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REPORT No. 460.

THE CHARACTERISTICS OF 78 RELATED AIRFOIL SECTIONS FROM TESTS IN THE VARIABLE-DENSITY WIND TUNNEL

By EASTMAN N. JACOBS, KENNETH E. WARD and ROBERT M. PINKERTON



REPRINT OF REPORT No. 460, ORIGINALLY PUBLISHED NOVEMBER 1933

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

		Metric		English			
	Symbol	Unit	Abbrevia- tion	Unit	Abbrevia- tion		
Length Time Force	l L F	meter. second weight of 1 kilogram	m s kg	foot (or mile) second (or hour) weight of 1 pound	ft. (or mi.) sec. (or hr.) lb.		
Power Speed	P V	horsepower (metric) {kilometers per hour meters per second	k.p.h. m.p.s.	horsepower miles per hour feet per second	hp. m.p.h. f.p.s.		

- W, Weight=mg
- g, Standard acceleration of gravity=9.80665 m/s² or 32.1740 ft./sec.²
- m, Mass = $\frac{W}{r}$
- I_i Moment of inertia = mk^2 . (Indicate axis of radius of gyration k by proper subscript.)
- μ , Coefficient of viscosity

2. GENERAL SYMBOLS

v,

Kinematic viscosity

 $\rho,$ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m⁻⁴-s² at 15° C. and 760 mm; or 0.002378 lb.-ft.⁻⁴ sec.²

Specific weight of "standard" air, 1.2255 kg/m³ or 0.07651 lb./cu.ft.

3. AERODYNAMIC SYMBOLS

<i>S</i> ,	Area	iw,	Angle of setting of wings (relative to thrust
Sw,	Area of wing		line)
Ø,	Gap	i,	Angle of stabilizer setting (relative to thrust
Ь,	Span		line)
с,	Chord	<i>Q</i> ,	Resultant moment
b²	Aspect ratio	Ω,	Resultant angular velocity
<u>s</u> '	Aspect 1840	N	Reynolds Number where I is a linear dimension
V,	True air speed	ľμ	(e g for a model airfoil 3 in chord 100
q,	Dynamic pressure $-\frac{1}{2}\rho V^2$	ĝe.	m.p.h. normal pressure at 15° C., the cor-
L,	Lift, absolute coefficient $C_L = \frac{L}{qS}$		of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)
D,	Drag, absolute coefficient $C_p = \frac{D}{qS}$	<i>O</i> _p ,	Center-of-pressure coefficient (ratio of distance of $c.v.$ from leading edge to chord length)
Det	Profile drag, absolute coefficient $C_n = \frac{D_o}{\Omega}$	α.	Angle of attack
		e.	Angle of downwash
Dy	Induced drag, absolute coefficient $C_{D_t} = \frac{D_t}{aS}$	α.,	Angle of attack, infinite aspect ratio
		α_{i_1}	Angle of attack, induced
D _{p1} .	Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$	α _a ,	Angle of attack, absolute (measured from zero- lift position)
С,	Cross-wind force, absolute coefficient $C_c = \frac{C}{c^{\alpha}}$	γ,	Flight-path angle
R,	Resultant force		

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SUMMARY

An investigation of a large group of related airfoils was made in the N.A.C.A. variable-density wind tunnel at a large value of the Reynolds Number. The tests were made to provide data that may be directly employed for a rational choice of the most suitable airfoil section for a given application. The variation of the aerodynamic characteristics with variations in thickness and mean-line form were therefore systematically studied.

The related airfoil profiles for this investigation were developed by combining certain profile thickness forms, obtained by varying the maximum thickness of a basic distribution, with certain mean lines, obtained by varying the length and the position of the maximum mean-line ordinate. A number of values of these shape variables were used to derive a family of airfoils. For the purposes of this investigation the construction and tests were limited to 68 airfoils of this family. In addition to these, several supplementary airfoils have been included in order to study the effects of certain other changes in the form of the mean line and in the thickness distribution.

The results are presented in the standard graphic form representing the airfoil characteristics for infinite aspect ratio and for aspect ratio 6. A table is also given by means of which the important characteristics of all the airfoils may be conveniently compared. The variation of the aerodynamic characteristics with changes in shape is shown by additional curves and tables. A comparison is made, where possible, with thin-airfoil theory, a summary of which is presented in an appendix.

INTRODUCTION

The forms of the airfoil sections that are in common use today are, directly or indirectly, the result of investigations made at Göttingen of a large number of airfoils. Previously, airfoils such as the R.A.F. 15 and the U.S.A. 27, developed from airfoil profiles investigated in England, were widely used. All these investigations, however, were made at low values of the Reynolds Number; therefore, the airfoils developed may not be the optimum ones for full-scale application. More recently a number of airfoils have been tested in the variable-density wind tunnel at values of the Reynolds Number approaching those of flight (reference 1) but, with the exception of the M-series and a series of propeller sections, the airfoils have not been systematically derived in such a way that the results could be satisfactorily correlated.

The design of an efficient airplane entails the careful balancing of many conflicting requirements. This statement is particularly true of the choice of the wing. Without a knowledge of the variations of the aerodynamic characteristics of the airfoil sections with the variations of shape that affect the weight of the structure, the designer cannot reach a satisfactory balance between the many conflicting requirements.

The purpose of the investigation reported herein was to obtain the characteristics at a large value of the Reynolds Numher of a wide variety of related airfoils. The benefits of such a systematic investigation are evident. The results will greatly facilitate the choice of the most satisfactory airfoil for a given application and should eliminate much routine airfoil testing. Finally, hecause the results may be correlated to indicate the trends of the aerodynamic characteristics with changes of shape, they may point the way to the design of new shapes having better characteristics.

Airfoil profiles may be considered as made up of certain profile-thickness forms disposed about certain mean lines. The major shape variables then become two, the thickness form and the mean-line form. The thickness form is of particular importance from a structural standpoint. On the other hand, the form of the mean line determines almost independently some of the most important aerodynamic properties of the airfoil section, e.g., the angle of zero lift and the pitching-moment characteristics.

The related airfoil profiles for this investigation were derived hy changing systematically these shape variahles. The symmetrical profiles were defined in terms of a basic thickness variation, symmetrical airfoils of varying thickness being obtained by the application of factors to the hasic ordinates. The camhered profiles were then developed by combining these thickness forms with various mean lines. The mean lines were obtained by varying the camber and by varying the shape of the mean line to alter the position of the maximum mean-line ordinate. The maximum ordinate

of the mean line is referred to throughout this report as the camber of the airfoil and the position of the maximum ordinate of the mean line as the position of the camber. An airfoil, produced as described above, is designated by a number of four digits: the first indicates the camber in percent of the chord; the second, the position of the camber in tenths of the chord from the leading edge; and the last two, the maximum thickness in percent of the chord. Thus the N.A.C.A. 2315 airfoil has a maximum camher of 2 percent of the chord at a position 0.3 of the chord from the leading edge, and a maximum thickness of 15 percent of the chord; the N.A.C.A. 0012 airfoil is a symmetrical airfoil having a maximum thickness of 12 percent of the chord.

In addition to the systematic series of airfoils, several supplementary airfoils have heen included in order to study the effects of a few changes in the form of the mean line and in the thickness distribution.

