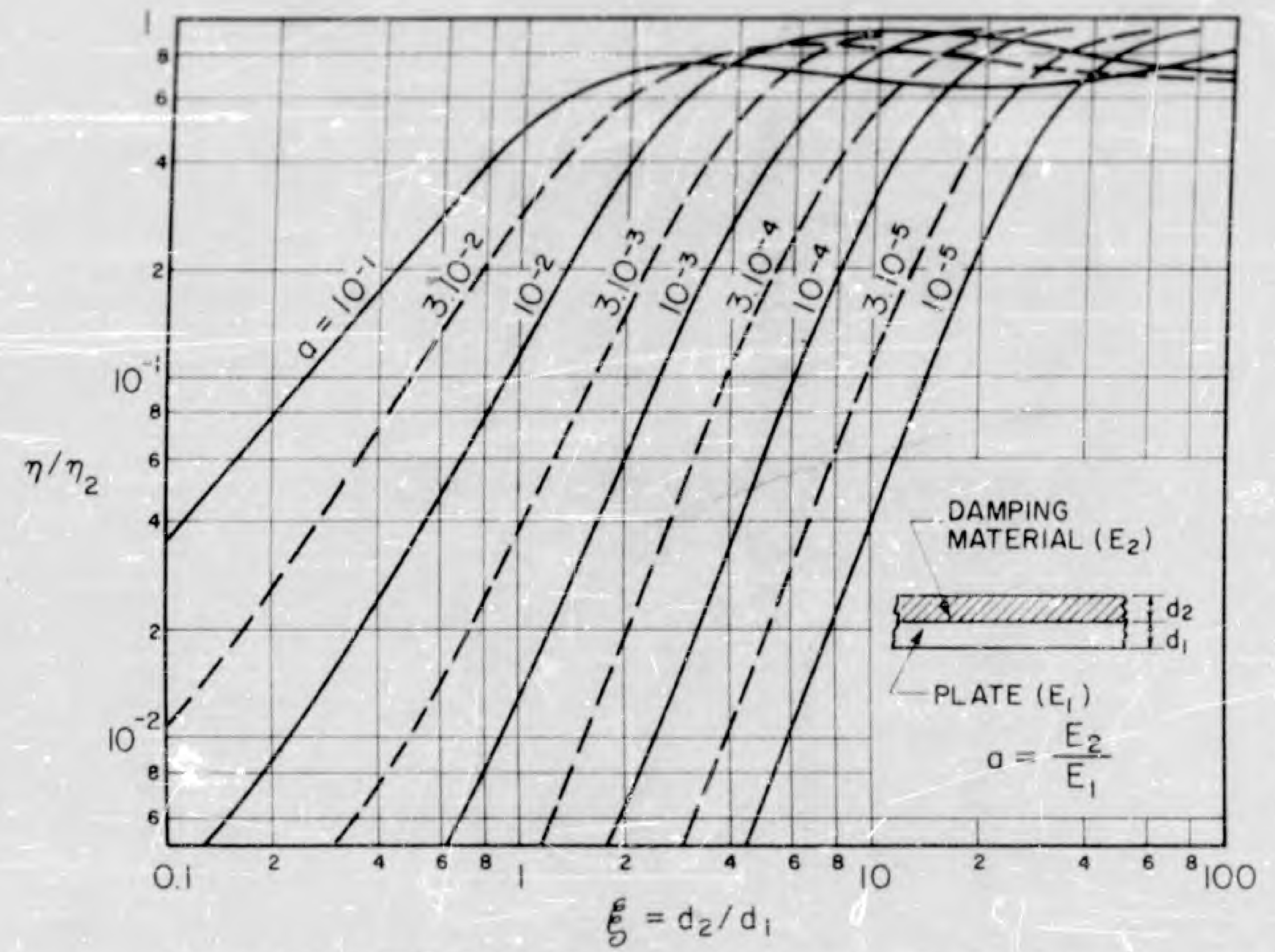


FIG. 4 DEPENDENCE OF DAMPING RATIO, η/η_2, FOR A DAMPED PLATE ON THICKNESS RATIO, $\xi = d_2/d_1$, AND ON RATIO OF ELASTIC MODULI, $a = E_2/E_1$

