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AND TECHNICAL DEVELOPMENT

FRAGMENTATION PANEL OF THE
STATIC DETONATION COMMITTEE.

A Theory of Fragmentation

by

N.F. Mott and E.H. Linfoot

Extra-mural Research No. F72/80

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A THEORY OF FRAGMENTATION

by
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Bristol University Extra-Mural Group.

SUMMARY. A tentative theory is given to account for the mean fragment sizes of certain types of bomb and shell, and for the relative numbers of large and small fragments.

1. THE MEAN FRAGMENT SIZE. The theory given here is applicable only to casings which expand plastically before rupture. This may not be the case for brittle materials such as cast iron.

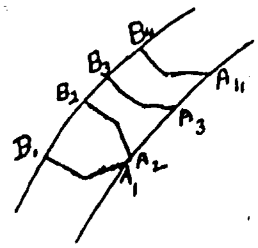


Fig. 1

We consider first fragmentation of the type occurring in the 3.7 inch A.A. shell. The larger fragments appear from inspection to be formed as shown in fig. 1, which represents a section through part of the casing. Cracks start on the inside, at such points as A_1, A_2, A_3 .... and spread outwards to B_1, B_2, B_3. This type of break-up has been discussed in Report No. 2232 from the Dept. of Metallurgy of the University of Sheffield, Ref. A.C.3098. The widths of typical fragments are of the order 1 cm; the length, parallel to the axis of the shell, is considerably greater.

At the moment of rupture, let r be the radius of the shell casing, t its thickness and V the velocity with which it is moving outwards. We suppose that rupture takes place when work-hardening has proceeded to such an extent that a crack will propagate itself with the expenditure of less energy than further plastic flow. Suppose that the casing then splits along two lines distant a apart; the cracks are represented by AB, A'B' in fig. 2, which, like fig. 1, represents a cross section through the shell casing. A splinter of cross section ABB'A' is then flying outwards with velocity V.

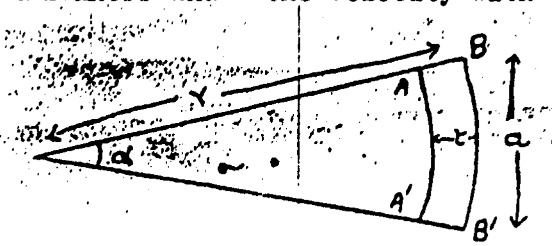


Fig. 2

The top surface AB of the fragment will have, in addition to the large outward velocity V, a velocity at right angles to it of amount $\frac{1}{2}V\alpha$, where $\alpha = a/r$. Similarly the bottom surface A'B' will have a downward velocity of the same amount. Referred to axes moving with the fragment, the metal will have kinetic energy, per unit length parallel to the axis of the shell, equal to

$$\frac{1}{2}\rho t V^2 \int_{-\frac{1}{2}\alpha}^{\frac{1}{2}\alpha} r\theta^2\, d\theta = \frac{1}{24}\rho t r \alpha^3 V^2$$

where ρ is the density of the metal. Since $r\alpha = a$, this becomes

$$\frac{1}{24} t V^2 \rho a^3/r^2 \qquad (1)$$

We now make the assumption that if the energy (1) is great enough to form a new crack through the fragment, it will do so, and the fragment will break into two. If W is the energy per unit area required to form a crack, the energy required for this is Wt. Thus no fragment will be formed with thickness a greater than that given by equating Wt to (1), which gives

$$a = \left[\frac{24 r^2 W}{\rho V^2}\right]^{\frac{1}{3}} \qquad (2)$$

For W we may take a value given by impact tests; according to Southwell (Trans. Manchester Assoc. of Engineers, 1937) this ranges from 70 to 800 ft/lbs. per sq. inch. We should take a value appropriate to the metal at the moment of rupture, i.e. after plastic deformation, when it will be very brittle. We therefore take the lower value, 70 ft/lbs. It is realised that the energy of rupture is not, in practice, proportional to the area, so our value will be very approximate. Moreover heating of the metal during its expansion may have an effect. Fortunately, since W occurs as $W^{1/3}$, the value of a is not very sensitive to the value of W. A measurement of the rupture energy for cold-worked H.E. steel would be of interest*.

