Coastal Engineering Technical Note

Even-Odd Function Analysis of Shoreline Position and Volume Change

by Julie Dean Rosati
and
Nicholas C. Kraus

Purpose: To present the background and methodology for separating shoreline position and volume change data into symmetric (even) and anti-symmetric (odd) functions. Applicability and interpretation of the even-odd function analysis for engineering application is also discussed. As applied to inlet data sets, the rates of shoreline and volume change which occur symmetrically about an inlet (e.g., storm erosion, erosion and accretion due to relative sea-level change) can be distinguished from those changes which are anti-symmetric (e.g., updrift impoundment, downdrift erosion).

Background: Even-odd function analysis is a direct and easily applied method for examining shoreline (or volume change) data. The power behind the method is its capability to unambiguously separate shoreline position and volume changes which are symmetric (even) about a chosen point along the shoreline from those that are anti-symmetric (odd). An even function $f_e(x)$ does not change sign if its argument changes sign, i.e., $f_e(-x) = f_e(+x)$; an odd function $f_o(x)$ does change sign $f_o(-x) = -f_o(+x)$. The sum of the even and odd functions reproduces the original data. Because of its ease in application, the method is becoming popular for determining the alongshore extent of inlet impacts, such as is required for Section 111 studies. However, the method requires engineering judgement in its interpretation, as discussed herein.

As an illustrative and idealized example, assume that a pair of impermeable jetties are constructed along a sandy beach with a significant net longshore transport. From time $t=0$ to $t=1$, ignoring possible inlet-induced adjacent beach losses, the left jetty impounds material updrift, with a commensurate erosion pattern downdrift of the right jetty (Figure 1a). The even-odd function method applied to this data set with the center point at the centerline of the inlet yields an odd function signature that returns to a negligible value at the alongshore limit of the jetty system's impact (Figure 1c). This example illustrates the most common interpretation of the even and odd function analysis, that the odd function is an indicator of the alongshore extent of a jetty system's impact. However, suppose that the same jetties were

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14. ABSTRACT
This Coastal Engineering Technical Note (CETN) presents the background and methodology for separating shoreline position and volume change data into symmetric (even) and anti-symmetric (odd) functions. Applicability and interpretation of the even-odd function analysis for engineering application are also discussed. As applied to inlet data sets, the rates of shoreline and volume change which occur symmetrically about an inlet (e.g., storm erosion, erosion and accretion due to relative sealevel change) can be distinguished from those changes which are anti-symmetric (e.g., updrift impoundment, downdrift erosion).

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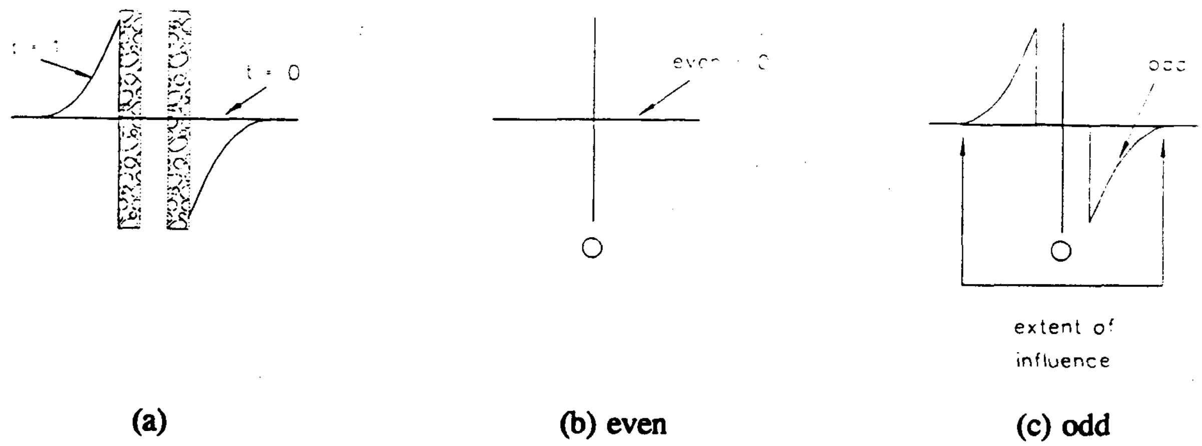

