UNCLASSIFIED

AD NUMBER
ADB816330
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to DoD only; Administrative/Operational Use; APR 1942. Other requests shall be referred to David Taylor Model Basin, Washington, DC. Pre-dates formal DoD distribution statements. Treat as DoD only.**
AUTHORITY
DWTNSRDC ltr dtd 4 Aug 1978

THIS PAGE IS UNCLASSIFIED

UNCLASSIFIED

AD NUMBER
ADB816330
CLASSIFICATION CHANGES
TO: **unclassified**
FROM: **restricted**
AUTHORITY
E.O. 10501 dtd 5 Nov 1953

THIS PAGE IS UNCLASSIFIED

REEL-C 621 A

A.T.I 16566

THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

EFFECTS OF IMPACT ON SIMPLE ELASTIC STRUCTURES

BY J. M. FRANKLAND, Ph.D.

APRIL 1942

REPORT 481

PERSONNEL

This report was prepared mainly by Dr. J.M. Frenkland as an attempt to unify numerous detailed investigations by the Applied Mechanics Section of the Taylor Model Basin made during the past two years. Contributions have been made by many members of the technical staff, of whom may be mentioned W.J. Sette in connection with the analysis of the impact accelerometer, M. Greenfield, who performed many of the detailed mathematical analyses, and F. Mintz, who calculated many of the response curves. The Section is under the direction of Commander W.P. Roop, USN, who reviewed the report and introduced some modifications which could not be given further consideration by Dr. Frankland. The appendices, however, remain as they were prepared by Dr. Frankland.

REPORT 481

EFFECTS OF IMPACT ON SIMPLE ELASTIC STRUCTURES

BY J. M. FRANKLAND, Ph.D.

APRIL 1942

TABLE OF CONTENTS

DIGEST

The Bureau of Ships and the David Taylor Model Basin have been increasingly concerned with the determination of stresses in ship structure arising from rapidly applied and brief loads such as are caused by gun blast or gun recoil. This report attempts to present a general picture of the characteristic features of this type of loading, referred to here as impact or shock loading.

All structures possess many degrees of freedom but frequently one is so preponderant as to determine the behavior of the system for all practical purposes. A structure is considered here which possesses a single degree of freedom; it is exemplified by a rigid mass supported without friction and attached to an inertialess spring, as shown in Figure 1a.

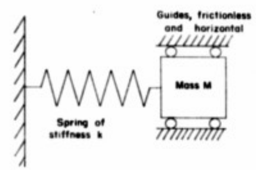


Figure 1a

System having one degree of freedom.

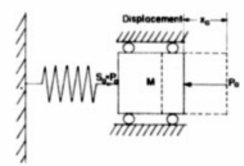


Figure 1b

System showing static displacement x_0 of the mass by a steady force P_0.

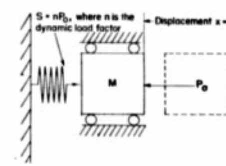


Figure 1c

System showing dynamic displacement x of the mass by a disturbing force P whose peak value is P_0.

Undamped Elastic System

Figure 1b illustrates the case of static loading on this system in which the force P_0 causes a displacement of the mass M and a shortening of the spring by the amount x_0. When the application of the force or load P is a function of time, as in Figure 1c, dynamic loading occurs; the displacement x and the spring force S can then be conveniently related to the corresponding values for static loading by the use of certain non-dimensional ratios.

The ratio between the applied force P at any moment during the dynamic loading period and the maximum value of the force P_0 applied to the system is designated as the ***disturbance factor*** or, more briefly, the ***disturbance***. As P never exceeds P_0, the maximum value of this factor is 1.

The static displacement x_0 of the mass M under the steady load P_0 can, as shown in Figure 1b, be used correspondingly as a unit of displacement.

Under sudden application of the force P, the displacement rises to a dynamic value z, as in Figure 1e; the ratio of this dynamic displacement z to the static displacement z_0 is called the ***response factor***, or, more briefly, the ***response***. As shown by a comparison of Figures 1b and 1c, the maximum value of this factor may greatly exceed **1**.

As the spring reaction S is assumed to follow Hooke's law, the response factor may be used to represent ratios of spring force or load as well as of displacement or deformation. Thus the reactive force S exerted by the spring at any time is the maximum force P_0 multiplied by the response factor at that inatant. The numerical maximum of the response factor, derived from the ratios z/z_0 or S/S_0, is the ***dynamic load factor*** ; it is the factor which, multiplied into the maximum load P_0, gives the maximum spring reactive force under the dynamic condition defined by the disturbance. Whereas S_0 always equals P_0 in static loading, the reactive force S exerted on and by the spring or equivalent supporting structure may greatly exceed the instantaneous value of the applied load P under dynamic loading conditions.

Tho factor **2**, to be found in all text books on mechanics for determining the force equivalent of a suddenly applied load, is a dynamic load factor. Knowledge of the dynamic load factor is a prerequisite to the deaign of a structure to resist a particular shock load.

If the maximum load on the spring is such that the elastic range of the material is not exceeded, the stress in the spring will be at all times proportional to the reactive force S. As it has previously been shown that the ratio S/S_0 is equal to the ratio z/z_0, the response factor, expressed by the latter ratio, can also be used as a ratio of stresses in the spring material. As the spring shown in Figure 1 can be replaced by an equivalent elastic structure having one degree of freedom, such as a long slender beam, it can be said that the maximum value of the response factor, equal to the dynamic load factor, givea the ratio of maximum stress in the beam under the shock or impact load to the stress set up in the beam by a static load P_0 equal to the peak load P in tho shock pulse, i.e., under the shock loading conditions.

Calculation of the response factor for several types of disturbance, by the formulas derived in Appendix 1, shows that the rate of application of the load and the duration of the load are of primary importance in a determination of the stresses set up in any particular structure. If the load is instantaneously applied and remains constant for a duration exceeding half the natural period T of vibration of the structure, the dynamic factor is **2**, that is, stresses are double those obtained on applying this load statically, i.e., very slowly. This corresponds to a duration of load P longer than that

required to give the mass M in Figure 1 a displacement of z_0, then to push it to a greater displacement z, and have it return to its initial position as shown in Figure 1a, as it will do under the influence of a maximum reactive force S much greater than P. If the duration t_1 falls below $T/2$, then in all cases the dynamic load factor decreases from **2** to **0** as the ratio t_1/T drops to zero.

If the load is not instantaneously applied and the time of rise t_0 to peak load is less than one-fourth the natural period T of the structure, then very nearly the maximum effect due to rate of application of load is realized; this is because the mass M in Figure 1 is traveling to the left for more than the entire duration of the increasing push exerted by the load. As t_0 becomes larger than $T/4$ the dynamic load factor progressively decreases.

For a disturbance similar to the first half-cycle of a sine wave, the largest dynamic load factor is for a duration t_1 of about one natural period, where the increase in stresses over the corresponding case of static loading is about 75 per cent. For the type of disturbance caused by gun blast, if it be assumed that the load rises instantaneously to its maximum, the dynamic load factor begins to decrease markedly if the duration of positive pressure falls bslow about $4T$. However, if the rise be not instantaneous, increasing to its maximum in the time t_0, then the dynamic load factor decreases at a rate determined by the ratios t_0/T and t_1/T.

In order that these results be applicable to an actual ship or other structure, it is necessary that the latter behave like a simple system with one degree of freedom of fundamental frequency. This requires that

(a) the duration of impact t_1 exceed 0.1 or more of the fundamental period T

(b) the impact load be distributed over the structure fairly uniformly

(c) the fundamental mode of the structure be uncoupled with the higher modes; in other words, the momentary deflection profile of the structure in its extreme position agrees with its profile under static load.

If the designer can satisfy himself that the nature of the structure and of the impact load is such as to make the use of a single dynamic load factor reasonably appropriate, the dynamic loading can be replaced by an equivalent static loading.

Although experimental evidence points to an increase in the yield and tensile strength of metals under extremely high rates of loading, these rates may be approached or reached only in particular parts of a structure, so that for the present a value of working stress based on the static strength properties of the material seems justified. A factor of safety on yield, depending upon the function of the structure, should be applied to the static yield strength to give such a working stress.

The measurement of impact loads is the converse of the problem so far discussed. Here a gage is acted upon by the disturbance and from the record of the response it is required to evaluate the disturbance. The interpretation of the response record depends upon the successful prediction of the dynamic behavior of the gage from its static characteristics; this can be achieved with a gage designed to approximate a system with one degree of freedom. The gage should have the highest natural frequency compatible with the desired sensitivity and its natural period of vibration should be as small as possible, certainly less than one-half the duration of the disturbance.

A structure which approximates a system with one degree of freedom will under a given impact experience at any point a combination of shears, bending moments, and axial forces, for all of which a single dynamic load factor suffices. When the system is one with many degrees of freedom, the dynamic load factor may vary with location in the structure and the factor for bending moment may not be the same as the factors for shear and axial loading. Appendix 2 considers a case of impact loading applied to a system having many degrees of freedom. The structure is a uniform, slender beam subjected to an instantaneously applied load. As the duration of the load and the area under load are varied it is found that the dynamic load factor may considerably exceed **2**, hence a careful preliminary analysis should form a part of the design of a structure of this kind.

EFFECTS OF IMPACT ON SIMPLE ELASTIC STRUCTURES

ABSTRACT

Elementary terms and factors concerned in the problem of impact loading are defined and a number of representative cases of impact loading and their effects upon a simple undamped system with one degree of freedom are discussed in Section 2. For non-oscillating disturbances these effects are governed mainly by the abruptness of original application and by the duration of the disturbance in terms of the natural period of vibration of the system. For oscillating disturbances there is an added influence similar to resonance in the steady-state condition.

Section 3 presents some means of judging to what degree the behavior of the idealized system of Section 2 is realized among actual ship and other structures.