Preliminary results which have been published include those for 12 symmetrical N.A.C.A. airfoils, the 00 series (reference 2) and other sections having different nose shapes (reference 3); and those for 42 cambered airfoils, the 43 and 63 series (reference 4), the 45 and 65 series (reference 5), the 44 and 64 series (reference 6), and the 24 series (reference 7). If the chord is taken along the x axis from 0 to 1, the ordinates y are given by an equation of the form

$$\pm y = a_0 \sqrt{x} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

The equation was adjusted to give the desired shape hy imposing the following conditions to determine the constants:

(1) Maximum ordinate 0.1 at 0.3 chord

$$\begin{array}{c} 0.3 & y = 0.1 \\ \frac{dy}{dx} = 0 \end{array}$$

(2) Ordinate at trailing edge x=1

x =

$$=1$$
 $y=0.002$

(3) Trailing-edge angle

x = 0.1

x = 1 dy/dx = -0.234

(4) Nose shape

$$y = 0.078$$

The following equation satisfying approximately the ahove-mentioned conditions represents a profile having a thickness of approximately 20 percent of the chord.

$$\begin{array}{l} \pm y = 0.29690 \sqrt{x} - 0.12600 x - 0.35160 x^2 + 0.28430 x^3 \\ - 0.10150 x^4 \end{array}$$



Sta Oro		8	1.25 3.157	2.5 4.358	5.0 5.925	7. 5 7. 000	10 7.805	15 8.909	20 9. 563	25 9.902	30 10. 003	40 9. 672	60 8. 323	60 7.606	70 6, 107	80 4. 872	90 2. 418	95 1.844	100 0. 210
T. F	redine	4.40																	

FIGURE 1.-Thickness variation.

The tests were made in the variable-density wind tunnel of the National Advisory Committee for Aeronautics during the period from April 1931 to February 1932.

DESCRIPTION OF AIRFOILS

Well-known airfoils of a certain class including the Göttingen 398 and the Clark Y, which have proved to be efficient, are nearly alike when their camher is removed (mean line straightened) and they are reduced to the same maximum thickness. A thickness variation similar to that of these airfoils was therefore chosen for the development of the N.A.C.A. airfoils. An equation defining the shape was used as a method of producing fair profiles. This equation was taken to define the hasic section. The hasic profile and a table of ordinates are given in figure 1. Points obtained by removing the camher from the Göttingen 398 and the Clark Y sections, and applying a factor to the ordinates of the resulting thickness curves to hring them to the same maximum thickness, are plotted on the above figure for comparison. Sections having any desired maximum thickness were obtained hy multiplying the hasic ordinates by the proper factor; that is

$$\pm y_t = \frac{t}{0.20} (0.29690 \sqrt{x} - 0.12600x - 0.35160x^2 + 0.28430x^3 - 0.10150x^4)$$

where t is the maximum thickness. The leading-edge and radius is found to be

$$r_t = \frac{1}{2} \left(\frac{t}{0.20} a_0 \right)^3 = 1.10t^2$$

When the mean lines of certain airfoils in common use were reduced to the same maximum ordinate and compared it was found that their shapes were quite different. It was observed, however, that the range of shapes could be well covered hy assuming some simple shape and varying the maximum ordinate and its position along the chord. The mean line was, therefore, arbitrarily defined by two parabolic equations of the form

$$y_c = b_0 + b_1 x + b_2 x^2$$

where the leading end of the mean line is at the origin and the trailing end is on the x axis at x=1. The values of the constants for hoth equations were then expressed in terms of the above variables; namely,

(1) Mean-line extremities

$$\begin{array}{ll} x=0 & y_c=0 \\ x=1 & y_c=0 \end{array}$$

(2) Maximum ordinate of mean line

x = p (position of maximum ordinate)

$$y_{e} = \frac{m}{(1-p)^{2}} [(1-2p)+2px-x^{2}]$$

(aft of maximum ordinate)

The method of combining the thickness forms with the mean-line forms is best described by means of the diagram in figure 2. The line joining the extremities of the mean line is chosen as the chord. Referring to the diagram, the ordinate y_i of the thickness form is measured along the perpendicular to the mean line from a point on the mean line at the station along the chord corresponding to the value of x for which y_i was computed. The resulting upper and lower surface points are then designated:

Stations x_u and x_i Ordinates y_u and y_i

where the subscripts u and l refer to upper and lower surfaces, respectively. In addition to these symbols, the symbol θ is employed to designate the angle between the tangent to the mean line and the x axis. This augle is given by

$$\theta = \tan^{-1} \frac{\mathrm{d} y_e}{\mathrm{d} x}$$



Sample calculations for derivation of N.A.C.A. 6321

Ŧ	¥1	¥+	tan 0	sln Ø	cos 0	yı sin o	yı cos ∂	<i>x</i> .	¥ n	x:	¥1
0 0,01250 .30000 .60000 1	0 0, 03314 , 10503 , 07986 , 00221	0 0, 00489 , 06000 , 04898 0	¹ 0. 40000 . 38333 0 07347 17143	0. 37140 . 35793 0 07327 16897	0, 92840 . 93375 1 . 99731 . 98562	0 0, 01186 0 00585 00037	0 0, 03094 . 10503 . 07965 . 00218	0.00064 .30000 .60585 1.00037	0, 03583 . 16503 . 12863 . 00218	0 0, 02436 . 30000 . 59415 . 99963	0 0.02605 04503 03067 00218

1 Slope of radius through end of chord.

FIGURE 2 .- Method of calculating ordinates of N.A.C.A. cambered airfoils.

$$y_c = m$$
(maximum ordinate)
 $dy_c/dx = 0$

The resulting equations defining the mean line then became

$$y_{\epsilon} = \frac{m}{p^2} \left[2px - x^2 \right]$$

(forward of maximum ordinate)

The following formulas for calculating the ordinates may now be derived from the diagram:

$$\begin{aligned} x_u &= x - y_i \sin \theta \\ y_u &= y_c + y_i \cos \theta \\ x_l &= x + y_i \sin \theta \\ y_l &= y_c - y_t \cos \theta \end{aligned}$$

Sample calculations are given in figure 2. The center for the leading-edge radius is placed on the tangent to the mean line at the leading edge. A family of related airfoile was derived in the manner described. Seven values of the maximum thickness, 0.06, 0.09, 0.12, 0.15, 0.18, 0.21, and 0.25; four values of the camber, 0.00, 0.02, 0.04, and 0.06; and six values of the position of the camber, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 were used to derive the related sections of this family. The profiles of the airfoils derived are shown collectively in figure 3.

For the purposes of this investigation the construction and tests were limited to 68 of the airfoils. Tables of ordinates at the standard stations are given in the figures presenting the aerodynamic characteristics. These ordinates were obtained graphically from the computed ordinates for all but the symmetrical secmodels, which are made of duralumin, have a chord of 5 inches and a span of 30 inches. They were constructed from the computed ordinates by the method described in reference 8.

Routine measuremente of lift, drag, and pitching moment about a point on the chord one quarter of the chord behind its forward end were made at a Reynolds Number of approximately 3,000,000 (tank pressure, approximately 20 atmospheres). Groupe of airfoils were first tested to study the variations with thickness, each group containing airfoils of different thicknesses but having the eame mean line. Finally, all airfoils having a thickness of 12 percent of the chord were

0006	2206	2306	2406	2506	2606	2706
0009	2209	2309	2409	2509	2609	2709
	2212	2312	2412	2512	2612	2712
			2415	2515	2615	2715
		2318	2418	2518	2618	2718
		\sim	\sim	\sim		
0021	2221	2321	2421	2521	2621	2721
	4206	4306	4406	4506	4606	4706
0025	4209	4309	4409	4509	4609	4709
	4212	4312	4412	4512	4612	4712
	4215	4315	4415	4515	4615	4715
	4218	4318	4418	4518	4618	4718
	4221	4321	4421	4521	4621	4721
	6206	6306	6406	6506	6606	6706
	6209	6309	6409	6509	6609	6709
	6212	6312	6412	6512	6612	6712
	6215	6315	6415	6515	6615	6715
	6218	6318	6418	6518	6618	6718
	6221	6321	6421	6521	6621	6721

FIGURE 3 .- N.A.O.A. airfoil profiles.

tions. Two sets of trailing-edge ordinates are given. Those inclosed by parentheses, which are given to facilitate construction, represent ordinates to which the surfaces are faired. In the construction of the models the trailing edges were rounded off.

Three groups of supplementary airfoils were also constructed and tested. The derivation of these airfoils will be considered later with the discussion.

APPARATUS AND METHODS

A description of the variable-density wind tunnel and the method of testing is given in reference 8. The tested to etudy the variations with changes in the mean line.