Figure 1. Example of jettied inlet on coast with significant net longshore sediment transport.

constructed along a sandy shore with a near-zero net longshore sediment transport, but a significant gross longshore sediment transport. The jetties would impound material on both the left and right, with erosion at some distance away outside their shadow zone (Figure 2a). The even-odd function method applied to this example yields a negligible odd function, because all shoreline changes were symmetric. However, the jetties obviously impacted the shoreline. For this case, determination of the extent of impact of the jetties is reflected by the zone at which the even function approaches a negligible value (Figure 2b). Thus, accurate interpretation of the analysis requires knowledge of site (e.g., direction and relative magnitude of net and gross longshore sediment transport rates) and structure (e.g., permeability) conditions. It is reiterated that, although the even-odd function analysis is unambiguous,

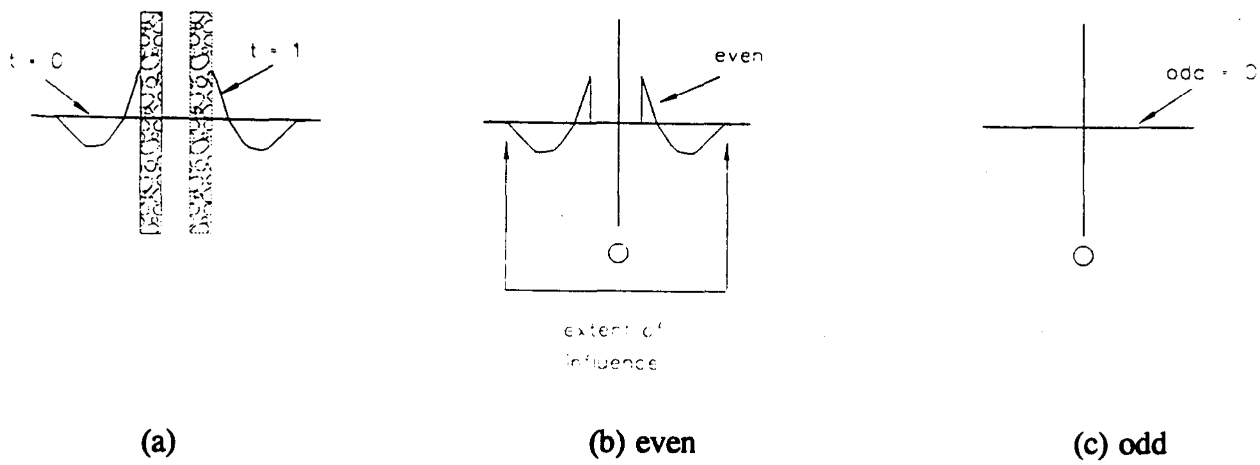

Figure 2. Example of jettied inlet on coast with zero net longshore sediment transport.

considerable coastal engineering judgement may be required in interpretation of the results. A listing of references discussing even-odd function applications is provided at the end of this Technical Note.

Procedure: Shoreline (or volume) change, denoted as $f(x)$, between two time periods at some alongshore position x, can be represented by even (symmetric) $f_e(x)$ and odd (anti-symmetric) $f_o(x)$ functions,

$$f(x) = f_e(x) + f_o(x) \tag{1}$$

The even function is determined as

$$f_e(x) = \frac{f(x) + f(-x)}{2} \tag{2}$$

and the odd function is given by

$$f_o(x) = \frac{f(x) - f(-x)}{2} \tag{3}$$

It is readily verified that substitution of Equations 2 and 3 into Equation 1 produces an identity. Note that the even-odd function method can only be applied for the length of shoreline for which both $f(x)$ and $f(-x)$ are available. Equations 2 and 3 are applied in the following example problem.