* It is of interest to compare the much smaller rupture energy for a brittle substance such as quartz, which from experiments on grinding sand appears to be of the order 61 ft/lbs. per sq. ft. (Martin, Trans. Ceramic Society, 23, 61, 1923).

For r we take 2.2 inches, and for V, the velocity of the fragments, 2500 ft/sec. We obtain for a

$$a = 0.55 \text{ inches}$$

in good agreement with the observed value.

For steels where fracture is due to shear we have no information from which the magnitude of W can be estimated.

We have not been able to find a theory to account for the average <u>length</u> of the splinters in this type of shell. For shells or bombs which bulge out in the middle before breaking up, the dimension parallel to the axis might be determined by the same mechanism, r being the radius of curvature of an axial section of the casing.

We may use formula (2) to compare the mean fragment sizes of bombs with different charge-weight ratios, sizes etc., Since, however, we have no theory of what determines the lengths of the splinters from a shell, we confine ourselves to a bomb which, at the moment of bursting, is roughly spherical. Then we can take the mean weight of a fragment to be proportional to $\rho a^2 t$, and thus to

$$\rho^{\frac{1}{3}} r^{\frac{4}{3}} W^{\frac{2}{3}} V^{-\frac{4}{3}} t .$$

If r_o, t_o refer to the bomb before expansion, and r, the radius at the moment of burst is equal to εr_o, then $t = t_o/\varepsilon^2$, so that the mean fragment weight is proportional to

$$r_o^{\frac{4}{3}} \rho^{\frac{1}{3}} t_o W^{\frac{2}{3}} / V^{\frac{4}{3}} \varepsilon^{\frac{2}{3}} \qquad (3)$$

If we keep the charge constant and vary the thickness t_o, we expect for heavy casings that V^2 will be proportional to $1/t_o$; thus the average weight of fragment is proportional to $t_o^{5/3}$ if ε is constant; actually, however, thick cased shells expand further than thin ones before breaking up, so we expect a rather less rapid variation with t_o than this.

2. <u>DISTRIBUTION OF FRAGMENT WEIGHTS.</u>

It was pointed out to the present authors by Dr. B.L.Welch (private communication dated 24th Sept.1941) that the distributions of fragments from two such different projectiles as the 3" U.P. (initial fragment velocity 4500 ft/sec.) and the 3.7" A.A. shell (fragment velocity about 2500 ft/sec) can be fitted approximately to the same law. This law is the following : if N(m)dm is the number of fragments with weights between m and m + dm, then

$$N(m)\, dm = C e^{-M/M_o}\, dM \qquad (4)$$

where $M = m^{1/3}$ and C and M_o are constants. For the shell and the U.P., M_o has respectively the values (in $(\text{ounces})^{1/3}$)

	3.7" shell	3" U.P.
M_o	0.33	0.15

The agreement is shown below :-

Wt. oz.	Shell		U.P.	
	obs.	calc.	obs.	calc.
1/50 - 1/25	not recovered		570	528
1/25 - 1/4	452	454	751	793
1/4 - ½	131	129	93	101
½ - 4	193	181	64	56
4 - 8	5	13	0	0
> 8	1	5		

The total number of fragments is CM_0 and the total weight $6M_0^4C$, so the average weight is $6M_0^3$, or 0.21 ounces for the 3.7 inch shell. The distribution is very skew, however, so that there are a large number of fragments with weights considerably greater than the average.

