Tables for Determining Expected Cost Per Unit under MIL-STD-105D Single Sampling Schemes

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Abstract: When a MIL-STD-105D sampling scheme is used for a long period, some lots will be subjected to normal, some to reduced, and some to tightened inspection. This paper provides for several single sampling plans and various quality levels, the expected fraction of lots rejected, the expected sample size per lot, and the expected number of lots to be processed before sampling inspection must be discontinued. Equations are given to calculate the long term cost of sampling inspection using these expected values and appropriate cost parameters.

Many private and government purchasers of manufactured products require that each lot submitted be subjected to sampling inspection by attributes. Lots which contain too many defectives may be returned to the manufacturer, purchased with a price concession, subjected to 100% screening, or scrapped. Clearly, there are substantial costs involved for inspection, disposal of rejected lots, and for the occurrence of defectives in accepted lots.

Dodge and Romig (2) have devised a set of attributes sampling plans based upon minimum cost, assuming a desired incoming quality. Hald (5) has greatly enlarged this idea, and developed plans which minimize cost for any prior distribution. However, neither the Dodge-Romig nor the Hald approach have achieved widespread popularity. Attributes sampling in the western world is dominated by the set of plans designated MIL-STD-105D (6) first published by the Department of Defense in 1963.

The MIL-STD-105D plans are not based upon cost concepts. Instead, the plans are indexed by lot size and by a number designated "acceptable quality level." The AQL is specified by the consumer, and is defined as the percent defective which will lead to a high probability of acceptance. This probability of acceptance is not a constant, but varies with lot size and AQL. The domain for probability

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of acceptance in the MIL-STD-105D plans is about 0.89-0.99.

Each plan in MIL-STD-105D provides a sample size, n, and an acceptance number, c, to be used for "normal" inspection of a lot. If c, or fewer defectives are found in the sample, the lot is accepted. The user is required to keep a historical record of lot-by-lot experience. Criteria are presented for an alteration of the values of n and/or c when the experience over several lots shows either unusually good or unusually bad quality. The rules are as follows (6):

- A switch from the normal values of n and c to "reduced" inspection is permissible when
 - a. Ten consecutive lots have been accepted.
 - b. The total number of defectives in the ten lots does not exceed a critical value supplied in Table VIII of MIL-STD-105D.
 - c. Production is continuous.
 - Reduced inspection is considered desirable by the responsible authority.

Under reduced inspection, n is substantially decreased to a value, n_R . Two numbers, c and r(>c) are supplied. Lots are accepted if the number of defectives is less than r. However, if a lot has more than c defectives, normal inspection must be resumed on the next lot.

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Standard Form 298 (Rev. 8-98) Prescribed by ANSI Std Z39-18 2. A switch to "tightened" from normal inspection is required when two of the most recent five lots have been rejected. Under tightened inspection, the sample size, n_T , is usually the same as for normal inspection, but c is reduced. A return to normal is permitted when five consecutive lots have been accepted. If tightened, inspection is still in use for ten consecutive lots; however, sampling inspection must be discontinued entirely.

It is clear that, if many lots from a process producing a fraction defective, p, are submitted under such a scheme, some lots will be subjected to normal inspection, some to reduced, and some to tightened inspection. If the probability of acceptance under reduced, normal, and tightened inspection is designated $P_{A,R}$, $P_{A,N}$, $P_{A,T}$ respectively, then this probability is progressively lower as we go from reduced to normal to tightened, so the fraction of lots rejected will depend upon the proportion inspected under each of the three plans. Furthermore, unless p is zero, it is inevitable that eventually, during one of the adoptions of tightened inspection, the criterion for return to normal inspection will not be met during the next 10 lots, so that sampling inspection ultimately will be abandoned.

The subject of this paper is the cost of lot-by-lot sampling inspection under the MIL-STD-105D plans. It is traditional in the literature of quality control to examine the performance of an attributes sampling plan under the assumption that, when the process is "in control," a stream of product is being produced with a fixed probability, p, that each item is defective [see Duncan (3), p. 147 or Grant and Leavenworth (4) p. 364]. The value of p for a particular kind of manufacturing process is usually well established. We may think of p as a parameter of a production process in control, a characteristic of the process. The purpose of attributes sampling inspection is, of course, to guard against sudden, "out of control" increases in p. However, if p is constant, and lots are formed and inspected under an attributes plan, there is a nonzero probability that each lot will be rejected, even though rejected and accepted lots have the same underlying quality. This "producer's risk" is an inherent consequence of acceptance sampling by attributes. We propose to answer the following question in the next section of this paper. If a manufacturer can consistently maintain a quality level, p, what will be the expected cost incurred per manufactured item as a result of exposure to MIL-STD-105D attributes sampling? In the third section, we explore the case where p is unknown, but where upper and lower bounds for p are available.

