NATIONAL DEFENSE RESEARCH COMMITTEE

REPORT NO. A-11 : PROGRESS REPORT

ON THE PROBABILITY OF PENETRATION

OF ARMOR PLATE

by

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Div 2

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Approved June 12,1941
    for submission to Division Chairman

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This problem was suggested to
Section S, Division A, by Dr. Herschel
Smith and Dr. C. Hudson of Frankford
Arsenal. It was later specifically
described as Experiment 2 of a group
of three experiments suggested to
Professor Smyth in a letter from
Major L.S. Fletcher, of Frankford Arsenal,
dated March 11, 1941, further in a
letter from Professor Smyth to Dr.
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ON THE PROBABILITY OF PENETRATION OF ARMOR PLATE

Abstract

The probability of penetration for projectiles with measured velocities near the ballistic limit has been experimentally determined for caliber .30 A.P. Mark II bullets fired against a particular plate of 1/2-inch armor of Brinell hardness 415 manufactured by Disston. Various ways of treating the data have little effect on either the ballistic limit or the standard deviation, which have values in the range 2315-2322 ft/sec and 23-27 ft/sec, respectively. In particular, corrections for variations in bullet weight are found unimportant.

It is shown that if the standard deviation for firings of this particular caliber, bullet type, method of measuring velocity, type of plate, and so forth, is known, the ballistic limit of a similar plate for similar conditions can be determined with good accuracy by two bracketing shots. Specifically, if the standard deviation is 25 ft/sec and two shots are fired that differ in velocity by 25 ft/sec, the higher velocity shot penetrating and the lower velocity shot failing to penetrate, the ballistic limit may be taken as half way between the two velocities with a probable error of 17 ft/sec. If the bracketing velocities differ by 50 ft/sec, the probable error is 19.5 ft/sec.

1. Object of the experiment

It is known that, if a number of bullets are fired with velocities in the neighborhood of the ballistic limit of a plate of armor, some of those having measured velocities below the ballistic limit will penetrate the plate while some of those having velocities above the limit will fail to penetrate. The problem is to determine the probability of such events from a large number of firings and to deduce from these results how many shots are required to measure with a prescribed accuracy the ballistic limit of a given armor for a given type of bullet; or, conversely, with what accuracy the ballistic

limit is determined by a certain number of shots.

2. Apparatus and procedure

The experiment was carried out in the NDRC Experimental Firing Range at Princeton University[1]. The gun used was a .30 caliber Mann type barrel on a mount similar to those in use at the Aberdeen Proving Ground and at the Frankford Arsenal. The bullets were standard A.P. Mark II, classified in groups according to weight (Table IV). To get the desired spread of velocities, the powder was loaded at the range with loads varying from 3.140 to 3.180 gm.

The plate was homogeneous armor, 1/2 X 30 X 30 in., of Brinell hardness 415, manufactured by Disston and supplied by Captain M.B. Chatfield, of Frankford Arsenal. The velocity of each shot was measured by means of an Aberdeen chronograph with two sets of sparks. Two screens were used, 50 ft apart, the second screen being 2 ft from the armor plate. Each screen was connected to two circuits in the chronograph.

Because of the spatter of the jackets and frequent backward bounce of the A.P. cores, no attempt was made to use yaw cards in front of the plate.

However, after a group of shots had been fired, the yaw of the holes was measured wherever possible by inserting a dummy core attached to a protractor. After each shot the plate was inspected with a flashlight to see whether the bullet had

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[1] H.D. Smyth, _Construction of the NDRC experimental firing range at Princeton University_, NDRC Report No. A-6 (June 11, 1941).

YAW IN DEGREES

VELOCITY IN FEET PER SECOND
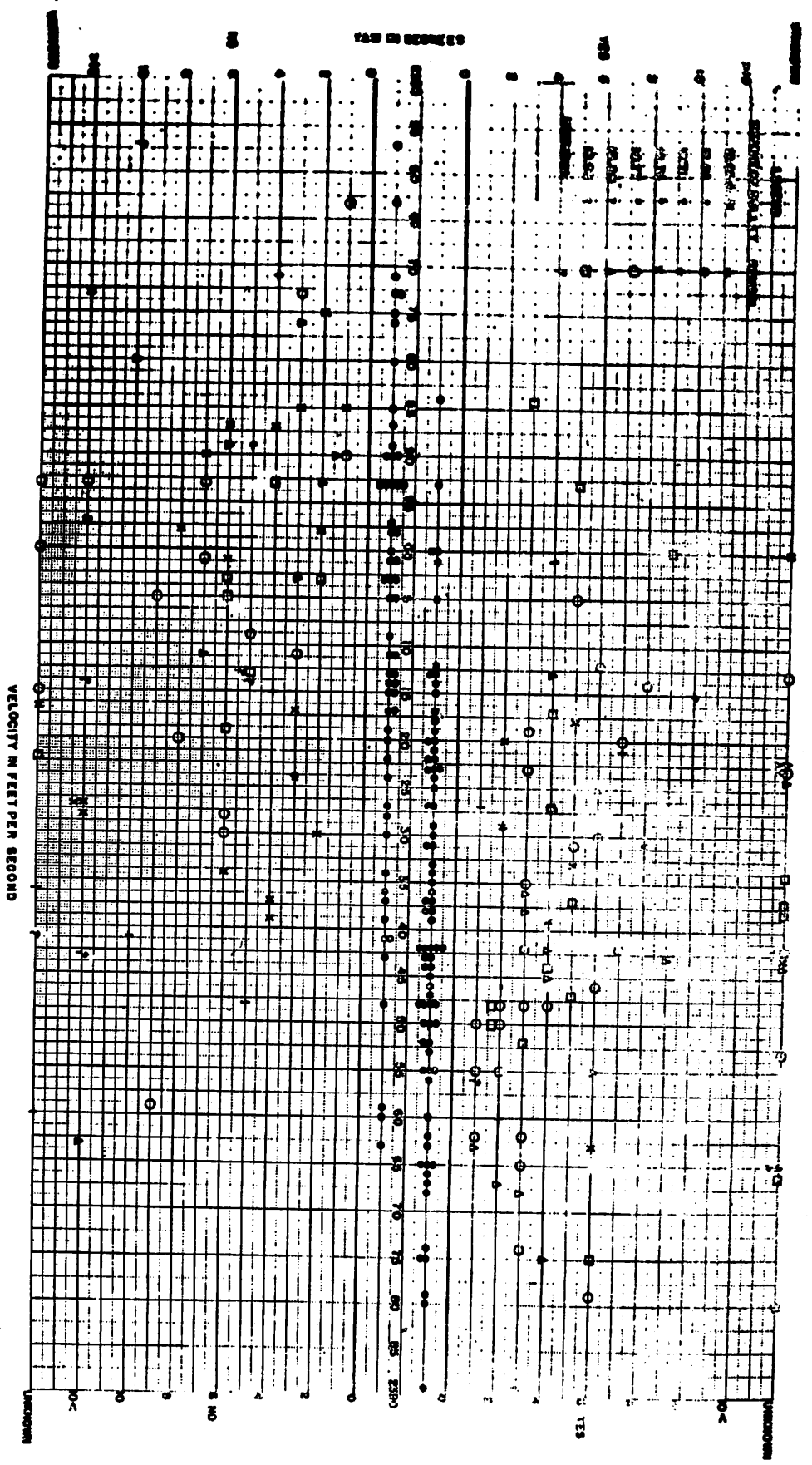
Fig. 1. Penetration chart, showing velocity, yaw and weight of bullet. The experimental data are plotted in two ways: (1) above and below the heavy horizontal zero yaw lines the shots are differentiated according to angle of yaw and weight of bullet as well as velocity and penetration; penetrating shots are above, nonpenetrating shots below; (2) between the zero yaw lines all the penetrating shots are plotted above and the nonpenetrating shots below, as functions of velocity only.

