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INSTITUTE FOR MATHEMATICS
AND MECHANICS

Underwater Explosion Bubbles

I. The Effect of Compressibility of the Water

by JOSEPH B. KELLER and IGNACE I. KOLODNER

Prepared under
Contract No. Nonr-285(02)
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I. The Effect of Compressibility of the Water

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This report represents results obtained at the Institute for Mathematics and Mechanics, New York University, under the auspices of Contract Nonr-285(02) with the Office of Naval Research. It was originally presented at the Fourth Conference on Progress in Research on Ship Protection against Underwater Explosions in December 1951.

NEW YORK, 1953

UNDERWATER EXPLOSION BUBBLES

I. The Effect of Compressibility of the Water

Joseph B. Keller and Ignace I. Kolodner

1. Introduction

As soon as an explosive detonates, it is converted into a gas at high pressure. In an underwater explosion this gas is called the "bubble". It expands rapidly until its pressure falls below that of the surrounding water and it is then compressed again to a high pressure, after which the cycle of expansions and contractions is repeated periodically with diminishing amplitude. Corresponding to the pressure variations in the bubble, there are pressure waves transmitted outward into the water. The first pressure wave is a shock. These waves are responsible for the damage caused by the explosion. As the bubble oscillates it also rises and flattens due to gravity and the influence of neighboring surfaces, and it ultimately breaks up or vents.

The phenomena described above have been the subject of extensive theoretical and experimental investigations [1]. In the series of reports of which this is the first, we intend to present various improvements which have been made in the theory of underwater explosion bubbles. At the same time we feel that these reports will present accurately the theory of this subject in its present form.

The simple theory of underwater explosion bubbles is based on three main assumptions: 1) The bubble is assumed to be spherical at all times, 2) the pressure is assumed to be the same at all points of the bubble at each instant of time, 3) the water is assumed to be incompressible. This theory yields two somewhat unsatisfactory results: 1) the bubble is found to perform undamped oscillations of constant period, 2) the pressure variations are found to propagate to all points of the water instantly.

In this report we eliminate these features by modifying the third assumption of the simple theory to take account of compressibility of the water. We then find damped oscillations of diminishing period, and finite propagation speed for the pressure waves in the water. The theory and results are essentially the same as those of Zoller [3], with some additions.

In section 2 the problem is formulated mathematically, and this formulation involves one novel feature which may be of more general interest. This is the use of the wave equation, rather than the usual Laplace equation, for the potential function in the water. It is surprising that although the wave equation is closer to the exact non-linear equations for the potential than is the Laplace equation, still it is widely held that the wave equation is applicable only to small motions, while Laplace's equation is employed for large motions.

A demonstration of the applicability of the wave equation to large motions has been given by G. I. Taylor [2], who compared the exact solution for the flow produc a uniformly expanding sphere with the approximate solu given by the wave equation. The agreement was excellent, even for very large velocities of the sphere, while the corresponding solution based on Laplace's equation misses some of the main features of the flow. Because of the similarity of Taylor's problem to the present one it seems clear that the wave equation should also yield a good solution in this case, and this expectation is borne out by the results.

In section 3 the mathematical problem is solved and in section 4 the direction field which characterizes the result, is analyzed. Sketches of this field, and the corresponding one obtained by specializing the result to an incompressible fluid, are given. A graph of bubble radius versus time based on a numerical integration for the compressible case, is also given and compared with an experimental curve. Graphs are also given from which the pressure in the water can be determined as a function of position and time.

In section 5 an analytic solution of the linearized equation for the bubble radius is given. This applies to a bubble whose pressure is not much different from the surrounding hydrostatic pressure.

Section 6, the conclusion, discusses the results in the light of other work on this subject. In the Appendix the wave equation is derived.

2. Formulation

We consider a sphere of gas (the bubble) of initial radius A_0 surrounded by an unbounded fluid (the water) initially at rest. We assume that the bubble remains spherical at all later times, that the pressure throughout the bubble is constant at each instant of time τ and that the pressure $\Pi(\tau)$ in the bubble is related to its radius $A(\tau)$ by the adiabatic relation

$$\Pi(\tau) = K\left(\frac{4\pi}{3} A^3(\tau)\right)^{-\gamma} \quad . \tag{1}$$

We further assume that the water velocity is derivable from a potential function $\Phi(R,\tau)$ which depends only upon radial distance R from the bubble center and time τ, and satisfies the wave equation (see Appendix)

$$\Delta\Phi - \frac{1}{C^2}\Phi_{\tau\tau} = 0 \quad . \tag{2}$$

Here C is the sound speed in the fluid, assumed to be constant. The pressure $P(r,\tau)$ in the water is given by the Bernoulli equation

$$P(r,\tau) = P_0 - \rho(\Phi_\tau + \frac{1}{2}\Phi_R^2) \quad . \tag{3}$$

Here ρ is the density of the fluid and P_0 is its initial pressure.

At the bubble surface, pressure must be continuous and the rate of change of bubble radius must equal the particle velocity in the water. Thus we have

$$(4) \qquad K\left(\frac{4\pi}{3}A^3(\tau)\right)^{-\gamma} = P_o - \rho\left(\Phi_\tau + \frac{1}{2}\Phi_R^2\right) \qquad \text{at} \quad R = A(\tau)$$

$$(5) \qquad \Phi_R = A_\tau \qquad \text{at} \quad R = A(\tau) \quad .$$

The problem is to solve (2), (4), (5) for $\Phi(R,\tau)$ and $\dot{A}(\tau)$ given that

$$(6) \qquad A_\tau(0) = \dot{A}_o, \; A(0) = A_o; \; \Phi(R,0) = 0, \; R \geq A_o; \; \Phi_\tau(R,0) = 0, \; R > A_o .$$

In the simple theory C is infinite so that $\frac{1}{C^2}\Phi_{\tau\tau}$ in (2) does not appear--the problem is otherwise the same. We will obtain the solution in that case as a specialization of our result.

Before solving this problem, we will introduce dimensionless variables as follows.

$$(7) \qquad A = aL, \quad \tau = Tt, \quad \Phi = L^2T^{-1}\phi, \quad R = Lr, \quad C = LT^{-1}c, \quad A_o = a_oL$$

where

$$L = \left[\frac{3}{4\pi}\left(\frac{P_o}{K}\right)^{-1/\gamma}\right]^{1/3}, \qquad T = L\sqrt{\rho P_o^{-1}} \; .$$

With these changes, (2), (4), (5), (6) become, with $\dot{a} \equiv a_t$.

$$(8) \qquad \Delta\phi - \frac{1}{c^2}\phi_{tt} = 0$$

$$(9) \qquad \phi_t + \frac{1}{2}\dot{a}^2 + a^{-3\gamma} = 1 \qquad \text{at} \quad r = a$$

$$(10) \qquad \phi_r = \dot{a} \qquad \text{at} \quad r = a$$

$$(11) \qquad \dot{a}(0) = \dot{a}_o, \; a(0) = a_o; \; \phi(r,0) = 0, \; r \geq a_o; \; \phi_t(r,0) = 0, \; r > a_o \; .$$