Brown and Rutemiller (1) have formulated a mathematical model of the MIL-STD-105D sampling scheme treating normal, reduced and tightened inspection as three stochastically coupled Markov chains. Using this analysis one may obtain the following information for any (lot size, AQL, p) combination:

- f_N = Expected fraction of lots under normal inspection; f_R = Expected fraction of lots under reduced inspection;
- f_T = Expected fraction of lots under tightened inspection;

- L = Expected number of lots inspected before sampling inspection must be abandoned;
- f = Expected fraction of lots rejected during sampling inspection.

The table results in the present paper were obtained using this model.

Assumptions

There are several costs which must be known or estimated to determine the total cost of sampling inspection under MIL-STD-105D.

- Let
- k₁ = cost of inspecting a single item under sampling inspection.
- k₂ = cost of inspecting a single item under 100% screening inspection;
- k₃ = cost of replacement for a single defective item detected under either sampling inspection or 100% screening;
- k₄ = cost of replacement for a single defective item detected later in the manufacturing process;
- $k_5 = \cos t$ of discontinuing sampling inspection completely.

The costs, k_1 and k_2 , will frequently differ since k_1 includes the cost of gathering a random sample. In addition, economies of scale occur when an entire lot is inspected.

The cost, k_3 , will often be substantially lower than k_4 because additional labor may be expended on items in accepted lots; when one of these is subsequently found to be defective, such additional labor costs are not recoverable. In addition, k_4 may include the cost of damage to a finished product of which the item is a component, customer reaction to a defective product, etc.

The cost, k_s , which occurs when sampling inspection is discontinued because too many consecutive lots have been on tightened inspection, will be generated by whatever remedial action is required to again institute sampling inspection. For example, this could involve a stopping of production for adjustments, frequently accompanied by a requirement that the next L lots be subjected to 100% screening before sampling inspection is resumed.

We define a cycle as the expected number of lots which will be subjected to sampling inspection until the tightened inspection rules of MIL-STD-105D require discontinuation of sampling inspection. For any (lot size, AQL, p) combination we define

- N = lot size;
- \overline{n} = expected sample size during sampling inspection;
- T = expected number of lots under sampling inspection during one cycle;
- C_{I} = expected cost per manufactured item incurred from MIL-STD-105D sampling.

We have

$$\overline{n} = f_N \cdot n + f_T \cdot n_T + f_R \cdot n_R;$$

$$f = 1 - [f_N \cdot P_{A,N} + f_T \cdot P_{A,T} + f_R \cdot P_{A,R}].$$

The value of T and the parameters needed to calculate nand f may be obtained from solution of the Markov matrices (1, p. 194).

The total cost incurred during one cycle will be

$$\begin{split} C &= T \{ k_1 \overline{n} + k_2 (N - \overline{n}) f + k_3 [\overline{n} + (N - \overline{n}) f] p \\ &+ k_4 (N - \overline{n}) (1 - f) p \} + k_5 ~. \end{split}$$

The number of items manufactured will be N(T+L), Hence,

$$C_I = \frac{1}{N(T+L)}$$

Discussion

The information in Table 1 will provide, for a particular lot size, AQL, and p, an estimate of the cost per unit attributable to defective items, when a "stream of product," each item having a probability, p, of being defective, is formed into lots. Several extensions of these calculations are possible.

If we have an accurate estimate of p, say from previous experience on similar products, then C_I may be computed for several of the AQL plans in MIL-STD-105D to find the sampling plan yielding minimum cost per unit. In many instances, lot size may also be set by the manufacturer. In this case, we could examine the various (AQL, lot size) combinations in MIL-STD-105D to ascertain the minimum cost combination.

An estimated domain for p may be available from prior experience on similar products. Clearly, it will be useful to employ the upper bound of this domain in conjunction with Table 1 to obtain an upper bound for costs attributable to sampling inspection. Even if p proves to be a random variable from lot to lot, costs from sampling will not exceed those calculated under "worst case" assumption that all lots are at the upper bound.

If p is completely unknown, we may still obtain valuable information from Table 1. For example, we can determine what quality level, p, must be maintained in production to hold the cost per item attributable to sampling inspection to X dollars, where X is a break-even value, or a value necessary to maintain a minimum profit level. Often, a knowledge of this *required* p is sufficient to ascertain whether a particular production method is practicable.

