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A THEORY OF UNSTABLE COMBUSTION IN LIQUID PROPELLANT ROCKET SYSTEMS

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A THEORY OF UNSTABLE COMBUSTION IN LIQUID PROPELLANT ROCKET SYSTEMS

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ABSTRACT

On the basis of an hypothesis that low frequency oscillations ("chugging") sometimes observed in liquid propellant rocket engines are the result of oscillatory propellant flow induced by a combustion time lag, conditions for the suppression of such oscillations are derived. It is found that stability can be achieved by increases in the length of feed line, the velocity of the propellant in the feed line, the ratio of feed pressure to chamber pressure, and the ratio of chamber volume to nozzle area. Equations are given for the frequencies of oscillation. Examination of the equation for stability indicates that self-igniting propellant combinations are likely to be more stable than non-self-igniting systems.

A THEORY OF UNSTABLE COMBUSTION IN LIQUID PROPELLANT ROCKET SYSTEMS[1]

Martin Summerfield[2]

INTRODUCTION

Liquid propellant rocket engines of all types are generally designed by the engineer to deliver a fairly constant, steady thrust for a duration that may extend from several seconds to as much as an hour. However, in many cases, it has been reported that the desired steady operation does not occur in actual test, and instead a condition variously described as "rough burning", "chugging", "screaming" or simply unstable combustion may take place. Frequencies ranging from 10 cycles per second to as much as 5000 cycles per second have been observed in oscillographic chamber pressure traces, amplitudes from a few percent to as much as 50 percent of the mean chamber pressure, and in many cases, the oscillation was not truly periodic but seemed to be merely a series of random fluctuations. Not only is the resulting thrust vibration undesirable from the standpoint of possible damage to the structural elements or instruments in the vehicle, but in extreme cases, failure of the power plant itself can occur. (Reference 1).

The theory presented here does not attempt to explain all cases of unstable combustion, but it may provide an explanation for one type of instability, perhaps the most serious type, sometimes called "chugging".

The phenomenon of "chugging" was first observed by the author in October 1941 during a series of tests of a 1000 lb. thrust nitric acid gasoline rocket motor at the Jet Propulsion Laboratory of the California Institute of Technology. Upon ignition, combustion would proceed smoothly at first, but would rapidly become rough, and in many instances, after five or ten seconds, severe fluctuations in chamber pressure would be taking place. Frequently, the bolts of the combustion chamber would rupture before the propellant flow could be turned off.

An explanation for this oscillation was advanced on the basis of an hypothesis that a time lag existed between an arbitrary fluctuation in propellant flow and its subsequent manifestation in the combustion chamber pressure. This time lag is the result of the finite rate

[1]Submitted in final form to Project SQUID April 11, 1951. (Original version dated January 26, 1951.)

[2]General Editor, Aeronautics Publication Program, Princeton University, Consultant to Project SQUID.

of the overall combustion process, and is determined therefore by the kinetic rates of mixing, vaporization, and chemical reaction. The argument was pursued to show that the instability could be suppressed by increasing the pressure drop across the injector. A numerical illustration of this argument may be helpful for an appreciation of the analysis below.

A particular rocket engine with a compressed gas type of feed system is designed to operate with a chamber pressure of 300 psia. In the first case, assume that the feed pressure is 500 psia. Now suppose a momentary decrease in chamber pressure occurs after steady operation is achieved, the decrease being from 300 to 200 psia. Assuming a square-law pressure drop across the injector, and neglecting the inertia of the liquid in the feed line, the flow rate will increase by 23% above the design value and will subsequently (after the time lag mentioned above) produce a 23% increase in chamber pressure above the design value, namely, 369 psia. The injector pressure drop is now only 131 psia, the corresponding flow rate is therefore reduced to 81% of the design value, and the chamber pressure will then fall (after the time lag), to 81% or 243 psia. The flow rate then increases again, and it is possible to continue the calculation in the same manner. In this case, after one cycle, it is apparent that the disturbance is decaying, the amplitude having decreased from 100 psia to 69 psia to 57 psia.

Next, consider a second case in which the design chamber pressure is also 300 psia, but the feed pressure is only 400 psia. If an arbitrary decrease of 100 psia is assumed as above, the sequence of chamber pressure values becomes 300, 200, 420, 0. In this case it is clear that the fluctuations will increase in strength as time proceeds. A third case, assuming a feed pressure of 450 psia, exhibits neutral stability; i.e., the amplitude of the fluctuation remains approximately constant.

It appears from such crude considerations that the injector pressure drop is a controlling parameter. It will be seen below that this numerical conclusion can be generalized in the following statement: instability is not possible if the injector pressure drop exceeds one half the mean chamber pressure. However, it will be seen that the converse is not always true. That is, instability can be suppressed even when the injector drop is less than half the chamber pressure.

Of course, even in its qualitative form, the theory suggests that instability can be eliminated by reducing the time lag in the combustion process. This was accomplished, in the 1941 program, by switching to aniline as a fuel instead of gasoline, and indeed, with no change in design or operating conditions, smooth running was achieved.