3. Solution

To solve (8)-(11) we choose a particular solution of (8), namely

$$(12) \qquad \phi = \frac{f(\zeta(t,r))}{r}, \qquad \zeta = t - \frac{r - a_o}{c} \quad .$$

Then (9), (10) become

$$(13)\quad a^{-1}f'(\zeta(t,a)) + \tfrac{1}{2}\dot{a}^2 + a^{-3\gamma} = 1$$

$$(14)\quad \dot{a} = -a^{-2}f(\zeta(t,a)) - (ac)^{-1}f'(\zeta(t,a)).$$

Eliminating $f'(\zeta(t,a))$ between (13), (14) and differentiating the result with respect to t yields

$$(15)\quad (\dot{a} - c)f'(\zeta(t,a)) + [(\tfrac{1}{2}\dot{a}^2 + a^{-3\gamma} - 1 - c\dot{a})a^2]^{\cdot} = 0 \quad .$$

Now eliminating $f'(\zeta(t,a))$ between (13), (15) and simplifying yields

$$(16)\quad (\dot{a} - c)(+a\ddot{a} + \tfrac{3}{2}\dot{a}^2 - a^{-3\gamma} + 1) - (\dot{a}^2 + (3\gamma - 2)a^{-3\gamma} + 2)\dot{a} = 0 \quad .$$

Equation (16) is a non linear second order ordinary differential equation for $a(t)$, and the initial data are given in (11). Once $a(t)$ is known, f and f' can be determined from (13), (14), which yield

$$(17)\quad f(\zeta(t,a)) = -a^2\dot{a} + \frac{a^2}{c}\left(\frac{\dot{a}^2}{2} + a^{-3\gamma} - 1\right)$$

$$f'(\zeta(t,a)) = -a\left(\frac{\dot{a}^2}{2} + a^{-3\gamma} - 1\right) \quad .$$

Since, from (11), $\phi(a_o, 0) = 0$ we must have $f(0) = 0$. Using this in (17) yields

$$(18)\quad \dot{a}_o = c \pm \sqrt{c^2 + 2(1 - a_o^{-3\gamma})} \quad .$$

As $\dot{a}_o$ must be less than c, we must choose the minus sign in (18). Then $\dot{a}_o \gtrless 0$ according as $a_o \lesseqgtr 1$. There is no real value of $\dot{a}_o$ if $a_o < \left(\frac{c^2}{2} + 1\right)^{-1/3\gamma}$, i.e. if a_o is too small, in which case the present theory fails since the initial bubble pressure is too high. If c is large compared to $2(1 - a_o^{-3\gamma})$ then (18) becomes

$$(19)\quad \dot{a}_o \approx \frac{-1 + a_o^{-3\gamma}}{c} \quad .$$

We note that f becomes $-a^2\dot{a}$ and $\dot{a}_o$ becomes 0 when c is infinite.

Since t does not occur explicitly in (16), we may reduce the equation to first order by defining

(20) $$v(a) = \dot{a} \quad .$$

Then (16) becomes

$$v'(a) = \frac{h(v,a)}{av(c-v)} \quad ,$$

(21)
$$h(v,a) = \tfrac{1}{2}(v^3 - 3v^2c - 2v - 2c) - a^{-3\gamma}(3\gamma - 1)(v - \frac{c}{3\gamma-1}) \quad .$$

In terms of v(a), the solution is given by

(22) $$t = \int_{a_0}^{a} \frac{da}{v(a)} \quad .$$

Thus we must solve (21) for v(a), then obtain a(t) from (22) and finally determine f from (17).

To calculate the pressure P(r,t) we have from (3), (12)

(23) $$P(r,t) = P_0[1 - (\frac{f'}{r} + \frac{f'^2}{2c^2r^2} + \frac{ff'}{cr^3} + \frac{f^2}{2r^4})] \quad .$$

The argument of f in (23) is $\zeta(t,s)$. However in (17) f is given in terms of argument $\zeta(t,a(t))$. To find $f(\zeta(t,r))$, we define $f(\zeta(t,a)) = g(t)$, and g is then given by the right side of (17). Then if we choose x such that

(24) $$a(x) - cx = r - ct$$

we have

$$f(\zeta(t,r)) = g(x) \quad .$$

Now (23) may be rewritten as

(25) $$P(r,t) = P_0 - \rho L^2 T^{-2} \left[\frac{-c\dot{g}}{(\dot{a} - c)r} - \frac{g\dot{g}}{(\dot{a} - c)r^3} + \frac{g^2}{2r^4} + \frac{\dot{g}^2}{2(\dot{a} - c)^2 r^2} \right] \quad .$$

In (25) the argument of g and of $\dot{a}$ is x, given by (24). When changes which occur during the time $\frac{a}{c}$, which is that required for a sound wave to travel one bubble radius, can be neglected, (24) yields

$$(26) \qquad x = t - \frac{r}{c} + \frac{a(x)}{c} \approx t - \frac{r}{c} \quad .$$

For large values of c, where only the first term in (17) need be retained for $g(t)$, (25) becomes

$$(27) \quad P(r,t) \approx P_o - \rho L^2 T^{-2} \left[\frac{-(a^2\dot{a})^{\cdot}}{r(1 - \frac{\dot{a}}{c})} + \frac{[(a^2\dot{a})^2]^{\cdot}}{2r^3(c-\dot{a})} + \frac{(a^2\dot{a})^2}{2r^4} + \frac{[(a^2\dot{a})^{\cdot}]^2}{2r^2(\dot{a}-c)^2} \right]$$

$$\approx P_o - \rho L^2 T^{-2} \left[\frac{-(a^2\dot{a})^{\cdot}}{r} + \frac{(a^2\dot{a})^2}{2r^4} \right] \quad .$$

The second expression in (27) has the same form as that given by the incompressible theory ($c = \infty$) but here the function $a(t)$ is different from that of the incompressible theory. Also the argument of a and $\dot{a}$ is x instead of t, showing that the pressure pulse spreads with finite velocity.

The first term in either bracket above is important when $\dot{a}$ is changing rapidly, which occurs near bubble minima, and thus is mainly responsible for the pressure in the primary shock wave and the subsequent pressure peaks. The factor $(1 - \frac{\dot{a}}{c})$ in the denominator of this term in the first form of (27) may increase this term somewhat over the value it would have in the incompressible theory, where this factor is unity. The last term in the above brackets is due to the motion of the water and is called the "afterflow" pressure. It is most important between pressure peaks, or between bubble minima.

4. The Direction Field; Results

To determine the qualitative behavior of the solution $v(a)$ of (21) we will now examine the direction field in the v,a plane determined by the right side of (21). We first note that this field has just two singular points--a saddle point at $a = 0$, $v = \frac{c}{3\gamma-1}$ and a spiral point at $a = 1$, $v = 0$. We next observe that $v'(a)$ becomes infinite on the lines $a = 0$, $v = 0$ and $v = c$ and it becomes zero on the line $h(v,a) = 0$. It also changes sign on crossing these lines. This information suffices to determine the topological character of the direction field. In order to present a more precise sketch of this field, we will first examine the curve $h(v,a) = 0$.