Examples

(1) MIL-STD-105D is to be instituted on lots of size 100, using general inspection level II, AQL = 4%. It is expected that the AQL level will be maintained in production. The

cost of inspecting a single item under sampling is estimated to be \$1.80. Rejected lots are to be 100% inspected, and the inspection cost is estimated at \$1.20 per item during 100% screening. Each defective item costs \$10.00 to replace if discovered during sampling or screening inspection. If discovered later in the production process, the cost is estimated at \$30.00. When sampling is discontinued, the cost incurred is estimated at \$600.00, including the cost of inspecting ten lots.

We have N = 100; — AQL = 4.0; 100p = 4.0; $k_1 = \$ 1.80;$ $k_2 = \$ 1.20;$ $k_3 = \$10.00;$ $k_4 = \$30.00;$ $k_5 = \$600.00;$ L = 10.

From Table 1,

$$n = 16.3;$$

 $100 f = 3.9;$
 $T = 661.$

Therefore,

$$C=661\{1.80(16.3) + 1,20(100-16.3)(.039) + 10.00[16.3 + (100-16.3)(.039)](.04) + 30(100-16.3)(1-.039)(.04)\} + 600: = 91,557.38;$$

 $C_I = 91,557.38/100(661+10)$

(2) Suppose that 100% inspection is used for this process at all times, in lieu of MIL-STD-105D inspection..

Then

12.11

$$C_I = 1.20 + 10(.04)$$

= \$1.60.

So that sampling inspection saves about \$0.23 per manufactured item in this case.

(3) Find the AQL plan which will minimize the cost of defective items for this process.

Using information from Table 1, and linear interpolation, we get

AQL	n	100 <i>f</i>	T	C_I
1.0	17.6	50.0	16	\$1.18
1.5	43.2	50.7	17	1.32
2.5	24.4	25.7	34	1.31
4.0	16.3	3.9	661	1.36
6.5	12.9	0.6	2.55 × 10 ⁶	1.33
10.0	8.7	0.0	6.60 × 10 ⁹	1.29

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
0.015	۵	0.00375 0.0075 0.01125 0.01875 0.0225 0.02625 0.03375 0.03375 0.03375 0.04125 0.04125 0.04875 0.0525 0.05625 0.06	827.8 856.4 884.6 911.5 936.4 959.1 979.6 997.7 1,013.9 1,028.1 1,040.7 1,051.8 1,061.6 1,070.3 1,078.1 1,085.0	3.1 6.2 9.5 12.7 16.0 19.3 22.5 25.7 28.8 31.7 34.6 37.4 40.0 42.6 45.0 47,4	3,309 535 203 109 70 51 40 33 28 25 23 21 19 18 18 18 17	0.25	K-L	0.0625 0.125 0.1875 0.25 0.3125 0.375 0.4375 0.5 0.625 0.625 0.625 0.6875 0.8125 0.8125 0.8125 0.9375 1.0	121.4 150.1 176.7 201.0 221.5 237.5 249.6 258.6 265.3 270.5 274.6 277.8 280.4 282.5 284.3 285.7	0.3 1.8 4.7 9.4 15.6 22.5 29.7 36.7 43.4 49.6 55.3 60.5 65.2 69.4 73.1 76.4	3.49 X 10° 15,686 982 194 .73 40 28 22 19 17 16 15 14 14 14 14 13