In formulating the problem in 1941 the author's analysis neglected the inertia of the liquid propellant in the feed lines.[3] The flexible coupling responsible for this self-excited

[3] The author is indebted to Dr. Theodore von Karman for profitable discussion that led to the basic hypothesis of a combustion time lag and for suggestions on the mathematical approach to the problem. (Private Communication, Nov., 1941.)

oscillation was assumed to be the capacitance of the combustion chamber. A recent paper by Gunder and Friant (Reference 2) presented an analysis of chugging in which the essential hypothesis was also the time lag described above (it was independently conceived by them), but instead of the chamber capacitance the important term was the inertia of the liquid in the feed lines. It is shown in their paper that instability is not possible when the pressure drop exceeds half the chamber pressure. Yachter and Waldinger, in an unpublished communication (1949), considered the case in which liquid inertia is neglected, and reached the same conclusion.

Since similar conclusions resulted either from a consideration of liquid inertia alone or chamber capacitance alone, it appeared logical to carry out the analysis with both effects present. This is done below, and not only does it include each of the above analyses as special cases, but certain conditions for stability can be derived in such simple form that physical conclusions suggest themselves immediately.

In passing, a few remarks concerning other types of non-steady operation are pertinent in order to clarify the assumptions of the present theory. It is assumed here that the pressure is uniform throughout the combustion chamber at any instant, that is, that the period of oscillation is long compared with the time required for a pressure wave to traverse the combustion chamber in any direction. However, high frequency "screaming" has been observed in some rocket motor tests, which indicates that chamber gas oscillations are possible, in addition to the system oscillations treated here. These high frequencies usually agree fairly well with those expected on the basis of axial resonance (organ pipe notes) or transverse resonance. What is not clear, however, is the nature of the process that energizes such oscillations and why they appear only under certain conditions of injection or chamber configuration. It is suspected that the oscillation is energized by a proper interaction between the variations in pressure and the rate of combustion, (that is, the time lag is some function of the instantaneous pressure) but the exact formulation of this interaction is not known.[4] The remedy is not clear unless it is to try arbitrary changes in the spatial combustion pattern. These high frequency vibrations are usually of small amplitude but are generally accompanied by significant increases in heat transfer to the chamber walls. The danger of this type of oscillation lies in the possibility of a burn-out of the chamber.

A third category of unsteady operation is connected with either mechanical or hydraulic oscillations in the rocket system. Aside from the well-known possibilities of structural vibration, valve flutter, feed pump pulsation, etc., there seems to be the additional possibility of non-steady hydraulic behavior of certain elements of the flow system, particularly the injector orifices. The danger of operating near the critical flow rate corresponding to the

[4]An analysis based on this hypothesis has been advanced by Dr. L. Crocco of Princeton University.

appearance of a vena-contracta in the injector orifice was first pointed out to the author by W. B. Powell[5] in 1942. The remedy for this type of instability lies clearly within the domain of the hydraulic engineer. The present theory is not concerned with such effects.

ANALYSIS

A schematic liquid propellant rocket system is shown in Figure 1. It is assumed that the system is either a monopropellant type or, if a bipropellant or multipropellant system is under consideration, that the feed lines and injector orifices for the separate liquid reactants have identical hydraulic characteristics so that it behaves like a monopropellant system. Although it is possible to repeat the monopropellant analysis (below) for the case of a multipropellant system, the additional complication is hardly worth while until the basic ideas are confirmed by experimental checks. The physical principles and the resulting rules for overcoming instability will be qualitatively the same for the bipropellant case.

At any instant during the oscillation, the rate of change of pressure in the combustion chamber is governed by the difference between the rate of evolution of combustion gas and the rate of gas flow through the nozzle.[6] In accordance with the fundamental hypothesis of this theory, the rate of evolution of gas at a given instant is equal to the rate of flow of propellant *at an instant τ seconds earlier*.

$$\frac{dp_c}{dt} = \frac{RT_c}{V_c} \rho A_2 \, v_{2(t-\tau)} - \frac{RT_c}{c^* L^*} p_c \qquad (1)$$

The effect of a changing chamber pressure on the flow of liquid propellant can be calculated by first setting up the general equation for the instantaneous flow rate with a specified difference between tank pressure and chamber pressure, and then differentiating this equation with respect to time. A convenient approach is to consider two control surfaces normal to the flow direction, one of area A_1 in the propellant tank upstream of the inlet to the feed line, and the other of area A_3 at the exit of the injector orifice, and then equate the total energy (work plus kinetic energy) entering the first control surface in a time interval dt to the energy leaving the second control surface plus the change of kinetic energy of the liquid contained between the two control surfaces plus the energy dissipated in the same time interval.

$$(p_1 A_1 v_1 + \tfrac{1}{2} \rho A_1 v_1^3)dt = (p_c A_3 v_3 + \tfrac{1}{2} \rho A_3 v_3^3)dt +$$
$$d(\tfrac{1}{2} A_1 l_1 \rho v_1^2 + \tfrac{1}{2} A_2 l_2 \rho v_2^2 + \tfrac{1}{2} A_3 l_3 \rho v_3^2) + \frac{\rho}{2}(k_1 v_1^2 + k_2 v_2^2 + k_3 v_3^2)dt$$

[5]Jet Propulsion Laboratory, California Institute of Technology.
[6]Symbols employed in the analysis are explained on page 14.