This observed distribution law suggested a theoretical explanation along the following lines : $m^{1/3}$ is proportional to the mean linear dimension of a fragment, and if this is written x, it suggests that the number of fragments with lengths between x and x + dx is given by

$$C e^{-x/x_0} dx$$

Such a formula can be derived for a rod or line broken up at random in one dimension only. Consider a line AB of length ℓ, cut at random into n + 1 pieces; each cut is independent of the positions of all the others and is equally likely to be at any point between A and B. Consider then any interval ξ of the line. The average number of cuts that it contained is $n\xi/\ell$ and the chance that it does not contain one at all is

$$e^{-n\xi/\ell}$$

Consider then any one cut, and let us calculate the chance that the next cut to the right is in an interval dx at a distance x; this is

$$e^{-nx/\ell} \frac{n\,dx}{\ell}$$

Thus the number intervals of lengths between x and x + dx is

$$\frac{\ell}{x_0^2} e^{-x/x_0} dx \quad , \quad x_0 = \frac{\ell}{n} \tag{5}$$

This immediately suggests that (4) is a three dimensional analogue of (5). We might expect that if a solid is broken up "at random", e.g. by planes cut at random through it, the distribution of fragment weights will be given, at any rate approximately, by (4). Unfortunately we have been unable to prove this; a mathematical discussion is given in Section 3.

Inspection shows, however, that for the 3.7" shell fragments of weight greater than about half an ounce usually have part of the original inner and outer surfaces on them; thus we should expect that, for the heavier fragments at any rate a distribution law of the type

$$N dm = C e^{-\alpha m^{1/2}} d(m^{1/2}) \tag{6}$$

would give a better fit than (4). It was in fact found that for this shell and for the 4.7" A.A. shell and 3" U.P., either formula (4) or (6) would give an equally good agreement for fragments of medium size, and that (6) was somewhat better for the largest fragments.

For a detailed comparison with experiment, Dr. Payman's results with model bombs are the most suitable, because they include an analysis of fragments down to one milligram. We should expect to get the most exact fit with (6), and the greatest divergence from (4), for very thin casings. Fig. 3 shows the fragmentation of a model bomb with casing of thickness 0.018" filled with tetryl (W. Payman, Fragmentation Report IV, R.C.276). The quantity v, of which the logarithm is plotted as ordinate, is the number of fragments between two given weights m_1 and m_2, divided by the interval $(m_2^{1/2} - m_1^{1/2})$, or $m_2^{1/3} - m_1^{1/3}$, according to the method of plotting; the abscissae are the mean of the extreme masses, namely $\frac{1}{2}(m_1^{1/2} + m_2^{1/2})$ or $\frac{1}{2}(m_1^{1/3} + m_2^{1/3})$. It will be seen that the fit with formula (6) is much better than with formula (4). The weights are here in grammes.

Fig. 4 shows what happens for a much thicker casing 0.3 inches thick. It will be seen that formula (6) gives fair agreement for the larger fragments, but that there are too many very small ones. This is to be expected, because small fragments will be broken off the ends or edges of the large ones.

The slopes of all these curves, plotted according to formula (6), give what seems to us the best indication of the mean linear size. The quantity a of formula (5) might be equated to $1/\alpha\sqrt{t}$.

We have not, however, attempted at this stage to compare formulae such as (3) with the mean fragment weight of any bomb or shell, because our theory is incomplete, as it does not account for the length of splinters from shells, but only for their breadth, and for bombs which do not give long splinters we have not been able to find experimental information about mean weights and speeds. Further, a direct comparison with theory would only be possible where most fragments are projected under the same conditions, e.g. from a long cylinder detonated from one end, or a spherical bomb detonated in the middle.

5. MATHEMATICAL DISCUSSION OF THE DISTRIBUTION LAW FOR FRAGMENT SIZES.

Distribution laws of the types (4) and (6) have been proposed in a number of papers for the weights or diameters of mineral particles after crushing, of sand particles and so on*. We do not know of any attempt to derive mathematically the two or three dimensional formulae.

* cf. Liemau. J. Franklin Inst. 1935, p.485, where other references are given.