0.04	M	0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14 0.15 0.16	327.0 339.4 351.6 363.1 373.7 399.2 405.8 411.6 421.1 425.0 428.5 431.5 434.3	3.2 6.6 10.0 13.5 17.0 20.4 23.8 27.1 30.4 33.5 36.4 39.3 42.0 44.7 47.2 49.6	2,814 462 178 97 63 46 36 30 26 23 21 20 19 18 17 16	0.25	Μ	0.0625 0.125 0.1875 0.25 0.3125 0.375 0.4375 0.5 0.625 0.625 0.625 0.625 0.6875 0.8125 0.8125 0.875 0.9375 1.0	130.4 148.1 192.4 251.8 292.9 309.3 313.8 314.8 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0	0.0 0.1 0.9 3.6 8.9 16.0 24.0 32.1 39.9 47.1 53.6 59.5 64.7 69.3 73.4 76.9	1.49 × 10° 800,001 10,379 753 158 63 36 26 21 18 16 15 15 15 15 14 14
0,065	L-M	0.01625 0.0325 0.4875 0.065 0.08125 0.0975 0.11375 0.13 0.14625 0.17875 0.195 0.21125 0.24375 0.26	207.7 215.6 223.4 230.7 237.4 243.4 248.7 253.4 257.5 261.0 264.2 266.9 269.3 271.4 274.9	3.3 6.8 10.3 13.9 17.5 21.0 24.5 27.9 31.1 34.3 37.3 40.2 43.0 45.6 48.1 50.6	2,622 433 168 92 60 44 35 29 26 23 21 19 18 17 17 16	0.40	G-H	0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6	33.2 34.4 35.6 36.7 37.8 39.5 40.3 40.9 41.5 42.0 42.4 42.8 43.1 43.4 43.7	3.3 6.6 10.1 13.6 17.1 20.6 24.0 27.4 30.6 33.7 36.7 39.5 42.3 44.9 47.4 49.8	2,774 456 176 95 62 46 30 26 23 21 20 19 18 17 16
0.15	J-К	0.0375 0.075 0.1125 0.15 0.225 0.2625 0.3 0.375 0.375 0.4125 0.45 0.4875 0.5625 0.5625 0.6	82.8 85.6 88.5 91.1 93.6 95.9 98.0 99.8 101.4 105.2 106.2 107.0 107.8 108.5	3.1 6.2 9.5 12.7 16.0 19.3 22.5 25.7 28.8 31.7 34.6 37.4 40.0 42.6 45.0 47.4	3,309 535 203 109 51 40 33 25 23 21 19 18 18 18 17	0.40	ŀΚ	0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6	75.9 93.9 110.6 126.1 139.3 149.6 157.4 163.2 167.6 171.0 173.6 175.7 177.4 178.8 179.9 180.9	0.3 1.8 4.7 9.5 15.7 22.8 30.1 37.2 43.9 50.2 55.9 61.1 65.8 69.9 73.6 76.9	3.29×10 ⁶ 14,916 941 188 71 39 27 22 19 17 16 15 14 14 14 13
0.25	ĻЧ	0.0625 0.125 0.1875 0.25 0.375 0.4375 0.5 0.5625 0.625 0.625 0.6875 0.8125 0.8125 0.8125 0.9375 1.0	51.9 53.9 55.9 57.8 59.5 61.4 63.6 64.7 66.4 67.1 67.8 68.8 69.3 69.3	3.2 6.5 9.9 13.4 16.9 23.7 27.0 30.2 33.3 36.3 39.2 41.9 44.6 47.1 49.5	2,835 465 179 97 63 46 37 31 26 24 21 20 19 18 17 16	0.40		0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6	83.6 95.3 124.7 162.6 187.5 196.9 199.4 199.9 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0	0.0 0.2 1.0 3.9 9.4 16.8 24.9 33.1 40.9 48.2 54.7 60.5 65.7 70.2 74.2 77.7	1,42 X 10 ⁹ 664,173 8,789 661 144 59 35 25 20 18 16 15 14 14 14 14 14

