UNCLASSIFIED

AD NUMBER
ADA801589
CLASSIFICATION CHANGES
TO: **UNCLASSIFIED**
FROM: **RESTRICTED**
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; FEB 1945. Other requests shall be referred to Office of Scientific Research and Development, Washington, DC 20301.**
AUTHORITY
OSRD list no. 18 dtd 22-26 Apr 1946; Declassified to "Open" (public) per cover page marking

THIS PAGE IS UNCLASSIFIED


TECHNICAL INTELLIGENCE
AMC

The U.S. GOVERNMENT

IS ABSOLVED

FROM ANY LITIGATION WHICH MAY ENSUE FROM THE CONTRACTORS INFRINGING ON THE FOREIGN PATENT RIGHTS WHICH MAY BE INVOLVED.

WRIGHT FIELD, DAYTON, OHIO

REEL – C

991

A.T.I.

25151

430

~~RESTRICTED~~

NATIONAL DEFENSE RESEARCH COMMITTEE

PROGRESS REPORT NO. A-29 (OSRD NO. 365)

ON THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

by

Theodor~~e~~ von Kármán

~~[illegible] contains information affecting the [illegible] defense [illegible] States within [illegible] of the Espionage Act, U.S.C., [illegible] transmission or the revelation [illegible] contents in any [illegible] person is prohibited by law.~~

Copy No. 178

Declassified to
OPEN
5/7/60

~~RESTRICTED~~

~~RESTRICTED~~

NATIONAL DEFENSE RESEARCH COMMITTEE

PROGRESS REPORT NO. A-29 (OSRD NO. 365)

ON THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

by

Theodore von Kármán

Approved January 28, 1942
for submission to the Section Chairman

Th. Kármán
Th. von Kármán, Author
Consultant, Section B

Approved January 30, 1942
for submission to the Division Chairman

John Burchard
J. Burchard, Chairman
Section B, Division A

Approved January 30, 1942
for submission to the Committee

Richard C Tolman
Richard C. Tolman
Chairman, Division A

~~RESTRICTED~~

Preface

The theory developed in this progress report is pertinent to the projects designated by the War Department Liaison Officer as CE-5 and CE-6.

The original draft of the report was submitted by the author on December 18, 1941.

Because of the large demand for copies of this report which was first issued in February 1942, it has been necessary to reissue it twice — in December 1943 and in February 1945.

Initial distribution of copies.

Copies No. 1 to 23, inclusive, to the Office of the Secretary of the Committee for distribution in the usual manner;

Copy No. 24 to the Chief of the Bureau of Ordnance, Navy Department (Attention: Research and Development Division);

Copy No. 25 to the Chief of Engineers, War Department (Attention: Lt. Col. C. H. Chorpening and Lt. Col. F. J. Wilson);

Copy No. 26 to the Chief, Bureau of Yards and Docks, Navy Department (Attention: Capt. C. A. Trexel);

Copy No. 27 to the Director, David Taylor Model Basin, Navy Department (Attention: Comdr. W. P. Roop);

Copies No. 28 and 29 to the Chief, Ordnance Department, U.S. Army, Frankford Arsenal (Attention: Lt. Col. L. S. Fletcher), Watertown Arsenal (Attention: Col. S. B. Ritchie);

Copy No. 30 to J. E. Burchard, Chairman, Section B, Division A;

Copy No. 31 to H. P. Robertson, Vice Chairman, Section B, Division A;

Copy No. 32 to T. von Kármán, Consultant, Section B, Division A;

Copy No. 33 to F. Seitz, Member, Section B, Division A;

Copy No. 34 to J. B. Wilbur, Member, Section B, Division A;

Copy No. 35 to W. M. Newmark, Consultant, Section B, Division A;

Copy No. 36 to L. T. E. Thompson, U.S. Naval Proving Ground, Dahlgren, Virginia.

ON THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

Abstract

Although the propagation of plastic deformation or permanent set is of fundamental importance for the interpretation of impact and penetration problems in which the stresses exceed the elastic limit of the bombarded material, the present treatment is perhaps the first attempt to compute the stress and strain caused by impact beyond the elastic limit. The method presented may possibly open the way to a systematic interpretation of a great many impact and penetration problems in which plastic deformations of beams, plates and armor are involved; and, if the stress-strain relation of the material is known, may lead to a prediction of the critical velocity causing rupture.

The propagation of plastic deformation or permanent set is of fundamental importance for the interpretation of impact phenomena. If an elastic body is hit by an impact load having a given velocity, the stress distribution can be determined by known theories, provided the stresses remain within the elastic limit of the material.

In many important applications the stresses are far beyond this limit; yet, within the knowledge of the author, no attempt has been made to compute the stress and strain caused by impact beyond the elastic limit of the material. In the present paper, such a treatment is provided for the simple case of longitudinal impact. However, this method can be extended to other types of sudden loading.