penetrated;[2/] theshot was then numbered with white paint.

3. The data

A total of 240 shots was fired at the plate. Of these, 84 were rejected for one or more of the following reasons: (1) a satisfactory velocity measurement was not obtained; (2) the core or a part of the core remained in the plate; (3) the shot came closer than two calibers to a previous shot; (4) the shot struck the frame of a screen or some other object. Figure 1 is a penetration-velocity chart in which the results obtained with most of the remaining 156 shots have been plotted in two ways: (1) grouped as to angle of yaw, weight of bullet and velocity; (2) with respect to velocity alone. A reduction of these data follows.

4. Theoretical analysis

If the values of any quantity x scatter about a mean value $\bar{x}$ according to a "normal" probability law, then the probability that x has a value between x and $x + dx$ is

$$p_{\bar{x}}(x)dx = (1/\sqrt{2\pi}\,\sigma)e^{-(x-\bar{x})^2/2\sigma^2}dx,$$

where σ is a constant, known as the standard deviation, the value of which characterizes the dispersion of values of x. In applying this equation to the problem of plate penetration as a function of velocity, we proceed as follows.

[2/] Throughout this report a bullet is said to have "penetrated" a plate if it has broken the back side of the plate sufficiently for light to be seen through the hole.

Let it be assumed that the plate has a certain mean ballistic limit velocity of u ft/sec and that the variation in the apparent ballistic limit for different shots can be attributed to some intrinsic variability -- inhomogeneity of the plate, inaccuracy of the velocity measurement, variation in bullets, and so forth -- of the individual shots which is described by the normal probability law. Then the probability that the apparent ballistic limit velocity for a given sample shot lies between v and v + dv is

$$p(v)dv = (1/2\pi\sigma)e^{-(v-u)^2/2\sigma^2}dv, \qquad (1)$$

where σ is the standard deviation. Essentially, our problem is to determine the value of σ from experimental firings and then to make deductions from this value.

However, Eq. (1) cannot be used directly. For a given shot we observe its velocity and whether it did or did not penetrate; that is, whether the apparent ballistic limit for that particular shot was above or below the measured velocity. The probability that a shot of velocity v will in fact penetrate the plate is the probability that the apparent ballistic limit will lie below v. This probability is given by

$$P(v) = \int_{-\infty}^{v} p(v)dv = \frac{1}{2}\left[1 + I\frac{u-v}{\sqrt{2}\sigma}\right], \qquad (2)$$

where, if $(v-u)/\sqrt{2}\sigma = z$, then

$$I(z) = (2/\sqrt{\pi})\int_{0}^{z} e^{-z^2}dz \qquad (3)$$

is the incomplete probability integral[3]

---

[3] For a table of values of I(z) see, for example, Peirce, A short table of integrals (3rd rev. ed., 1929), pp. 116-120.

Curves of functions of this form are shown in Figs. 3, 4 and 5.

In order to determine $\underline{u}$ and the standard deviation σ (or the probable error, 0.6745σ) from the data, use will be made of the **probability** **of the** **unexpected** -- that is, the probability that a shot for which $\underline{v} \ll \underline{u}$ will penetrate or that a shot for which $\underline{v} \gg \underline{u}$ will not penetrate. These probabilities are given respectively by $P(\underline{v})$ of Eq. (2) for $\underline{v}$ below $\underline{u}$, and by $1-P(\underline{v})$ for $\underline{v}$ above $\underline{u}$, or, over the whole range of $\underline{v}$ from $-\infty$ to $+\infty$, by

$$\prod (v) = \tfrac{1}{2}\left[1 - I \, \frac{|v - u|}{\sqrt{2}\,\sigma} \right]. \qquad (4)$$
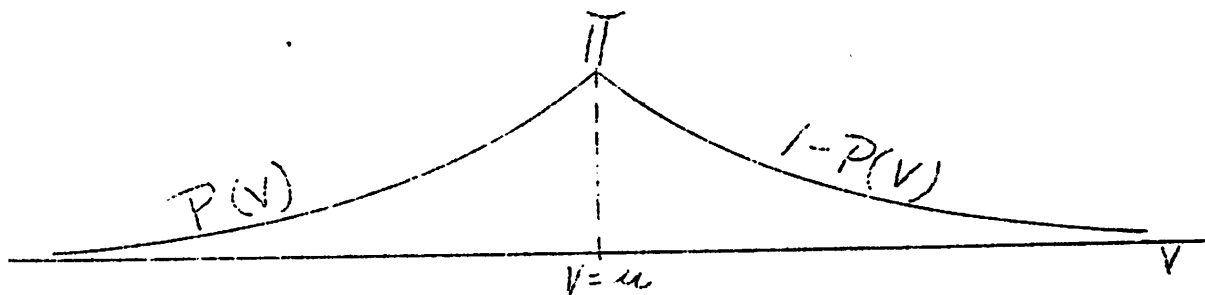
Fig. 2. Schematic graph of $\prod(v)$ as a function of $\underline{v}$.

The graph of this function, which is represented schematically in Fig. 2, is symmetric about the line $v=u$. The moment,

$$M = \int_{u}^{\infty} (v-u)\,\prod (v)\,dv = \tfrac{1}{4}\,\sigma^{2}, \qquad (5)$$

of the area under the curve on either side of the limit about the line $v=u$ is a measure of the dispersion of the distribution.