Table 1: Expected sample size, expected percentage of lots rejected, and expected number of lots before Discontinuation of compliant inspection for some representative MU-STD-105D single-compliant plane and quality levels

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lo rejected	Expected ots lots unde sampling
0.40	М	0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6	137.8 167.5 231.4 283.8 307.1 313.7 314.8 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0 315.0	0.0 0.1 0.9 3.6 8.7 16.3 25.5 35.1 44.4 52.9 60.4 67.0 72.6 77.5 81.5 84.9	2.50 × 10 ¹⁰ 7.34 × 10 ⁶ 30,529 1,303 198 65 34 23 18 16 15 14 14 14 13 13 13	1.0	G-H	0.25 0.5 0.75 1.0 1.25 1.5 1.75 2.0 2.25 2.5 2.75 3.0 3.25 3.75 4.0	30.4 37.5 44.2 50.4 55.7 59.9 63.0 65.3 67.1 68.4 69.5 70.3 71.0 71.5 72.0 72.4	0.3 1.8 4.7 9.5 15.7 22.8 30.1 37.2 43.9 55.9 61.1 65.8 69.9 73.6 76.9	3.30 x 10° 14,916 941 188 71 39 27 22 19 17 16 15 14 14 14 13
0.65	F-G	0.1625 0.325 0.4875 0.65 0.8125 0.975 1.1375 1.3 1.4625 1.625 1.7875 1.95 2.1125 2.275 2.4375 2.6	20.8 21.6 22.5 23.2 23.9 24.5 25.1 25.6 26.0 26.4 26.7 27.0 27.2 27.5 27.6 27.8	3.3 6.8 10.3 14.0 17.6 24.7 28.1 31.4 34.5 37.6 40.5 43.3 46.0 48.5 50.9	2,548 422 164 90 59 44 35 29 25 23 21 19 18 17 17 16	1.0		0.25 0.5 0.75 1.0 1.5 1.5 2.0 2.25 2.5 2.5 3.0 3.25 3.75 3.75 4.0	33.4 38.1 49.9 65.0 75.0 78.8 79.8 80.0 80.0 80.0 80.0 80.0 80.0 80.0 8	0.0 0.2 1.0 3.9 9.4 16.8 24.9 33.1 40.9 48.2 54.7 60.5 65.7 70.2 74.2 77.7	1,42 × 10° 664,173 8,789 661 144 59 35 25 20 18 16 15 14 14 14 14 14
0.65	Ŧ	0.1625 0.325 0.4875 0.65 0.8125 0.975 1.1375 1.37 1.4625 1.625 1.7875 1.95 2.1125 2.275 2.4375 2.6	49.1 60.9 71.9 81.7 89.8 95.9 100.5 103.8 106.3 108.3 109.8 110.9 111.9 112.7 113.3 113.9	0.4 1.9 5.2 10.3 16.8 24.1 31.5 38.7 45.5 51.7 57.4 62.5 67.1 71.2 74.9 78.1	2.61 × 10 ⁶ 12,090 787 163 64 37 26 21 18 16 15 15 15 14 14 14 13	1.0	К	0.25 0.5 0.75 1.0 1.25 1.5 1.75 2.0 2.25 2.5 2.5 2.75 3.0 3.25 3.5 3.75 4.0	55.0 68.0 98.8 116.7 123.2 124.7 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0	0.0 0.1 1.0 3.7 8.6 15.9 25.0 34.5 43.8 52.2 59.8 66.4 72.1 77.0 81.1 84.5	2.50 × 10 ⁹ 7.00 × 10 ⁶ 28,432 1,314 205 67 35 23 19 16 15 14 14 13 13
0.65	к	0.1625 0.325 0.4875 0.65 0.8125 0.975 1.375 1.375 1.375 1.4625 1.625 1.7875 1.95 2.1125 2.275 2.4375 2.6	52.3 60.0 79,1 102.9 117.9 123.3 124.7 124.9 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0	0.0 0.2 1.1 4.2 9.9 17.5 25.8 34.1 42.0 49.2 55.7 61.5 66.7 71.2 75.1 78.6	1.11 X 10 ⁹ 560,102 7,541 586 132 56 33 24 20 17 16 15 14 14 13 13	1.5	D-E	0.375 0.75 1.125 1.875 2.25 € 2.625 3.0 3.375 3.75 4.125 4.5 4.875 5.625 6.0	8.3 8.6 9.2 9.5 9.8 10.0 10.2 10.4 10.5 10.7 10.8 10.9 11.0 11.1 11.2	3.1 6.3 9.6 12.9 16.3 19.6 22.9 26.2 29.3 32.4 35.3 38.1 40.8 43.4 45.9 48.3	3,080 507 192 103 67 49 38 32 27 24 22 20 19 18 17 17
1.0	E-F	0.25 0.5 0.75 1.0 1.25 1.5 1.75 2.0 2.25 2.75 3.0 3.25 3.75 3.75 4.0	13.5 13.9 14.4 14.9 15.6 16.0 16.2 16.5 16.7 16.9 17.1 17.2 17.3 17.5 17.6	3.3 6.7 10.2 13.8 17.3 20.8 24.2 27.6 30.8 33.9 36.9 39.7 42.5 45.1 45.6 50.0	2,735 450 173 94 62 45 36 30 26 23 21 20 18 17 17 16	1.5	F-G	0.375 0.75 1.125 1.5 2.25 2.625 3.0 3.375 4.125 4.5 4.875 5.25 5.625 6.0	20,0 25,5 29,8 33,5 37,7 39,4 40,9 42,7 43,4 43,9 44,4 43,9 44,4 45,0 45,3	0.3 1.8 4.7 9.1 14.6 20.9 27.7 34.4 40.9 47.0 52.6 57.8 62.5 66.8 70.6 74.1	4.45 × 10 ⁶ 186,217 1,145 226 84 45 30 23 20 18 16 15 15 15 14 14 14