The equation $h(v,a) = 0$ may be rewritten as

$$a^{3\gamma} = 2(3\gamma-1)\left(v - \frac{c}{3\gamma-1}\right)\left[(v-c)^3 - (2+3c^2)(v-c) - 4c - 2c^3\right]^{-1} \quad (28)$$

The denominator has only one real root, and thus a becomes infinite for only one value of v, provided that $-(\frac{2}{3} + c^2)^3 + (2c+c^3)^2 > 0$. This is satisfied if $c^2 > \frac{4}{9}(\sqrt{3} - \frac{3}{2}) = .103$, which we assume to be the case. The root is then easily seen to lie in the interval $3c < v < 3c(1 + \frac{8}{27c^2})$. Furthermore a, given by (28) has only one maximum when $\gamma < \frac{4}{3}$. With this information, assuming that $c^2 > .103$ and $\frac{4}{3} > \gamma \geq 1$, the direction field has the topological behavior indicated in figure 1. We see that all solutions with initial velocity $v = 0$ and $0 < a < 1$ increase to a maximum, then decrease to a minimum, and perform damped oscillations approaching $a = 1$, $y = 0$. This is exactly the behavior observed in practice. When c is infinite (19) becomes the equation of the simple incompressible theory, and has only a vortex point at $v = 0$, $a = 1$. The direction field for this case is shown in figure 2. All solutions are periodic, undamped oscillations.

We have determined $a(t)$ for a particular set of initial data by numerical integration of (16). The result of this calculation is shown in figure 3 along with a set of measured values of the bubble radius obtained experimentally. These experimental points were taken from [1], p. 271, fig. 8.1, and apply to a .55 lb tetryl charge fired at 300 feet below the surface of water.

To carry out the integration we must know A_0, P_0, K, γ, ρ and C, or an equivalent set of data, see eqs. (7) and (18). From the experimental data we estimated that the ultimate bubble radius in our case would be 6 inches which we therefore chose as the unit of length L. The first maximum bubble radius was measured to be 18 inches, which becomes 3 in terms of our unit of length. Since the measured data was poor initially, we started the numerical integration from the first maximum using initial velocity 0 and initial radius 3, integrating both forwards and backwards in time from this point.

The values $\gamma = 1.25$, $\rho = 1$ g/cm^3, $C = 1{,}485$ m/sec were used. These yield,

$$c = 46.5$$

$$T = 4.83 \text{ milliseconds} \quad .$$

From the figure it seems that the theory agrees satisfactorily with the observations. It is in particular worthwhile to note that the calculated first oscillation period is 27.9 milli-seconds as compared with the reported experimental value of 28 milliseconds. Some of the discrepancy in the radius time curve is undoubtedly due to error in estimating the ultimate radius from the experimental curve. This not only affects the unit of length but also the initial radius.

Using the computed values of $a(t)$, the functions $f(\zeta)$ and $f'(\zeta)$ were computed from (17). Graphs of these functions are shown in Figures 4 and 5 respectively. These functions can be used to determine the pressure $P(r,t)$ which is given in (23). In fact, for large distances r, the excess-pressure vs. time curve is essentially a multiple of the f' curve. For very small distances, however, the other terms in eq. (23) may become important, especially between the bubble minima. Yet at $r = 6$, i.e. at a distance of two maximum bubble radii, the correction due to the neglected terms is at most .85 percent which occurs at $\zeta = 1.0$.

5. Small Oscillations

If the initial radius is nearly equal to 1, which is the equilibrium radius, we may write

(29) $$a(t) = 1 + b(t) \quad .$$

If we assume that b(t) is small, insert (29) into (16) and retain only linear terms in b, we obtain

(30) $$\ddot{b} + \frac{3\gamma}{c}\dot{b} + 3\gamma b = 0 \quad .$$

Thus

(31) $$b(t) = b(0)e^{-\frac{3\gamma}{2c}t}\cos\left(\sqrt{3\gamma - \left(\frac{3\gamma}{2c}\right)^2}\; t\right) \quad .$$

The solution (31) yields exponentially damped sinusoidal oscillations. During one period b decreases by the factor $e^{-\frac{\pi\sqrt{3\gamma}}{c}}$, if $\left(\frac{3\gamma}{2c}\right)^2$ is small compared to 3γ.

The energy in the bubble consists of internal energy and potential energy and is given by

(32) $$E = \frac{4\pi}{3}A^3\left[P_0 + \frac{K}{\gamma-1}\left(\frac{4\pi}{3}A^3\right)^{-\gamma}\right] = P_0\left(\frac{K}{P_0}\right)^{\frac{1}{\gamma}} a^3\left(1 + \frac{a^{-3\gamma}}{\gamma-1}\right) \quad .$$

Let us call the excess energy the difference between the actual energy and that corresponding to the equilibrium radius a = 1. Then for small oscillations, using (29) in (32), the excess energy is given by

(33) $$E = P_0\left(\frac{K}{P_0}\right)^{\frac{1}{\gamma}} \cdot \frac{9\gamma}{2}\, b^2(t) \quad .$$

From (31) and (33) we see that the excess energy decreases by the factor $e^{-\frac{2\pi\sqrt{3\gamma}}{c}}$ during each period.

In Table I, are presented some calculations based on the small oscillation theory just described. The dimensionless sound speed $c = C\sqrt{P_0^{-1}\rho}$ was computed for three depths 100, 200, and 1000 feet respectively. The sound speed C = 1465 meters/sec and density ρ = 1 gm/cm^3 were used.

Table I

Calculations Based on Small Oscillation Theory

$\gamma = 1.25$

Dimensionless sound speed c	Decrease in amplitude per period $e^{-\frac{\pi\sqrt{3\gamma}}{c}}$	Number of periods for amplitude to be halved	Period correction factor $(1 - \frac{3\gamma}{4c^2})^{-1/2}$
72	.919	8.2	1.00009
55	.894	6.3	1.00016
26	.791	3.0	1.0074

6. Conclusion

The present theory of a spherical bubble takes account of the compressibility of the water. Consequently it yields damped radial oscillations of the bubble and finite propagation speed for the pressure pulses in the water, in agreement with observation. The theory is essentially the same as that of Zoller [3], from which it differs in minor details. The feature which makes the theory simple analytically is the use of the wave equation for the water motion.

Zoller computed the pressure in the water and compared it with more exact calculations based on numerical integration of the non linear fluid dynamic equations in the water. These calculations were also based on the assumption of uniform pressure through the bubble. The agreement was excellent near the bubble, which was the only region considered. He also compared the radius-time curves obtained by these two different calculations and they also agreed well, but the time over which they were extended was not very long.

Our calculations constitute another check on this theory. We have computed the radius-time curve for nearly four oscillations, and have found it to check very well with an experimental curve corresponding to the same initial conditions. Thus we may conclude that in water this theory is adequate to describe the bubble oscillations and the pressure in the water near the bubble.

It is particularly important to point out that in these calculations all the energy is accounted for. Since the radius-time curve checks with experiment, we conclude that all the energy in the experimental situation is similarly accounted for. Thus the question of "missing energy", which was raised in recent years seems unnecessary.