Consider a rod or wire extending from $x = -\infty$ to $x = 0$, and assume that the endpoint at $x = 0$ is suddenly put in motion with a constant velocity v_0. Let the stress-strain relation for the material be given by a function of the form $\sigma = \sigma(\varepsilon)$ where σ is the stress* and ε the strain. We shall neglect stresses that depend on the time-rate of strain; for, with the exception of the case of extremely high velocities, such stresses are small in comparison with the stresses that depend on the strain itself. To be sure, the relation $\sigma = \sigma(\varepsilon)$ holds only for the first deformation of the material beyond the

---

*It has been shown that this stress is in reality the apparent, or engineering, stress and not the true stress as was originally assumed. See P. E. Duwez, Preliminary experiments on the propagation of plastic deformation, NDRC Report A-244 (OSRD No. 3207), Feb. 1944.

elastic limit; in the case of load reversal, another functional relation which takes the hysteresis into account has to be used. However, for the problem considered in this paper — namely, the initial effect of an impact load — the stresses σ can be considered to be a given, unique, function of the strains ε. The lateral contraction of the material — that is, the contribution of the lateral contraction to the kinetic energy — is neglected in the following calculation.

With these simplifications, the equation of motion for an element of the rod or wire can be written in the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{d\sigma}{d\varepsilon}\frac{\partial \varepsilon}{\partial x}, \qquad (1)$$

where u is the displacement of the element in the longitudinal direction, ρ is the density of the material and t is the time. Since $\varepsilon = \partial u/\partial x$, Eq. (1) can also be written in the form[1/]

$$\rho \frac{\partial^2 u}{\partial t^2} = T \frac{\partial^2 u}{\partial x^2}, \qquad (2)$$

where $T[= d\sigma/d\varepsilon]$ is the modulus of deformation, elastic or plastic. The quantity T is considered to be a given function of the strain $\varepsilon[= \partial u/\partial x]$.

The boundary conditions are $u = v_0 t$ for $x = 0$, and $u = 0$ for $x = \infty$.

It is easily seen that a solution of the form

$$u = v_0\left(t + \frac{x}{v_1}\right), \qquad (3)$$

with an arbitrary value of the velocity of propagation v_1, satisfies Eq. (2) and the boundary condition at $x = 0$. For this solution the strain ε is constant and is equal to v_0/v_1.

A second solution is obtained by putting

$$T/\rho = x^2/t^2. \qquad (4)$$

[1/] A nonlinear wave equation was treated by a method similar to that used in this paper by M. A. Biot in his paper, "Quadratic wave equation — flood waves in a channel with quadratic friction," Proc. Nat. Acad. Sci. 21, No. 7, 436 (1935).

Since $T[=d\sigma/d\varepsilon]$ is a given function of ε, Eq. (4) represents a solution for which $\varepsilon[=\partial u/\partial x]$ is a function only of the variable $\xi = x/t$.

Assume $\varepsilon = f(\xi)$; then the displacement u has the form

$$u = \int_{-\infty}^{x} \frac{\partial u}{\partial x}\,dx = \int_{-\infty}^{x} f(\xi)\,dx = t\int_{-\infty}^{\xi} f(\xi)\,d\xi, \tag{5}$$

since $dx = t\,d\xi$. By differentiation one readily obtains

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \frac{\xi^2}{t} f'(\xi), \\ \frac{\partial^2 u}{\partial x^2} &= \frac{1}{t} f'(\xi); \end{aligned} \tag{6}$$

and substitution of Eqs. (6) in Eq. (2) shows that one of the two equations,

$$\rho\xi^2 = T \tag{7}$$

or

$$f'(\xi) = 0, \tag{8}$$

must hold. Equation (8) leads to the solution expressed by Eq. (3) whereas Eq. (7) gives the solution of Eq. (4).

The complete solution is obtained as follows:

(a) For $x < v_1 t$, the strain ε is constant and equal to ε_1;

(b) For $v_1 t < x < ct$, where c is the velocity of propagation of the elastic wave,

$$T(\varepsilon) = \rho x^2/t^2; \tag{9}$$

(c) For $x > ct$, $\varepsilon = 0$.

The distribution of ε as a function of $\xi[=x/t]$ is shown schematically in Fig. 1. The value of T for small values of ε -- that is, within the elastic limit -- is equal to E, Young's modulus of elasticity for the material. The elastic wave propagates with the velocity $c = \sqrt{E/\rho}$. Between the plastic wave front, which is propagated with the velocity v_1, and the elastic wave front the strain is variable, since every strain-increase from ε to $\varepsilon + d\varepsilon$ proceeds with a velocity equal to the specific value of $\sqrt{T/\rho}$ corresponding to the strain ε.