RESULTS

The results are presented in the standard graphic form (figs. 4 to 80) as coefficients corrected after the method of reference 8 to give airfoil characteristics for infinite aspect ratio and aspect ratio 6. Where more than one test has been used for the analysis, the infinite aspect ratio characteristics from the earlier test have heen indicated by additional points on the figure. Table I gives the important characteristics of all the airfoils.



FIGURE 5 .- N.A.C.A. 0009 sirfoli.









FIGURE 11.-N.A.C.A. 2212 airfoil.



FIGURE 18 .- N.A.C.A. 2309 airfoll.



FIGURE 15.-N.A.C.A, 2315 airfoll,







FIGURE 19 .- N.A.C.A. 2415 airfoll.



FIGURE 21 .- N.A.C.A. 2421 airfoil.



FIGURE 23 .- N.A.C.A. 2509 airfoil.





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FIGURE 27,-N.A.C.A. 2521 airfoil.







FIGURE 31.-N.A.C.A. 4306 airfoll.







FIGURE 35 .- N.A.C.A. 4318 airfoll.





FIGURE 87.-N.A.C.A. 4406 airfoil.



FIGURE 39 .- N.A.C.A. 4412 airfoil.



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FIGURE 45.-N.A.C.A. 4512 airfoil.



FIGURE 47 .--- N.A.C.A. 4518 airfoil.





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FIGURE 51.-N.A.C.A. 6212 airfoil,



















FIGURE 61.-N.A.C.A. 6415 airfoil.



FIGURE 63 .- N.A.C.A. 6421 airfoil.

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FIGURE 65.-N.A.C.A. 6500 airfoil.



FIGURE 67 .- N.A.C.A. 6515 sirfold.



FIGURE 69.-N.A.O.A. 6521 airfoil.



FIGURE 71 .--- N.A.C.A. 6712 airfoil.



FIGURE 73.-N.A.C.A. 0006B airfoil.



FIGURE 75 .- N.A.C.A. 0012B airfoil.



FIGURE 77 .- N.A.C.A. 0018B airfoil.



FIGURE 79 .- N.A.C.A. 2R:12 airfoil.



FIGURE 80.-N.A.O.A. 0012Fo and 0012F1 airfoils.

PRECISION

A general discussion of the errors and corrections involved in airfoil testing in the variable-density tunnel is included in reference 8. In connection with this report, it was hoped that a more specific discussion of the various sources of error and separate estimates of the various errors might be given. However, after a careful study of all the measurements it became apparent that practically all the errors may be regarded as accidental; that is, of the type the magnitude of which may best be estimated from the dispersion of the results of independent repeat measurements. The major portion of these errors is caused by insufficient sensitivity of the balance and manometers, by the personal error involved in reading mean values of slightly fluctuating quantities, and by the error due to slight surface imperfections in the model. The last is perhaps the most serious source of error. The models were carefully finished before each test, but the presence of particles of hard foreign matter in the air stream tended to cause a slight pitting of the leading edge of the model during each test. This pitting was probably the major source of error in connection with the earlier tests, but it was reduced for the later tests when the necessity of a more careful inspection of each model was appreciated. After a considerable period of running the particles in the tunnel were found to become lodged, permitting this source of error to be

largely eliminated during the later tests. For this report, however, the effect of the error from this source has been minimized by repeating the tests of many of the airfoils, including all of the symmetrical series originally reported in reference 2.

The magnitude of all such accidental errors was judged from the results of repeat tests of many airfoils, and from the results of approximately 25 tests of one airfoil that were made periodically throughout the investigation to check the consistency of the measurements. The accidental errors in the results presented in this report are believed to be within the limits indicated in the following table:

α	$\pm 0.15^{\circ}$
$C_{L_{max}}$	$\left\{ \begin{array}{c} 0.01 \\ -0.03 \end{array} \right.$
$C_{m_{c/4}}$	± 0.003
$C_{D_0}(C_L = 0)$	$\left\{ egin{array}{c} 0.0006 \\ - 0.0002 \end{array} ight.$
$C_{D_0}(C_L = 1)$	$\left\{ \begin{array}{c} 0.0015 \\ -0.0008 \end{array} \right.$

In addition to the consideration of the accidental errors, all measurements were carefully analyzed to consider possible sources of errors of the type that would not be apparent from the dispersion of the results of repeat tests. A rather large (approximately 1.5 percent) error of this type is present in all the airvelocity measurements resulting from a reduction in the apparent weight of the manometer liquid when the density of the air in the tunnel is raised to that corresponding to a pressure of 20 atmospheres. The effects of this error, however, are reduced by the presence of another error in the air-velocity measurements due to the blocking effects of the model in the tunnel. The measured coefficients, obtained by dividing the measured forces by $\frac{1}{2}\rho V^2$, as well as the derived coefficients are, of course, affected by errors in the airvelocity measurement. Aside from this source of error, it is believed that only two other sources need be considered: first, the deflection of the model and supports under the air load; and second, the interference of the airfoil supports on the airfoil. The angle of attack and the moment coefficient are affected by the deflection of the airfoil and supports. The error in angle of attack, which is proportional to $C_{me_{14}}$, was found to be approximately -0.1° for an



airfoil having a moment coefficient of -0.075. The error from this source in the moment coefficient is inappreciable at zero lift, but at a lift coefficient of 1 may amount to -0.001. The errors resulting from the support interference are more difficult to evaluate, but tests of airfoils with different support arrangements lead to the belief that they are within the limits indicated in the following table:

α	$\pm 0.05^{\circ}$
a	∫ 0.00
OL max	1 - 0.02
C_{max}	± 0.001
α (α = 0)	0.0002
$C_{D_0}(C_L=0)$	ነ 0.0000
$C_{D_0}(C_L = 1)$	± 0.0010

The tunnel-wall and induced-drag corrections applied to obtain the airfoil section characteristics might also be treated as sources of systematic errors. Such errore need not be considered, however, if the section characteristics are defined as the measured characteristics with certain calculated corrections applied. Errors in the tunnel-wall corrections, however, should be considered when the results from different wind tunnels are compared. For consideration of these errors, the reader is referred to references 9 and 10.

For the purpose of comparing the results from different wind tunnels and of applying these results to airplanes in flight, it is also necessary to consider the effects of air-stream turbulence. In air streams baving different degrees of turbulence, the value of the Revnolds Number cannot be considered as a sufficient measure of the effective dynamic scale of the flow The airfoil characteristics presented in this report were obtained at a value of the Revnolds Number of approximately 3,000,000, which corresponds roughly to the Reynolds Number attained in flight by a mediumsized airplane flying near its stalling speed. Consideration of the effects of the turbulence present in the variable-density tunnel (see references 11 and 12) leads, however, to the belief that these results are more nearly directly applicable to the characteristics that would be obtained in flight at larger values of the Reynolds Number.





DISCUSSION

The results of this investigation are here discussed and analyzed to indicate the variation of the aerodynamic characteristics with variations in thickness and in mean-line form. For the analysis of the effect of thickness, test data from consecutive tests of airfoils having different thicknesses and the same mean-line form are used. The analysis of the effect of the meanline form is made with respect to consecutive tests of airfoils of the same thickness (12 percent of the chord) and related mean-line forms. The results are compared, where possible, with the results predicted by thin-airfoil theory, a summary of which is presented in the appendix.

LIFT

Lift curve.—In the usual working range of an airfoil section the lift coefficient may be expressed as a linear function of the angle of attack

$$C_L = a_0 \ (\alpha_0 - \alpha_{L_0})$$

where a_0 is the slope of the lift curve for the wing of infinite aspect ratio and α_{L_0} is the angle of attack at zero lift.