Carrying out the differentiation, replacing v_1 and v_3 by v_2 through the continuity equation, the equation takes the following form:

$$(p_1-p_c) + \frac{1}{2}\rho v_2^2 \left[\left(\frac{A_2^2}{A_1^2} - \frac{A_2^2}{A_3^2}\right) - K\right] = \rho\left(l_1 \frac{A_2}{A_1} + l_2 + l_3 \frac{A_2}{A_3}\right) \frac{dv_2}{dt} \qquad (2)$$

In practical designs, $\frac{A_2}{A_1} << \frac{A_2}{A_3}$, $l_1 \frac{A_2}{A_1} << l_2$, and $l_3 \frac{A_2}{A_3} << l_2$. Hence, without appreciable error, these terms may be neglected. The energy dissipations due to pipe-line friction, valve losses, and imperfect flow in the injector orifices are contained in the term $K \frac{1}{2} \rho v_2^2$.

Differentiating equation (2) with respect to time, the desired equation is:

$$\frac{dp_c}{dt} + \rho\left(\frac{A_2^2}{A_3^2} + K\right) v_2 \frac{dv_2}{dt} + \rho l_2 \frac{d^2 v_2}{dt^2} = 0. \qquad (3)$$

Equations (1) and (3) can be combined to eliminate p_c and provide a single equation with a single dependent variable, v_2. Thus, differentiate equation (1) to produce an equation containing the first and second derivative of p_c. The first derivative can be replaced by equation (3) and an expression for the second derivative is obtainable by differentiating equation (3). The resulting equation for v_2 is the following (dropping subscript "2" of v_2, and writing prime for the derivative with respect to time):

$$l_2 v''' + \frac{RT_c l_2}{c^* L^*} v'' + \left(\frac{A_2^2}{A_3^2} + K\right) v v'' + \left(\frac{A_2^2}{A_3^2} + K\right)(v')^2 \qquad (4)$$

$$+ \frac{RT_c}{c^* L^*}\left(\frac{A_2^2}{A_3^2} + K\right) v v' + \frac{A_2 RT_c}{V_c} v'_{(t-\tau)} = 0$$

This non-linear differential equation can be linearized by the following argument. If the amplitude of oscillation Δv is small compared with the equilibrium value of v (which is a plausible assumption in testing for instability), then v_e can replace v in equation (4). From equation (2):

$$v_e = \left(\frac{A_2^2}{A_3^2} + K\right)^{-\frac{1}{2}} \left(\frac{2\Delta p}{\rho}\right)^{\frac{1}{2}} \qquad (5)$$

The same assumption that Δv is small compared with v_e makes it legitimate to throw out the $(v')^2$ term in comparison with either the v'' terms or the v' terms. The latter comparison leads to this conclusion provided RT_c/c^*L^* is identified as the maximum value of the frequency when conditions are near neutral stability. This identification appears later and provides a posteriori justification for this approximation.

Making the substitution, $u = v'$, the differential equation becomes:

$$u'' + Au' + Bu + Cu_{(t-\tau)} = 0 \tag{6}$$

In this equation:

$$A = \frac{RT_c}{c^*L^*} + \left(\frac{A_2^2}{A_3^2} + K\right)\frac{v_e}{l_2} = \left(\frac{RT_c}{c^*L^*}\right) + \left(\frac{2\Delta p}{p_c}\right)\left(\frac{p_c A_2}{l_2 \dot{m}}\right)$$

$$B = \frac{RT_c}{c^*L^*}\cdot\left(\frac{A_2^2}{A_3^2} + K\right)\frac{v_e}{l_2} = \left(\frac{RT_c}{c^*L^*}\right)\left(\frac{2\Delta p}{p_c}\right)\left(\frac{p_c A_2}{l_2 \dot{m}}\right) \tag{7}$$

$$C = \frac{RT_c}{V_c}\cdot\frac{A_2}{l_2} = \left(\frac{RT_c}{c^*L^*}\right)\left(\frac{p_c A_2}{l_2 \dot{m}}\right)$$

To solve equation (6), it is assumed that the general solution can be represented in the form of a linear sum of particular solutions, each of these being a sinusoidal oscillation modified by a damping factor.[7]

$$u = \sum_n U_n\, e^{(\lambda_n + i\omega_n)t} \tag{8}$$

The successive values (λ_n, ω_n) are the roots of the characteristic equations obtained by inserting the above solution into equation (6), and grouping real and imaginary terms:

$$\lambda^2 + A\lambda + B - \omega^2 + C\, e^{-\lambda\tau}\cos(\omega\tau) = 0 \tag{9}$$

$$2\lambda\omega + A\omega - C\, e^{-\lambda\tau}\sin(\omega\tau) = 0 \tag{10}$$

It is convenient to transform these equations by introducing new variables:

$$\begin{aligned} \omega\tau &= \theta \\ \lambda\tau &= x \\ A\tau &= a \\ B\tau^2 &= b \\ C\tau^2 &= c \end{aligned} \tag{11}$$

Equations (9) and (10) become

$$x^2 + ax + b - \theta^2 + c\, e^{-x}\cos\theta = 0 \tag{12}$$

$$(2x + a)\,\theta - c\, e^{-x}\sin\theta = 0 \tag{13}$$

[7] The method of solution employed here follows that of N. Minorsky for time lag problems. (Reference 4).