The method adopted in the following analysis is based

upon these considerations. From the firing data the probability
P of penetration within each velocity interval of 5 (or 10)
ft/sec is computed as the ratio of the number of penetrations
in that interval to the total number of shots in the interval.
The mean ballistic limit u for the plate is then obtained by
the requirement that the moment of P to the left of v=u shall
equal the moment of 1-P to the right of v=u; the common value
of the moment M then yields the standard deviation $\sigma = 2M^{\frac{1}{2}}$
of the individual shots.

5. Analysis of the data

The data on the plate in question, which are plotted in
Fig. 1, are represented in Table I in a form suitable for re-
production in accordance with the scheme presented at the
end of Sec. 4. The 12 shots with velocities less than 2283
ft/sec, none of which penetrated the plate, and the 18 above
2362 ft/sec, all of which did, are not included in the tabulation.
Also rejected are the 9 shots of measured yaw $> 10^{\circ}$, none of
which penetrated the plate.

The 117 tabulated shots have been lumped in 5-ft/sec
intervals, as shown in the first column of Table I. The next
group of four columns gives the number and percentage of shots
in each velocity interval that did and that did not penetrate
the plate; in the second group of four columns these shots have
been further consolidated into 10-ft/sec intervals. Finally,
in the last group of four columns is listed the corresponding
analysis for those shots, 64 in number, with a known yaw $\leq 5^{\circ}$.

Table I. Data in form for reduction.

Velocity Interval (ft/sec)	Yaw ≤ 10° or unknown								Yaw ≤ 5%			
	Penetrated		Not Penetrated		Penetrated		Not Penetrated		Penetrated		Not Penetrated	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
2283-87	1	20	4	80)	1	10	9	90	1	14	6	86
88-92	0	0	5	100)								
93-97	1	20	4	80)	4	31	9	69	2	40	3	60
98-02	3	37	5	63)								
2303-07	1	17	5	83)	2	20	8	80	1	20	4	80
08-12	1	25	3	75)								
13-17	5	42	7	58)	12	56	10	44	5	50	5	50
18-22	7	70	3	30)								
25-27	7	87	1	13)	11	73	4	27	5	56	4	44
28-32	4	57	3	43)								
33-37	6	67	3	33)	14	70	6	30	8	80	2	20
38-42	8	73	3	27)								
43-47	7	100	0	0)	16	94	1	6	11	92	1	8
48-52	9	90	1	10)								
53-57	5	100	0	0)	8	80	2	20	6	100	0	0
58-62	3	60	2	40)								
Total	68		49		68		49		39		25	

Yaw = 10° or unknown, 5-ft/sec intervals, u = 2322.0 ft/sec, σ = 24.9

(Alternate 10-ft/sec interval
yields u = 2324.0 ft/sec,
σ = 24.9)