The variation of the lift-curve slope with thickness is shown in figure 81. The points on the figure represent the deduced slopes as measured in the angular range of low profile drag. These results confirm previous results (reference 1) in that they show the lift-curve slope to decrease with increasing thickness. The camher has very little effect on the slope, as indicated in figure 82, although a rearward movement of the position of the camher tends to decrease the given mean line without altering the camber position. The theory also predicts an increased negative angle as the position of the camher moves back along the chord. The experimental values are compared with the theoretical values in figures 83 and 84. The ex-



FIGURE 23.—Variation of angle of zero lift with camber. Points shown are for 12 percent thick airfoils. Curves indicate general trends for the different thicknesses.



FIGURE 84.-Variation of angle of zero lift with thickness. Numbers refer to meancamber designation.





slope slightly. Table II gives the numerical values of the slope in convenient form for noting the general trends with respect to variations in thickness and in camher. It will be noted that all values of the slope lie below the approximate theoretical value for thin wings, 2π per radian; the measured values lie between 95 and 81 percent, approximately, of the theoretical.

The angle of zero lift is best analyzed by means of a comparison with that predicted hy the theory. Thinairfoil theory states that the angle of zero lift is proportional to the camber if the camber is varied, as with these related airfoils, hy scaling the ordinates of a perimental values lie between 100 and 75 percent, approximately, of the theoretical values, the departure becoming greater with a rearward movement of the position of the camber and with increased thickness (above 9 to 12 percent of the chord). Numerical values of the angle of zero lift are given in table III.

Maximum lift.—The variation of the maximum lift coefficient with thickness is shown in figure 85. It will he noted that the highest values are obtained with moderately thick sections (9 to 12 percent of the chord thick, except for the symmetrical sections for which the highest values are obtained with somewhat thicker sections). The variation with camber, shown in figure 86, confirms the expected increase in maximum lift with camber. The gain is small, however, for the normal positions of the camber, but becomes larger as the camber moves either rearward or forward. It will be seen hy reference to figure 85 that the camber becomes less effective as the thickness is increased. This reduced effectiveness of the camber is in agreement with a conclusion reacbed in reference 13 that for airfoils having a thickness ratio of approximately 20 percent of the cbord, camber is of questionable value. Numerical values of the maximum lift coefficient are given in table IV.

Air-flow discontinuities.—These and other windtunnel tests indicate that at the attitude of maximum



FIGURE 86.-Variation of maximum lift with camber. Results for 12 percent thick airfoils.

lift the air forces on certain airfoils exhibit sudden changes which in many instances result in a serious loss of lift. The probable cause of these air-flow discontinuities is discussed briefly in reference 13. The stability or instability of the air flow at maximum lift may be judged by the character of the lift-curve peaks indicated for the various airfoils. The curves are classified into three general types as noted in table IV, but the degree of stability is difficult to judge. It may be generally concluded that improved stability may he obtained by (1) having a small leading-edge radius, which causes an early breakdown of the flow with a consequent low value of the maximum lift, (2) increasing the thickness (beyond the normal thickness ratios), or (3) increasing the camber (for airfoils baving normal camber positions; i.e., 0.3c to 0.5c).

MOMENT

Thin-airfoil theory separates the air forces acting on any airfoil into two parts: First, the forces that produce a couple but no lift (they are dependent only on the shape of the mean line); second, the forces that produce the lift only, the resultant of which acts at



FIGURE 57.—Variation of moment at zero lift with camber. Points shown are for 12 parcent thick airfoils. Curves indicate general trends for the different thicknesses.



FIGURE 88.-Variation of moment at zero lift with thickness. Numbers refer to mean-camber designation.

a fixed point. We then have in the working range an expression for the total moment taken about any point

$$C_m = C_{m_0} + nC_L$$

where C_{m_0} is the moment coefficient at zero lift and nC_L is the additional moment due to lift.

As with the angle of zero lift, the theory states that the moment at zero lift is proportional to the camber and predicts an increase in the magnitude of the moment as the camber moves back along the chord. Figures 87 and 88 show the values of the moment coefficient as affected by variations of camber and thickness compared with the theoretical values. Referring to figure 87, the plotted data indicate that the moment coefficients are nearly proportional to the camher. It will also be noted that the curves representing the ratios of the experimental coefficients to the camber are nearly parallel to the equivalent curve representing the theoretical ratios except that the curves tend to diverge for positions of the camher well back. Figure 88 shows that the experimental values lie between 87 and 64 percent, approximately, of the theoretical. Numerical values of the moment coefficient at zero lift are given in table V.



FIGURE 80.—Variation of position of constant moment with thickness. Values of n for equation $C_{w_{i,i}} = C_{w_{i}} + n \mathcal{L}_{L}$. Results for sirfolls having normal camber positions (0.3c to 0.5c).

If the resultant of the lift forces acted exactly through the quarter-chord point, as predicted hy the theory of thin airfoils, there would be no additional moment due to the lift when the moments are taken ahout this point. The curves of $C_{mc/4}$ against C_{L_r} , however, show a slope in the working range which indicates that the axis of constant moment is displaced somewhat from the quarter-chord point. The factor nrepresents the amount of this displacement as obtained from the deduced slopes of the moment curves in the





normal working range. The variation of this displacement with thickness and with camber is shown in figures 89 and 90. Table VI gives the numerical values. Beyond the stall all the airfoils show a sharp increase in the magnitude of the pitching moment. The suddenness of this increase follows the degree of stability at the etall as indicated by the type of the lift-curve peak.

DRAG

The total drag of an airfoil is considered as made up of the induced drag and the profile drag. Considering the profile drag as the minimum value plus an additional drag dependent upon the attitude of the airfoil, we have in coefficient form

$$C_D = C_{D_i} + (C_{D_0 \min} + \Delta C_{D_0})$$

The induced-drag coefficient G_{D_1} , which is computed by means of the formula given in reference 8, is considered to he independent of the airfoil section. The variation of the profile-drag coefficient with the shape variables of the airfoil section is analyzed with respect



FIGURE 91.---Variation of minimum profile drag with thickness for the symmetrical airfoils.

to the variations of the two components of the profile drag.

Minimum profile drag.—The variation of the minimum profile-drag coefficient with thickness for the symmetrical sections is shown in figure 91. The cambered sections show the same general variation with thickness but, to avoid confusion, the results are not plotted. The variation of the minimum profile-drag



FIGURE 92.—Increase in minimum profile drag due to camber. Results for 12 percent thick atfolia. Values of k for equation $C_{D_{pum}} = k + 0.006 + 0.01 k + 0.1 k$, where k is the increase in $C_{D_{pum}}$ due to camber and t is the maximum thlokness in fraction of chord.

coefficient with the profile thickness may be expressed by the empirical relation

$$C_{Domin} = k + 0.0056 + 0.01t + 0.1t^2$$

where t is the thickness ratio and k (which is approxihately constant for sections having the same mean line) represents the increase in D_{D_0min} above that computed for the symmetrical section of corresponding thickness. The variation of D_{D_0min} with camber is indicated by the variation of k as shown in figure 92. The effect of camber ie email except for the highly cambered sections having the maximum camber well hack. Numerical values of C_{Domin} are given in table VII.

Additional profile drag.—The additional profile drag, which is dependent upon the attitude of the airfoil, has previously been expressed as a function of the lift (reference 4) by the equation

$$\Delta C_{D_0} = C_{D_0} - C_{D_0 min} = 0.0062 (C_L - C_{L_{out}})^2$$

where $C_{L_{opt}}$ may be called the optimum lift coefficient; that is, the lift coefficient corresponding to the minimum profile-drag coefficient. This equation holds This function is represented in figure 93 as the curve determined from the results for the symmetrical airfoils and for the airfoile having a camber of 2 percent of the chord. As the camher is increased, the dispersion of the plotted points from the curve hecomes greater. In general the points above the curve correepond to thick elections and sections in which the maximum camher is well hack. The departure from the curve becomes greater with increased thickness and with a rearward movement of the maximum-camher position. The points well helow the curve correspond to the thin airfoils.



FIGURE 93 .- Additional profile drag.

reasonably well for the normally shaped airfoile at values of the lift coefficient below unity.