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
1.5	н	0.375 0.75 1.125 1.5 1.875 2.25 2.625 3.0 3.375 3.75 4.125 4.5 4.875 5.625 6.0	20.8 24.0 32.8 42.4 49.9 50.0 50.0 50.0 50.0 50.0 50.0 50.0 5	0.0 0.1 1.0 3.5 8.0 14.2 21.4 29.1 36.6 43.7 50.3 56.2 61.5 66.3 70.5 74.2	2.15 × 10 ⁹ 1.07 × 10 ⁶ 12,457 922 196 76 42 29 23 19 17 16 15 14 14	2.5	G	0.625 1.25 1.875 2.5 3.75 4.375 5.0 5.625 6.25 6.25 6.875 7.5 8.125 8.75 9.375 10.0	13.6 15.5 20.2 26.2 30.1 31.5 31.9 32.0 32.0 32.0 32.0 32.0 32.0 32.0 32.0	0.0 0.2 1.1 3.9 9.4 16.8 24.9 33.1 40.9 48.2 54.7 60.5 65.7 70.2 74.3 77.7	1.38 × 10 ⁹ 647,412 8,633 656 143 59 35 25 20 18 16 15 14 14 14 14
1.5	J	$\begin{array}{c} 0.375\\ 0.75\\ 1.125\\ 1.5\\ 1.875\\ 2.625\\ 3.0\\ 3.375\\ 3.75\\ 4.125\\ 4.5\\ 4.5\\ 5.625\\ 5.625\\ 6.0\\ \end{array}$	35.0 42.5 61.2 73.4 78.4 79.7 80.0 80.0 80.0 80.0 80.0 80.0 80.0 80	0.0 0.1 0.9 3.1 7.4 14.0 22.3 31.5 40.5 49.0 56.6 63.3 69.2 74.3 78.7 82.4	2.69 × 10 ⁹ 1.32 × 10 ⁷ 46,481 1.946 277 84 40 26 20 17 15 14 14 13 13 13	2.5	н	0.625 1.25 1.875 2.5 3.75 4.375 5.0 5.625 6.875 7.5 8.125 8.75 9.375 10.0	22.0 27.2 39.5 46.7 49.3 49.9 50.0 50.0 50.0 50.0 50.0 50.0 50.0 5	0.0 0.1 1.0 3.7 8.6 15.9 25.0 34.5 43.8 52.2 59.8 66.4 72.1 77.0 81.1 84.5	2.50 × 10 ⁹ 7.00 × 10 ⁶ 28,432 1,314 205 67 35 23 19 16 15 14 14 13 13
1.5	ĸ	$\begin{array}{c} 0.375\\ 0.75\\ 1.125\\ 1.5\\ 2.25\\ 2.625\\ 3.0\\ 3.375\\ 3.75\\ 4.125\\ 4.5\\ 4.5\\ 5.625\\ 5.625\\ 6.0\\ \end{array}$	50.7 54.8 85.8 114.3 122.8 124.7 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0	0.0 0.2 1.1 3.6 9.1 18.4 30.5 42.9 54.2 63.8 71.7 78.1 83.2 87.2 90.3	1.77 × 10 ¹⁰ 2.69 × 10 ⁹ 996,132 12,153 797 144 51 29 21 17 15 14 17 15 14 13 13 13	2.5	J	0.625 1.25 1.875 2.5 3.125 3.75 4.375 5.0 5.625 6.25 6.25 6.875 7.5 8.125 8.75 9.375 10.0	32.5 35.6 55.0 73.1 78.7 79.9 80.0 80.0 80.0 80.0 80.0 80.0 80.0 8	0.0 0.2 1.5 5.0 12.3 23.9 37.2 49.9 60.8 69.8 77.0 82.6 86.9 90.3 92.8	1.48 × 10 ¹⁰ 1.14 × 10 ⁹ 410,188 5,454 415 89 37 23 18 16 14 14 13 13 12
2.5	C-D	$\begin{array}{c} 0.625\\ 1.25\\ 1.875\\ 2.5\\ 3.75\\ 4.375\\ 5.0\\ 5.625\\ 6.25\\ 6.875\\ 7.5\\ 8.125\\ 8.75\\ 9.375\\ 10.0\\ \end{array}$	5.2 5.4 5.8 5.1 6.2 4 5.6 6.6 7 8.8 9 6.6 6.6 7 8.8 9 6.9 6.9 6.9 6.9 6.9 6.9 6.9 6.9 6.9 6	3.2 6.5 9.9 13.4 16.9 20.3 23.7 27.0 30.2 33.3 36.3 39.2 41.9 44.6 47.1 49.5	2,835 465 179 97 63 46 37 31 26 24 21 20 19 18 17 16	2.5	к	0.625 1.25 1.875 2.5 3.125 4.375 5.0 5.625 6.25 6.25 6.875 7.5 8.75 9.375 10.0	50.2 52.8 73.5 113.6 123.8 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0 125.0	0.0 0.0 0.1 1.4 7.3 14.6 29.6 45.8 60.0 71.4 80.1 86.4 90.0 94.0 96.1 97.5	5.43 X 10 ¹⁰ 4.50 X 10 ⁹ 4.95 X 10 ⁶ 12,909 459 72 29 19 15 15 14 13 13 13 13 12 12
2.5	E-F	0.625 1.25 1.875 2.5 3.75 4.375 5.0 5.625 6.875 7.5 8.125 8.75 9.375 10.0	12.1 15.0 17.7 20.2 22.3 25.2 26.1 26.8 27.4 27.8 28.1 28.4 28.4 28.6 28.8 28.9	0.3 1.8 4.7 9.5 15.7 22.8 30.1 37.2 43.9 50.2 55.9 61.1 65.8 69.9 73.6 76.9	3.30 × 10 ⁶ 14,916 941 188 71 39 27 22 19 17 16 15 14 14 14 14	4.0	B-C	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	3.1 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4.0 4.1 4.1 4.2 4.2 4.2 4.3	3.1 6.3 9.6 13.0 16.4 19.8 23.2 26.5 29.7 32.8 35.7 38.6 41.3 44.0 46.5 48.9	2,942 481 185 100 65 48 38 31 27 24 22 20 19 18 17 17

