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2
3 A METHOD FOR DETERMINING THE APPROXIMATE
4 RESONANCE FREQUENCY OF A STRUCTURE
5 SURROUNDED BY A COMPRESSIBLE FLUID

6
7 STATEMENT OF GOVERNMENT INTEREST

8 The invention described herein may be manufactured and used
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11 therefor.

12
13 BACKGROUND OF THE INVENTION

14 (1) Field of the Invention

15 This invention relates to methods for determining the
16 approximate resonance frequencies of underwater structures, and
17 is directed more particularly to such a method as is more
18 efficient than current harmonic sweep methods and more accurate
19 than modal finite element/boundary element methods, the latter
20 being based on an assumption of fluid incompressibility.

21
22 (2) Description of the Prior Art

23 Usually when modeling a structure in water, one is
24 interested in either 1) how the structure behaves in a certain
25 frequency range, or 2) at what frequency one obtains a desired
26 mode shape. This invention is concerned with the latter. The

1 classical approach is to perform an in-water modal analysis to
2 determine the in-air resonance frequencies. After displaying the
3 mode shapes, one determines a mode shape of interest, and
4 therefore a resonance frequency of interest. One then performs
5 an in-water harmonic analysis by first estimating a coarse
6 frequency range and then refining the range to capture the
7 resonance frequency within a desired tolerance. For large models
8 this is a very time consuming process.

9 There is thus a need for a technique for determining the
10 resonance frequencies of such structures in a more efficient
11 manner.

12 The behavior of complex fluid-loaded structures typically is
13 modeled using one of the following methods: 1) mathematically
14 describing the structure and surrounding fluid with finite
15 elements, that is, regional subdivisions of the structure in each
16 of which the behavior is described by a separate set of assumed
17 functions representing the stresses or displacements in a given
18 region; 2) describing the structure with finite elements and the
19 fluid with boundary elements, that is, two-dimensional elements
20 located on the wetted surface of the structure, such elements
21 describing the acoustic loading of the structure; and 3)
22 describing the structure with finite elements and the fluid with
23 a combination of finite elements and infinite elements, that is,
24 elements used in conjunction with finite elements to define the
25 exterior fluid. These infinite elements allow the domain of a
26 finite fluid element to extend to infinity in one or more

1 directions by mapping an infinite domain into a finite domain.
2 For all three methods, the fluid-loaded behavior is generally
3 determined using a harmonic, i.e., forced, analysis involving
4 computations at each frequency over a specified range of
5 frequencies. Since the in-water resonance frequency, that is,
6 the frequency at which the input reactance vanishes and the input
7 resistance is small, is not known a priori, this procedure can
8 involve multiple frequency sweeps. Some finite element programs
9 do offer a fluid-loaded modal analysis, but the use of finite
10 elements to model the fluid implies many additional degrees of
11 freedom in the system of equations. See *The Finite Element*
12 *Method*, by O.C. Zienkiewicz (McGraw-Hill UK, London, 1977), 3rd
13 ed. A combination of finite and infinite elements to model the
14 fluid results in fewer additional degrees of freedom, but, at
15 present, in-fluid modal analysis is not offered in the major
16 codes possessing infinite elements. As with infinite elements,
17 modeling the fluid with boundary elements does not result in any
18 additional degrees of freedom for the fluid, which is desirable.
19 However, because this method is based on a frequency-dependent
20 influence matrix to describe the fluid, either one must make the
21 assumption that the fluid is incompressible, in which case the
22 fluid influence matrix is approximated by its low frequency
23 limit, see "Solution of Elasto-Acoustic Problems using a
24 Variational Finite Element/Boundary Element Technique," by J.P.
25 Coyette and K.R. Fyfe, in *Numerical Techniques in Acoustic*
26 *Radiation*, edited by R.J. Bernhard and R.F. Keltie, NCA-Vol. 6

1 (ASME, New York, 1989), or one must know the in-water
2 eigenfrequency a priori in order to accurately compute the fluid
3 load. The fluid influence matrix is a complex, frequency
4 dependent, symmetric matrix that represents the mass loading
5 (imaginary part) and damping (real part) effects of an exterior
6 or interior fluid on the vibration of a structure. In the case
7 of an enclosed interior fluid, the real part of the influence
8 matrix is zero.

9 The usual procedure for determining the resonance frequency
10 of structures, using boundary elements to represent the fluid, is
11 as follows:

12 1) Perform an in vacuo eigenvalue analysis using a finite
13 element model to determine the frequencies and modes of the
14 structure in question. The eigenvalue equation is:

$$[K] - \omega_i^2 [M] \phi_i = 0 \quad (1)$$

15 where $[K]$ is the stiffness matrix, $[M]$ is the consistent mass
16 matrix, ω_i^2 is the set of eigenvalues, and ϕ_i is the set of
17 orthogonal eigenvectors. This equation is written for elastic
18 structures, although the structure can also be piezoelectric or
19 magnetostrictive.

20 2) Use boundary elements to compute the influence matrix
21 over a selected frequency range to attempt to encompass the in-
22 fluid resonance frequency of interest. Usually, a coarse
23 frequency resolution is used initially since the in-fluid
24 frequency is not known a priori.

1 3) Compute the forced fluid-loaded behavior of the
2 structure by performing a harmonic analysis using mass and
3 stiffness matrices (computed by a finite element program) and the
4 influence matrix. The harmonic equation is

$$[K] - \omega^2 [M] + j\omega [Z(\omega)] u = F, \quad (2)$$

5 the frequency of excitation, $Z(\omega)$ is the influence matrix, $\{u\}$ is
6 the vector of computed nodal displacements, and $\{F\}$ is the vector
7 of specified applied forces. The derivation of this equation may
8 be found in "A Coupled Finite Element/Boundary Element Approach
9 for Predicting the Performance of High-Powered, Low-Frequency
10 Projectors with Two Applications (U)," by R.E. Montgomery, C.M.
11 Siders, and T.A. Henriquez, Journal Underwater Acoustics, Special
12 Issue on Transducers, January 1991, and in "Numerical Solution of
13 Acoustic-Structure Interaction Problems," by H.A. Schenck and
14 G.W. Benthien, Naval Ocean Systems Center Technical Report 1263,
15 April 1989. If $[Z]$ is computed using a nodal boundary element
16 code, then it is brought into the equation unmodified. If it is
17 computed using a "patch" boundary element code, then the third
18 term in Equation (2) becomes: $j\omega [X] [Z(\omega)] [X]^T$, where $[X]$ is a
19 matrix that translates between field variables at the centroid of
20 a patch and the equivalent variables at the nodes.

21 4) Examine the computed nodal displacements to determine
22 if the behavior of the structure at any frequency in the selected
23 frequency range matches the behavior desired mode. If the
24 desired mode is not present in the frequency range, a new

1 frequency range must be chosen and the user must go back to step

2 2. If the displaced shape of interest is present in the
3 frequency range that was run, proceed to the next step.

4 5) The approximate in-fluid resonance frequency is the
5 frequency at which the structural displacement is a maximum. To
6 determine this frequency, one must examine the displacement at a
7 node that is significant for the mode of interest. Specifically,
8 one must not select a node whose displacement is zero or nearly
9 zero for the mode of interest.

10 6) Refine the approximation of the in-fluid resonance
11 frequency by choosing a smaller frequency step in a shorter range
12 of frequencies about the frequency identified in step 5, and
13 recompute the in-water displacements.

14 7) Repeat steps 2-6 until the resonance frequency is
15 determined with the desired accuracy.

16 The above computations can be lengthy and time consuming
17 inasmuch as step (6) can require repeated choices or
18 approximations of in-fluid resonance frequencies about the
19 frequency identified in step (5). To shorten such computations,
20 knowledge of the approximate in-water frequency is required
21 (usually not known). The above procedure can provide an accurate
22 calculation of the fluid-loaded behavior of the structure;
23 however, it can be computationally prohibitive for realistic
24 structures for which the fluid-loaded resonance frequency is not
25 known a priori.

There is thus a need for a method for determining the resonance frequency of a structure surrounded by a compressible fluid, wherein the compressibility of the fluid is a factor and wherein the in-water resonance frequency can be computed based upon a known frequency of the structure.

SUMMARY OF THE INVENTION

It is, then, an object of the invention to provide a method for using boundary elements in a fluid-loaded modal analysis that eliminates the need for a priori knowledge of the in-water eigenfrequency, and that does not require the assumption of incompressibility.