Example Problem: Table 1 presents shoreline change rate data from July 1929 and May 1933 (pre-inlet) to May 1996 (post-inlet and jetty construction) for the shorelines adjacent to Ocean City Inlet, Maryland, which is located on the east coast of the United States. The inlet was created when a hurricane broke through the existing barrier island on August 23, 1933, and jetties were constructed from September 1933 through May 1935. This site has a significant net longshore sediment transport to the south (from top to bottom in Table 1), with impoundment at the updrift jetty and significant erosion of the downdrift beach occurring during this time period. Shoreline change data are given in 500-m increments, extending from 15 km updrift (north) of the inlet to 14 km downdrift (south) of the inlet; however, the even-odd function analysis was conducted for the region for which both $f(x)$ and $f(-x)$ were available ($\pm$ 14 km). The July 1929 and May 1933 shorelines were digitized from topographic maps (T-sheets) created through a cooperative National Ocean Service (NOS)- Coastal Engineering Research Center study. The May 1996 shoreline was derived from a Global Positioning System-controlled field survey of the foreshore. Shoreline data sources and their relative accuracy are discussed in CETN-II-39 (Kraus and Rosati 1997).

Table 1
Shoreline Change Rate Data for Shorelines Adjacent to Ocean City Inlet, Maryland: Nov 1929 and May 1933 to May 1996