Yaw = 10° or unknown, 10-ft/sec intervals, u = 2321.2 ft/sec, σ = 24.8

Yaw ≤ 5°, 10-ft/sec intervals, u = 2315.8 ft/sec, σ = 27.1
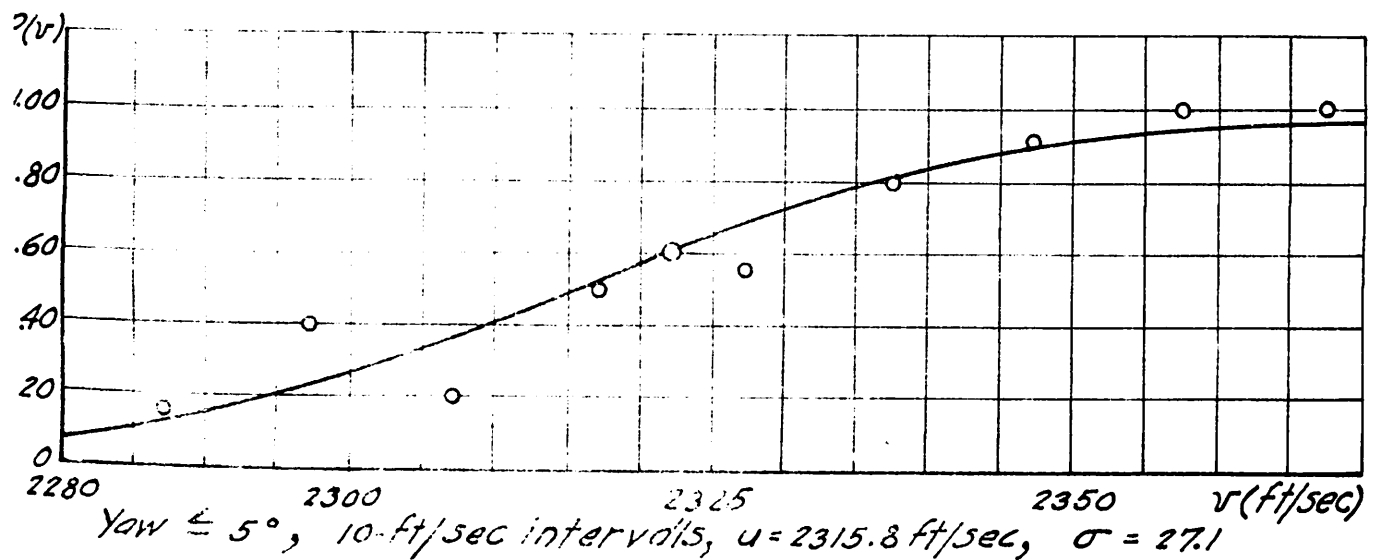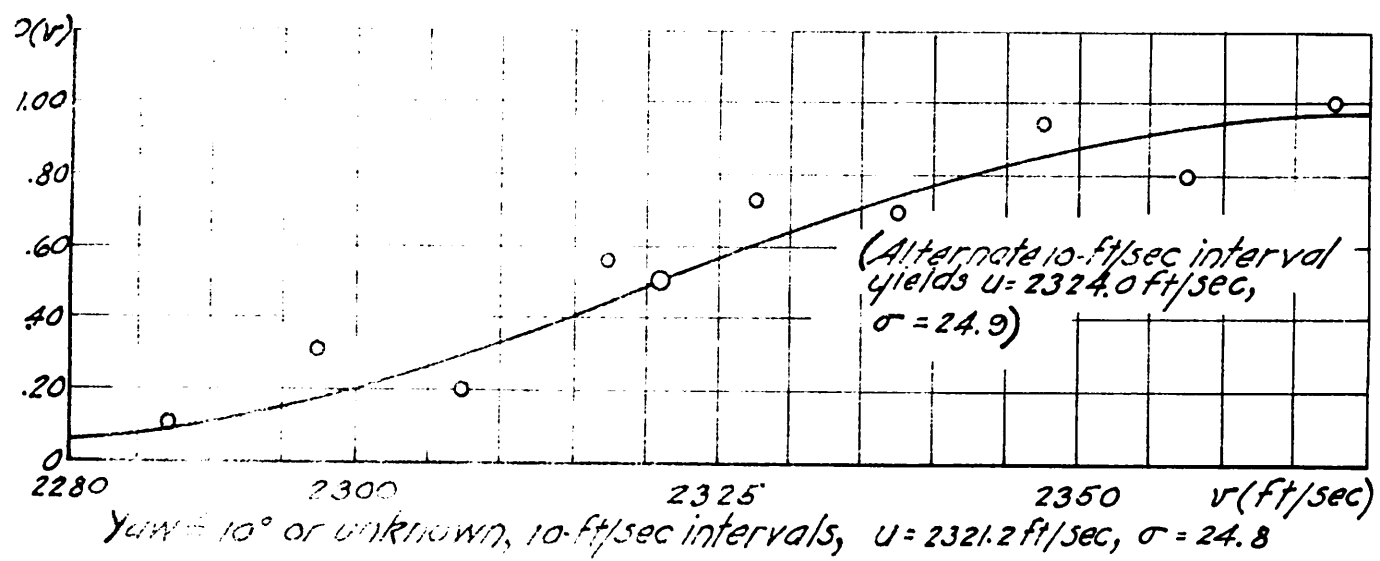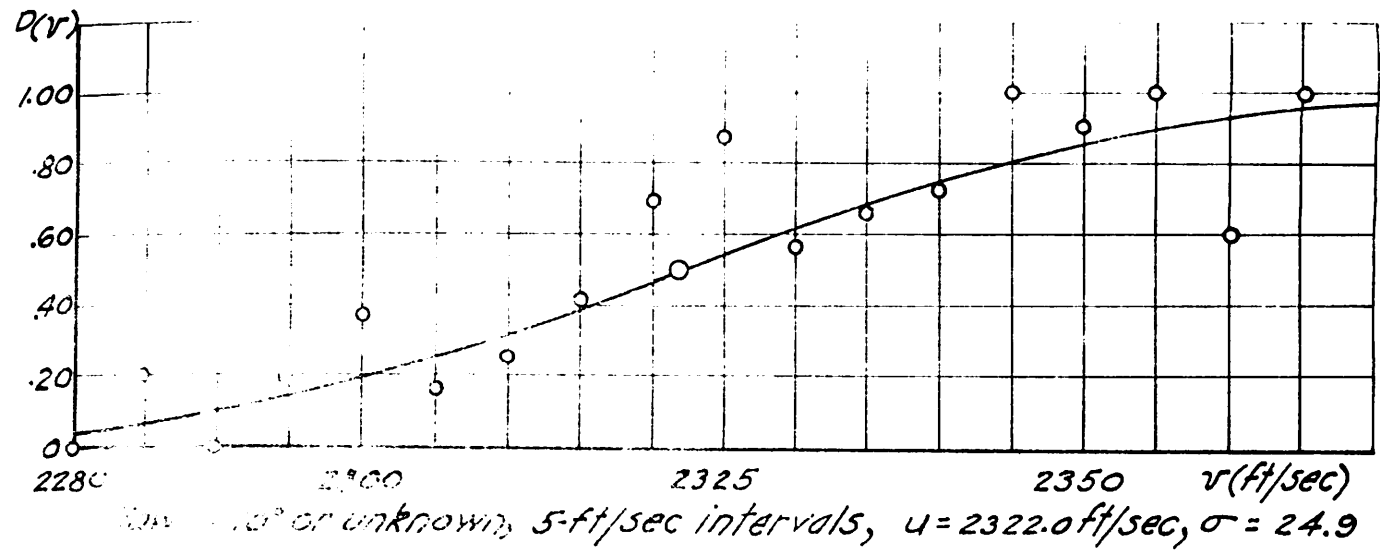
Fig. 3. Probability of penetration.

The data given in this table are used to plot the "observed" probability $P(\underline{v})$ that a shot of measured velocity $\underline{v}$ will penetrate the plate, as shown by the three series of points in Fig. 3. In consonance with the lumping procedure proposed in Sec. 4, the curve representing the observed accumulated probability $P(\underline{v})$ would be obtained by drawing horizontal segments 5 ft/sec (10 ft/sec) in length, centered on the observed points; in order not to confuse the presentation, this broken line curve has not been drawn in.

The inferred accumulated probability curves shown in full line in Fig. 3 have been found by the method outlined above: the mean $\underline{u}$ has been determined by the requirement that the moment of the area under that part of the curve of the observed $P(\underline{v})$ to the left of $\underline{u}$ shall equal the moment of the area under the graph of the observed $1-P(\underline{v})$ to the right of $\underline{u}$; the common value of this moment is then set equal to $\frac{1}{4}\sigma^2$, where σ is the standard deviation of the basic probability function $p(\underline{v})$. The resulting values of $\underline{u}$ and σ for the three curves · listed in Table II. The decrease in $\underline{u}$ in the

Table II. Values of u and σ for the three curves of Fig. 3.

Yaw	Velocity Interval (ft/sec)	u (ft/sec)	σ (ft/sec)
$\leq 10°$ or unknown	5	2322	25
	10	2321	25
$\leq 5°$	10	2316	27

case of shots of small yaw is to be expected, but it is surprising that the dispersion has in this case seemed actually to have increased; true,the difference is quite negligible, but one should expect a significant decrease in dispersion instead of an insignificant increase!

The three curves in Fig. 3, inferred from the observed points, have been plotted with σ = 25 ft/sec, centered on the proper value of $\underline{u}$. The function

$$P(v) = \frac{1}{2} \left[1 + I\left(\frac{v-u}{2}\right) \right] \qquad \underline{4/}$$

is readily computed with the aid of the tabulated values of the incomplete probability integral $I(\underline{x})$; the results are given in Table III.