By plotting these equations in the x - θ plane it is possible to locate all the intersections of (12) and (13), and thus, for any given values of A, B, C, τ, to determine the behavior of the system. Those solutions involving positive values of x lead to instability; those solutions with negative x represent disturbances that decay. It is the objective of the designer to select values of A, B, and C such that *all* the terms in (8) are damped (or at least the low frequency terms, since the high frequencies are generally suppressed by viscous dissipation).

A plot of equations (12) and (13) would look like Figure 2 for arbitrary values of a, b, and c. Only the positive θ half of the plane need be examined: both equations are invariant with respect to a change in the sign of θ. Equation (12) yields two branches: the positive one gradually converges upon the line $x = \theta$; the negative one is a broken curve drifting off to more negative values of x. Equation (13) describes a broken curve that converges upon the line $x = - a/2$. Examination of the three curves reveals that, in general, intersections can occur only a finite number of times, since the curves separate from each other at large values of θ. This indicates that, for any given system, there is an upper limit to the frequencies that can occur, whether positively or negatively damped, and these can be calculated graphically in the manner of Figure 2.

Certain conditions can be derived for which solutions with positive values of x are impossible. Thus, if $a > c$, the curve given by equation (13) would lie entirely in the negative x half of the plane. Therefore, this inequality is a sufficient condition for stability. The inequality can be expressed in terms of the appropriate design parameters by means of the defining equations (7) and (11) and the following relationships:

$$\begin{aligned} \dot{m} &= \rho v_2 A_2 \\ \Delta p &= \tfrac{1}{2} \rho v_2{}^2 \left(\frac{A_2{}^2}{A_3{}^2} + K \right) \\ c^* &= p_c A_t / \dot{m} \end{aligned} \qquad (14)$$

The condition for stability then becomes:

$$\frac{l_2}{p_c} \frac{\dot{m}}{A_2} + \frac{c^* L^*}{R T_c} \frac{2\Delta p}{p_c} > \tau \qquad (15)$$

Another useful form of this equation can be obtained by using the theoretical expression for c^*:

$$c^* = f(\gamma) \sqrt{R T_c} \; ; \; f(\gamma) = \gamma^{-\frac{1}{2}} \left[\frac{\gamma + 1}{2} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \qquad (16)$$

$$f(\gamma) \approx 1.55 \text{ for } 1.1 < \gamma < 1.3$$

By substitution in (15) the inequality takes the form:

$$\frac{l_2}{p_c}\frac{\dot{m}}{A_2} + 2.4\,\frac{L^*}{c^*}\frac{2\Delta p}{p_c} > \tau \qquad (17)$$

Although this inequality expresses merely a *sufficient condition* for stability, it will appear in the following section, when the magnitudes of the terms are examined for typical cases, that the condition is sufficiently close to be useful as a design criterion.

Another general condition for stability can be derived by combining equation (12) and (13) and examining the possible roots (θ_n, x_n):

$$\begin{aligned} &\theta^4 + \theta^2 f(x) + g(x) = 0 \\ &f(x) = 2x^2 + 2ax + (a^2-2b) \\ &g(x) = (x^2 + ax + b)^2 - c^2\, e^{-2x} \end{aligned} \qquad (18)$$

The signs of $f(x)$ and $g(x)$ are of interest. The term (a^2-2b) can be evaluated in terms of the defining equations (7) and (11), and it can be seen that it is inherently positive. Consequently, in the positive x domain where our attention is focused, $f(x)$ is positive. Then, θ^2 can be positive only if $g(x)$ is negative, and conversely, it is possible to eliminate real, positive values of θ^2, and hence of $\dot{\theta}$, by setting $g(x) > 0$ in the positive x domain. This condition can be assured by setting $b > c$. This inequality can be expressed in terms of the design parameters through equations (7) and (11), yielding the following condition for stability:

$$\frac{2\Delta p}{p_c} > 1 \qquad (19)$$

As in the case of equation (15), this inequality represents merely a *sufficient condition* for stability. Clearly, stability is possible even if $2\Delta p/p_c$ is less than unity, by reference to equation (15).

The frequencies of the oscillations can be evaluated in principle by solving equation (18) together with either of equations (12) or (13). However, consideration of equation (18) alone can indicate the magnitude of the frequency of the particular mode in series (8) that makes its appearance as the stability conditions (15) and (19) are gradually relaxed. Thus, equation (18) being quadratic in θ^2, it can be solved explicitly for θ_{cr}, the critically damped frequency:

$$\theta_{cr} = \sqrt{-\tfrac{1}{2} f(0) + \tfrac{1}{2}\sqrt{f(0)^2 - 4g(0)}} \qquad (20)$$

If it is supposed that the conditions of critical damping occurs when $(2\Delta p/p_c)$ is near to unity, then by setting $(c-b) \ll 1$ and expanding the square root accordingly it can be shown that:

$$\omega_{cr} \approx \frac{\left(1 - \frac{2\Delta p}{p_c}\right)^{1/2}}{\left[\left(\frac{l_2 \dot{m}}{p_c A_2}\right)^2 + \left(\frac{c^* L^*}{RT_c}\right)^2\right]^{1/2}} \tag{21}$$