With the above and other objects in view, as will hereinafter appear, a feature of the present invention is the provision of a method for determining the approximate resonance frequency of a structure surrounded by a compressible fluid, the method comprising the steps of: (1) using a finite element model, performing an in-vacuo eigenvalue analysis to determine in-vacuo frequencies and mode shapes of the structure; (2) selecting one of the mode shapes as an in-vacuo mode of interest, along with an attendant eigenfrequency thereof; (3) using a boundary element program, computing an influence matrix at the eigenfrequency of the selected mode of interest; (4) combining the computed influence matrix with structural stiffness and mass matrices from a finite element program, and using the modified matrices, computing eigenvalues of the structure, as well as

1 eigenvectors; (5) selecting from the computed eigenvectors a
2 computed mode having substantially the same displaced shape as
3 the in-vacuo mode of interest; (6) determining the eigenfrequency
4 of the selected computed mode; (7) determining any difference
5 between the computed eigenfrequency of the selected computed mode
6 and the in-vacuo eigenfrequency of the selected in-vacuo mode of
7 interest; wherein (8) if the difference is equal to, or less
8 than, a selected tolerance, the computed eigenfrequency of the
9 computed mode is the approximate resonance frequency of the
10 structure surrounded by compressible fluid; and (9) if the
11 difference is greater than the tolerance, repeating step (3)
12 substituting the computed eigenfrequency of the selected computed
13 mode for the in-vacuo eigenfrequency for the in-vacuo mode of
14 interest, and thereafter, repeating steps (4) - (9), substituting
15 in step (3) a most recent computed eigenfrequency of the selected
16 computed mode for the previously used computed eigenfrequency of
17 the selected computed mode.

18 The above and other features of the invention will now be
19 more particularly described with reference to the accompanying
20 drawings and pointed out in the claims. It will be understood
21 that the particular method embodying the invention is described
22 and shown by way of illustration only and not as a limitation of
23 the invention. The principles and features of this invention may
24 be employed in various and numerous embodiments without departing
25 from the scope of the invention.

BRIEF DESCRIPTION OF THE DRAWINGS

Reference is made to the accompanying drawings in which is shown an illustrative embodiment of the invention, from which its novel features and advantages will be apparent.

In the drawings:

FIGS. 1-11 are charts illustrative of numerical values determined in Examples #1 and #2 set forth in the specification;

FIG. 12 is a graph illustrative of numerical values determined in Example #1;

FIG. 13 is a side elevational view of an acoustic projector which is the structure under study in Example #2;

FIG. 14 is a sectional view of the acoustic projector of FIG. 13;

FIG. 15 depicts an axisymmetric finite element model of the projector of FIGS. 13 and 14; and

FIG. 16 depicts a 3-D model of the projector of FIGS. 13 and 14.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

The objective of the inventive method is to compute in-fluid modal frequencies using boundary elements to describe the fluid, without assuming the fluid to be incompressible. This approach eliminates both the hit-and-miss of the harmonic boundary element procedure described above, and the necessity of meshing a portion of the infinite fluid domain, as in the finite element approach. As stated before, the reason this is not possible with the usual

1 boundary element description is that the computation of the fluid
2 influence matrix requires knowledge of the in-fluid frequency of
3 the mode of interest. Of course, in the absence of the
4 experimental measurements, the designer does not know this
5 frequency. What is known, or is readily ascertainable, is the in
6 vacuo frequency of the mode. In the method described below, the
7 in vacuo frequency is used as the starting point of an iterative
8 eigenvalue analysis.

9 The equation for a standard in-vacuo finite element
10 eigenvalue analysis is as follows:

$$[K] - \omega_i^2 [M] \phi_i = 0 \quad (3)$$

11 where ω_i and ϕ_i ($i = 1$ to n) represent the eigenvalues and
12 eigenvectors, respectively, of the in- vacuo structure. For the
13 examples presented herein, the stiffness matrix is assumed to be
14 real; that is, there is no structural damping. Structural
15 damping can be included, leading to a complex eigenvalue
16 analysis, but its effect on the eigenvalues is usually small.

17 The idea of the present analysis is to include the fluid
18 loading in the eigenvalue equation. As stated hereinabove, the
19 real part of the influence matrix $[R]$, describes the radiation
20 damping, and the imaginary part divided by the radian frequency
21 describes the added mass of the fluid. The radiation damping and
22 mass can be added to the structural stiffness and mass matrices,

respectively. These additional terms are frequency dependent; however, choosing a specific angular frequency, ω_0 , we have the following relation:

$$([K] + j\omega_0) - \omega_i^2([M] + \frac{[X(\omega_0)]}{\omega_0})\{\phi_i\} = \{0\} \quad (4)$$

For simplicity, we can rewrite as:

$$([K'] - \omega_i^2[M'])\{\phi_i\} = \{0\} \quad (5)$$

where

$$[K'] = [K] + j\omega_0[R(\omega_0)], \quad (6)$$

and

$$[M'] = [M] + \frac{[X(\omega_0)]}{\omega_0}. \quad (7)$$

Note that $[R(\omega_0)]$ and $[X(\omega_0)]/\omega_0$ each comprises a matrix of constants; therefore Equation (5) is a mathematically valid eigenvalue equation. (The form of an eigenvalue equation requires that $[K]$ and $[M]$ not depend on ω .) Nevertheless, Equation (5) only represents the physical behavior of one in-fluid eigenvector. That is, if $\omega_i = \omega_0$ for some i , and $[Z(\omega)]$ is computed at ω_0 , then Equation (5) will give the correct eigenvector corresponding to the eigenvalue ω_i . There are $(n-1)$ other eigenvalues and eigenvectors, but none of them has the correct fluid loading since $[Z]$ is computed at the frequency corresponding to mode i . For this reason, one can only use this method for determining one in-fluid eigenvalue at a time. For

1 practical design problems, this is not a serious limitation
2 because the designer is generally only interested in one mode.

3 The problem remains of finding the value of the
4 eigenfrequency for the particular mode of interest. To do this,
5 we begin with the only information that is available, that is,
6 the in-vacuo eigenvalue of the mode of interest. We compute the
7 influence matrix at this frequency, modify the mass matrix (and
8 the stiffness matrix if desired), and solve the resulting
9 eigensystem. If the modes remain uncoupled under fluid loading,
10 we will find that there is a mode in the set of in-fluid
11 eigenvectors that matches the shape of the in vacuo mode of
12 interest. We then obtain the frequency corresponding to this
13 mode, recompute the influence matrix at this frequency, and so
14 on, until two consecutive in-fluid eigenvalue computations match,
15 for the mode of interest, within a specified tolerance.

16 This new procedure for determining the fluid-loaded
17 resonance frequency of a structure is as follows:

18 Perform an in-vacuo eigenvalue analysis using a finite
19 element model to determine the in vacuo frequencies, and mode
20 shapes. This step is the same as step 1 in the above described
21 prior art method. Select the mode of interest, i , and its
22 eigenfrequency. The in vacuo eigenfrequency of interest is
23 designated f_0 .

24 Compute the influence matrix, $[Z]$, at f_0 using a boundary
25 element program.

1 Solve a modified eigenvalue problem using the mass and
2 stiffness matrices from a finite element program and the computed
3 influence matrix. The equation used here is:

$$\{[K'] - \omega_i^2 [M']\} \{\phi_i\} = \{0\} \quad (8)$$

4 where

$$[K'] = [K] + j\omega_o \text{Re}[(\omega_o)], \quad (9)$$

5 and

$$[M'] = [M] + \frac{\text{Im}[Z(\omega_o)]}{\omega_o}, \quad (10)$$

6 and $\omega_o = 2\pi f_o$. "Re" and "Im" represent the real part and
7 imaginary part, respectively, of the complex $[Z]$. In Equation
8 (8), the structural mass matrix has been modified to include the
9 entrained fluid mass (radiation mass), and the structural
10 stiffness matrix has been modified to include the radiation
11 damping. In practice, the effect of radiation damping on the in-
12 fluid eigenfrequency is much smaller than that of the radiation
13 mass, so that the stiffness matrix can be left unmodified. With
14 this simplification, the complex eigenvalue problem, Equation
15 (8), becomes purely real.

16 Examine the computed eigenvectors to determine which mode
17 has the same displaced shape as the selected in vacuo mode of
18 interest. The computed frequency of the selected mode is
19 designated f .

1 Compare f and f_0 . If the difference is less than a
2 specified tolerance, then f is the approximate fluid-loaded
3 resonance frequency. If the difference is greater than the
4 specified tolerance, then set $f_0=f$ and return to computation of
5 the influence matrix and repeat the steps thereafter.

6 The procedure outlined above is based on complex
7 eigenfrequency computations, which require significantly more
8 computation time than real eigensolutions. However, for most
9 problems, the in-fluid eigenfrequency is determined primarily by
10 the added mass of the fluid, so that the effects of the radiation
11 damping can be ignored. In this case, $[K']=[K]$ and the
12 eigensolution becomes purely real. The differences between the
13 in-water resonance frequencies with and without radiation damping
14 will be examined hereinbelow.

15 At this point, an operator may be concerned with the number
16 of iterations required to determine the in-fluid eigenfrequency.
17 If the procedure requires a large number of iterations, it might
18 be faster just to perform a harmonic frequency sweep over a very
19 wide frequency range. This is a valid concern, but it turns out
20 that at low ka , where ka is the non dimensional wave number (we
21 are restricted to low ka for mode preservation, at least in
22 theory), the solution converges very quickly. In fact, the first
23 iteration in the modified eigensolution, that is, the solution
24 for which the $[Z]$ matrix is computed at the in vacuo frequency,
25 results in a reasonable estimate of the in-fluid eigenfrequency.
26 The reason for this can be understood by studying the classical

1 problem of the fluid loading on a piston in an infinite baffle.