A convenient practical method of allowing for the increased valuee of G_{B_0} at moderately high values of the lift coefficient is to include the additional profile drag with the induced drag, as suggested in reference 2. For the symmetrical airfoils of moderate thickness the term to be added to the induced-drag coefficient was given as $0.0062 \ C_L^2$. The relative importance of this term may he hetter appreciated by coneidering that it represents 11.7 percent of the induced drag of an elliptical airfoil of aspect ratio 6. The eame method may also he applied to other airfoils if the value of the optimum lift is not too large.

Andrews (reference 14), using the part of these data publiched in references 2, 4, and 5, euggests for the additional profile drag the form

$$\Delta C_{D_0} = f\left(\frac{C_L - C_{L_{opt}}}{C_{L_{max}} - C_{L_{opt}}}\right)$$

Because the additional profile drag is not a eimple function of the lift, and also because the results as presented in figure 93 are difficult to follow, generalized curves for the relation

$$\Delta C_{D_0} = f(C_L - C_{L_{opt}})$$

are given in figure 94. These curves are given to represent more accurately the additional profile drag for the normally shaped sections.

Optimum lift.—The optimum lift, as defined above, is the value of the lift corresponding to the minimum profile drag. As the determination of this value of the lift is largely dependent upon the fairing of the profiledrag curves, special curves were faired for this purpose on enlarged-scale plote corresponding to certain related airfoils grouped together. The valuee of the optimum lift coefficients obtained in this manner are given in tahle VIII. It may be noted hy reference to this table that the optimum lift coefficient increasee with camber

.02 Mear Max, thickness 6% 0% comber 0 .01 2% Δ 4% 62 .02 30 Maximum thickness 92 .01 C .02 Moximum thickness 12% .01 + 3 0 Maximum thickness 15% .01 0 0 .02 Maximum thickness 18% .01 0 .02 30 Maximum thickness 21% .01 1.0 1.2 1.4 .8 · CLas C.

and for the highly cambered sections a definite increase

accompanies a forward movement of the camber.

FIGURE 94.-Additional profile drag as a function of Ct.-Ct.ent. Results are for airfoils having normal camber positions (0.3c to 0.5c).

it is not primarily dependent upon the shape of the mean line. Nevertheless it is interesting to compare the optimum lift coefficients with the values included in table VIII representing the theoretical lift coefficients at the "ideal" angle of attack for the mean line; i.e., the angle of attack for which the thin-airfoil theory gives a finite velocity at the nose. (See the appendix.)

GENERAL EFFICIENCY

The general efficiency of an airfoil cannot be expressed by means of a single number. The ratio of



thick sirfoils.

the maximum lift to the minimum profile drag is, however, of some value as the measure of the efficiency of an airfoil section. The variation of this ratio with thickness is shown in figure 95. The curves of this figure indicate that the highest values of the ratio are





More important than these variations, however, is the variation with thickness. The rapid decrease in the optimum lift with increased thickness indicates that | figure 96, is less important. An increase in the camber

given by the sections between 9 and 12 percent of the chord thick. The variation with camber, shown in

above 2 percent of the chord and a rearward movement of the camber (for the highly cambered sections) tend to decrease the value of $C_{L_{mes}}/C_{D_0^{min}}$. The numerical values of the ratio are given in table IX.

SUPPLEMENTARY AIRFOILS

For the purpose of investigating briefly the effects of certain shape variables other than those discussed in the main body of the report, 10 supplementary airfoils were tested. The airfoil sections were as follows: 6 symmetrical sections with modified nose shapes, 2 sections with reflexed mean lines, and 2 sections simulating those of a wing having a flexible trailing edge.



Airfoils with modified nose shapes .- The airfoils of the first supplementary group investigated were developed from three of the symmetrical N.A.C.A. family airfoils: The N.A.C.A. 0006, the N.A.C.A. 0012, and the N.A.C.A. 0018. For each of these basic (or normal) sections one thinner-nosed section, denoted by the suffix T, and one blunter-nosed section, denoted by the suffix B, were developed and tested. The derivation of each modified section was similar to that of the normal section and was accomplished by a systematic change in the equation that defines the normal section. This change is principally a change in the nose radius, but it also results in modifications to the profile throughout its length, except at the maximum ordinate and at the trailing edge. The nose radii of the sections in percent of the chord are as follows:

Section	T series	Normal	B series
0006	0. 10	0.40	1. 19
0012	. 40	1.58	3. 80
0018	. 89	3.56	7. 15

The aerodynamic characteristics of the modified sections are given in figures 72 to 77. These may be compared with the characteristics of the normal sections given in figures 4, 6, and 8. The maximum lift coefficients of the modified and the normal sections are plotted against the leading-edge radii in figure 97. It is interesting to note that the leading-edge radius is very critical in its effect on the maximum lift when the radius is small. This critical effect is also indicated by the rapid increase in the maximum lift with increasing thickness for the thin sections as shown in figure 85.

Airfoils with reflexed mean lines.—Previous investigations have shown that the pitching moment of cambered airfoils can be reduced by altering the form of the mean line toward the trailing edge, with a consequent loss of maximum lift but only a small reduction in drag. In order to compare the characteristics of sections of this type with those of the related sections of normal form, two airfoils were developed with the basic thickness distribution of the N.A.C.A. 0012 disposed about certain mean lines of the form given in reference 15

$$y_c = hx(1-x) (1-\lambda x)$$

The values of h in this equation were chosen to give a camber of 0.02 and the values of h were chosen to give the airfoil designated the N.A.C.A. 2R₁12 a small negative moment and the airfoil designated the N.A.C.A. 2R₂12 a small positive moment. Characteristic curves for the two airfoils are given in figures 78 and 79. The principal characteristics of the sections may be conveniently compared with those of the related symmetrical section, the N.A.C.A. 0012, and a related normal section having a camber of 2 percent of the chord, the N.A.C.A. 2412, by means of the following table arranged in the order of increasing pitching-moment coefficients.

Section	C _{Lmas}	Cognia	CL _{mas} CD ₀ min	C=0
2R ₂ 12 0012 2R ₁ 12 2412	1.47 1.53 1.53 1.62	0.0086 .0083 .0083 .0083 .0085	171 184 184 190	0.004 002 020 044

These results indicate that airfoils having reflexed mean lines may be of questionable value because of the adverse effect of this mean-line shape on the maximum lift coefficient.

Thickness and camber modifications near the trailing edge.-Two airfoils were developed to simulate an airfoil having a flexible trailing edge in a straight and in a given deflected position. The thickness distribution is composed of three parts: the forward portion (0 to 0.3c) having the same distribution as the N.A.C.A. 0012, the rear portion (from 0.7c to the trailing edge) having a thin, uniform value, and the central portion joining these two with fair curves. As shown in figure 80, the two sirfoils differ only in the rear portion, the section designated N.A.C.A. 0012F, simulating that of a wing having the trailing edge deformed for the high-speed condition, and the section designated N.A.C.A. 0012F1 simulating that of the same wing with the trailing edge bent down in a circular arc. Curves of the aerodynamic characteristics for both conditions are compared in figure 80. Considering the results given by both airfoils as two conditions for one airfoil, a very high maximum lift with a reasonably low minimum drag is obtained.

On this assumption the ratio $\frac{C_{L_{max}}}{C_{D_0min}}$ is 197, slightly

higher than the value of this ratio given by the N.A.C.A. 2412.

In order to study the effects of an extreme change in the thickness distribution, the principal characteristics of the two sections may be compared with those of the related normal sections, the N.A.C.A. 0012 and the N.A.C.A. 6712. The maximum lift coefficient is little affected by the change in the thickness distribution, but it is of interest to note (table I) that the slope of the lift curve of the N.A.C.A. 0012Fe is slightly greater than 2π per radian, as compared with an appreciably lower slope for the N.A.C.A. 0012. The profile drag is also affected by the change in the thickness distribution. Of the two symmetrical sections, the profile drag of the N.A.C.A. 0012F₀ is much higher than that of the N.A.C.A. 0012 over the entire lift range. This is not true, however, for the two cambered sections. Comparing the characteristics of the N.A.C.A. 0012F₁ with those of the N.A.C.A. 6712, we find that at low values of the lift the profile drag of the former is much higher, but as the lift increases this difference becomes less, and in the high-lift range the profile drag of the N.A.C.A. 0012F, is considerably less than that of the N.A.C.A. 6712.