As expected, $\omega_{cr} \rightarrow 0$ as $\left(\frac{2\Delta p}{p_c}\right)_{cr} \rightarrow 1$

As a second case, it may be supposed that the condition of critical damping is reached by relaxing condition (15). Then, c may be replaced by a in equation (20), and if it is further assumed that $l_2\dot{m}/p_c A_2$ is of the same order as $\frac{c^*L^*}{RT_c} \cdot \frac{2\Delta p}{p_c}$, it can be shown that:

$$\omega_{cr} \approx \left[\left(\frac{RT_c}{c^*L^*}\right)\left(\frac{p_c A_2}{l_2 \dot{m}}\right)\left(\frac{2\Delta p}{p_c} + 1\right)\right]^{1/2} \tag{22}$$

By comparison with condition (15), it is apparent that in this case ω_{cr} is of the order of $2\tau^{-1}$.

A third case of interest may be developed by assuming that $\tau \ll \omega_{cr}^{-1}$. This assumption makes it possible to reconsider the basic differential equation (6). Thus, $u(t-\tau)$ can be expanded in a Taylor series:

$$u(t-\tau) = u(t) - \tau u'(t) + \ldots \tag{23}$$

and the differential equation takes a well-known form:

$$u'' + (A-C\tau)\, u' + (B+C)\, u = 0 \tag{24}$$

The condition for stability is that the coefficient $(A-C\tau)$ shall be positive; this provides the same relation as equation (15), but in this case it is a *necessary condition* whereas (15) was merely a sufficient condition. The frequency in the critically damped situation is simply $\sqrt{B+C}$ which reduces exactly to equation (22). (This result requires careful examination in each specific case, however, since $\sqrt{B+C}$ is often of the order of τ^{-1}.)

By inserting typical design values in equations (20), (21), and (22), it may be concluded that frequencies greater than 200 cycles/second are unlikely. This supports the opinion expressed in the Introduction, that this type of instability ("chugging") is a different phenomenon from the high frequency oscillations ("screaming") sometimes observed.

It is of interest to observe that the monopropellant analysis of Reference (2) appears as a special case of this analysis. In equation (6) allow $L^* \rightarrow 0$:

$$\frac{l_2 \dot{m}}{p_c A_2} u' + \frac{2\Delta p}{p_c} u + u_{(t-\tau)} = 0 \qquad (25)$$

This equation can be identified exactly with equation (5) of Reference (2). Being of lower order, the differential equation for this limiting case can be solved for the critical condition in simple closed form (in contrast to the system of equations (12), (13), (20) which do not yield so simple a solution); thus, it is unnecessary to resort to the relatively weaker sufficient conditions. The critical condition for stability then becomes:

$$\frac{l_2 \dot{m}}{p_c A_2} \cdot \frac{\pi - \cos^{-1}\left(\frac{2\Delta p}{p_c}\right)}{\sqrt{1 - \left(\frac{2\Delta p}{p_c}\right)^2}} > \tau \qquad (26)$$

This is equivalent to equation (13) of Reference (2). This stability condition is useful whenever $(l_2\dot{m}/p_c A_2)$ is much greater than $(c^* L^*/RT_c)\ (2\Delta p/p_c)$. It has the advantage of permitting the use of smaller values of $(l_2\dot{m}/p_c A_2)$ than the weaker condition (15).

DISCUSSION OF RESULTS

Equations (15) and (19) lead to the following rules for overcoming instability in a liquid propellant rocket system:

(a) Increase the pressure difference between supply tank and combustion chamber, either by reducing the area of the injector orifices or by inserting resistance elements in the feed lines.

(b) Increase the volume or L^* of the combustion chamber.

(c) Increase the length of tubing from supply tank to combustion chamber.

(d) Reduce the cross-sectional area of the propellant flow passages leading to the injector (or increase the mass flow per unit area).

(e) In a bipropellant system, instability may occur in only one of the two feed systems. Therefore, an important step in eliminating instability is to increase the value of the left side of equation (15) pertaining to the weaker of the two.

(f) Reduce the combustion time lag of the propellant. This may require changing over to a more reactive propellant or adding catalytic or combustion-promoting substances. This may be outside the bounds of permissible changes, in practice. However, it is possible to change the configuration of the injector (somewhat empirically) or to preheat the propellants to accomplish the same result. One way to accelerate combustion of the propellant is to provide a flow pattern in the combustion chamber in which hot gases are recirculated vigorously to mix with the incoming propellant.

There is experimental evidence that some of these recommendations are in the right direction. It has been reported that an increase in Δp or an increase in L^* has been effective in certain instances in eliminating instability (Reference 1). In the author's experience, switching from nitric acid-gasoline to nitric acid-aniline in the identical rocket system removed the instability that had been encountered with the former. (See the remarks in the Introduction above.) In the case of nitromethane, a redesign of the combustion chamber and relocation of the injectors so as to promote recirculation of hot gases and more prompt vaporization and reaction of the injected liquid has been found to suppress instability.