2 The added mass of a piston on an infinite baffle is given by:

$$M_p = \frac{X_p(\omega)}{\omega} = \frac{\pi \rho_o a^3 H_1(2ka)}{k^2 a^2} \quad (11)$$

3 where a is the radius of the piston, ρ_o is the density of the
4 medium, k is the fluid wave number, and H_1 is the first order
5 Struve function. For $ka \ll 1$, this becomes:

$$M_p \approx \frac{8 \rho_o a^3}{3} \quad (12)$$

6 so the added mass is constant for small values of ka . Moreover,
7 as ka increases to values near one, the added mass of Equation
8 (11) changes slowly, as shown in FIG. 12. The implication of
9 this is that for low ka , the added mass of the fluid computed at
10 the in vacuo eigenfrequency is not very much different from the
11 correct added mass computed at the in-fluid eigenfrequency. The
12 in-fluid frequency is always lower than the in vacuo value, so we
13 are headed in the direction of lower ka . This concept will be
14 demonstrated for a realistic underwater projector hereinbelow.

15 The modified eigenvalue procedure described above has been
16 applied to several problems. In each case the ATILA (Analyse de
17 Transducteurs par Integration des equations de Laplace) finite
18 element and the CHIEF boundary element codes were used. The
19 modified modal analysis was implemented in two ways. In both
20 implementations, the CHIEF code was used to generate the fluid
21 influence matrix. In the first implementation, the stiffness and

1 mass matrices were computed by ATILA and written to an external
2 file. This file and the file created by CHIEF were read into an
3 external program where they were combined and solved using
4 standard LAPACK modal routines. Both real and complex
5 eigensolutions (the latter included radiation, but not
6 structural, damping) were performed, but the use of the external
7 program was limited to very small problems because of computer
8 memory. In the second implementation, the CHIEF influence matrix
9 was brought into ATILA, and used to modify the mass matrix. Then
10 a standard ATILA modal analysis was performed. The reason for
11 modifying only the mass matrix is that the ATILA code performs a
12 real eigenvalue solution.

13 Two examples will be presented here: a water-loaded
14 spherical shell, and a mechanically driven projector in water.
15 In the latter case, the method has been applied to both a two-
16 dimensional (2-D) and a three-dimensional (3-D) model. Results
17 will include in vacuo and in-water eigenfrequencies and modes,
18 and the harmonic response with and without radiation damping. In
19 addition, the error introduced by assuming incompressibility of
20 the fluid will be presented.

21 22 EXAMPLE 1

23 The first example is a water-loaded spherical steel shell,
24 the geometry of which has been chosen to match that of Junger and
25 Feit in "Sound, Structures, and their Interaction" by M. C.
26 Junger and D. Feit (MIT Press, Cambridge, MA) 1972. The behavior

1 of the shell is assumed to be axisymmetric. The ratio of the
2 thickness to radius of the shell is 0.01, the Poisson's ratio is
3 0.30, the ratio of structural to fluid density is 7.67, and the
4 ratio of plate speed in the shell to sound speed in the fluid is
5 3.53. In "Sound, Structures ..." there are presented a
6 transcendental equation and a table of results for undamped
7 eigenfrequencies, neglecting flexural effects. The modes of the
8 shell are characterized by a value of $n \geq 0$. For the $n=0$ mode,
9 only one real eigenfrequency exists; this is the breathing mode.
10 For each mode $n > 0$ there are two eigenfrequencies. The two sets
11 or branches of modes represent two different types of behavior.
12 The $n \geq 2$ modes of the lower branch are characterized by
13 predominantly radial motion, while the $n \geq 1$ modes in the upper
14 branch have more tangential motion.

15 The axisymmetric ATILA finite element model of the spherical
16 shell comprises 64 eight-noded quadrilateral elements, with each
17 node having two displacement degrees of freedom. These elements
18 represent the complete axisymmetric equations of motion, so that
19 flexural effects are included. This model was used to compute
20 the in vacuo eigenfrequencies and modes. The corresponding
21 boundary element model consists of 64 line elements which
22 coincide with the external boundary of the finite element mesh.
23 The CHIEF code generates a 3-D mesh from the 2-D geometry, using
24 a specified number of rotational symmetry blocks, n_{blks} , and
25 solves the equation set in three dimensions. In this case, the
26 number of rotational symmetry blocks is 100. Because the

1 resulting influence matrix is computed for the full 3-D mesh, it
2 must be scaled by the factor $nblks/(2\pi)$ so that it represents the
3 fluid impedance per radian, making it compatible with the
4 axisymmetric finite element matrices.

5 FIG. 2 provides the in vacuo eigenfrequencies from "Sound,
6 Structures ...", which is incorporated by reference herein, and
7 those from the ATILA finite element model, for the first few
8 modes of the lower branch and the first mode of the upper branch.
9 All frequencies are rounded to the nearest hertz. The percent
10 differences between the two sets of results are computed relative
11 to the analytic result. The fact that the two sets of
12 frequencies are in such close agreement indicates that for this
13 thickness-to-radius ratio and frequency range, flexural effects
14 are negligible, and that the structural finite element model
15 accurately represents the elastic behavior of the sphere.

16 FIG. 3 provides the undamped in-fluid eigenfrequencies from
17 "Sound, Structural ..." and those from the modified modal
18 analysis implemented in the ATILA code, for the first three
19 nonzero eigenfrequencies of the lower branch ($ka \sim 1.1$ to 1.6), and
20 the first eigenfrequency of the upper branch ($ka \sim 4$). The
21 radiation damping is neglected for both sets of results. The
22 differences are again computed relative to the analytic result.
23 This part of the analysis was limited to low order modes to avoid
24 the need for a finer numerical mesh. In FIG. 3 the largest

1 difference between the undamped in-water modal frequencies for
2 the modified finite element/boundary element (FE/BE) method and
3 the analytic method is 4.0%.

4 The number of iterations required to reach convergence to
5 within one hertz for the results in FIG. 3 varied between four
6 and seven. Had the convergence tolerance been specified as a
7 percentage of the eigenvalue, it is assumed that roughly four
8 iterations would have been required for any of the modes. This
9 result was obtained without using any of the various techniques
10 available for speeding up the convergence process, for example,
11 the bisection method. The low number of iterations is,
12 therefore, somewhat surprising, especially as the mode order and
13 ka increase. As ka increases, the variation of the added mass
14 with frequency becomes more significant, so that the initial
15 estimate would be expected to be worse, requiring more iterations
16 before convergence. For realistic problems, this discussion is
17 moot because the modes are not likely to be preserved at such
18 high values of ka .

19 The differences shown in FIG. 3 are not entirely a result of
20 the in-fluid modal method. A more appropriate test is to compare
21 the results of the modified modal method with the results of a
22 harmonic analysis using the same FE/BE model, since the peak in
23 the harmonic response of the numerical model is what we are
24 trying to determine. FIG. 4 presents this comparison under the
25 conditions that the harmonic response is computed using only the
26 imaginary part of the complex influence matrix. The percent

1 differences are computed relative to the harmonic results. . . .
2 Comparing FIGS. 2, 3, and 4 we see that the differences between
3 the in-water eigenfrequencies obtained from the analytic and
4 FE/BE models are primarily a result of the differences in the way
5 the fluid and the fluid/structure interaction are described. The
6 fact that the in vacuo eigenfrequencies are in close agreement
7 (FIG. 2) indicates that the structural models are equivalent. In
8 any case, a difference of 4% is not considered to be significant.

9 The error caused by neglecting the effects of radiation
10 damping can be determined by comparing the peaks in the harmonic
11 response computed using only the imaginary part of the influence
12 matrix with those computed using the complex influence matrix
13 (see FIG. 5). The percent differences are computed relative to
14 the latter results. The most significant effect of radiation
15 damping is on the lowest mode, for which the resonance frequency
16 is lowered by 1.5%. For the higher modes, the effect is
17 indiscernible for all practical purposes.

18 Having shown the accuracy of the iterative in-water method,
19 there remains one point of interest relative to the spherical
20 shell problem. It is stated above that if finite elements are
21 used to describe the fluid, one must contend with a large number
22 of additional equations in the eigenvalue solution. It is also
23 pointed out that an in-water eigenvalue solution previously was
24 not possible with boundary elements, unless one assumed
25 incompressibility. We will now quantify the error introduced by
26 the assumption of incompressibility, for the spherical shell in

1 water. This assumption implies that there is no radiation
2 damping, and that the added mass of the fluid is computed at
3 $ka=0$.

4 FIG. 6 compares the results of the iterative in-water modal
5 technique with those obtained when the fluid influence matrix,
6 $[Z]$, is computed at $ka=0$, i.e., at $\omega=0$. For the modes in the
7 lower branch, we see that the differences in the computed
8 eigenvalues are between 3% and 7%, and that the eigenvalues from
9 the incompressible solution are higher than the values obtained
10 from the iterative technique. For the breathing mode, however,
11 the error in the incompressible value is 66%, and it is lower
12 than the correct value. The differences in the magnitude and
13 sign of the error between the two branches can be explained by
14 studying FIG. 13. Focusing on the $n=2$ /lower mode, we see that
15 the added mass at $ka=0$ is lower than the value at $ka=1.1$ (the
16 value of ka corresponding to iterative eigensolution). Since the
17 added mass is too low, the computed eigenfrequency is too high.
18 The same is true for modes $n=3$ and $n=4$, although the eigenvalue
19 error is progressively lower since the error in the added mass
20 decreases with increasing mode order. If we now consider the
21 added mass of the $n=0$ /upper mode (breathing mode) in FIG. 13, we
22 see that the value at $ka=0$ is much greater than that at $ka=4.5$
23 (the ka value at the true in-water eigenfrequency). This leads
24 to an incompressible eigenfrequency that is much lower than the

1 true value. These results demonstrate that while the
2 incompressibility approximation may be satisfactory at very low
3 ka values, the error increases as ka increases.