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lot rejected	Expected s lots unde sampling
4.0	D-E	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	7.8 9,7 11.6 13.2 14,5 16.2 16.7 17.1 17.4 17.6 17.8 18.0 18.1 18.2 18.3	0.3 1.9 5.1 10.2 16.6 23.8 31.1 38.3 50.0 51.2 56.9 62.0 66.6 70.7 74.4 77.6	2.84 × 10 ⁶ 12,897 827 169 66 37 26 21 18 16 15 15 15 14 14 14 14	6.5	C-D	1.625 3.25 4.875 6.5 8.125 9.75 11.375 13.0 14.625 16.25 17.875 19.5 21.125 22.75 24.375 26.0	4.7 5.9 7.1 8.2 9.1 9.8 10.3 10.7 10.9 11.1 11.3 11.4 11.5 11.6 11.7 11.8	0.3 1.9 5.2 10.4 17.2 24.8 32.4 39.8 46.7 53.1 58.8 63.9 68.5 72.6 76.1 79.3	2.36 × 10 ⁶ 10,958 720 151 60 35 25 20 18 16 15 15 15 14 14 14 13
4.0	E	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	8.4 9.5 12.5 16.3 18.8 19.7 19.9 20.0 20.0 20.0 20.0 20.0 20.0 20.0 2	0.0 0.2 1.0 3.9 9.4 16.8 24.9 33.1 40.9 48.2 54.5 60.5 65.7 70.2 74.2 77.7	1.42 × 10 ⁹ 664,173 8,789 661 144 59 35 25 20 18 16 15 14 14 14 14	6.5	E	$\begin{array}{c} 1.625\\ 3.25\\ 4.875\\ 6.5\\ 8.125\\ 9.75\\ 11.375\\ 13.0\\ 14.625\\ 16.25\\ 17.875\\ 19.5\\ 21.125\\ 22.75\\ 24.375\\ 26.0\\ \end{array}$	5.2 6.1 8.3 10.9 12.4 12.9 13.0 13.0 13.0 13.0 13.0 13.0 13.0 13.0	0.0 0.2 1.3 4.8 11.3 19.4 28.3 36.7 44.7 51.9 58.4 64.1 69.1 73.4 77.2 80.5	7.91 X 10 ⁸ 387,112 5,370 443 107 48 30 23 19 17 15 15 15 14 14 13 13
4.0	G	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	14.4 17.5 24.1 29.2 31.3 31.9 32.0 32.0 32.0 32.0 32.0 32.0 32.0 32.0	0.0 0.1 1.0 3.8 9.2 17.1 26.6 36.4 45.7 54.1 61.6 68.1 73.7 78.4 82.4 85.7	2.33 × 10 ⁹ 5.72 × 10 ⁶ 24,785 1,114 177 60 32 22 18 16 15 14 13 13 13 13	6.5	F	$\begin{array}{c} 1.625\\ 3.25\\ 4.875\\ 6.5\\ 8.125\\ 9.75\\ 11.375\\ 13.0\\ 14.625\\ 16.25\\ 17.875\\ 19.5\\ 21.125\\ 22.75\\ 24.375\\ 26.0 \end{array}$	8.9 10.9 15.2 18.3 19.6 19.9 20.0 20.0 20.0 20.0 20.0 20.0 20.0 2	0.0 0.1 1.1 9.7 17.9 27.6 37.6 46.9 55.4 69.2 74.7 79.3 83.2 86.4	2.33 × 10° 4.72 × 10° 20,813 967 159 56 30 22 18 16 14 14 13 13 13
4.0	н	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	20.3 22.3 34.4 45.7 49.2 49.9 50.0 50.0 50.0 50.0 50.0 50.0 50.0 5	0.0 0.2 1.5 5.0 12.3 23.9 37.2 49.9 60.8 69.8 77.0 82.6 86.9 90.3 92.8	1.48 × 10 ¹⁰ 1.14 × 10 ⁹ 410,188 5,454 415 89 37 23 18 16 14 14 13 13 13 12	6.5	G	$\begin{array}{c} 1.625\\ 3.25\\ 4.875\\ 6.5\\ 8.125\\ 9.75\\ 9.75\\ 11.375\\ 13.0\\ 14.625\\ 16.25\\ 17.875\\ 19.5\\ 21.125\\ 22.65\\ 24.375\\ 26.0\\ \end{array}$	13.3 14.6 23.6 29.9 31.7 32.0 32.0 32.0 32.0 32.0 32.0 32.0 32.0	0.0 0.3 1.9 6.1 14.8 27.6 41.4 54.1 64.6 73.1 79.8 85.0 88.9 91.9 94.1	1.27 × 10 ¹⁰ 6.66 × 10 ⁸ 207,269 3,286 69 32 21 17 15 14 13 13 13 13
4.0	J	1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0	32,2 34.0 49,8 73.9 79.5 80.0 80.0 80.0 80.0 80.0 80.0 80.0 80	0.0 0.2 1.5 6.0 16.5 32.4 48.7 62.7 73.8 82.0 87.9 92.0 94.8 96.7 97.9	4.97 × 10 ¹⁰ 4.15 × 10 ⁹ 2.74 × 10 ⁸ 8,612 344 60 26 18 15 14 13 13 12 12 12 12	6.5	н	$\begin{array}{c} 1.625\\ 3.25\\ 4.875\\ 6.5\\ 8.125\\ 9.75\\ 11.375\\ 13.0\\ 14.625\\ 16.25\\ 17.875\\ 19.5\\ 21.125\\ 22.75\\ 24.375\\ 26.0 \end{array}$	$\begin{array}{c} 20.1 \\ 21.3 \\ 29.7 \\ 45.4 \\ 49.6 \\ 50.0 \\ 50$	0.0 0.2 1.6 6.6 17.8 34.2 50.6 64.5 75.2 83.1 88.8 92.7 95.3 97.1 98.2	4.69 × 10 ⁴⁰ 3.94 × 10 ⁹ 2.40 × 10 ⁶ 6.944 289 54 25 17 15 14 13 13 13 12 12 12