It is of interest to consider the numerical magnitudes of the terms in the stability equation (15) by inserting typical values of the parameters involved: For example, consider a 5000 lb. thrust rocket airplane installation:

Length of tubing from tanks to motor	=	5	feet
Equivalent diameter of tubing	=	2.5	inches
Mass flow of propellant = 25lbs/sec.	=	.78	slugs/sec.
Chamber pressure	=	400	psia
Tank pressure	=	550	psia
Characteristic velocity (c^*)	=	5,000	ft/sec.
Gas constant R of chamber gas	=	2,000	ft-lbs/slug ^oR
Adiabatic flame temperature	=	5,500	^oR
Characteristic length of motor (L^*)	=	60	inches

$$\tau_{cr} = \frac{5 \times .78}{400 \times 4.9} + \frac{5000 \times 5}{2000 \times 5500}\,\frac{300}{400} = .002 + .002 = .004 \text{ sec.}$$

In this case, the system will be stable if the combustion time lag is less than 4 milliseconds. On the other hand, if the same motor is installed in a similar system, but with 30 feet of tubing, the tolerable limit for the time lag is increased to 14 milliseconds. A propellant combination and combustion chamber that chugs in the first installation might be stable in the latter.

It is not unreasonable to expect the actual time lags of practical propellant systems to fall within this range. Although the process of ignition is not quite like that of combustion, the observed ignition lag of self-igniting propellants may be expected to indicate an upper estimate of combustion time lags. Observed ignition lags, under conditions similar to the injection process in a rocket motor, are of the order of 5 to 30 milliseconds, i.e., in the sensitive range. (Reference 3.)

An interesting interpretation of equation (15) can be developed by considering the overall combustion reaction. The time lag τ may be defined as the interval between the instant of entry of an elementary mass of liquid propellant and the later "instant" when this mass is converted to high temperature gas, that is, when it exerts its full effect upon a pressure gauge connected to the combustion chamber. This time interval τ includes the time required (on the average) for the gasification of the elementary liquid mass and the time required for the gas phase reaction. The gasification time is not calculable on simple grounds, particularly since the process of vaporization is always accompanied by gas phase or liquid phase reactions of unknown character. The gas phase reaction time, on the other hand, can be estimated (roughly) by equating it to the gas phase residence time in a combustion chamber of *minimum* volume, i.e., an empirically determined volume such that further reduction results in a serious loss in specific impulse. This residence time is given by:

$$\tau_g \approx \frac{(V_c)_{\min}}{\dot{m}/\rho_{g\ \mathrm{mean}}} = \frac{L^*_{\min}\ c^*}{p_c/\rho_{g\ \mathrm{mean}}} = \frac{\rho_{g\ \mathrm{mean}}}{\rho_{g\ \min}} \cdot \frac{c^* L^*_{\min}}{RT_c} \tag{27}$$

A reasonable assumption is that $\rho_{g\ \mathrm{mean}} \sim \rho_{g\ \min}$; then,

$$\tau_g \approx \frac{c^* L^*_{\min}}{RT_c} \tag{28}$$

Denoting the time in the liquid phase as τ_l, equation (15) takes the following form as a condition for stability:

$$\frac{l_2\ \dot{m}}{p_c A_2} + \frac{c^* L^*}{RT_c} \cdot \frac{2\Delta p}{p_c} > \tau_l + \frac{c^* L^*_{\min}}{RT_c} \tag{29}$$

The second term on the left side and the corresponding term on the right side are usually of the same order of magnitude, since, in practical designs, $2\Delta p/p_c$ is less than unity and L^* is generally made greater than $L^*_{\min}$. There are sound reasons for such design practice (based upon considerations other than the control of chugging); therefore, the inequality becomes sensitive to the relative magnitudes of the term on the left describing the momentum of the liquid in the feed lines and the term τ_l on the right.

This comparison indicates that in the case of self-igniting propellants (e.g., nitric acid and hydrazine) chugging is less likely to occur than in the case of non-self-igniting propellants (e.g., nitric acid-kerosene) by virtue of the relatively small gasification time of the former. In fact, chugging may be impossible with certain self-igniting combinations even if l_2 is very small, since the L^* term may ensure the inequality. On the other hand, care may be necessary in rockets burning less reactive liquids to provide sufficient momentum in the feed lines.

Equation (15) indicates that, in the trend toward compact liquid rocket installations with short pipe lines, the appearance of instability will become more probable. Of the remedies available to the designer (listed above), those involving changes in the propellant feed system are simpler than those involving changes in the configuration of the rocket motor. The pressure difference Δp may be increased by inserting flow restrictors in the lines, but this has the disadvantage of requiring a higher supply pressure. A more attractive alternative is to increase the length or decrease the area of the feed lines: this would require a smaller increase in supply pressure to achieve the same degree of stability.

In computing the magnitude of the momentum term, if the feed system consists of several passages in series that term in equation (15) becomes $\frac{\dot{m}}{p_c}\sum_i \frac{l_i}{A_i}$. In such calculations, the momentum of the liquid in the regenerative cooling passages should not be overlooked. In systems having centrifugal pumps instead of pressurized propellant tanks, the theory is directly applicable, but the momentum term should include the suction lines as well as the high pressure lines. In the case of a system with positive displacement pumps, this theory would deny the possibility of unstable flow (unless a compressible air cavity or elastic tubing is present to permit flow oscillations).