5 EXAMPLE #2

6 The second example is a low frequency projector 20 that
7 operates by mechanically driving two opposing flexural disks 22
8 (see FIGS 14 and 15). The disks 22 have a linearly varying
9 thickness, with the greatest thickness being at the center. A
10 circular piston 24 drives a finite area at the center of each
11 disk 22.

12 The behavior of the projector 20 was described first with an
13 axisymmetric finite element model (using the ATILA code) to
14 determine the appropriate boundary conditions and the general
15 character of the operational mode, which corresponds to the
16 fundamental mode of a circular plate. Then a 3-D model was
17 developed to study the influence of parasitic modes, i.e.,
18 undesirable modes that might interfere with the mode of interest.
19 For both models, the fluid loading was described using the CHIEF
20 boundary element program. At the time of the original analysis,
21 the method described herein was not available. Therefore, the
22 in-water behavior of the projector was determined using a
23 harmonic analysis with an applied force at the center of the
24 disks. For the mode of interest the flexural disks vibrate in
25 the fundamental mode of a circular plate, which has significant
26 accession to inertia (large added mass) under fluid loading. The

1 result is that the in vacuo modal frequency is greatly reduced
2 when the projector is submerged. Therefore, a broad frequency
3 sweep and a great deal of computation time were required to
4 determine the in-water resonance frequency using the harmonic
5 method. This problem led to the conception of the idea for the
6 in-fluid modal method for finite element/boundary element models.

7 The axisymmetric finite element model of the projector is
8 shown in FIG. 16. In addition to the axial symmetry, there is a
9 plane of symmetry through the center plane of the housing. The
10 mechanical structure, comprising the disks, housing, and
11 compliant pad, is modeled with eight-noded quadrilateral
12 elements. Each node has two translational degrees of freedom.
13 For this 2-D representation, the boundary element model is
14 generated using line elements, with a one-to-one correspondence
15 between the structural and boundary elements. The CHIEF code
16 then generates a 3-D model for the computation of the fluid
17 influence matrix. Because CHIEF cannot combine both rotational
18 and planar symmetry, the [Z] matrix was computed for a
19 rotationally symmetric geometry with no planar symmetry. The
20 computed matrix was then reduced to account for planar symmetry.

21 The results for the in vacuo and in-water eigenfrequencies
22 and the in-water harmonic resonance frequencies are presented in
23 FIG. 7. The in-water ka values are also given. Mode 3 has been
24 omitted from the results because it is a mode of the housing, and
25 we are interested in the disk modes. In FIG. 7, we see that the
26 in-water eigenfrequencies computed with the modified modal method

1 are very close to the peaks in the harmonic response computed
2 with a purely imaginary influence matrix. The effect of
3 radiation damping on the harmonic response peaks is demonstrated
4 by comparing these harmonic response peaks to those computed with
5 the complex influence matrix. For the mode of primary interest
6 in this study (mode 1), the effect is negligible, while for mode
7 5, there is a 1.6% difference between the two results. It is
8 surprising to note that the modes are preserved in water for ka
9 values at least as high as 7, but this likely is a result of the
10 axisymmetry of the model. The nonaxisymmetric modes are
11 eliminated, thus reducing the possibility of modal coupling.

12 The results presented thus far have demonstrated the
13 accuracy of the modified modal method for a spherical shell and a
14 flexural disk projector. It remains to show that the proposed
15 method is also efficient. FIG. 8 presents the progression of the
16 iterative eigenvalue calculation for the axisymmetric projector
17 model. These results were obtained without any schemes for
18 improving the convergence efficiency. For each of the first four
19 disk modes, the procedure begins with the in vacuo modal
20 frequency (column 2). For mode 1, only two in-water eigenvalue
21 calculations are required. Modes 2 and 4 each require three
22 iterations. Mode 5 converges to within 2 Hz after three
23 iterations, but then begins to jump back and forth between two
24 values. This problem would be eliminated by any of the available
25 convergence improvement schemes, even the simple bisection
26 method. The first and last columns of FIG. 8 give the starting

1 and ending mode numbers for the four modes of interest. Note
2 that the eigenvector that was fifth in the list of in vacuo
3 modes, becomes the sixth mode in water. This switching of modal
4 order is always a possibility because the in vacuo eigenfrequency
5 of a higher mode whose motion is predominantly radial can be
6 lowered more by fluid loading than that of a lower mode whose
7 motion is mainly tangential. This is why it is very important to
8 examine the eigenvectors, at least after the first in-water
9 eigenvalue computation. For most problems, the largest change in
10 the influence matrix occurs in the first iteration, so the modal
11 order does not usually change after this step.

12 The full 3-D model of the flexural disk projector is shown
13 in FIG. 17. This model has two planes of symmetry. Only the
14 fundamental disk mode of the projector (mode 5 for the 3-D model)
15 was studied using this model, because more elements would be
16 required for higher values of ka . FIG. 9 provides the in vacuo
17 eigenfrequency and the in-water eigenfrequencies and resonance
18 frequencies for the fundamental mode. Note that the in vacuo
19 eigenfrequency computed using the 3-D model is slightly higher
20 than that obtained with the axisymmetric model. This is not
21 unexpected since the two models have different mesh densities.
22 The in-water modal frequency computed with the modified modal
23 method is 43 Hz, while the peak in the in-water harmonic response
24 is at 41 Hz (using either the imaginary or complex influence
25 matrix). This difference is thought to be a result of the large
26 number of equations in the eigensystem. While the difference

1 corresponds to an error of nearly 5%, in practical terms 2 Hz is.
2 not significant. FIG. 10 shows the progression of the iterative
3 procedure for mode 5.

4 In FIG. 10, note that the in-water eigenvalue converges
5 after only two iterations. FIG. 11 compares the CPU time
6 required for two in-fluid eigenvalue computations to the time
7 required for a typical harmonic frequency sweep, both performed
8 in ATILA on a DEC 3000/400 Alpha workstation. It is assumed
9 (conservatively) that the first harmonic sweep is done over ten
10 frequencies, and that the in-water eigenvalue lies within the
11 first range of frequencies selected. The harmonic sweep requires
12 over six hours of CPU time, while the modified modal method
13 requires only one-half hour. If one is not so fortunate, one
14 could spend days finding the peak in the frequency response.

15 The computation of in-water modal frequencies of FE/BE
16 models using an iterative eigenvalue computation has been shown
17 to be both accurate and efficient. The results have been
18 compared to those of a closed-form solution for a spherical shell
19 and those of a standard harmonic analysis for a 2-D and a 3-D
20 FE/BE model of a flexural disk projector. For smaller numerical
21 models, for which a harmonic analysis is not inordinately time
22 consuming, the main advantage of the method is to eliminate the
23 guess work in isolating the in-water eigenfrequency of a
24 particular mode of interest. For large models, the method
25 additionally provides a tremendous savings in computation time
26 compared to the standard harmonic method. Furthermore, it has

1 been demonstrated that while the error introduced by neglecting
2 radiation damping is small, the error arising from the assumption
3 of fluid incompressibility can be quite significant.

4 There is thus provided a method which is significantly
5 faster than the prior art approach set forth hereinabove,
6 eliminates the guesswork involved therein in locating the
7 appropriate fluid-loaded resonance frequency, and retains the
8 compressibility of the fluid, through the frequency dependence of
9 the influence matrix.

10 It is to be understood that the present invention is by no
11 means limited to the particular steps herein disclosed and/or
12 shown in the drawings, but also comprises any modifications or
13 equivalents.

1 N.C. 77230

2
3 A METHOD FOR DETERMINING THE APPROXIMATE
4 RESONANCE FREQUENCY OF A STRUCTURE
5 SURROUNDED BY A COMPRESSIBLE FLUID
6

7 ABSTRACT OF THE DISCLOSURE

8 A method for determining the approximate resonance frequency
9 of a structure surrounded by a compressible fluid includes the
10 steps of: (1) performing an in-vacuo eigenvalue analysis to
11 determine in-vacuo frequencies and mode shapes of the structure;
12 (2) selecting one of the mode shapes as an in-vacuo mode of
13 interest, (3) computing an influence matrix at the eigenfrequency
14 of the selected mode of interest; (4) combining the computed
15 influence matrix with structural stiffness and mass matrices from
16 a finite element program, and using the modified matrices
17 computing eigenvalues of the structure, including eigenvectors;
18 (5) selecting from the computed eigenvectors a computed mode
19 having substantially the same displaced shape as the in-vacuo
20 mode of interest; (6) determining the eigenfrequency of the
21 selected computed mode; (7) determining any difference between
22 the computed eigenfrequency of the selected computed mode and the
23 in-vacuo eigenfrequency of the selected in-vacuo mode of
24 interest; wherein (8) if the difference is equal to, or less
25 than, a selected tolerance, the computed eigenfrequency of the
26 computed mode is the approximate resonance frequency of the

1 structure; and (9) if the difference is greater than the
2 tolerance, repeating step (3) substituting the computed
3 eigenfrequency of the selected computed mode for the in-vacuo
4 eigenfrequency for the in-vacuo mode of interest, and thereafter,
5 repeating steps (4) - (9), substituting in step (3) a most recent
6 computed eigenfrequency of the selected computed mode for the
7 previously used computed eigenfrequency of the selected computed
8 mode.