We discuss first the case where a thin sheet is broken up into rectangular fragments by two sets of parallel lines. The analysis will be appropriate if a shell casing is broken up by cracks parallel to the axis at an average distance, say, x_0 apart, and the lengths have an average value y_0 independent of the breadth and are distributed according to the usual law. According to our assumptions, the number with breadths between x and x + dx is proportional to $\exp(-x/x_0)dx$, and the number with lengths between y and y + dy proportional to $\exp(-y/y_0)dy$. Thus the number per unit area with area greater than a^2 is given by

$$\frac{1}{(x_0 y_0)^2} \iint \exp\left[-\frac{x}{x_0} - \frac{y}{y_0}\right] dx\,dy$$

where the integration is for all positive values of xy for which $xy > a^2$. Integrating with respect to y we obtain

$$\frac{1}{x_0^2 y_0} \int_0^\infty \exp\left[-\frac{x}{x_0} - \frac{a^2}{x y_0}\right] dx$$

Putting $x = a\sqrt{\frac{x_0}{y_0}}\, e^{\theta}$, the integral becomes

$$\frac{1}{x_0 y_0}\, z \int_0^\infty e^{-z\cosh\theta} \cosh\theta \, d\theta$$

where $z = \frac{2a}{\sqrt{x_0 y_0}}$

This is equal to $-\frac{z}{x_0 y_0} K_1(z)$

Differentiating with respect to a, we find for the number of fragments for which a lies between a and a + da

$$\frac{2}{(x_0 y_0)^{3/2}}\, z K_0(z)\, da$$

for large z this behaves like

$$(\tfrac{1}{2}\pi z)^{\frac{1}{2}} e^{-z}$$

and for small z like

$$z \log z$$

The function $\log(z K_0(z))$ is plotted against z in fig. 5; it will be seen that it is nearly linear except for small z.

If a thin shell casing is broken up at random, and a denotes the square root of the area, and νda the number of fragments such that a lies between a and $a + da$, then a plot of $\log \nu$ against a should give a closer approximation to a straight line than fig. 5. The proof is as follows :

We must first define what we mean by "at random". We suppose that the sheet is cut by a large number of straight lines, of which the directions are random. Consider any one of these lines; then we may take it that a length L of this line is cut by L/x_0 other lines, and that the number of intervals of length between x and $x + dx$ is $L\, dx\, e^{-x/x_0}/x_0^2$. Also that $L \sin\theta\, d\theta/2x_0$ of these lines make an angle with it between θ and $\theta + d\theta$ $(0 \leq \theta \leq \pi)$.

If the fragments were all of the same shape, then we should have $\nu = \exp(-a/a_0)$ exactly. They are, however, much more nearly all the same shape than when the sheet is cut up by two parallel sets of lines, as was assumed above. Then, if one side of a fragment is very small, there is no particular likelihood that the other one is. On the other hand, with random fragmentation, any very small interval a on one of the lines cutting the sheet is probably one side of a small triangle of area of the order a^2. Thus we have more very small fragments than with the above distribution. A similar argument shows that we should have more very large ones. Thus if $\log \nu$ is plotted against a, a straighter line should be obtained than that shown in fig. 7.

We can prove that ν tends to a constant non-zero value as $a \to 0$. The very small fragments will nearly all be triangles. If θ, ϕ are the two angles of one of these triangles adjacent to a side of base x the area is