Column 1 (x) Distance from Center of Inlet (km)	Column 2 S(x) Ocean Shoreline Change Rate (m/yr)	Column 3 Dc Ocean Depth of Closure (m NGVD)	Column 4 B Ocean Active Berm Elev (m NGVD)	Column 5 Ad Ocean Active Depth (m)	Column 6 f(x) Ocean Vol Chg Rate (m3/m/yr)	Column 7 (-x) Inverted Distance from Inlet (km)	Column 8 f(-x) Inverted Vol Chg Rate (m3/m/yr)	Column 9 [f(x)+f(-x)]/2 Even Function (m3/m/yr)	Column 10 [f(x)-f(-x)]/2 Odd Function (m3/m/yr)
-15	-0.47	6.1	3	9.1	-4.3				
-14.5	-0.32	6.1	3	9.1	-2.9				
-14	0.11	6.1	3	9.1	1.0	14	-2.0	-0.5	1.5
-13.5	0.17	6.1	3	9.1	1.5	13.5	-2.6	-0.5	2.1
-13	0.27	6.1	3	9.1	2.5	13	-2.2	0.2	2.3
-12.5	0.24	6.1	3	9.1	2.2	12.5	-4.0	-0.9	3.1
-12	0.07	6.1	3	9.1	0.6	12	-6.4	-2.9	3.5
-11.5	0.05	6.1	3	9.1	0.5	11.5	-8.4	-4.0	4.4
-11	-0.46	6.1	3	9.1	-4.2	11	-6.8	-5.5	1.3
-10.5	-0.18	6.1	3	9.1	-1.6	10.5	-6.2	-3.9	2.3
-10	-0.02	6.1	3	9.1	-0.2	10	-6.5	-3.4	3.2
-9.5	0.57	6.1	3	9.1	5.2	9.5	-7.3	-1.1	6.2
-9	0.07	6.1	3	9.1	0.6	9	-8.9	-4.1	4.7
-8.5	-0.49	6.1	3	9.1	-4.5	8.5	-14.3	-9.4	4.9
-8	-0.4	6.1	3	9.1	-3.6	8	-16.3	-10.0	6.4
-7.5	0.41	6.1	3	9.1	3.7	7.5	-20.1	-8.2	11.9
-7	0.5	6.1	3	9.1	4.6	7	-21.2	-8.3	12.9
-6.5	0.37	6.1	3	9.1	3.4	6.5	-19.4	-8.0	11.4
-6	0.55	6.1	3	9.1	5.0	6	-24.1	-9.5	14.5
-5.5	0.63	6.1	3	9.1	5.7	5.5	-35.7	-15.0	20.7
-5	0.37	6.1	3	9.1	3.4	5	-38.6	-17.6	21.0
-4.5	0.44	6.1	3	9.1	4.0	4.5	-40.1	-18.1	22.1
-4	0.74	6.1	3	9.1	6.7	4	-46.1	-19.7	26.4
-3.5	0.88	6.1	3	9.1	8.0	3.5	-49.0	-20.5	28.5
-3	0.58	6.1	3	9.1	5.3	3	-51.9	-23.3	28.6
-2.5	0.92	6.1	3	9.1	8.4	2.5	-56.3	-23.9	32.3
-2	0.97	6.1	3	9.1	8.8	2	-56.7	-23.9	32.8
-1.5	1.12	6.1	3	9.1	10.2	1.5	-51.6	-20.7	30.9
-1	2	6.1	3	9.1	18.2	1	-41.6	-11.7	29.9
-0.5	3.42	6.1	3	9.1	31.1	0.5	-33.4	-1.1	32.3
0	0	6.1	3	9.1	0.0	0	0.0	0.0	0.0
0.5	-3.93	6.1	2.4	8.5	-33.4	-0.5	31.1	-1.1	-32.3
1	-4.89	6.1	2.4	8.5	-41.6	-1	18.2	-11.7	-29.9
1.5	-6.07	6.1	2.4	8.5	-51.6	-1.5	10.2	-20.7	-30.9
2	-6.67	6.1	2.4	8.5	-56.7	-2	8.8	-23.9	-32.8
2.5	-6.62	6.1	2.4	8.5	-56.3	-2.5	8.4	-23.9	-32.3
3	-6.11	6.1	2.4	8.5	-51.9	-3	5.3	-23.3	-28.6
3.5	-5.76	6.1	2.4	8.5	-49.0	-3.5	8.0	-20.5	-28.5
4	-5.42	6.1	2.4	8.5	-46.1	-4	6.7	-19.7	-26.4
4.5	-4.72	6.1	2.4	8.5	-40.1	-4.5	4.0	-18.1	-22.1
5	-4.54	6.1	2.4	8.5	-38.6	-5	3.4	-17.6	-21.0
5.5	-4.15	6.1	2.5	8.6	-35.7	-5.5	5.7	-15.0	-20.7
6	-2.8	6.1	2.5	8.6	-24.1	-6	5.0	-9.5	-14.5
6.5	-2.25	6.1	2.5	8.6	-19.4	-6.5	3.4	-8.0	-11.4
7	-2.47	6.1	2.5	8.6	-21.2	-7	4.6	-8.3	-12.9
7.5	-2.34	6.1	2.5	8.6	-20.1	-7.5	3.7	-8.2	-11.9
8	-1.9	6.1	2.5	8.6	-16.3	-8	-3.6	-10.0	-6.4
8.5	-1.66	6.1	2.5	8.6	-14.3	-8.5	-4.5	-9.4	-4.9
9	-1.03	6.1	2.5	8.6	-8.9	-9	0.6	-4.1	-4.7
9.5	-0.85	6.1	2.5	8.6	-7.3	-9.5	5.2	-1.1	-6.2
10	-0.76	6.1	2.5	8.6	-6.5	-10	-0.2	-3.4	-3.2
10.5	-0.72	6.1	2.5	8.6	-6.2	-10.5	-1.6	-3.9	-2.3
11	-0.79	6.1	2.5	8.6	-6.8	-11	-4.2	-5.5	-1.3
11.5	-0.98	6.1	2.5	8.6	-8.4	-11.5	0.5	-4.0	-4.4
12	-0.74	6.1	2.5	8.6	-6.4	-12	0.6	-2.9	-3.5
12.5	-0.47	6.1	2.5	8.6	-4.0	-12.5	2.2	-0.9	-3.1
13	-0.25	6.1	2.5	8.6	-2.2	-13	2.5	0.2	-2.3
13.5	-0.3	6.1	2.5	8.6	-2.6	-13.5	1.5	-0.5	-2.1
14	-0.23	6.1	2.5	8.6	-2.0	-14	1.0	-0.5	-1.5
						-14.5	-2.9		
						-15	-4.3		