Table III. The accumulated probability
function $P(v)$, for σ = 25 ft/sec.

v-u	P(v)	v-u	P(v)
-50	0.023	0	0.500
-45	.036	5	.579
-40	.055	10	.655
-35	.081	15	.726
-30	.115	20	.789
-25	.159	25	.841
-20	.212	30	.885
-15	.274	35	.919
-10	.345	40	.945
-5	.421	45	.964
0	.500	50	.977

The procedure adopted above is subject, in principle, to the criticism that it does not take into account the fact

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$\underline{4/}$ See footnote 3, Sec. 4.

that the lumped points are of different weight; for example,
in the three curves the number of shots per interval varies
in the ranges 4-11, 10-22 and 5-12, respectively. This factor
could be taken into account if it seems desirable; from a super-
ficial examination of the plot it would seem doubtful whether
this refinement would have any significant effect on the results.

The number n_- of tabulated shots of velocity <u>less</u> than
<u>u</u> which penetrated, and the number n_+ of velocity <u>greater</u> than
<u>u</u> which did not penetrate, are, for the two groups:

Yaw $\leq 10^{\circ}$ or unknown $n_- = 17, \; n_+ = 14$

Yaw $\leq 5^{\circ}$ $n_- = 5, \; n_+ = 8.$

6. <u>Reduction to common projectile weight</u>

An attempt was made to allow for the scattering in weight
of the bullets used. Of the tabulated shots, 112 had the weight
distribution, in 0.03-gm intervals, given in Table IV.

<u>Table IV. Weight distribution of the bullets.</u>

Designation on Fig. 1	Weight (gm)	Penetrated No.	%	Not Penetrated No.	%	v (ft/sec)
⚹	10.65	0	0	1	100	-13
⚹	.68	2	40	3	60	-10
+	.71	5	45	6	55	- 6
×	.74	9	45	11	55	- 3
○	.77	25	66	13	34	0
△	.80	11	69	5	31	+ 3
▢	.83	14	67	7	33	+ 6
Total		66		46		

It would appear from Table IV that the expectation of penetration increases with the weight of the bullet. It is therefore desirable to reduce the data to a common weight -- say 10.77 gm, which is the weight of the bullets designated (O) and is also the mean of the distribution given in Table IV. At first sight it might be expected that the core weight would be of more significance for this purpose than the bullet weight, and only the latter is available for the shots in question. An attempt was therefore made to find a correlation between core and bullet weights in the case of 12 caliber .30 Mark II bullets; but, contrary to expectation, no correlation was found. The attempt was therefore made to reduce the data on the basis of the known bullet weights; the most promising hypothesis considered was to assume that the total energy of the bullet is the principal factor which determines the influence of weight on penetration -- a hypothesis that has been found valid in reducing slight weight variations in data on solid projectiles. If, then, a projectile of weight $w = w_o + \delta w$ produces a certain effect when given a velocity v, the velocity $v + \delta v$ to be assigned to it in the reduction is determined by the equation

$$(1/2g)\ (w_o + \delta w)\ v^2 = (w_o/2g)\ (v + \delta v)^2 \ ,$$

or

$$\delta v = v(\delta w/2w_o) = 108\ \delta w,$$

where δw is measured in grams. The resulting corrections for the bullets used are listed in the last column of Table IV. The standard deviation of these bullet weights is found to be 0.042 gm and, in accordance with the aforementioned hypothesis,

the dispersion in velocity due to scattering in bullet weight

should therefore be given by the standard deviation

$$\sigma_w = 0.042v/2w_o = 4.5 \text{ ft/sec.}$$

This should have but little effect on the distribution, for

the deviation σ' in velocity due to all other causes should,

theoretically, be given by

$$\sigma'^2 = \sigma^2 - \sigma_w^2 = (25)^2 - (4.5)^2 = 605 \text{ ft}^2/\text{sec}^2,$$

whence the decrease in $\underline{\sigma}$ is only of the order of 0.5 ft/sec.

$\underline{\text{If}}$ the entire scattering in weight were due to the cores alone,

then σ_w would be increased in the ratio $\underline{\text{bullet weight}}/\underline{\text{core}}$

$\underline{\text{weight}} = 2$; that is,

$$\sigma'^2 = (25)^2 - (9)^2 = 544 \text{ ft}^2/\text{sec}^2,$$

or $\underline{\sigma}$ is decreased by approximately 1.5 ft/sec.

On reducing the data given in Fig. 1 for mass variation

in accordance with the foregoing considerations, a new distribution

is obtained; the resulting data are summarized in Table V and

plotted in Fig. 4. The treatment of these data by the method

used in Sec. 5 on the raw data yields the results listed in

Table VI.

It is difficult to see how to account for the apparent

decrease in limit velocity under this reduction -- it is true

that the n_+ nonpenetrating shots for which $\underline{v} > \underline{u}$ lose an

average of 3 ft/sec on reduction, but the n_- penetrating shots

for which $\underline{v} < \underline{u}$ maintain their average. There does seem to

be some decrease in dispersion, of the order contemplated in

the foregoing theoretical discussion, but one·should hesitate

to draw any conclusions concerning the mechanism of penetration

from such slender statistical data. The theoretical curves in

Table V. Penetration data corrected for weight.

Velocity Interval (ft/sec)	Yaw ≤ 10° Unknown								Yaw ≤ 5°			
	Penetrated		Not Penetrated		Penetrated		Not Penetrated		Penetrated		Not Penetrated	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
2283-87	0	0	5	100)								
88-92	2	40	3	60)	2	20	8	80	1	25	3	75
93-97	1	17	5	83)								
98-02	1	20	4	80)	2	18	9	82	2	33	4	67
2303-07	2	50	2	50)								
08-12	1	12	7	88)	3	25	9	75	1	14	6	86
13-17	7	58	5	42)								
18-22	6	75	2	25)	13	65	7	35	6	60	4	40
23-27	6	67	3	33)								
28-32	4	40	6	60)	10	53	9	47	4	50	4	50
33-37	4	80	1	20)								
38-42	7	78	2	22)	11	79	3	21	6	75	2	25
43-47	7	100	0	0)								
48-52	7	87	1	13)	14	93	1	7	8	100	0	0
53-57	8	100	0	0)								
58-62	5	83	1	17)	13	93	1	7	11	100	0	0
Total	68		47		68		47		39		23	