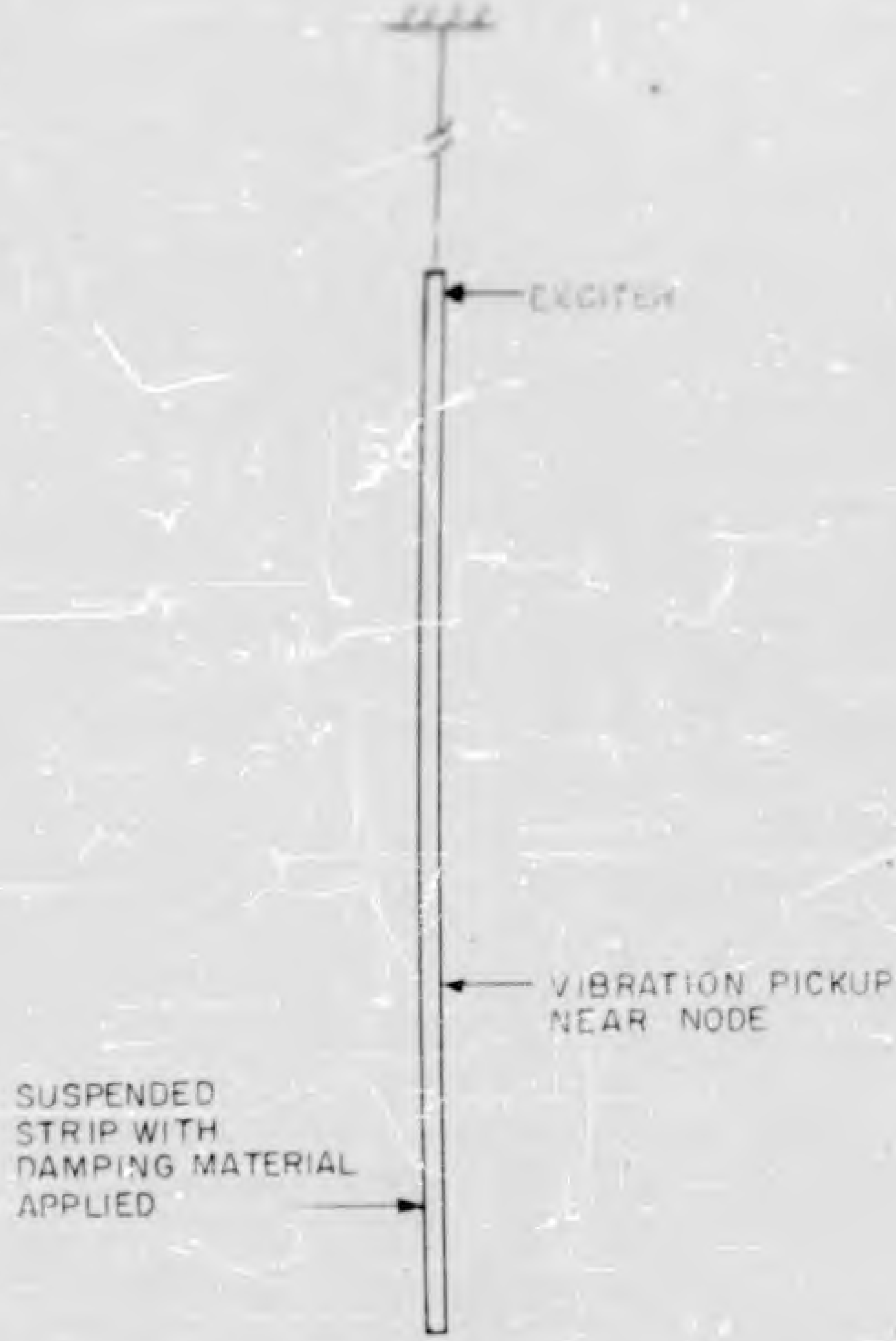
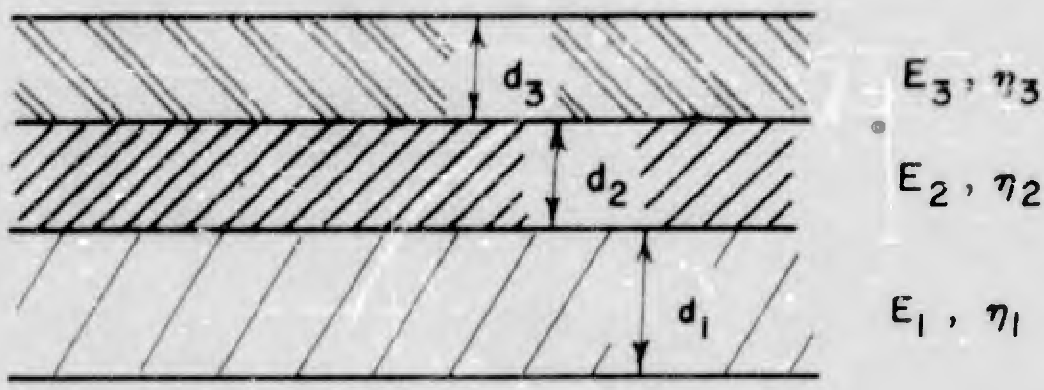
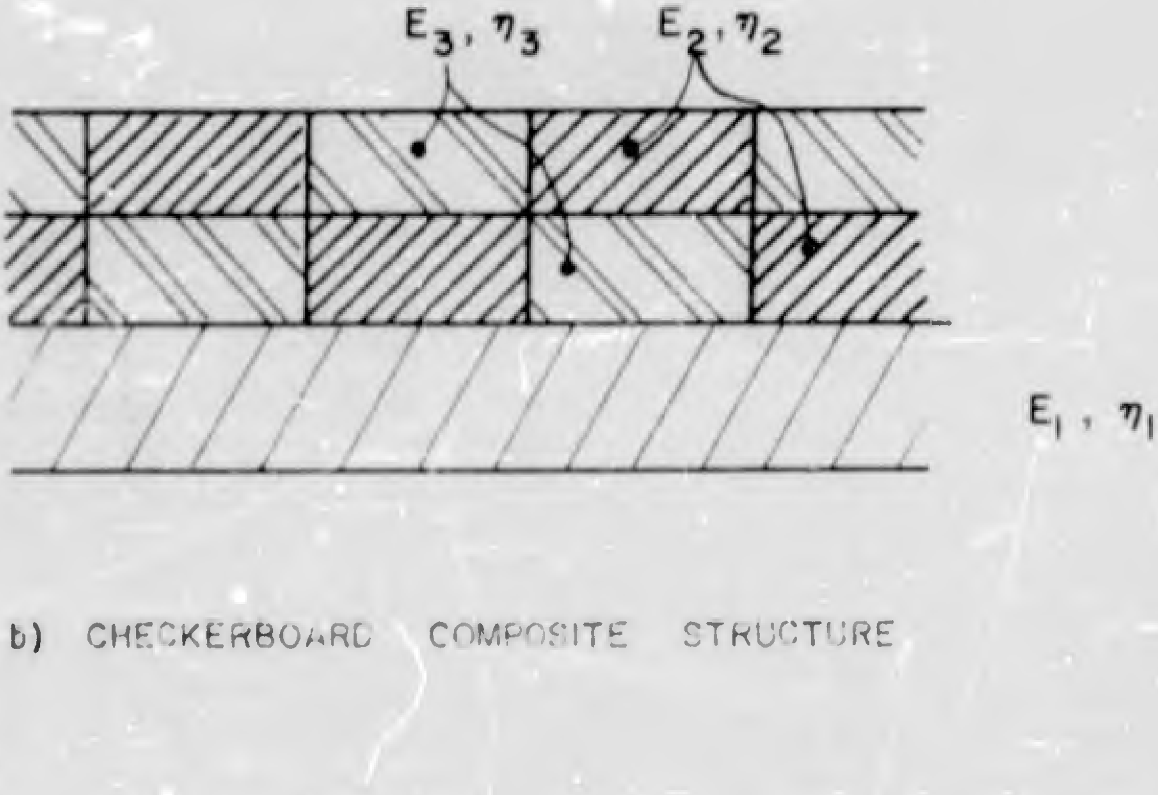


FIG. 5 STANDING WAVE APPARATUS FOR STUDYING DAMPING MATERIALS ON A METAL STRIP

FIG. 6 ARRANGEMENT FOR MEASURING ATTENUATION OF PROGRESSIVE BENDING WAVES IN A BAR



a) THREE-LAYER COMPOSITE STRUCTURE



b) CHECKERBOARD COMPOSITE STRUCTURE

FIG. 7 COMPOSITE DAMPING TREATMENTS