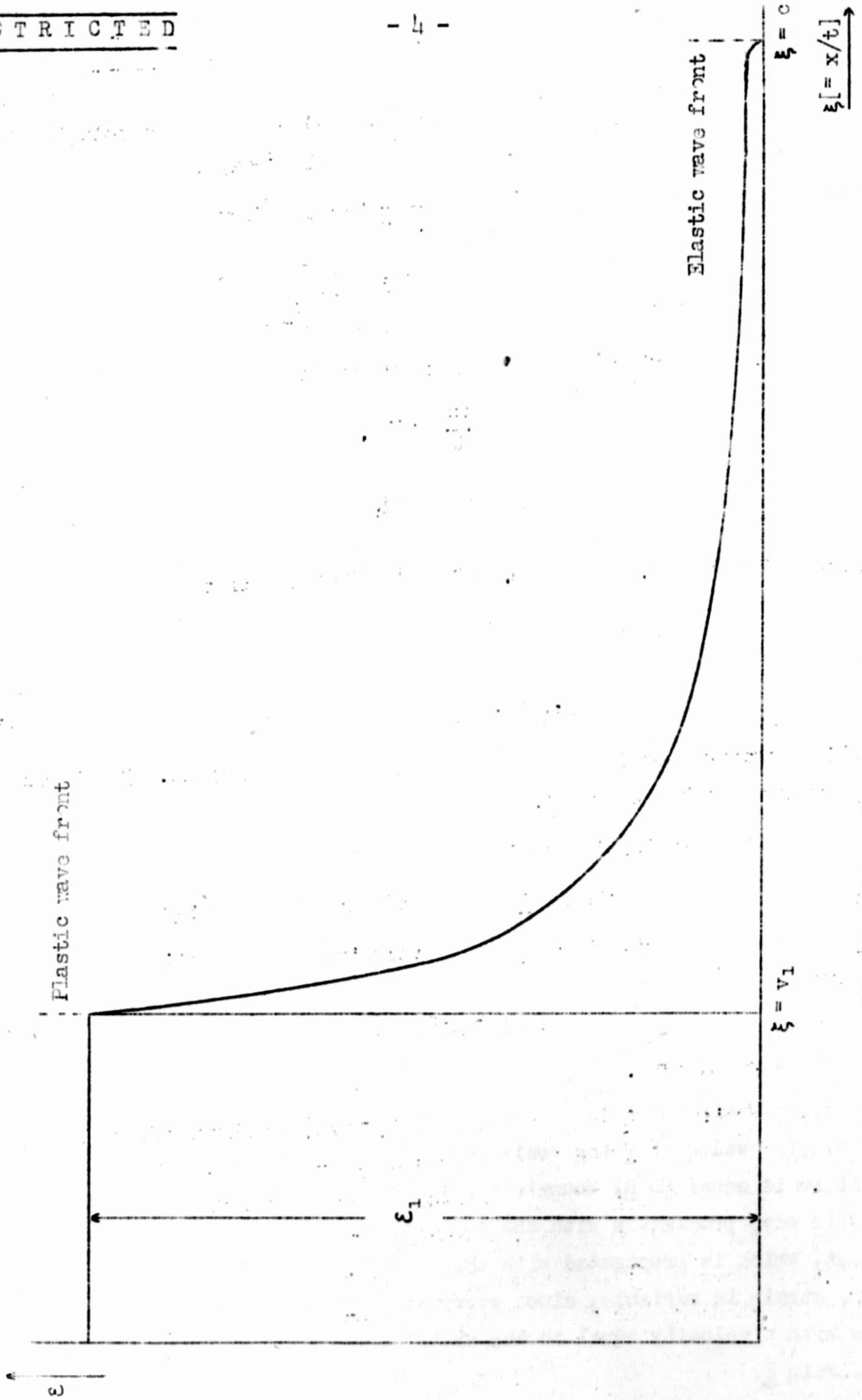

Fig. 1. Schematic representation of ε versus ξ.

The main problem is to determine the velocity v_1 of the plastic wave and the maximum strain ε_1 as a function of the velocity of impact v_0.

Since $u(0,t) = v_0 t$, it is obvious from Eq. (5) that

$$v_0 = \int_{-\infty}^{0} f(\xi)\, d\xi. \tag{10}$$

It is seen from Fig. 1 that the right-hand member of Eq. (10) can be written in the form

$$\int_0^{\varepsilon_1} \xi \, d\varepsilon\, ;$$

thus, upon substituting for ξ from Eq. (7), Eq. (10) takes the form

$$v_0 = \int_0^{\varepsilon_1} \sqrt{(T/\rho)}\, d\varepsilon. \tag{11}$$

Since T is a given function of ε, Eq. (11) determines ε_1 as a function of v_0.

