TITLE: Eigenoscillations Near Cascade of Thin Disks Between the Pair of Parallel Planes

EIGENOSCILLATIONS NEAR CASCADE OF THIN DISKS BETWEEN THE PAIR OF PARALLEL PLANES

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ABSTRACT

Eigenoscillations near cascade of thin disks between the pair of parallel planes are studied. The sufficient conditions of existence for eigenoscillations are found. The dependence of eigenfrequencies on the geometric parameters of structure is investigated. It is shown that the frequencies of eigenoscillations of given structure are discrete and they formed a bundle of resonance frequencies which is under specific conditions act as a continuous band of resonance frequencies. The number of resonance bundles is calculated.

PROBLEM FORMULATION

The present work generalized the results obtained in [1] for periodic chain of thin disks to the structure which is shown on Fig.1. Mathematical statement of the problem based on the Helmholtz equation in cylindrical coordinate system, the assumption of rigidity of all boundaries together with the condition of local energy finiteness and could be write down in the following form:

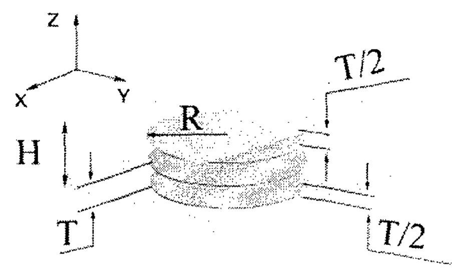

Fig.1. Region of oscillations and basic designations.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}+\lambda^2 u=0,\quad \left.\frac{\partial u}{\partial \bar{n}}\right|_{\Gamma}=0 \quad E(u)=\int\limits_{\Omega}\left[|u|^2+|\nabla u|^2\right]d\Omega<\infty \tag{1}$$

Here we assume that the process is steady-state, so the time dependence is taken to be *exp(-iωt)* and the solution is independent on polar angle, Γ is the boundary of domain of oscillations including walls and cascade of thin disks. The meaning of all other designations is evident form Fig.1.

GENERAL APPROACH

Owing to the fact that we have set the cascade of disks as a symmetrical array with respect to channel walls we can expand given problem (1) and solve it in all 3D space but with additional conditions (2) on imagine boundaries set up by the collection of parallel planes at $\Pi_k=\left\{(x,y,z): z=H/_{2T}+k\right\},\ k\in Z$

$$\frac{\partial u}{\partial \bar{n}}=0,\ \text{at}\ \Pi_k, k\in Z \tag{2}$$

The general form of a solution for this new problem in a free space ($x^2+y^2>R^2$) could be written in the form (3) with help of Floquet theorem:

$$u(x,y,z)=v(x,y,z)\cos(\xi z),\ v(x,y,z+1)=v(x,y,z),\ \xi = n\pi/N,\ n=0\ldots(N-1) \qquad (3)$$

This follows that the solution of the problem could be understand as a stationary wave in case $\xi \neq 0$ with phase shift in each interdisk space and as a stationary wave which is synphase in each interdisk space when $\xi=0$.

In [2] it was shown that if $R \geq \theta_0/\pi$, here θ_0 is the first zero of Bessel function of zero order than the periodic chain of disks possesses the waveguiding properties for any $\xi \neq 0$. In case $\xi=0$ the existence of resonance frequencies is proved for R>0.342. Coupled with existence proof the formulas (4) for dependencies of waveguiding and resonance frequencies on the geometric parameters of the chain were obtained.

$$\sin[\chi(\lambda,\xi)]=0\text{, here}$$

$$\chi(\lambda,\xi)=-\frac{\pi}{2}-arctg\frac{J_1(\lambda R)}{J_0(\lambda R)}+\frac{\lambda}{\pi}\ln(2)-\arcsin\frac{\lambda}{|\xi|}--\sum_{n=1}^{N}\left\{\arcsin\left(\frac{\lambda}{\xi+2n\pi}\right)+\arcsin\left(\frac{\lambda}{\xi-2n\pi}\right)\right\}+\sum_{n=1}^{2N}\arcsin\left(\frac{\lambda}{n\pi}\right)$$

$$\cos\sigma\ (\lambda,R)=0\text{, here} \qquad (4)$$

$$\sigma\ (\lambda,R)=-arctg\frac{J_1\left(\sqrt{\lambda^2-\pi^2}R\right)}{J_0\left(\sqrt{\lambda^2-\pi^2}R\right)}-\sum_{n=1}^{N}\arcsin\left(\frac{\sqrt{\lambda^2-\pi^2}}{\sqrt{(2n\pi)^2-\pi^2}}\right)+\sum_{n=2}^{N}\arcsin\left(\frac{\sqrt{\lambda^2-\pi^2}}{\sqrt{((2n-1)\pi)^2-\pi^2}}\right)$$

here N is a parameter, natural number, which control the approximation accuracy

The fact is that the waveguiding and resonance frequencies of the problem with additional condition (2) correspond to the eigenfrequencies of the problem (1) if one chose the appropriate values for ξ parameter (3), so the eigenfrequencies of problem (1) are discrete.