Applications of the foregoing to the strength of structures under impact are considered in Section 4, with a discussion of the variations in yield and ultimate strength of metals under high rates of loading. Suggestions are made for a basis of design of decks, superstructure, and gun foundations under blast and gun-recoil loads.

The application of these principles to the design of instruments for making observations during impact tests and to an interpretation of their records is considered briefly in Section 5. In the appendices a formal mathematical treatment of the subject is presented.

1. INTRODUCTION

During recent years the Bureau of Ships and the David Taylor Model Basin have been increasingly concerned with problems involving impact loading, such as gun-blast and gun-recoil effects. The reaction of structures to these loads can be analyzed by well known methods (1) (2)* but the descriptive literature on the subject is scattered and scanty. Textbooks on mechanical vibrations treat mainly of the steady state finally reached under periodic disturbing forces, and the concern there is often with factors quite other than those which govern when the disturbance has the character of a blow. In consequence it is difficult to obtain a broad physical picture of the characteristic features of impact phenomena without recourse to the rather tedious calculation of a large number of specific problems. This report attempts to present such a general picture as obtained from the analysis of several typical cases.

* Numbers in parentheses indicate references at the end of this report.

Attention will be focused mainly on simple systems of one degree of freedom which are subjected to impacts not severe enough to exceed the elastic range of the system. Although, strictly speaking, all physical structures possess many degrees of freedom, it frequently occurs that one of these is so preponderant that it determines the behavior of the system for all practical purposes; the problem can then be greatly simplified by analysis on the basis of a single degree of freedom. This is not always the case, however, and gross errors may be made if this point is not investigated. This question is given attention in the last part of the report.

Though some degree of damping exists in all structures, the amount of damping commonly encountered is so small that it may be neglected for most purposes in calculating the response of the system.

The purpose of these simplifications in the present study is to exhibit more clearly some principal characteristics of the effect of impact; nevertheless the principles which will be brought out in this report describe many simple structures with sufficient accuracy to permit their application as a basis for structural design and their use in the development of instruments to measure impact effects.

2. INVESTIGATION OF SOME CHARACTERISTIC CASES

The essential features of the undamped elastic system with one degree of freedom are shown in Figure 1a. A rigid mass M is supported from an

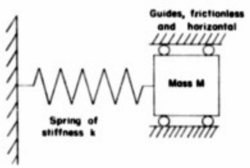


Figure 1a

System having one degree of freedom.

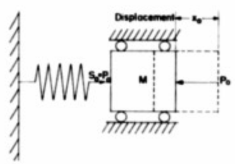


Figure 1b

System showing static displacement x_0 of the mass by a steady force P_0.

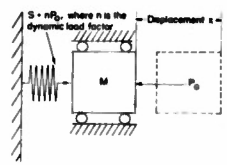


Figure 1c

System showing dynamic displacement x of the mass by a disturbing force P whose peak value is P_0.

Undamped Elastic System

unyielding base by an inertialess spring and is constrained by a frictionless guide to move only in the horizontal direction. An external disturbing force P is applied to the mass. It is desired to know the resulting reactive force S in the spring as a function of time. Since this equals the product of the displacement of the mass x by the stiffness constant of the spring k, x may

be adopted as the single parameter denoting the configuration of the system. All of the cases considered here are of the foregoing type.

It is convenient to express the results in non-dimensional terms obtained from the comparison with the corresponding case of static loading. The disturbing force P can be put in non-dimensional form by dividing by some reference value P_0, say the peak value, of the load. The non-dimensional ratio between the applied force at any moment and the maximum value of the force applied to the system, P/P_0, will be designated as the ***disturbance factor*** or, more briefly, the ***disturbance***. For present purposes, variation of this ratio with time does not affect its dimensionless nature.

The static displacement x_0 of the mass under a steady load equal to P_0 can, as shown on Figure 1b, be used as a unit of displacement. Under sudden application of the force P, the displacement rises to a dynamic value x, as in Figure 1c. The ratio of this dynamic displacement x to the static displacement x_0, is called the ***response factor***, or, more briefly, the ***response***.

As the spring reaction S is assumed to follow Hooke's law, the response factor may be used to represent ratios of spring force as well as of displacements. Thus the reactive force exerted by the spring at any time is P_0 multiplied by the response factor n* at that time. The numerical maximum of the response factor is the ***dynamic load factor***; it is the factor which, multiplied into P_0, gives the maximum spring reactive force under the dynamic condition defined by the disturbance.

CASE 1. SUDDENLY APPLIED LOAD

A disturbing force or load applied instantaneously at zero time and remaining on the structure indefinitely may be represented by the diagram of Figure 2. The response, either in terms of force (load) or displacement ratio, is shown by the solid line of Figure 3.

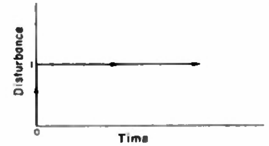


Figure 2 - Disturbing Load Applied Instantaneously and Maintained

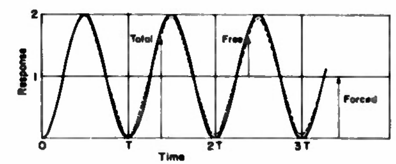


Figure 3 - Response to Disturbance of Figure 2

* n stands in general for any dimensionless number. Elsewhere (10) it is sometimes used in a reciprocal sense, with values less than unity, to measure the static equivalent of a peak transient load.

The unit of time chosen in Figure 3 is the natural period T of free vibration of the disturbed mass (or structure).

The response may be considered as made up of two components, a constant or ***forced response*** and a varying response, in this case, the superimposed free vibration upon it; the latter may be called the ***free response***. In this case the constant or forced response is the static displacement z_0, which forms the denominator for all response factor ratios. In Figure 3 this denominator is represented by the horizontal line with unit ordinate starting at zero time. The combined or resultant response factor variation may be referred to briefly as the total response.

The amplitude and phase of the free vibration of the mass are governed by the fact that the system starts from rest, as in Figure 1a; hence the curve of total response in Figure 3 must start from the origin with a horizontal tangent. The maximum value of the response factor, or dynamic load factor, in this case has the value 2. The spring must consequently be designed to carry double the external load P. The excess load in the spring is due to the fact that the spring must bring the mass to rest as indicated in Figure 1c, after it has passed through the position of static equilibrium, Figure 1b.

The curve represented by the broken line in Figure 3 shows how a small amount of natural damping would affect the response. The amount of damping shown is of the order that may be encountered in ordinary steel structures. It will be observed that the effects of the damping are trivial in the early stages of the phenomenon, but that these effects are cumulative with time.

In Appendix 1 are given formulas for the response factor curve of Figure 3 as well as for responses to other disturbances considered later in this report.

If the load is not applied instantaneously, a response differing from that in Figure 3 is obtained. For example, let the disturbance be as shown in Figure 4, in which the load increases uniformly with time and reaches its full value only after a lapse of time t_0. Since the condition of static loading is approached as the rate of loading is reduced, it is clear that the dynamic load factor will approach unity when the time t_0 is sufficiently increased.

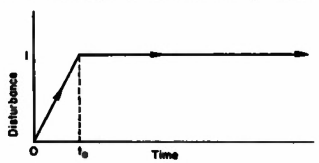


Figure 4 - Disturbing Load Applied Gradually and Maintained Indefinitely

The responses to some typical disturbances of the type of Figure 4 are shown in Figure 5, where the disturbance is plotted as a broken line. Except for

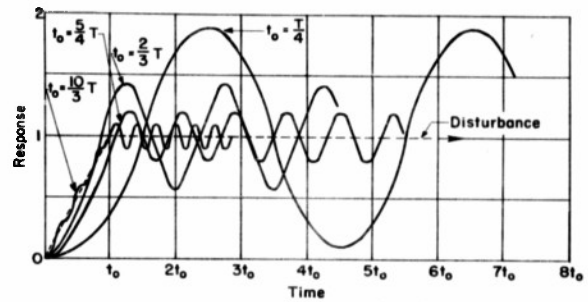


Figure 5 - Response of Various Systems to Gradually Applied Loads

The disturbances are of the type shown in Figure 4.
The natural period of the vibrating system is represented by T, as in Figure 3.

a slight phase shift along the time scale, the response is practically the same for any time of rise t_0 which is less than a quarter of the natural period of the responding system ($t_0 < 0.25T$). This is shown in Figure 6, in which the dynamic load factor is plotted against the ratio of time of rise of disturbance to the natural period of the system.

If the time of rise t_0 is an integral multiple of the natural period T, there is no permanent free vibration after the disturbance has reached its full value. This fact, however, is mainly of academic interest, since any time of rise which does not exactly fulfill this condition will cause a free response superimposed on the forced response, as may be seen in Figure 6.* Times of rise t_0 greater than $4T$ will produce dynamic load factors not exceeding 1.07, so that Figure 6 covers the significant range of excess loading due to dynamic effects.

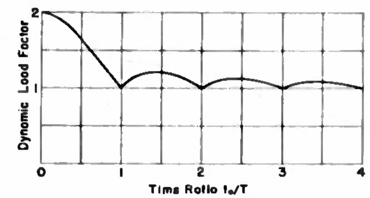


Figure 6 - Dynamic Load Factor

The disturbances are of the type of Figure 4, varying as to rate of application of load.

It appears from the preceding considerations that time rate of application of load has a significant effect in determining the effects of shock loading. This is true for other types of disturbance, as will appear later.

* Substantially the same curve is given by Conrad in Figure 25 of Experimental Model Basin Report 458.

CASE 2. THE SINGLE PULSE

A fundamental element in impact loading is the duration of the shock. This was not a factor in the considerations of Case 1, since there the load remained on the structure indefinitely and the only variations concerned the initial phases of application of the load.