CONCLUSIONS

The variation of the aerodynamic characteristics of the related airfoils with the geometric characteristics investigated may be summarized as follows:

Variation with thickness ratio:

1. The slope of the lift curve in the normal working range decreases with increased thickness, varying from 95 to 81 percent, approximately, of the theoretical slope for thin airfoils $(2\pi \text{ per radian})$.

2. The angle of zero lift moves toward zero with increased thickness (above 9 to 12 percent of the chord thickness ratios).

3. The highest values of the maximum lift are obtained with sections of normal thickness ratios (9 to 15 percent).

4. The greatest instability of the air flow at maximum lift is encountered with the moderately thick, low-cambered sections.

5. The magnitude of the moment at zero lift decreases with increased thickness, varying from 87 to 64 percent, approximately (for normally shaped airfoils), of the values obtained by thin-airfoil theory.

6. The axis of constant moment usually passes slightly forward of the guarter-chord point, the displacement increasing with increased thickness.

7. The minimum profile drag varies with thickness approximately in accordance with the expression

$$C_{D_{0}min} = k + 0.0056 + 0.01t + 0.1t^2$$

where the value of k depends upon the camber and t is the ratio of the maximum thickness to the chord.

8. The optimum lift coefficient (the lift coefficient corresponding to the minimum profile-drag coefficient) approaches zero as the thickness is increased.

9. The ratio of the maximum lift to the minimum profile drag is highest for airfoils of medium thickness ratios (9 to 12 percent).

Variation with camber:

1. The slope of the lift curve in the normal working range is little affected by the camber; a slight decrease in the slope is indicated as the position of the camber moves back.

2. The angle of zero lift is between 100 and 75 percent, approximately, of the value given by thin-airfoil theory, the smaller departures being for airfoils with the normal camber positions.

3. The maximum lift increases with increased camber, the increase being more rapid as the camber moves forward or back from a point near the 0.3c position.

4. Greater stability of the air flow at maximum lift is obtained with increased camber if the camber is in the normal positions (0.3c to 0.5c).

5. The moment at zero lift is nearly proportional to the camber. For any given thickness, the difference between the experimental value of the constant of proportionality and the value predicted by thin-airfoil theory is not appreciably affected by the position of the camber except for the sections having the maximum camber well back, where the difference becomes slightly greater.

6. The axis of constant moment moves forward as the camber moves back.

7. The minimum profile drag increases with increased camber, and also with a rearward movement of the camber.

8. The optimum lift cofficient increases with the camber and for the highly cambered sections a definite increase accompanies a forward movement of the camber.

9. The ratio of the maximum lift to the minimum profile drag tends to decrease with increased camher (above 2 percent of the chord) and with a rearward movement of the camber (for the highly camhered sections).

LANGLEY MEMORIAL AERONAUTICAL LABRATORY, NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS, LANGLEY FIELD, VA., December 20, 1932.

APPENDIX

It is proposed in this section of the report to present, hriefly, a summary of the results of the existing thinairfoil theory (hased on the section mean line) as applied to the prediction of certain section characteristics. Such a summary is desirable because at present the results must he obtained from several different sources which give them in a form not easily applied. Three characteristics are considered; namely, (1) the angle of zero lift $\alpha_{L_{0}}$, (2) the pitching-moment coefficient $C_{m_{e'}}$, and (3) the "ideal" angle of attack α_{I} , or the corresponding lift coefficient $C_{L_{I}}$, that is, values corresponding to the unique condition for which the theory gives a finite velocity at the nose of the airfoil. (See reference 16.)

Expressions for lift and moment coefficients may be written as follows if the angles are measured in radians:

$$C_L = 2\pi (\alpha - \alpha_{L_0}) \tag{1}$$

$$C_{L_l=2\pi(\alpha_l-\alpha_{L_0})} \tag{2}$$

$$C_{m_c/4} = \frac{\pi}{2} (\beta + \alpha_{L_0})$$
(3)

If the leading end of the mean line is chosen as the origin of coordinates and the trailing end is taken on the x axis at x=1, then the parameters α_{L_0} , α_I , and β are given by the following integrals

$$\alpha_{L_0} = \int_0^1 y \, f_1(x) \, \mathrm{d}x \tag{4}$$

$$\alpha_I = \int_0^1 y \, f_2(x) \, \mathrm{d}x \tag{5}$$

$$\beta = \int_0^1 y \, f_3(x) \, \mathrm{d}x$$

$$f_1(x) = \frac{-1}{\pi (1-x) [x(1-x)]^{\frac{1}{2}}}$$
(7)

(8)

$$f_2(x) = \frac{(1-2x)}{2\pi [x(1-x)]^{\frac{3}{2}}}$$

$$f_3(x) = \frac{4(1-2x)}{\pi [x(1-x)]^{\frac{1}{2}}}$$
(9)

and y is the ordinate of the mean line at a given abscissa x. The integrals (4) and (6) may be shown to be identical with the corresponding integrals given hy Glauert (reference 15) and by Munk (reference 17), and integral (5) is given hy Theodorsen (reference 16).

The evaluation of these integrals for the N.A.C.A. airfoil sections given in this report was accomplished analytically. The values of α_{L_0} (changed from radians to degrees), $C_{m_{d_e}}$ and C_{L_t} , so computed, are given in tahles III, V, and VIII, respectively, in the main hody of the report. This method of evaluation, however, cannot he applied to many of the commonly used sections because they do not have analytically defined mean lines; hence, an approximate method must be used. A graphical determination gives good results and for convenience the values of the three functions, (7), (8), and (9), at several values of x, are given in the following tahle:

x	f1(x)	f2(X)	f3(x)	x	f ₁ (x)	f2(x)	h(x)
0 0.0125 .0250 .0500 .0750 .1000 .15 .20 .25		⁰⁰ 113, 15 39, 73 13, 84 7, 403 4, 716 2, 447 1, 492 . 980	00 11.17 7.747 5.258 4.109 3.395 2.406 1.910 1.470	0.30 .40 .50 .50 .70 .90 .95 1.00	$\begin{array}{r} -0.992 \\ -1.083 \\ -1.273 \\ -1.624 \\ -2.315 \\ -3.979 \\ -10.61 \\ -29.21 \\ -\infty \end{array}$	$\begin{array}{c} 0.662\\ .271\\ 0\\271\\662\\ -1.492\\ -4.716\\ -13.84\\ -\infty\end{array}$	1.111 .520 0.520 -1.111 -1.910 -3.395 -5.258 -∞

In general, some difficulty would be expected with the graphical method because the values of the above functions tend to infinity at the leading and trailing edges. Actually, because the ordinates of the meanline extremities are zero, the integrand may approach zero, and does at the leading edge for the integral (4). and at the leading and trailing edges for the integral (6). Difficulty, however, is encountered at the trailing edge for the integral (4) and at the leading and trailing edges for the integral (5). In order to avoid this difficulty, integral (4) is evaluated graphically from x=0 to x=0.95, and the increment contributed by the portion from x=0.95 to x=1 is determined analytically. Likewise, integral (5) is evaluated graphically from x = 0.05 to x = 0.95 and analytically for the extremities. The analytical determination of the increments is accomplished by assuming the mean line near the ends to be of the form

$$y = a + bx + cx^2$$

(6) Evaluating the integrals gives

$$\Delta \alpha_{L_0} = -0.964 y_{0.95} + 0.0954 y_1' \qquad (x = 0.95 \text{ to } x = 1)$$

$$\Delta \alpha_{I} = \begin{cases} +0.467y_{0.05} + 0.0472y'_{0} & (x = 0 \text{ to } x = 0.05) \\ -0.467y_{0.05} + 0.0472y'_{1} & (x = 0.95 \text{ to } x = 1) \end{cases}$$

where y'_0 and y'_1 are the mean-line slopes at the leading and trailing edges, respectively.