Yaw ≦ 10° or unknown, 5ft/sec intervals, u=2318.7ft/sec, σ = 24.2

Yaw ≦ 10° or unknown, 10-ft/sec intervals, u=2318.3 ft/sec, σ = 22.9

(Alternate 10-ft/sec interval
yields u=2319.1 ft/sec, σ=25.0)
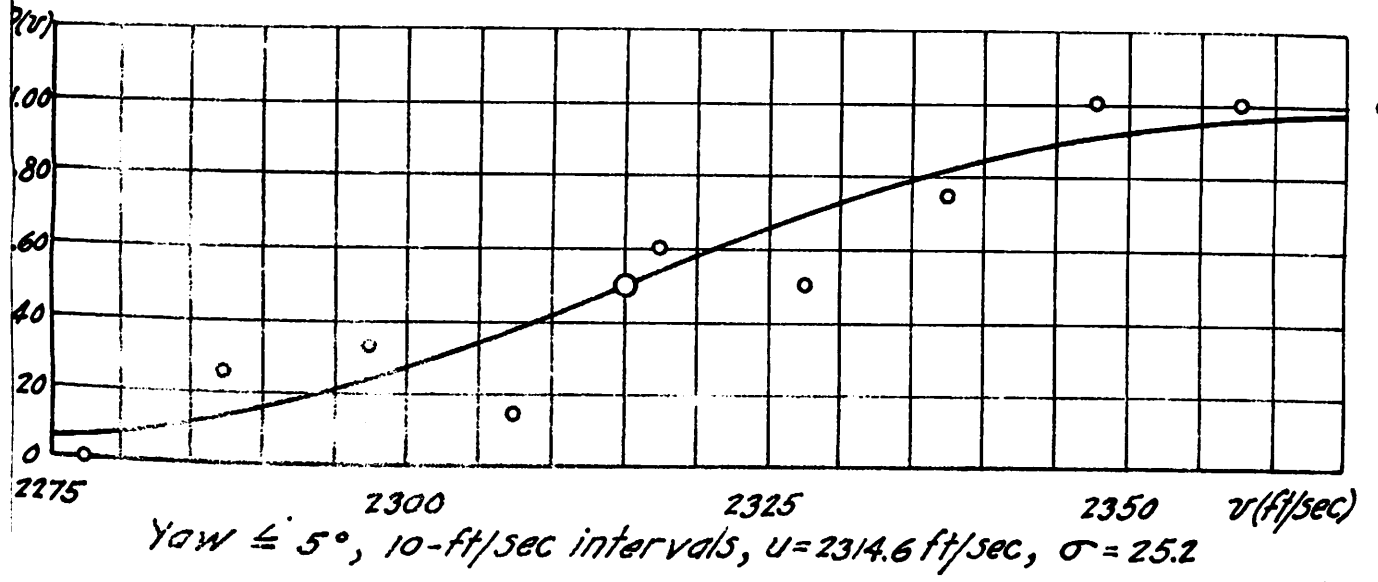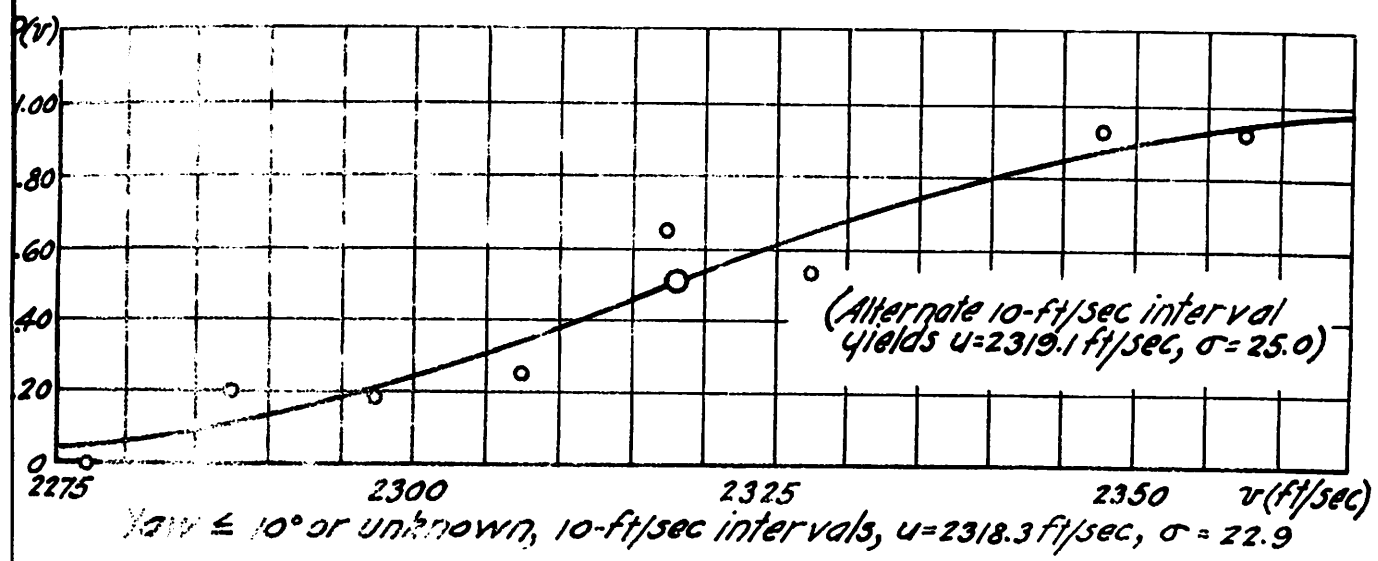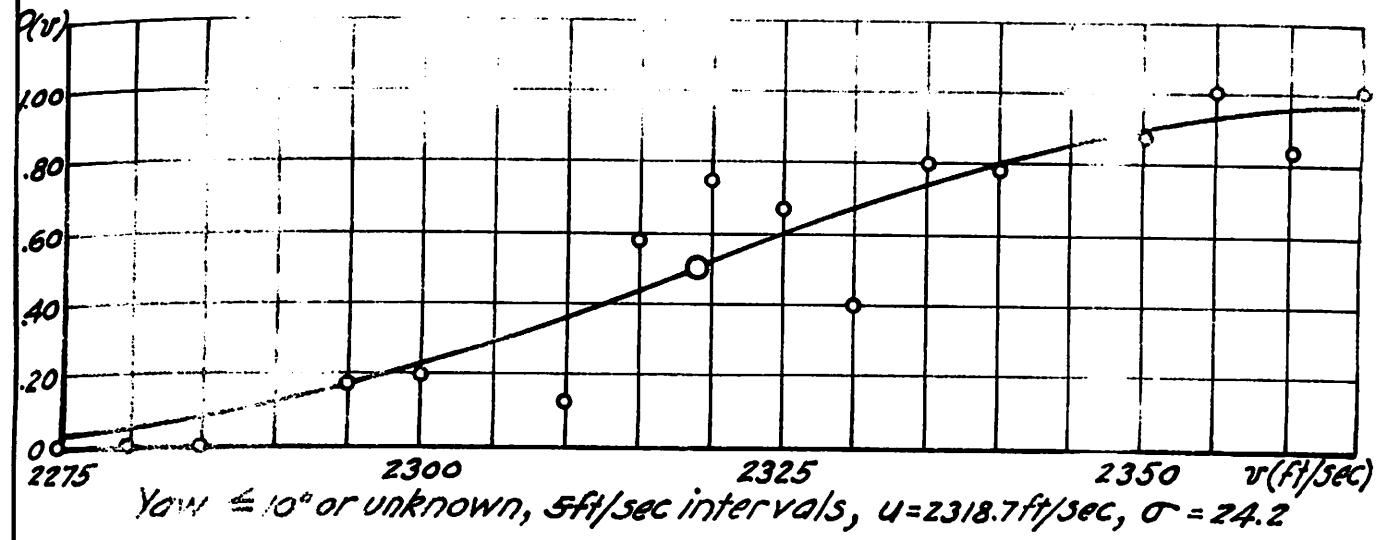
Yaw ≦ 5°, 10-ft/sec intervals, u=2314.6 ft/sec, σ = 25.2