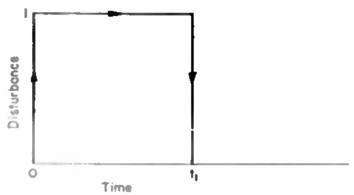


Figure 7 - The Rectangular Pulse

The simplest case illustrating the effect of duration of loading is given by the disturbance shown in Figure 7, where the load is instantaneously applied at zero time, remains constant, and is removed instantaneously at the time t_1. Such a disturbance is described as a pulse of duration t_1. Characteristic responses to this type of disturbance are shown in Figure 8, where the disturbance is indicated by the broken curve.

If the duration t_1 of the pulse is greater than half of the natural period T, the dynamic load factor is 2. In other words, the maximum

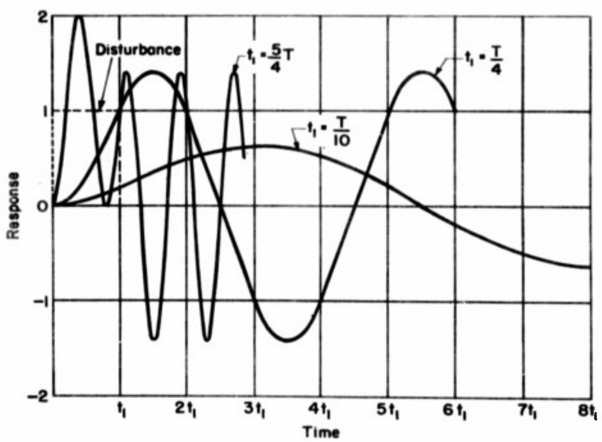


Figure 8 - Response to Disturbance of the Type of Figure 7 (Rectangular Pulse)

effect of a rectangular pulse lasting more than half a natural period is independent of the duration of the pulse and is the same as though the load were not removed. Should the pulse last for a shorter time, however, the effect of duration becomes evident and the dynamic load factor decreases with the duration t_1, as shown in Figure 9. It should be noted that the pulse can be made so short that the dynamic effect is less than the static effect under the same load. Any rectangular pulse lasting less than one-sixth of the natural period will give a dynamic load factor less than unity.

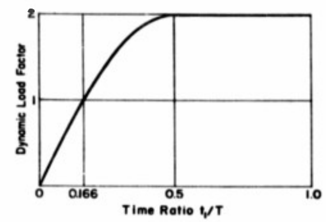


Figure 9 - Dynamic Load Factor

The disturbance is of the rectangular pulse type of Figure 7, and varies with duration of load.

The previous examples have shown how duration of pulse and initial rate of rise can be important factors in determining the response of the structure. Two more typical cases of single pulses will now be investigated to show how these factors may combine with the factor of shape of pulse-profile in determining the response.

The first case is that of the triangular pulse where the disturbance rises to its peak of unity at a time of 0.5 t_1, then falls to zero, with a total duration of t_1. This disturbance is shown in broken lines in Figure 10, together with a number of responses for representative values of t_1/T, the

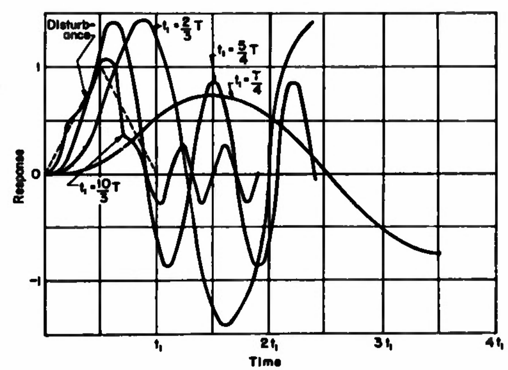


Figure 10 - Response to a Triangular Pulse

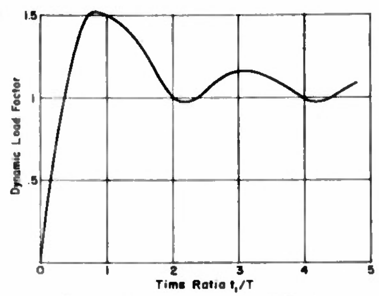


Figure 11 - Dynamic Load Factor

The disturbance is of the triangular pulse type of Figure 10 and varies with duration of load.

ratio of duration of pulse to natural period of the responding system. As the duration of pulse is increased up to a little less than one natural period, $t_1 < T$, the peak response increases, since the effect of duration of pulse is predominant as in Figure 9. For pulses lasting longer than this, the opposing effect of reduced rate of rise becomes the predominating factor and the peak response falls off, as for $t_1 = 10T/3$. These features are illustrated in the dynamic load factor curve of Figure 11. The maximum dynamic load factor is less than that obtained for the rectangular pulse because of the effect of finite rate of rise; it is 1.52 for the triangular pulse compared to 2.00 for the rectangular pulse. For disturbances of greater duration than those shown in Figure 11, the dynamic load factor will not exceed 1.11.

Similar features are to be observed in the response to a pulse having the shape of a half sine-wave, as for the disturbance shown in the broken line of Figure 12. The response curves shown in this figure are rather

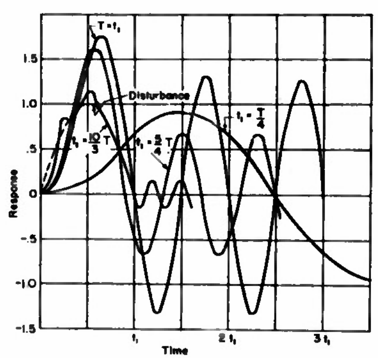


Figure 12 - Response to a Sinusoidal Pulse

similar to those for the triangular pulse, except that the corresponding peaks are a little higher. The dynamic load factor curve of Figure 13 illustrates this point. The greatest load factor is reached again at a duration of a little less than one natural period and has the value 1.75. For durations greater than shown in Figure 13, the dynamic load factor will not exceed 1.10.

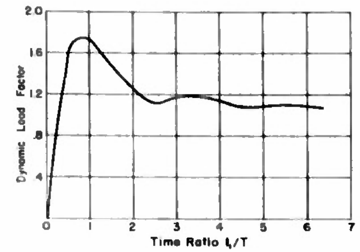


Figure 13 - Dynamic Load Factor for Sinusoidal Pulse

Disturbance varies with duration of load.

CASE 3. THE BLAST PULSE

The general features of the disturbance due to gun blast can be represented by the curve of Figure 14. An equation for this curve and the formulas for the response are given in Appendix 1, Equations [21] to [24]. The duration of the pulse is characterized by the time of positive pressure, shown as t_1 in Figure 14. This disturbance differs from the preceding cases in that the load reverses; it becomes negative from the time t_1 after which it gradually approaches zero.

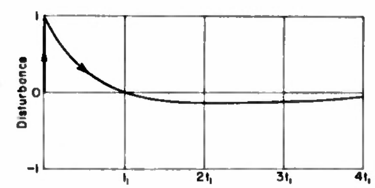
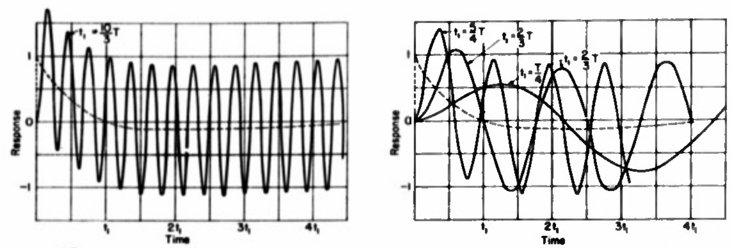


Figure 14 - Disturbance due to Gun Blast

The character of the response for several values of the duration t_1 are shown in Figure 15, together with the disturbance producing them. Figure 15a gives the response of a structure which has a natural period small with respect to the time t_1 of positive pressure. The initial abrupt rise of the disturbance provokes a large free vibration of the structure, while the forced response is substantially identical with the disturbance. In Figure 15b responses are shown for structures having natural periods nearly t_1 or greater. It will be observed that the forced response departs more and more from the disturbance as the blast load is applied to structures of increasing natural period.

In case of blast loading the greatest numerical value of the response factor is reached at the first maximum when the natural period of the structure is less than $4/3\ t_1$. When the natural period is greater than $4/3\ t_1$, the minimum following the first maximum gives the greatest numerical value



15a 15b

Figure 15 - Response to Blast Load

The disturbance is shown by a broken line.

of response; that is, the displacement of the structure at peak response is in a direction opposite to that which would be produced by a static positive pressure.

The dynamic load factor curve for the disturbance of Figure 14 is given in Figure 16. It is similar in general character to the dynamic load factor for the rectangular pulse given in Figure 9. The initial rate of rise is an important factor in both cases. If the blast disturbance of Figure 14 is modified to incorporate a finite time of rise t_0 at the start, as shown in Figure 17, the dynamic load factor curve of Figure 18 is obtained. This shows the effect of increasing the time of rise t_0 while the duration t_1 is held constant. The curve of Figure 18 was drawn for a duration t_1 of 70 times the natural period, but applies with only minor changes down to much shorter durations. The variation of dynamic load factor with time of rise is notably like that shown in Figure 6 for a load rapidly applied in the manner of Figure 4.

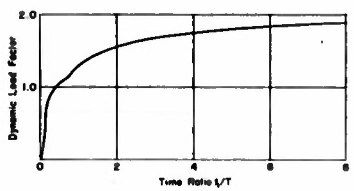


Figure 16 - Dynamic Load Factor

Gun blast disturbance as shown in Figure 14 varies with respect to duration.