If the deformation remains within the elastic limit, $T = E =$ constant, and $v_0 = \varepsilon_1 c = \varepsilon_1 \sqrt{E/\rho}$. Hence the stress $\sigma_1 [= E\varepsilon_1]$ is given by

$$\sigma_1 = v_0 E/c = \rho v_0 c. \tag{12}$$

Equation (12) is universally used for the calculation of the stress produced in an elastic body when hit by an impact body having a velocity v_0. It appears that Eq. (11) replaces Eq. (12) in the case of a deformation beyond the elastic limit. If the stress remains within the elastic limit, there are two regions: for $x < ct$, $\sigma = \rho v_0 c$; and for $x > ct$, $\sigma = 0$. In the case of plastic deformation, there are two fronts. Beyond the front of the elastic wave, $\sigma = 0$; between the fronts of the elastic and plastic waves, σ increases gradually from $\sigma = 0$ to a maximum value, $\sigma = \sigma_1$; and behind the front of the plastic wave, σ has the constant value σ_1, corresponding to a total strain ε_1 -- elastic plus permanent -- where ε_1 is given by Eq. (11).

For most materials $d\sigma/d\varepsilon$ approaches zero for large values of ε, and at some particular value of ε, the material breaks. Hence the integral constituting the right-hand number of Eq. (11) has a maximum value, and one obtains a critical value of the velocity v_0. It can be expected that an impact with a velocity larger than v_0 will cause an instantaneous breakdown of the material.

At the suggestion of the author, Dr. Pol E. Duwez has carried out certain experiments on the propagation of the permanent set in a copper wire, with the object of checking the assumptions of this simple theory. The results he has obtained thus far seem to confirm the theory.[2/]

It is believed that the method presented in this paper opens the way to a systematic interpretation of a large number of impact and penetration problems in which plastic deformations of beams, plates, armor, and so forth, are involved. It is also believed that the theory will reveal the relation between the behavior of the material at the yield point and the discontinuities observed in the plastic deformation of certain materials.

[2/] The results are described in NDRC Report A-244 (OSRD No. 3207).

REEL – C

991

A.T.I.

25151

ATI- 25151

REVISION
(None)

ORIG. AGENCY NO.
A-29

PUBLISHING AGENCY NO.
OSRD-365

U

OVER

TITLE: On the Propagation of Plastic Deformation in Solids

AUTHOR(S): von Karman, T.
ORIGINATING AGENCY: Office of Scientific Research and Development, NDRC, Div. A
PUBLISHED BY: (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Jan '42	Unclass.	U.S.	Eng.	8	diagr

ABSTRACT:

Although the propagation of plastic deformation or permanent set is of fundamental importance for the interpretation of impact and penetration problems in which the stresses exceed the elastic limit of the bombarded material, the present treatment is perhaps the first attempt to compute the stress, and strain caused by impact beyond the elastic limit. The method presented may possibly open the way to a systematic interpretation of a great many impact, and penetration problems in which plastic deformations of beams, plates, and armor are involved; and, if the stress-strain relation of the material is known, may lead to a prediction of the critical velocity causing rupture.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Ordnance and Armament ~~(22)~~ 22
SECTION: Armor ~~(5)~~ 2

SUBJECT HEADINGS: Armor - Plastic deformation (11399)

ATI SHEET NO.: R-22-5-14

[redacted]

TITLE: On the Propagation of Plastic Deformation in Solids

AUTHOR(S): von Karman, Th.
ORIGINATING AGENCY: Calif. Inst., of Technology, Pasadena, Calif.
PUBLISHED BY: Office of Scientific Research and Development, NDRC, Div.2

ATI- 27772
REVISION (None)
ORIG. AGENCY NO. (None)
PUBLISHING AGENCY NO. (None)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Jan '42	[redacted]	U.S.	Eng.	9	graph

ABSTRACT:

The theoretical data on propagation of plastic deformations in solids presents the first attempt to compute stress and strains caused by impact beyond the elastic limit of the material. Such a treatment is provided in the simple case of longitudinal impact. However, this method may be extended to other types of sudden loading. It is believed that the method opens the way to systematic interpretation of a large oumber of impact and penetration problems in which plastic deformations of beams, plates, and armor are involved. It is also believed that the theory will reveal the relation between behavior of the material at the yield point and discontinuities observed in plastic deformation of certain materials.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Stress Analysis and Structures (7)
SECTION: Structure Theory and Analysis Methods (2)

ATI SHEET NO.: R-7-2-41

SUBJECT HEADINGS: Metals - Plastic deformation (61072.5)

Air Documents Division, Intelligence Department
Air Materiel Command

AIR TECHNICAL INDEX

Wright-Patterson Air Force Base
Dayton, Ohio

[redacted]

UNCLASSIFIED PER AUTHORITY:
OSRD LIST # 18 Dated 22-26 April 1946