In systems with long feed lines it is possible that the effective length may be somewhat less than the measured length due to compressibility of the liquid, particularly if bubbles are present. A rough calculation indicates that the maximum effective length is of the order $(\dot{m}/A_2\rho\beta p_c\omega)$, where β is the volume compressibility of the liquid. At 100 cycles/sec., this would be of the order of 10 to 20 feet for the usual steel tubing filled with air-free nitric acid. (However, a more precise analysis including the role of the elasticity of the liquid should be carried out for such cases.)

Caution is required in the application of these stability criteria to cases in which the linearization performed herein may be invalid. In addition to the mathematical linearization above, a physical linearization is tacitly assumed in the statement that τ is a constant independent of pressure.

In conclusion, it is suggested that experiments be conducted to test the correctness of this theory, first, to determine the magnitude of the hypothetical time lag and its dependence on pressure, injector configuration, etc., and second, to determine whether equations (15) and (19) are valid conditions for stability.

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NOMENCLATURE

Physical Quantities

A cross-sectional area (normal to the flow direction)

c^* characteristic velocity of the propellant

K overall frictional pressure-loss coefficient

l length of flow passage

L^* characteristic length of the rocket motor

$\dot{m}$ mass flow (mean value) through the rocket system

p pressure (absolute)

$\Delta p = p_1 - p_c$ = pressure difference from tank to chamber

R gas constant of combustion gas (based on unit mass)

T temperature (absolute)

v velocity of flow

V volume of chamber

β volume compressibility of liquid

λ damping factor of oscillation

ω frequency of flow oscillation (radians/sec)

ρ density of the liquid

τ combustion time lag

Subscripts

$_1$ refers to propellant tank

$_2$ refers to feed line

$_3$ refers to injector orifice

c refers to combustion chamber

t refers to exhaust nozzle throat

e refers to equilibrium value of ν

g refers to combustion gas

Parameters

$A, B, C, a, b, c, U, u, x, \theta$ are employed in the analysis to simplify algebraic manipulation and are defined in the text.

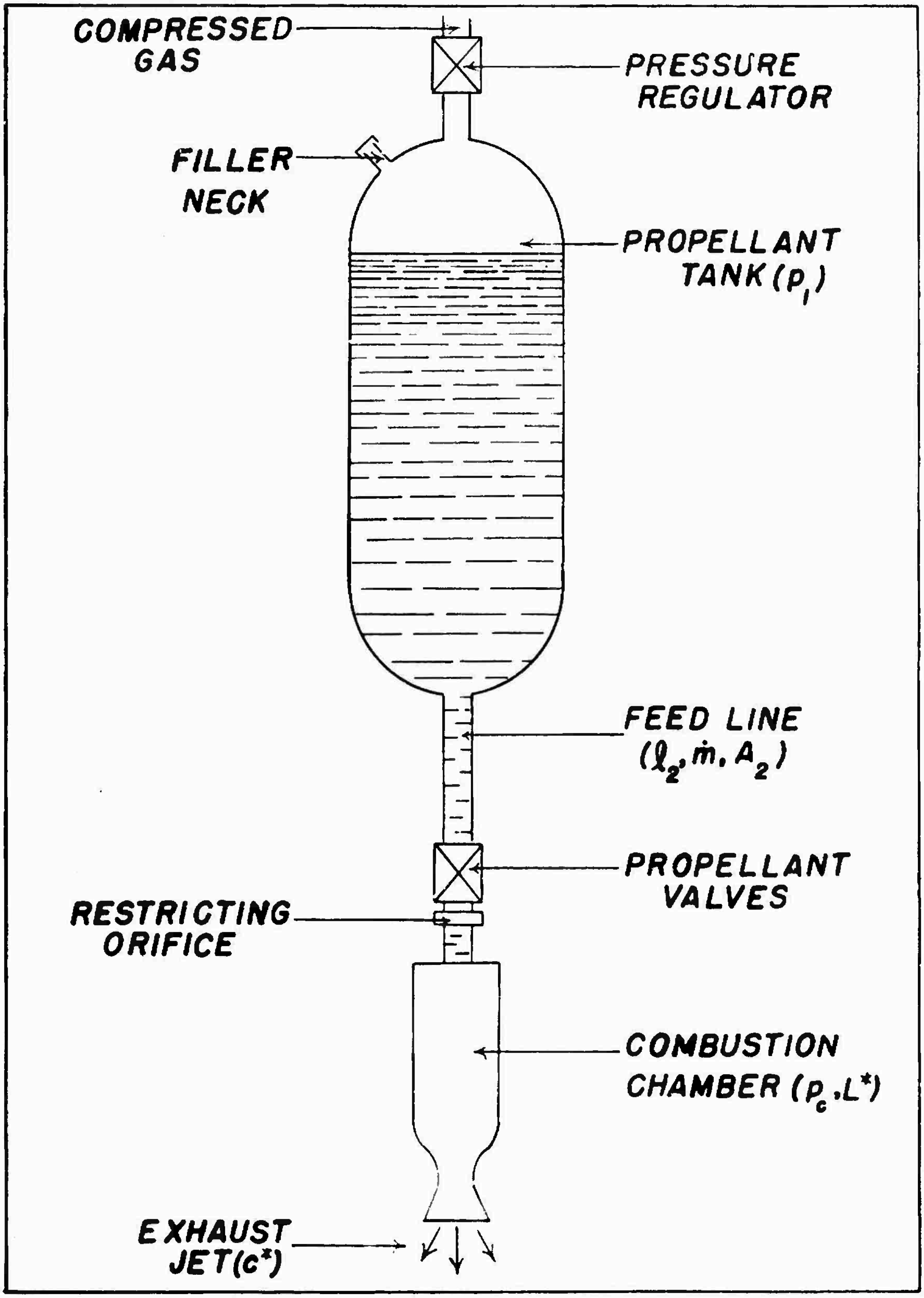
COMPRESSED
GAS
PRESSURE
REGULATOR
FILLER
NECK
PROPELLANT
TANK (p1)
FEED LINE
(l2, ṁ, A2)
PROPELLANT
VALVES
RESTRICTING
ORIFICE
COMBUSTION
CHAMBER (pc, L*)
EXHAUST
JET (c*)