$$\tfrac{1}{2} x^2 / (\cot\theta + \cot\phi)$$

Thus the number of fragments with area less than a^2 is proportional to

$$\iiint e^{-x/x_0} \sin\theta \sin\phi \, dx\, d\theta\, d\phi$$

the integral being over all values of x, θ, ϕ such that

$$\tfrac{1}{2} x^2 < a^2 (\cot\theta + \cot\phi)$$

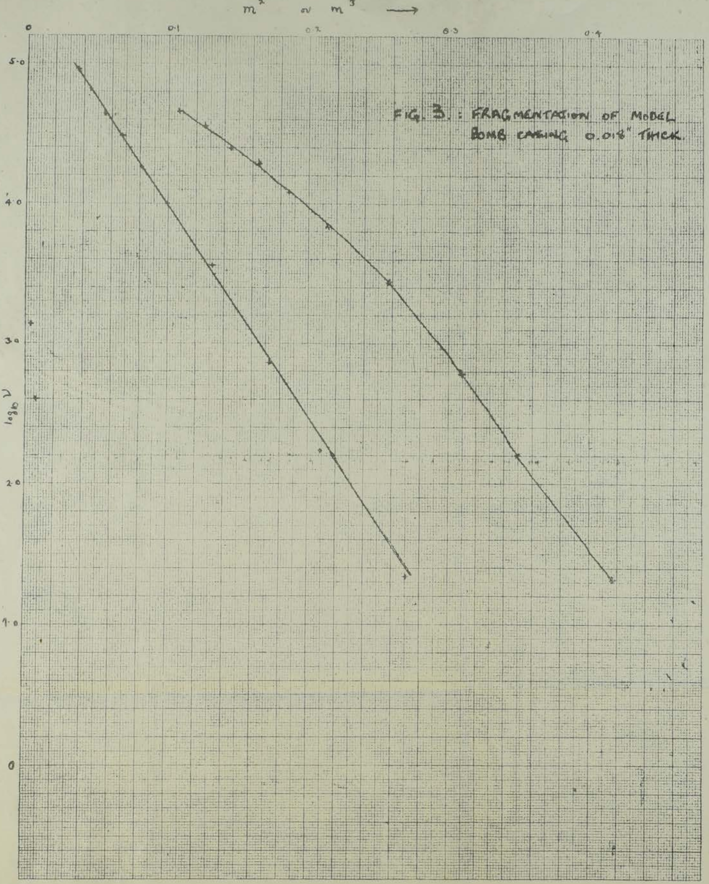
The integral becomes, on integrating with respect to x,

$$\int_0^{\pi} \int_0^{\pi-\theta} x_0 \left\{ 1 - e^{-\frac{a}{x_0}\left[2(\cot\theta + \cot\phi)\right]^{\frac{1}{2}}} \right\} \sin\theta\, d\theta \sin\phi\, d\phi$$

The first term in the expansion of this function in ascending powers of a is

$$a \iint \sqrt{2(\cot\theta + \cot\phi)} \sin\theta \sin\phi \, d\theta\, d\phi$$

which does not vanish. Thus $\nu(a)$ tends to a constant value as a tends to zero.



$m^{\frac{1}{2}}$ or $m^{\frac{1}{3}}$ ⟶
0
0·1
0·2
0·3
0·4
5·0
4·0
3·0
2·0
1·0
0
$\log_{10} \nu$
FIG. 3. : FRAGMENTATION OF MODEL
BOMB CASING 0.018" THICK.

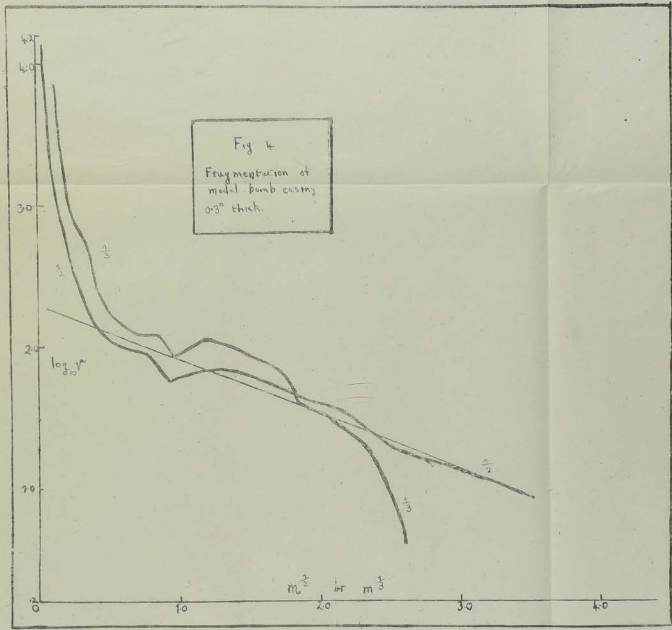


Fig 4
Fragmentation of
model bomb casing
0·3" thick.
4·2
4·0
3·0
2·0
1·0
·2
$\log_{10} \nu$
$\frac{1}{2}$
$\frac{1}{3}$
$m^{\frac{1}{2}}$ or $m^{\frac{1}{3}}$
0
1·0
2·0
3·0
4·0