The alongshore coordinate $x = 0$ was chosen as the center of the inlet, with negative values indicating distances updrift and positive values indicating distances downdrift from the inlet, respectively (column 1). The shoreline change rates have not been adjusted to remove the volume of beach fill placed during this time period. The active depth (A_d in m, column 5) is given as the sum of the depth of closure (D_c in m, column 3) and the active berm crest (B in m, column 4):

$$A_d = D_c + B \tag{4}$$

The shoreline change rate ($S(x)$ in m/year, column 2) has been converted to a volume change rate per unit length of beach ($f(x)$ in m^3/m/year, column 6) by multiplying by A_d, which ranges from 8.5 to 9.1 m (see column 5):

$$f(x) = S(x)\, A_d \tag{5}$$

For the Ocean City data set, volumetric change rate data are preferred over shoreline change rate data to account for A_d, which varies alongshore, and to account for beach fills (discussed in the next section). Note that the volume change rate per unit length of beach, $f(x)$ (in units of m^3/m/year), is readily converted into a volume change rate at each cell (in units of m^3/year) by multiplying by the cell width, $W = 500$ m. The first step in the even-odd function analysis procedure is to invert x and $f(x)$ about $x = 0$, producing values for $-x$ (column 7) and $f(-x)$ (column 8). The even (column 9) and odd (column 10) functions can then be calculated using Equations 2 and 3, respectively. Figure 3 shows $f(x)$ (noted as "total" in the legend), $f_e(x)$, and $f_o(x)$.

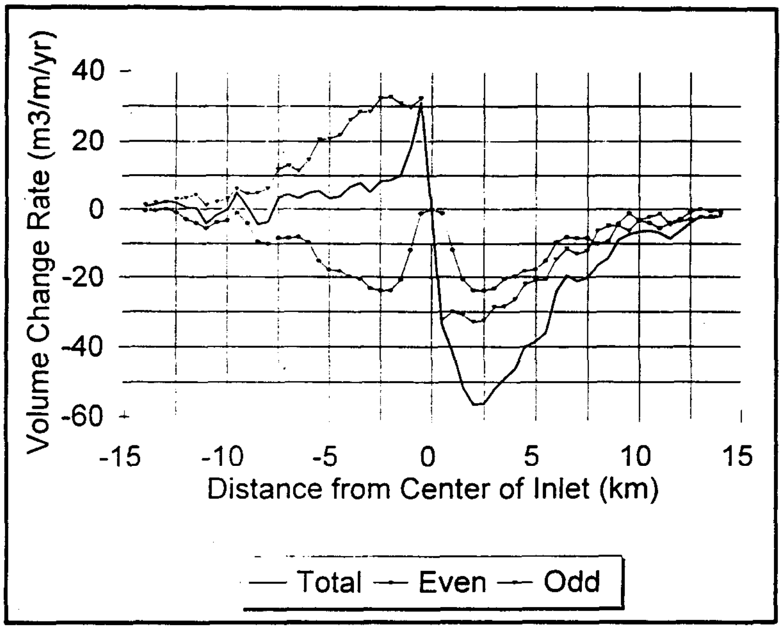

Figure 3. Example problem: Ocean City even-odd function analysis, 1929/1933 to 1966