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TABLE I.--IMPORTANT CHARACTERISTICS

				Bect	ion charact	eristics			Wing el	aracterist	los A.R. 6	Thick	ness at	.c.p.	st	
Airfoll	Pege	CL.mez	α _e et CL _{max} (deg.)	α _{L0} (deg.)	$a_0 = \frac{dC_L}{d\alpha_0}$ (per deg.)	Co _{lmin}	$rac{C_{L_{max}}}{C \mu_{0_{min}}}$	C _{m₀}	C _{Dmin}	(L/D)***	$C_L \text{ at}$ $(L/D)_{max}$	0.15 chord	0.65 chord	Maxi- mum forward position	C ¹⁴ C _{Lmer}	4(<i>C</i> / <i>B</i>)s
			les vol									Percen	t chord	Percent	chord	
0006 0002 0018 0023 0023 2212 2315 2315 2315 2315 2315 2315 23	7 7 8 8 0 0 0 0 1 1 1 1 1 2 2 1 3 3 4 4 1 5 1 5 1 5 1 1 7 7 1 8 8 9 1 9 2 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2	0.272830032322323232323232323232323232323232			. 102 . 101 . 101 . 101 . 101 . 101 . 103 . 105 . 005 . 00	0,0005,0000000000000000000000000000000	1365 1366 1367 1844 1845 1846 1847 1848 1848 1848 1848 1848 1848 1848			2210 (1) (2) (2) (2) (2) (2) (2) (2) (2) (2) (2	$\begin{array}{c} 0.31\\ 0.31\\ 0.31\\ 0.31\\ 0.31\\ 0.31\\ 0.31\\ 0.32\\$	$\begin{smallmatrix} 5.56\\ 5.02\\ 0.058\\ $	400008094022114078410404050114468021114788474827841442518844944028804777828128921411412885281127844680214468021446804462814114178446802144680214468044628144114178446804462844804468044680446804468044680	**************************************	257224742828287444428858544407782888448498844844488840488569488555441555509257588555474209885574655	-0. 052 -0. 052 -0. 253 -1. 2255 -1. 2255
0006B 0012T 0012B 0018T 0013B	41 42 43 43	1,06 1,03 1,50 1,29 1,40	11 12 16 15 16	1 .0 .1	. 104 . 099 . 101 . 098 . 097	. 0076 . 0092 . 0095 . 0102 . 0122	130 126 158 127 116	.005 .001 .009 .000 .001	.0076 .0082 .0095 .0102 .0122	22.6 21.5 20.8 19.3 18.2	.34 .38 .40 .41 .41	5, 74 10, 19 11, 48 15, 28 17, 23	4.57 7.68 9.13 11.52 13.70	1 24 1 25 1 24 1 25 1 23	22 24 23 24 22	162 178 275 238 272
2R:12 2R:12	44	1.53 1.47	16 16	-1.6 6	. 101 . 102	. 0063 . 0086	184 171	020	. 0086 . 0087	22.3 22.2	. 38 . 38	10, 70 10, 71	8,26 8,25	1 25 1 25	29 28	264 257
0012Fe 0012F1	45 45	1, 53 2, 05	15 12	~11.7	. 111 . 099	. 0104	147 147	.000	. 0104 . 0184	19.5 17.6	. 42	10, 70 10, 70		1 26 84	25 68	238 180

Based on 34 CL_{max}. Based on straight portion of lift curve extended. See curve for actual value. Based on straight portion of moment curve extended. See curve for actual value. Ratio of the chord component to the heam component of the air forces to be used for the high angle of attack condition if the plane of the drag truss is parallel to the airfoll obord. C/B should be excluded when the plane of the drag truss is not parallel to the chord.

 $C/B = \tan [\tan^{-1} (C/B)_0 - i_i]$ or

where f_i is the angle of in-cidence of the drag truss with respect to the airfoil chord.

 $C/B = \frac{(C/B)_0 + k}{1 - k (C/B)_0}$ where k is the slope of the drag truss with respect to the airfoll chord.

à ŝ

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 TABLE II.—SLOPE OF LIFT CURVE, $a_0 = \frac{dC_L}{da_0}$ (PER DEG.)
 TABLE IV.—MAXIMUM LIFT COEFFICIENT, $C_{L_{mag}}$

Thickness designa- tion Cam- bar des- ignation	06	09	12	15	16	21	25	112
00	0.102	0, 101	0. 101	0.100	0.098	0.094	0.089	0.101
22 23 24 25 26 27	.104 .103 .103	.103 .103 .102	.102 .103 .102	. 102 . 101 . 099	. 098 . 096	.097 .095		.103 .101 .101 .102 .100 .100
42 43 44 45 46 47	. 108 . 104 . 104	.108 .108 .108	. 102 . 100 . 101	. 103 . 101 . 101	. 099 . 096 . 096	. 095 . 093 . 095		.102 .100 .100 .007 .098
62 63 64 75 66	. 105 . 104 . 101	-104 .101 .108	. 102 . 102 . 101	. 101 . 099 . 099	. 098 . 099 . 095	. 096 . 095 . 094		. 100 . 101 . 101 . 101 . 099
67								. 097

Additional tests to determine variation with camber.



Thickness designa- tion Cam- ber des- ignation	06	09	12	16	18	21	25	12	Theor.
00	-0.1	0.9	0.0	0.0	0.0	-0.1	0.0	9.0	0
22	-1.8 -1.7 -2.9	-20 -1.7 -2.9	-1.7 -1.7 -2.9	-1.7 -1.7 -2.9	-1.9 -2.0	-1.7 -1.8		-1.8 -1.9 -2.1 -2.1 -2.0	-1.30 -1.92 -2.08 -2.29 -2.59 -3.04
42 43 46 46 47	13.8 13.9 14.8	3,6 3,6 4,1	3.7 3.9 4.0	-3.8 -3.8 -4.1	-3.6 -3.7 -3.9	-3.6 -3.4 -3.4		-3.4 -3.9 -4.2 -4.6 -6.9	8,60 3.84 4.16 4.58 5.18 5.09
62 63 64 65 65 67	1-5.2 -5.6 1-5.3	-5.4 -5.9 -5.8	-5.4 -5.7 -5.8	5.4 5.7 5.0	5.2 6.7 6.7	-6.2 -6.2 -5.3		-6.2 -5.6 -5.7 -5.2 -5.8 -7.9	-5.40 -6.76 -6.23 -5.88 -7.78 -9.13

Based on straight portion of lift curve extended. See curve for actual value.

Thickness designa- tion Cam ber des- ignation	06	09	12	15	16	21	25	12
00	• 0. 88	è 1. 27	* 1. 58	¥ 1. 53	¥1.49	* 1. 38	٥ 1. 20	• 1. 53
22 23 24 25 26 26 27	• 1. 04 • 1. 01 • 1. 08	*1.51 *1.51 *1.38	* 1. 60 * 1. 59 * 1. 60	^b 1. 64 ^b 1. 55 ^b 1. 53	* 1, 43 * 1, 48	* 1. 35 * 1. 38		 ^b 1. 60 ^b 1. 61 ^b 1. 62 ^b 1. 62 ^b 1. 66 ^b 1. 68
42 48 44 45 46 47	• 1. 20 • 1. 23 • 1. 15	* 1, 60 * 1, 60 * 1, 56	* 1. 68 * 1. 61 * 1. 69	* 1. 56 * 1. 57 * 1. 62	*1.48 *1.47 *1.54	•1.29 •1.37 •1.46		= 1.71 * 1.63 = 1.65 * 1.69 * 1.76 = 1.62
62 63 64 65 86 86 67	• 1. 64 • 1. 43 • 1. 29	* 1. 67 * 1. 68 * 1. 71	* 1. 64 * 1. 85 * 1. 75	* 1. 55 * 1. 69 * 1. 67	* 1. 43 * 1. 51 * 1. 61	*1.37 *1.41 *1.49		* 1.75 * 1.66 * 1.67 * 1.75 * 1.83 * 1.95



NOTE .- Letter indicates type of lift curve peak.