COMPARISON OF IN VACUO EIGENFREQUENCIES OF A SPHERICAL SHELL,
COMPUTED FROM ANALYTIC AND FINITE ELEMENT MODELS

MODE NUMBER/BRANCH	ANALYTIC (HZ)	FINITE ELEMENT (HZ)	% DIFFERENCE
2/LOWER	591	591	0.0
3/LOWER	700	700	0.0
4/LOWER	742	744	0.3
0/UPPER	1357	1360	0.2

FIG. 1

COMPARISON OF UNDAMPED IN-WATER EIGENFREQUENCIES OF A SPHERICAL
SHELL, COMPUTED FROM ANALYTIC AND FINITE ELEMENT/
BOUNDARY ELEMENT (FE/BE) MODELS

MODE NUMBER/BRANCH	ANALYTIC EIGENFREQUENCY (HZ)	FE/BE EIGENFREQUENCY (HZ)	% DIFFERENCE	% AT ANALYTIC EIGENFREQUENCY
2/LOWER	268	272	1.5	1.1
3/LOWER	330	342	3.6	1.4
4/LOWER	389	391	0.5	1.6
0/UPPER	1028	1069	4.0	4.3

FIG. 2

COMPARISON OF UNDAMPED IN-WATER EIGENFREQUENCIES WITH THE UNDAMPED IN-WATER
HARMONIC RESONANCE FREQUENCIES FOR A SPHERICAL SHELL,
BOTH COMPUTED WITH THE FE/BE MODEL

MODE NUMBER/BRANCH	EIGENFREQUENCY (HZ) [Re{Z}=0]	RESONANCE FREQUENCY (HZ) [Re{Z}=0]	% DIFFERENCE
2/LOWER	272	272	0.4
3/LOWER	342	342	0.0
4/LOWER	391	391	0.0
0/UPPER	1069	1049	1.9

FIG. 3

COMPARISON OF IN-WATER HARMONIC RESONANCE FREQUENCIES FOR A SPHERICAL SHELL,
COMPUTED USING THE FE/BE MODEL, WITH AND WITHOUT RADIATION DAMPING

MODE NUMBER/BRANCH	RESONANCE FREQUENCY (HZ) [Re{Z}=0]	RESONANCE FREQUENCY (HZ) [COMPLEX Z]	% DIFFERENCE
2/LOWER	271	267	1.5
3/LOWER	342	341	0.3
4/LOWER	391	391	0.0
0/UPPER	1049	1049	0.0

FIG. 4

COMPARISON OF IN-WATER EIGENVALUES FOR A SPHERICAL SHELL COMPUTED USING THE FE/BE MODEL, WITH THE IN-WATER MODEL TECHNIQUE, NEGLECTING RADIATION DAMPING; AND THE INCOMPRESSIBLE APPROXIMATION (NO RADIATION DAMPING AND THE MASS LOADING COMPUTED AT $ka=0$)

MODE NUMBER/BRANCH	IN-WATER EIGENFREQUENCY (HZ) [ITERATIVE METHOD]	IN-WATER EIGENFREQUENCY (HZ) [INCOMPRESSIBLE, $ka=0$]	% DIFFERENCE	ka OF IN-WATER EIGENVALUE [ITERATIVE METHOD]
2/LOWER	272	291	+7.0	1.1
3/LOWER	342	359	+5.0	1.4
4/LOWER	391	404	+3.3	1.6
0/UPPER	1069	361	-66.0	4.5

FIG. 5

COMPARISON OF IN VACUO AND IN-WATER EIGENFREQUENCIES, AND IN-WATER HARMONIC RESONANCE FREQUENCIES, FOR AXISYMMETRIC FE(/BE) PROJECTOR MODEL

MODE NUMBER	IN VACUO EIGENFREQUENCY (HZ)	IN-WATER EIGENFREQUENCY (HZ)	IN-WATER HARMONIC PEAK FREQUENCY (HZ) [$\text{Re}\{Z\}=0$] [COMPLEX Z]	ka OF IN-WATER EIGENVALUE
1	84	38	38	0.16
2	456	307	303	1.30
4	1092	852	851	2.80
5	1904	1685	1682	7.00

FIG. 6

CONVERGENCE OF IN-WATER EIGENVALUES USING AXISYMMETRIC FE/BE PROJECTOR MODEL

STARTING MODE NUMBER	STARTING VALUE (HZ)	ITERATION 1 (HZ)	ITERATION 2 (HZ)	ITERATION 3 (HZ)	ENDING MODE NUMBER
1	84.0	38.6	38.4	---	1
2	456.0	310.6	307.3	307.4	2
4	1092.0	846.6	851.7	851.5	4
5	1904.0	1666.7	1686.2	1684.2	6

FIG. 7

IN VACUO AND IN-WATER EIGENFREQUENCIES FOR 3-D FE/BE PROJECTOR MODEL

MODE NUMBER	IN VACUO EIGENFREQUENCY (HZ)	IN-WATER EIGENFREQUENCY (HZ)	IN-WATER HARMONIC PEAK FREQUENCY (HZ) [Re{Z}=0] [COMPLEX Z]	ka OF IN-WATER EIGENVALUE
5	90	43	41	0.18

FIG. 8

CONVERGENCE OF FUNDAMENTAL DISK MODE USING 3-D PROJECTOR MODEL

STARTING VALUE (HZ)	ITERATION 1 (HZ)	ITERATION 2 (HZ)
90.0	42.9	43.0

FIG. 9

COMPARISON OF CPU TIME FOR HARMONIC AND MODIFIED MODAL COMPUTATIONS FOR FUNDAMENTAL DISK MODE OF 3-D PROJECTOR MODEL

ANALYSIS	CPU SECONDS PER FREQUENCY/ITERATION	NUMBER OR FREQUENCIES/ITERATIONS	TOTAL CPU SECONDS
HARMONIC	2234	10	22,340
MODIFIED MODAL	907	2	1,814