It should also be pointed out that certain compressibility effects were taken into account by Herring [4] in an attempt to estimate energy loss from the bubble. More recently Trilling [5] also considered compressibility of the water in showing that wave motion within the bubble did not modify the bubble motion during the first half-period from that computed on the basis of uniform pressure throughout the bubble.

By examining the period for the first oscillation, we find that it does not differ appreciably from that given by the incompressible theory since c is large. Therefore calculations of the period based on the incompressible theory, such as those of Friedman [6], can still be employed for the first period. The subsequent periods will differ because of energy losses due to compressibility.

Appendix: Derivation of the Wave Equation

We consider the non-viscous motion of a compressible fluid in which pressure is a function of density alone.*
The equations of motion for such a fluid are

(A1) $$\rho_t + \nabla \cdot (\rho u) = 0$$

(A2) $$u_t + (u \cdot \nabla) u + \rho^{-1} \nabla p = - \nabla \Omega$$

(A3) $$\rho = \rho(p) \quad .$$

In these equations ρ is the density, p the pressure, u the velocity vector, and Ω the potential of external forces per unit mass. We further assume that the motion is irrotational, which implies the existence of a velocity potential ϕ such that

(A4) $$u = \nabla \phi \quad .$$

From (A2), (A4) we have the Bernoulli equation, in which $F(t)$ is arbitrary:

(A5) $$\tfrac{1}{2}(\nabla\phi)^2 + \int_{p_o}^{p} \frac{dp}{\rho} + \phi_t + \Omega = F(t) \quad .$$

Now differentiating (A5) with respect to t yields

(A6) $$\nabla\phi \cdot \nabla\phi_t + \rho^{-1} p_t + \phi_{tt} + \Omega_t = F_t \quad .$$

With the use of (A3), the derivatives of ρ appearing in (A1) may be eliminated in terms of derivatives of p. These in turn may be eliminated by means of (A2) and (A6). We have, in fact

(A7) $$\rho^{-1}\rho_t = \rho^{-1}\rho_p p_t = \rho_p [F_t - \Omega_t - \phi_{tt} - \nabla\phi \cdot \nabla\phi_t]$$

---

*If pressure is a function of density alone, the flow is called barotropic. An isentropic flow is always barotropic. Thus if the entropy is initially constant, and if viscosity and heat-conduction are absent, the flow is isentropic and thus barotropic, in the absence of shocks.

(A8) $\rho^{-1}\nabla p = \rho^{-1}\rho_p \nabla P = \rho_p[-\nabla\Omega - u_t - (u\cdot\nabla)u]$.

Using (A7), (A8) in (A1) yields

(A9) $\nabla^2\phi - \rho_p\phi_{tt}$

$$= \rho_p[\Omega_t - F_t + \nabla\phi\cdot\nabla\Omega + \frac{1}{2}\frac{d}{dt}(\nabla\phi)^2 + \frac{1}{2}(\nabla\phi)^2_t] \; .$$

Equation (A9) is the exact non-linear equation for the velocity potential, in which ρ_p is to be eliminated by means of (A3) and (A5). When the right side is negligible and ρ_p constant we see that ϕ satisfies the linear wave equation with sound speed $c = \rho_p^{-1/2}$. If $\rho_p\phi_{tt}$ is also negligible, ϕ satisfies Laplace's equation.

In the present problem $\Omega = 0$, F_t can be set equal to zero, and the terms remaining on the right are derivatives of $\frac{\nabla\phi^2}{c^2}$. If we assume the speed $(\nabla\phi)$ is small compared to sound speed c, these terms can be neglected, yielding the wave equation. We also assume that ρ_p is constant which seems valid for the range of pressures we consider when the medium is water.

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Fig. 1

Direction field in the compressible case.