TABLE V.---MOMENT COEFFICIENT AT ZERO LIFT, Cm0

Thickness designa- tion Oam- ber des- ignation	06 .	09	12	15	18	21	25	12	Theor.
00	-0.002	-0.003	-0.002	0.000	-0.002	0.001	-0.003	- 0. 002	0
22 23 24 24 25 26 27	036 039 048	036 044 052	036 042 050	034 040 049	037 047	036 043		-0.029 038 044 054 060 075	-0.0370 0447 0531 0628 0749 0912
42 43 44 45 46 47	075 087 109	073 066 100	072 087 102	068 083 097	065 078 094	057 071 062		059 075 089 105 124 143	0739 0894 1002 1257 1497 1825
62 63 64 66 67	¹ 109 ¹ 129 ¹ 159	110 133 155	108 129 154	~. 105 –. 125 1–_150	097 118 139	090 110 129		1 087 110 132 1 159 1 188 1 206	, 1109 ,1342 , 1594 , 1885 , 2246 , 2737

¹ Based on straight portion of moment curve extended. See curve for actual value.

TABLE VI.—DISPLACEMENT OF CONSTANT MOMENT POSITION IN PERCENT CHORD AHEAD OF QUAR-TER-CHORD POINT (100 TIMES VALUES OF *n* FOR EQUATION $C_{m_{e}/4} = C_{m_0} + nC_L$)

Thiokness designa- tion Cam- her des- ignation	06	09	12	15	18	21	25	12
00	0.7	0.7	0.9	1.1	1.4	1.8	2.6	.0.9
23 24 24 25 25	-8 -1 .0	.8 .2	.3 .4 .3	-6 .7 .6	1.0 1.0	1.5 1.7		-6 -0 -7 -8
42								-8
43 44 45 46	.8 .3 .3	.4 .3 .8	.5 .8	1.0	1.2 1.4 1.4	1.5 1.7 1.7		-4 .5 1.0 1.1
4 7 62								1.8
63 84 65 66	4 7 .0	.1 .0 .0	.6 .7	.9 1.6	1.1 1.8 1.8	1.5 1.7 1.9		.5 .8 1.5 2.0
67								2.1

TABLE VII.---MINIMUM PROFILE-DRAG COEFFI-CIENT, C_{Domin}

Thickness designs- tion Cam- ber des- ignation	06	09	12	16	18	21	25	12
00	0.0065	0.0074	0.0083	0.0093	0. 0106	0.0120	0.0143	0.0083
22	.0078 .0070 .0073 .0080 .0080 .0076 .0087	.0083 .0080 .0081	.0068 .0090 .0089 .0101 .0101 .0095 .0103	.0100 .0099 .0103 .0107 .0107 .0105 .0113	.0112 .0112 .0113 .0119 .0116 .0125	.0127 .0126 .0134 .0134 .0132 .0138		. 0087 . 0089 . 0085 . 0088 . 0089 . 0089 . 0099 . 0092 . 0095 . 0092 . 0095 . 0092 . 0095 . 0092
62 63 54 06 67	.0092 .0068 .0093	.0101 .0094 .0100	.0106 .0104 .0111	.0120 .0120 .0127	.0130 .0132 .0141	.0144 .0146 .0154		.0101 .0102 .0104 .0106 .0114 .0126

TABLE VIII.-OPTIMUM LIFT COEFFICIENT, CLeant

Thickness designa- tion Cam- ber des- ignation	08	09	12	15	18	21	25	12	1C _{L7}
00	0.00	0.00	0, 00	0. 00	0.00	0. 00	0.00	0.00	0
22 23 24 25 26 27	.17 .23 .18	.18 .17 .16	.10 .15 .15	.10 .12 .11	.11 .08	.07 .03		.17 .15 .20 .23 .20 .20 .20	0.308 .972 .256 .251 .256 .272
42 43 44 45 46 47	.35 .40 .40	.30 .36 .34	.23 .38 .27	-12 .22 .22	. 20 . 16 . 16	.05 .10 .08		.85 .84 .32 .30 .30 .30 .33	. 616 . 544 . 512 . 502 . 512 . 512 . 544
62 63 64 65 66 67	.63 .60 .80	.45 .55 .58	.40 .42 .42	.83 .33 .33	. 24 . 24 . 24	-13 .15 .10		.55 .47 .45 .38 .30	.923 .816 .767 .754 .767 .816

¹ Theoretical lift coefficient at "ideal" angla of attack.

TABLE IX.—RATIO OF MAXIMUM LIFT COEFFICIENT TO MINIMUM PROFILE-DRAG COEFFICIENT, C_{Lmax}/C_{D0min}

Thickness designa- tion Oam- ber des- ignation	06	09	13	15	18	21	25	12
00	185	172	184	164	138	115	84	184
22 23 24 25 26 27	142 144 141	182 189 170	182 177 180	154 156 148	128 132	108 109		184 181 190 184 187 187
42 43 44 45 46 47	150 162 132	180 168 168	161 170 184	146 150 143	123 127 123	96 104 100		186 172 179 178 178
62 63 64 65 87	187 165 139	108 179 171	162 159 158	129 133 132	110 114 114	96 97 97		178 163 160 165 161
						· · · · · · ·		200

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Positive directions of axes and angles (forces and moments) are shown by arrows

Axis	Force	Mome	Moment about axis			ə	Velocities		
Designation	Sym- bol	(parallel to axis) symbol	Designation	Sym- bol	Positive direction	Designa- tion *	Sym- bol	Linear (compo- nent along axis)	Angular
Longitudinal Lateral Normal	X Y Z	X Y Z	Rolling Pitching Yawing	L M N	$\begin{array}{c} Y \longrightarrow Z \\ Z \longrightarrow X \\ X \longrightarrow Y \end{array}$	Roll Pitch.2 Yaw	ф 8 • у		p g r

Absolute coefficients of moment

$C_i = \frac{L}{qbS} \qquad C_m = \frac{M}{qcS}$ (rolling) (pitching)

Geometric pitch

Inflow velocity

Slipstream velocity

Thrust, absolute coefficient $C_r = \frac{T}{\rho n^2 D^2}$

Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^s D^6}$

Diameter

Pitch ratio

p/D,

Ŵ,

V,

T.

Q,

Angle of set of control surface (relative to neutral position), 5. (Indicate surface by proper subscript.)

 $C_n = \frac{N}{qbS}$ (yawing)

4. PROPELLER SYMBOLS

η,

n,

- P_r , Power, absolute coefficient $C_P = \frac{P}{p n^3 D^3}$
- C_{ij} . Speed-power coefficient = $\sqrt[6]{\frac{\rho V^5}{Pn^2}}$
 - Efficiency
 - Revolutions per second, r.p.s.
- Φ , Effective helix angle = $\tan^{=1}\left(\frac{1}{2}\right)$

5. NUMERICAL RELATIONS

- 1 lb.=0.4536 kg.
 - 1 kg = 2,2046 lb.
 - 1 mi. = 1,609.35 m = 5.71 m = 2.2008 ft
 - 1 m=3.2808 ft.
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1 hp. = 76.04 kg-m/s = 550 ft-lb./sec. 1 metric horsepower = 1.0132 hp. 1 m.p.h. = 0.4470 m.p.s. 1 m.p.s. = 2.2369 m.p.h.

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THE CHARACTERISTICS OF 78 RELATED AIRFOIL SECTIONS FROM TESTS IN THE VARIABLE DENSITY WIND TUNNEL 1946, by Eastman N. Jacobs, Kenneth E. Ward and Robert M Pinkerton, 1946, 60 pp.

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