FIG. 10

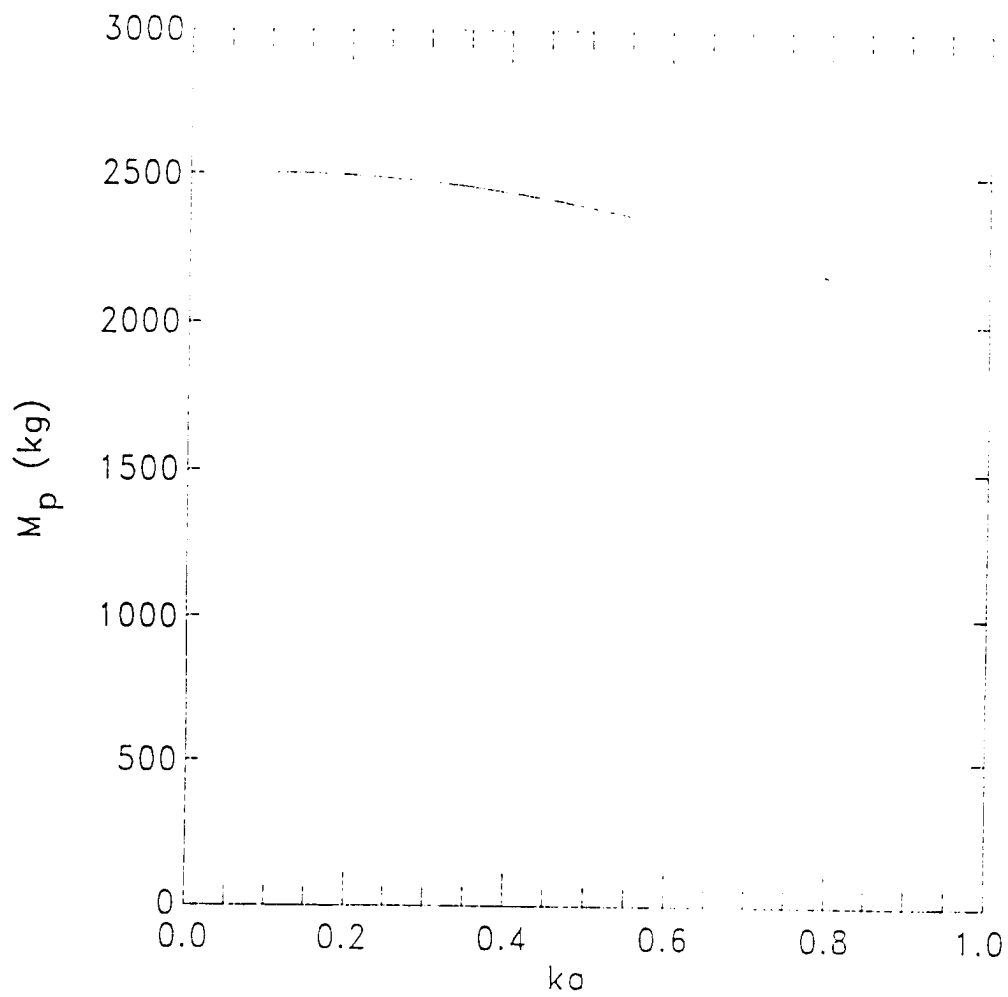


FIG. 11

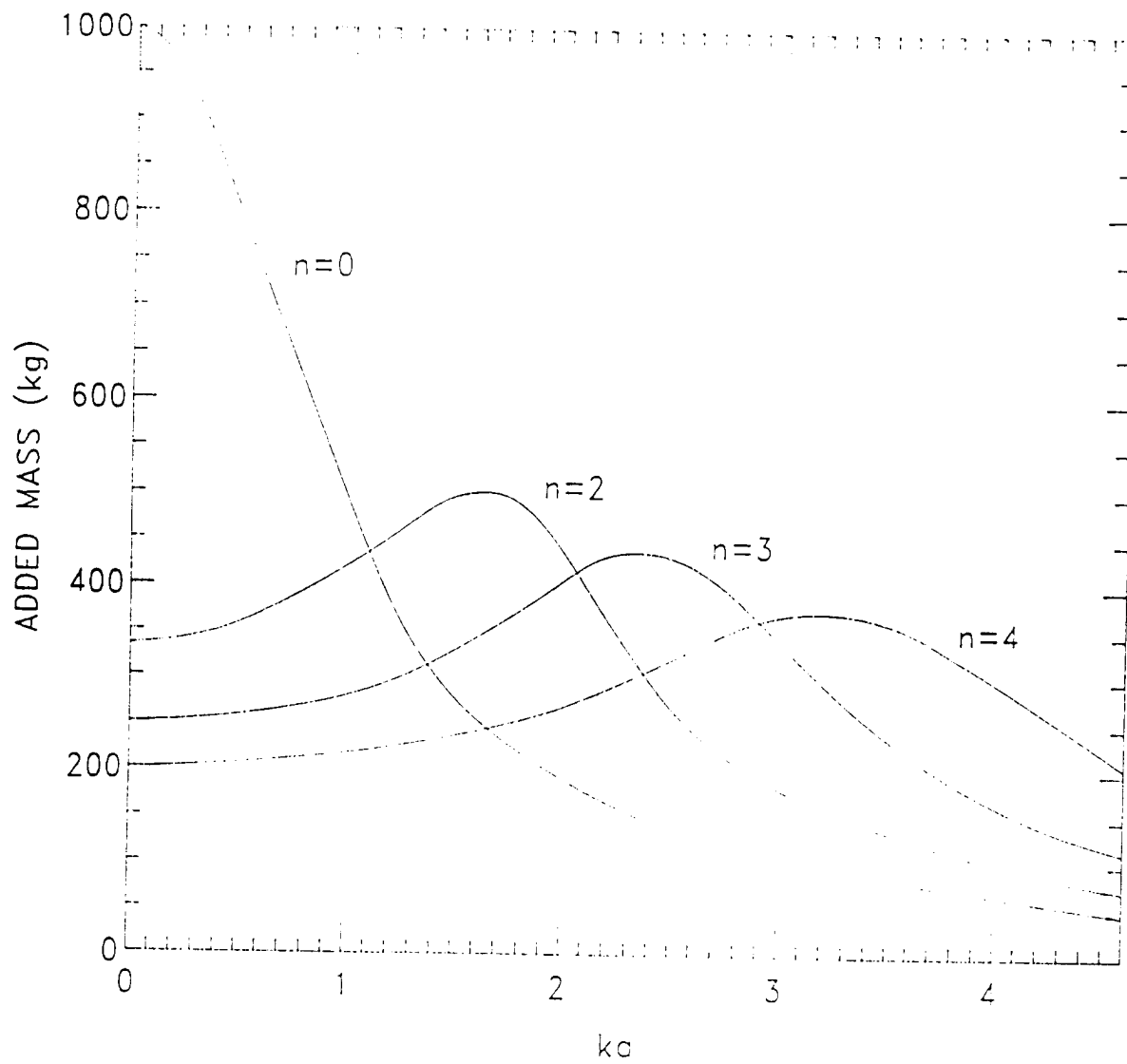


FIG. 12

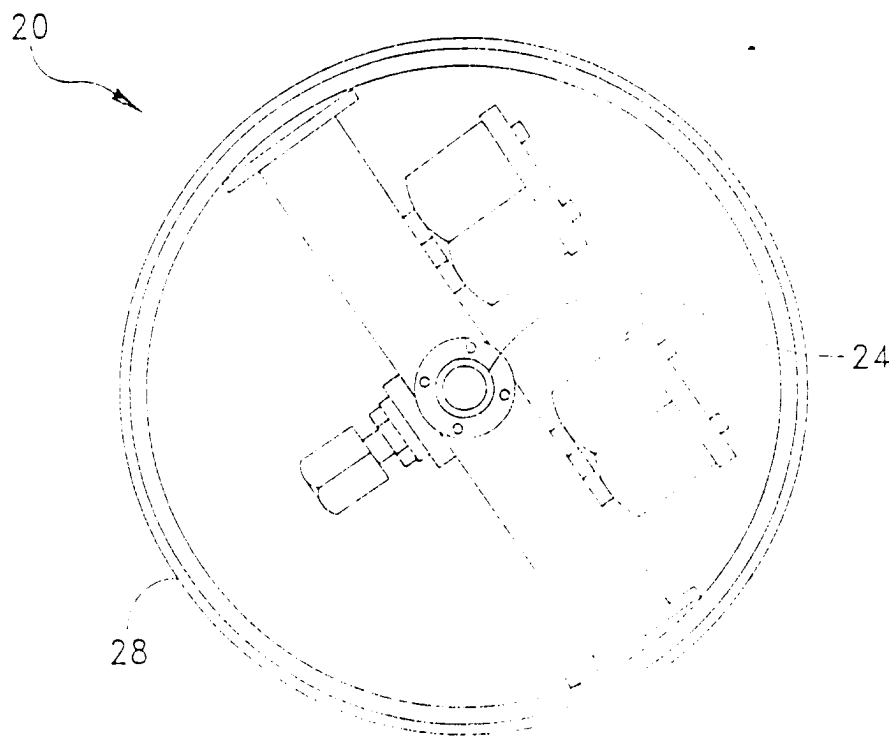


FIG. 13

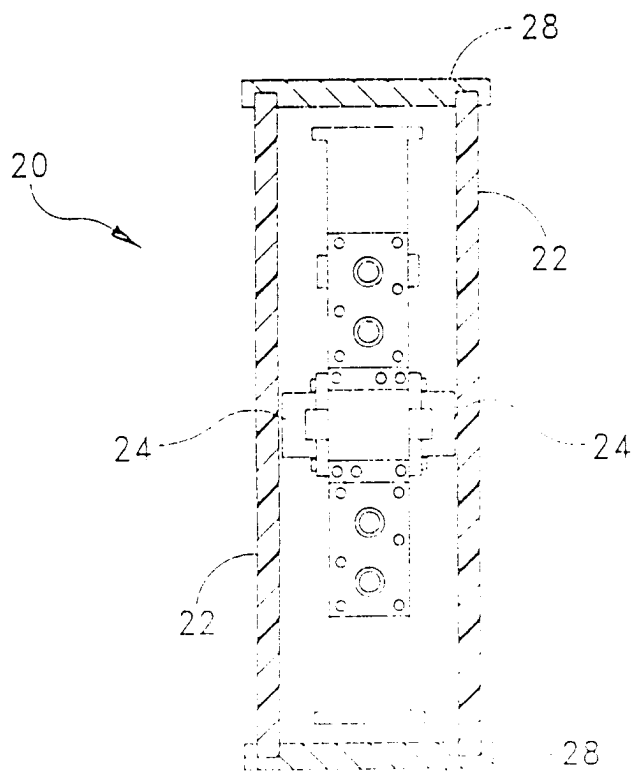


FIG. 14

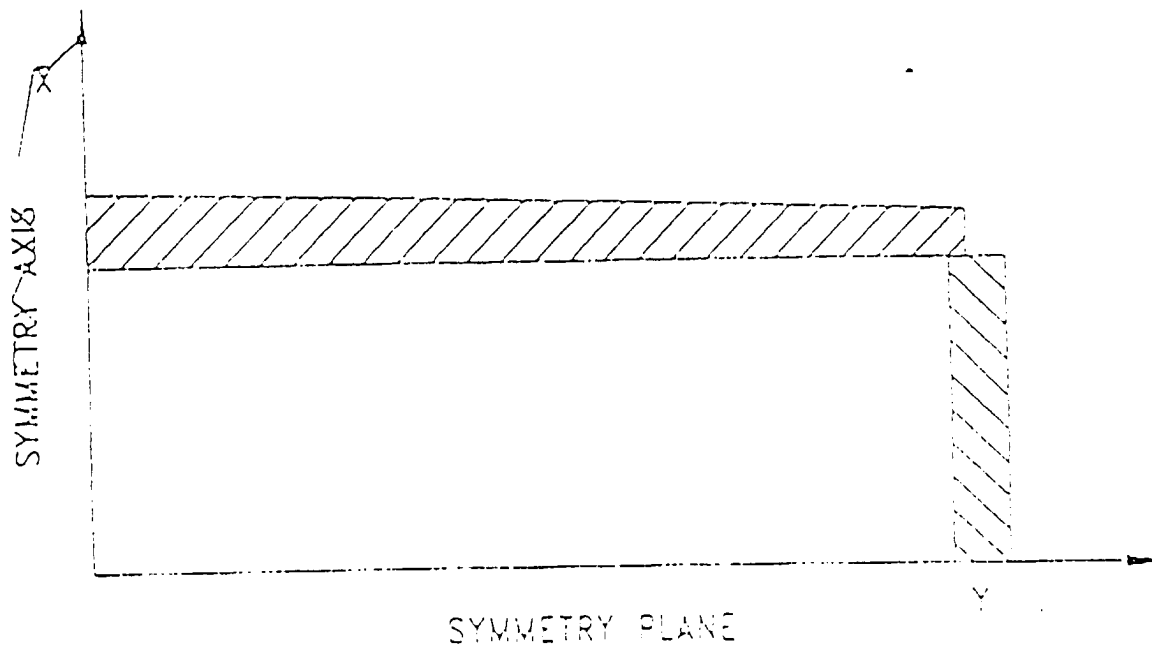


FIG. 15

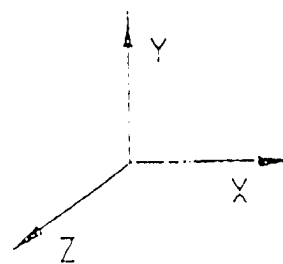
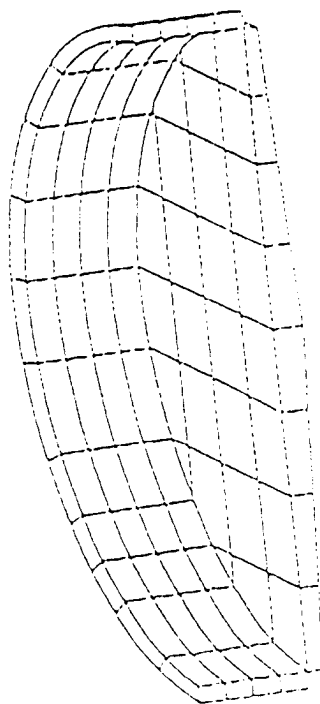


FIG. 16

COMPARISON OF IN VACUO EIGENFREQUENCIES OF A SPHERICAL SHELL,
COMPUTED FROM ANALYTIC AND FINITE ELEMENT MODELS

MODE NUMBER/BRANCH	ANALYTIC (HZ)	FINITE ELEMENT (HZ)	% DIFFERENCE
2/LOWER	591	591	0.0
3/LOWER	700	700	0.0
4/LOWER	742	744	0.3
0/UPPER	1357	1360	0.2

FIG. 1

COMPARISON OF UNDAMPED IN-WATER EIGENFREQUENCIES OF A SPHERICAL
SHELL, COMPUTED FROM ANALYTIC AND FINITE ELEMENT/
BOUNDARY ELEMENT (FE/BE) MODELS

MODE NUMBER/BRANCH	ANALYTIC EIGENFREQUENCY (HZ)	FE/BE EIGENFREQUENCY (HZ)	% DIFFERENCE	ka AT ANALYTIC EIGENFREQUENCY
2/LOWER	268	272	1.5	1.1
3/LOWER	330	342	3.6	1.4
4/LOWER	389	391	0.5	1.6
0/UPPER	1028	1069	4.0	4.3

FIG. 2

COMPARISON OF UNDAMPED IN-WATER EIGENFREQUENCIES WITH THE UNDAMPED IN-WATER
HARMONIC RESONANCE FREQUENCIES FOR A SPHERICAL SHELL,
BOTH COMPUTED WITH THE FE/BE MODEL

MODE NUMBER/BRANCH	EIGENFREQUENCY (HZ) [Re{Z}=0]	RESONANCE FREQUENCY (HZ) [Re{Z}=0]	% DIFFERENCE
2/LOWER	272	272	0.4
3/LOWER	342	342	0.0
4/LOWER	391	391	0.0
0/UPPER	1069	1049	1.9

FIG. 3

COMPARISON OF IN-WATER HARMONIC RESONANCE FREQUENCIES FOR A SPHERICAL SHELL,
COMPUTED USING THE FE/BE MODEL, WITH AND WITHOUT RADIATION DAMPING

MODE NUMBER/BRANCH	RESONANCE FREQUENCY (HZ) [Re{Z}=0]	RESONANCE FREQUENCY (HZ) [COMPLEX Z]	% DIFFERENCE
2/LOWER	271	267	1.5
3/LOWER	342	341	0.3
4/LOWER	391	391	0.0
0/UPPER	1049	1049	0.0

FIG. 4