RESULTS

From analytical and numerical investigations of expressions (4) follows that the eigenfrequencies are group into certain number of bundles in case $\xi \neq 0$ and forms a finite set of isolated frequencies in case $\xi=0$. The distant between the boundary of these bundles depends on geometrical parameters of disks cascade. Using the Neumann-Dirichlet bracket we were able to prove the following statements.

Statement 1. The number of resonance frequency bundles K is determined by the expression $\mu_0 \leq K \leq \mu_1$ where μ_i is the number of the roots for Bessel function J_i which are contained in the segment $[0,\pi R]$.

Statement 2. The number of isolated resonance frequency A can be estimate from the expression $\mu_0 \leq A \leq \mu_1$ where μ_i is the number of the roots for Bessel function J_i which are contained in the segment $[0, R\pi\sqrt{3}]$.

The boundary of resonance frequency bundles and isolated resonance frequencies are show on Fig.2. The dependence of eigenfrequencies on phase shift parameter (ξ) is illustrated on Fig.3.

In terms of this investigation one could conclude that since the position of the boundary of resonance bundles continuously depends on the geometrical parameters of the disks cascade the methods of reduction of objectionable resonance phenomena which are based on breaking symmetry of the structure given had no fundamental basis. The second conclusion could be the following. Since the increasing of the number of disks in cascade lead to gaining of the number of resonance frequencies per bundle and due to non-linear phenomenon called "Capturing of resonance frequencies" we could finally get the continuous band of resonance frequencies instead of set discrete bundles of resonance frequencies.

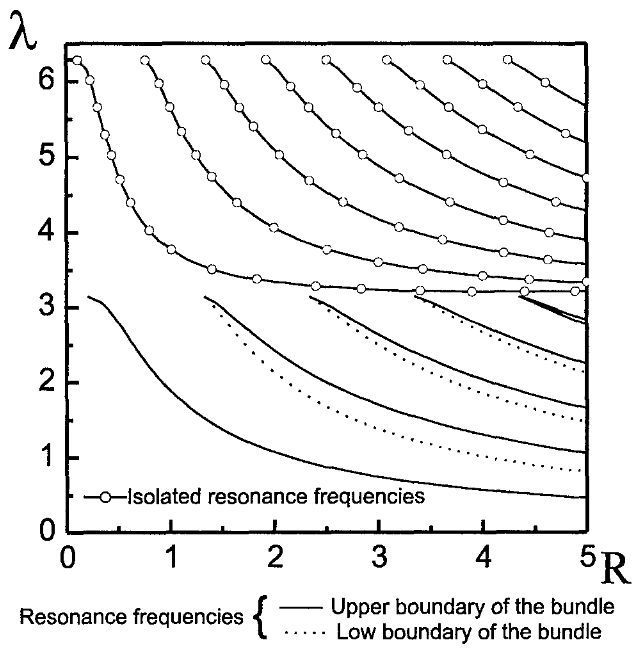

Fig.2. The dependence of boundaries of the bundles of resonance frequencies and isolated resonance frequencies on the disk radius

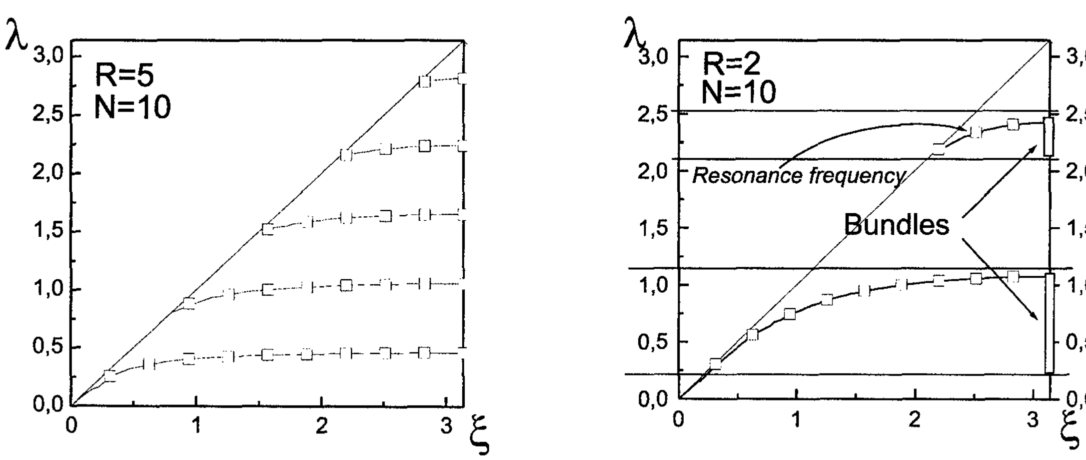

Fig.3. The dependence of eigenfrequencies on phase shift parameter (ξ). Here R is the disk radius, N is the number of disks in cascade.

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