The previous examples make it clear that load produces greater peak deflections and stresses in a structure as the time of rise to the peak value of load is reduced. However, there is a limit to this effect. The maximum deflection caused by a given load is reached when the peak value of load is

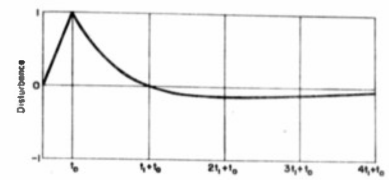


Figure 17 - Modified Blast Disturbance

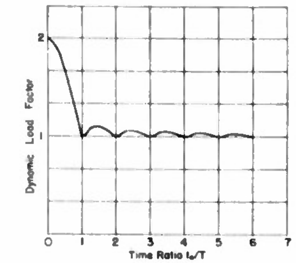


Figure 18 - Dynamic Load Factor

Gun blast disturbance as in Figure 17, where $t_1 = 70T$, varying as to duration of initial rise, t_0.

attained instantaneously, but the load must remain on the structure for a sufficient length of time for its full effect to be realized. Thus the severity of an abruptly applied impact load can be offset if the impact is very brief. In this connection, a load which reaches its peak in less than one-fourth the natural period of the structure will produce for practical purposes as much effect as though its peak value were reached instantaneously.

Most of the details of the particular cases thus far investigated can be explained on the basis of these two facts. There appears to be little other specific effect of shape of pulse profile, at least for the simple shapes considered previously.

When the duration of the pulse is very short, the effect on the structure is determined solely by the impulse, that is, by the product of the mean force and the duration of the impact. The structure behaves as though it had experienced an instantaneous increase in momentum, equal to the impulse of the impact. In this case the response of the system is independent of the details of shape of the pulse profile and is determined only by the area under the disturbance curve. If a mean duration of pulse is defined as the impulse divided by the peak load, examination of the dynamic load factor curves of Figures 9, 11, and 13 reveals that they become identical for practical purposes when the mean duration is about one-fourth the natural period. Thus for pulses of this character with a mean duration of less than one-fourth of a natural period, the response of the system can be considered as determined solely by impulse.

This matter is of especial importance because the response of the system, under these circumstances, is independent of the peak load; the effect of a given load acting for a given time is the same as that of twice the load for half the time, provided the time is short enough.

This fact must be borne in mind in interpretation of the response curves and dynamic load factors. To the casual view it might appear that since by this theory the stress caused by an impact is limited to twice that of the corresponding static load, the theory fails to account for actual phenomena. Such a view is to this extent correct, that the theory gives no clue to the actual peak value which the load may attain; it gives the value of the dynamic load factor, but leaves the term to which this factor is to be applied to be determined by other means.

Peak value of load is actually difficult to evaluate, either by calculation or test. It is therefore fortunate if the governing parameter in pulses of brief duration is the momentum transfer, or impulse value.

A further discussion of the separate effects of peak and duration of load on mode of response will be found on page 13.

CASE 4. DAMPED OSCILLATING DISTURBANCE

A type of impact loading which presents certain features not found in the cases already discussed is the oscillating disturbance. Impact loading of this kind is experienced by the foundations of elastic structures subjected to impact and by structures loaded by the acceleration of a freely vibrating support. Such a disturbance will be slightly damped, as it is excited by decaying free vibration.

An excitation of this kind produces effects which have certain features in common with steady-state forced vibrations. For instance, when the period of the disturbance approaches that of the structure, a condition of quasi-resonance appears and the amplitude of the response will at first increase with time, then decrease later as the amplitude of the disturbance dies away. The forced response will be geometrically similar to the disturbance, but displaced slightly in phase and multiplied by the resonant gain factor of the steady-state response.* The free response produced depends on the time of rise to the first peak of the disturbance.

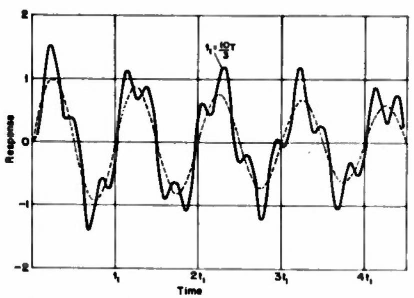


Figure 19 - Response to Damped Oscillating Disturbance, Initial Rise Sinusoidal

Disturbance is shown by the broken curve.

* The resonant gain factor for the undamped structure is $\frac{1}{1 - r^2}$, where r is the ratio of the natural period of the structure T to the period of the disturbance t_1. See Equations [25] to [29] of Appendix 1.

The complete discussion of this case is beyond the scope of this report and would call for consideration of the damping present in the structure upon which the disturbance acts. Two illustrative examples of the response to this kind of disturbance are shown in Figures 19 and 20. The first is the response to a disturbance represented by a slightly damped sine curve of a structure whose natural period T is 0.3 of the period of the disturbance t_1. The disturbance for Figure 20 is a cosine curve, slightly damped as before, and with the same ratio of periods. The abrupt rise to an initial peak load in the latter case causes a free response which is much greater than in the case of Figure 19.

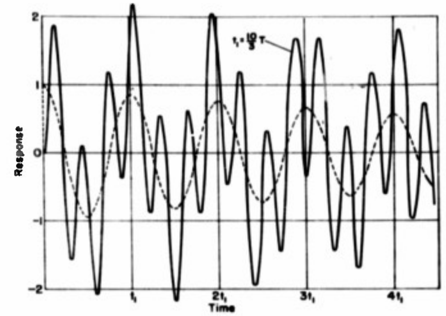


Figure 20 - Response to Damped Oscillating Disturbance with Abrupt Initial Rise

Disturbance is shown by the broken curve.

Some quantitative discussion of these cases will be found in Appendix 1.

3. ACTUAL STRUCTURES AND THE IDEALIZED SYSTEM OF ONE DEGREE OF FREEDOM

The behavior of the nominal system discussed in the preceding section is far simpler than that of any real structure. To what extent is it permissible to apply these results to the behavior of actual structures under service loading? A practical answer requires some data on the character of the impact and on the type of structure.

Since there are no perfectly rigid bodies and no massless springs in nature, it is clear that there is no exact counterpart among real structures of the simple nominal system with one degree of freedom of Figure 1a. One can hope only to find an approximation to this simple system.

Dr. N. Minorsky has pointed out in an unpublished memorandum* that a duration of impact can always be chosen short enough so that the behavior of any actual system subjected to it becomes complicated by the propagation of stress waves and other phenomena essentially foreign to the system of one

* Available at the Taylor Model Basin.

degree of freedom. This is true no matter how closely actual structure is designed to simulate a system of one degree of freedom. It is clear that some limitation on the character of the impact is required if the structural response is to be of the kind discussed in the preceding section.

An elastic structure subjected to impact can respond in many modes, as shown by the tones given off by a bell. All of the modes of vibration will in general be excited to some degree. The relative amplitudes in these different modes depend in part on the manner of loading. Under a given load, applied statically, a given profile of deflection occurs. When this load, with the same distribution, is applied as a pulse of brief duration, the structure may be said to respond as a system of one degree of freedom if the momentary deflection profile in the extreme position agrees* with that under static load. When the load is not too brief in duration and is distributed uniformly over the structure response in a single mode is favored, ordinarily the gravest** mode. Concentration of the load in space or time will produce an increased contribution from higher modes, and the behavior will lose resemblance to a system with one degree of freedom.

Two cases which illustrate the points made in the foregoing receive a more detailed treatment in Appendix 2.

A further condition for application of the nominal theory is that the various degrees of freedom should be uncoupled; that is, that the motions in the various modes should be independent. This condition will in most cases be realized satisfactorily if the frequencies of higher modes lie well above the fundamental frequency.

To summarize, an actual structure will behave like a simple system with one degree of freedom of fundamental frequency provided that

(a) the duration of impact t_1 is sufficiently long, say a tenth*** or more of the fundamental natural period T

(b) the impact load is distributed fairly uniformly over the structure

(c) the fundamental mode of the structure is uncoupled with higher modes.

---

* That is, if it is geometrically similar.

** Mode of lowest or fundamental frequency.

*** This limiting value is tentative, since investigations have been carried through for only a few cases.

4. THE STRENGTH OF SIMPLE STRUCTURES UNDER IMPACT LOADS

The analysis of Section 2 may be used in designing structures subjected to impact loading. A static load equivalent to the dynamic action can be derived by multiplying the peak impact load by the dynamic load factor for the type of impact involved. This method has become well established in dealing with turret foundations; although the impact in this case is not of high order, the peak value and duration of load are well defined.

Application of the same method to other cases in which the impact is of higher order is also being made, as in cases of gun blast, projectile impact, and underwater explosion, and experimental data on peak value and duration of loads are being gathered.

A structure which under a given impact behaves approximately as a system of one degree of freedom will experience at any point a combination of shears, bending moments, and axial forces which may be determined to a sufficient degree of accuracy by observing or calculating the corresponding shears, etc., under the equivalent static load. When the case can no longer be described in terms of a single degree of freedom, the problem of design becomes more involved, since there is no longer a single factor by which the dynamic loading can be reduced to an equivalent static loading. If the concept of dynamic load factor is to be retained for a system differing appreciably from the simple one-degree-of-freedom case, the load factor will vary with location in the body, and the factor for bending moment may not be the same as the factors for shear or axial loading. Some preliminary investigation of conditions in boundary-line cases has indicated, however, that there exists the possibility of a load factor for ***maximum*** bending moment or ***maximum*** shear which is given satisfactorily by the considerations of Section 2, even though the distribution of dynamic bending moments and shears may differ appreciably from the static case.*

These remarks serve to indicate that the boundary line between one degree of freedom and many degrees is not well defined and may shift with the type of information desired. Further experience in this field is needed before precise criteria can be set up.

In spite of the lack of precise knowledge of the limitations indicated by the foregoing, the use of the model with one degree of freedom to represent many actual structures for the purpose of deriving design criteria seems to be justified at the present time. The limitations of this method are expressed in Section 3 as explicitly as current information permits.

---

* See Appendix 2.

The designer, then, should first satisfy himself that the nature of the structure and of the impact load is such as to make the use of a single dynamic load factor reasonably appropriate. If this is the case, the dynamic loading can be replaced by an equivalent static loading. To check the strength of the design, it remains only to determine a suitable working stress.