COMPARISON OF IN-WATER EIGENVALUES FOR A SPHERICAL SHELL COMPUTED USING THE FE/BE MODEL, WITH THE IN-WATER MODEL TECHNIQUE, NEGLECTING RADIATION DAMPING; AND THE INCOMPRESSIBLE APPROXIMATION (NO RADIATION DAMPING AND THE MASS LOADING COMPUTED AT $k\alpha=0$)

MODE NUMBER/BRANCH	IN-WATER EIGENFREQUENCY (HZ) [ITERATIVE METHOD]	IN-WATER EIGENFREQUENCY (HZ) [INCOMPRESSIBLE, $k\alpha=0$]	% DIFFERENCE	$k\alpha$ OF IN-WATER EIGENVALUE [ITERATIVE METHOD]
2/LOWER	272	291	+7.0	1.1
3/LOWER	342	359	+5.0	1.4
4/LOWER	391	404	+3.3	1.6
0/UPPER	1069	361	-66.0	4.5

FIG. 5

COMPARISON OF IN VACUO AND IN-WATER EIGENFREQUENCIES, AND IN-WATER HARMONIC RESONANCE FREQUENCIES, FOR AXISYMMETRIC FE(/BE) PROJECTOR MODEL

MODE NUMBER	IN VACUO EIGENFREQUENCY (HZ)	IN-WATER EIGENFREQUENCY (HZ)	IN-WATER HARMONIC PEAK FREQUENCY (HZ) [$\text{Re}\{Z\}=0$] [COMPLEX Z]	$k\alpha$ OF IN-WATER EIGENVALUE
1	84	38	38	0.16
2	456	307	307	1.30
4	1092	852	851	2.80
5	1904	1685	1682	7.00

FIG. 6

CONVERGENCE OF IN-WATER EIGENVALUES USING AXISYMMETRIC FE/BE PROJECTOR MODEL

STARTING MODE NUMBER	STARTING VALUE (HZ)	ITERATION 1 (HZ)	ITERATION 2 (HZ)	ITERATION 3 (HZ)	ENDING MODE NUMBER
1	84.0	38.6	38.4	---	1
2	456.0	310.6	307.3	307.4	2
4	1092.0	846.6	851.7	851.5	4
5	1904.0	1666.7	1686.2	1684.2	6

FIG. 7

IN VACOU AND IN-WATER EIGENFREQUENCIES FOR 3-D FE/BE PROJECTOR MODEL

MODE NUMBER	IN VACOU EIGENFREQUENCY (HZ)	IN-WATER EIGENFREQUENCY (HZ)	IN-WATER HARMONIC PEAK FREQUENCY (HZ) [$\text{Re}\{Z\}=0$] [COMPLEX Z]	k_a OF IN-WATER EIGENVALUE
5	90	43	41	0.18

FIG. 8

CONVERGENCE OF FUNDAMENTAL DISK MODE USING 3-D PROJECTOR MODEL

STARTING VALUE (HZ)	ITERATION 1 (HZ)	ITERATION 2 (HZ)
90.0	42.9	43.0

FIG. 9

COMPARISON OF CPU TIME FOR HARMONIC AND MODIFIED MODAL COMPUTATIONS FOR FUNDAMENTAL DISK MODE OF 3-D PROJECTOR MODEL

ANALYSIS	CPU SECONDS PER FREQUENCY/ITERATION	NUMBER OR FREQUENCIES/ITERATIONS	TOTAL CPU SECONDS
HARMONIC	2234	10	22,340
MODIFIED MODAL	907	2	1,814