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AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
10.0	C	2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 22.5 25.0 27.5 30.0 32.5 35.0 37.5 40.0	3.0 3.8 4.4 5.6 6.0 6.3 6.5 6.5 6.5 6.5 6.7 7.0 7.1 7.2 7.2 7.2	0.3 1.8 4.7 9.5 15.7 22.8 30.1 37.2 43.9 50.2 55.9 61.1 65.8 69.9 73.6 76.9	3.30 X 10 ⁶ 14,916 941 188 71 39 27 22 19 17 16 15 14 14 14 13	10.0	F	2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 22.5 25.0 27.5 30.0 32.5 35.0 37.5 40.0	8.1 8.9 13.8 18.3 19.7 20.0 20.0 20.0 20.0 20.0 20.0 20.0 20	0.0 0.0 0.2 1.5 5.0 12.3 23.9 37.2 49.9 60.1 69.8 77.0 82.6 86.9 90.3 92.8	1.48 X 10 ¹⁰ 1.14 X 10 ⁹ 410,188 5,454 415 89 37 23 18 16 14 14 14 13 13 13 12
10.0	E	2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 22.5 25.0 27.5 30.0 32.5 35.0 37.5 40.0	5.5 7.0 10.4 12.2 13.0 13.0 13.0 13.0 13.0 13.0 13.0 13.0	0.0 2 0.1 2 1.2 4.3 9.9 18.0 27.6 37.6 37.6 46.9 55.4 62.8 69.2 74.7 79.3 83.2 86.4	2.50 x 10 ⁹ 4.07 x 10 ⁶ 18,352 919 157 55 30 22 18 16 14 14 13 13 13 13	10.0	G	2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 22.5 25.0 27.5 30.0 32.5 35.0 37.5 40.0	13.1 13.8 20.3 29.7 31.8 32.0 32.0 32.0 32.0 32.0 32.0 32.0 32.0	0.0 0.2 1.6 6.0 16.5 32.4 48.7 62.7 73.8 82.0 88.0 92.0 94.8 92.0 94.8 92.0 94.8	4.69×10^{10} 3.94×10^{9} 2.66×10^{6} 8,562 344 60 26 18 15 14 13 12 12 12 12

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