With respect to the design of superstructure and decks under severe conditions of gun blast, it would appear that these loads can be considered in the nature of emergency loads, rather than routine service loads. If bulkheads and decks are strong enough to stand the loading without excessive permanent deformation, this will probably suffice in most cases. In case of working doors, machinery attached to bulkheads, and the like, even a small permanent set may cause failure of the normal functioning of the structure. As a matter of practical experience it is known that momentarily high values of deflection may be accepted though the same deflection if maintained would result in permanent set. This may be due at least in part to differences in the behavior of the steel itself at different rates of loading.

References (3), (4), (5), (6), (7), and (8), are illustrative of the present state of knowledge with regard to the dynamic strength properties of metals. There is a general similarity in behavior of all metals in common use, though steel is possibly the most anomalous of all. The information obtained from tension tests at rates of strain from static to 1000 inches per inch per second may be summarized broadly as follows: Ductility is not notably affected by high speeds of loading, but the yield strength may be appreciably increased at even moderate rates; tensile strength behaves less regularly, increasing on the whole with speed of loading. In none of the experimental work of the references is there indication of any marked loss of ductility under simple tensile impact load. There is ground to fear, however, that brittle failure may occur under impact conditions if the stress system is not simple, that is, in the case of multi-axial stress (8). Such conditions may occur in connections of members, particularly in riveted connections. Insufficient evidence is available to assess this factor quantitatively, but it is probable that connections well designed to withstand repeated static stressing will serve reasonably well in this regard under the conditions of gun-blast and gun-recoil loads.

It should be realized, however, that the rate of change of strain in a deck, bulkhead, or gun-foundation structure is influenced by the characteristics of the structure as well as by the duration of the impact load. Structures with a natural frequency of 50 CPS or less cannot experience a rate of change of strain much greater than about 0.1 inch per inch per second

unless the impact load is severely concentrated in space and time. The effect of high rate of loading on the strength properties of the material is not marked at such strain rates. From extrapolation of the data of Reference (6), it appears likely that the dynamic yield strength of medium steel exceeds the static yield strength by less than 20 per cent in this range of strain rates. The effect on tensile strength and ductility is even less.

The effects of impact, especially if of high order, may be more pronounced near the point of loading than in more distant parts of the structure. This suggests the matter of delayed transmission of stress. The calculations of this paper ignore completely the possibility that stress of high intensity may occur in parts of the structure distant from the point of loading as a consequence of the delay in transmission of abruptly generated stress. If a transient load is maintained for so brief a period that a stress wave moving at the speed of sound in steel is followed closely by a cancellation of the stress condition, then the pulse will detach itself and move off independently of the load in a way made familiar by the more easily observed case of slow surface waves on water. In general such an effect will be attenuated, as it goes, by its extension over greater volume of steel and by dissipative action. Under certain circumstances, however, it is possible that the high intensities may be maintained in such a detached stress pulse, or even increased, as in the case of geometrical configurations such as cause stress concentrations. It is hoped that work now underway may clarify understanding of this phenomenon, but it is still too imperfectly understood to make even nominal calculations feasible.

With regard to failures by instability, yielding must progress to an appreciable degree before failures of this kind can occur, so that if the necessary time is not available, liability to failure of this type is diminished. The rather scanty evidence available is in accordance with this point of view. It is suggested that the same design stress be used both for tensile and for compressive loading.

A value for the working stress under impact conditions due to gun-blast and gun-recoil loads, based on the static strength properties of the material, is quite justified in the light of existing knowledge; it is perhaps slightly, but not unduly, conservative. A factor of safety on yield, depending on the functions of the structure, should be applied to the static yield strength to give such a working stress.

The reader is explicitly warned not to confuse the results in this report with those obtained when the load is applied by a blow as from a hammer. In this case the peak load rises to very large but mostly unknown values. The accompanying large deflections and stresses are the result of high values of P_0, not of the dynamic load factor n.

5. DESIGN OF IMPACT INSTRUMENTS AND INTERPRETATION OF RECORDS

Many instruments to measure impact effects can be imagined as simple structures designed to function as much as possible as systems of one degree of freedom and equipped with some means of recording or indicating the magnitude of the response. With such an instrument, the mechanical behavior is predictable by calculation to a satisfactory accuracy when the static calibrations are known. The experimental difficulties of producing an impact disturbance known quantitatively in detail are such that absolute calibrations under impact loads are either out of the question or are of insufficient accuracy. Understanding of the behavior of the instrument is consequently dependent on reliable predictions of the dynamic behavior from static characteristics. This can be achieved by a gage designed to approximate, as closely as possible, a system of a single degree of freedom.

The problem of interpreting records from gages of this type is the converse of that discussed in Section 2; it is to obtain the disturbance from a known response, which is the record to be analyzed.

It has been pointed out in Section 2, and it will be seen mors clearly in the equations of Appendix 1, that the forced response of a structure of one degree of freedom tends to reproduce the disturbance, and will do so more accurately when the natural period of the structure is short with respect to the duration of the disturbance. The free response thus appears as an error to be eliminated. Its amount depends less on the general shape of the disturbance than on the abruptness with which the disturbance begins and the length of time it lasts. Hence if the forced response can be derived from the total response, a reasonable picture of the disturbance can be obtained from a gage of this typs if it is suitably designed for the type of impact to which it is to be exposed.

One way to suppress the free response of a gage is to introduce appreciable damping, as by means of viscous material with dashpots, or by friction. This procedure is common in instruments intended to measure steady-state phenomena, which is the usual ease in vibration problems, but this approach meets difficulties in its application to impact instruments. It is usually the case that the impact instrument is intended to measure effects which change rapidly in time. To follow these, gages with a high natural frequency are required, and the introduction of appreciable damping into a high-frequency mechanical system raises difficult design problems. Furthermore, the presence of damping introduces some undesirable features, such as time lags in the response, which may lead to distortion of the record. Finally, the detailed response of even the simplest gage is made more complicated by the presence of damping to such a degree that it may prevent the user from

obtaining any trustworthy understanding of the mechanical behavior of the gage at all.

For the reasons just outlined, the recent development of impact gages at the Taylor Model Basin has been in the direction of simplicity of design and a natural frequency as high as is compatible with the sensitivity needed. No artificial damping has been introduced.

A sample oscillogram from one such gage is shown in Figure 21. This shows the response of a thin circular clamped diaphragm to the blast

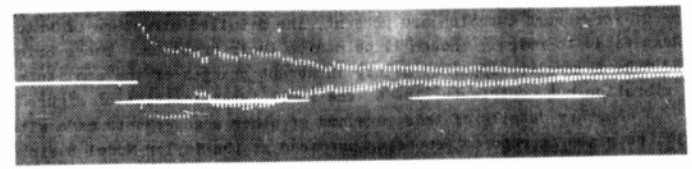


Figure 21 - Oscillogram from Diaphragm Blast Gage

pressure produced by the explosion of a small charge of high explosive. The natural frequency of the gage is 4000 CPS. At first glance practically nothing can be seen but the free vibrations of the diaphragm at its natural frequency. The forced response can be separated out of the record without difficulty, however, and is shown plotted in Figure 22. The method by which this is done consists in plotting the successive midpoints of the up and down strokes of Figure 21. About four cycles of free vibration occur during the initial phase of positive pressure. Referring to Equation [23] of Appendix 1, where the second term on the right-hand side gives the forced response, it is evident that a disturbance of the type of Equation [21] should be reproduced quite faithfully by the forced response, since the constant c of Equation [23] is approximately equal to 25, and phase angle ϕ, by Equation [24], is nearly $\pi/2$. The measured disturbance (Figure 22) is of the same general character except that it is complicated after about 13 cycles by multiple reflections. It can be considered

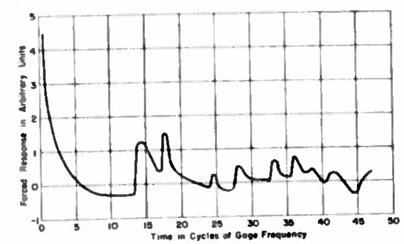


Figure 22 - Forced Response from Blast-Pressure Record

as the superposition of several disturbances like that of Equation [23], but with various time lags between them. Hence it may be concluded that Figure 22 presents a picture, possibly smoothed in detail, of the actual pressure variations experienced by the diaphragm.

The qualification with regard to the details of the picture presented by Figure 22 is necessary, as will be seen by examination of the original record shown in Figure 21. Abrupt changes in amplitude of free vibration occur many times in the record. These could not be present if the disturbance or pressure-time curve were smooth. Undoubtedly superposed pulses of very short duration are present, the detailed structure of which the gage fails to reveal, though it points to their presence. These brief pulses make themselves apparent mainly in abrupt changes of the amplitude of free vibration but contribute also to the local irregularities of Figure 22.

Further details of this gage and of other measurements made with it will be given in reports now being prepared at the Taylor Model Basin.

An accelerometer for use on a structure under severe impact has been develooed at the Taylor Model Basin along similar lines. The type of disturbance experienced by this instrument is of the general character described in Section 2 as a damped oscillating disturbance. The technique just described for the diaphragm gage may also be used for evaluating the records obtained from the accelerometer, but an added practical difficulty occurs owing to the fact that a higher gage frequency is needed to obtain the time resolution required in the oscillograms where the disturbance has an oscillating character. This conflicts with the need to obtain a record of appreciable duration to follow the later phases of the phenomenon. The compromise between these features has led to the use of the accelerometer under conditions where the wave form of the response could not be clearly distinguished on the record. As Figures 19 and 20 make clear, the amplitude of the total response is markedly affected by the steepness of the original rise of the disturbance. However even under these conditions it has been possible to infer the acceleration amplitudes from the total response to within estimated error of 25 per cent.

In the light of these two examples and the foregoing sections, the requirements for gages of this type to measure impact forces and accelerations may be specified as follows:

(a) The gage should have the highest natural frequency which will permit the desired sensitivity to be obtained. The natural period of the gage should be short in comparison with the duration or period of the initial disturbance, certainly less than one-half the latter.