Figure 1.

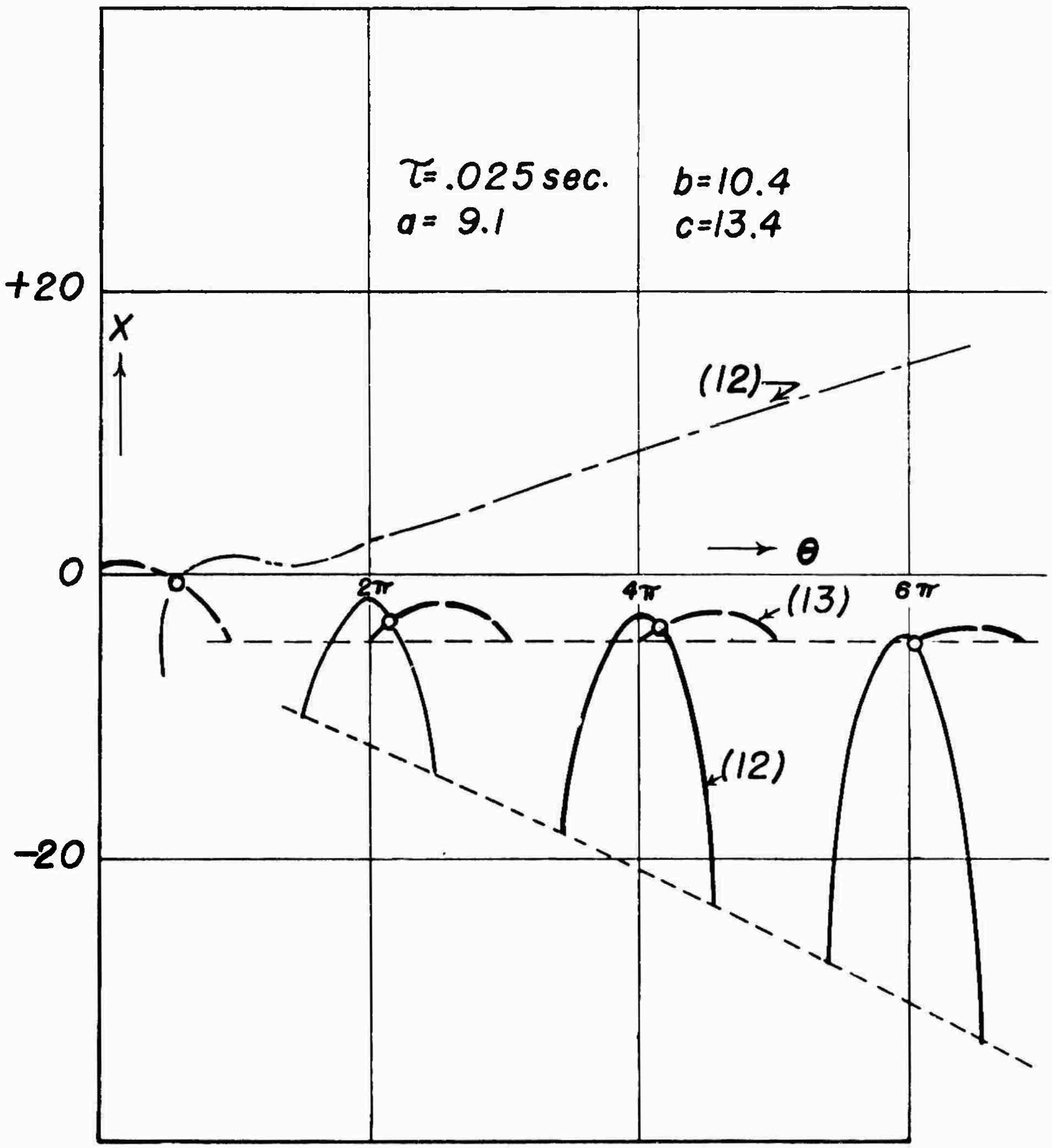

Figure 2.

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