Fig. 4. Probability of penetration, corrected for bullet weights.

Table VI.

Yaw n_-, n_+	Velocity Interval (ft/sec)	u (ft/sec)	σ' (ft/sec)
$\leq 10°$ or unknown $n_- = 15$, $n_+ = 16$	5 10	2319 2318	24 23
$\leq 5°$ $n_- = n_+ = 7$	10	2315	23

Fig. 4 are taken with $\sigma' \doteq 25$ ft/sec, as it was felt that the small difference did not warrant the extra computation.

7. Deductions from data

In practice a piece of armor plate is never tested by firing 200 shots at it. Hence, our problem is to deduce from the results we have obtained some conclusion as to the reliability of tests made with a very few shots. Specifically, suppose a plate is tested with _two_ shots, of different velocities, such that the shot of smaller velocity does not penetrate but the shot of larger velocity does penetrate. The usual conclusion is that the ballistic limit is half way between the velocity of the two shots. But the results already given show that it is possible for the ballistic limit of the plate to be less than the velocity of the low-velocity shot or more than the velocity of the high-velocity shot. The question is, how probable is this, or, in another form, how accurately is the ballistic limit determined by two bracketing shots.

Let two shots be fired with velocities a ft/sec apart, say with velocities $u - \frac{1}{2}a$ and $u + \frac{1}{2}a$; let it be supposed that N, the first shot, does not penetrate the plate and that Y, the second shot, does penetrate. The ballistic limit inferred from these data alone is u. Our problem is to determine the probable error of this inference on the assumption that, if many shots were fired at the plate, they would scatter with a standard deviation σ as in the experiments we have described in Sec. 5; or, to state the problem more generally, from the occurrence of the events N and Y and the general knowledge of penetration probability from the experiments here reported, we want to determine the probability $q(v)dv$ that the ballistic limit of the plate lies between v and $v+dv$.