(b) The structural design should be simple and direct and should approximate as closely as possible to a system with a single degree of freedom, as outlined in Section 3.

6. REFERENCES

(1) "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Company, New York, N. Y., 1928.

(2) "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Company, New York, N. Y., 1934.

(3) "Stress-Strain Relations under Tension Impact Loading," by D. S. Clark and G. Dätwyler, Proceedings of American Society for Testing Materials, volume 38, 1938, Part II, pp. 98-111.

(4) "High-Speed Tension Tests at Elevated Temperatures," by Manjoine and Nadai, Proceedings of American Society for Testing Materials, volume 40, 1940, Part 1, pp. 822-837; preprint 44; Parts II and III, Scientific Paper 1011, Research Laboratories, Westinghouse Electric and Manufacturing Company, 1940.

(5) "Report on Impact Research," by A. V. DeForest, A. R. Anderson and R. Fanning. Report submitted May 1941 to Director, David W. Taylor Model Basin, from Research Laboratory for Dynamics of Materials, Massachusetts Institute of Technology.

(6) "An Investigation of the Effect of Rate of Strain on the Results of Tension Tests of Metals," P. G. Jones and H. F. Moore, American Society for Testing Materials, 1940, 15 pages.

(7) "Effect of Impact Velocity on Tensile Characteristics," LeVan Griffis, Progress Report 6 of the California Institute of Technology Research Project, May 1941.

(8) "Some Complexities of Impact Strength," A. V. DeForest, Howe Memorial Lecture, Transactions of the American Institute of Mining and Metallurgical Engineers, 1941.

(9) "Impact Stresses in a Freely Supported Beam," R. N. Arnold, Proceedings of the Institution of Mechanical Engineers, volume 137, 1937, pp. 217-281

(10) Additional related references of confidential nature may be consulted by authorized persons at the Bureau of Ships or the David Taylor Model Basin.

APPENDIX 1
FORMULAS FOR RESPONSE TO IMPACT DISTURBANCE

NOTATION

m is the mass, pounds divided by 386 inches per second per second
k is the spring constant, pounds per inch
P is the applied force, or load, pounds
P_0 is the maximum value of applied force
$p = P/P_0$ is the disturbance factor, or disturbance
x is the displacement in inches
$x_0 = P_0/k$ is the static displacement due to P_0
$u = x/x_0$ is the response factor, or response
T is the natural period in seconds
$f = 1/T$ is the natural frequency in cycles per second
$\kappa = 2\pi f$ is the circular natural frequency in radians per second
t is the time in seconds
t_0 is the time of rise of gradually applied load (see Figure 4)
t_1 is the duration of pulse
$\dot{u}, \ddot{u}$ are the first and second derivatives of u with respect to time
n is the dynamic load factor
ω is the circular frequency of cyclic part of disturbance in radians per second
$r = \omega/\kappa$ is the frequency ratio
ϕ is the phase angle
c, β are constants

GENERAL EQUATIONS

The system of Figure 1a acted upon by a force P gives rise to the following differential equation:

$$m\ddot{x}(t) + kx(t) = P(t) \qquad [1]$$

where $\ddot{x}$ is the acceleration. In non-dimensional terms, this equation is

$$\kappa^{-2}\ddot{u}(t) + u(t) = p(t) \qquad [2]$$

where $\kappa^2 = k/m$.

The differential equation is to be solved subject to the condition that the system starts from rest at zero time, that is,

$$u(0) = 0 \text{ and } \dot{u}(0) = 0 \qquad [3]$$

A solution may be obtained in the form of a definite integral as follows:

Multiply Equation [2] by $\kappa^2 \sin \kappa t$; this gives

$$\ddot{u} \sin \kappa t + \kappa^2 u \sin \kappa t = \kappa^2 p(t) \sin \kappa t \qquad [3a]$$

This can be written

$$\frac{d}{dt}(\dot{u} \sin \kappa t) - \frac{d}{dt}(\kappa u \cos \kappa t) = \kappa^2 p(t) \sin \kappa t \qquad [3b]$$

as may be verified by performing the differentiations and comparing with [3a]. Integrating between **0** and t gives

$$\dot{u} \sin \kappa t - \kappa u \cos \kappa t = \kappa^2 \int_0^t p(t) \sin \kappa t \, dt = \kappa^2 \int_0^t p(\alpha) \sin \kappa \alpha \, d\alpha \qquad [3c]$$

where α is used as the variable of integration to distinguish from t in the limits of integration. Likewise multiplying [2] by $\kappa^2 \cos \kappa t$ and repeating the steps as above, there is obtained

$$\dot{u} \cos \kappa t + \kappa u \sin \kappa t = \kappa^2 \int_0^t p(\alpha) \cos \kappa \alpha \, d\alpha \qquad [3d]$$

Eliminating $\dot{u}$ between [3c] and [3d], gives

$$\kappa u = \kappa^2 \sin \kappa t \int_0^t p(\alpha) \cos \kappa \alpha \, d\alpha - \kappa^2 \cos \kappa t \int_0^t p(\alpha) \sin \kappa \alpha \, d\alpha$$

or

$$u = \kappa \int_0^t [\sin \kappa t \cos \kappa \alpha - \cos \kappa t \sin \kappa \alpha] \, d\alpha$$

$$u(t) = \kappa \int_0^t p(\alpha) \sin \kappa (t - \alpha) \, d\alpha \qquad [4]$$

An alternative form can be obtained by integrating [4] by parts; thus

$$\int_0^t p(\alpha) [\sin \kappa (t - \alpha)] \, d\alpha = \frac{p(\alpha) \cos \kappa (t - \alpha)}{\kappa} \Big|_0^t - \frac{1}{\kappa} \int_0^t \cos \kappa (t - \alpha) \, \dot{p}(\alpha) \, d\alpha$$

$$u(t) = p(t) - p(0) \cos \kappa t - \int_0^t \dot{p}(\alpha) \cos \kappa (t - \alpha) \, d\alpha \qquad [5]$$

If the disturbance reaches a final steady value $\bar{p}$ at the time $\bar{t}$, the final response is

$$u(t) = \bar{p} - \cos \kappa t \left[\int_0^{\bar{t}} \dot{p}(\alpha) \cos \kappa \alpha \, d\alpha + p(0) \right] - \sin \kappa t \int_0^{\bar{t}} \dot{p}(\alpha) \sin \kappa \alpha \, d\alpha \qquad [6]$$

for any $t > \bar{t}$.

The final response is cyclic, with a mean value $\bar{p}$ and an amplitude A of the cyclic component given by

$$A^2 = \left[p(0) + \int_0^{\bar{t}} \dot{p}(\alpha) \cos \kappa \alpha \, d\alpha \right]^2 + \left[\int_0^{\bar{t}} \dot{p}(\alpha) \sin \kappa \alpha \, d\alpha \right]^2 \qquad [7]$$

RAPIDLY APPLIED LOAD

If the load is instantaneously applied, the disturbance is as shown in Figure 2, and

$$p = 0 \quad \text{for } t < 0; \qquad p = 1 \quad \text{for } t \geq 0 \qquad [8]$$

The response is

$$u = 1 - \cos \kappa t \qquad \text{for } t \geq 0 \qquad [9]$$

Equation [9] is plotted in Figure 3.

If the disturbance is of the type of Figure 4, that is:

$$p = 0 \quad \text{for } t < 0; \qquad p = \frac{t}{t_0} \quad \text{for } 0 \leq t \leq t_0; \qquad p = 1 \quad \text{for } t \geq t_0 \qquad [10]$$

The response is given by

$$u = \frac{t}{t_0} - \frac{1}{\kappa t_0} \sin \kappa t \qquad \text{for } 0 \leq t \leq t_0$$
$$= 1 + \frac{1}{\kappa t_0} [\sin \kappa (t - t_0) - \sin \kappa t] \quad \text{for } t \geq t_0 \qquad [11]$$

which is plotted in Figure 5.

The dynamic load factor is

$$n = 1 + \left| \frac{2}{\kappa t_0} \sin \frac{\kappa t_0}{2} \right| \qquad [12]$$

and is plotted in Figure 6.

THE RECTANGULAR PULSE

The disturbance shown in Figure 7 is

$$p = 0 \quad \text{for } t < 0; \qquad p = 1 \quad \text{for } 0 \leq t \leq t_1; \qquad p = 0 \quad \text{for } t > t_1 \qquad [13]$$

The response to this disturbance is

$$u = 1 - \cos \kappa t \quad \text{in } 0 \leq t \leq t_1$$
$$u = 2 \sin \frac{\kappa t_1}{2} \sin \kappa \left(t - \frac{t_1}{2} \right) \quad \text{in } t \geq t_1 \qquad [14]$$

which is given in Figure 8.

The dynamic load factor is

$$n = 2\sin\frac{\kappa t_1}{2} \qquad \text{for } t_1 \leq 0.5T$$
$$n = 2 \qquad \text{for } t_1 \geq 0.5T \qquad [15]$$

THE TRIANGULAR PULSE

The disturbance is

$$\begin{aligned} p &= 0 && \text{for } t < 0 \\ p &= 2\frac{t}{t_1} && \text{for } 0 \leq t \leq 0.5t_1 \\ p &= 2 - 2\frac{t}{t_1} && \text{for } 0.5t_1 \leq t \leq t_1 \\ p &= 0 && \text{for } t \geq t_1 \end{aligned} \qquad [16]$$

The response is

$$u = 2\frac{t}{t_1} - \frac{2}{\kappa t_1}\sin\kappa t \qquad \text{in } 0 \leq t \leq 0.5t_1$$

$$u = 2 - 2\frac{t}{t_1} + \frac{2}{\kappa t_1}\left[2\sin\kappa\left(t - \frac{t_1}{2}\right) - \sin\kappa t\right] \qquad \text{in } 0.5t_1 \leq t \leq t_1 \qquad [17]$$

$$u = \frac{2}{\kappa t_1}\left[-\sin\kappa(t - t_1) + 2\sin\kappa\left(t - \frac{t_1}{2}\right) - \sin\kappa t\right] \qquad \text{in } t \geq t_1$$

These are plotted in Figure 10.