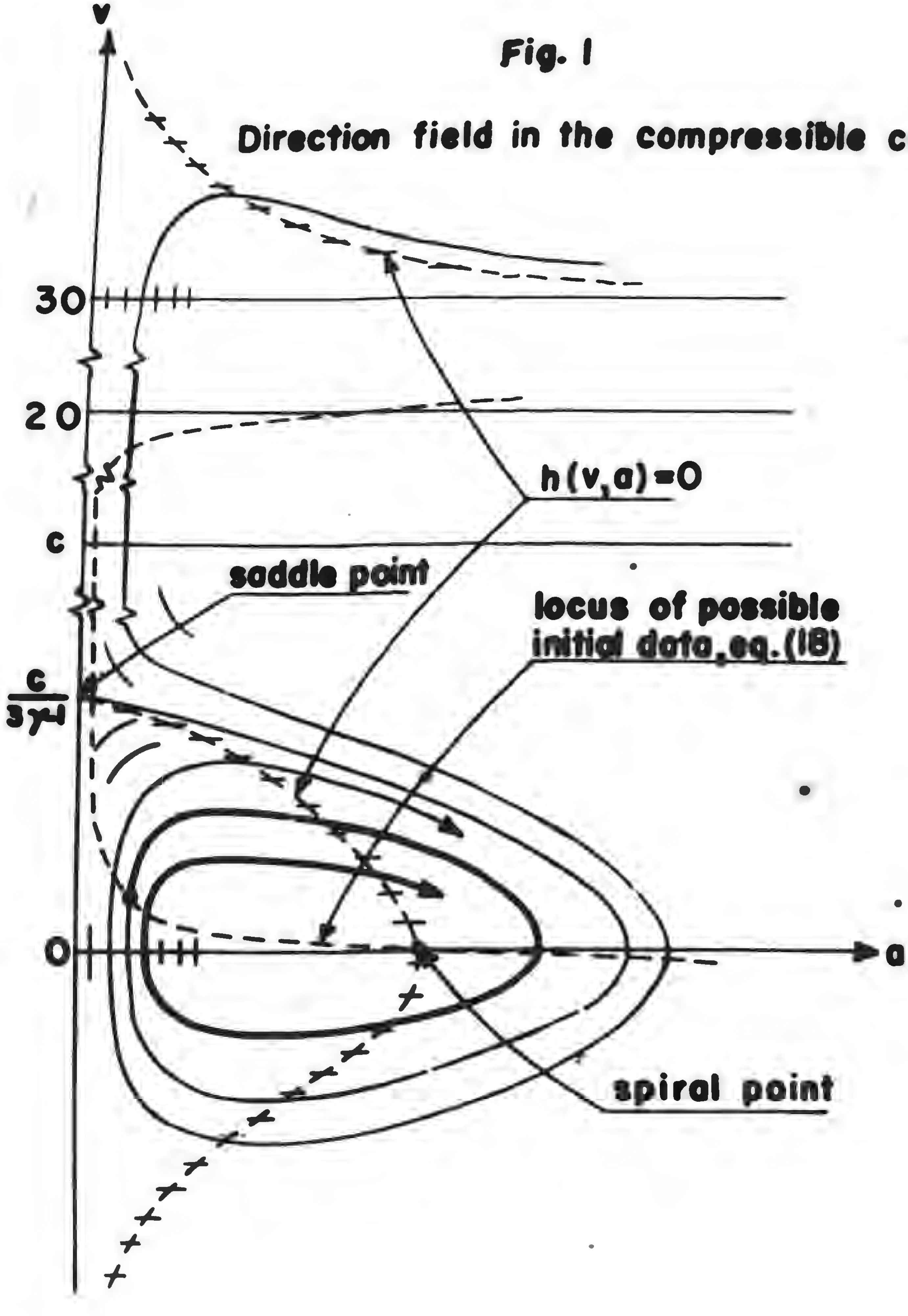
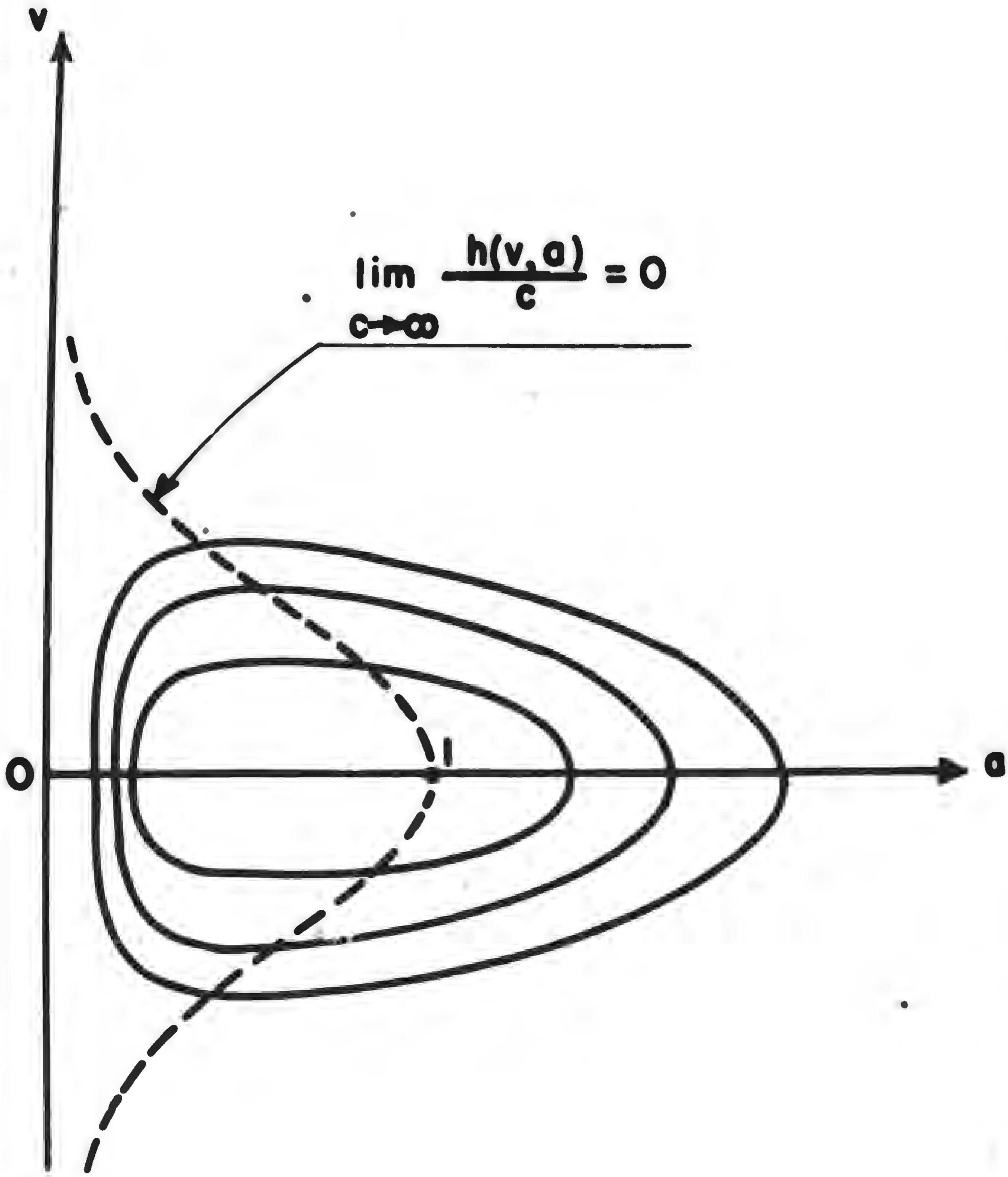
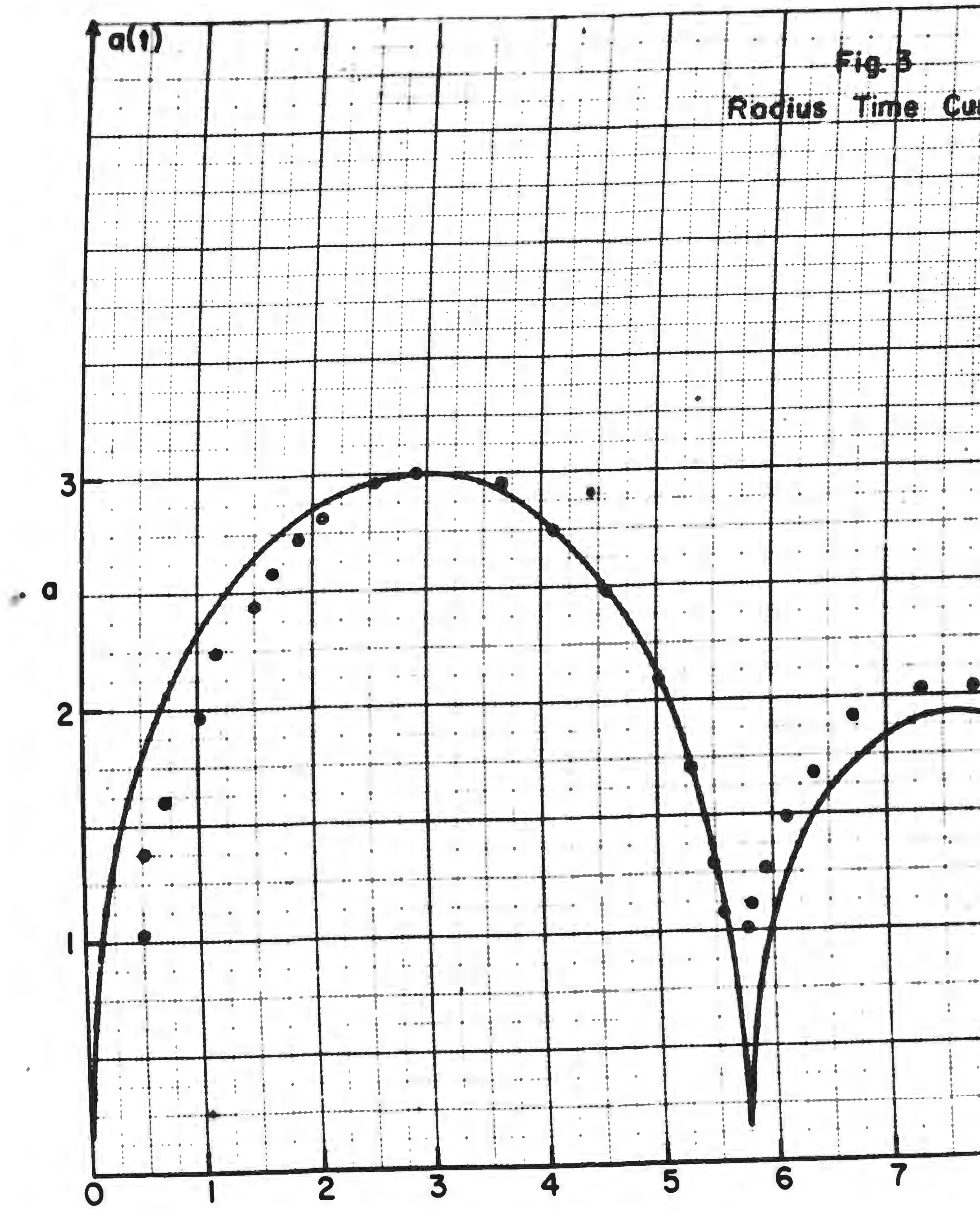
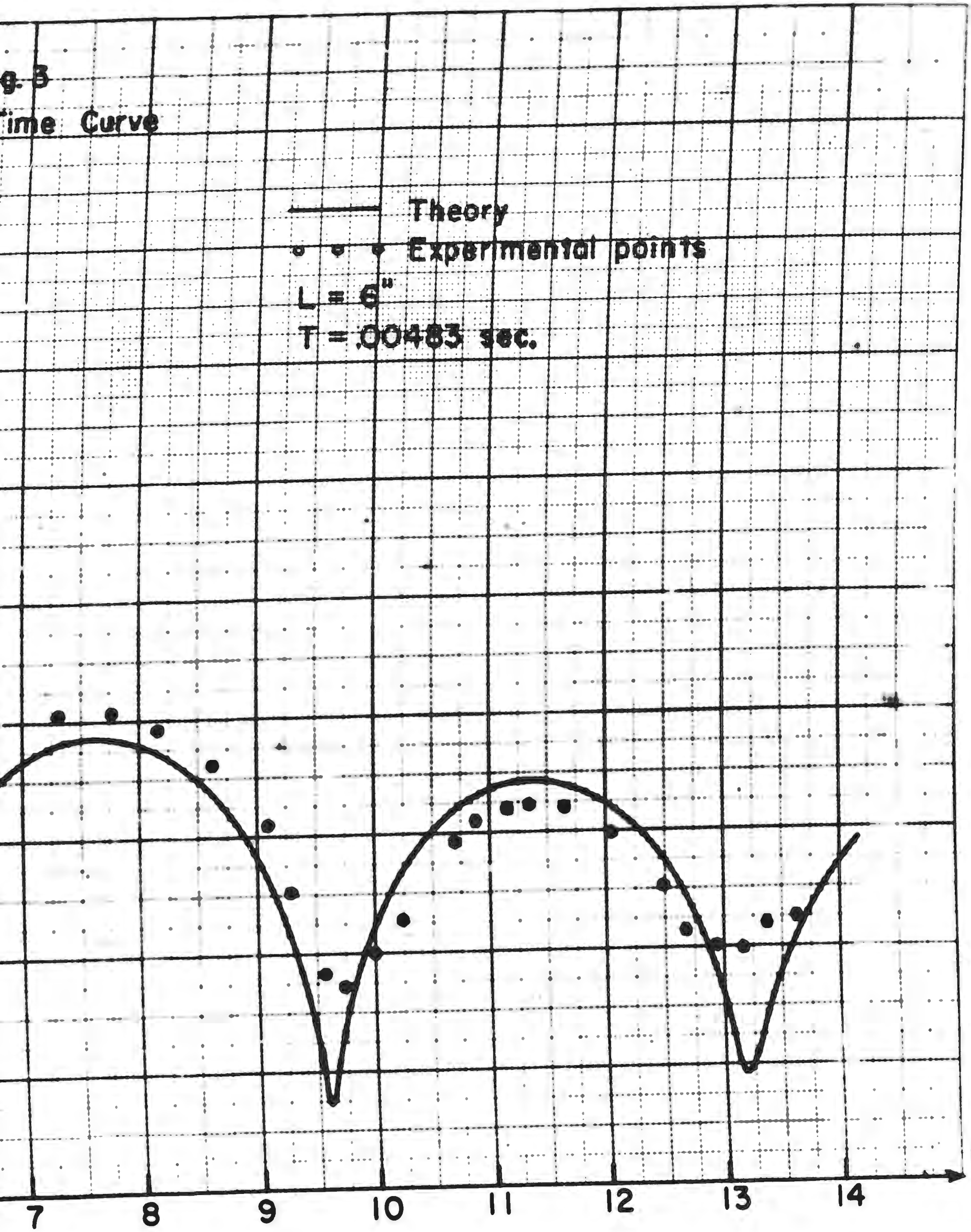