For this site with a significant net longshore sediment transport rate (approximately 115,000 m^3/year at present), the alongshore point at which the odd function returns to a zero value is an indicator of the alongshore distance influenced by anti-symmetric processes, such as impoundment, which is a project-induced impact. The alongshore point at which the even function approaches a constant value is an indication of the alongshore distance influenced by symmetric processes, such as storm impacts and inlet-induced impacts such as shoreline retreat due to feeding the ebb and flood tidal shoals. For the region of shoreline for which the even function approaches a constant value, the even function can be interpreted to represent the "background" shoreline (or volumetric) change rate (e.g., sea level rise, influence of storms, etc.). From a cursory observation, because the odd function does not return to a zero value, it appears that the inlet's extent of influence in terms of impoundment extends beyond the longshore extent of the data limits. Qualitatively, the even function appears to approach a constant value at x = 9 to 12 km from the center of the inlet.

However, two other factors must be considered. First, as mentioned previously, the data presented in Table 1 and Figure 3 were not corrected to adjust the 1996 shoreline position for beach fill. For this particular site, this adjustment is important due to the volume of dredged material placed on the downdrift beach (a total volume of approximately 2.9 million m^3), and the June 1988 through April 1995 beach fill project on the updrift beach (a total volume of approximately 7.4 million m^3 placed from the north jetty at -0.2 km to -14 km). To account for the volume of beach fill placed, a three-step process was used. First, the 1929/1933 to 1996 volume change rate (in units of m^3/year) was converted to a volume change (units of m^3) by multiplying by the number of years (63 years). Second, the volume of beach fill estimated to be remaining in 1996 at each alongshore cell was subtracted from the 1929/33 to 1996 volume change. This estimation was based on analysis of seven profiles from June 1988 (immediately pre-fill) and May 1996 (present-day condition) for the updrift beach. From this analysis, it was estimated that the updrift beach fill lost material at a rate of approximately 2 percent per year. This rate was used for all beach fill placed, resulting in estimates of 0.72 million m^3 remaining on the downdrift beach, and 5.8 million m^3 remaining on the updrift beach. Finally, the shoreline change rate accounting for beach fill was calculated by subtracting the fill remaining from the volume change at each cell, which was then converted back into a volume change rate (units of m^3/year). Figure 4 shows the total, even, and odd functions for the 1929/33 to 1996 volumetric change rate as

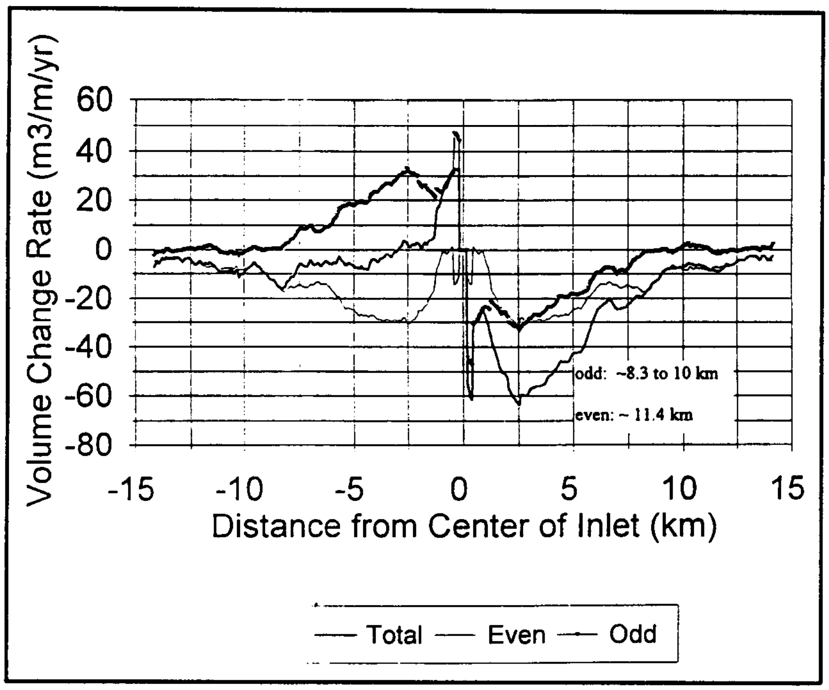

Figure 4. Ocean City even-odd function analysis, 1929/1933 to1996, adjusted for beach fill

adjusted to remove the volume of beach fill placed (note that these data are presented for the full data set with cell widths $W=50$ m).