FIG. 10

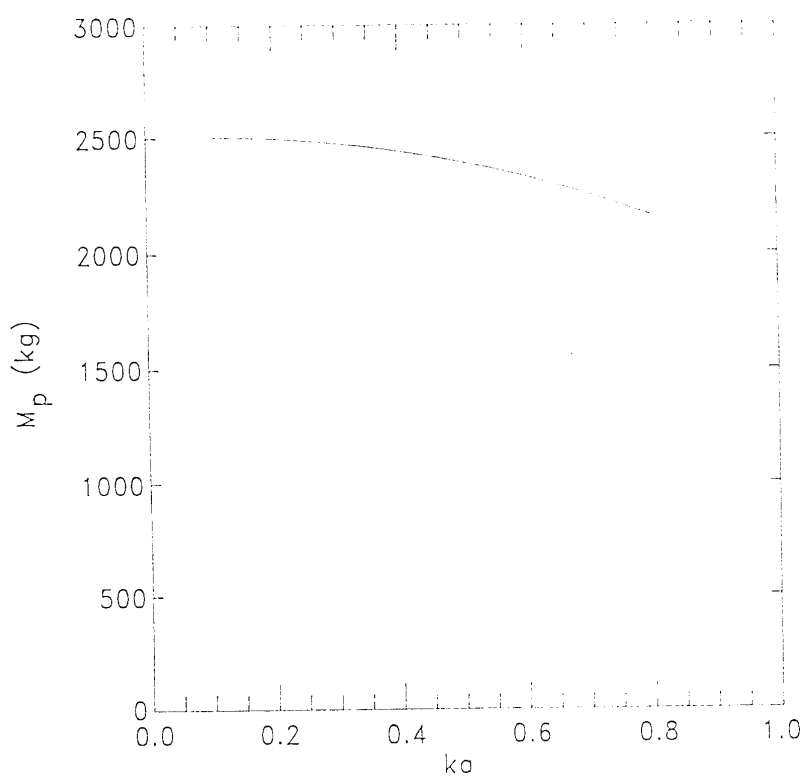


FIG. 11

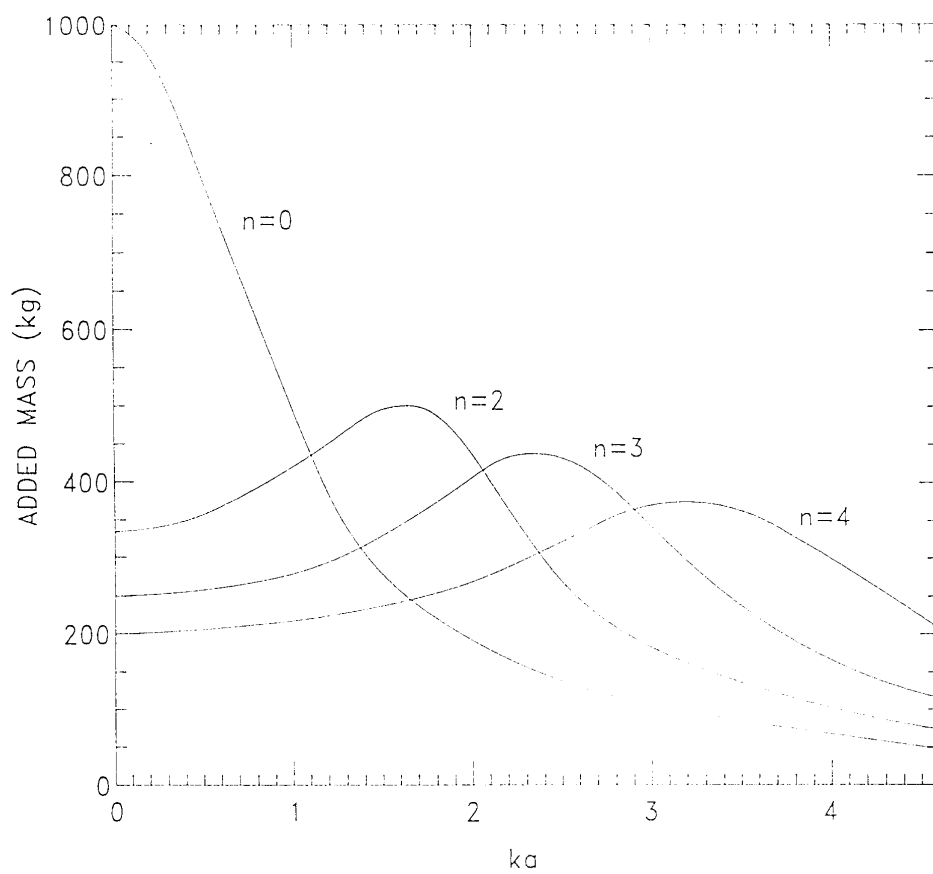


FIG. 12

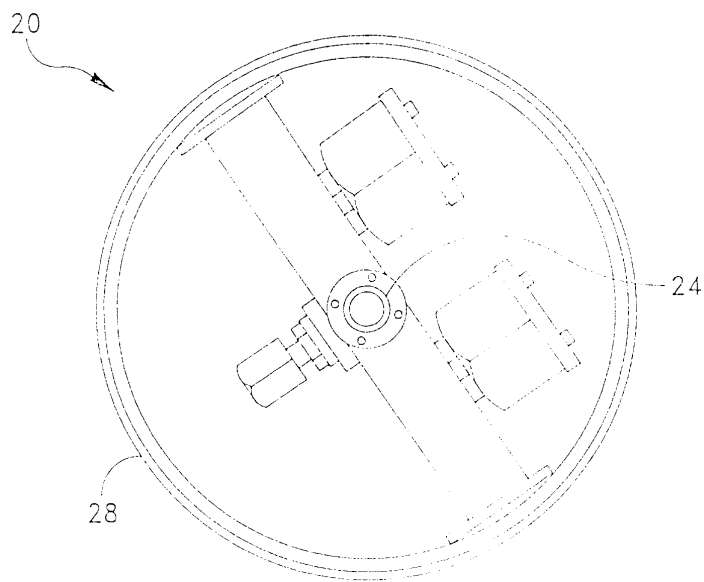


FIG. 13

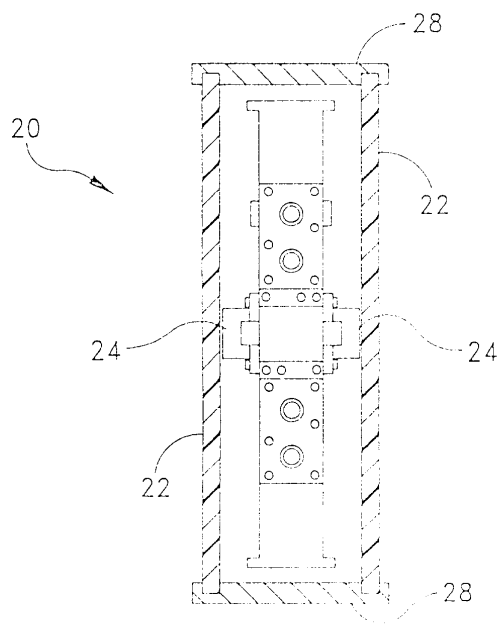


FIG. 14

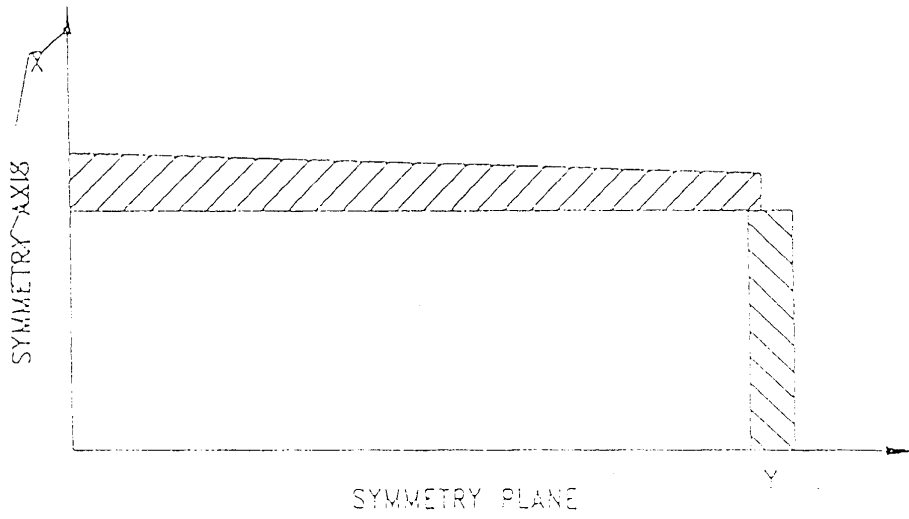


FIG. 15

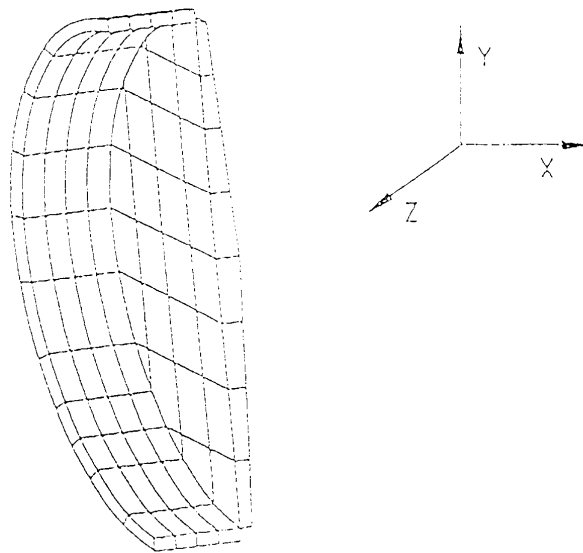


FIG. 16