Fig. 2

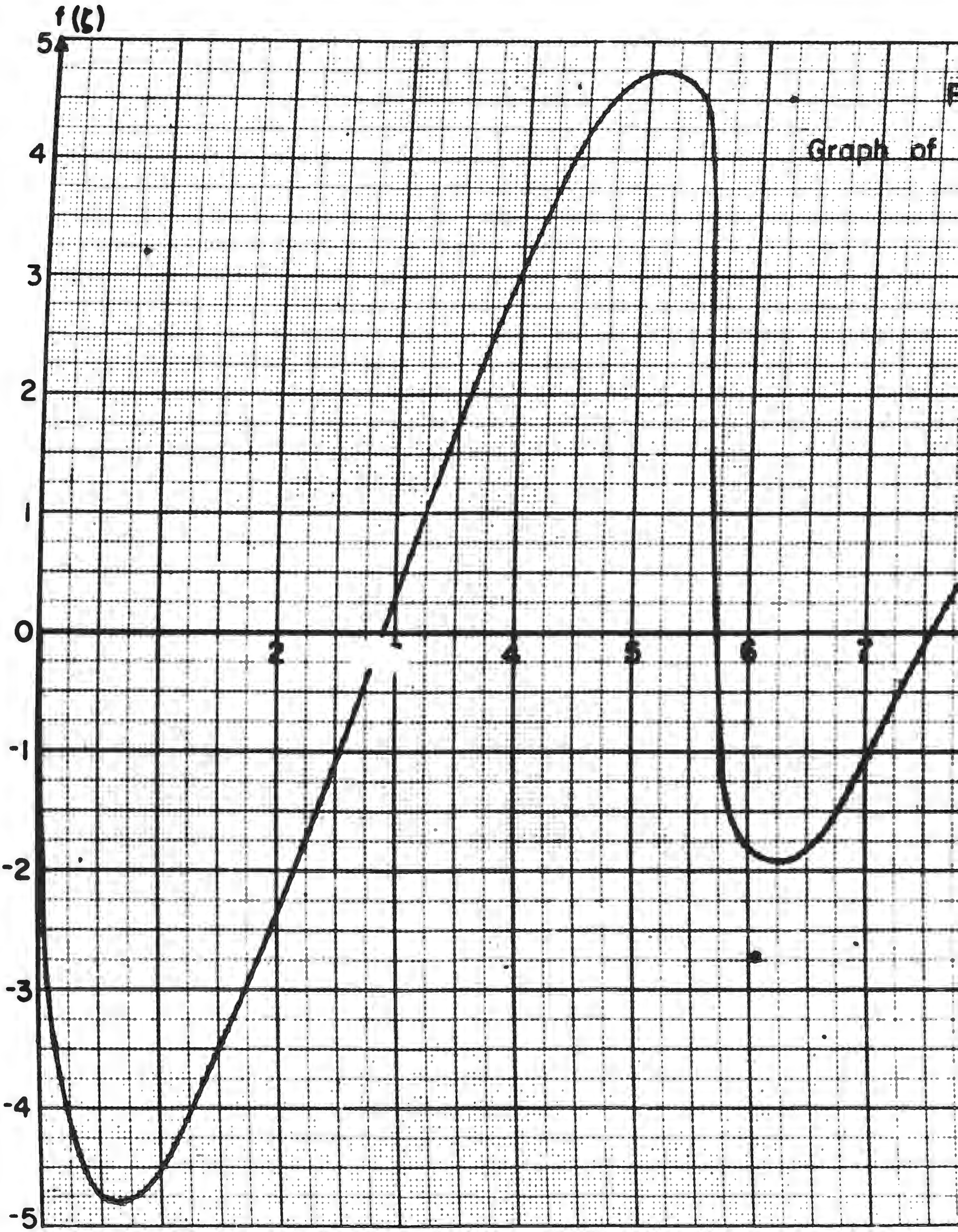
Direction field in the incompressible case.