The second consideration concerns confidence in the accuracy of the shoreline data set (see CETN-II-39). The shoreline positions are estimated to have an uncertainty or tolerance of $\pm$ 10 m, based on consideration of four potential error sources: accuracy of shorelines plotted on NOS T-sheets, photo-interpretation of the high-water line, line thickness of shorelines on the maps, and equipment/operator error incurred during the digitization process. Placing the uncertainty estimate into Equation 5 results in a "data-capture estimate" of $\pm$ 1.4 m^3/m/year. The minimum distance at which the odd function equals the negative value of the data-capture estimate (-1.4 m^3/m/year) is approximately 8.3 km from the center line of the inlet, and the distance represented by the positive value (+1.4 m^3/m/yr) is approximately 10 km from the center line of the inlet. The even function approaches a constant value at the distance for which the standard deviation of this function at greater distances equals the data-capture error estimate. For the 1929/33-1996 data set, the even function approaches a constant value approximately 11.4 km from the center line of the inlet (i.e., the standard deviation of the even function from 11.4 km to the data limit equals $\pm$ 1.4 m^3/m/year). Thus, analysis of shoreline response data from 1929/33 to 1996 indicates that the influence of the Ocean City Inlet has extended from 8.3 to 11.4 km from the center of the inlet.

Additional Information: A comprehensive bibliography is provided for the reader to consult for additional discussion of interpretation of the even-odd function method. Questions about this CETN can be addressed to Ms. Julie Dean Rosati (601-634-3005, Fax 601-634-4314 email: jd.rosati@cerc.wes.army.mil) or Dr. Nicholas C. Kraus (601-634-2016, Fax 601-634-2055, email: n.kraus@cerc.wes.army.mil). A spreadsheet version of the example problem is available from the authors.

Bibliography

Berek, E. P., and Dean, R. G. (1982). "Field investigation of longshore transport distribution." *Proc. 18th Coastal Engineering Conference*, ASCE, NY, 1620-1638.

Dean, J. L., and Pope, J. (1987). "The Redington Shores breakwater project: Initial response." *Proceedings Coastal Sediments '87*. ASCE, NY, 1369-1384.

Dean, R. G., and Work, P. A. (1993). Interaction of navigational entrances with adjacent shorelines," *Journal of Coastal Research* 18: 91-110.

Douglas, B. D., and Walther, M. P. (1994). "Odd/even analysis modified to account for sediment losses to ebb and flood shoals." *Proceedings 1994 National Conference on Beach Preservation Technology*. FSBPA, Tallahassee, FL, 354-373.

Kraus, N. C. "Distinguishing cross-shore and longshore processes in shoreline change evaluation." *Proceedings 1997 National Conference on Beach Preservation Technology*. In press, FSBPA, Tallahassee, FL.

Kraus, N. C., and Rosati, J. D. 1997. "Interpretation of Shoreline-Position Data for Coastal Engineering Analysis," Coastal Engineering Technical Note CETN-II-39, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS.

Rosati, J. D., and Ebersole, B. A. "Littoral impacts of Ocean City Inlet, Maryland, USA." *Proceedings 25th Coastal Engineering Conference*. In press, ASCE, NY.

Work, P. A., and Dean, R. G. (1990). Even/odd analysis of shoreline changes adjacent to Florida's tidal inlets." *Proceedings 20th Coastal Engineering Conference*. ASCE, NY, 2522-2535.