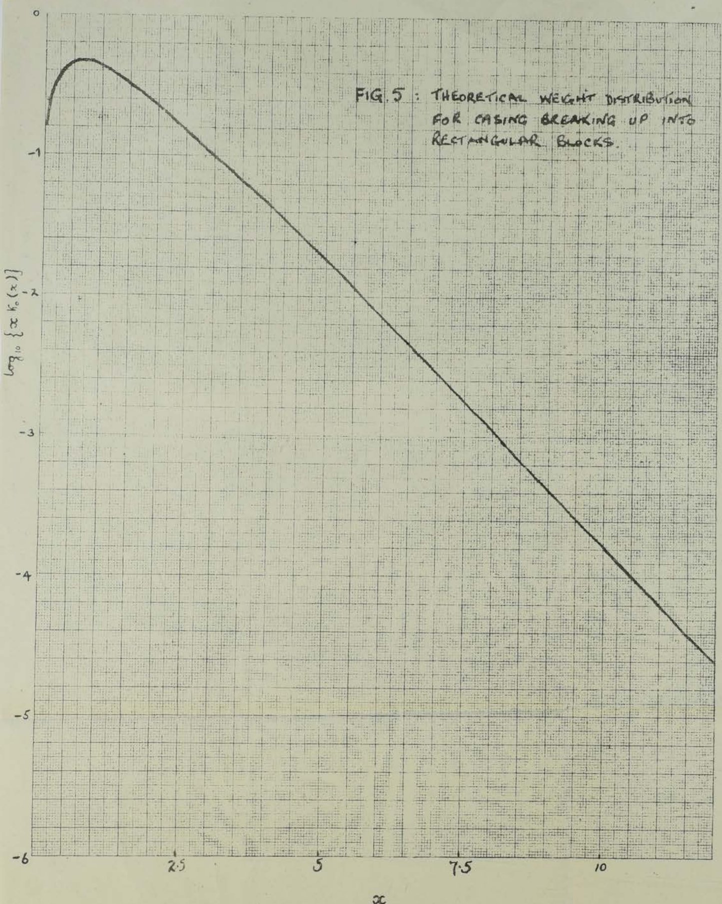


FIG. 5: THEORETICAL WEIGHT DISTRIBUTION FOR CASING BREAKING UP INTO RECTANGULAR BLOCKS.

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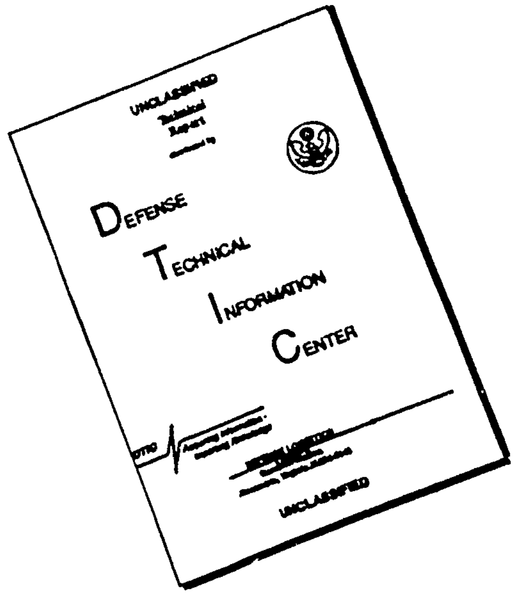

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