a(t)
Fig. 3
Radius Time Cu
a
3
2
1
0
1
2
3
4
5
6
7



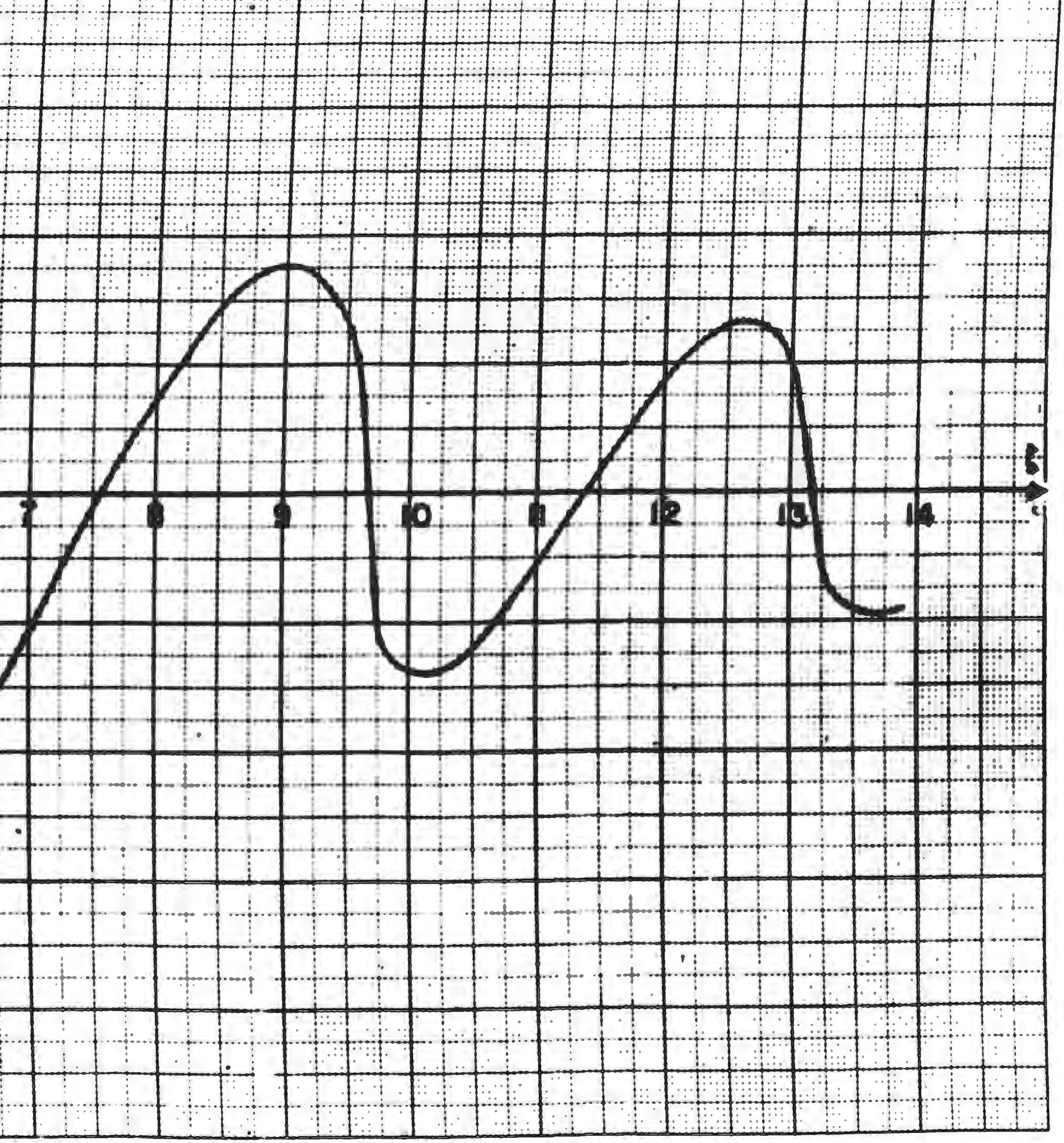
g. 3
Time Curve
Theory
Experimental points
L = 6"
T = .00483 sec.
7
8
9
10
11
12
13
14
t