The theory of probability gives an answer to this question by a direct application of Bayes' theorem. According to this theorem, if it be assumed that all values of the ballistic limit v are a priori equally probable but that we do know that events N and Y occur, then $q(v)$ at every value of v will be proportional to the probability $Q(v)$ that the events N and Y would both occur in testing a plate whose ballistic limit was in fact that particular value v. This probability $Q(v)$ can easily be expressed in general terms. It is the product of the probability P that a shot of velocity $u + \frac{1}{2}a$ will penetrate a plate of ballistic limit v and the probability $(1 - P)$ that a shot of velocity $u - \frac{1}{2}a$ will not penetrate such a plate. Thus, using Eqs. (2),
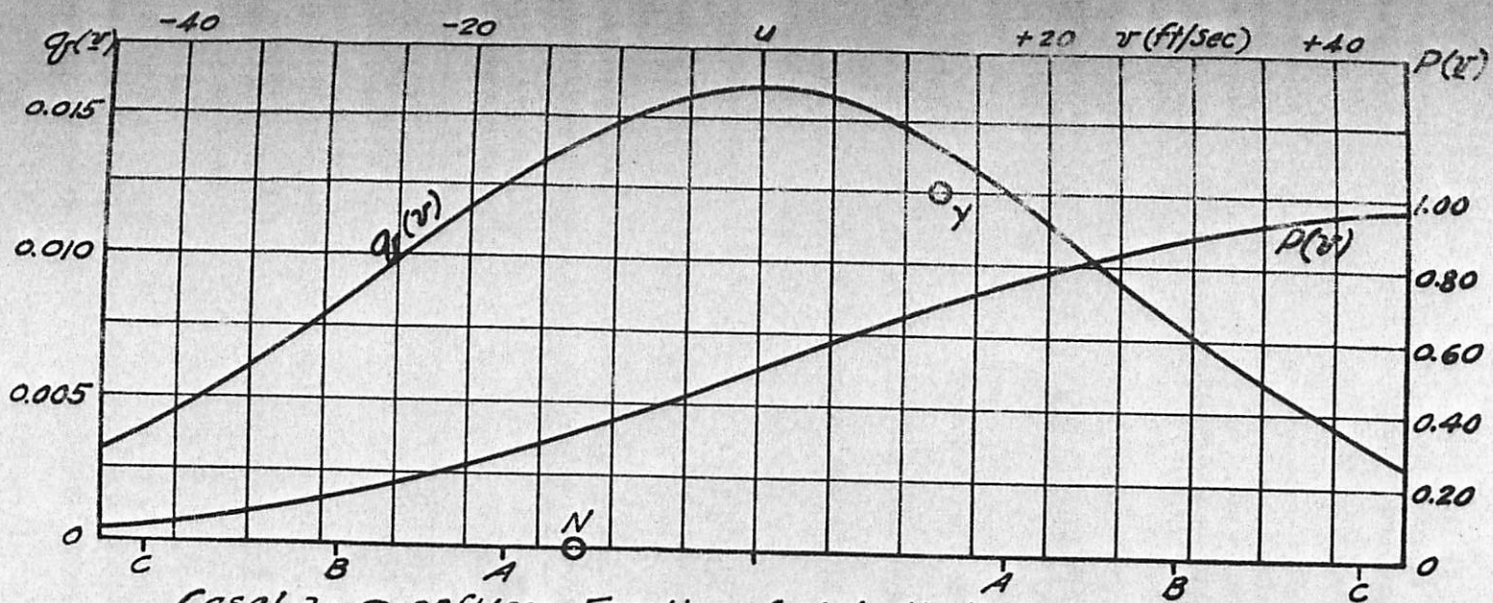
Case I. $a = \sigma = 25\,ft/sec$. Fraction of plate limits between: $N-Y(\pm 12.5\,ft/sec) = 0.38$; $B-B(\pm 29.0\,ft/sec) = 0.75$; $A-A(P.E. = \pm 17.1\,ft/sec) = 0.50$; $C-C(\pm 41.8\,ft/sec) = 0.90$.
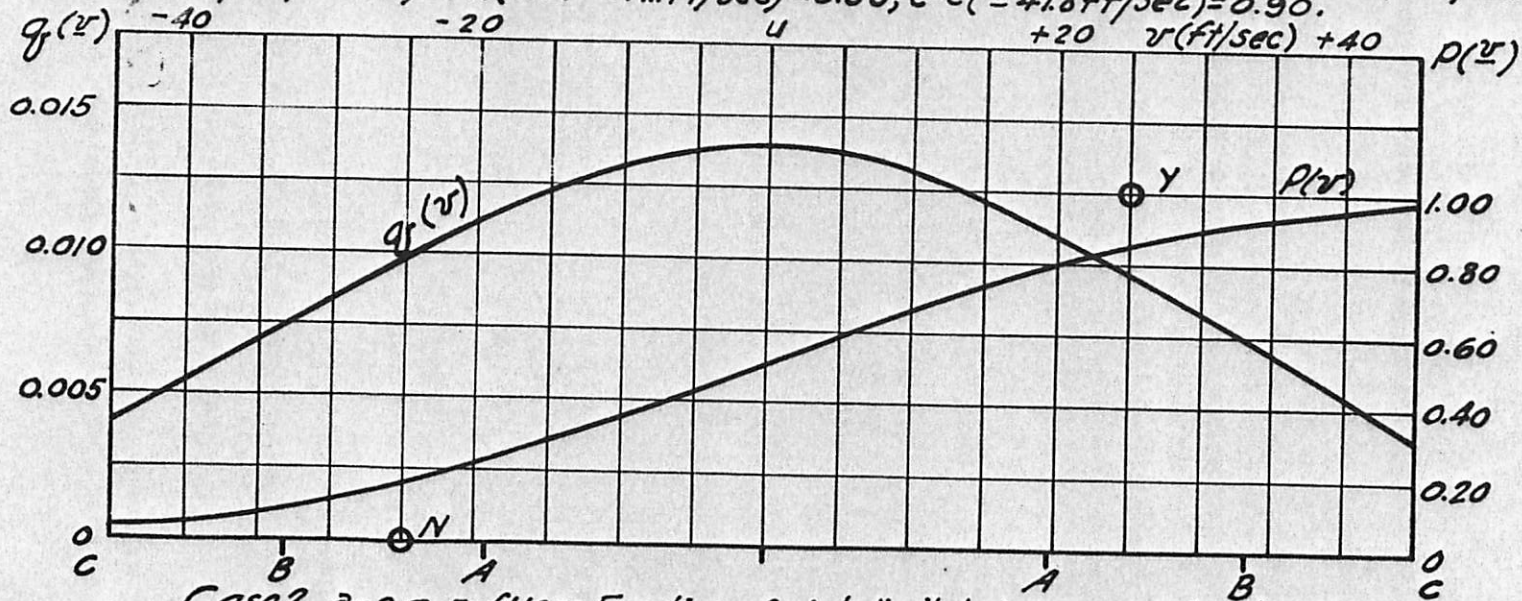
Case 2. $a = 2\sigma = 50\,ft/sec$. Fraction of plate limits between: $A-A(P.E. \pm 19.5\,ft/sec) = 0.50$; $B-B(\pm 32.2\,ft/sec) = 0.75$; $N-Y(\pm 25.0\,ft/sec) = 0.61$; $C-C(\pm 47.3\,ft/sec) = 0.90$.

Fig. 5 Distribution of $q(v)$ and $P(v)$, as inferred from plate data.

$$Q(v) = P_v(u + \tfrac{1}{2}a) \cdot \left[1 - P_v(u - \tfrac{1}{2}a)\right]$$

$$= \tfrac{1}{2}\left[1 + I\left(\frac{u-v + \tfrac{1}{2}a}{\sqrt{2}\,\sigma}\right)\right] \cdot \tfrac{1}{2}\left[1 + I\left(\frac{v-u + \tfrac{1}{2}a}{\sqrt{2}\,\sigma}\right)\right],$$

where $I(z)$ is again Eq. (3), the incomplete probability integral. According to Bayes' theorem, then, the required probability is given by

$$q(v) = kQ(v),$$

where k is a constant of proportionality and must have a value such that

$$k \int_{-\infty}^{+\infty} Q(v)dv = 1.$$

Actual values of the distribution $q(v)$ will depend on σ and a. The standard deviation σ is taken as 25 ft/sec, based on the experimental firings (see Table II). Computations have been made for two values of a: case 1, a = σ = 25 ft/sec; case 2, a = 2σ = 50 ft/sec. The results are given in Table VII and are plotted in Fig. 5.

Table VII. Values of $q(v)$.

$v - u$ (ft/sec)	q_1	q_2	$v - u$ (ft/sec)	q_1	q_2
0	0.0159	0.0138	30	0.0077	0.0081
5	.0156	.0136	35	.0060	.0067
10	.0147	.0130	40	.0045	.0053
15	.0133	.0121	45	.0032	.0041
20	.0115	.0109	50	.0022	.0031
25	.0096	.0095	75	.0002	.0004

The probable errors of these distributions have been computed and are found to be 17 and 20 ft/sec, respectively; that is, half of the plates thus tested should have actual ballistic limits within the intervals

(u $\pm$ 17) ft/sec (case 1) and (u $\pm$ 20)ft/sec. (case 2)
Other points of interest are indicated in Fig. 5.

The foregoing considerations are based, to repeat, on the assumption that nothing more than the assumed standard deviation σ of the plate limit was known a priori; the a posteriori probability q(v) was then deduced from the two observations N and Y. But, in practice, more is actually known a priori; indeed, previous experience must be responsible for the decision to place the test shots for such a plate in the neighborhood of the velocity u, and this a priori knowledge should tend to reduce the probable error in the plate limit thus determined. However, this influence is so slight that it may be disregarded; thus in case 2 an assumed a priori probable error about u of 67 ft/sec reduces the inferred a posteriori probable error of 19.5 ft/sec by only 1 percent.