THE SINUSOIDAL PULSE

The disturbance is

$$\begin{aligned} p &= 0 && \text{for } t < 0 \\ p &= \sin\frac{\pi t}{t_1} && \text{for } 0 \leq t \leq t_1 \\ p &= 0 && \text{for } t > t_1 \end{aligned} \qquad [18]$$

The response is

$$u = \frac{2}{\frac{T}{t_1} - \frac{4t_1}{T}}\left(\sin\kappa t - 2\frac{t_1}{T}\sin\frac{\pi t}{t_1}\right) \qquad \text{in } 0 \leq t \leq t_1$$

$$u = \frac{2}{\frac{T}{t_1} - \frac{4t_1}{T}}\left[(1 + \cos\kappa t_1)\sin\kappa t - \sin\kappa t_1\cos\kappa t\right] \qquad \text{in } t \geq t_1 \qquad [19]$$

The preceding formulas are indeterminate for $t_1 = 0.5T$. For this particular case, the response is

$$u = \frac{1}{2}(\sin\kappa t - \kappa t\cos\kappa t) \qquad \text{in } 0 \leq t \leq 0.5T$$

$$u = -\frac{\pi}{2}\cos\kappa t \qquad \text{in } t \geq 0.5T \qquad [20]$$

These are plotted in Figure 12.

BLAST DISTURBANCE

The disturbance shown in Figure 14 is

$$\begin{aligned} p &= 0 && \text{for } t < 0 \\ p &= e^{-\frac{t}{t_1}}\left(1 - \frac{t}{t_1}\right) && \text{for } t \geq 0 \end{aligned} \qquad [21]$$

The response is

$$u = \frac{1}{(1+c^2)^2}\Big[2c^3\sin\kappa t + c^3(1 - c^2)\cos\kappa t$$
$$+ c^2 e^{-\frac{t}{t_1}}\left[c^2 - 1 - (1+c^2)\frac{t}{t_1}\right]\Big] \quad \text{for } t \geq 0 \qquad [22]$$

where $c = \kappa t_1$.

An alternative form which is sometimes convenient is

$$u = \frac{c^2}{1+c^2}\sin(\kappa t - \phi) + \frac{c^2}{(1+c^2)^2}e^{-\frac{t}{t_1}}\left[c^2 - 1 - (1+c^2)\frac{t}{t_1}\right] \quad \text{for } t \geq 0 \qquad [23]$$

in which the phase angle ϕ is given by

$$\sin\phi = \frac{c^2 - 1}{c^2 + 1}$$
$$\cos\phi = \frac{2c}{c^2 + 1} \qquad [24]$$

Representative responses are plotted in Figure 15.

DAMPED OSCILLATING DISTURBANCE

The disturbance considered is of the form

$$\begin{aligned} p &= 0 && \text{for } t < 0 \\ p &= e^{\beta\left(\frac{\pi}{2} - \phi - \omega t\right)}\sin(\omega t + \phi) && \text{for } t \geq 0 \end{aligned} \qquad [25]$$

The phase angle ϕ is considered to lie in the range $0 \leq \phi \leq \frac{\pi}{2}$ and the damping constant is small enough so that β^2 is negligible in comparison with unity. Under these conditions the first peak of the disturbance function p has the value unity.

The response is

$$u = \frac{e^{\beta\left(\frac{\pi}{2} - \phi\right)}}{1 - r^2}\left(\left[\frac{1 + r^2}{1 - r^2}r\beta\sin\phi - r\cos\phi\right]\sin\kappa t - \left[\sin\phi\right.\right.$$
$$\left.\left. + \frac{2\beta r^2}{1 - r^2}\cos\phi\right]\cos\kappa t + e^{-\beta\omega t}\left[\sin(\omega t + \phi) + \frac{2\beta r^2}{1 - r^2}\cos(\omega t + \phi)\right]\right) \qquad [26]$$

where $r = \frac{\omega}{\kappa} \neq 1$.

For $\omega = \kappa$,

$$u = e^{\beta(\frac{\pi}{2}-\phi)}\left(\frac{1}{4}\beta\sin\phi\sin\kappa t - \frac{1}{2}\cos\kappa t\sin\phi\right.$$
$$\left. + \frac{1}{2}e^{-\beta\kappa t}\left[1+\frac{1}{2}\beta\kappa t\right]\left[\sin(\kappa t+\phi) - \kappa t\cos(\kappa t+\phi)\right]\right) \qquad [27]$$

Equation [26] can be written as

$$u = \frac{e^{\beta(\frac{\pi}{2}-\phi)}}{1-r^2}\left(a\sin(\kappa t+\psi) + e^{-\beta\omega t}\left[\sin(\omega t+\phi) + \frac{2\beta r^2}{1-r^2}\cos(\omega t+\phi)\right]\right) \qquad [28]$$

The amplitude a of the free vibration term inside the large parentheses is given by

$$a^2 = r^2 + (1-r^2)\sin^2\phi + \beta r^2\sin 2\phi \qquad [29]$$

The amplitude a varies monotonically from r to 1 as ϕ increases from zero to $\frac{\pi}{2}$.

The response of Figure 19 is plotted for $r = 0.3$, $\beta = 0.02$, and $\phi = 0$. The same parameters are used for the curve of Figure 20, with the exception that $\phi = \frac{\pi}{2}$. The differences in disturbance are shown in the broken lines of the two figures.

APPENDIX 2

IMPACT LOADING ON A SYSTEM OF MANY DEGREES OF FREEDOM

The system to be considered here is the uniform slender beam subjected to suddenly applied load so that the disturbance has the form shown in Figure 2. It is necessary also to specify the distribution of load along the length of the beam. If x is a coordinate along the length of the beam, being zero at one end and equal to 1 at the other end, and if t is the time, the load may be described by

$$\begin{aligned} P(x,t) &= 0 && \text{for } t<0 \\ P(x,t) &= q(x) && \text{for } t\geq 0 \end{aligned} \qquad [30]$$

The common differential equation for the transverse vibration of a beam (Equation [116] of Reference (1)) is an approximation valid only for the lower modes. Secondary effects neglected there will have a notable effect wherever the contribution of the higher modes is of importance, as, for example, in analyzing the propagation of a transverse disturbance along a bar. For the purposes of this Appendix, however, it is unimportant whether the system chosen corresponds wholly to a real structure, as the interest is directed toward the effects of many degrees of freedom rather than at the behavior of an actual beam under impact.

If E is Young's modulus,
- I is the moment of inertia,
- m is the mass per unit length, and
- y is the deflection of the beam in the direction of loading, a function of both space and time,

then the simple differential equation mentioned above is

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = P(x,t) \qquad [31]$$

If the beam is initially at rest, and the load is given by Equation [30], the solution of Equation [31] is

$$y(x,t) = \sum_{n=1}^{\infty} a_n f_n(x)(1-\cos\omega_n t) \qquad [32]$$

In [32] the $f_n(x)$ are the normal functions of the problem (Reference (1), page 201), the ω_n are the natural frequencies corresponding to the normal modes, and the a_n are the coefficients in the expansion of the static deflection y_s under the given load distribution in terms of the normal functions, or

$$y_s(x) = \sum_{n=1}^{\infty} a_n f_n(x) \qquad [33]$$

For a pin-ended (simple) beam,

$$f_n(x) = \sin\frac{n\pi x}{l}$$

and

$$\omega_n = n^2\omega_1$$

The constants a_n are simply the Fourier coefficients of the expansion of $y_s(x)$ in a sine series.

If the loading is uniform and equal to p per unit length, but distributed only over a central portion of length a,

$$\begin{aligned} q(x) &= p \qquad \text{for } \frac{l-a}{2} \leq x \leq \frac{l+a}{2} \\ q(x) &= 0 \qquad \text{elsewhere} \end{aligned} \qquad [34]$$

and

$$a_n = \frac{4pl^4}{EI\pi^5}\cdot\frac{1}{n^5}\sin\frac{n\pi}{2}\sin\frac{n\pi a}{2l}$$

The dynamic deflection of the beam under the load $P(x,t)$ is therefore

$$y(x,t) = \frac{4pl^4}{EI\pi^5}\sum_{n=1}^{\infty}\frac{1}{n^5}\sin\frac{n\pi}{2}\sin\frac{n\pi a}{2l}\sin\frac{n\pi x}{l}(1-\cos n^2\omega_1 t) \quad \text{for } t \geq 0 \qquad [35]$$

The bending moment is

$$M = -EI\frac{\partial^2 y}{\partial x^2} = \frac{4pl^2}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^3}\sin\frac{n\pi}{2}\sin\frac{n\pi a}{2l}\sin\frac{n\pi x}{l}(1-\cos n^2\omega_1 t) \qquad [36]$$

and the shear is

$$V = \frac{\partial M}{\partial x} = \frac{4pl}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi a}{2l}\cos\frac{n\pi x}{l}(1-\cos n^2\omega_1 t) \qquad [37]$$

The corresponding formulas for uniform loading over the entire span can be obtained from the foregoing by substituting l for a in Equations [34] - [37].

Equations analogous to [4] and [5] of Appendix 1 can be used for obtaining the dynamic deflection under values of load other than the suddenly applied load of Equation [30] provided that the distribution of load does not change.

The coefficients a_n must now be considered as functions of t, $a_n(t)$. Then

$$y(x,t) = \sum_{n=1}^{\infty} f_n(x)\int_0^t a_n(\alpha)\sin\omega_n(t-\alpha)\,d\alpha \qquad [38]$$

where the impact begins at $t = 0$.