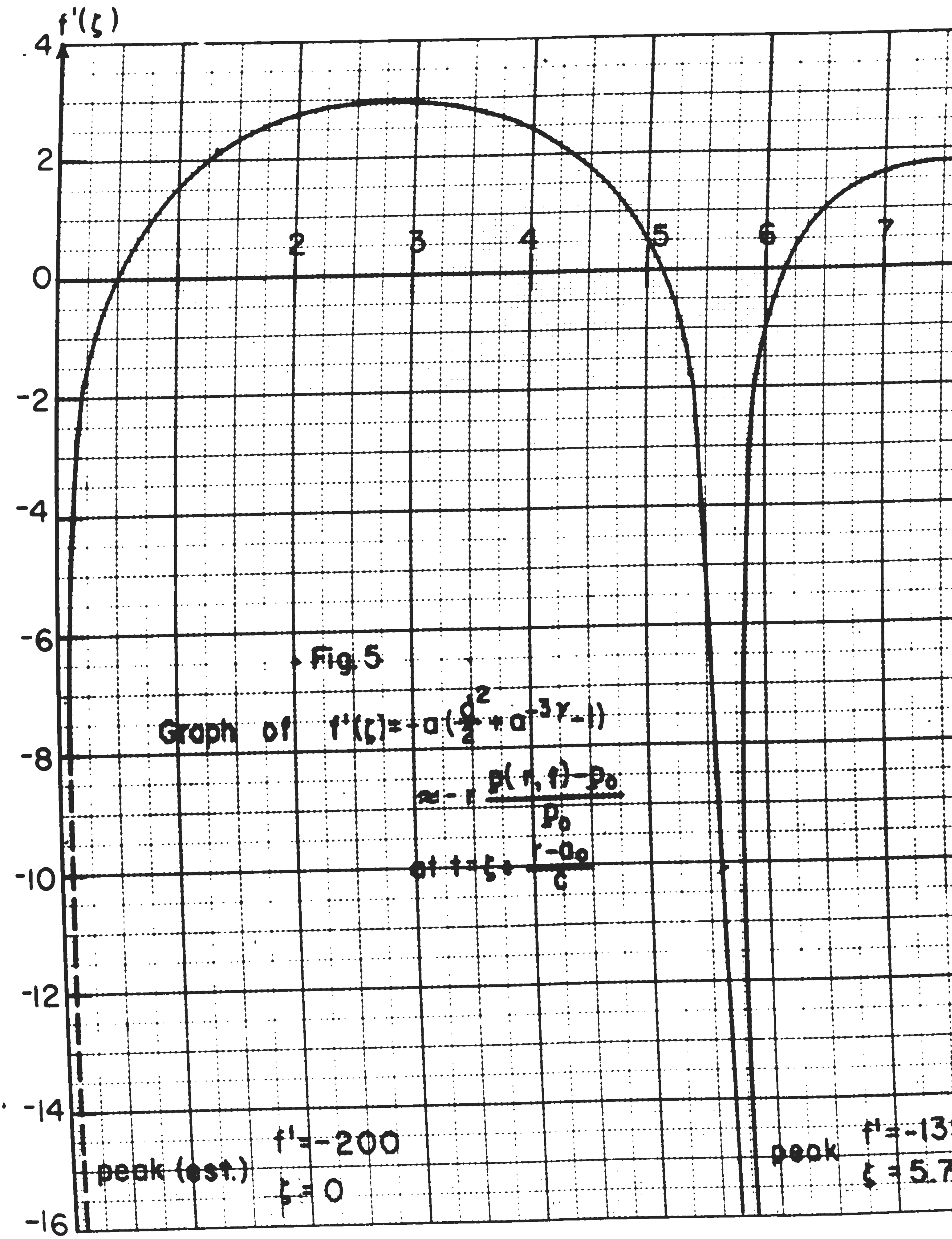


f(ζ)
5
4
3
2
1
0
-1
-2
-3
-4
-5
2
4
5
6
7
Graph of

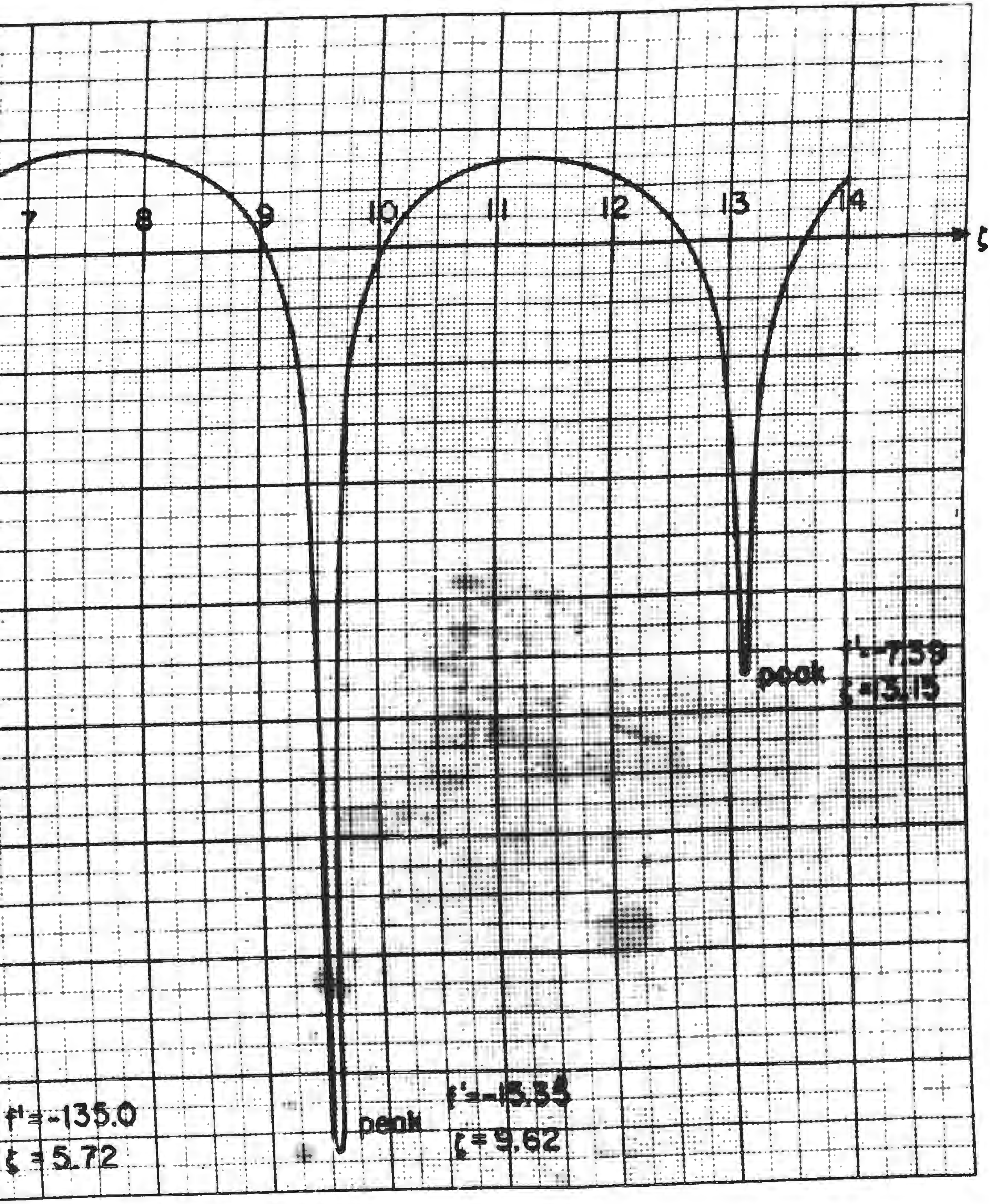
Fig. 4

...aph of $f(\xi) = a^2\dot{a} + \frac{a}{c}\left(\frac{1}{2}\dot{a}^2 + a^{-3\gamma} - 1\right)$





f'(ζ)
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
2
3
4
5
6
7
Fig. 5
Graph of $f'(\zeta) = -a\left(\frac{a^2}{2} + a^{-3\gamma} - 1\right)$
$\approx -r\,\frac{p(r,t)-p_0}{p_0}$
at $t = \zeta + \frac{r-a_0}{c}$
peak (est.)
f' = -200
ζ = 0
peak
f' = -13
ζ = 5.7



7
8
9
10
11
12
13
14
ζ
peak
ζ' = -7.39
ζ = 13.15
ζ' = -135.0
ζ = 5.72
peak
ζ' = -15.34
ζ = 9.62

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