For a rectangular pulse as in Figure 7, lasting from $t = 0$ to $t = t_1$ and consisting of a uniform load p over the central length a,

$$y(x,t) = \frac{4pl^4}{EI\pi^5}\sum_{n=1}^{\infty}\frac{2}{n^5}\sin\frac{n\pi}{2}\sin\frac{n\pi a}{2l}\sin\frac{n\pi x}{l}\sin\frac{n^2\omega_1 t_1}{2}\sin n^2\omega_1\left(t-\frac{t_1}{2}\right) \quad \text{in } t \geq t_1 \qquad [39]$$

For $0 \leq t \leq t_1$, the dynamic deflection is given by [35].

Equation [39] shows that the relative excitation of higher modes increases when t_1 becomes less than half of a fundamental period.

An analogous point can be made about the spatial distribution of load. The presence of the factor $\sin\frac{n\pi a}{2l}$ in Equations [35] and [39] shows that concentration of load, that is, small values of a/l, lead to an increasing excitation of the higher modes. Thus there can always be found values of $\omega_1 t_1$ and a/l below which the dynamic behavior of the beam is certainly quite different from that of a system with a single degree of freedom.

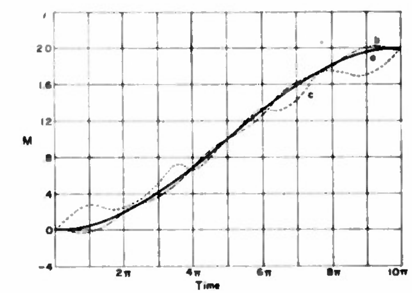


Figure 23 - Bending Moment at Center of Bar

The effect of concentration of load in time and in space may be illustrated graphically. Figure 23 shows how concentration of load in space affects the response; the bending moment at the center of the bar (expressed in units of equivalent static bending moment) is plotted as a function of time. Curve (a) represents a system with a single degree of freedom; Curve (b) the bar loaded uniformly along the entire length; and Curve (c) the bar loaded uniformly along the central 10 per cent. In all three cases the load is the step function (see Figure 2). The dynamic load factor in these cases is 2.

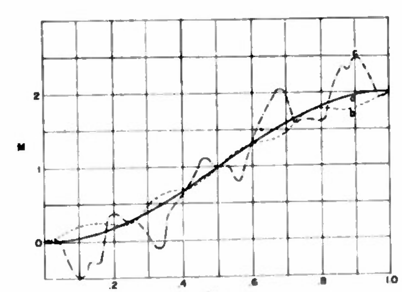


Figure 24 - Bending Moment near End of Bar

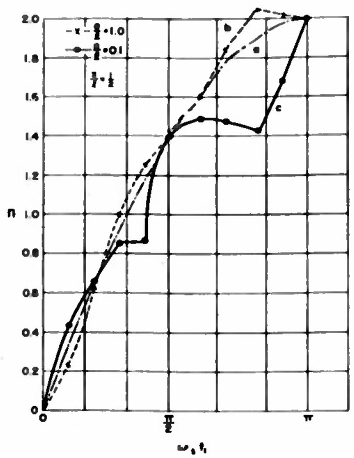


Figure 25 - Dynamic Load Factor in Bending Moment Against Duration of Pulse

However, if a similar plot is made of the bending moment near the end of the bar x/l = 1/8 (see Figure 24), a dynamic load factor of 2.5 is obtained in Curve (c). The initial negative portion indicates that in the early stages of the motion, the portions of the bar near the supporta deflect in the direction opposite that of the blow. The elastic curve taken up by the bar at that instant ia therefore not equivalent to the eurve that would be taken by a system of a single degree of freedom. The various parts of the bar in the latter case would always be in phase.

The effect of concentration of load in time is shown by Figures 25 and 26. Figure 25 is a plot of the dynamic load factor, with respect to the bending moment at the center of the bar, aa a function of $\omega_1 t_1$ where t_1 is the duration of the step pulse (see Figure 7). Figure 26 is a plot of the dynamic load factor, with respect to the shear at the end of the bar, as a function of $\omega_1 t_1$. As in Figure 23 the Curvea (a), (b), (c), refer respectively to the case of one degree of freedom, uniform loading along the bar, and concentrated loading .ı the central portion of the bar.

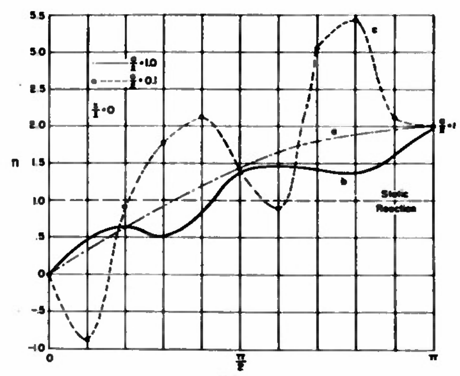


Figure 26 - Dynamic Load Factor in Shear Against Duration of Pulse

It is seen that in all instances the behavior of a beam with concentrated space loading deviates from that of

a system with a single degree of freedom. This deviation is very pronounced in the case of shear at the bar's end (Figure 26), in which case a dynamic load factor of approximately 3.5 is reached [Curve (c)].

In the case of uniform loading along the entire bar [Curve (b)], the deviation from the single-degree-of-freedom case is not as marked. It appears from the foregoing that when a real structure may not be approximated by a system of a single degree of freedom, the dynamic load factor may exceed 2 considerably.

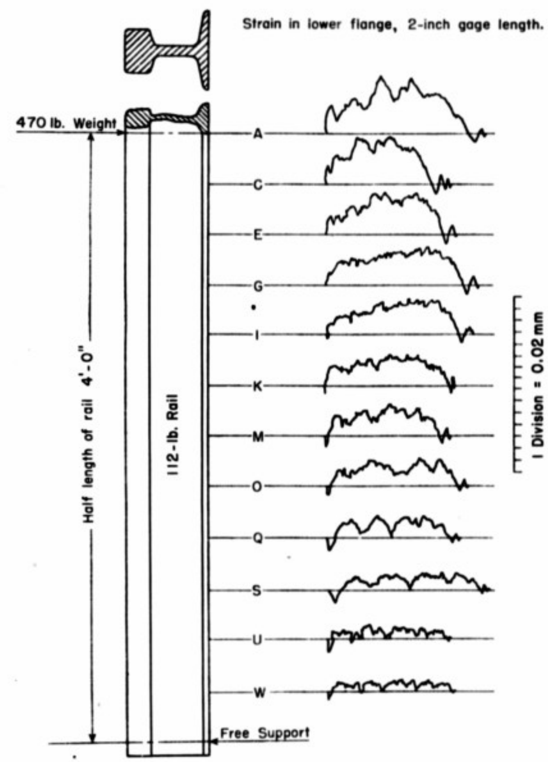


Figure 27 - Bending Strains due to a Single Impact

It is interesting to compare the foregoing statements with some experimental findings of R. N. Arnold (9). His apparatus consisted of a simply supported beam subject to impacts at mid length. Figure 27 consists of scratch photographs indicating variation of bending moment along the beam. Though the horizontal scale in this figure is not linear with time, the record does indicate how the bending moment varies. These records should be compared with the theoretical curves in Figure 23(c), and Figure 24(c). Curve S in Figure 27 corresponds to $a/l = 1/8$ and curve A to $a/l = 1/2$. The author states that the impact loads at the center of the bar (obtained by four independent means) are in all cases 2.5 times those predicted by the assumption of statical equivalence.

TSDIN FORM 69 A (13 MAR 47)

RESTRICTED

P-7-1-2

ATI- 16566

ORIG. AGENCY NUMBER

R-481

REVISION

AUTHOR(S)

Frankland, J. M.

DIVISION: Stress Analysis and Structures (7)

SECTION: Loads, Criteria and Specifications (1)

CROSS REFERENCES: Impact loads (50924); Structures, Aircraft - Gun blast resistance (90864.5)

AMER. TITLE: Effects of impact on simple elastic structures

FORG'N. TITLE:

ORIGINATING AGENCY: David Taylor Model Basin, Washington, D. C.

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N.CLASS	U. S.CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Restr.	Apr'42	41	33	diagrs, graphs

ABSTRACT

The characteristic features of stresses arising from rapidly applied or brief loads such as gun blast or gun recoil, referred to as impact loading, are presented. Representative cases of impact loading and their effects upon a simple undamped system with one degree of freedom are investigated for oscillating and nonoscillating disturbances. Means of judging to what degree the behavior of the idealized system is realized among actual ships and other structures is discussed. Application of this to the strength of structures under impact is considered.

TSDIN FORM 69 A (13 MAR 47)

RESTRICTED

R-7-1-2

ATI- 16566

Frankland, J. M.

AUTHOR(S)

DIVISION: Stress Analysis and Structures (7)

SECTION: Loads, Criteria and Specifications (1) 3

CROSS REFERENCES: Impact loads (50924); Structures, Aircraft - Gun blast resistance (90864.5)

ORIG. AGENCY NUMBER

R-481

REVISION

U R

AMER. TITLE: Effects of impact on simple elastic structures

FORG'N. TITLE:

ORIGINATING AGENCY: David Taylor Model Basin, Washington, D. C.

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N. CLASS	U. S. CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Restr.	Apr'42	41	33	diagrs, graphs

ABSTRACT

The characteristic features of stresses arising from rapidly applied or brief loads such as gun blast or gun recoil, referred to as impact loading, are presented. Representative cases of impact loading and their effects upon a simple undamped system with one degree of freedom are investigated for oscillating and nonoscillating disturbances. Means of judging to what degree the behavior of the idealized system is realized among actual ships and other structures is discussed. Application of this to the strength of structures under impact is considered.

EO 10501 dd 5 NOV 1953

T-2, HQ., AIR MATERIEL COMMAND

AIR TECHNICAL INDEX

WRIGHT FIELD, OHIO, USAAF

RESTRICTED

WF-O-21 MAR 47 22,500